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1.1 Installing scikit-learn

Note: If you wish to contribute to the project, it’s recommended you install the latest development version.

1.1.1 Installing the latest release

Scikit-learn requires:

- Python (>= 2.6 or >= 3.3),
- NumPy (>= 1.6.1),
- SciPy (>= 0.9).

If you already have a working installation of numpy and scipy, the easiest way to install scikit-learn is using pip:

```
pip install -U scikit-learn
```

or conda:

```
conda install scikit-learn
```

If you have not installed NumPy or SciPy yet, you can also install these using conda or pip. When using pip, please ensure that binary wheels are used, and NumPy and SciPy are not recompiled from source, which can happen when using particular configurations of operating system and hardware (such as Linux on a Raspberry Pi). Building numpy and scipy from source can be complex (especially on Windows) and requires careful configuration to ensure that they link against an optimized implementation of linear algebra routines. Instead, use a third-party distribution as described below.

If you must install scikit-learn and its dependencies with pip, you can install it as scikit-learn[alldeps]. The most common use case for this is in a requirements.txt file used as part of an automated build process for a PaaS application or a Docker image. This option is not intended for manual installation from the command line.

1.1.2 Third-party Distributions

If you don’t already have a python installation with numpy and scipy, we recommend to install either via your package manager or via a python bundle. These come with numpy, scipy, scikit-learn, matplotlib and many other helpful
scientific and data processing libraries.
Available options are:

**Canopy and Anaconda for all supported platforms**

Canopy and Anaconda both ship a recent version of scikit-learn, in addition to a large set of scientific python library for Windows, Mac OSX and Linux.

Anaconda offers scikit-learn as part of its free distribution.

---

**Warning:** To upgrade or uninstall scikit-learn installed with Anaconda or conda you should not use the pip command. Instead:

To upgrade scikit-learn:

```
conda update scikit-learn
```

To uninstall scikit-learn:

```
conda remove scikit-learn
```

Upgrading with `pip install -U scikit-learn` or uninstalling `pip uninstall scikit-learn` is likely to fail to properly remove files installed by the conda command.

Pip upgrade and uninstall operations only work on packages installed via `pip install`.

---

**WinPython for Windows**

The WinPython project distributes scikit-learn as an additional plugin.

For installation instructions for particular operating systems or for compiling the bleeding edge version, see the Advanced installation instructions.

---

**1.2 Frequently Asked Questions**

Here we try to give some answers to questions that regularly pop up on the mailing list.

**1.2.1 What is the project name (a lot of people get it wrong)?**

scikit-learn, but not scikit or SciKit nor sci-kit learn. Also not scikits.learn or scikits-learn, which were previously used.

**1.2.2 How do you pronounce the project name?**

sy-kit learn. sci stands for science!

**1.2.3 Why scikit?**

There are multiple scikits, which are scientific toolboxes build around SciPy. You can find a list at https://scikits.appspot.com/scikits. Apart from scikit-learn, another popular one is scikit-image.
1.2.4 How can I contribute to scikit-learn?

See Contributing. Before wanting to add a new algorithm, which is usually a major and lengthy undertaking, it is recommended to start with known issues.

1.2.5 What’s the best way to get help on scikit-learn usage?

For general machine learning questions, please use Cross Validated with the [machine-learning] tag.

For scikit-learn usage questions, please use Stack Overflow with the [scikit-learn] and [python] tags. You can alternatively use the mailing list.

Please make sure to include a minimal reproduction code snippet (ideally shorter than 10 lines) that highlights your problem on a toy dataset (for instance from sklearn.datasets or randomly generated with functions of numpy.random with a fixed random seed). Please remove any line of code that is not necessary to reproduce your problem.

The problem should be reproducible by simply copy-pasting your code snippet in a Python shell with scikit-learn installed. Do not forget to include the import statements.

More guidance to write good reproduction code snippets can be found at:
http://stackoverflow.com/help/mcve

If your problem raises an exception that you do not understand (even after googling it), please make sure to include the full traceback that you obtain when running the reproduction script.

For bug reports or feature requests, please make use of the issue tracker on Github.

There is also a scikit-learn Gitter channel where some users and developers might be found.

Please do not email any authors directly to ask for assistance, report bugs, or for any other issue related to scikit-learn.

1.2.6 How can I create a bunch object?

Don’t make a bunch object! They are not part of the scikit-learn API. Bunch objects are just a way to package some numpy arrays. As a scikit-learn user you only ever need numpy arrays to feed your model with data.

For instance to train a classifier, all you need is a 2D array $X$ for the input variables and a 1D array $y$ for the target variables. The array $X$ holds the features as columns and samples as rows. The array $y$ contains integer values to encode the class membership of each sample in $X$.

1.2.7 How can I load my own datasets into a format usable by scikit-learn?

Generally, scikit-learn works on any numeric data stored as numpy arrays or scipy sparse matrices. Other types that are convertible to numeric arrays such as pandas DataFrame are also acceptable.

For more information on loading your data files into these usable data structures, please refer to loading external datasets.

1.2.8 What are the inclusion criteria for new algorithms?

We only consider well-established algorithms for inclusion. A rule of thumb is at least 3 years since publication, 200+ citations and wide use and usefulness. A technique that provides a clear-cut improvement (e.g. an enhanced data structure or a more efficient approximation technique) on a widely-used method will also be considered for inclusion.
From the algorithms or techniques that meet the above criteria, only those which fit well within the current API of scikit-learn, that is a `fit, predict/transform` interface and ordinarily having input/output that is a numpy array or sparse matrix, are accepted.

The contributor should support the importance of the proposed addition with research papers and/or implementations in other similar packages, demonstrate its usefulness via common use-cases/applications and corroborate performance improvements, if any, with benchmarks and/or plots. It is expected that the proposed algorithm should outperform the methods that are already implemented in scikit-learn at least in some areas.

Also note that your implementation need not be in scikit-learn to be used together with scikit-learn tools. You can implement your favorite algorithm in a scikit-learn compatible way, upload it to github and let us know. We will list it under Related Projects.

### 1.2.9 Why are you so selective on what algorithms you include in scikit-learn?

Code is maintenance cost, and we need to balance the amount of code we have with the size of the team (and add to this the fact that complexity scales non linearly with the number of features). The package relies on core developers using their free time to fix bugs, maintain code and review contributions. Any algorithm that is added needs future attention by the developers, at which point the original author might long have lost interest. Also see this thread on the mailing list.

### 1.2.10 Why did you remove HMMs from scikit-learn?

See *Will you add graphical models or sequence prediction to scikit-learn?*.

### 1.2.11 Will you add graphical models or sequence prediction to scikit-learn?

Not in the foreseeable future. scikit-learn tries to provide a unified API for the basic tasks in machine learning, with pipelines and meta-algorithms like grid search to tie everything together. The required concepts, APIs, algorithms and expertise required for structured learning are different from what scikit-learn has to offer. If we started doing arbitrary structured learning, we’d need to redesign the whole package and the project would likely collapse under its own weight.

There are two project with API similar to scikit-learn that do structured prediction:

- **pystruct** handles general structured learning (focuses on SSVMs on arbitrary graph structures with approximate inference; defines the notion of sample as an instance of the graph structure)
- **seqlearn** handles sequences only (focuses on exact inference; has HMMs, but mostly for the sake of completeness; treats a feature vector as a sample and uses an offset encoding for the dependencies between feature vectors)

### 1.2.12 Will you add GPU support?

No, or at least not in the near future. The main reason is that GPU support will introduce many software dependencies and introduce platform specific issues. scikit-learn is designed to be easy to install on a wide variety of platforms. Outside of neural networks, GPUs don’t play a large role in machine learning today, and much larger gains in speed can often be achieved by a careful choice of algorithms.
1.2.13 Do you support PyPy?

In case you didn’t know, PyPy is the new, fast, just-in-time compiling Python implementation. We don’t support it. When the NumPy support in PyPy is complete or near-complete, and SciPy is ported over as well, we can start thinking of a port. We use too much of NumPy to work with a partial implementation.

1.2.14 How do I deal with string data (or trees, graphs...)?

scikit-learn estimators assume you’ll feed them real-valued feature vectors. This assumption is hard-coded in pretty much all of the library. However, you can feed non-numerical inputs to estimators in several ways.

If you have text documents, you can use a term frequency features; see Text feature extraction for the built-in text vectorizers. For more general feature extraction from any kind of data, see Loading features from dicts and Feature hashing.

Another common case is when you have non-numerical data and a custom distance (or similarity) metric on these data. Examples include strings with edit distance (aka. Levenshtein distance; e.g., DNA or RNA sequences). These can be encoded as numbers, but doing so is painful and error-prone. Working with distance metrics on arbitrary data can be done in two ways.

Firstly, many estimators take precomputed distance/similarity matrices, so if the dataset is not too large, you can compute distances for all pairs of inputs. If the dataset is large, you can use feature vectors with only one “feature”, which is an index into a separate data structure, and supply a custom metric function that looks up the actual data in this data structure. E.g., to use DBSCAN with Levenshtein distances:

```python
>>> from leven import levenshtein
>>> from sklearn.cluster import dbscan
>>> data = ["ACCTCTAGAAG", "ACCTACTAGAAGTT", "GAATATTAGGCCGA"]
>>> def lev_metric(x, y):
...     i, j = int(x[0]), int(y[0])  # extract indices
...     return levenshtein(data[i], data[j])
...
>>> X = np.arange(len(data)).reshape(-1, 1)
>>> X
array([[0],
       [1],
       [2]])
>>> dbscan(X, metric=lev_metric, eps=5, min_samples=2)
([0, 1], array([[ 0, 0, -1]]))
```

(This uses the third-party edit distance package leven.)

Similar tricks can be used, with some care, for tree kernels, graph kernels, etc.

1.2.15 Why do I sometime get a crash/freeze with n_jobs > 1 under OSX or Linux?

Several scikit-learn tools such as GridSearchCV and cross_val_score rely internally on Python’s multiprocessing module to parallelize execution onto several Python processes by passing n_jobs > 1 as argument.

The problem is that Python multiprocessing does a fork system call without following it with an exec system call for performance reasons. Many libraries like (some versions of) Accelerate / vecLib under OSX, (some versions of) MKL, the OpenMP runtime of GCC, nvidia’s Cuda (and probably many others), manage their own internal thread pool. Upon a call to fork, the thread pool state in the child process is corrupted: the thread pool believes it has many threads while only the main thread state has been forked. It is possible to change the libraries to make them detect
when a fork happens and reinitialize the thread pool in that case: we did that for OpenBLAS (merged upstream in master since 0.2.10) and we contributed a patch to GCC’s OpenMP runtime (not yet reviewed).

But in the end the real culprit is Python’s multiprocessing that does fork without exec to reduce the overhead of starting and using new Python processes for parallel computing. Unfortunately this is a violation of the POSIX standard and therefore some software editors like Apple refuse to consider the lack of fork-safety in Accelerate / vecLib as a bug.

In Python 3.4+ it is now possible to configure multiprocessing to use the ‘forkserver’ or ‘spawn’ start methods (instead of the default ‘fork’) to manage the process pools. To work around this issue when using scikit-learn, you can set the JOBLIB_START_METHOD environment variable to ‘forkserver’. However the user should be aware that using the ‘forkserver’ method prevents joblib.Parallel to call function interactively defined in a shell session.

If you have custom code that uses multiprocessing directly instead of using it via joblib you can enable the ‘forkserver’ mode globally for your program: Insert the following instructions in your main script:

```python
import multiprocessing

# other imports, custom code, load data, define model...

if __name__ == '__main__':
    multiprocessing.set_start_method('forkserver')

    # call scikit-learn utils with n_jobs > 1 here
```

You can find more default on the new start methods in the multiprocessing documentation.

### 1.2.16 Why is there no support for deep or reinforcement learning / Will there be support for deep or reinforcement learning in scikit-learn?

Deep learning and reinforcement learning both require a rich vocabulary to define an architecture, with deep learning additionally requiring GPUs for efficient computing. However, neither of these fit within the design constraints of scikit-learn; as a result, deep learning and reinforcement learning are currently out of scope for what scikit-learn seeks to achieve.

### 1.2.17 Why is my pull request not getting any attention?

The scikit-learn review process takes a significant amount of time, and contributors should not be discouraged by a lack of activity or review on their pull request. We care a lot about getting things right the first time, as maintenance and later change comes at a high cost. We rarely release any “experimental” code, so all of our contributions will be subject to high use immediately and should be of the highest quality possible initially.

Beyond that, scikit-learn is limited in its reviewing bandwidth; many of the reviewers and core developers are working on scikit-learn on their own time. If a review of your pull request comes slowly, it is likely because the reviewers are busy. We ask for your understanding and request that you not close your pull request or discontinue your work solely because of this reason.

### 1.2.18 How do I set a random_state for an entire execution?

For testing and replicability, it is often important to have the entire execution controlled by a single seed for the pseudo-random number generator used in algorithms that have a randomized component. Scikit-learn does not use its own global random state; whenever a RandomState instance or an integer random seed is not provided as an argument, it relies on the numpy global random state, which can be set using `numpy.random.seed`. For example, to set an execution’s numpy global random state to 42, one could execute the following in his or her script:
However, a global random state is prone to modification by other code during execution. Thus, the only way to ensure replicability is to pass `RandomState` instances everywhere and ensure that both estimators and cross-validation splitters have their `random_state` parameter set.

### 1.3 Support

There are several ways to get in touch with the developers.

#### 1.3.1 Mailing List

- The main mailing list is scikit-learn.
- There is also a commit list scikit-learn-commits, where updates to the main repository and test failures get notified.

#### 1.3.2 User questions

- Some scikit-learn developers support users on StackOverflow using the [scikit-learn] tag.
- For general theoretical or methodological Machine Learning questions stack exchange is probably a more suitable venue.

In both cases please use a descriptive question in the title field (e.g. no “Please help with scikit-learn!” as this is not a question) and put details on what you tried to achieve, what were the expected results and what you observed instead in the details field.

Code and data snippets are welcome. Minimalistic (up to ~20 lines long) reproduction script very helpful.

Please describe the nature of your data and the how you preprocessed it: what is the number of samples, what is the number and type of features (i.e. categorical or numerical) and for supervised learning tasks, what target are you trying to predict: binary, multiclass (1 out of `n_classes`) or multilabel (k out of `n_classes`) classification or continuous variable regression.

#### 1.3.3 Bug tracker

If you think you’ve encountered a bug, please report it to the issue tracker:

https://github.com/scikit-learn/scikit-learn/issues

Don’t forget to include:

- steps (or better script) to reproduce,
- expected outcome,
- observed outcome or python (or gdb) tracebacks

To help developers fix your bug faster, please link to a https://gist.github.com holding a standalone minimalistic python script that reproduces your bug and optionally a minimalistic subsample of your dataset (for instance exported as CSV files using `numpy.savetxt`).

Note: gists are git cloneable repositories and thus you can use git to push datafiles to them.
1.3.4 IRC

Some developers like to hang out on channel #scikit-learn on irc.freenode.net.
If you do not have an IRC client or are behind a firewall this web client works fine: http://webchat.freenode.net

1.3.5 Documentation resources

This documentation is relative to 0.18.2. Documentation for other versions can be found here:

- 0.17
- 0.16
- 0.15

Printable pdf documentation for all versions can be found here.

1.4 Related Projects

Below is a list of sister-projects, extensions and domain specific packages.

1.4.1 Interoperability and framework enhancements

These tools adapt scikit-learn for use with other technologies or otherwise enhance the functionality of scikit-learn’s estimators.

- ML Frontend provides dataset management and SVM fitting/prediction through web-based and programmatic interfaces.
- sklearn_pandas bridge for scikit-learn pipelines and pandas data frame with dedicated transformers.
- Scikit-Learn Laboratory A command-line wrapper around scikit-learn that makes it easy to run machine learning experiments with multiple learners and large feature sets.
- auto-sklearn An automated machine learning toolkit and a drop-in replacement for a scikit-learn estimator
- TPOT An automated machine learning toolkit that optimizes a series of scikit-learn operators to design a machine learning pipeline, including data and feature preprocessors as well as the estimators. Works as a drop-in replacement for a scikit-learn estimator.
- sklearn-pmml Serialization of (some) scikit-learn estimators into PMML.
- sklearn2pmml Serialization of a wide variety of scikit-learn estimators and transformers into PMML with the help of JPMML-SkLearn library.

1.4.2 Other estimators and tasks

Not everything belongs or is mature enough for the central scikit-learn project. The following are projects providing interfaces similar to scikit-learn for additional learning algorithms, infrastructures and tasks.

- pylearn2 A deep learning and neural network library build on theano with scikit-learn like interface.
- sklearn_theano scikit-learn compatible estimators, transformers, and datasets which use Theano internally
- lightning Fast state-of-the-art linear model solvers (SDCA, AdaGrad, SVRG, SAG, etc...).
- Seqlearn Sequence classification using HMMs or structured perceptron.
• **HMMLearn** Implementation of hidden markov models that was previously part of scikit-learn.
• **PyStruct** General conditional random fields and structured prediction.
• **pomegranate** Probabilistic modelling for Python, with an emphasis on hidden Markov models.
• **py-earth** Multivariate adaptive regression splines
• **sklearn-compiledtrees** Generate a C++ implementation of the predict function for decision trees (and ensembles) trained by sklearn. Useful for latency-sensitive production environments.
• **lda** Fast implementation of Latent Dirichlet Allocation in Cython.
• **Sparse Filtering** Unsupervised feature learning based on sparse-filtering
• **Kernel Regression** Implementation of Nadaraya-Watson kernel regression with automatic bandwidth selection
• **gplearn** Genetic Programming for symbolic regression tasks.
• **nolearn** A number of wrappers and abstractions around existing neural network libraries
• **sparkit-learn** Scikit-learn functionality and API on PySpark.
• **keras** Theano-based Deep Learning library.
• **mlxtend** Includes a number of additional estimators as well as model visualization utilities.
• **kmodes** k-modes clustering algorithm for categorical data, and several of its variations.
• **hdbscan** HDBSCAN and Robust Single Linkage clustering algorithms for robust variable density clustering.
• **lasagne** A lightweight library to build and train neural networks in Theano.
• **multisotonic** Isotonic regression on multidimensional features.
• **spherecluster** Spherical K-means and mixture of von Mises Fisher clustering routines for data on the unit hypersphere.

### 1.4.3 Statistical learning with Python

Other packages useful for data analysis and machine learning.

• **Pandas** Tools for working with heterogeneous and columnar data, relational queries, time series and basic statistics.
• **theano** A CPU/GPU array processing framework geared towards deep learning research.
• **statsmodels** Estimating and analysing statistical models. More focused on statistical tests and less on prediction than scikit-learn.
• **PyMC** Bayesian statistical models and fitting algorithms.
• **REP** Environment for conducting data-driven research in a consistent and reproducible way
• **Sacred** Tool to help you configure, organize, log and reproduce experiments
• **gensim** A library for topic modelling, document indexing and similarity retrieval
• **Seaborn** Visualization library based on matplotlib. It provides a high-level interface for drawing attractive statistical graphics.
• **Deep Learning** A curated list of deep learning software libraries.
Domain specific packages

- scikit-image Image processing and computer vision in python.
- Natural language toolkit (nltk) Natural language processing and some machine learning.
- NiLearn Machine learning for neuro-imaging.
- AstroML Machine learning for astronomy.
- MSMBuilder Machine learning for protein conformational dynamics time series.

1.4.4 Snippets and tidbits

The wiki has more!

1.5 About us

This is a community effort, and as such many people have contributed to it over the years.

1.5.1 History

This project was started in 2007 as a Google Summer of Code project by David Cournapeau. Later that year, Matthieu Brucher started work on this project as part of his thesis.

In 2010 Fabian Pedregosa, Gael Varoquaux, Alexandre Gramfort and Vincent Michel of INRIA took leadership of the project and made the first public release, February the 1st 2010. Since then, several releases have appeared following a ~3 month cycle, and a thriving international community has been leading the development.

1.5.2 People

The following people have been core contributors to scikit-learn’s development and maintenance:

- Mathieu Blondel
- Matthieu Brucher
- Lars Buitinck
- David Cournapeau
- Noel Dawe
- Vincent Dubourg
- Edouard Duchesnay
- Tom Dupré la Tour
- Alexander Fabisch
- Virgile Fritsch
- Satra Ghosh
- Angel Soler Gollonet
- Chris Filo Gorgolewski
- Alexandre Gramfort
- Olivier Grisel
- Jaques Grobler
- Yaroslav Halchenko
- Brian Holt
- Arnaud Joly
- Thouis (Ray) Jones
Please do not email the authors directly to ask for assistance or report issues. Instead, please see What’s the best way to ask questions about scikit-learn in the FAQ.

See also:
How you can contribute to the project

1.5.3 Citing scikit-learn

If you use scikit-learn in a scientific publication, we would appreciate citations to the following paper:


Bibtex entry:

```
@article{scikit-learn,
  title={Scikit-learn: Machine Learning in {P}ython},
  journal={Journal of Machine Learning Research},
  volume={12},
  pages={2825--2830},
  year={2011}
}
```

If you want to cite scikit-learn for its API or design, you may also want to consider the following paper:

API design for machine learning software: experiences from the scikit-learn project, Buitinck et al., 2013.

Bibtex entry:

```
@inproceedings{sklearn_api,
  author = {Lars Buitinck and Gilles Louppe and Mathieu Blondel and Fabian Pedregosa and Andreas Mueller and Olivier Grisel and Vlad Niculae and Peter Prettenhofer and Alexandre Gramfort
}
```
1.5.4 Artwork

High quality PNG and SVG logos are available in the doc/logos/ source directory.

1.5.5 Funding

INRIA actively supports this project. It has provided funding for Fabian Pedregosa (2010-2012), Jaques Grobler (2012-2013) and Olivier Grisel (2013-2015) to work on this project full-time. It also hosts coding sprints and other events.

NYU Moore-Sloan Data Science Environment funded Andreas Mueller (2014-2016) to work on this project. The Moore-Sloan Data Science Environment also funds several students to work on the project part-time.

Télécom Paristech funds Manoj Kumar (2014), Tom Dupré la Tour
(2015), Raghav RV (2015-2016) and Thierry Guillemot (2016) to work on scikit-learn.

Columbia University funds Andreas Mueller since 2016. The following students were sponsored by Google to work on scikit-learn through the Google Summer of Code program.

- 2007 - David Cournapeau
- 2011 - Vlad Niculae
- 2012 - Vlad Niculae, Immanuel Bayer.
- 2013 - Kemal Eren, Nicolas Trésegnié
- 2014 - Hamzeh Alsalhi, Issam Laradji, Maheshakya Wijewardena, Manoj Kumar.
- 2015 - Raghav RV, Wei Xue
- 2016 - Nelson Liu, YenChen Lin

It also provided funding for sprints and events around scikit-learn. If you would like to participate in the next Google Summer of code program, please see this page.

The NeuroDebian project providing Debian packaging and contributions is supported by Dr. James V. Haxby (Dartmouth College).

The PSF helped find and manage funding for our 2011 Granada sprint. More information can be found here.

tiny clues funded the 2011 international Granada sprint.

Donating to the project

If you are interested in donating to the project or to one of our code-sprints, you can use the Paypal button below or the NumFOCUS Donations Page (if you use the latter, please indicate that you are donating for the scikit-learn project).

All donations will be handled by NumFOCUS, a non-profit-organization which is managed by a board of Scipy community members. NumFOCUS’s mission is to foster scientific computing software, in particular in Python. As a fiscal home of scikit-learn, it ensures that money is available when needed to keep the project funded and available while in compliance with tax regulations.

The received donations for the scikit-learn project mostly will go towards covering travel-expenses for code sprints, as well as towards the organization budget of the project.

1 Regarding the organization budget in particular, we might use some of the donated funds to pay for other project expenses such as DNS, hosting or continuous integration services.
Notes

The 2013 Paris international sprint

Fig. 1.1: IAP VII/19 - DYSCO

For more information on this sprint, see here

1.5.6 Infrastructure support

- We would like to thank Rackspace for providing us with a free Rackspace Cloud account to automatically build the documentation and the example gallery from for the development version of scikit-learn using this tool.
- We would also like to thank Shining Panda for free CPU time on their Continuous Integration server.

1.6 Who is using scikit-learn?

1.6.1 Spotify

Scikit-learn provides a toolbox with solid implementations of a bunch of state-of-the-art models and makes it easy to plug them into existing applications. We’ve been using it quite a lot for music recommendations at Spotify and I think it’s the most well-designed ML package I’ve seen so far.

Erik Bernhardsson, Engineering Manager Music Discovery & Machine Learning, Spotify
1.6.2 Inria

At INRIA, we use scikit-learn to support leading-edge basic research in many teams: Parietal for neuroimaging, Lear for computer vision, Visages for medical image analysis, Privatics for security. The project is a fantastic tool to address difficult applications of machine learning in an academic environment as it is performant and versatile, but all easy-to-use and well documented, which makes it well suited to grad students.

Gaël Varoquaux, research at Parietal

1.6.3 betaworks

Betaworks is a NYC-based startup studio that builds new products, grows companies, and invests in others. Over the past 8 years we’ve launched a handful of social data analytics-driven services, such as Bitly, Chartbeat, digg and Scale Model. Consistently the betaworks data science team uses Scikit-learn for a variety of tasks. From exploratory analysis, to product development, it is an essential part of our toolkit. Recent uses are included in digg’s new video recommender system, and Poncho’s dynamic heuristic subspace clustering.

Gilad Lotan, Chief Data Scientist

1.6.4 Evernote

Building a classifier is typically an iterative process of exploring the data, selecting the features (the attributes of the data believed to be predictive in some way), training the models, and finally evaluating them. For many of these tasks, we relied on the excellent scikit-learn package for Python.

Mark Ayzenshtat, VP, Augmented Intelligence

1.6.5 Télécom ParisTech

At Telecom ParisTech, scikit-learn is used for hands-on sessions and home assignments in introductory and advanced machine learning courses. The classes are for undergrads and masters students. The
great benefit of scikit-learn is its fast learning curve that allows students to quickly start working on interesting and motivating problems.
Alexandre Gramfort, Assistant Professor

1.6.6 Booking.com

At Booking.com, we use machine learning algorithms for many different applications, such as recommending hotels and destinations to our customers, detecting fraudulent reservations, or scheduling our customer service agents. Scikit-learn is one of the tools we use when implementing standard algorithms for prediction tasks. Its API and documentations are excellent and make it easy to use. The scikit-learn developers do a great job of incorporating state of the art implementations and new algorithms into the package. Thus, scikit-learn provides convenient access to a wide spectrum of algorithms, and allows us to readily find the right tool for the right job.
Melanie Mueller, Data Scientist

1.6.7 AWeber

The scikit-learn toolkit is indispensable for the Data Analysis and Management team at AWeber. It allows us to do AWesome stuff we would not otherwise have the time or resources to accomplish. The documentation is excellent, allowing new engineers to quickly evaluate and apply many different algorithms to our data. The text feature extraction utilities are useful when working with the large volume of email content we have at AWeber. The RandomizedPCA implementation, along with Pipelining and FeatureUnions, allows us to develop complex machine learning algorithms efficiently and reliably.
Anyone interested in learning more about how AWeber deploys scikit-learn in a production environment should check out talks from PyData Boston by AWeber’s Michael Becker available at https://github.com/mdbecker/pydata_2013
Michael Becker, Software Engineer, Data Analysis and Management Ninjas

1.6.8 Yhat

The combination of consistent APIs, thorough documentation, and top notch implementation make scikit-learn our favorite machine learning package in Python. scikit-learn makes doing advanced analysis in Python accessible to anyone. At Yhat, we make it easy to integrate these models into your production applications. Thus eliminating the unnecessary dev time encountered productionizing analytical work.
Greg Lamp, Co-founder Yhat
1.6.9 Rangespan

Rangespan is a core tool in the data science group at Rangespan. Its large collection of well documented models and algorithms allow our team of data scientists to prototype fast and quickly iterate to find the right solution to our learning problems. We find that scikit-learn is not only the right tool for prototyping, but its careful and well tested implementation give us the confidence to run scikit-learn models in production.

Jurgen Van Gael, Data Science Director at Rangespan Ltd

1.6.10 Birchbox

At Birchbox, we face a range of machine learning problems typical to E-commerce: product recommendation, user clustering, inventory prediction, trends detection, etc. Scikit-learn lets us experiment with many models, especially in the exploration phase of a new project: the data can be passed around in a consistent way; models are easy to save and reuse; updates keep us informed of new developments from the pattern discovery research community. Scikit-learn is an important tool for our team, built the right way in the right language.

Thierry Bertin-Mahieux, Birchbox, Data Scientist

1.6.11 Bestofmedia Group

Best of Media Group is our #1 toolkit for all things machine learning at Bestofmedia. We use it for a variety of tasks (e.g. spam fighting, ad click prediction, various ranking models) thanks to the varied, state-of-the-art algorithm implementations packaged into it. In the lab it accelerates prototyping of complex pipelines. In production I can say it has proven to be robust and efficient enough to be deployed for business critical components.

Eustache Diemert, Lead Scientist Bestofmedia Group

1.6. Who is using scikit-learn?
1.6.12 Change.org

At change.org we automate the use of scikit-learn’s RandomForestClassifier in our production systems to drive email targeting that reaches millions of users across the world each week. In the lab, scikit-learn’s ease-of-use, performance, and overall variety of algorithms implemented has proved invaluable in giving us a single reliable source to turn to for our machine-learning needs.

Vijay Ramesh, Software Engineer in Data/science at Change.org

1.6.13 PHIMECA Engineering

At PHIMECA Engineering, we use scikit-learn estimators as surrogates for expensive-to-evaluate numerical models (mostly but not exclusively finite-element mechanical models) for speeding up the intensive post-processing operations involved in our simulation-based decision making framework. Scikit-learn’s fit/predict API together with its efficient cross-validation tools considerably eases the task of selecting the best-fit estimator. We are also using scikit-learn for illustrating concepts in our training sessions. Trainees are always impressed by the ease-of-use of scikit-learn despite the apparent theoretical complexity of machine learning.

Vincent Dubourg, PHIMECA Engineering, PhD Engineer

1.6.14 HowAboutWe

At HowAboutWe, scikit-learn lets us implement a wide array of machine learning techniques in analysis and in production, despite having a small team. We use scikit-learn’s classification algorithms to predict user behavior, enabling us to (for example) estimate the value of leads from a given traffic source early in the lead’s tenure on our site. Also, our users’ profiles consist of primarily unstructured data (answers to open-ended questions), so we use scikit-learn’s feature extraction and dimensionality reduction tools to translate these unstructured data into inputs for our matchmaking system.

Daniel Weitzenfeld, Senior Data Scientist at HowAboutWe
1.6.15 PeerIndex

At PeerIndex we use scientific methodology to build the Influence Graph - a unique dataset that allows us to identify who’s really influential and in which context. To do this, we have to tackle a range of machine learning and predictive modeling problems. Scikit-learn has emerged as our primary tool for developing prototypes and making quick progress. From predicting missing data and classifying tweets to clustering communities of social media users, scikit-learn proved useful in a variety of applications. Its very intuitive interface and excellent compatibility with other python tools makes it and indispensable tool in our daily research efforts.

Ferenc Huszar - Senior Data Scientist at Peerindex

1.6.16 DataRobot

DataRobot is building next generation predictive analytics software to make data scientists more productive, and scikit-learn is an integral part of our system. The variety of machine learning techniques in combination with the solid implementations that scikit-learn offers makes it a one-stop-shopping library for machine learning in Python. Moreover, its consistent API, well-tested code and permissive licensing allow us to use it in a production environment. Scikit-learn has literally saved us years of work we would have had to do ourselves to bring our product to market.

Jeremy Achin, CEO & Co-founder DataRobot Inc.

1.6.17 OkCupid

We’re using scikit-learn at OkCupid to evaluate and improve our matchmaking system. The range of features it has, especially preprocessing utilities, means we can use it for a wide variety of projects, and it’s performant enough to handle the volume of data that we need to sort through. The documentation is really thorough, as well, which makes the library quite easy to use.

David Koh - Senior Data Scientist at OkCupid

1.6.18 Lovely

At Lovely, we strive to deliver the best apartment marketplace, with respect to our users and our listings. From understanding user behavior, improving data quality, and detecting fraud, scikit-learn is a
regular tool for gathering insights, predictive modeling and improving our product. The easy-to-read documentation and intuitive architecture of the API makes machine learning both explorable and accessible to a wide range of python developers. I’m constantly recommending that more developers and scientists try scikit-learn.

Simon Frid - Data Scientist, Lead at Lovely

1.6.19 Data Publica

Data Publica builds a new predictive sales tool for commercial and marketing teams called C-Radar. We extensively use scikit-learn to build segmentations of customers through clustering, and to predict future customers based on past partnerships success or failure. We also categorize companies using their website communication thanks to scikit-learn and its machine learning algorithm implementations. Eventually, machine learning makes it possible to detect weak signals that traditional tools cannot see. All these complex tasks are performed in an easy and straightforward way thanks to the great quality of the scikit-learn framework.

Guillaume Lebourgeois & Samuel Charron - Data Scientists at Data Publica

1.6.20 Machinalis

Scikit-learn is the cornerstone of all the machine learning projects carried at Machinalis. It has a consistent API, a wide selection of algorithms and lots of auxiliary tools to deal with the boilerplate. We have used it in production environments on a variety of projects including click-through rate prediction, information extraction, and even counting sheep!

In fact, we use it so much that we’ve started to freeze our common use cases into Python packages, some of them open-sourced, like FeatureForge. Scikit-learn in one word: Awesome.

Rafael Carrascosa, Lead developer

1.6.21 solido

Scikit-learn is helping to drive Moore’s Law, via Solido. Solido creates computer-aided design tools used by the majority of top-20 semiconductor companies and fabs, to design the bleeding-edge chips inside smartphones, automobiles, and more. Scikit-learn helps to power Solido’s algorithms for rare-event estimation, worst-case verification, optimization, and more. At Solido, we are particularly fond of scikit-learn’s libraries for Gaussian Process models, large-scale regularized linear regression, and classification. Scikit-learn has increased our productivity, because for many ML problems we no longer need to “roll our own” code. This PyData 2014 talk has details.

Trent McConaghy, founder, Solido Design Automation Inc.
1.6.22 INFONEA

INFONEA®

We employ scikit-learn for rapid prototyping and custom-made Data Science solutions within our in-memory based Business Intelligence Software INFONEA®. As a well-documented and comprehensive collection of state-of-the-art algorithms and pipelining methods, scikit-learn enables us to provide flexible and scalable scientific analysis solutions. Thus, scikit-learn is immensely valuable in realizing a powerful integration of Data Science technology within self-service business analytics.

Thorsten Kranz, Data Scientist, Coma Soft AG.

1.6.23 Dataiku

Dataiku

Our software, Data Science Studio (DSS), enables users to create data services that combine ETL with Machine Learning. Our Machine Learning module integrates many scikit-learn algorithms. The scikit-learn library is a perfect integration with DSS because it offers algorithms for virtually all business cases. Our goal is to offer a transparent and flexible tool that makes it easier to optimize time consuming aspects of building a data service, preparing data, and training machine learning algorithms on all types of data.

Florian Douetteau, CEO, Dataiku

1.6.24 Otto Group

Otto Group

Here at Otto Group, one of global Big Five B2C online retailers, we are using scikit-learn in all aspects of our daily work from data exploration to development of machine learning application to the productive deployment of those services. It helps us to tackle machine learning problems ranging from e-commerce to logistics. It consistent APIs enabled us to build the Palladium REST-API framework around it and continuously deliver scikit-learn based services.

Christian Rammig, Head of Data Science, Otto Group

1.7 Release history

1.7.1 Version 0.18.2

June 20, 2017

Last release with Python 2.6 support

Scikit-learn 0.18 is the last major release of scikit-learn to support Python 2.6. Later versions of scikit-learn will require Python 2.7 or above.

Changelog

- Fixes for compatibility with NumPy 1.13.0: #7946 #8355 by Loic Esteve.
Minor compatibility changes in the examples #9010 #8040 #9149.

Code Contributors

Aman Dalmia, Loic Esteve, Nate Guerin, Sergei Lebedev

1.7.2 Version 0.18.1

November 11, 2016

Last release with Python 2.6 support

Scikit-learn 0.18 is the last major release of scikit-learn to support Python 2.6. Later versions of scikit-learn will require Python 2.7 or above.

Changelog

Enhancements

- Improved `sample_without_replacement` speed by utilizing `numpy.random.permutation` for most cases. As a result, samples may differ in this release for a fixed random state. Affected estimators:
  - `ensemble.BaggingClassifier`
  - `ensemble.BaggingRegressor`
  - `linear_model.RANSACRegressor`
  - `model_selection.RandomizedSearchCV`
  - `random_projection.SparseRandomProjection`

  This also affects the `datasets.make_classification` method.

Bug fixes

- Fix issue where `min_grad_norm` and `n_iter_without_progress` parameters were not being utilised by `manifold.TSNE`. #6497 by Sebastian Säger
- Fix bug for svm’s decision values when `decision_function_shape` is `ovr` in `svm.SVC`. svm.SVC’s decision function was incorrect from versions 0.17.0 through 0.18.0. #7724 by Bing Tian Dai
- Attribute `explained_variance_ratio` of `discriminant_analysis.LinearDiscriminantAnalysis` calculated with SVD and Eigen solver are now of the same length. #7632 by JPFrancoia
- Fixes issue in Univariate feature selection where score functions were not accepting multi-label targets. #7676 by ‘Mohammed Affan’
- Fixed setting parameters when calling `fit` multiple times on `feature_selection.SelectFromModel`. #7756 by Andreas Müller
- Fixes issue in partial_fit method of `multiclass.OneVsRestClassifier` when number of classes used in partial_fit was less than the total number of classes in the data. #7786 by Srivatsan Ramesh
• Fixes issue in `calibration.CalibratedClassifierCV` where the sum of probabilities of each class for a data was not 1, and `CalibratedClassifierCV` now handles the case where the training set has less number of classes than the total data. #7799 by Srivatsan Ramesh

• Fix a bug where `sklearn.feature_selection.SelectFdr` did not exactly implement Benjamini-Hochberg procedure. It formerly may have selected fewer features than it should. #7490 by Peng Meng.

• `sklearn.manifold.LocallyLinearEmbedding` now correctly handles integer inputs. #6282 by Jake Vanderplas.

• The `min_weight_fraction_leaf` parameter of tree-based classifiers and regressors now assumes uniform sample weights by default if the `sample_weight` argument is not passed to the `fit` function. Previously, the parameter was silently ignored. #7301 by Nelson Liu.

• Numerical issue with `linear_model.RidgeCV` on centered data when `n_features > n_samples`. #6178 by Bertrand Thirion

• Tree splitting criterion classes’ cloning/pickling is now memory safe #7680 by Ibraim Ganiev.

• Fixed a bug where `decomposition.NMF` sets its `n_iters_` attribute in `transform()`. #7553 by Ekaterina Krivich.

• `sklearn.linear_model.LogisticRegressionCV` now correctly handles string labels. #5874 by Raghav RV.

• Fixed a bug where `sklearn.model_selection.train_test_split` raised an error when `stratify` is a list of string labels. #7593 by Raghav RV.

• Fixed a bug where `sklearn.model_selection.GridSearchCV` and `sklearn.model_selection.RandomizedSearchCV` were not pickleable because of a pickling bug in `np.ma.MaskedArray`. #7594 by Raghav RV.

• All cross-validation utilities in `sklearn.model_selection` now permit one time cross-validation splitters for the `cv` parameter. Also non-deterministic cross-validation splitters (where multiple calls to `split` produce dissimilar splits) can be used as `cv` parameter. The `sklearn.model_selection.GridSearchCV` will cross-validate each parameter setting on the split produced by the first `split` call to the cross-validation splitter. #7660 by Raghav RV.

**API changes summary**

**Trees and forests**

• The `min_weight_fraction_leaf` parameter of tree-based classifiers and regressors now assumes uniform sample weights by default if the `sample_weight` argument is not passed to the `fit` function. Previously, the parameter was silently ignored. (#7301) by ‘Nelson Liu’.

• Tree splitting criterion classes’ cloning/pickling is now memory safe (#7680). By ‘Ibraim Ganiev’.

**Linear, kernelized and related models**

• Length of `explained_variance_ratio` of `discriminant_analysis.LinearDiscriminantAnalysis` changed for both Eigen and SVD solvers. The attribute has now a length of `min(n_components, n_classes - 1)`. #7632 by JPFrancoia

• Numerical issue with `linear_model.RidgeCV` on centered data when `n_features > n_samples`. (#6178) by Bertrand Thirion

**1.7. Release history**
1.7.3 Version 0.18

September 28, 2016

The new module \texttt{sklearn.model_selection}, which groups together the functionalities of formerly \texttt{sklearn.cross_validation}, \texttt{sklearn.grid_search} and \texttt{sklearn.learning_curve}, introduces new possibilities such as nested cross-validation and better manipulation of parameter searches with Pandas.

Many things will stay the same but there are some key differences. Read below to know more about the changes.

- **Data-independent CV splitters enabling nested cross-validation**

The new cross-validation splitters, defined in the \texttt{sklearn.model_selection}, are no longer initialized with any data-dependent parameters such as \texttt{y}. Instead they expose a \texttt{split} method that takes in the data and yields a generator for the different splits.

This change makes it possible to use the cross-validation splitters to perform nested cross-validation, facilitated by \texttt{model_selection.GridSearchCV} and \texttt{model_selection.RandomizedSearchCV} utilities.

- **The enhanced cv_results\_ attribute**

The new \texttt{cv_results\_} attribute (of \texttt{model_selection.GridSearchCV} and \texttt{model_selection.RandomizedSearchCV}) introduced in lieu of the \texttt{grid_scores\_} attribute is a dict of 1D arrays with elements in each array corresponding to the parameter settings (i.e. search candidates).

The \texttt{cv_results\_} dict can be easily imported into pandas as a DataFrame for exploring the search results. The \texttt{cv_results\_} arrays include scores for each cross-validation split (with keys such as 'split0_test_score'), as well as their mean ('mean_test_score') and standard deviation ('std_test_score').
The ranks for the search candidates (based on their mean cross-validation score) is available at `cv_results_['rank_test_score']`.

The parameter values for each parameter is stored separately as numpy masked object arrays. The value, for that search candidate, is masked if the corresponding parameter is not applicable. Additionally a list of all the parameter dicts are stored at `cv_results_['params']`.

- **Parameters `n_folds` and `n_iter` renamed to `n_splits`**

Some parameter names have changed: The `n_folds` parameter in new `model_selection.KFold`, `model_selection.GroupKFold` (see below for the name change), and `model_selection.StratifiedKFold` is now renamed to `n_splits`. The `n_iter` parameter in `model_selection.ShuffleSplit`, the new class `model_selection.GroupShuffleSplit` and `model_selection.StratifiedShuffleSplit` is now renamed to `n_splits`.

- **Rename of splitter classes which accepts group labels along with data**


Note the change from singular to plural form in `model_selection.LeavePGroupsOut`.

- **Fit parameter labels renamed to groups**

The `labels` parameter in the `split` method of the newly renamed splitters `model_selection.GroupKFold`, `model_selection.LeaveOneGroupOut`, `model_selection.LeavePGroupsOut`, `model_selection.GroupShuffleSplit` is renamed to `groups` following the new nomenclature of their class names.

- **Parameter `n_labels` renamed to `n_groups`**

The parameter `n_labels` in the newly renamed `model_selection.LeavePGroupsOut` is changed to `n_groups`.

- **Training scores and Timing information**

`cv_results_` also includes the training scores for each cross-validation split (with keys such as 'split0_train_score'), as well as their mean ('mean_train_score') and standard deviation ('std_train_score'). To avoid the cost of evaluating training score, set `return_train_score=False`.

Additionally the mean and standard deviation of the times taken to split, train and score the model across all the cross-validation splits is available at the key 'mean_time' and 'std_time' respectively.

**Changelog**

**New features**

Classifiers and Regressors

- The Gaussian Process module has been reimplemented and now offers classification and regression estimators through `gaussian_process.GaussianProcessClassifier` and `gaussian_process.GaussianProcessRegressor`. Among other things, the new implementation supports kernel engineering, gradient-based hyperparameter optimization or sampling of functions from GP prior and GP posterior. Extensive documentation and examples are provided. By Jan Hendrik Metzen.

- Added new supervised learning algorithm: `Multi-layer Perceptron` #3204 by Issam H. Laradji

- Added `linear_model.HuberRegressor`, a linear model robust to outliers. #5291 by Manoj Kumar.
• Added the `multioutput.MultiOutputRegressor` meta-estimator. It converts single output regressors to multi-output regressors by fitting one regressor per output. By Tim Head.

Other estimators

• New `mixture.GaussianMixture` and `mixture.BayesianGaussianMixture` replace former mixture models, employing faster inference for sounder results. #7295 by Wei Xue and Thierry Guillemot.

• Class `decomposition.RandomizedPCA` is now factored into `decomposition.PCA` and it is available calling with parameter `svd_solver='randomized'`. The default number of `n_iter` for 'randomized' has changed to 4. The old behavior of PCA is recovered by `svd_solver='full'`. An additional solver calls `arpack` and performs truncated (non-randomized) SVD. By default, the best solver is selected depending on the size of the input and the number of components requested. #5299 by Giorgio Patrini.

• Added two functions for mutual information estimation: `feature_selection.mutual_info_classif` and `feature_selection.mutual_info_regression`. These functions can be used in `feature_selection.SelectKBest` and `feature_selection.SelectPercentile` as score functions. By Andrea Bravi and Nikolay Mayorov.

• Added the `ensemble.IsolationForest` class for anomaly detection based on random forests. By Nicolas Goix.

• Added `algorithm='elkan'` to `cluster.KMeans` implementing Elkan’s fast K-Means algorithm. By Andreas Müller.

Model selection and evaluation

• Added `metrics.cluster.fowlkes_mallows_score`, the Fowlkes Mallows Index which measures the similarity of two clusterings of a set of points By Arnaud Fouchet and Thierry Guillemot.

• Added `metrics.calinski_harabaz_score`, which computes the Calinski and Harabaz score to evaluate the resulting clustering of a set of points. By Arnaud Fouchet and Thierry Guillemot.

• Added new cross-validation splitter `model_selection.TimeSeriesSplit` to handle time series data. #6586 by YenChen Lin

• The cross-validation iterators are replaced by cross-validation splitters available from `sklearn.model_selection`, allowing for nested cross-validation. See Model Selection Enhancements and API Changes for more information. #4294 by Raghav RV.

Enhancements

Trees and ensembles

• Added a new splitting criterion for `tree.DecisionTreeRegressor`, the mean absolute error. This criterion can also be used in `ensemble.ExtraTreesRegressor`, `ensemble.RandomForestRegressor`, and the gradient boosting estimators. #6667 by Nelson Liu.

• Added weighted impurity-based early stopping criterion for decision tree growth. #6954 by Nelson Liu

• The random forest, extra tree and decision tree estimators now have a method `decision_path` which returns the decision path of samples in the tree. By Arnaud Joly.

• A new example has been added unveiling the decision tree structure. By Arnaud Joly.

• Random forest, extra trees, decision trees and gradient boosting estimator accept the parameter `min_samples_split` and `min_samples_leaf` provided as a percentage of the training samples. By yelitl and Arnaud Joly.
• Gradient boosting estimators accept the parameter criterion to specify to splitting criterion used in built decision trees. #6667 by Nelson Liu.

• The memory footprint is reduced (sometimes greatly) for ensemble.bagging.BaseBagging and classes that inherit from it, i.e., ensemble.BaggingClassifier, ensemble.BaggingRegressor, and ensemble.IsolationForest, by dynamically generating attribute estimators_samples_ only when it is needed. By David Staub.

• Added n_jobs and sample_weight parameters for ensemble.VotingClassifier to fit underlying estimators in parallel. #5805 by Ibraim Ganiev.

Linear, kernelized and related models

• In linear_model.LogisticRegression, the SAG solver is now available in the multinomial case. #5251 by Tom Dupre la Tour.

• linear_model.RANSACRegressor, svm.LinearSVC and svm.LinearSVR now support sample_weight. By Imaculate.

• Add parameter loss to linear_model.RANSACRegressor to measure the error on the samples for every trial. By Manoj Kumar.

• Prediction of out-of-sample events with Isotonic Regression (isotonic.IsotonicRegression) is now much faster (over 1000x in tests with synthetic data). By Jonathan Arfa.

• Isotonic regression (isotonic.IsotonicRegression) now uses a better algorithm to avoid $O(n^2)$ behavior in pathological cases, and is also generally faster (##6691). By Antony Lee.

• naive_bayes.GaussianNB now accepts data-independent class-priors through the parameter priors. By Guillaume Lemaitre.

• linear_model.ElasticNet and linear_model.Lasso now works with np.float32 input data without converting it into np.float64. This allows to reduce the memory consumption. #6913 by YenChen Lin.

• semi_supervised.LabelPropagation and semi_supervised.LabelSpreading now accept arbitrary kernel functions in addition to strings knn and rbf. #5762 by Utkarsh Upadhyay.

Decomposition, manifold learning and clustering

• Added inverse_transform function to decomposition.NMF to compute data matrix of original shape. By Anish Shah.

• cluster.KMeans and cluster.MiniBatchKMeans now works with np.float32 and np.float64 input data without converting it. This allows to reduce the memory consumption by using np.float32. #6846 by Sebastian Sager and YenChen Lin.

Preprocessing and feature selection

• preprocessing.RobustScaler now accepts quantile_range parameter. #5929 by Konstantin Podshumok.

• feature_extraction.FeatureHasher now accepts string values. #6173 by Ryad Zenine and Devashish Deshpande.

• Keyword arguments can now be supplied to func in preprocessing.FunctionTransformer by means of the kw_args parameter. By Brian McFee.

• feature_selection.SelectKBest and feature_selection.SelectPercentile now accept score functions that take X, y as input and return only the scores. By Nikolay Mayorov.

Model evaluation and meta-estimators
• `multiclass.OneVsOneClassifier` and `multiclass.OneVsRestClassifier` now support partial_fit. By Asish Panda and Philipp Dowling.

• Added support for substituting or disabling `pipeline.Pipeline` and `pipeline.FeatureUnion` components using the set_params interface that powers sklearn.grid_search. See sphx_glr_plot_compare_reduction.py. By Joel Nothman and Robert McGibbon.

• The new `cv_results_` attribute of `model_selection.GridSearchCV` (and `model_selection.RandomizedSearchCV`) can be easily imported into pandas as a DataFrame. Ref Model Selection Enhancements and API Changes for more information. #6697 by Raghav RV.

• Generalization of `model_selection.cross_val_predict`. One can pass method names such as `predict_proba` to be used in the cross validation framework instead of the default `predict`. By Ori Ziv and Sears Merritt.

• The training scores and time taken for training followed by scoring for each search candidate are now available at the `cv_results_` dict. See Model Selection Enhancements and API Changes for more information. #7325 by Eugene Chen and Raghav RV.

Metrics

• Added `labels` flag to `metrics.log_loss` to to explicitly provide the labels when the number of classes in `y_true` and `y_pred` differ. #7239 by Hong Guangguo with help from Mads Jensen and Nelson Liu.

• Support sparse contingency matrices in cluster evaluation (metrics.cluster.supervised) to scale to a large number of clusters. #7419 by Gregory Stupp and Joel Nothman.

• Add `sample_weight` parameter to `metrics.matthews_corrcoef`. By Jatin Shah and Raghav RV.

• Speed up `metrics.silhouette_score` by using vectorized operations. By Manoj Kumar.

• Add `sample_weight` parameter to `metrics.confusion_matrix`. By Bernardo Stein.

Miscellaneous

• Added `n_jobs` parameter to `feature_selection.RFECV` to compute the score on the test folds in parallel. By Manoj Kumar.

• Codebase does not contain C/C++ cython generated files: they are generated during build. Distribution packages will still contain generated C/C++ files. By Arthur Mensch.

• Reduce the memory usage for 32-bit float input arrays of `utils.sparse_func.mean_variance_axis` and `utils.sparse_func.incr_mean_variance_axis` by supporting cython fused types. By YenChen Lin.

• The `ignore_warnings` now accept a category argument to ignore only the warnings of a specified type. By Thierry Guillemot.

• Added parameter `return_X_y` and return type `(data, target) : tuple` option to `load_iris` dataset #7049, `load_breast_cancer` dataset #7152, `load_digits` dataset, `load_diabetes` dataset, `load_linnerud` dataset, `load_boston` dataset #7154 by Manvendra Singh.

• Simplification of the `clone` function, deprecate support for estimators that modify parameters in `__init__`. #5340 by Andreas Müller.

• When unpickling a scikit-learn estimator in a different version than the one the estimator was trained with, a `UserWarning` is raised, see the documentation on model persistence for more details. (#7248) By Andreas Müller.

Bug fixes

Trees and ensembles
Random forest, extra trees, decision trees and gradient boosting won’t accept anymore min_samples_split=1 as at least 2 samples are required to split a decision tree node. By Arnaud Joly.

ensemble.VotingClassifier now raises NotFittedError if predict, transform or predict_proba are called on the non-fitted estimator. by Sebastian Raschka.

Fix bug where ensemble.AdaBoostClassifier and ensemble.AdaBoostRegressor would perform poorly if the random_state was fixed (#7411). By Joel Nothman.

Fix bug in ensembles with randomization where the ensemble would not set random_state on base estimators in a pipeline or similar nesting. (#7411). Note, results for ensemble.BaggingClassifier ensemble.BaggingRegressor, ensemble.AdaBoostClassifier and ensemble.AdaBoostRegressor will now differ from previous versions. By Joel Nothman.

Linear, kernelized and related models

Fixed incorrect gradient computation for loss='squared_epsilon_insensitive' in linear_model.SGDClassifier and linear_model.SGDRegressor (#6764). By Wenhua Yang.

Fix bug in linear_model.LogisticRegressionCV where solver='liblinear' did not accept class_weights='balanced'. (#6817). By Tom Dupre la Tour.

Fix bug in neighbors.RadiusNeighborsClassifier where an error occurred when there were outliers being labelled and a weight function specified (#6902). By LeonieBorne.

Fix linear_model.ElasticNet sparse decision function to match output with dense in the multioutput case.

Decomposition, manifold learning and clustering

decomposition.RandomizedPCA default number of iterated_power is 4 instead of 3. #5141 by Giorgio Patrini.

utils.extmath.randomized_svd performs 4 power iterations by default, instead of 0. In practice this is enough for obtaining a good approximation of the true eigenvalues/vectors in the presence of noise. When n_components is small(< .1 * min(X.shape)) n_iter is set to 7, unless the user specifies a higher number. This improves precision with few components. #5299 by Giorgio Patrini.

Whiten/non-whiten inconsistency between components of decomposition.PCA and decomposition.RandomizedPCA (now factored into PCA, see the New features) is fixed. components_ are stored with no whitening. #5299 by Giorgio Patrini.

Fixed bug in manifold.spectral_embedding where diagonal of unnormalized Laplacian matrix was incorrectly set to 1. #4995 by Peter Fischer.


Attribute explained_variance_ratio_ calculated with the SVD solver of discriminant_analysis.LinearDiscriminantAnalysis now returns correct results. By JPFrancoia

Preprocessing and feature selection

preprocessing.data._transform_selected now always passes a copy of X to transform function when copy=True (#7194). By Caio Oliveira.

Model evaluation and meta-estimators
• `model_selection.StratifiedKFold` now raises error if all n_labels for individual classes is less than n_folds. #6182 by Devashish Deshpande.

• Fixed bug in `model_selection.StratifiedShuffleSplit` where train and test sample could overlap in some edge cases, see #6121 for more details. By Loic Esteve.

• Fix in sklearn.model_selection.StratifiedShuffleSplit to return splits of size train_size and test_size in all cases (#6472). By Andreas Müller.

• Cross-validation of OneVsOneClassifier and OneVsRestClassifier now works with precomputed kernels. #7350 by Russell Smith.

• Fix incomplete `predict_proba` method delegation from `model_selection.GridSearchCV` to `linear_model.SGDClassifier` (#7159) by Yichuan Liu.

Metrics

• Fix bug in `metrics.silhouette_score` in which clusters of size 1 were incorrectly scored. They should get a score of 0. By Joel Nothman.

• Fix bug in `metrics.silhouette_samples` so that it now works with arbitrary labels, not just those ranging from 0 to n_clusters - 1.

• Fix bug where expected and adjusted mutual information were incorrect if cluster contingency cells exceeded 2**16. By Joel Nothman.

• `metrics.pairwise.pairwise_distances` now converts arrays to boolean arrays when required in scipy.spatial.distance. #5460 by Tom Dupre la Tour.

• Fix sparse input support in `metrics.silhouette_score` as well as example examples/text/document_clustering.py. By YenChen Lin.

• `metrics.roc_curve` and `metrics.precision_recall_curve` no longer round `y_score` values when creating ROC curves; this was causing problems for users with very small differences in scores (#7353).

Miscellaneous

• `model_selection.tests._search._check_param_grid` now works correctly with all types that extends/implements Sequence (except string), including range (Python 3.x) and xrange (Python 2.x). #7323 by Viacheslav Kovalevskyi.

• `utils.extmath.randomized_range_finder` is more numerically stable when many power iterations are requested, since it applies LU normalization by default. If n_iter<2 numerical issues are unlikely, thus no normalization is applied. Other normalization options are available: 'none', 'LU' and 'QR'. #5141 by Giorgio Patrini.

• Fix a bug where some formats of `scipy.sparse` matrix, and estimators with them as parameters, could not be passed to base.clone. By Loic Esteve.

• `datasets.load_svmlight_file` now is able to read long int QID values. #7101 by Ibraim Ganiev.

API changes summary

Linear, kernelized and related models

• `residual_metric` has been deprecated in `linear_model.RANSACRegressor`. Use `loss` instead. By Manoj Kumar.

• Access to public attributes `.X_` and `.y_` has been deprecated in `isotonic.IsotonicRegression`. By Jonathan Arfa.

Decomposition, manifold learning and clustering
• The old mixture.DPGMM is deprecated in favor of the new mixture.BayesianGaussianMixture (with the parameter weight_concentration_prior_type='dirichlet_process'). The new class solves the computational problems of the old class and computes the Gaussian mixture with a Dirichlet process prior faster than before. #7295 by Wei Xue and Thierry Guillemot.

• The old mixture.VBGMM is deprecated in favor of the new mixture.BayesianGaussianMixture (with the parameter weight_concentration_prior_type='dirichlet_distribution'). The new class solves the computational problems of the old class and computes the Variational Bayesian Gaussian mixture faster than before. #6651 by Wei Xue and Thierry Guillemot.

• The old mixture.GMM is deprecated in favor of the new mixture.GaussianMixture. The new class computes the Gaussian mixture faster than before and some of computational problems have been solved. #6666 by Wei Xue and Thierry Guillemot.

Model evaluation and meta-estimators

• The sklearn.cross_validation, sklearn.grid_search and sklearn.learning_curve have been deprecated and the classes and functions have been reorganized into the sklearn.model_selection module. Ref Model Selection Enhancements and API Changes for more information. #4294 by Raghav RV.

• The grid_scores_ attribute of model_selection.GridSearchCV and model_selection.RandomizedSearchCV is deprecated in favor of the attribute cv_results_. Ref Model Selection Enhancements and API Changes for more information. #6697 by Raghav RV.

• The parameters n_iter or n_folds in old CV splitters are replaced by the new parameter n_splits since it can provide a consistent and unambiguous interface to represent the number of train-test splits. #7187 by YenChen Lin.

• classes parameter was renamed to labels in metrics.hamming_loss. #7260 by Sebastián Vanrrell.

• The splitter classes LabelKFold, LabelShuffleSplit, LeaveOneLabelOut and LeavePLabelOut are renamed to model_selection.GroupKFold, model_selection.GroupShuffleSplit, model_selection.LeaveOneGroupOut and model_selection.LeavePGroupsOut respectively. Also the parameter labels in the split method of the newly renamed splitters model_selection.LeaveOneGroupOut and model_selection.LeavePGroupsOut is renamed to groups. Additionally in model_selection.LeavePGroupsOut, the parameter n_labels is renamed to n_groups. #6660 by Raghav RV.

Code Contributors
1.7.4 Version 0.17.1

February 18, 2016

Changelog

Bug fixes

- Upgrade vendored joblib to version 0.9.4 that fixes an important bug in `joblib.Parallel` that can silently yield to wrong results when working on datasets larger than 1MB: https://github.com/joblib/joblib/blob/0.9.4/CHANGES.rst
- Fixed reading of Bunch pickles generated with scikit-learn version <= 0.16. This can affect users who have already downloaded a dataset with scikit-learn 0.16 and are loading it with scikit-learn 0.17. See #6196 for how this affected `datasets.fetch_20newsgroups`. By Loic Esteve.
- Fixed a bug that prevented using ROC AUC score to perform grid search on several CPU / cores on large arrays. See #6147 By Olivier Grisel.
- Fixed a bug that prevented to properly set the `presort` parameter in `ensemble.GradientBoostingRegressor`. See #5857 By Andrew McCulloh.
- Fixed a joblib error when evaluating the perplexity of a `decomposition.LatentDirichletAllocation` model. See #6258 By Chyi-Kwei Yau.

1.7.5 Version 0.17

November 5, 2015
Changelog

New features

- All the Scaler classes but `preprocessing.RobustScaler` can be fitted online by calling `partial_fit`. By Giorgio Patrini.

- The new class `ensemble.VotingClassifier` implements a “majority rule” / “soft voting” ensemble classifier to combine estimators for classification. By Sebastian Raschka.

- The new class `preprocessing.RobustScaler` provides an alternative to `preprocessing.StandardScaler` for feature-wise centering and range normalization that is robust to outliers. By Thomas Unterthiner.

- The new class `preprocessing.MaxAbsScaler` provides an alternative to `preprocessing.MinMaxScaler` for feature-wise range normalization when the data is already centered or sparse. By Thomas Unterthiner.

- The new class `preprocessing.FunctionTransformer` turns a Python function into a Pipeline-compatible transformer object. By Joe Jevnik.

- The new classes `cross_validation.LabelKFold` and `cross_validation.LabelShuffleSplit` generate train-test folds, respectively similar to `cross_validation.KFold` and `cross_validation.ShuffleSplit`, except that the folds are conditioned on a label array. By Brian McFee, Jean Kossaifi and Gilles Louppe.

- `decomposition.LatentDirichletAllocation` implements the Latent Dirichlet Allocation topic model with online variational inference. By Chyi-Kwei Yau, with code based on an implementation by Matt Hoffman. (#3659)

- The new solver `sag` implements a Stochastic Average Gradient descent and is available in both `linear_model.LogisticRegression` and `linear_model.Ridge`. This solver is very efficient for large datasets. By Danny Sullivan and Tom Dupre la Tour. (#4738)

- The new solver `cd` implements a Coordinate Descent in `decomposition.NMF`. Previous solver based on Projected Gradient is still available setting new parameter `solver` to `pg`, but is deprecated and will be removed in 0.19, along with `decompositionProjectedGradientNMF` and parameters `sparseness`, `eta`, `beta` and `nls_max_iter`. New parameters `alpha` and `l1_ratio` control L1 and L2 regularization, and `shuffle` adds a shuffling step in the `cd` solver. By Tom Dupre la Tour and Mathieu Blondel.

Enhancements

- `manifold.TSNE` now supports approximate optimization via the Barnes-Hut method, leading to much faster fitting. By Christopher Erick Moody. (#4025)

- `cluster.mean_shift_.MeanShift` now supports parallel execution, as implemented in the `mean_shift` function. By Martino Sorbaro.

- `naive_bayes.GaussianNB` now supports fitting with `sample_weight`. By Jan Hendrik Metzen.

- `dummy.DummyClassifier` now supports a prior fitting strategy. By Arnaud Joly.

- Added a `fit_predict` method for `mixture.GMM` and subclasses. By Cory Lorenz.

- Added the `metrics.label_ranking_loss` metric. By Arnaud Joly.

- Added the `metrics.cohen_kappa_score` metric.

- Added a `warm_start` constructor parameter to the bagging ensemble models to increase the size of the ensemble. By Tim Head.
• Added option to use multi-output regression metrics without averaging. By Konstantin Shmelkov and Michael Eickenberg.

• Added stratify option to cross_validation.train_test_split for stratified splitting. By Miroslav Batchkarov.

• The tree.export_graphviz function now supports aesthetic improvements for tree.DecisionTreeClassifier and tree.DecisionTreeRegressor, including options for coloring nodes by their majority class or impurity, showing variable names, and using node proportions instead of raw sample counts. By Trevor Stephens.

• Improved speed of newton-cg solver in linear_model.LogisticRegression, by avoiding loss computation. By Mathieu Blondel and Tom Dupre la Tour.

• The class_weight="auto" heuristic in classifiers supporting class_weight was deprecated and replaced by the class_weight="balanced" option, which has a simpler formula and interpretation. By Hanna Wallach and Andreas Müller.

• Add class_weight parameter to automatically weight samples by class frequency for linear_model.PassiveAgressiveClassifier. By Trevor Stephens.

• Added backlinks from the API reference pages to the user guide. By Andreas Müller.

• The labels parameter to sklearn.metrics.f1_score, sklearn.metrics.fbeta_score, sklearn.metrics.recall_score and sklearn.metrics.precision_score has been extended. It is now possible to ignore one or more labels, such as where a multiclass problem has a majority class to ignore. By Joel Nothman.

• Add sample_weight support to linear_model.RidgeClassifier. By Trevor Stephens.

• Provide an option for sparse output from sklearn.metrics.pairwise.cosine_similarity. By Jaidev Deshpande.

• Add minmax_scale to provide a function interface for MinMaxScaler. By Thomas Unterthiner.

• RCV1 dataset loader (sklearn.datasets.fetch_rcv1). By Tom Dupre la Tour.

• The “Wisconsin Breast Cancer” classical two-class classification dataset is now included in scikit-learn, available with sklearn.dataset.load_breast_cancer.

• Upgraded to joblib 0.9.3 to benefit from the new automatic batching of short tasks. This makes it possible for scikit-learn to benefit from parallelism when many very short tasks are executed in parallel, for instance by the grid_search.GridSearchCV meta-estimator with n_jobs > 1 used with a large grid of parameters on a small dataset. By Vlad Niculae, Olivier Grisel and Loic Estève.

• For more details about changes in joblib 0.9.3 see the release notes: https://github.com/joblib/joblib/blob/master/CHANGES.rst#release-093

• Improved speed (3 times per iteration) of decomposition.DictLearning with coordinate descent method from linear_model.Lasso. By Arthur Mensch.

• Parallel processing (threaded) for queries of nearest neighbors (using the ball-tree) by Nikolay Mayorov.

• Allow datasets.make_multilabel_classification to output a sparse y. By Kashif Rasul.

• cluster.DBSCAN now accepts a sparse matrix of precomputed distances, allowing memory-efficient distance precomputation. By Joel Nothman.

• tree.DecisionTreeClassifier now exposes an apply method for retrieving the leaf indices samples are predicted as. By Daniel Galvez and Gilles Louppe.
• Speed up decision tree regressors, random forest regressors, extra trees regressors and gradient boosting estimators by computing a proxy of the impurity improvement during the tree growth. The proxy quantity is such that the split that maximizes this value also maximizes the impurity improvement. By Arnaud Joly, Jacob Schreiber and Gilles Louppe.

• Speed up tree based methods by reducing the number of computations needed when computing the impurity measure taking into account linear relationship of the computed statistics. The effect is particularly visible with extra trees and on datasets with categorical or sparse features. By Arnaud Joly.

• `ensemble.GradientBoostingRegressor` and `ensemble.GradientBoostingClassifier` now expose an apply method for retrieving the leaf indices each sample ends up in under each try. By Jacob Schreiber.

• Add `sample_weight` support to `linear_model.LinearRegression`. By Sonny Hu. (#4881)

• Add `n_iter_without_progress` to `manifold.TSNE` to control the stopping criterion. By Santi Vil-lalba. (#5186)

• Added optional parameter `random_state` in `linear_model.Ridge`, to set the seed of the pseudo random generator used in `sag` solver. By Tom Dupre la Tour.

• Added optional parameter `warm_start` in `linear_model.LogisticRegression`. If set to True, the solvers `lbfgs`, `newton-cg` and `sag` will be initialized with the coefficients computed in the previous fit. By Tom Dupre la Tour.

• Added `sample_weight` support to `linear_model.LogisticRegression` for the `lbfgs`, `newton-cg`, and `sag` solvers. By Valentin Stolbunov. Support added to the `liblinear` solver. By Manoj Kumar.

• Added optional parameter `presort` to `ensemble.GradientBoostingRegressor` and `ensemble.GradientBoostingClassifier`, keeping default behavior the same. This allows gradient boosters to turn off presorting when building deep trees or using sparse data. By Jacob Schreiber.

• Altered `metrics.roc_curve` to drop unnecessary thresholds by default. By Graham Clenaghan.

• Added `feature_selection.SelectFromModel` meta-transformer which can be used along with estimators that have `coef_` or `feature_importances_` attribute to select important features of the input data. By Maheshakya Wijewardena, Joel Nothman and Manoj Kumar.

• Added `metrics.pairwise.laplacian_kernel`. By Clyde Fare.

• `covariance.GraphLasso` allows separate control of the convergence criterion for the Elastic-Net subproblem via the `enet_tol` parameter.

• Improved verbosity in `decomposition.DictionaryLearning`.

• `ensemble.RandomForestClassifier` and `ensemble.RandomForestRegressor` no longer explicitly store the samples used in bagging, resulting in a much reduced memory footprint for storing random forest models.

• Added positive option to `linear_model.Lars` and `linear_model.lars_path` to force coefficients to be positive. (#5131)

• Added the `X_norm_squared` parameter to `metrics.pairwise.euclidean_distances` to provide precomputed squared norms for `X`.

• Added the `fit_predict` method to `pipeline.Pipeline`.

• Added the `preprocessing.min_max_scale` function.
Bug fixes

- Fixed the output shape of `linear_model.RANSACRegressor` to `(n_samples,)`. By Andreas Müller.
- Fixed bug in `decomposition.DictLearning` when `n_jobs < 0`. By Andreas Müller.
- Fixed bug where `grid_search.RandomizedSearchCV` could consume a lot of memory for large discrete grids. By Joel Nothman.
- Fixed bug in `linear_model.LogisticRegressionCV` where `penalty` was ignored in the final fit. By Manoj Kumar.
- Fixed bug in `ensemble.forest.ForestClassifier` while computing `oob_score` and `X` is a sparse.csc_matrix. By Ankur Ankan.
- All regressors now consistently handle and warn when given `y` that is of shape `(n_samples,1)`. By Andreas Müller and Henry Lin. (#5431)
- Fix in `cluster.KMeans` cluster reassignment for sparse input by Lars Buitinck.
- Fixed a bug in `lda.LDA` that could cause asymmetric covariance matrices when using shrinkage. By Martin Billinger.
- Fixed `cross_validation.cross_val_predict` for estimators with sparse predictions. By Buddha Prakash.
- Fixed the `predict_proba` method of `linear_model.LogisticRegression` to use soft-max instead of one-vs-rest normalization. By Manoj Kumar. (#5182)
- Fixed the `partial_fit` method of `linear_model.SGDClassifier` when called with `average=True`. By Andrew Lamb. (#5282)
- Dataset fetchers use different filenames under Python 2 and Python 3 to avoid pickling compatibility issues. By Olivier Grisel. (#5355)
- Fixed a bug in `naive_bayes.GaussianNB` which caused classification results to depend on scale. By Jake Vanderplas.
- Fixed temporarily `linear_model.Ridge`, which was incorrect when fitting the intercept in the case of sparse data. The fix automatically changes the solver to ‘sag’ in this case. #5360 by Tom Dupre la Tour.
- Fixed a performance bug in `decomposition.RandomizedPCA` on data with a large number of features and fewer samples. (#4478) By Andreas Müller, Loic Esteve and Giorgio Patrini.
- Fixed bug in `cross_decomposition.PLS` that yielded unstable and platform dependent output, and failed on `fit_transform`. By Arthur Mensch.
- Fixes to the Bunch class used to store datasets.
- Fixed `ensemble.plot_partial_dependence` ignoring the percentiles parameter.
- Providing a set as vocabulary in `CountVectorizer` no longer leads to inconsistent results when pickling.
- Fixed the conditions on when a precomputed Gram matrix needs to be recomputed in `linear_model.LinearRegression`, `linear_model.OrthogonalMatchingPursuit`, `linear_model.Lasso` and `linear_model.ElasticNet`.
- Fixed inconsistent memory layout in the coordinate descent solver that affected `linear_model.DictionaryLearning` and `covariance.GraphLasso`. (#5337) By Olivier Grisel.
- `manifold.LocallyLinearEmbedding` no longer ignores the `reg` parameter.
• Nearest Neighbor estimators with custom distance metrics can now be pickled. (#4362)

• Fixed a bug in `pipeline.FeatureUnion` where `transformer_weights` were not properly handled when performing grid-searches.

• Fixed a bug in `linear_model.LogisticRegression` and `linear_model.LogisticRegressionCV` when using `class_weight='balanced'` or `class_weight='auto'`. By Tom Dupre la Tour.

• Fixed bug #5495 when doing OVR(SVC(decision_function_shape="ovr")). Fixed by Elvis Dohmatob.

API changes summary

• Attribute `data_min`, `data_max` and `data_range` in `preprocessing.MinMaxScaler` are deprecated and won’t be available from 0.19. Instead, the class now exposes `data_min_`, `data_max_` and `data_range_`. By Giorgio Patrini.

• All Scaler classes now have an `scale_` attribute, the feature-wise rescaling applied by their `transform` methods. The old attribute `std_` in `preprocessing.StandardScaler` is deprecated and superseded by `scale_`; it won’t be available in 0.19. By Giorgio Patrini.

• `svm.SVC` and `svm.NuSVC` now have an `decision_function_shape` parameter to make their decision function of shape `(n_samples,n_classes)` by setting `decision_function_shape='ovr'`. This will be the default behavior starting in 0.19. By Andreas Müller.

• Passing 1D data arrays as input to estimators is now deprecated as it caused confusion in how the array elements should be interpreted as features or as samples. All data arrays are now expected to be explicitly shaped `(n_samples,n_features)`. By Vighnesh Birodkar.

• `lda.LDA` and `qda.QDA` have been moved to `discriminant_analysis.LinearDiscriminantAnalysis` and `discriminant_analysis.QuadraticDiscriminantAnalysis`.

• The `store_covariance` and `tol` parameters have been moved from the fit method to the constructor in `discriminant_analysis.LinearDiscriminantAnalysis` and the `store_covariances` and `tol` parameters have been moved from the fit method to the constructor in `discriminant_analysis.QuadraticDiscriminantAnalysis`.

• Models inheriting from `_LearntSelectorMixin` will no longer support the transform methods. (i.e, RandomForests, GradientBoosting, LogisticRegression, DecisionTrees, SVMs and SGD related models). Wrap these models around the metatransformer `feature_selection.SelectFromModel` to remove features (according to `coefs_` or `feature_importances_`) which are below a certain threshold value instead.

• `cluster.KMeans` re-runs cluster-assignments in case of non-convergence, to ensure consistency of `predict(X)` and `labels_`. By Vighnesh Birodkar.

• Classifier and Regressor models are now tagged as such using the `_estimator_type` attribute.

• Cross-validation iterators always provide indices into training and test set, not boolean masks.

• The `decision_function` on all regressors was deprecated and will be removed in 0.19. Use `predict` instead.

• `datasets.load_lfw_pairs` is deprecated and will be removed in 0.19. Use `datasets.fetch_lfw_pairs` instead.

• The deprecated `hmm` module was removed.

• The deprecated `Bootstrap` cross-validation iterator was removed.

• The deprecated `Ward` and `WardAgglomerative` classes have been removed. Use `clustering.AgglomerativeClustering` instead.

• `cross_validation.check_cv` is now a public function.
The property `residues_` of `linear_model.LinearRegression` is deprecated and will be removed in 0.19.

The deprecated `n_jobs` parameter of `linear_model.LinearRegression` has been moved to the constructor.

Removed deprecated `class_weight` parameter from `linear_model.SGDClassifier`'s `fit` method. Use the constructor parameter instead.

The deprecated support for the sequence of sequences (or list of lists) multilabel format was removed. To convert to and from the supported binary indicator matrix format, use `MultiLabelBinarizer`.

The behavior of calling the `inverse_transform` method of `Pipeline.pipeline` will change in 0.19. It will no longer reshape one-dimensional input to two-dimensional input.

The deprecated attributes `indicator_matrix_`, `multilabel_` and `classes_` of `preprocessing.LabelBinarizer` were removed.

Using `gamma=0` in `svm.SVC` and `svm.SVR` to automatically set the gamma to 1. / `n_features` is deprecated and will be removed in 0.19. Use `gamma="auto"` instead.

### Code Contributors


### 1.7.6 Version 0.16.1

April 14, 2015
Changelog

Bug fixes

- Allow input data larger than `block_size` in `covariance.LedoitWolf` by Andreas Müller.
- Fix a bug in `isotonic.IsotonicRegression` deduplication that caused unstable result in `calibration.CalibratedClassifierCV` by Jan Hendrik Metzen.
- Fix sorting of labels in `preprocessing.label_binarize` by Michael Heilman.
- Fix several stability and convergence issues in `cross_decomposition.CCA` and `cross_decomposition.PLSCanonical` by Andreas Müller
- Fix a bug in `cluster.KMeans` when `precompute_distances=False` on fortran-ordered data.
- Fix a speed regression in `ensemble.RandomForestClassifier`'s `predict` and `predict_proba` by Andreas Müller.
- Fix a regression where `utils.shuffle` converted lists and dataframes to arrays, by Olivier Grisel

1.7.7 Version 0.16

March 26, 2015

Highlights

- Speed improvements (notably in `cluster.DBSCAN`), reduced memory requirements, bug-fixes and better default settings.
- Multinomial Logistic regression and a path algorithm in `linear_model.LogisticRegressionCV`.
- `cluster.Birch` clustering method for large-scale datasets.
- Scalable approximate nearest neighbors search with Locality-sensitive hashing forests in `neighbors.LSHForest`.
- Improved error messages and better validation when using malformed input data.
- More robust integration with pandas dataframes.

Changelog

New features

- The new `neighbors.LSHForest` implements locality-sensitive hashing for approximate nearest neighbors search. By Maheshakya Wijewardena.
- Added `svm.LinearSVR`. This class uses the liblinear implementation of Support Vector Regression which is much faster for large sample sizes than `svm.SVR` with linear kernel. By Fabian Pedregosa and Qiang Luo.
- Incremental fit for `GaussianNB`.
• Added the `metrics.label_ranking_average_precision_score` metrics. By Arnaud Joly.
• Add the `metrics.coverage_error` metrics. By Arnaud Joly.
• Added `linear_model.LogisticRegressionCV`. By Manoj Kumar, Fabian Pedregosa, Gael Varoquaux and Alexandre Gramfort.
• Added `warm_start` constructor parameter to make it possible for any trained forest model to grow additional trees incrementally. By Laurent Djer.
• Added `sample_weight` support to `ensemble.GradientBoostingClassifier` and `ensemble.GradientBoostingRegressor`. By Peter Prettenhofer.
• Added `decomposition.IncrementalPCA`, an implementation of the PCA algorithm that supports out-of-core learning with a `partial_fit` method. By Kyle Kastner.
• Averaged SGD for `SGDClassifier` and `SGDRegressor` By Danny Sullivan.
• Added `cross_val_predict` function which computes cross-validated estimates. By Luis Pedro Coelho
• Added `linear_model.TheilSenRegressor`, a robust generalized-median-based estimator. By Florian Wilhelm.
• Added `metrics.median_absolute_error`, a robust metric. By Gael Varoquaux and Florian Wilhelm.
• Add `cluster.Birch`, an online clustering algorithm. By Manoj Kumar, Alexandre Gramfort and Joel Nothman.
• Added shrinkage support to `discriminant_analysis.LinearDiscriminantAnalysis` using two new solvers. By Clemens Brunner and Martin Billinger.
• Added `kernel_ridge.KernelRidge`, an implementation of kernelized ridge regression. By Mathieu Blondel and Jan Hendrik Metzen.
• All solvers in `linear_model.Ridge` now support `sample_weight`. By Mathieu Blondel.
• Added `cross_validation.PredefinedSplit` cross-validation for fixed user-provided cross-validation folds. By Thomas Unterthiner.
• Added `calibration.CalibratedClassifierCV`, an approach for calibrating the predicted probabilities of a classifier. By Alexandre Gramfort, Jan Hendrik Metzen, Mathieu Blondel and Balazs Kegl.

**Enhancements**

• Add option `return_distance` in `hierarchical.ward_tree` to return distances between nodes for both structured and unstructured versions of the algorithm. By Matteo Visconti di Oleggio Castello. The same option was added in `hierarchical.linkage_tree`. By Manoj Kumar
• Add support for sample weights in scorer objects. Metrics with sample weight support will automatically benefit from it. By Noel Dawe and Vlad Niculae.
• Added `newton-cg` and `lbfgs` solver support in `linear_model.LogisticRegression`. By Manoj Kumar.
• Add `selection="random"` parameter to implement stochastic coordinate descent for `linear_model.Lasso`, `linear_model.ElasticNet` and related. By Manoj Kumar.
• Add `sample_weight` parameter to `metrics.jaccard_similarity_score` and `metrics.log_loss`. By Jatin Shah.
• Support sparse multilabel indicator representation in `preprocessing.LabelBinarizer` and `multiclass.OneVsRestClassifier` (by Hamzeh Alsalhi with thanks to Rohit Sivaprasad), as well as evaluation metrics (by Joel Nothman).
• Add `sample_weight` parameter to `metrics.jaccard_similarity_score`. By Jatin Shah.

• Add support for multiclass in `metrics.hinge_loss`. Added `labels=None` as optional parameter. By Saurabh Jha.

• Add `sample_weight` parameter to `metrics.hinge_loss`. By Saurabh Jha.

• Add `multi_class=\"multinomial\"` option in `linear_model.LogisticRegression` to implement a Logistic Regression solver that minimizes the cross-entropy or multinomial loss instead of the default One-vs-Rest setting. Supports `lbfgs` and `newton-cg` solvers. By Lars Buitinck and Manoj Kumar. Solver option `newton-cg` by Simon Wu.

• DictVectorizer can now perform `fit_transform` on an iterable in a single pass, when giving the option `sort=False`. By Dan Blanchard.

• GridSearchCV and RandomizedSearchCV can now be configured to work with estimators that may fail and raise errors on individual folds. This option is controlled by the `error_score` parameter. This does not affect errors raised on re-fit. By Michal Romaniuk.

• Add `digits` parameter to `metrics.classification_report` to allow report to show different precision of floating point numbers. By Ian Gilmore.

• Add a quantile prediction strategy to the `dummy.DummyRegressor`. By Aaron Staple.

• Add `handle_unknown` option to `preprocessing.OneHotEncoder` to handle unknown categorical features more gracefully during transform. By Manoj Kumar.

• Added support for sparse input data to decision trees and their ensembles. By Fares Hedyati and Arnaud Joly.

• Optimized `cluster.AffinityPropagation` by reducing the number of memory allocations of large temporary data-structures. By Antony Lee.

• Parellization of the computation of feature importances in random forest. By Olivier Grisel and Arnaud Joly.

• Add `n_iter_` attribute to estimators that accept a `max_iter` attribute in their constructor. By Manoj Kumar.

• Added decision function for `multiclass.OneVsOneClassifier` By Raghav RV and Kyle Beauchamp.

• `neighbors.kneighbors_graph` and `radius_neighbors_graph` support non-Euclidean metrics. By Manoj Kumar.

• Parameter `connectivity` in `cluster.AgglomerativeClustering` and family now accept callables that return a connectivity matrix. By Manoj Kumar.

• Sparse support for `paired_distances`. By Joel Nothman.

• `cluster.DBSCAN` now supports sparse input and sample weights and has been optimized: the inner loop has been rewritten in Cython and radius neighbors queries are now computed in batch. By Joel Nothman and Lars Buitinck.

• Add `class_weight` parameter to automatically weight samples by class frequency for `ensemble.RandomForestClassifier`, `tree.DecisionTreeClassifier`, `ensemble.ExtraTreesClassifier` and `tree.ExtraTreeClassifier`. By Trevor Stephens.

• `grid_search.RandomizedSearchCV` now does sampling without replacement if all parameters are given as lists. By Andreas Müller.

• Parallelized calculation of `pairwise_distances` is now supported for scipy metrics and custom callables. By Joel Nothman.

• Allow the fitting and scoring of all clustering algorithms in `pipeline.Pipeline`. By Andreas Müller.

• More robust seeding and improved error messages in `cluster.MeanShift` by Andreas Müller.
• Make the stopping criterion for mixture.GMM, mixture.DPGMM and mixture.VBGMM less dependent on the number of samples by thresholding the average log-likelihood change instead of its sum over all samples. By Hervé Bredin.

• The outcome of manifold.spectral_embedding was made deterministic by flipping the sign of eigenvectors. By Hasil Sharma.


• Numerical stability improvements for preprocessing.StandardScaler and preprocessing.scale. By Nicolas Goix

• svm.SVC fitted on sparse input now implements decision_function. By Rob Zinkov and Andreas Müller.

• cross_validation.train_test_split now preserves the input type, instead of converting to numpy arrays.

Documentation improvements

• Added example of using FeatureUnion for heterogeneous input. By Matt Terry

• Documentation on scorers was improved, to highlight the handling of loss functions. By Matt Pico.

• A discrepancy between liblinear output and scikit-learn’s wrappers is now noted. By Manoj Kumar.

• Improved documentation generation: examples referring to a class or function are now shown in a gallery on the class/function’s API reference page. By Joel Nothman.

• More explicit documentation of sample generators and of data transformation. By Joel Nothman.

• sklearn.neighbors.BallTree and sklearn.neighbors.KDTree used to point to empty pages stating that they are aliases of BinaryTree. This has been fixed to show the correct class docs. By Manoj Kumar.

• Added silhouette plots for analysis of KMeans clustering using metrics.silhouette_samples and metrics.silhouette_score. See Selecting the number of clusters with silhouette analysis on KMeans clustering

Bug fixes

• Metaestimators now support ducktyping for the presence of decision_function, predict_proba and other methods. This fixes behavior of grid_search.GridSearchCV, grid_search.RandomizedSearchCV, pipeline.Pipeline, feature_selection.RFE, feature_selection.RFECV when nested. By Joel Nothman

• The scoring attribute of grid-search and cross-validation methods is no longer ignored when a grid_search.GridSearchCV is given as a base estimator or the base estimator doesn’t have predict.

• The function hierarchical.ward_tree now returns the children in the same order for both the structured and unstructured versions. By Matteo Visconti di Oleggio Castello.

• feature_selection.RFECV now correctly handles cases when step is not equal to 1. By Nikolay Mayorov

• The decomposition.PCA now undoes whitening in its inverse_transform. Also, its components_ now always have unit length. By Michael Eickenberg.

• Fix incomplete download of the dataset when datasets.download_20newsgroups is called. By Manoj Kumar.
• Various fixes to the Gaussian processes subpackage by Vincent Dubourg and Jan Hendrik Metzen.

• Calling `partial_fit` with `class_weight='auto'` throws an appropriate error message and suggests a work around. By Danny Sullivan.

• `RBFSampler` with `gamma=g` formerly approximated `rbf_kernel` with `gamma=g/2`: the definition of `gamma` is now consistent, which may substantially change your results if you use a fixed value. (If you cross-validated over `gamma`, it probably doesn’t matter too much.) By Dougal Sutherland.

• Pipeline object delegate the `classes_` attribute to the underlying estimator. It allows, for instance, to make bagging of a pipeline object. By Arnaud Joly

• `neighbors.NearestCentroid` now uses the median as the centroid when metric is set to `manhattan`. It was using the mean before. By Manoj Kumar

• Fix numerical stability issues in `linear_model.SGDClassifier` and `linear_model.SGDRegressor` by clipping large gradients and ensuring that weight decay rescaling is always positive (for large l2 regularization and large learning rate values). By Olivier Grisel

• When `compute_full_tree` is set to “auto”, the full tree is built when `n_clusters` is high and is early stopped when `n_clusters` is low, while the behavior should be vice-versa in `cluster.AgglomerativeClustering` (and friends). This has been fixed by Manoj Kumar

• Fix lazy centering of data in `linear_model.enet_path` and `linear_model.lasso_path`. It was centered around one. It has been changed to be centered around the origin. By Manoj Kumar

• Fix handling of precomputed affinity matrices in `cluster.AgglomerativeClustering` when using connectivity constraints. By Cathy Deng


• Fixed a crash in `metrics.precision_recall_fscore_support` when using unsorted labels in the multi-label setting. By Andreas Müller.

• Avoid skipping the first nearest neighbor in the methods `radius_neighbors`, `kneighbors`, `kneighbors_graph` and `radius_neighbors_graph` in `sklearn.neighbors.NearestNeighbors` and family, when the query data is not the same as fit data. By Manoj Kumar.

• Fix log-density calculation in the `mixture.GMM` with tied covariance. By Will Dawson

• Fixed a scaling error in `feature_selection.SelectFdr` where a factor `n_features` was missing. By Andrew Tulloch

• Fix zero division in `neighbors.KNeighborsRegressor` and related classes when using distance weighting and having identical data points. By Garret-R.

• Fixed round off errors with non positive-definite covariance matrices in GMM. By Alexis Mignon.

• Fixed an error in the computation of conditional probabilities in `naive_bayes.BernoulliNB`. By Hanna Wallach.

• Make the method `radius_neighbors` of `neighbors.NearestNeighbors` return the samples lying on the boundary for `algorithm='brute'`. By Yan Yi.

• Flip sign of `dual_coef_` of `svm.SVC` to make it consistent with the documentation and `decision_function`. By Artem Sobolev.

• Fixed handling of ties in `isotonic.IsotonicRegression`. We now use the weighted average of targets (secondary method). By Andreas Müller and Michael Bommarito.
API changes summary

- GridSearchCV and cross_val_score and other meta-estimators don’t convert pandas DataFrames into arrays any more, allowing DataFrame specific operations in custom estimators.

- multiclass.fit_ovr, multiclass.predict_ovr, predict_proba_ovr, multiclass.fit_ovo, multiclass.predict_ovo, multiclass.fit_ecoc and multiclass.predict_ecoc are deprecated. Use the underlying estimators instead.

- Nearest neighbors estimators used to take arbitrary keyword arguments and pass these to their distance metric. This will no longer be supported in scikit-learn 0.18; use the metric_params argument instead.

- n_jobs parameter of the fit method shifted to the constructor of the LinearRegression class.

- The predict_proba method of multiclass.OneVsRestClassifier now returns two probabilities per sample in the multiclass case; this is consistent with other estimators and with the method’s documentation, but previous versions accidentally returned only the positive probability. Fixed by Will Lamond and Lars Buitinck.

- Change default value of precompute in ElasticNet and Lasso to False. Setting precompute to “auto” was found to be slower when n_samples > n_features since the computation of the Gram matrix is computationally expensive and outweighs the benefit of fitting the Gram for just one alpha. precompute="auto" is now deprecated and will be removed in 0.18 By Manoj Kumar.

- Expose positive option in linear_model.enet_path and linear_model.enet_path which constrains coefficients to be positive. By Manoj Kumar.

- Users should now supply an explicit average parameter to sklearn.metrics.f1_score, sklearn.metrics.fbeta_score, sklearn.metrics.recall_score and sklearn.metrics.precision_score when performing multiclass or multilabel (i.e. not binary) classification. By Joel Nothman.

- scoring parameter for cross validation now accepts ‘f1_micro’, ‘f1_macro’ or ‘f1_weighted’. ‘f1’ is now for binary classification only. Similar changes apply to ‘precision’ and ‘recall’. By Joel Nothman.

- The fit_intercept, normalize and return_models parameters in linear_model.enet_path and linear_model.lasso_path have been removed. They were deprecated since 0.14

- From now onwards, all estimators will uniformly raise NotFittedError (utils.validation.NotFittedError), when any of the predict like methods are called before the model is fit. By Raghav RV.

- Input data validation was refactored for more consistent input validation. The check_arrays function was replaced by check_array and check_X_y. By Andreas Müller.

- Allow X=None in the methods radius_neighbors, kneighbors, kneighbors_graph and radius_neighbors_graph in sklearn.neighbors.NearestNeighbors and family. If set to None, then for every sample this avoids setting the sample itself as the first nearest neighbor. By Manoj Kumar.

- Add parameter include_self in neighbors.kneighbors_graph and neighbors.radius_neighbors_graph which has to be explicitly set by the user. If set to True, then the sample itself is considered as the first nearest neighbor.

- thresh parameter is deprecated in favor of new tol parameter in GMM, DPGMM and VBGMM. See Enhancements section for details. By Hervé Bredin.

- Estimators will treat input with dtype object as numeric when possible. By Andreas Müller

- Estimators now raise ValueError consistently when fitted on empty data (less than 1 sample or less than 1 feature for 2D input). By Olivier Grisel.

• `cluster.DBSCAN` now uses a deterministic initialization. The `random_state` parameter is deprecated. By Erich Schubert.

Code Contributors


1.7.8 Version 0.15.2

September 4, 2014

Bug fixes

• Fixed handling of the p parameter of the Minkowski distance that was previously ignored in nearest neighbors models. By Nikolay Mayorov.

• Fixed duplicated alphas in `linear_model.LassoLars` with early stopping on 32 bit Python. By Olivier Grisel and Fabian Pedregosa.

• Fixed the build under Windows when scikit-learn is built with MSVC while NumPy is built with MinGW. By Olivier Grisel and Federico Vaggi.

• Fixed an array index overflow bug in the coordinate descent solver. By Gael Varoquaux.

• Better handling of numpy 1.9 deprecation warnings. By Gael Varoquaux.

• Removed unnecessary data copy in `cluster.KMeans`. By Gael Varoquaux.

• Explicitly close open files to avoid `ResourceWarning` under Python 3. By Calvin Giles.
• The transform of `discriminant_analysis.LinearDiscriminantAnalysis` now projects the input on the most discriminant directions. By Martin Billinger.

• Fixed potential overflow in `_tree.safe_realloc` by Lars Buitinck.

• Performance optimization in `isotonic.IsotonicRegression` by Robert Bradshaw.

• `nose` is non-longer a runtime dependency to import `sklearn`, only for running the tests. By Joel Nothman.

• Many documentation and website fixes by Joel Nothman, Lars Buitinck Matt Pico, and others.

1.7.9 Version 0.15.1

August 1, 2014

Bug fixes

• Made `cross_validation.cross_val_score` use `cross_validation.KFold` instead of `cross_validation.StratifiedKFold` on multi-output classification problems. By Nikolay Mayorov.

• Support unseen labels `preprocessing.LabelBinarizer` to restore the default behavior of 0.14.1 for backward compatibility. By Hamzeh Alsalhi.

• Fixed the `cluster.KMeans` stopping criterion that prevented early convergence detection. By Edward Raff and Gael Varoquaux.

• Fixed the behavior of `multiclass.OneVsOneClassifier` in case of ties at the per-class vote level by computing the correct per-class sum of prediction scores. By Andreas Müller.

• Made `cross_validation.cross_val_score` and `grid_search.GridSearchCV` accept Python lists as input data. This is especially useful for cross-validation and model selection of text processing pipelines. By Andreas Müller.

• Fixed data input checks of most estimators to accept input data that implements the NumPy `__array__` protocol. This is the case for `pandas.Series` and `pandas.DataFrame` in recent versions of pandas. By Gael Varoquaux.

• Fixed a regression for `linear_model.SGDClassifier` with `class_weight="auto"` on data with non-contiguous labels. By Olivier Grisel.

1.7.10 Version 0.15

July 15, 2014

Highlights

• Many speed and memory improvements all across the code

• Huge speed and memory improvements to random forests (and extra trees) that also benefit better from parallel computing.

• Incremental fit to `BernoulliRBM`

• Added `cluster.AgglomerativeClustering` for hierarchical agglomerative clustering with average linkage, complete linkage and ward strategies.

• Added `linear_model.RANSACRegressor` for robust regression models.
• Added dimensionality reduction with manifold.TSNE which can be used to visualize high-dimensional data.

Changelog

New features

• Added ensemble.BaggingClassifier and ensemble.BaggingRegressor meta-estimators for ensembling any kind of base estimator. See the Bagging section of the user guide for details and examples. By Gilles Louppe.


• Added linear_model.RANSACRegressor meta-estimator for the robust fitting of regression models. By Johannes Schönberger.

• Added cluster.AgglomerativeClustering for hierarchical agglomerative clustering with average linkage, complete linkage and ward strategies, by Nelle Varoquaux and Gael Varoquaux.

• Shorthand constructors pipeline.make_pipeline and pipeline.make_union were added by Lars Buitinck.

• Shuffle option for cross_validation.StratifiedKFold. By Jeffrey Blackburne.

• Incremental learning (partial_fit) for Gaussian Naive Bayes by Imran Haque.

• Added partial_fit to BernoulliRBM By Danny Sullivan.

• Added learning_curve utility to chart performance with respect to training size. See Plotting Learning Curves. By Alexander Fabisch.

• Add positive option in LassoCV and ElasticNetCV. By Brian Wignall and Alexandre Gramfort.

• Added linear_model.MultiTaskElasticNetCV and linear_model.MultiTaskLassoCV. By Manoj Kumar.

• Added manifold.TSNE. By Alexander Fabisch.

Enhancements


• Memory improvements of decision trees, by Arnaud Joly.

• Decision trees can now be built in best-first manner by using max_leaf_nodes as the stopping criteria. Refactored the tree code to use either a stack or a priority queue for tree building. By Peter Prettenhofer and Gilles Louppe.

• Decision trees can now be fitted on fortran- and c-style arrays, and non-continuous arrays without the need to make a copy. If the input array has a different dtype than np.float32, a fortran- style copy will be made since fortran-style memory layout has speed advantages. By Peter Prettenhofer and Gilles Louppe.

• Speed improvement of regression trees by optimizing the the computation of the mean square error criterion. This lead to speed improvement of the tree, forest and gradient boosting tree modules. By Arnaud Joly

• The img_to_graph and grid_tograph functions in sklearn.feature_extraction.image now return np.ndarray instead of np.matrix when return_as=np.ndarray. See the Notes section for more information on compatibility.
• Changed the internal storage of decision trees to use a struct array. This fixed some small bugs, while improving code and providing a small speed gain. By Joel Nothman.

• Reduce memory usage and overhead when fitting and predicting with forests of randomized trees in parallel with n_jobs != 1 by leveraging new threading backend of joblib 0.8 and releasing the GIL in the tree fitting Cython code. By Olivier Grisel and Gilles Louppe.

• Speed improvement of the sklearn.ensemble.gradient_boosting module. By Gilles Louppe and Peter Prettenhofer.

• Various enhancements to the sklearn.ensemble.gradient_boosting module: a warm_start argument to fit additional trees, a max_leaf_nodes argument to fit GBM style trees, a monitor fit argument to inspect the estimator during training, and refactoring of the verbose code. By Peter Prettenhofer.

• Faster sklearn.ensemble.ExtraTrees by caching feature values. By Arnaud Joly.

• Faster depth-based tree building algorithm such as decision tree, random forest, extra trees or gradient tree boosting (with depth based growing strategy) by avoiding trying to split on found constant features in the sample subset. By Arnaud Joly.

• Add min_weight_fraction_leaf pre-pruning parameter to tree-based methods: the minimum weighted fraction of the input samples required to be at a leaf node. By Noel Dawe.

• Added metrics.pairwise_distances_argmin_min, by Philippe Gervais.

• Added predict method to cluster.AffinityPropagation and cluster.MeanShift, by Mathieu Blondel.

• Vector and matrix multiplications have been optimised throughout the library by Denis Engemann, and Alexandre Gramfort. In particular, they should take less memory with older NumPy versions (prior to 1.7.2).

• Precision-recall and ROC examples now use train_test_split, and have more explanation of why these metrics are useful. By Kyle Kastner

• The training algorithm for decomposition.NMF is faster for sparse matrices and has much lower memory complexity, meaning it will scale up gracefully to large datasets. By Lars Buitinck.

• Added svd_method option with default value to “randomized” to decomposition.FactorAnalysis to save memory and significantly speedup computation by Denis Engemann, and Alexandre Gramfort.

• Changed cross_validation.StratifiedKFold to try and preserve as much of the original ordering of samples as possible so as not to hide overfitting on datasets with a non-negligible level of samples dependency. By Daniel Nouri and Olivier Grisel.

• Add multi-output support to gaussian_process.GaussianProcess by John Novak.

• Support for precomputed distance matrices in nearest neighbor estimators by Robert Layton and Joel Nothman.

• Norm computations optimized for NumPy 1.6 and later versions by Lars Buitinck. In particular, the k-means algorithm no longer needs a temporary data structure the size of its input.

• dummy.DummyClassifier can now be used to predict a constant output value. By Manoj Kumar.

• dummy.DummyRegressor has now a strategy parameter which allows to predict the mean, the median of the training set or a constant output value. By Maheshakya Wijewardena.

• Multi-label classification output in multilabel indicator format is now supported by metrics.roc_auc_score and metrics.average_precision_score by Arnaud Joly.

• Significant performance improvements (more than 100x speedup for large problems) in isotonic.IsotonicRegression by Andrew Tulloch.

• Speed and memory usage improvements to the SGD algorithm for linear models: it now uses threads, not separate processes, when n_jobs>1. By Lars Buitinck.
• Grid search and cross validation allow NaNs in the input arrays so that preprocessors such as `preprocessing.Imputer` can be trained within the cross validation loop, avoiding potentially skewed results.

• Ridge regression can now deal with sample weights in feature space (only sample space until then). By Michael Eickenberg. Both solutions are provided by the Cholesky solver.

• Several classification and regression metrics now support weighted samples with the new `sample_weight` argument: `metrics.accuracy_score`, `metrics.zero_one_loss`, `metrics.precision_score`, `metrics.average_precision_score`, `metrics.f1_score`, `metrics.fbeta_score`, `metrics.recall_score`, `metrics.roc_auc_score`, `metrics.explained_variance_score`, `metrics.mean_squared_error`, `metrics.mean_absolute_error`, `metrics.r2_score`. By Noel Dawe.

• Speed up of the sample generator `datasets.make_multilabel_classification`. By Joel Nothman.

Documentation improvements

• The Working With Text Data tutorial has now been worked in to the main documentation’s tutorial section. Includes exercises and skeletons for tutorial presentation. Original tutorial created by several authors including Olivier Grisel, Lars Buitinck and many others. Tutorial integration into the scikit-learn documentation by Jaques Grobler

• Added Computational Performance documentation. Discussion and examples of prediction latency / throughput and different factors that have influence over speed. Additional tips for building faster models and choosing a relevant compromise between speed and predictive power. By Eustache Diemert.

Bug fixes

• Fixed bug in `decomposition.MiniBatchDictionaryLearning` : `partial_fit` was not working properly.

• Fixed bug in `linear_model.stochastic_gradient` : `l1_ratio` was used as `(1.0 -l1_ratio)`.

• Fixed bug in `multiclass.OneVsOneClassifier` with string labels.

• Fixed a bug in `LassoCV` and `ElasticNetCV`: they would not pre-compute the Gram matrix with `precompute=True or precompute="auto" and n_samples > n_features`. By Manoj Kumar.

• Fixed incorrect estimation of the degrees of freedom in `feature_selection.f_regression` when variables are not centered. By Virgile Fritsch.

• Fixed a race condition in parallel processing with `pre_dispatch !="all"` (for instance, in `cross_val_score`). By Olivier Grisel.

• Raise error in `cluster.FeatureAgglomeration` and `cluster.WardAgglomeration` when no samples are given, rather than returning meaningless clustering.

• Fixed bug in `gradient_boosting.GradientBoostingRegressor` with `loss='huber'`: gamma might have not been initialized.

• Fixed feature importances as computed with a forest of randomized trees when fit with `sample_weight != None` and/or with `bootstrap=True`. By Gilles Louppe.
API changes summary

- `sklearn.hmm` is deprecated. Its removal is planned for the 0.17 release.
- Use of `covariance.EllipticEnvelop` has now been removed after deprecation. Please use `covariance.EllipticEnvelope` instead.
- `cluster.Ward` is deprecated. Use `cluster.AgglomerativeClustering` instead.
- `cluster.WardClustering` is deprecated. Use `cluster.AgglomerativeClustering` instead.
- `cross_validation.Bootstrap` is deprecated. `cross_validation.KFold` or `cross_validation.ShuffleSplit` are recommended instead.
- Direct support for the sequence of sequences (or list of lists) multilabel format is deprecated. To convert to and from the supported binary indicator matrix format, use `MultiLabelBinarizer`. By Joel Nothman.
- Add score method to `PCA` following the model of probabilistic PCA and deprecate `ProbabilisticPCA` model whose score implementation is not correct. The computation now also exploits the matrix inversion lemma for faster computation. By Alexandre Gramfort.
- The score method of `FactorAnalysis` now returns the average log-likelihood of the samples. Use `score_samples` to get log-likelihood of each sample. By Alexandre Gramfort.
- Generating boolean masks (the setting `indices=False`) from cross-validation generators is deprecated. Support for masks will be removed in 0.17. The generators have produced arrays of indices by default since 0.10. By Joel Nothman.
- 1-d arrays containing strings with `dtype=object` (as used in Pandas) are now considered valid classification targets. This fixes a regression from version 0.13 in some classifiers. By Joel Nothman.
- Fix wrong `explained_variance_ratio_` attribute in `RandomizedPCA`. By Alexandre Gramfort.
- Fit alphas for each `l1_ratio` instead of mean `l1_ratio` in `linear_model.ElasticNetCV` and `linear_model.LassoCV`. This changes the shape of `alphas_` from `(n_alphas,)` to `(n_l1_ratio, n_alphas)` if the `l1_ratio` provided is a 1-D array like object of length greater than one. By Manoj Kumar.
- Fix `linear_model.ElasticNetCV` and `linear_model.LassoCV` when fitting intercept and input data is sparse. The automatic grid of alphas was not computed correctly and the scaling with normalize was wrong. By Manoj Kumar.
- Fix wrong maximal number of features drawn (`max_features`) at each split for decision trees, random forests and gradient tree boosting. Previously, the count for the number of drawn features started only after one non constant features in the split. This bug fix will affect computational and generalization performance of those algorithms in the presence of constant features. To get back previous generalization performance, you should modify the value of `max_features`. By Arnaud Joly.
- Fix wrong maximal number of features drawn (`max_features`) at each split for `ensemble.ExtraTreesClassifier` and `ensemble.ExtraTreesRegressor`. Previously, only non constant features in the split was counted as drawn. Now constant features are counted as drawn. Furthermore at least one feature must be non constant in order to make a valid split. This bug fix will affect computational and generalization performance of extra trees in the presence of constant features. To get back previous generalization performance, you should modify the value of `max_features`. By Arnaud Joly.
- Fix `utils.compute_class_weight` when `class_weight="auto"`. Previously it was broken for input of non-integer `dtype` and the weighted array that was returned was wrong. By Manoj Kumar.
People

List of contributors for release 0.15 by number of commits.

- 312 Olivier Grisel
- 275 Lars Buitinck
- 221 Gael Varoquaux
- 148 Arnaud Joly
- 134 Johannes Schönberger
- 119 Gilles Louppe
- 113 Joel Nothman
- 111 Alexandre Gramfort
- 95 Jaques Grobler
- 89 Denis Engemann
- 83 Peter Prettenhofer
- 83 Alexander Fabisch
- 62 Mathieu Blondel
- 60 Eustache Diemert
- 60 Nelle Varoquaux
- 49 Michael Bommarito
- 45 Manoj-Kumar-S
- 28 Kyle Kastner
- 26 Andreas Mueller
- 22 Noel Dawe
- 21 Maheshakya Wijewardena
- 21 Brooke Osborn
- 21 Hamzeh Alsahlhi
- 21 Jake VanderPlas
- 21 Philippe Gervais
- 19 Bala Subrahmaniam Varanasi
- 12 Ronald Phlypo
- 10 Mikhail Korobov
- 8 Thomas Unterthiner
- 8 Jeffrey Blackburne
- 8 eltermann
- 8 bwignall
- 7 Ankit Agrawal
- 7 CJ Carey
• 6 Daniel Nouri
• 6 Chen Liu
• 6 Michael Eickenberg
• 6 ugurthemaster
• 5 Aaron Schumacher
• 5 Baptiste Lagarde
• 5 Rajat Khanduja
• 5 Robert McGibbon
• 5 Sergio Pascual
• 4 Alexis Metaireau
• 4 Ignacio Rossi
• 4 Virgile Fritsch
• 4 Sebastian Säger
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• 3 Andrew Tulloch
• 3 murad
• 3 Steven Maude
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• 3 cjlin
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• 2 LK
• 2 Vlad Niculae
• 2 Laurent Dierer
• 2 Erik Shilts
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• 2 jess010
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• 2 Andrew Clegg
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• 2 Simon Frid
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• 1 Ralf Gommers
• 1 t-aft
• 1 Ronan Amicel
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• 1 Ryan Wang
• 1 Samuel Charron
• 1 Samuel St-Jean
• 1 Fabian Pedregosa
• 1 Skipper Seabold
• 1 Stefan Walk
• 1 Stefan van der Walt
• 1 Stephan Hoyer
• 1 Allen Riddell
• 1 Valentin Haenel
• 1 Vijay Ramesh
• 1 Will Myers
• 1 Yaroslav Halchenko
• 1 Yoni Ben-Meshulam
• 1 Yury V. Zaytsev
Changelog

- Missing values with sparse and dense matrices can be imputed with the transformer `preprocessing.Imputer` by Nicolas Trésegnie.
- The core implementation of decisions trees has been rewritten from scratch, allowing for faster tree induction and lower memory consumption in all tree-based estimators. By Gilles Louppe.
- Added `ensemble.AdaBoostClassifier` and `ensemble.AdaBoostRegressor`, by Noel Dawe and Gilles Louppe. See the `AdaBoost` section of the user guide for details and examples.
- Added biclustering algorithms (`sklearn.cluster.bicluster.SpectralCoclustering` and `sklearn.cluster.bicluster.SpectralBiclustering`), data generation methods (`sklearn.datasets.make_biclusters` and `sklearn.datasets.make_checkerboard`), and scoring metrics (`sklearn.metrics.consensus_score`). By Kemal Eren.
- Added Restricted Boltzmann Machines (`neural_network.BernoulliRBM`), by Yann Dauphin.
• Python 3 support by Justin Vincent, Lars Buitinck, Subhodeep Moitra and Olivier Grisel. All tests now pass under Python 3.3.

• Ability to pass one penalty (alpha value) per target in linear_model.Ridge, by @eickenberg and Mathieu Blondel.

• Fixed sklearn.linear_model.stochastic_gradient.py L2 regularization issue (minor practical significance). By Norbert Crombach and Mathieu Blondel.

• Added an interactive version of Andreas Müller’s Machine Learning Cheat Sheet (for scikit-learn) to the documentation. See Choosing the right estimator. By Jaques Grobler.

• grid_search.GridSearchCV and cross_validation.cross_val_score now support the use of advanced scoring function such as area under the ROC curve and f-beta scores. See The scoring parameter: defining model evaluation rules for details. By Andreas Müller and Lars Buitinck. Passing a function from sklearn.metrics as score_func is deprecated.

• Multi-label classification output is now supported by metrics.accuracy_score, metrics.zero_one_loss, metrics.f1_score, metrics.fbeta_score, metrics.classification_report, metrics.precision_score and metrics.recall_score by Arnaud Joly.

• Two new metrics metrics.hamming_loss and metrics.jaccard_similarity_score are added with multi-label support by Arnaud Joly.

• Speed and memory usage improvements in feature_extraction.text.CountVectorizer and feature_extraction.text.TfidfVectorizer, by Jochen Wersdörfer and Roman Sinayev.

• The min_df parameter in feature_extraction.text.CountVectorizer and feature_extraction.text.TfidfVectorizer, which used to be 2, has been reset to 1 to avoid unpleasant surprises (empty vocabularies) for novice users who try it out on tiny document collections. A value of at least 2 is still recommended for practical use.

• svm.LinearSVC, linear_model.SGDClassifier and linear_model.SGDRegressor now have a sparsify method that converts their coef_ into a sparse matrix, meaning stored models trained using these estimators can be made much more compact.

• linear_model.SGDClassifier now produces multiclass probability estimates when trained under log loss or modified Huber loss.

• Hyperlinks to documentation in example code on the website by Martin Luessi.

• Fixed bug in preprocessing.MinMaxScaler causing incorrect scaling of the features for non-default feature_range settings. By Andreas Müller.

• max_features in tree.DecisionTreeClassifier, tree.DecisionTreeRegressor and all derived ensemble estimators now supports percentage values. By Gilles Louppe.

• Performance improvements in isotonic.IsotonicRegression by Nelle Varoquaux.

• metrics.accuracy_score has an option normalize to return the fraction or the number of correctly classified sample by Arnaud Joly.

• Added metrics.log_loss that computes log loss, aka cross-entropy loss. By Jochen Wersdörfer and Lars Buitinck.

• A bug that caused ensemble.AdaBoostClassifier’s to output incorrect probabilities has been fixed.

• Feature selectors now share a mixin providing consistent transform, inverse_transform and get_support methods. By Joel Nothman.

• A fitted grid_search.GridSearchCV or grid_search.RandomizedSearchCV can now generally be pickled. By Joel Nothman.
• Refactored and vectorized implementation of `metrics.roc_curve` and `metrics.precision_recall_curve`. By Joel Nothman.

• The new estimator `sklearn.decomposition.TruncatedSVD` performs dimensionality reduction using SVD on sparse matrices, and can be used for latent semantic analysis (LSA). By Lars Buitinck.


• The default number of components for `sklearn.decomposition.RandomizedPCA` is now correctly documented to be `n_features`. This was the default behavior, so programs using it will continue to work as they did.

• `sklearn.cluster.KMeans` now fits several orders of magnitude faster on sparse data (the speedup depends on the sparsity). By Lars Buitinck.

• Reduce memory footprint of FastICA by Denis Engemann and Alexandre Gramfort.

• Verbose output in `sklearn.ensemble.gradient_boosting` now uses a column format and prints progress in decreasing frequency. It also shows the remaining time. By Peter Prettenhofer.

• `sklearn.ensemble.gradient_boosting` provides out-of-bag improvement `oob_improvement_` rather than the OOB score for model selection. An example that shows how to use OOB estimates to select the number of trees was added. By Peter Prettenhofer.

• Most metrics now support string labels for multiclass classification by Arnaud Joly and Lars Buitinck.

• New `sklearn.cluster.KMeans` class by Alexandre Gramfort and Vlad Niculae.

• Fixed a bug in `sklearn.covariance.GraphLassoCV`: the ‘alphas’ parameter now works as expected when given a list of values. By Philippe Gervais.

• Fixed an important bug in `sklearn.covariance.GraphLassoCV` that prevented all folds provided by a CV object to be used (only the first 3 were used). When providing a CV object, execution time may thus increase significantly compared to the previous version (bug results are correct now). By Philippe Gervais.

• `cross_validation.cross_val_score` and the `grid_search` module is now tested with multi-output data by Arnaud Joly.

• `datasets.make_multilabel_classification` can now return the output in label indicator multi-label format by Arnaud Joly.


• Random state in LibSVM-based estimators (`svm.SVC`, `NuSVC`, `OneClassSVM`, `svm.SVR`, `svm.NuSVR`) can now be controlled. This is useful to ensure consistency in the probability estimates for the classifiers trained with `probability=True`. By Vlad Niculae.

• Out-of-core learning support for discrete naïve Bayes classifiers `sklearn.naive_bayes.MultinomialNB` and `sklearn.naive_bayes.BernoulliNB` by adding the `partial_fit` method by Olivier Grisel.

• New website design and navigation by Gilles Louppe, Nelle Varoquaux, Vincent Michel and Andreas Müller.

• Improved documentation on multi-class, multi-label and multi-output classification by Yannick Schwartz and Arnaud Joly.

• Better input and error handling in the `metrics` module by Arnaud Joly and Joel Nothman.

• Speed optimization of the `hmm` module by Mikhail Korobov.

• Significant speed improvements for `sklearn.cluster.DBSCAN` by cleverless.
API changes summary

- The `auc_score` was renamed `roc_auc_score`.
- Testing scikit-learn with `sklearn.test()` is deprecated. Use `nose tests sklearn` from the command line.
- Feature importances in `tree.DecisionTreeClassifier`, `tree.DecisionTreeRegressor` and all derived ensemble estimators are now computed on the fly when accessing the `feature_importances_` attribute. Setting `compute_importances=True` is no longer required. By Gilles Louppe.
- `linear_model.lasso_path` and `linear_model.enet_path` can return its results in the same format as that of `linear_model.lars_path`. This is done by setting the `return_models` parameter to False. By Jaques Grobler and Alexandre Gramfort.
- `grid_search.IterGrid` was renamed to `grid_search.ParameterGrid`.
- Fixed bug in `KFold` causing imperfect class balance in some cases. By Alexandre Gramfort and Tadej Janež.
- `sklearn.neighbors.BallTree` has been refactored, and a `sklearn.neighbors.KDTree` has been added which shares the same interface. The Ball Tree now works with a wide variety of distance metrics. Both classes have many new methods, including single-tree and dual-tree queries, breadth-first and depth-first searching, and more advanced queries such as kernel density estimation and 2-point correlation functions. By Jake Vanderplas.
- Support for scipy.spatial.cKDTree within neighbors queries has been removed, and the functionality replaced with the new KDTree class.
- `sklearn.neighbors.KernelDensity` has been added, which performs efficient kernel density estimation with a variety of kernels.
- `sklearn.decomposition.KernelPCA` now always returns output with `n_components` components, unless the new parameter `remove_zero_eig` is set to True. This new behavior is consistent with the way kernel PCA was always documented; previously, the removal of components with zero eigenvalues was tacitly performed on all data.
- `gcv_mode="auto"` no longer tries to perform SVD on a densified sparse matrix in `sklearn.linear_model.RidgeCV`.
- Sparse matrix support in `sklearn.decomposition.RandomizedPCA` is now deprecated in favor of the new TruncatedSVD.
- `cross_validation.KFold` and `cross_validation.StratifiedKFold` now enforce `n_folds >= 2` otherwise a `ValueError` is raised. By Olivier Grisel.
- `datasets.load_files`'s charset and charset_errors parameters were renamed encoding and decode_errors.
- Attribute `oob_score_` in `sklearn.ensemble.GradientBoostingRegressor` and `sklearn.ensemble.GradientBoostingClassifier` is deprecated and has been replaced by `oob_improvement_`.
- Attributes in OrthogonalMatchingPursuit have been deprecated (copy_X, Gram, ...) and precompute_gram renamed precompute for consistency. See #2224.
- `sklearn.preprocessing.StandardScaler` now converts integer input to float, and raises a warning. Previously it rounded for dense integer input.
- `sklearn.multiclass.OneVsRestClassifier` now has a decision_function method. This will return the distance of each sample from the decision boundary for each class, as long as the underlying estimators implement the decision_function method. By Kyle Kastner.
- Better input validation, warning on unexpected shapes for y.
People

List of contributors for release 0.14 by number of commits.

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- 245 Lars Buitinck
- 187 Andreas Mueller
- 124 Arnaud Joly
- 112 Jaques Grobler
- 109 Gael Varoquaux
- 107 Olivier Grisel
- 102 Noel Dawe
- 99 Kemal Eren
- 79 Joel Nothman
- 75 Jake VanderPlas
- 73 Nelle Varoquaux
- 71 Vlad Niculae
- 65 Peter Prettenhofer
- 64 Alexandre Gramfort
- 54 Mathieu Blondel
- 38 Nicolas Trésegnie
- 35 eustache
- 27 Denis Engemann
- 25 Yann N. Dauphin
- 19 Justin Vincent
- 17 Robert Layton
- 15 Doug Coleman
- 14 Michael Eickenberg
- 13 Robert Marchman
- 11 Fabian Pedregosa
- 11 Philippe Gervais
- 10 Jim Holmström
- 10 Tadej Janež
- 10 syhw
- 9 Mikhail Korobov
- 9 Steven De Gryze
- 8 sergeyf
- 7 Ben Root

1.7. Release history
1.7.12 Version 0.13.1

February 23, 2013

The 0.13.1 release only fixes some bugs and does not add any new functionality.

Changelog

- Fixed a testing error caused by the function `cross_validation.train_test_split` being interpreted as a test by Yaroslav Halchenko.
- Fixed a bug in the reassignment of small clusters in the `cluster.MiniBatchKMeans` by Gael Varoquaux.
- Fixed default value of `gamma` in `decomposition.KernelPCA` by Lars Buitinck.
- Updated joblib to 0.7.0d by Gael Varoquaux.
- Fixed scaling of the deviance in `ensemble.GradientBoostingClassifier` by Peter Prettenhofer.
- Better tie-breaking in `multiclass.OneVsOneClassifier` by Andreas Müller.
- Other small improvements to tests and documentation.
People

List of contributors for release 0.13.1 by number of commits.

- 16 Lars Buitinck
- 12 Andreas Müller
- 8 Gael Varoquaux
- 5 Robert Marchman
- 3 Peter Prettenhofer
- 2 Hrishikesh Huilgolkar
- 1 Bastiaan van den Berg
- 1 Diego Molla
- 1 Gilles Louppe
- 1 Mathieu Blondel
- 1 Nelle Varoquaux
- 1 Rafael Cunha de Almeida
- 1 Rolando Espinoza La fuente
- 1 Vlad Niculae
- 1 Yaroslav Halchenko

1.7.13 Version 0.13

January 21, 2013

New Estimator Classes


- `decomposition.FactorAnalysis`, a transformer implementing the classical factor analysis, by Christian Osendorfer and Alexandre Gramfort. See `Factor Analysis` in the user guide.

- `feature_extraction.FeatureHasher`, a transformer implementing the “hashing trick” for fast, low-memory feature extraction from string fields by Lars Buitinck and `feature_extraction.text.HashingVectorizer` for text documents by Olivier Grisel See `Feature hashing` and `Vectorizing a large text corpus with the hashing trick` for the documentation and sample usage.

- `pipeline.FeatureUnion`, a transformer that concatenates results of several other transformers by Andreas Müller. See `FeatureUnion: composite feature spaces` in the user guide.

- `random_projection.GaussianRandomProjection`, `random_projection.SparseRandomProjection` and the function `random_projection.johnson_lindenstrauss_min_dim`. The first two are transformers implementing Gaussian and sparse random projection matrix by Olivier Grisel and Arnaud Joly. See `Random Projection` in the user guide.

- `preprocessing.OneHotEncoder`, a transformer that computes binary encodings of categorical features by Andreas Müller. See *Encoding categorical features* in the user guide.


- `ensemble.RandomTreesEmbedding`, a transformer for creating high-dimensional sparse representations using ensembles of totally random trees by Andreas Müller. See *Totally Random Trees Embedding* in the user guide.

- `manifold.SpectralEmbedding` and function `manifold.spectral_embedding`, implementing the “laplacian eigenmaps” transformation for non-linear dimensionality reduction by Wei Li. See *Spectral Embedding* in the user guide.

- `isotonic.IsotonicRegression` by Fabian Pedregosa, Alexandre Gramfort and Nelle Varoquaux.

**Changelog**

- `metrics.zero_one_loss` (formerly `metrics.zero_one`) now has option for normalized output that reports the fraction of misclassifications, rather than the raw number of misclassifications. By Kyle Beauchamp.

- `tree.DecisionTreeClassifier` and all derived ensemble models now support sample weighting, by Noel Dawe and Gilles Louppe.

- Speedup improvement when using bootstrap samples in forests of randomized trees, by Peter Prettenhofer and Gilles Louppe.

- Partial dependence plots for *Gradient Tree Boosting* in `ensemble.partial_dependence.partial_dependence` by Peter Prettenhofer. See *Partial Dependence Plots* for an example.

- The table of contents on the website has now been made expandable by Jaques Grobler.

- `feature_selection.SelectPercentile` now breaks ties deterministically instead of returning all equally ranked features.

- `feature_selection.SelectKBest` and `feature_selection.SelectPercentile` are more numerically stable since they use scores, rather than p-values, to rank results. This means that they might sometimes select different features than they did previously.

- Ridge regression and ridge classification fitting with `sparse_cg` solver no longer has quadratic memory complexity, by Lars Buitinck and Fabian Pedregosa.

- Ridge regression and ridge classification now support a new fast solver called `lsqr`, by Mathieu Blondel.

- Speed up of `metrics.precision_recall_curve` by Conrad Lee.

- Added support for reading/writing `svmlight` files with pairwise preference attribute (qid in `svmlight` file format) in `datasets.dump_svmlight_file` and `datasets.load_svmlight_file` by Fabian Pedregosa.

- Faster and more robust `metrics.confusion_matrix` and *Clustering performance evaluation* by Wei Li.

- `cross_validation.cross_val_score` now works with precomputed kernels and affinity matrices, by Andreas Müller.

- LARS algorithm made more numerically stable with heuristics to drop regressors too correlated as well as to stop the path when numerical noise becomes predominant, by Gael Varoquaux.

- Faster implementation of `metrics.precision_recall_curve` by Conrad Lee.
• New kernel metrics.chi2_kernel by Andreas Müller, often used in computer vision applications.
• Fix of longstanding bug in naive_bayes.BernoulliNB fixed by Shaun Jackman.
• Implemented predict_proba in multiclass.OneVsRestClassifier, by Andrew Winterman.
• Improve consistency in gradient boosting: estimators ensemble.GradientBoostingRegressor and ensemble.GradientBoostingClassifier use the estimator tree.DecisionTreeRegressor instead of the tree._tree.Tree data structure by Arnaud Joly.
• Fixed a floating point exception in the decision trees module, by Seberg.
• Fix metrics.roc_curve fails when y_true has only one class by Wei Li.
• Add the metrics.mean_absolute_error function which computes the mean absolute error. The metrics.mean_squared_error, metrics.mean_absolute_error and metrics.r2_score metrics support multioutput by Arnaud Joly.
• Fixed class_weight support in svm.LinearSVC and linear_model.LogisticRegression by Andreas Müller. The meaning of class_weight was reversed as erroneously higher weight meant less positives of a given class in earlier releases.
• Improve narrative documentation and consistency in sklearn.metrics for regression and classification metrics by Arnaud Joly.
• Fixed a bug in sklearn.svm.SVC when using csr-matrices with unsorted indices by Xinfan Meng and Andreas Müller.
• MiniBatchKMeans: Add random reassignment of cluster centers with little observations attached to them, by Gael Varoquaux.

API changes summary

• Renamed all occurrences of n_atoms to n_components for consistency. This applies to decomposition.DictionaryLearning, decomposition.MiniBatchDictionaryLearning, decomposition.dict_learning, decomposition.dict_learning_online.
• Renamed all occurrences of max_iters to max_iter for consistency. This applies to semi_supervised.LabelPropagation and semi_supervised.label_propagation.LabelSpreading.
• Renamed all occurrences of learn_rate to learning_rate for consistency in ensemble.BaseGradientBoosting and ensemble.GradientBoostingRegressor.
• The module sklearn.linear_model.sparse is gone. Sparse matrix support was already integrated into the “regular” linear models.
• sklearn.metrics.mean_square_error, which incorrectly returned the accumulated error, was removed. Use mean_squared_error instead.
• Passing class_weight parameters to fit methods is no longer supported. Pass them to estimator constructors instead.
• GMMs no longer have decode and rvs methods. Use the score, predict or sample methods instead.
• The solver fit option in Ridge regression and classification is now deprecated and will be removed in v0.14. Use the constructor option instead.
• feature_extraction.text.DictVectorizer now returns sparse matrices in the CSR format, instead of COO.
• Renamed \texttt{k} in \texttt{cross_validation.KFold} and \texttt{cross_validation.StratifiedKFold} to \texttt{n\_folds}, renamed \texttt{n\_bootstraps} to \texttt{n\_iter} in \texttt{cross_validation.Bootstrap}.

• Renamed all occurrences of \texttt{n\_iterations} to \texttt{n\_iter} for consistency. This applies to \texttt{cross_validation.ShuffleSplit}, \texttt{cross_validation.StratifiedShuffleSplit}, \texttt{utils.randomized_range_finder} and \texttt{utils.randomized_svd}.

• Replaced \texttt{rho} in \texttt{linear_model.ElasticNet} and \texttt{linear_model.SGDClassifier} by \texttt{l1\_ratio}. The \texttt{rho} parameter had different meanings; \texttt{l1\_ratio} was introduced to avoid confusion. It has the same meaning as previously \texttt{rho} in \texttt{linear_model.ElasticNet} and \texttt{(1-rho)} in \texttt{linear_model.SGDClassifier}.

• \texttt{linear_model.LassoLars} and \texttt{linear_model.Lars} now store a list of paths in the case of multiple targets, rather than an array of paths.

• The attribute \texttt{gmm} of \texttt{hmm.GMMHMM} was renamed to \texttt{gmm\_} to adhere more strictly with the API.

• \texttt{cluster.spectral\_embedding} was moved to \texttt{manifold.spectral\_embedding}.

• Renamed \texttt{eig\_tol} in \texttt{manifold.spectral\_embedding}, \texttt{cluster.SpectralClustering} to \texttt{eigen\_tol}, renamed \texttt{mode} to \texttt{eigen\_solver}.

• Renamed \texttt{mode} in \texttt{manifold.spectral\_embedding} and \texttt{cluster.SpectralClustering} to \texttt{eigen\_solver}.

• \texttt{classes\_} and \texttt{n\_classes\_} attributes of \texttt{tree.DecisionTreeClassifier} and all derived ensemble models are now flat in case of single output problems and nested in case of multi-output problems.

• The \texttt{estimators\_} attribute of \texttt{ensemble.gradient\_boosting.GradientBoostingRegressor} and \texttt{ensemble.gradient\_boosting.GradientBoostingClassifier} is now an array of :class:`tree.DecisionTreeRegressor`.

• Renamed \texttt{chunk\_size} to \texttt{batch\_size} in \texttt{decomposition.MiniBatchDictionaryLearning} and \texttt{decomposition.MiniBatchSparsePCA} for consistency.

• \texttt{svm.SVC} and \texttt{svm.NuSVC} now provide a \texttt{classes\_} attribute and support arbitrary dtypes for labels \texttt{y}. Also, the dtype returned by \texttt{predict} now reflects the dtype of \texttt{y} during \texttt{fit} (used to be \texttt{np.float}).

• Changed default \texttt{test\_size} in \texttt{cross_validation.train\_test\_split} to \texttt{None}, added possibility to infer \texttt{test\_size} from \texttt{train\_size} in \texttt{cross_validation.ShuffleSplit} and \texttt{cross_validation.StratifiedShuffleSplit}.

• Renamed function \texttt{sklearn.metrics.zero\_one} to \texttt{sklearn.metrics.zero\_one\_loss}. Be aware that the default behavior in \texttt{sklearn.metrics.zero\_one\_loss} is different from \texttt{sklearn.metrics.zero\_one}: \texttt{normalize=False} is changed to \texttt{normalize=True}.

• Renamed function \texttt{metrics.zero\_one\_score} to \texttt{metrics.accuracy\_score}.

• \texttt{datasets.make\_circles} now has the same number of inner and outer points.

• In the Naive Bayes classifiers, the \texttt{class\_prior} parameter was moved from \texttt{fit} to \texttt{__init__}.

**People**

List of contributors for release 0.13 by number of commits.

- 364 Andreas Müller
- 143 Arnaud Joly
- 137 Peter Prettenhofer
- 131 Gael Varoquaux
• 117 Mathieu Blondel
• 108 Lars Buitinck
• 106 Wei Li
• 101 Olivier Grisel
• 65 Vlad Niculae
• 54 Gilles Louppe
• 40 Jaques Grobler
• 38 Alexandre Gramfort
• 30 Rob Zinkov
• 19 Aymeric Masurelle
• 18 Andrew Winterman
• 17 Fabian Pedregosa
• 17 Nelle Varoquaux
• 16 Christian Osendorfer
• 14 Daniel Nouri
• 13 Virgile Fritsch
• 13 syhw
• 12 Satrajit Ghosh
• 10 Corey Lynch
• 10 Kyle Beauchamp
• 9 Brian Cheung
• 9 Immanuel Bayer
• 9 mr.Shu
• 8 Conrad Lee
• 8 James Bergstra
• 7 Tadej Janež
• 6 Brian Cajes
• 6 Jake Vanderplas
• 6 Michael
• 6 Noel Dawe
• 6 Tiago Nunes
• 6 cow
• 5 Anze
• 5 Shiqiao Du
• 4 Christian Jauvin
• 4 Jacques Kvam
1.7.14  Version 0.12.1

October 8, 2012

The 0.12.1 release is a bug-fix release with no additional features, but is instead a set of bug fixes

Changelog

- Improved numerical stability in spectral embedding by Gael Varoquaux
- Doctest under windows 64bit by Gael Varoquaux
- Documentation fixes for elastic net by Andreas Müller and Alexandre Gramfort
- Proper behavior with fortran-ordered NumPy arrays by Gael Varoquaux
• Make GridSearchCV work with non-CSR sparse matrix by Lars Buitinck
• Fix parallel computing in MDS by Gael Varoquaux
• Fix Unicode support in count vectorizer by Andreas Müller
• Fix MinCovDet breaking with X.shape = (3, 1) by Virgile Fritsch
• Fix clone of SGD objects by Peter Prettenhofer
• Stabilize GMM by Virgile Fritsch

People

• 14 Peter Prettenhofer
• 12 Gael Varoquaux
• 10 Andreas Müller
• 5 Lars Buitinck
• 3 Virgile Fritsch
• 1 Alexandre Gramfort
• 1 Gilles Louppe
• 1 Mathieu Blondel

1.7.15 Version 0.12

September 4, 2012

Changelog

• Various speed improvements of the decision trees module, by Gilles Louppe.
• ensemble.GradientBoostingRegressor and ensemble.GradientBoostingClassifier now support feature subsampling via the max_features argument, by Peter Prettenhofer.
• Added Huber and Quantile loss functions to ensemble.GradientBoostingRegressor, by Peter Prettenhofer.
• Decision trees and forests of randomized trees now support multi-output classification and regression problems, by Gilles Louppe.
• Added preprocessing.LabelEncoder, a simple utility class to normalize labels or transform non-numerical labels, by Mathieu Blondel.
• Added the epsilon-insensitive loss and the ability to make probabilistic predictions with the modified huber loss in Stochastic Gradient Descent, by Mathieu Blondel.
• Added Multi-dimensional Scaling (MDS), by Nelle Varoquaux.
• SVMlight file format loader now detects compressed (gzip/bzip2) files and decompresses them on the fly, by Lars Buitinck.
• SVMlight file format serializer now preserves double precision floating point values, by Olivier Grisel.
• A common testing framework for all estimators was added, by Andreas Müller.
• Understandable error messages for estimators that do not accept sparse input by Gael Varoquaux
• Speedups in hierarchical clustering by Gael Varoquaux. In particular building the tree now supports early stopping. This is useful when the number of clusters is not small compared to the number of samples.

• Add MultiTaskLasso and MultiTaskElasticNet for joint feature selection, by Alexandre Gramfort.

• Added metrics.auc_score and metrics.average_precision_score convenience functions by Andreas Müller.

• Improved sparse matrix support in the Feature selection module by Andreas Müller.

• New word boundaries-aware character n-gram analyzer for the Text feature extraction module by @kernc.

• Fixed bug in spectral clustering that led to single point clusters by Andreas Müller.

• In feature_extraction.text.CountVectorizer, added an option to ignore infrequent words, min_df by Andreas Müller.

• Add support for multiple targets in some linear models (ElasticNet, Lasso and OrthogonalMatchingPursuit) by Vlad Niculae and Alexandre Gramfort.

• Fixes in decomposition.ProbabilisticPCA score function by Wei Li.

• Fixed feature importance computation in Gradient Tree Boosting.

API changes summary

• The old scikits.learn package has disappeared; all code should import from sklearn instead, which was introduced in 0.9.

• In metrics.roc_curve, the thresholds array is now returned with it’s order reversed, in order to keep it consistent with the order of the returned fpr and tpr.

• In hmm objects, like hmm.GaussianHMM, hmm.MultinomialHMM, etc., all parameters must be passed to the object when initialising it and not through fit. Now fit will only accept the data as an input parameter.

• For all SVM classes, a faulty behavior of gamma was fixed. Previously, the default gamma value was only computed the first time fit was called and then stored. It is now recalculated on every call to fit.

• All Base classes are now abstract meta classes so that they can not be instantiated.

• cluster.ward_tree now also returns the parent array. This is necessary for early-stopping in which case the tree is not completely built.

• In feature_extraction.text.CountVectorizer the parameters min_n and max_n were joined to the parameter n_gram_range to enable grid-searching both at once.

• In feature_extraction.text.CountVectorizer, words that appear only in one document are now ignored by default. To reproduce the previous behavior, set min_df=1.

• Fixed API inconsistency: linear_model.SGDClassifier.predict_proba now returns 2d array when fit on two classes.

• Fixed API inconsistency: discriminant_analysis.QuadraticDiscriminantAnalysis.decision_function and discriminant_analysis.LinearDiscriminantAnalysis.decision_function now return 1d arrays when fit on two classes.

• Grid of alphas used for fitting linear_model.LassoCV and linear_model.ElasticNetCV is now stored in the attribute alphas_ rather than overriding the init parameter alphas.

• Linear models when alpha is estimated by cross-validation store the estimated value in the alpha_ attribute rather than just alpha or best_alpha.

• `svm.sparse.SVC` and other sparse SVM classes are now deprecated. The all classes in the `Support Vector Machines` module now automatically select the sparse or dense representation base on the input.

• All clustering algorithms now interpret the array \( X \) given to `fit` as input data, in particular `cluster.SpectralClustering` and `cluster.AffinityPropagation` which previously expected affinity matrices.

• For clustering algorithms that take the desired number of clusters as a parameter, this parameter is now called `n_clusters`.

People

• 267 Andreas Müller
• 94 Gilles Louppe
• 89 Gael Varoquaux
• 79 Peter Prettenhofer
• 60 Mathieu Blondel
• 57 Alexandre Gramfort
• 52 Vlad Niculae
• 45 Lars Buitinck
• 44 Nelle Varoquaux
• 37 Jaques Grobler
• 30 Alexis Mignon
• 30 Immanuel Bayer
• 27 Olivier Grisel
• 16 Subhodeep Moitra
• 13 Yannick Schwartz
• 12 @kernc
• 11 Virgile Fritsch
• 9 Daniel Duckworth
• 9 Fabian Pedregosa
• 9 Robert Layton
• 8 John Benediktsson
• 7 Marko Burjek
• 5 Nicolas Pinto
• 4 Alexandre Abraham
• 4 Jake Vanderplas
• 3 Brian Holt
• 3 Edouard Duchesnay
• 3 Florian Hoenig
• Gradient boosted regression trees (Gradient Tree Boosting) for classification and regression by Peter Prettenhofer and Scott White.

• Simple dict-based feature loader with support for categorical variables (feature_extraction.DictVectorizer) by Lars Buitinck.

• Added Matthews correlation coefficient (metrics.matthews_corrcoef) and added macro and micro average options to metrics.precision_score, metrics.recall_score and metrics.f1_score by Satrajit Ghosh.

• Out of Bag Estimates of generalization error for Ensemble methods by Andreas Müller.

• Randomized sparse models: Randomized sparse linear models for feature selection, by Alexandre Gramfort and Gael Varoquaux

• Label Propagation for semi-supervised learning, by Clay Woolam. Note the semi-supervised API is still work in progress, and may change.
scikit-learn user guide, Release 0.18.2

- Added BIC/AIC model selection to classical Gaussian mixture models and unified the API with the remainder of scikit-learn, by Bertrand Thirion
- Added `sklearn.cross_validation.StratifiedShuffleSplit`, which is a `sklearn.cross_validation.ShuffleSplit` with balanced splits, by Yannick Schwartz.
- `sklearn.neighbors.NearestCentroid` classifier added, along with a `shrink_threshold` parameter, which implements shrunken centroid classification, by Robert Layton.

Other changes

- Merged dense and sparse implementations of Stochastic Gradient Descent module and exposed utility extension types for sequential datasets `seq_dataset` and weight vectors `weight_vector` by Peter Prettenhofer.
- Added `partial_fit` (support for online/minibatch learning) and `warm_start` to the Stochastic Gradient Descent module by Mathieu Blondel.
- Dense and sparse implementations of Support Vector Machines classes and `linear_model.LogisticRegression` merged by Lars Buitinck.
- Regressors can now be used as base estimator in the Multiclass and multilabel algorithms module by Mathieu Blondel.
- Added `n_jobs` option to `metrics.pairwise.pairwise_distances` and `metrics.pairwise.pairwise_kernels` for parallel computation, by Mathieu Blondel.
- K-means can now be run in parallel, using the `n_jobs` argument to either K-means or kMeans, by Robert Layton.
- Improved Cross-validation: evaluating estimator performance and Tuning the hyper-parameters of an estimator documentation and introduced the new `cross_validation.train_test_split` helper function by Olivier Grisel
- `svm.SVC` members `coef_` and `intercept_` changed sign for consistency with `decision_function`; for `kernel==linear`, `coef_` was fixed in the one-vs-one case, by Andreas Müller.
- Performance improvements to efficient leave-one-out cross-validated Ridge regression, esp. for the `n_samples > n_features` case, in `linear_model.RidgeCV`, by Reuben Fletcher-Costin.
- Refactoring and simplification of the Text feature extraction API and fixed a bug that caused possible negative IDF, by Olivier Grisel.
- Beam pruning option in `_BaseHMM` module has been removed since it is difficult to Cythonize. If you are interested in contributing a Cython version, you can use the python version in the git history as a reference.
- Classes in Nearest Neighbors now support arbitrary Minkowski metric for nearest neighbors searches. The metric can be specified by argument `p`.

API changes summary

- `covariance.EllipticEnvelop` is now deprecated - Please use `covariance.EllipticEnvelope` instead.
- NeighborsClassifier and NeighborsRegressor are gone in the module Nearest Neighbors. Use the classes KNeighborsClassifier, RadiusNeighborsClassifier, KNeighborsRegressor and/or RadiusNeighborsRegressor instead.
- Sparse classes in the Stochastic Gradient Descent module are now deprecated.
• In `mixture.GMM`, `mixture.DPGMM` and `mixture.VBGMM`, parameters must be passed to an object when initialising it and not through `fit`. Now `fit` will only accept the data as an input parameter.

• methods `rvs` and `decode` in GMM module are now deprecated. `sample` and `score` or `predict` should be used instead.

• attribute `_scores` and `_pvalues` in univariate feature selection objects are now deprecated. `scores_` or `pvalues_` should be used instead.

• In `LogisticRegression`, `LinearSVC`, `SVC` and `NuSVC`, the `class_weight` parameter is now an initialization parameter, not a parameter to fit. This makes grid searches over this parameter possible.

• LFW data is now always shape `(n_samples, n_features)` to be consistent with the Olivetti faces dataset. Use `images` and `pairs` attribute to access the natural images shapes instead.

• In `svm.LinearSVC`, the meaning of the `multi_class` parameter changed. Options now are `'ovr'` and `'crammer_singer'`, with `'ovr'` being the default. This does not change the default behavior but hopefully is less confusing.

• Class `feature_selection.textVectorizer` is deprecated and replaced by `feature_selection.text.TfidfVectorizer`.

• The preprocessor / analyzer nested structure for text feature extraction has been removed. All those features are now directly passed as flat constructor arguments to `feature_selection.text.TfidfVectorizer` and `feature_selection.text.CountVectorizer`, in particular the following parameters are now used:

  – `analyzer` can be `'word'` or `'char'` to switch the default analysis scheme, or use a specific python callable (as previously).

  – `tokenizer` and `preprocessor` have been introduced to make it still possible to customize those steps with the new API.

  – `input` explicitly control how to interpret the sequence passed to `fit` and `predict`: filenames, file objects or direct (byte or Unicode) strings.

  – `charset decoding` is explicit and strict by default.

  – the `vocabulary`, fitted or not is now stored in the `vocabulary_` attribute to be consistent with the project conventions.

• Class `feature_selection.text.TfidfVectorizer` now derives directly from `feature_selection.text.CountVectorizer` to make grid search trivial.

• methods `rvs` in `_BaseHMM` module are now deprecated. `sample` should be used instead.

• Beam pruning option in `_BaseHMM` module is removed since it is difficult to be Cythonized. If you are interested, you can look in the history codes by git.

• The SVMlight format loader now supports files with both zero-based and one-based column indices, since both occur “in the wild”.

• Arguments in class `ShuffleSplit` are now consistent with `StratifiedShuffleSplit`. Arguments `test_fraction` and `train_fraction` are deprecated and renamed to `test_size` and `train_size` and can accept both float and int.

• Arguments in class `Bootstrap` are now consistent with `StratifiedShuffleSplit`. Arguments `n_test` and `n_train` are deprecated and renamed to `test_size` and `train_size` and can accept both float and int.

• Argument `p` added to classes in `Nearest Neighbors` to specify an arbitrary Minkowski metric for nearest neighbors searches.
People

- 282 Andreas Müller
- 239 Peter Prettenhofer
- 198 Gael Varoquaux
- 129 Olivier Grisel
- 114 Mathieu Blondel
- 103 Clay Woolam
- 96 Lars Buitinck
- 88 Jaques Grobler
- 82 Alexandre Gramfort
- 50 Bertrand Thirion
- 42 Robert Layton
- 28 flyingimmidev
- 26 Jake Vanderplas
- 26 Shiqiao Du
- 21 Satrajit Ghosh
- 17 David Marek
- 17 Gilles Louppe
- 14 Vlad Niculae
- 11 Yannick Schwartz
- 10 Fabian Pedregosa
- 9 fcostin
- 7 Nick Wilson
- 5 Adrien Gaidon
- 5 Nicolas Pinto
- 4 David Warde-Farley
- 5 Nelle Varoquaux
- 5 Emmanuelle Gouillart
- 3 Joonas Sillanpää
- 3 Paolo Losi
- 2 Charles McCarthy
- 2 Roy Hyunjin Han
- 2 Scott White
- 2 ibayer
- 1 Brandyn White
- 1 Carlos Scheidegger
1.7.17 Version 0.10

January 11, 2012

Changelog

- Python 2.5 compatibility was dropped; the minimum Python version needed to use scikit-learn is now 2.6.
- Sparse inverse covariance estimation using the graph Lasso, with associated cross-validated estimator, by Gael Varoquaux
- New Tree module by Brian Holt, Peter Prettenhofer, Satrajit Ghosh and Gilles Louppe. The module comes with complete documentation and examples.
- Fixed a bug in the RFE module by Gilles Louppe (issue #378).
- Fixed a memory leak in Support Vector Machines module by Brian Holt (issue #367).
- Faster tests by Fabian Pedregosa and others.
- Silhouette Coefficient cluster analysis evaluation metric added as sklearn.metrics.silhouette_score by Robert Layton.
- Fixed a bug in K-means in the handling of the n_init parameter: the clustering algorithm used to be run n_init times but the last solution was retained instead of the best solution by Olivier Grisel.
- Minor refactoring in Stochastic Gradient Descent module; consolidated dense and sparse predict methods; Enhanced test time performance by converting model parameters to fortran-style arrays after fitting (only multi-class).
- Adjusted Mutual Information metric added as sklearn.metrics.adjusted_mutual_info_score by Robert Layton.
- Models like SVC/SVR/LinearSVC/LogisticRegression from libsvm/liblinear now support scaling of C regularization parameter by the number of samples by Alexandre Gramfort.
- New Ensemble Methods module by Gilles Louppe and Brian Holt. The module comes with the random forest algorithm and the extra-trees method, along with documentation and examples.
Novelty and Outlier Detection: outlier and novelty detection, by Virgile Fritsch.

Kernel Approximation: a transform implementing kernel approximation for fast SGD on non-linear kernels by Andreas Müller.

Fixed a bug due to atom swapping in Orthogonal Matching Pursuit (OMP) by Vlad Niculae.

Sparse coding with a precomputed dictionary by Vlad Niculae.

Mini Batch K-Means performance improvements by Olivier Grisel.

K-means support for sparse matrices by Mathieu Blondel.

Improved documentation for developers and for the sklearn.utils module, by Jake Vanderplas.

Vectorized 20newsgroups dataset loader (sklearn.datasets.fetch_20newsgroups_vectorized) by Mathieu Blondel.

Multiclass and multilabel algorithms by Lars Buitinck.

Utilities for fast computation of mean and variance for sparse matrices by Mathieu Blondel.

Make sklearn.preprocessing.scale and sklearn.preprocessing.Scaler work on sparse matrices by Olivier Grisel.

Feature importances using decision trees and/or forest of trees, by Gilles Louppe.

Parallel implementation of forests of randomized trees by Gilles Louppe.

sklearn.cross_validation.ShuffleSplit can subsample the train sets as well as the test sets by Olivier Grisel.

Errors in the build of the documentation fixed by Andreas Müller.

API changes summary

Here are the code migration instructions when upgrading from scikit-learn version 0.9:

- Some estimators that may overwrite their inputs to save memory previously had overwrite_ parameters; these have been replaced with copy_ parameters with exactly the opposite meaning.
  
  This particularly affects some of the estimators in linear_model. The default behavior is still to copy everything passed in.

- The SVMlight dataset loader sklearn.datasets.load_svmlight_file no longer supports loading two files at once; use load_svmlight_files instead. Also, the (unused) buffer_mb parameter is gone.

- Sparse estimators in the Stochastic Gradient Descent module use dense parameter vector coef_ instead of sparse_coef_. This significantly improves test time performance.

- The Covariance estimation module now has a robust estimator of covariance, the Minimum Covariance Determinant estimator.

- Cluster evaluation metrics in metrics.cluster have been refactored but the changes are backwards compatible. They have been moved to the metrics.cluster.supervised, along with metrics.cluster.unsupervised which contains the Silhouette Coefficient.

- The permutation_test_score function now behaves the same way as cross_val_score (i.e. uses the mean score across the folds.)

- Cross Validation generators now use integer indices (indices=True) by default instead of boolean masks. This make it more intuitive to use with sparse matrix data.
• The functions used for sparse coding, `sparse_encode` and `sparse_encode_parallel` have been combined into `sklearn.decomposition.sparse_encode`, and the shapes of the arrays have been transposed for consistency with the matrix factorization setting, as opposed to the regression setting.

• Fixed an off-by-one error in the SVMlight/LibSVM file format handling; files generated using `sklearn.datasets.dump_svmlight_file` should be re-generated. (They should continue to work, but accidentally had one extra column of zeros prepended.)

• `BaseDictionaryLearning` class replaced by `SparseCodingMixin`.

• `sklearn.utils.extmath.fast_svd` has been renamed `sklearn.utils.extmath.randomized_svd` and the default oversampling is now fixed to 10 additional random vectors instead of doubling the number of components to extract. The new behavior follows the reference paper.

### People

The following people contributed to scikit-learn since last release:

• 246 Andreas Müller
• 242 Olivier Grisel
• 220 Gilles Louppe
• 183 Brian Holt
• 166 Gael Varoquaux
• 144 Lars Buitinck
• 73 Vlad Niculae
• 65 Peter Prettenhofer
• 64 Fabian Pedregosa
• 60 Robert Layton
• 55 Mathieu Blondel
• 52 Jake Vanderplas
• 44 Noel Dawe
• 38 Alexandre Gramfort
• 24 Virgile Fritsch
• 23 Satrajit Ghosh
• 3 Jan Hendrik Metzen
• 3 Kenneth C. Arnold
• 3 Shiqiao Du
• 3 Tim Sheerman-Chase
• 3 Yaroslav Halchenko
• 2 Bala Subrahmaniam Varanasi
• 2 DraXus
• 2 Michael Eickenberg
• 1 Bogdan Trach
1.7.18 Version 0.9

September 21, 2011

scikit-learn 0.9 was released on September 2011, three months after the 0.8 release and includes the new modules
Manifold learning, The Dirichlet Process as well as several new algorithms and documentation improvements.

This release also includes the dictionary-learning work developed by Vlad Niculae as part of the Google Summer of
Code program.
Changelog

- New *Manifold learning* module by Jake Vanderplas and Fabian Pedregosa.
- New *Dirichlet Process* Gaussian Mixture Model by Alexandre Passos
• **Nearest Neighbors** module refactoring by Jake Vanderplas: general refactoring, support for sparse matrices in input, speed and documentation improvements. See the next section for a full list of API changes.

• Improvements on the **Feature selection** module by Gilles Louppe: refactoring of the RFE classes, documentation rewrite, increased efficiency and minor API changes.

• **Sparse principal components analysis (SparsePCA and MiniBatchSparsePCA)** by Vlad Niculae, Gael Varoquaux and Alexandre Gramfort

• Printing an estimator now behaves independently of architectures and Python version thanks to Jean Kossaifi.

• **Loader for libsvm/svmlight format** by Mathieu Blondel and Lars Buitinck

• Documentation improvements: thumbnails in example gallery by Fabian Pedregosa.

• Important bugfixes in **Support Vector Machines** module (segfaults, bad performance) by Fabian Pedregosa.

• Added **Multinomial Naive Bayes** and **Bernoulli Naive Bayes** by Lars Buitinck

• Text feature extraction optimizations by Lars Buitinck

• Chi-Square feature selection (feature_selection.univariate_selection.chi2) by Lars Buitinck.

• **Sample generators** module refactoring by Gilles Louppe

• **Multiclass and multilabel algorithms** by Mathieu Blondel

• Ball tree rewrite by Jake Vanderplas

• Implementation of **DBSCAN** algorithm by Robert Layton

• Kmeans predict and transform by Robert Layton

• Preprocessing module refactoring by Olivier Grisel

• Faster mean shift by Conrad Lee

• New **Bootstrap, Random permutations cross-validation a.k.a. Shuffle & Split** and various other improvements in cross validation schemes by Olivier Grisel and Gael Varoquaux

• Adjusted Rand index and V-Measure clustering evaluation metrics by Olivier Grisel

• Added **Orthogonal Matching Pursuit** by Vlad Niculae

• Added 2D-patch extractor utilities in the **Feature extraction** module by Vlad Niculae

• Implementation of linear_model.LassoLarsCV (cross-validated Lasso solver using the Lars algorithm) and linear_model.LassoLarsIC (BIC/AIC model selection in Lars) by Gael Varoquaux and Alexandre Gramfort

• Scalability improvements to metrics.roc_curve by Olivier Hervieu

• Distance helper functions metrics.pairwise.pairwise_distances and metrics.pairwise.pairwise_kernels by Robert Layton

• **Mini-Batch K-Means** by Nelle Varoquaux and Peter Prettenhofer.

• **Downloading datasets from the mldata.org repository** utilities by Pietro Berkes.

• **The Olivetti faces dataset** by David Warde-Farley.

**API changes summary**

Here are the code migration instructions when upgrading from scikit-learn version 0.8:
• The scikits.learn package was renamed sklearn. There is still a scikits.learn package alias for backward compatibility.

Third-party projects with a dependency on scikit-learn 0.9+ should upgrade their codebase. For instance, under Linux / MacOSX just run (make a backup first):

```
find -name '*.py' | xargs sed -i 's/bscikits.learn\b\sklearn/g'
```

• Estimators no longer accept model parameters as fit arguments: instead all parameters must be only be passed as constructor arguments or using the now public set_params method inherited from base.BaseEstimator.

Some estimators can still accept keyword arguments on the fit but this is restricted to data-dependent values (e.g. a Gram matrix or an affinity matrix that are precomputed from the X data matrix.

• The cross_val package has been renamed to cross_validation although there is also a cross_val package alias in place for backward compatibility.

Third-party projects with a dependency on scikit-learn 0.9+ should upgrade their codebase. For instance, under Linux / MacOSX just run (make a backup first):

```
find -name '*.py' | xargs sed -i 's/bcross_val\bcross_validation/g'
```

• The score_func argument of the sklearn.cross_validation.cross_val_score function is now expected to accept y_test and y_predicted as only arguments for classification and regression tasks or X_test for unsupervised estimators.

• gamma parameter for support vector machine algorithms is set to $1 / n_{features}$ by default, instead of $1 / n_{samples}$.

• The sklearn.hmm has been marked as orphaned: it will be removed from scikit-learn in version 0.11 unless someone steps up to contribute documentation, examples and fix lurking numerical stability issues.

• sklearn.neighbors has been made into a submodule. The two previously available estimators, NeighborsClassifier and NeighborsRegressor have been marked as deprecated. Their functionality has been divided among five new classes: NearestNeighbors for unsupervised neighbors searches, KNeighborsClassifier & RadiusNeighborsClassifier for supervised classification problems, and KNeighborsRegressor & RadiusNeighborsRegressor for supervised regression problems.

• sklearn.ball_tree.BallTree has been moved to sklearn.neighbors.BallTree. Using the former will generate a warning.

• sklearn.linear_model.LARS() and related classes (LassoLARS, LassoLARSCV, etc.) have been renamed to sklearn.linear_model.Lars().

• All distance metrics and kernels in sklearn.metrics.pairwise now have a Y parameter, which by default is None. If not given, the result is the distance (or kernel similarity) between each sample in Y. If given, the result is the pairwise distance (or kernel similarity) between samples in X to Y.

• sklearn.metrics.pairwise.11_distance is now called manhattan_distance, and by default returns the pairwise distance. For the component wise distance, set the parameter sum_over_features to False.

Backward compatibility package aliases and other deprecated classes and functions will be removed in version 0.11.

People

38 people contributed to this release.

• 387 Vlad Niculae
1.7.19 Version 0.8

May 11, 2011

scikit-learn 0.8 was released on May 2011, one month after the first “international” scikit-learn coding sprint and is marked by the inclusion of important modules: Hierarchical clustering, Cross decomposition, Non-negative matrix factorization (NMF or NNMF), initial support for Python 3 and by important enhancements and bug fixes.

Changelog

Several new modules where introduced during this release:

- New Hierarchical clustering module by Vincent Michel, Bertrand Thirion, Alexandre Gramfort and Gael Varoquaux.
- Kernel PCA implementation by Mathieu Blondel
- The Labeled Faces in the Wild face recognition dataset by Olivier Grisel.
- New Cross decomposition module by Edouard Duchesnay.
- Non-negative matrix factorization (NMF or NNMF) module Vlad Niculae
- Implementation of the Oracle Approximating Shrinkage algorithm by Virgile Fritsch in the Covariance estimation module.

Some other modules benefited from significant improvements or cleanups.

- Initial support for Python 3: builds and imports cleanly, some modules are usable while others have failing tests by Fabian Pedregosa.
- decomposition.PCA is now usable from the Pipeline object by Olivier Grisel.
- Guide How to optimize for speed by Olivier Grisel.
- Fixes for memory leaks in libsvm bindings, 64-bit safer BallTree by Lars Buitinck.
- bug and style fixing in K-means algorithm by Jan Schlüter.
- Add attribute converged to Gaussian Mixture Models by Vincent Schut.
- Implemented transform, predict_log_proba in discriminant_analysis.LinearDiscriminantAnalysis by Mathieu Blondel.
- Refactoring in the Support Vector Machines module and bug fixes by Fabian Pedregosa, Gael Varoquaux and Amit Aides.
- Refactored SGD module (removed code duplication, better variable naming), added interface for sample weight by Peter Prettenhofer.
- Wrapped BallTree with Cython by Thouis (Ray) Jones.
- Added function svm.l1_min_c by Paolo Losi.
- Typos, doc style, etc. by Yaroslav Halchenko, Gael Varoquaux, Olivier Grisel, Yann Malet, Nicolas Pinto, Lars Buitinck and Fabian Pedregosa.

People

People that made this release possible preceded by number of commits:

- 159 Olivier Grisel
1.7.20 Version 0.7

March 2, 2011

scikit-learn 0.7 was released in March 2011, roughly three months after the 0.6 release. This release is marked by the speed improvements in existing algorithms like k-Nearest Neighbors and K-Means algorithm and by the inclusion of an efficient algorithm for computing the Ridge Generalized Cross Validation solution. Unlike the preceding release, no new modules were added to this release.

Changelog

- Performance improvements for Gaussian Mixture Model sampling [Jan Schlüter].
- Implementation of efficient leave-one-out cross-validated Ridge in `linear_model.RidgeCV` [Mathieu Blondel]
• Better handling of collinearity and early stopping in `linear_model.lars_path` [Alexandre Gramfort and Fabian Pedregosa].

• Fixes for liblinear ordering of labels and sign of coefficients [Dan Yamins, Paolo Losi, Mathieu Blondel and Fabian Pedregosa].

• Performance improvements for Nearest Neighbors algorithm in high-dimensional spaces [Fabian Pedregosa].

• Performance improvements for `cluster.kMeans` [Gael Varoquaux and James Bergstra].

• Sanity checks for SVM-based classes [Mathieu Blondel].

• Refactoring of `neighbors.NeighborsClassifier` and `neighbors.kneighbors_graph`: added different algorithms for the k-Nearest Neighbor Search and implemented a more stable algorithm for finding barycenter weights. Also added some developer documentation for this module, see `notes_neighbors` for more information [Fabian Pedregosa].

• Documentation improvements: Added `pca.RandomizedPCA` and `linear_model.LogisticRegression` to the class reference. Also added references of matrices used for clustering and other fixes [Gael Varoquaux, Fabian Pedregosa, Mathieu Blondel, Olivier Grisel, Virgile Fritsch, Emmanuelle Gouillart].

• Binded decision_function in classes that make use of liblinear, dense and sparse variants, like `svm.LinearSVC` or `linear_model.LogisticRegression` [Fabian Pedregosa].

• Performance and API improvements to `metrics.euclidean_distances` and to `pca.RandomizedPCA` [James Bergstra].

• Fix compilation issues under NetBSD [Kamel Ibn Hassen Derouiche].

• Allow input sequences of different lengths in `hmm.GaussianHMM` [Ron Weiss].

• Fix bug in affinity propagation caused by incorrect indexing [Xinfan Meng].

People

People that made this release possible preceded by number of commits:

• 85 Fabian Pedregosa
• 67 Mathieu Blondel
• 20 Alexandre Gramfort
• 19 James Bergstra
• 14 Dan Yamins
• 13 Olivier Grisel
• 12 Gael Varoquaux
• 4 Edouard Duchesnay
• 4 Ron Weiss
• 2 Satrajit Ghosh
• 2 Vincent Dubourg
• 1 Emmanuelle Gouillart
• 1 Kamel Ibn Hassen Derouiche
• 1 Paolo Losi
• 1 VirgileFritsch
scikit-learn user guide, Release 0.18.2

• 1 Yaroslav Halchenko
• 1 Xinfan Meng

1.7.21 Version 0.6

December 21, 2010

scikit-learn 0.6 was released on December 2010. It is marked by the inclusion of several new modules and a general renaming of old ones. It is also marked by the inclusion of new example, including applications to real-world datasets.

Changelog

• New stochastic gradient descent module by Peter Prettenhofer. The module comes with complete documentation and examples.

• Improved svm module: memory consumption has been reduced by 50%, heuristic to automatically set class weights, possibility to assign weights to samples (see SVM: Weighted samples for an example).

• New Gaussian Processes module by Vincent Dubourg. This module also has great documentation and some very neat examples. See example_gaussian_process_plot_gp_regression.py or example_gaussian_process_plot_gp_probabilistic_classification_after_regression.py for a taste of what can be done.

• It is now possible to use liblinear’s Multi-class SVC (option multi_class in svm.LinearSVC)

• New features and performance improvements of text feature extraction.

• Improved sparse matrix support, both in main classes (grid_search.GridSearchCV) as in modules sklearn.svm.sparse and sklearn.linear_model.sparse.

• Lots of cool new examples and a new section that uses real-world datasets was created. These include: Faces recognition example using eigenfaces and SVMs, Species distribution modeling, Libsvm GUI, Wikipedia principal eigenvector and others.

• Faster Least Angle Regression algorithm. It is now 2x faster than the R version on worst case and up to 10x times faster on some cases.

• Faster coordinate descent algorithm. In particular, the full path version of lasso (linear_model.lasso_path) is more than 200x times faster than before.

• It is now possible to get probability estimates from a linear_model.LogisticRegression model.

• module renaming: the glm module has been renamed to linear_model, the gmm module has been included into the more general mixture model and the sgd module has been included in linear_model.

• Lots of bug fixes and documentation improvements.

People

People that made this release possible preceded by number of commits:

• 207 Olivier Grisel
• 167 Fabian Pedregosa
• 97 Peter Prettenhofer
• 68 Alexandre Gramfort
• 59 Mathieu Blondel
New classes

- New `pipeline.Pipeline` object to compose different estimators.
- Recursive Feature Elimination routines in module `Feature selection`.
- Addition of various classes capable of cross validation in the `linear_model` module (`linear_model.LassoCV`, `linear_model.ElasticNetCV`, etc.).
- New, more efficient LARS algorithm implementation. The Lasso variant of the algorithm is also implemented. See `linear_model.lars_path`, `linear_model.Lars` and `linear_model.LassoLars`.
- New module `feature_extraction` (see `class reference`).
- New FastICA algorithm in module `sklearn.fastica`.

Documentation

- Improved documentation for many modules, now separating narrative documentation from the class reference. As an example, see documentation for the SVM module and the complete class reference.

Fixes

- API changes: adhere variable names to PEP-8, give more meaningful names.
- Fixes for svm module to run on a shared memory context (multiprocessing).
- It is again possible to generate latex (and thus PDF) from the sphinx docs.
Examples

- new examples using some of the mlcomp datasets: sphx_glr_auto_examples_mlcomp_sparse_document_classification.py (since removed) and Classification of text documents using sparse features
- Many more examples. See here the full list of examples.

External dependencies

- Joblib is now a dependency of this package, although it is shipped with (sklearn.externals.joblib).

Removed modules

- Module ann (Artificial Neural Networks) has been removed from the distribution. Users wanting this sort of algorithms should take a look into pybrain.

Misc

- New sphinx theme for the web page.

Authors

The following is a list of authors for this release, preceded by number of commits:

- 262 Fabian Pedregosa
- 240 Gael Varoquaux
- 149 Alexandre Gramfort
- 116 Olivier Grisel
- 40 Vincent Michel
- 38 Ron Weiss
- 23 Matthieu Perrot
- 10 Bertrand Thirion
- 7 Yaroslav Halchenko
- 9 VirgileFritsch
- 6 Edouard Duchesnay
- 4 Mathieu Blondel
- 1 Ariel Rokem
- 1 Matthieu Brucher

1.7.23 Version 0.4

August 26, 2010
Changelog

Major changes in this release include:

- Coordinate Descent algorithm (Lasso, ElasticNet) refactoring & speed improvements (roughly 100x times faster).
- Coordinate Descent Refactoring (and bug fixing) for consistency with R’s package GLMNET.
- New metrics module.
- New GMM module contributed by Ron Weiss.
- Implementation of the LARS algorithm (without Lasso variant for now).
- feature_selection module redesign.
- Migration to GIT as version control system.
- Removal of obsolete attrselect module.
- Rename of private compiled extensions (added underscore).
- Removal of legacy unmaintained code.
- Documentation improvements (both docstring and rst).
- Improvement of the build system to (optionally) link with MKL. Also, provide a lite BLAS implementation in case no system-wide BLAS is found.
- Lots of new examples.
- Many, many bug fixes ...

Authors

The committer list for this release is the following (preceded by number of commits):

- 143 Fabian Pedregosa
- 35 Alexandre Gramfort
- 34 Olivier Grisel
- 11 Gael Varoquaux
- 5 Yaroslav Halchenko
- 2 Vincent Michel
- 1 Chris Filo Gorgolewski

1.7.24 Earlier versions

Earlier versions included contributions by Fred Mailhot, David Cooke, David Huard, Dave Morrill, Ed Schofield, Travis Oliphant, Pearu Peterson.
2.1 An introduction to machine learning with scikit-learn

Section contents
In this section, we introduce the machine learning vocabulary that we use throughout scikit-learn and give a simple learning example.

2.1.1 Machine learning: the problem setting

In general, a learning problem considers a set of \( n \) samples of data and then tries to predict properties of unknown data. If each sample is more than a single number and, for instance, a multi-dimensional entry (aka multivariate data), it is said to have several attributes or features.

We can separate learning problems in a few large categories:

- **supervised learning**, in which the data comes with additional attributes that we want to predict (Click here to go to the scikit-learn supervised learning page). This problem can be either:
  - **classification**: samples belong to two or more classes and we want to learn from already labeled data how to predict the class of unlabeled data. An example of classification problem would be the handwritten digit recognition example, in which the aim is to assign each input vector to one of a finite number of discrete categories. Another way to think of classification is as a discrete (as opposed to continuous) form of supervised learning where one has a limited number of categories and for each of the \( n \) samples provided, one is to try to label them with the correct category or class.
  - **regression**: if the desired output consists of one or more continuous variables, then the task is called regression. An example of a regression problem would be the prediction of the length of a salmon as a function of its age and weight.

- **unsupervised learning**, in which the training data consists of a set of input vectors \( x \) without any corresponding target values. The goal in such problems may be to discover groups of similar examples within the data, where it is called clustering, or to determine the distribution of data within the input space, known as density estimation, or to project the data from a high-dimensional space down to two or three dimensions for the purpose of visualization (Click here to go to the Scikit-Learn unsupervised learning page).
Training set and testing set

Machine learning is about learning some properties of a data set and applying them to new data. This is why a common practice in machine learning to evaluate an algorithm is to split the data at hand into two sets, one that we call the **training set** on which we learn data properties and one that we call the **testing set** on which we test these properties.

### 2.1.2 Loading an example dataset

`scikit-learn` comes with a few standard datasets, for instance the `iris` and `digits` datasets for classification and the `boston house prices` dataset for regression.

In the following, we start a Python interpreter from our shell and then load the `iris` and `digits` datasets. Our notational convention is that `$` denotes the shell prompt while `>>>` denotes the Python interpreter prompt:

```
$ python
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> digits = datasets.load_digits()
```

A dataset is a dictionary-like object that holds all the data and some metadata about the data. This data is stored in the `.data` member, which is a `n_samples,n_features` array. In the case of supervised problem, one or more response variables are stored in the `.target` member. More details on the different datasets can be found in the dedicated section.

For instance, in the case of the `digits` dataset, `digits.data` gives access to the features that can be used to classify the digits samples:

```
>>> print(digits.data)
[[ 0.  0.  5. ...,  0.  0.  0.]
 [ 0.  0. 10. ...,  0.  0.  0.]
 [ 0.  0.  6. ..., 16.  9.  0.]
 ...,
 [ 0.  0.  1. ...,  6.  0.  0.]
 [ 0.  0.  2. ..., 12.  0.  0.]
 [ 0.  0. 10. ..., 12.  1.  0.]]
```

and `digits.target` gives the ground truth for the digit dataset, that is the number corresponding to each digit image that we are trying to learn:

```
>>> digits.target
array([0, 1, 2, ..., 8, 9, 8])
```

### Shape of the data arrays

The data is always a 2D array, shape `(n_samples,n_features)`, although the original data may have had a different shape. In the case of the `digits` dataset, each original sample is an image of shape `(8,8)` and can be accessed using:

```
>>> digits.images[0]
array([[ 0.,  0.,  5.,  ...,  0.,  0.,  0.],
       [ 0.,  0., 13.,  ...,  0.,  0.,  0.],
       [ 0.,  3., 15.,  ...,  0.,  11.,  8.,  0.],
       ...,
       [ 0.,  4., 12.,  ...,  0.,  0.,  8.,  8.,  0.],
       [ 0.,  5.,  8.,  ...,  0.,  0.,  9.,  8.,  0.],
       [ 0.,  4., 11.,  ...,  0.,  1., 12.,  ...,  0.],
       [ 0.,  2., 14.,  ..., 10., 12.,  0.,  0.],
       [ 0.,  0.,  6., 13., 10.,  0.,  0.,  0.]])
```
The *simple example on this dataset* illustrates how starting from the original problem one can shape the data for consumption in scikit-learn.

**Loading from external datasets**

To load from an external dataset, please refer to *loading external datasets*.

### 2.1.3 Learning and predicting

In the case of the digits dataset, the task is to predict, given an image, which digit it represents. We are given samples of each of the 10 possible classes (the digits zero through nine) on which we fit an estimator to be able to predict the classes to which unseen samples belong.

In scikit-learn, an estimator for classification is a Python object that implements the methods `fit` (`X`, `y`) and `predict` (`T`).

An example of an estimator is the class `sklearn.svm.SVC` that implements support vector classification. The constructor of an estimator takes as arguments the parameters of the model, but for the time being, we will consider the estimator as a black box:

```python
>>> from sklearn import svm

>>> clf = svm.SVC(gamma=0.001, C=100.)
```

#### Choosing the parameters of the model

In this example we set the value of `gamma` manually. It is possible to automatically find good values for the parameters by using tools such as *grid search* and *cross validation*.

We call our estimator instance `clf`, as it is a classifier. It now must be fitted to the model, that is, it must learn from the model. This is done by passing our training set to the `fit` method. As a training set, let us use all the images of our dataset apart from the last one. We select this training set with the `[:-1]` Python syntax, which produces a new array that contains all but the last entry of `digits.data`:

```python
>>> clf.fit(digits.data[:-1], digits.target[:-1])
SVC(C=100.0, cache_size=200, class_weight=None, coef0=0.0,
   decision_function_shape='ovr', degree=3, gamma=0.001, kernel='rbf',
   max_iter=1000, probability=False, random_state=None, shrinking=True,
   tol=0.001, verbose=False)
```

Now you can predict new values, in particular, we can ask to the classifier what is the digit of our last image in the `digits` dataset, which we have not used to train the classifier:

```python
>>> clf.predict(digits.data[-1:])
array([8])
```
As you can see, it is a challenging task: the images are of poor resolution. Do you agree with the classifier?

A complete example of this classification problem is available as an example that you can run and study: Recognizing hand-written digits.

2.1.4 Model persistence

It is possible to save a model in the scikit by using Python’s built-in persistence model, namely pickle:

```python
>>> from sklearn import svm
>>> from sklearn import datasets
>>> clf = svm.SVC()
>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target
>>> clf.fit(X, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
    decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
    max_iter=-1, probability=False, random_state=None, shrinking=True,
    tol=0.001, verbose=False)
>>> import pickle

>>> s = pickle.dumps(clf)

>>> clf2 = pickle.loads(s)

>>> clf2.predict(X[0:1])
array([0])

>>> y[0]
0
```

In the specific case of the scikit, it may be more interesting to use joblib’s replacement of pickle (joblib.dump & joblib.load), which is more efficient on big data, but can only pickle to the disk and not to a string:

```python
>>> from sklearn.externals import joblib

>>> joblib.dump(clf, 'filename.pkl')

Later you can load back the pickled model (possibly in another Python process) with:

```python
>>> clf = joblib.load('filename.pkl')
```

Note: joblib.dump and joblib.load functions also accept file-like object instead of filenames. More information on data persistence with Joblib is available here.

Note that pickle has some security and maintainability issues. Please refer to section Model persistence for more detailed information about model persistence with scikit-learn.
2.1.5 Conventions

scikit-learn estimators follow certain rules to make their behavior more predictive.

Type casting

Unless otherwise specified, input will be cast to `float64`:

```python
>>> import numpy as np
>>> from sklearn import random_projection

>>> rng = np.random.RandomState(0)
>>> X = rng.rand(10, 2000)
>>> X = np.array(X, dtype='float32')
>>> X.dtype
dtype('float32')
>>> transformer = random_projection.GaussianRandomProjection()
>>> X_new = transformer.fit_transform(X)
>>> X_new.dtype
dtype('float64')
```

In this example, `X` is `float32`, which is cast to `float64` by `fit_transform(X)`.

Regression targets are cast to `float64`, classification targets are maintained:

```python
>>> from sklearn import datasets
>>> from sklearn.svm import SVC

>>> iris = datasets.load_iris()
>>> clf = SVC()
>>> clf.fit(iris.data, iris.target)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
   decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
   max_iter=-1, probability=False, random_state=None, shrinking=True,
   tol=0.001, verbose=False)

>>> list(clf.predict(iris.data[:3]))
[0, 0, 0]

>>> clf.fit(iris.data, iris.target_names[iris.target])
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
   decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
   max_iter=-1, probability=False, random_state=None, shrinking=True,
   tol=0.001, verbose=False)

>>> list(clf.predict(iris.data[:3]))
['setosa', 'setosa', 'setosa']
```

Here, the first `predict()` returns an integer array, since `iris.target` (an integer array) was used in `fit`. The second `predict()` returns a string array, since `iris.target_names` was for fitting.

Refitting and updating parameters

Hyper-parameters of an estimator can be updated after it has been constructed via the `sklearn.pipeline.Pipeline.set_params` method. Calling `fit()` more than once will overwrite what was learned by any previous `fit()`:
Here, the default kernel rbf is first changed to linear after the estimator has been constructed via SVC(), and changed back to rbf to refit the estimator and to make a second prediction.

### Multiclass vs. multilabel fitting

When using multiclass classifiers, the learning and prediction task that is performed is dependent on the format of the target data fit upon:

```python
>>> from sklearn.svm import SVC
>>> from sklearn.multiclass import OneVsRestClassifier
>>> from sklearn.preprocessing import LabelBinarizer

>>> X = [[1, 2], [2, 4], [4, 5], [3, 2], [3, 1]]
>>> y = [0, 0, 1, 1, 2]

>>> classif = OneVsRestClassifier(estimator=SVC(random_state=0))
>>> classif.fit(X, y).predict(X)
array([0, 0, 1, 1, 2])
```

In the above case, the classifier is fit on a 1d array of multiclass labels and the predict() method therefore provides corresponding multiclass predictions. It is also possible to fit upon a 2d array of binary label indicators:

```python
>>> y = LabelBinarizer().fit_transform(y)
>>> classif.fit(X, y).predict(X)
array([[0, 0, 0, 1, 1],
       [0, 0, 0, 1, 1],
       [0, 0, 0, 1, 1],
       [0, 0, 0, 1, 1],
       [0, 0, 0, 1, 1]])
```

Here, the classifier is fit() on a 2d binary label representation of y, using the LabelBinarizer. In this case predict() returns a 2d array representing the corresponding multilabel predictions.
Note that the fourth and fifth instances returned all zeroes, indicating that they matched none of the three labels fit upon. With multilabel outputs, it is similarly possible for an instance to be assigned multiple labels:

```python
>> from sklearn.preprocessing import MultiLabelBinarizer
>> y = [[0, 1], [0, 2], [1, 3], [0, 2, 3], [2, 4]]
>> y = preprocessing.MultiLabelBinarizer().fit_transform(y)
>> clasif.fit(X, y).predict(X)
array([[1, 1, 0, 0, 0],
       [1, 0, 1, 0, 0],
       [0, 1, 0, 1, 0],
       [1, 0, 1, 1, 0],
       [0, 0, 1, 0, 1]])
```

In this case, the classifier is fit upon instances each assigned multiple labels. The `MultiLabelBinarizer` is used to binarize the 2d array of multilabels to fit upon. As a result, `predict()` returns a 2d array with multiple predicted labels for each instance.

### 2.2 A tutorial on statistical-learning for scientific data processing

#### Statistical learning

Machine learning is a technique with a growing importance, as the size of the datasets experimental sciences are facing is rapidly growing. Problems it tackles range from building a prediction function linking different observations, to classifying observations, or learning the structure in an unlabeled dataset.

This tutorial will explore statistical learning, the use of machine learning techniques with the goal of statistical inference: drawing conclusions on the data at hand.

Scikit-learn is a Python module integrating classic machine learning algorithms in the tightly-knit world of scientific Python packages (NumPy, SciPy, matplotlib).

#### 2.2.1 Statistical learning: the setting and the estimator object in scikit-learn

##### Datasets

Scikit-learn deals with learning information from one or more datasets that are represented as 2D arrays. They can be understood as a list of multi-dimensional observations. We say that the first axis of these arrays is the samples axis, while the second is the features axis.

A simple example shipped with the scikit: iris dataset

```python
>>> from sklearn.datasets import datasets
>>> iris = datasets.load_iris()
>>> data = iris.data
>>> data.shape
(150, 4)
```

It is made of 150 observations of irises, each described by 4 features: their sepal and petal length and width, as detailed in `iris.DESCRIPTOR`.
When the data is not initially in the \((n_{\text{samples}}, n_{\text{features}})\) shape, it needs to be preprocessed in order to be used by scikit-learn.

### An example of reshaping data would be the digits dataset

```
>>> digits = datasets.load_digits()
```

```
>>> digits.images.shape
(1797, 8, 8)
```

```
>>> import matplotlib.pyplot as plt
```

```
>>> plt.imshow(digits.images[-1], cmap=plt.cm.gray_r)
```

The digits dataset is made of 1797 8x8 images of hand-written digits

```
>>> data = digits.images.reshape((digits.images.shape[0], -1))
```

### Estimators objects

**Fitting data:** the main API implemented by scikit-learn is that of the estimator. An estimator is any object that learns from data; it may be a classification, regression or clustering algorithm or a transformer that extracts/filters useful features from raw data.

All estimator objects expose a `fit` method that takes a dataset (usually a 2-d array):

```
>>> estimator.fit(data)
```

**Estimator parameters:** All the parameters of an estimator can be set when it is instantiated or by modifying the corresponding attribute:

```
>>> estimator = Estimator(param1=1, param2=2)
>>> estimator.param1
```

**Estimated parameters:** When data is fitted with an estimator, parameters are estimated from the data at hand. All the estimated parameters are attributes of the estimator object ending by an underscore:

```
>>> estimator.estimated_param_
```
2.2.2 Supervised learning: predicting an output variable from high-dimensional observations

The problem solved in supervised learning

_Supervised learning_ consists in learning the link between two datasets: the observed data $X$ and an external variable $y$ that we are trying to predict, usually called “target” or “labels”. Most often, $y$ is a 1D array of length $n_{\text{samples}}$.

All supervised estimators in scikit-learn implement a `fit(X, y)` method to fit the model and a `predict(X)` method that, given unlabeled observations $X$, returns the predicted labels $y$.

Vocabulary: classification and regression

If the prediction task is to classify the observations in a set of finite labels, in other words to “name” the objects observed, the task is said to be a _classification_ task. On the other hand, if the goal is to predict a continuous target variable, it is said to be a _regression_ task.

When doing classification in scikit-learn, $y$ is a vector of integers or strings.

Note: See the _Introduction to machine learning with scikit-learn Tutorial_ for a quick run-through on the basic machine learning vocabulary used within scikit-learn.

Nearest neighbor and the curse of dimensionality

Classifying irises:

The iris dataset is a classification task consisting in identifying 3 different types of irises (Setosa, Versicolour, and Virginica) from their petal and sepal length and width:
>>> import numpy as np
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> iris_X = iris.data
>>> iris_y = iris.target
>>> np.unique(iris_y)
array([0, 1, 2])

k-Nearest neighbors classifier

The simplest possible classifier is the nearest neighbor: given a new observation $X_{test}$, find in the training set (i.e. the data used to train the estimator) the observation with the closest feature vector. (Please see the Nearest Neighbors section of the online Scikit-learn documentation for more information about this type of classifier.)

Training set and testing set

While experimenting with any learning algorithm, it is important not to test the prediction of an estimator on the data used to fit the estimator as this would not be evaluating the performance of the estimator on new data. This is why datasets are often split into train and test data.

KNN (k nearest neighbors) classification example:

```python
>>> # Split iris data in train and test data
>>> # A random permutation, to split the data randomly
>>> np.random.seed(0)
>>> indices = np.random.permutation(len(iris_X))
>>> iris_X_train = iris_X[indices[:-10]]
>>> iris_y_train = iris_y[indices[:-10]]
>>> iris_X_test = iris_X[indices[-10:]]
>>> iris_y_test = iris_y[indices[-10:]]
>>> # Create and fit a nearest-neighbor classifier
>>> from sklearn.neighbors import KNeighborsClassifier
```
```python
>>> knn = KNeighborsClassifier()
>>> knn.fit(iris_X_train, iris_y_train)
KNeighborsClassifier(algorithm='auto', leaf_size=30, metric='minkowski',
                     metric_params=None, n_jobs=1, n_neighbors=5, p=2,
                     weights='uniform')
>>> knn.predict(iris_X_test)
array([1, 2, 1, 0, 0, 0, 2, 1, 2, 0])
>>> iris_y_test
array([1, 1, 1, 0, 0, 0, 2, 1, 2, 0])
```

## The curse of dimensionality

For an estimator to be effective, you need the distance between neighboring points to be less than some value $d$, which depends on the problem. In one dimension, this requires on average $n \frac{1}{d}$ points. In the context of the above $k$-NN example, if the data is described by just one feature with values ranging from 0 to 1 and with $n$ training observations, then new data will be no further away than $1/n$. Therefore, the nearest neighbor decision rule will be efficient as soon as $1/n$ is small compared to the scale of between-class feature variations.

If the number of features is $p$, you now require $n \frac{1}{d^p}$ points. Let’s say that we require 10 points in one dimension: now $10^p$ points are required in $p$ dimensions to pave the $[0, 1]$ space. As $p$ becomes large, the number of training points required for a good estimator grows exponentially.

For example, if each point is just a single number (8 bytes), then an effective $k$-NN estimator in a paltry $p$ 20 dimensions would require more training data than the current estimated size of the entire internet ($\pm 1000$ Exabytes or so).

This is called the **curse of dimensionality** and is a core problem that machine learning addresses.

### Linear model: from regression to sparsity

#### Diabetes dataset

The diabetes dataset consists of 10 physiological variables (age, sex, weight, blood pressure) measured on 442 patients, and an indication of disease progression after one year:

```python
>>> diabetes = datasets.load_diabetes()
>>> diabetes_X_train = diabetes.data[:-20]
>>> diabetes_X_test = diabetes.data[-20:]
>>> diabetes_y_train = diabetes.target[:-20]
>>> diabetes_y_test = diabetes.target[-20:]
```

The task at hand is to predict disease progression from physiological variables.

#### Linear regression

LinearRegression, in its simplest form, fits a linear model to the data set by adjusting a set of parameters in order to make the sum of the squared residuals of the model as small as possible.
Linear models: $y = X\beta + \epsilon$

- $X$: data
- $y$: target variable
- $\beta$: Coefficients
- $\epsilon$: Observation noise

```python
>>> from sklearn import linear_model
data: https://unl.iq

>>> regr = linear_model.LinearRegression()
>>> regr.fit(diabetes_X_train, diabetes_y_train)
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)

>>> print(regr.coef_)
[ 0.30349955 -237.63931533 510.53060544 327.73698041 -814.13170937
  492.81458798 102.84845219 184.60648906 743.51961675 76.09517222]

>>> # The mean square error
>>> np.mean((regr.predict(diabetes_X_test)-diabetes_y_test)**2)
2004.56760268...

>>> # Explained variance score: 1 is perfect prediction
>>> # and 0 means that there is no linear relationship
>>> # between X and y.
>>> regr.score(diabetes_X_test, diabetes_y_test)
0.5850753022690...
```

Shrinkage

If there are few data points per dimension, noise in the observations induces high variance:
A solution in high-dimensional statistical learning is to *shrink* the regression coefficients to zero: any two randomly chosen set of observations are likely to be uncorrelated. This is called *Ridge* regression:

![Ridge regression diagram]

This is an example of bias/variance tradeoff: the larger the ridge alpha parameter, the higher the bias and the lower the variance.

We can choose alpha to minimize left out error, this time using the diabetes dataset rather than our synthetic data:
Note: Capturing in the fitted parameters noise that prevents the model to generalize to new data is called overfitting. The bias introduced by the ridge regression is called a regularization.

### Sparsity

**Fitting only features 1 and 2**

Note: A representation of the full diabetes dataset would involve 11 dimensions (10 feature dimensions and one of the target variable). It is hard to develop an intuition on such representation, but it may be useful to keep in mind that it would be a fairly empty space.

We can see that, although feature 2 has a strong coefficient on the full model, it conveys little information on $y$ when considered with feature 1.

To improve the conditioning of the problem (i.e. mitigating the *The curse of dimensionality*), it would be interesting to select only the informative features and set non-informative ones, like feature 2 to 0. Ridge regression will decrease their contribution, but not set them to zero. Another penalization approach, called *Lasso* (least absolute shrinkage and selection operator), can set some coefficients to zero. Such methods are called sparse method and sparsity can be seen as an application of Occam’s razor: prefer simpler models.

```python
>>> regr = linear_model.Lasso()
>>> scores = [regr.set_params(alpha=alpha
...       ).fit(diabetes_X_train, diabetes_y_train
...       ).score(diabetes_X_test, diabetes_y_test)
...       for alpha in alphas]
>>> best_alpha = alphas[scores.index(max(scores))]
>>> regr.alpha = best_alpha
```
Different algorithms can be used to solve the same mathematical problem. For instance, the Lasso object in scikit-learn solves the lasso regression problem using a coordinate decent method, that is efficient on large datasets. However, scikit-learn also provides the LassoLars object using the LARS algorithm, which is very efficient for problems in which the weight vector estimated is very sparse (i.e. problems with very few observations).

Classification

For classification, as in the labeling iris task, linear regression is not the right approach as it will give too much weight to data far from the decision frontier. A linear approach is to fit a sigmoid function or logistic function:

\[ y = \frac{1}{1 + \exp(-X\beta + \text{offset})} + \epsilon \]

```python
>>> logistic = linear_model.LogisticRegression(C=1e5)
>>> logistic.fit(iris_X_train, iris_y_train)
LogisticRegression(C=100000.0, class_weight=None, dual=False, 
fit_intercept=True, intercept_scaling=1, max_iter=100, 
multi_class='ovr', n_jobs=1, penalty='l2', random_state=None, 
solver='liblinear', tol=0.0001, verbose=0, warm_start=False)
```
This is known as `LogisticRegression`.

### Multiclass classification

If you have several classes to predict, an option often used is to fit one-versus-all classifiers and then use a voting heuristic for the final decision.

### Shrinkage and sparsity with logistic regression

The `C` parameter controls the amount of regularization in the `LogisticRegression` object: a large value for `C` results in less regularization. `penalty="l2"` gives **Shrinkage** (i.e. non-sparse coefficients), while `penalty="l1"` gives **Sparsity**.

### Exercise

Try classifying the digits dataset with nearest neighbors and a linear model. Leave out the last 10% and test prediction performance on these observations.

```python
from sklearn import datasets, neighbors, linear_model
digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target
```

Solution: `../../auto_examples/exercises/digits_classification_exercise.py`

### Support vector machines (SVMs)

#### Linear SVMs

Support Vector Machines belong to the discriminant model family: they try to find a combination of samples to build a plane maximizing the margin between the two classes. Regularization is set by the `C` parameter: a small value for `C` means the margin is calculated using many or all of the observations around the separating line (more regularization); a large value for `C` means the margin is calculated on observations close to the separating line (less regularization).
Example:

- Plot different SVM classifiers in the iris dataset

SVMs can be used in regression – SVR (Support Vector Regression) –, or in classification – SVC (Support Vector Classification).

```python
>>> from sklearn import svm

>>> svc = svm.SVC(kernel='linear')
>>> svc.fit(iris_X_train, iris_y_train)
```

```
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
    decision_function_shape=None, degree=3, gamma='auto', kernel='linear',
    max_iter=-1, probability=False, random_state=None, shrinking=True,
    tol=0.001, verbose=False)
```

**Warning: Normalizing data**

For many estimators, including the SVMs, having datasets with unit standard deviation for each feature is important to get good prediction.

**Using kernels**

Classes are not always linearly separable in feature space. The solution is to build a decision function that is not linear but may be polynomial instead. This is done using the *kernel trick* that can be seen as creating a decision energy by positioning kernels on observations:

```python
>>> svc = svm.SVC(kernel='linear')

>>> svc = svm.SVC(kernel='poly',
    degree=3)

>>> # degree: polynomial degree
```
>>> svc = svm.SVC(kernel='rbf')
>>> # gamma: inverse of size of
>>> # radial kernel

Interactive example

See the SVM GUI to download svm_gui.py; add data points of both classes with right and left button, fit the model and change parameters and data.

Exercise

Try classifying classes 1 and 2 from the iris dataset with SVMs, with the 2 first features. Leave out 10% of each class and test prediction performance on these observations.

Warning: the classes are ordered, do not leave out the last 10%, you would be testing on only one class.

Hint: You can use the decision_function method on a grid to get intuitions.
2.2.3 Model selection: choosing estimators and their parameters

Score, and cross-validated scores

As we have seen, every estimator exposes a `score` method that can judge the quality of the fit (or the prediction) on new data. Bigger is better.

```python
>>> from sklearn import datasets, svm
>>> digits = datasets.load_digits()
>>> X_digits = digits.data
>>> y_digits = digits.target
>>> svc = svm.SVC(C=1, kernel='linear')
>>> svc.fit(X_digits[:-100], y_digits[:-100]).score(X_digits[-100:], y_digits[-100:])
0.97999999999999998
```

To get a better measure of prediction accuracy (which we can use as a proxy for goodness of fit of the model), we can successively split the data in `folds` that we use for training and testing:

```python
>>> import numpy as np

>>> X_folds = np.array_split(X_digits, 3)
>>> y_folds = np.array_split(y_digits, 3)
>>> scores = list()
>>> for k in range(3):
...     # We use 'list' to copy, in order to 'pop' later on
...     X_train = list(X_folds)
...     X_test = X_train.pop(k)
...     X_train = np.concatenate(X_train)
...     y_train = list(y_folds)
...     y_test = y_train.pop(k)
...     y_train = np.concatenate(y_train)
...     scores.append(svc.fit(X_train, y_train).score(X_test, y_test))
```

```python
>>> print(scores)
[0.93489148580968284, 0.95659432387312182, 0.93989983305509184]
```

This is called a `KFold` cross-validation.

Cross-validation generators

Scikit-learn has a collection of classes which can be used to generate lists of train/test indices for popular cross-validation strategies.

They expose a `split` method which accepts the input dataset to be split and yields the train/test set indices for each iteration of the chosen cross-validation strategy.

This example shows an example usage of the `split` method.
>>> from sklearn.model_selection import KFold, cross_val_score
>>> X = ["a", "a", "b", "c", "c", "c"]
>>> k_fold = KFold(n_splits=3)
>>> for train_indices, test_indices in k_fold.split(X):
...     print('Train: %s | test: %s' % (train_indices, test_indices))
Train: [2 3 4 5] | test: [0 1]
Train: [0 1 4 5] | test: [2 3]
Train: [0 1 2 3] | test: [4 5]

The cross-validation can then be performed easily:

>>> kfold = KFold(n_splits=3)
>>> [svc.fit(X_digits[train], y_digits[train]).score(X_digits[test], y_digits[test])
...     for train, test in k_fold.split(X_digits)]
[0.93489148580968284, 0.95659432387312182, 0.93989983305509184]

The cross-validation score can be directly calculated using the `cross_val_score` helper. Given an estimator, the cross-validation object and the input dataset, the `cross_val_score` splits the data repeatedly into a training and a testing set, trains the estimator using the training set and computes the scores based on the testing set for each iteration of cross-validation.

By default the estimator’s `score` method is used to compute the individual scores.

Refer the `metrics module` to learn more on the available scoring methods.

>>> cross_val_score(svc, X_digits, y_digits, cv=k_fold, n_jobs=-1)
array([ 0.93489149, 0.95659432, 0.93989983])

`n_jobs=-1` means that the computation will be dispatched on all the CPUs of the computer.

Alternatively, the `scoring argument can be provided to specify an alternative scoring method.

>>> cross_val_score(svc, X_digits, y_digits, cv=k_fold, scoring='precision_macro')
array([ 0.93969761, 0.95911415, 0.94041254])

Cross-validation generators

<table>
<thead>
<tr>
<th>KFold (n_splits, shuffle, random_state)</th>
<th>StratifiedKFold (n_iter, test_size, train_size, random_state)</th>
<th>GroupKFold (n_splits, shuffle, random_state)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Splits it into K folds, trains on K-1 and then tests on the left-out.</td>
<td>Same as K-Fold but preserves the class distribution within each fold.</td>
<td>Ensures that the same group is not in both testing and training sets.</td>
</tr>
<tr>
<td>ShuffleSplit (n_iter, test_size, train_size, random_state)</td>
<td>StratifiedShuffleSplit</td>
<td>GroupShuffleSplit</td>
</tr>
<tr>
<td>Generates train/test indices based on random permutation.</td>
<td>Same as shuffle split but preserves the class distribution within each iteration.</td>
<td>Ensures that the same group is not in both testing and training sets.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LeaveOneGroupOut ()</th>
<th>LeavePGroupsOut (p)</th>
<th>LeaveOneOut ()</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takes a group array to group observations.</td>
<td>Leave P groups out.</td>
<td>Leave one observation out.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LeavePOut (p)</th>
<th>PredefinedSplit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leave P observations out.</td>
<td>Generates train/test indices based on predefined splits.</td>
</tr>
</tbody>
</table>
Exercise

On the digits dataset, plot the cross-validation score of a \textit{SVC} estimator with an linear kernel as a function of parameter $C$ (use a logarithmic grid of points, from 1 to 10).

import numpy as np
from sklearn.model_selection import cross_val_score
from sklearn import datasets, svm

digits = datasets.load_digits()
X = digits.data
y = digits.target

svc = svm.SVC(kernel='linear')
C_s = np.logspace(-10, 0, 10)

Solution: Cross-validation on Digits Dataset Exercise

Grid-search and cross-validated estimators

Grid-search

scikit-learn provides an object that, given data, computes the score during the fit of an estimator on a parameter grid and chooses the parameters to maximize the cross-validation score. This object takes an estimator during the construction and exposes an estimator API:

>>> from sklearn.model_selection import GridSearchCV, cross_val_score
>>> Cs = np.logspace(-6, -1, 10)
>>> clf = GridSearchCV(estimator=svc, param_grid=dict(C=Cs),
...                     n_jobs=-1)
>>> clf.fit(X_digits[:1000], y_digits[:1000])
GridSearchCV(cv=None,...
>>> clf.best_score_
0.925...
>>> clf.best_estimator_.C
0.0077...
By default, the `GridSearchCV` uses a 3-fold cross-validation. However, if it detects that a classifier is passed, rather than a regressor, it uses a stratified 3-fold.

## Nested cross-validation

```python
>>> cross_val_score(clf, X_digits, y_digits)...
array([ 0.938...,  0.963...,  0.944...])
```

Two cross-validation loops are performed in parallel: one by the `GridSearchCV` estimator to set `gamma` and the other one by `cross_val_score` to measure the prediction performance of the estimator. The resulting scores are unbiased estimates of the prediction score on new data.

**Warning:** You cannot nest objects with parallel computing (`n_jobs` different than 1).

## Cross-validated estimators

Cross-validation to set a parameter can be done more efficiently on an algorithm-by-algorithm basis. This is why, for certain estimators, scikit-learn exposes `Cross-validation: evaluating estimator performance` estimators that set their parameter automatically by cross-validation:

```python
>>> from sklearn import linear_model, datasets
>>> diabetes = datasets.load_diabetes()
>>> X_diabetes = diabetes.data
>>> y_diabetes = diabetes.target
>>> lasso.fit(X_diabetes, y_diabetes)
LassoCV(alphas=None, copy_X=True, cv=None, eps=0.001, fit_intercept=True,
       max_iter=1000, n_alphas=100, n_jobs=1, normalize=False, positive=False,
       precompute='auto', random_state=None, selection='cyclic', tol=0.0001,
       verbose=False)
>>> # The estimator chose automatically its lambda:
>>> lasso.alpha_
0.01229...
```

These estimators are called similarly to their counterparts, with ‘CV’ appended to their name.

### Exercise

On the diabetes dataset, find the optimal regularization parameter alpha.

**Bonus:** How much can you trust the selection of alpha?
Solution: Cross-validation on diabetes Dataset Exercise

2.2.4 Unsupervised learning: seeking representations of the data

Clustering: grouping observations together

The problem solved in clustering

Given the iris dataset, if we knew that there were 3 types of iris, but did not have access to a taxonomist to label them: we could try a clustering task: split the observations into well-separated group called clusters.

K-means clustering

Note that there exist a lot of different clustering criteria and associated algorithms. The simplest clustering algorithm is K-means.

```python
>>> from sklearn import cluster, datasets
>>> iris = datasets.load_iris()
>>> X_iris = iris.data
>>> y_iris = iris.target

>>> k_means = cluster.KMeans(n_clusters=3)
>>> k_means.fit(X_iris)
KMeans(algorithm='auto', copy_x=True, init='k-means++', ...

>>> print(k_means.labels_[::10])
[1 1 1 1 0 0 0 0 2 2 2 2 2 2 2]

>>> print(y_iris[::10])
[0 0 0 0 1 1 1 1 2 2 2 2 2 2 2]
```

Warning: There is absolutely no guarantee of recovering a ground truth. First, choosing the right number of clusters is hard. Second, the algorithm is sensitive to initialization, and can fall into local minima, although scikit-learn employs several tricks to mitigate this issue.
Don’t over-interpret clustering results

Application example: vector quantization

Clustering in general and KMeans, in particular, can be seen as a way of choosing a small number of exemplars to compress the information. The problem is sometimes known as vector quantization. For instance, this can be used to posterize an image:

```python
>>> import scipy as sp
>>> try
...    face = sp.face(gray=True)
... except AttributeError:
...    from scipy import misc
...    face = misc.face(gray=True)
>>> X = face.reshape((-1, 1))  # We need an (n_sample, n_feature) array
>>> k_means = cluster.KMeans(n_clusters=5, n_init=1)
>>> k_means.fit(X)
KMeans(algorithm='auto', copy_x=True, init='k-means++', ...)
>>> values = k_means.cluster_centers_.squeeze()
>>> labels = k_means.labels_
>>> face_compressed = np.choose(labels, values)
>>> face_compressed.shape = face.shape
```

Hierarchical agglomerative clustering: Ward

A Hierarchical clustering method is a type of cluster analysis that aims to build a hierarchy of clusters. In general, the various approaches of this technique are either:

- **Agglomerative** - bottom-up approaches: each observation starts in its own cluster, and clusters are iteratively merged in such a way to minimize a linkage criterion. This approach is particularly interesting when the clusters of interest are made of only a few observations. When the number of clusters is large, it is much more computationally efficient than k-means.

- **Divisive** - top-down approaches: all observations start in one cluster, which is iteratively split as one moves down the hierarchy. For estimating large numbers of clusters, this approach is both slow (due to all observations starting as one cluster, which it splits recursively) and statistically ill-posed.

Connectivity-constrained clustering

With agglomerative clustering, it is possible to specify which samples can be clustered together by giving a connectivity graph. Graphs in the scikit are represented by their adjacency matrix. Often, a sparse matrix is used. This can be useful, for instance, to retrieve connected regions (sometimes also referred to as connected components) when
clustering an image:

```python
import matplotlib.pyplot as plt
from sklearn.feature_extraction.image import grid_to_graph
from sklearn.cluster import AgglomerativeClustering
from sklearn.utils.testing import SkipTest
from sklearn.utils.fixes import sp_version

if sp_version < (0, 12):
    raise SkipTest("Skipping because SciPy version earlier than 0.12.0 and "
    "thus does not include the scipy.misc.face() image.")

# Generate data
try:
    face = sp.face(gray=True)
except AttributeError:
    # Newer versions of scipy have face in misc
    from scipy import misc
    face = misc.face(gray=True)

# Resize it to 10% of the original size to speed up the processing
face = sp.misc.imresize(face, 0.10) / 255.
```

Feature agglomeration

We have seen that sparsity could be used to mitigate the curse of dimensionality, *i.e.* an insufficient amount of observations compared to the number of features. Another approach is to merge together similar features: feature agglomeration. This approach can be implemented by clustering in the feature direction, in other words clustering
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the transposed data.

```python
>>> digits = datasets.load_digits()
>>> images = digits.images
>>> X = np.reshape(images, (len(images), -1))
>>> connectivity = grid_to_graph(*images[0].shape)

>>> agglo = cluster.FeatureAgglomeration(connectivity=connectivity,
...                                        n_clusters=32)
>>> agglo.fit(X)
FeatureAgglomeration(affinity='euclidean', compute_full_tree='auto',...)

>>> X_reduced = agglo.transform(X)

>>> X_approx = agglo.inverse_transform(X_reduced)

transform and inverse_transform methods

Some estimators expose a transform method, for instance to reduce the dimensionality of the dataset.

Decompositions: from a signal to components and loadings

Components and loadings

If X is our multivariate data, then the problem that we are trying to solve is to rewrite it on a different observational basis: we want to learn loadings $L$ and a set of components $C$ such that $X = L C$. Different criteria exist to choose the components

Principal component analysis: PCA

Principal component analysis (PCA) selects the successive components that explain the maximum variance in the signal.
The point cloud spanned by the observations above is very flat in one direction: one of the three univariate features can almost be exactly computed using the other two. PCA finds the directions in which the data is not flat.

When used to transform data, PCA can reduce the dimensionality of the data by projecting on a principal subspace.

```python
>>> # Create a signal with only 2 useful dimensions
>>> x1 = np.random.normal(size=100)
>>> x2 = np.random.normal(size=100)
>>> x3 = x1 + x2
>>> X = np.c_[x1, x2, x3]

>>> from sklearn import decomposition
>>> pca = decomposition.PCA()
>>> pca.fit(X)
<<<
PCA(copy=True, iterated_power='auto', n_components=None, random_state=None, svd_solver='auto', tol=0.0, whiten=False)

>>> print(pca.explained_variance_)
[ 2.18565811e+00 1.19346747e+00 8.43026679e-32]

>>> # As we can see, only the 2 first components are useful
>>> pca.n_components = 2
>>> X_reduced = pca.fit_transform(X)
>>> X_reduced.shape
(100, 2)
```

**Independent Component Analysis: ICA**

*Independent component analysis (ICA)* selects components so that the distribution of their loadings carries a maximum amount of independent information. It is able to recover **non-Gaussian** independent signals:
# Generate sample data

time = np.linspace(0, 10, 2000)

s1 = np.sin(2 * time)  # Signal 1: sinusoidal signal
s2 = np.sign(np.sin(3 * time))  # Signal 2: square signal
S = np.c_[s1, s2]
S += 0.2 * np.random.normal(size=S.shape)  # Add noise
S /= S.std(axis=0)  # Standardize data

# Mix data
A = np.array([[1, 1], [0.5, 2]])  # Mixing matrix
X = np.dot(S, A.T)  # Generate observations

# Compute ICA
ica = decomposition.FastICA()
S_ = ica.fit_transform(X)  # Get the estimated sources
A_ = ica.mixing_.T
np.allclose(X, np.dot(S_, A_) + ica.mean_)  # True
2.2.5 Putting it all together

Pipelining

We have seen that some estimators can transform data and that some estimators can predict variables. We can also create combined estimators:

```python
from sklearn import linear_model, decomposition, datasets
from sklearn.pipeline import Pipeline
from sklearn.model_selection import GridSearchCV

logistic = linear_model.LogisticRegression()

pca = decomposition.PCA()
pipe = Pipeline(steps=[('pca', pca), ('logistic', logistic)])

digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target

#############################################################################
# Plot the PCA spectrum
pca.fit(X_digits)
plt.figure(1, figsize=(4, 3))
plt.clf()
plt.axes([.2, .2, .7, .7])
plt.plot(pca.explained_variance_, linewidth=2)
plt.axis('tight')
plt.xlabel('n_components')
plt.ylabel('explained_variance_')

#############################################################################
# Prediction

n_components = [20, 40, 64]
Cs = np.logspace(-4, 4, 3)

#Parameters of pipelines can be set using '__' separated parameter names:

estimator = GridSearchCV(pipe,
                         dict(pca__n_components=n_components,
                              logistic__C=Cs))

estimator.fit(X_digits, y_digits)

plt.axvline(estimator.best_estimator_.named_steps['pca'].n_components,
            2.2. A tutorial on statistical-learning for scientific data processing 119
Faces recognition with eigenfaces

The dataset used in this example is a preprocessed excerpt of the “Labeled Faces in the Wild”, also known as LFW:


Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Person</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-score</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ariel Sharon</td>
<td>0.67</td>
<td>0.92</td>
<td>0.77</td>
<td>13</td>
</tr>
<tr>
<td>Colin Powell</td>
<td>0.75</td>
<td>0.78</td>
<td>0.76</td>
<td>60</td>
</tr>
<tr>
<td>Donald Rumsfeld</td>
<td>0.78</td>
<td>0.67</td>
<td>0.72</td>
<td>27</td>
</tr>
<tr>
<td>George W Bush</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>146</td>
</tr>
<tr>
<td>Gerhard Schroeder</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
<td>25</td>
</tr>
<tr>
<td>Hugo Chavez</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
<td>15</td>
</tr>
<tr>
<td>Tony Blair</td>
<td>0.81</td>
<td>0.69</td>
<td>0.75</td>
<td>36</td>
</tr>
<tr>
<td>Avg / total</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>322</td>
</tr>
</tbody>
</table>

---

print(__doc__)

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO, format='%(asctime)s %(message)s')
# Download the data, if not already on disk and load it as numpy arrays

```python
lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)
```

# introspect the images arrays to find the shapes (for plotting)
```python
n_samples, h, w = lfw_people.images.shape
```

# for machine learning we use the 2 data directly (as relative pixel positions info is ignored by this model)
```python
X = lfw_people.data
n_features = X.shape[1]
```

# the label to predict is the id of the person
```python
y = lfw_people.target
target_names = lfw_people.target_names	n_classes = target_names.shape[0]
```

```python
print("Total dataset size:")
print("n_samples: %d" % n_samples)
print("n_features: %d" % n_features)
print("n_classes: %d" % n_classes)
```

# Split into a training set and a test set using a stratified k fold

```python
# split into a training and testing set
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.25, random_state=42)
```

# Compute a PCA (eigenfaces) on the face dataset (treated as unlabeled dataset): unsupervised feature extraction / dimensionality reduction
```python
n_components = 150
```

```python
print("Extracting the top %d eigenfaces from %d faces" % (n_components, X_train.shape[0]))
t0 = time()
pca = PCA(n_components=n_components, svd_solver='randomized',
          whiten=True).fit(X_train)
print("done in %0.3fs" % (time() - t0))
```

```python
eigenfaces = pca.components_.reshape((n_components, h, w))
```

```python
print("Projecting the input data on the eigenfaces orthonormal basis")
t0 = time()
X_train_pca = pca.transform(X_train)
X_test_pca = pca.transform(X_test)
print("done in %0.3fs" % (time() - t0))
```

# Train a SVM classification model

```python
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```
print("Fitting the classifier to the training set")
t0 = time()
param_grid = { 'C': [1e3, 5e3, 1e4, 5e4, 1e5],
               'gamma': [0.0001, 0.0005, 0.001, 0.005, 0.01, 0.1], }
clf = GridSearchCV(SVC(kernel='rbf', class_weight='balanced'), param_grid)
clf = clf.fit(X_train_pca, y_train)
print("done in %0.3fs" % (time() - t0))
print("Best estimator found by grid search:")
print(clf.best_estimator_)

# Quantitative evaluation of the model quality on the test set
print("Predicting people's names on the test set")
t0 = time()
y_pred = clf.predict(X_test_pca)
print("done in %0.3fs" % (time() - t0))
print(classification_report(y_test, y_pred, target_names=target_names))
print(confusion_matrix(y_test, y_pred, labels=range(n_classes)))

# Qualitative evaluation of the predictions using matplotlib

def plot_gallery(images, titles, h, w, n_row=3, n_col=4):
    """Helper function to plot a gallery of portraits""
    plt.figure(figsize=(1.8 * n_col, 2.4 * n_row))
    plt.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.35)
    for i in range(n_row * n_col):
        plt.subplot(n_row, n_col, i + 1)
        plt.imshow(images[i].reshape((h, w)), cmap=plt.cm.gray)
        plt.title(titles[i], size=12)
        plt.xticks(())
        plt.yticks(())

    # plot the result of the prediction on a portion of the test set

def title(y_pred, y_test, target_names, i):
    pred_name = target_names[y_pred[i]].rsplit(' ', 1)[-1]
    true_name = target_names[y_test[i]].rsplit(' ', 1)[-1]
    return 'predicted: %s
true: %s' % (pred_name, true_name)

prediction_titles = [title(y_pred, y_test, target_names, i)
                     for i in range(y_pred.shape[0])]

plot_gallery(X_test, prediction_titles, h, w)

# plot the gallery of the most significative eigenfaces

eigenface_titles = ['eigenface %d' % i for i in range(eigenfaces.shape[0])]
plot_gallery(eigenfaces, eigenface_titles, h, w)

plt.show()
Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Name</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-score</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerhard_Schroeder</td>
<td>0.91</td>
<td>0.75</td>
<td>0.82</td>
<td>28</td>
</tr>
<tr>
<td>Donald_Rumsfeld</td>
<td>0.84</td>
<td>0.82</td>
<td>0.83</td>
<td>33</td>
</tr>
<tr>
<td>Tony_Blair</td>
<td>0.65</td>
<td>0.82</td>
<td>0.73</td>
<td>34</td>
</tr>
<tr>
<td>Colin_Powell</td>
<td>0.78</td>
<td>0.88</td>
<td>0.83</td>
<td>58</td>
</tr>
<tr>
<td>George_W_Bush</td>
<td>0.93</td>
<td>0.86</td>
<td>0.90</td>
<td>129</td>
</tr>
<tr>
<td><strong>avg / total</strong></td>
<td><strong>0.86</strong></td>
<td><strong>0.84</strong></td>
<td><strong>0.85</strong></td>
<td><strong>282</strong></td>
</tr>
</tbody>
</table>

**Open problem: Stock Market Structure**

Can we predict the variation in stock prices for Google over a given time frame?

*Learning a graph structure*

### 2.2.6 Finding help

**The project mailing list**

If you encounter a bug with scikit-learn or something that needs clarification in the docstring or the online documentation, please feel free to ask on the Mailing List

**Q&A communities with Machine Learning practitioners**

- **Quora.com** Quora has a topic for Machine Learning related questions that also features some interesting discussions: [https://www.quora.com/topic/Machine-Learning](https://www.quora.com/topic/Machine-Learning)

- **Stack Exchange** The Stack Exchange family of sites hosts multiple subdomains for Machine Learning questions.

  – _‘An excellent free online course for Machine Learning taught by Professor Andrew Ng of Stanford’: [https://www.coursera.org/learn/machine-learning](https://www.coursera.org/learn/machine-learning)_

  – _‘Another excellent free online course that takes a more general approach to Artificial Intelligence’: [https://www.udacity.com/course/intro-to-artificial-intelligence--cs271](https://www.udacity.com/course/intro-to-artificial-intelligence--cs271)_
2.3 Working With Text Data

The goal of this guide is to explore some of the main scikit-learn tools on a single practical task: analysing a collection of text documents (newsgroups posts) on twenty different topics.

In this section we will see how to:

- load the file contents and the categories
- extract feature vectors suitable for machine learning
- train a linear model to perform categorization
- use a grid search strategy to find a good configuration of both the feature extraction components and the classifier

2.3.1 Tutorial setup

To get started with this tutorial, you firstly must have the scikit-learn and all of its required dependencies installed. Please refer to the installation instructions page for more information and for per-system instructions.

The source of this tutorial can be found within your scikit-learn folder:

```text
scikit-learn/doc/tutorial/text_analytics/
```

The tutorial folder, should contain the following folders:

- *.rst files - the source of the tutorial document written with sphinx
- data - folder to put the datasets used during the tutorial
- skeletons - sample incomplete scripts for the exercises
- solutions - solutions of the exercises

You can already copy the skeletons into a new folder somewhere on your hard-drive named sklearn_tut_workspace where you will edit your own files for the exercises while keeping the original skeletons intact:

```bash
% cp -r skeletons work_directory/sklearn_tut_workspace
```

Machine Learning algorithms need data. Go to each $TUTORIAL_HOME/data sub-folder and run the fetch_data.py script from there (after having read them first).

For instance:

```bash
% cd $TUTORIAL_HOME/data/languages
% less fetch_data.py
% python fetch_data.py
```

2.3.2 Loading the 20 newsgroups dataset

The dataset is called “Twenty Newsgroups”. Here is the official description, quoted from the website:

The 20 Newsgroups data set is a collection of approximately 20,000 newsgroup documents, partitioned (nearly) evenly across 20 different newsgroups. To the best of our knowledge, it was originally collected by Ken Lang, probably for his paper “Newsweeder: Learning to filter netnews,” though he does not explicitly mention this collection. The 20 newsgroups collection has become a popular data set for experiments in text applications of machine learning techniques, such as text classification and text clustering.
In the following we will use the built-in dataset loader for 20 newsgroups from scikit-learn. Alternatively, it is possible to download the dataset manually from the web-site and use the `sklearn.datasets.load_files` function by pointing it to the 20news-bydate-train subfolder of the uncompressed archive folder.

In order to get faster execution times for this first example we will work on a partial dataset with only 4 categories out of the 20 available in the dataset:

```python
>>> categories = ['alt.atheism', 'soc.religion.christian',
... 'comp.graphics', 'sci.med']
```

We can now load the list of files matching those categories as follows:

```python
>>> from sklearn.datasets import fetch_20newsgroups
...
>>> twenty_train = fetch_20newsgroups(subset='train',
... categories=categories, shuffle=True, random_state=42)
```

The returned dataset is a scikit-learn “bunch”: a simple holder object with fields that can be both accessed as python `dict` keys or `object` attributes for convenience, for instance the `target_names` holds the list of the requested category names:

```python
>>> twenty_train.target_names
['alt.atheism', 'comp.graphics', 'sci.med', 'soc.religion.christian']
```

The files themselves are loaded in memory in the `data` attribute. For reference the filenames are also available:

```python
>>> len(twenty_train.data)
2257
>>> len(twenty_train.filenames)
2257
```

Let’s print the first lines of the first loaded file:

```python
>>> print("\n ").join(twenty_train.data[0].split("\n")[0:3])
From: sd345@city.ac.uk (Michael Collier)
Subject: Converting images to HP LaserJet III?
Nntp-Posting-Host: hampton
>>> print(twenty_train.target_names[twenty_train.target[0]])
comp.graphics
```

Supervised learning algorithms will require a category label for each document in the training set. In this case the category is the name of the newsgroup which also happens to be the name of the folder holding the individual documents.

For speed and space efficiency reasons scikit-learn loads the target attribute as an array of integers that corresponds to the index of the category name in the `target_names` list. The category integer id of each sample is stored in the `target` attribute:

```python
>>> twenty_train.target[:10]
array([1, 1, 3, 3, 3, 3, 3, 2, 2, 2])
```

It is possible to get back the category names as follows:

```python
>>> for t in twenty_train.target[:10]:
...     print(twenty_train.target_names[t])
...
comp.graphics
comp.graphics
soc.religion.christian
```
You can notice that the samples have been shuffled randomly (with a fixed RNG seed): this is useful if you select only
the first samples to quickly train a model and get a first idea of the results before re-training on the complete dataset
later.

## 2.3.3 Extracting features from text files

In order to perform machine learning on text documents, we first need to turn the text content into numerical feature
vectors.

### Bags of words

The most intuitive way to do so is the bags of words representation:

1. assign a fixed integer id to each word occurring in any document of the training set (for instance by building a
dictionary from words to integer indices).
2. for each document \( #_i \), count the number of occurrences of each word \( w \) and store it in \( X[#_i, #_j] \) as the value of
feature \( #_j \) where \( #_j \) is the index of word \( w \) in the dictionary

The bags of words representation implies that \( n\text{-features} \) is the number of distinct words in the corpus: this
number is typically larger than 100,000.

If \( n\text{-samples} == 10000 \), storing \( X \) as a numpy array of type float32 would require 10000 x 100000 x 4 bytes =
4GB in RAM which is barely manageable on today’s computers.

Fortunately, most values in \( X \) will be zeros since for a given document less than a couple thousands of distinct words
will be used. For this reason we say that bags of words are typically high-dimensional sparse datasets. We can save
a lot of memory by only storing the non-zero parts of the feature vectors in memory.

scipy.sparse matrices are data structures that do exactly this, and scikit-learn has built-in support for these
structures.

### Tokenizing text with scikit-learn

Text preprocessing, tokenizing and filtering of stopwords are included in a high level component that is able to build a
dictionary of features and transform documents to feature vectors:

```python
>>> from sklearn.feature_extraction.text import CountVectorizer
>>> count_vect = CountVectorizer()
>>> X_train_counts = count_vect.fit_transform(twenty_train.data)
>>> X_train_counts.shape
(2257, 35788)
```

CountVectorizer supports counts of N-grams of words or consecutive characters. Once fitted, the vectorizer has
built a dictionary of feature indices:
The index value of a word in the vocabulary is linked to its frequency in the whole training corpus.

**From occurrences to frequencies**

Occurrence count is a good start but there is an issue: longer documents will have higher average count values than shorter documents, even though they might talk about the same topics.

To avoid these potential discrepancies it suffices to divide the number of occurrences of each word in a document by the total number of words in the document: these new features are called \( tf \) for Term Frequencies.

Another refinement on top of \( tf \) is to downscale weights for words that occur in many documents in the corpus and are therefore less informative than those that occur only in a smaller portion of the corpus.

This downscaling is called \( tf\text{-}idf \) for “Term Frequency times Inverse Document Frequency”.

Both \( tf \) and \( tf\text{-}idf \) can be computed as follows:

```python
>>> from sklearn.feature_extraction.text import TfidfTransformer
>>> tf_transformer = TfidfTransformer(use_idf=False).fit(X_train_counts)
>>> X_train_tf = tf_transformer.transform(X_train_counts)
>>> X_train_tf.shape
(2257, 35788)
```

In the above example-code, we firstly use the `fit(..)` method to fit our estimator to the data and secondly the `transform(..)` method to transform our count-matrix to a tf-idf representation. These two steps can be combined to achieve the same end result faster by skipping redundant processing. This is done through using the `fit_transform(..)` method as shown below, and as mentioned in the note in the previous section:

```python
>>> tfidf_transformer = TfidfTransformer()
>>> X_train_tfidf = tfidf_transformer.fit_transform(X_train_counts)
>>> X_train_tfidf.shape
(2257, 35788)
```

### 2.3.4 Training a classifier

Now that we have our features, we can train a classifier to try to predict the category of a post. Let’s start with a naïve Bayes classifier, which provides a nice baseline for this task. scikit-learn includes several variants of this classifier; the one most suitable for word counts is the multinomial variant:

```python
>>> from sklearn.naive_bayes import MultinomialNB
>>> clf = MultinomialNB().fit(X_train_tfidf, twenty_train.target)
```

To try to predict the outcome on a new document we need to extract the features using almost the same feature extracting chain as before. The difference is that we call `transform` instead of `fit_transform` on the transformers, since they have already been fit to the training set:

```python
>>> docs_new = ['God is love', 'OpenGL on the GPU is fast']
>>> X_new_counts = count_vect.transform(docs_new)
>>> X_new_tfidf = tfidf_transformer.transform(X_new_counts)
>>> predicted = clf.predict(X_new_tfidf)
>>> for doc, category in zip(docs_new, predicted):
```

---

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2.3.5 Building a pipeline

In order to make the vectorizer => transformer => classifier easier to work with, scikit-learn provides a `Pipeline` class that behaves like a compound classifier:

```python
>>> from sklearn.pipeline import Pipeline

>>> text_clf = Pipeline([('vect', CountVectorizer()),
                         ('tfidf', TfidfTransformer()),
                         ('clf', MultinomialNB())])
```

The names `vect`, `tfidf` and `clf` (classifier) are arbitrary. We shall see their use in the section on grid search, below. We can now train the model with a single command:

```python
>>> text_clf = text_clf.fit(twenty_train.data, twenty_train.target)
```

2.3.6 Evaluation of the performance on the test set

Evaluating the predictive accuracy of the model is equally easy:

```python
>>> import numpy as np

>>> twenty_test = fetch_20newsgroups(subset='test',
                                      categories=categories, shuffle=True, random_state=42)

>>> docs_test = twenty_test.data

>>> predicted = text_clf.predict(docs_test)

>>> np.mean(predicted == twenty_test.target)
0.834...
```

I.e., we achieved 83.4% accuracy. Let’s see if we can do better with a linear support vector machine (SVM), which is widely regarded as one of the best text classification algorithms (although it’s also a bit slower than naïve Bayes). We can change the learner by just plugging a different classifier object into our pipeline:

```python
>>> from sklearn.linear_model import SGDClassifier

>>> text_clf = Pipeline([('vect', CountVectorizer()),
                         ('tfidf', TfidfTransformer()),
                         ('clf', SGDClassifier(loss='hinge', penalty='l2',
                                              alpha=1e-3, n_iter=5, random_state=42)),
                         ...
                         ...
                         ...
                        ])

>>> _ = text_clf.fit(twenty_train.data, twenty_train.target)

>>> predicted = text_clf.predict(docs_test)

>>> np.mean(predicted == twenty_test.target)
0.912...
```

scikit-learn further provides utilities for more detailed performance analysis of the results:

```python
>>> from sklearn import metrics

>>> print(metrics.classification_report(twenty_test.target, predicted,
                                      target_names=twenty_test.target_names))
```
As expected the confusion matrix shows that posts from the newsgroups on atheism and christian are more often confused for one another than with computer graphics.

### 2.3.7 Parameter tuning using grid search

We’ve already encountered some parameters such as `use_idf` in the `TfidfTransformer`. Classifiers tend to have many parameters as well; e.g., `MultinomialNB` includes a smoothing parameter `alpha` and `SGDClassifier` has a penalty parameter `alpha` and configurable loss and penalty terms in the objective function (see the module documentation, or use the Python `help` function, to get a description of these).

Instead of tweaking the parameters of the various components of the chain, it is possible to run an exhaustive search of the best parameters on a grid of possible values. We try out all classifiers on either words or bigrams, with or without idf, and with a penalty parameter of either 0.01 or 0.001 for the linear SVM:

```python
>>> from sklearn.model_selection import GridSearchCV

>>> parameters = {'vect__ngram_range': [(1, 1), (1, 2)],
                'tfidf__use_idf': (True, False),
                'clf__alpha': (1e-2, 1e-3),
                ...
}
```

Obviously, such an exhaustive search can be expensive. If we have multiple CPU cores at our disposal, we can tell the grid searcher to try these eight parameter combinations in parallel with the `n_jobs` parameter. If we give this parameter a value of -1, grid search will detect how many cores are installed and uses them all:

```python
>>> gs_clf = GridSearchCV(text_clf, parameters, n_jobs=-1)
```

The grid search instance behaves like a normal `scikit-learn` model. Let’s perform the search on a smaller subset of the training data to speed up the computation:

```python
>>> gs_clf = gs_clf.fit(twenty_train.data[:400], twenty_train.target[:400])
```

The result of calling `fit` on a `GridSearchCV` object is a classifier that we can use to predict:

```python
>>> twenty_train.target_names[gs_clf.predict(['God is love'])[0]]
'soc.religion.christian'
```

The object’s `best_score_` and `best_params_` attributes store the best mean score and the parameters setting corresponding to that score.
A more detailed summary of the search is available at `gs_clf.cv_results_`.
The `cv_results_` parameter can be easily imported into pandas as a DataFrame for further inspection.

**Exercises**

To do the exercises, copy the content of the ‘skeletons’ folder as a new folder named ‘workspace’:

```
% cp -r skeletons workspace
```

You can then edit the content of the workspace without fear of losing the original exercise instructions.

Then fire an ipython shell and run the work-in-progress script with:

```
[1] %run workspace/exercise_XX_script.py arg1 arg2 arg3
```

If an exception is triggered, use `%debug` to fire-up a post mortem ipdb session.

Refine the implementation and iterate until the exercise is solved.

For each exercise, the skeleton file provides all the necessary import statements, boilerplate code to load the data and sample code to evaluate the predictive accuracy of the model.

### 2.3.8 Exercise 1: Language identification

- Write a text classification pipeline using a custom preprocessor and CharNGramAnalyzer using data from Wikipedia articles as training set.
- Evaluate the performance on some held out test set.

```
%run workspace/exercise_01_language_train_model.py data/languages/paragraphs/
```

### 2.3.9 Exercise 2: Sentiment Analysis on movie reviews

- Write a text classification pipeline to classify movie reviews as either positive or negative.
- Find a good set of parameters using grid search.
- Evaluate the performance on a held out test set.

```
%run workspace/exercise_02_sentiment.py data/movie_reviews/txt_sentoken/
```
2.3.10 Exercise 3: CLI text classification utility

Using the results of the previous exercises and the cPickle module of the standard library, write a command line utility that detects the language of some text provided on stdin and estimate the polarity (positive or negative) if the text is written in English.

Bonus point if the utility is able to give a confidence level for its predictions.

2.3.11 Where to from here

Here are a few suggestions to help further your scikit-learn intuition upon the completion of this tutorial:

- Try playing around with the analyzer and token normalisation under CountVectorizer
- If you don’t have labels, try using Clustering on your problem.
- If you have multiple labels per document, e.g categories, have a look at the Multiclass and multilabel section
- Try using Truncated SVD for latent semantic analysis.
- Have a look at using Out-of-core Classification to learn from data that would not fit into the computer main memory.
- Have a look at the HashingVectorizer as a memory efficient alternative to CountVectorizer.

2.4 Choosing the right estimator

Often the hardest part of solving a machine learning problem can be finding the right estimator for the job. Different estimators are better suited for different types of data and different problems.

The flowchart below is designed to give users a bit of a rough guide on how to approach problems with regard to which estimators to try on your data.

Click on any estimator in the chart below to see its documentation.

2.5 External Resources, Videos and Talks

For written tutorials, see the Tutorial section of the documentation.

2.5.1 New to Scientific Python?

For those that are still new to the scientific Python ecosystem, we highly recommend the Python Scientific Lecture Notes. This will help you find your footing a bit and will definitely improve your scikit-learn experience. A basic understanding of NumPy arrays is recommended to make the most of scikit-learn.

2.5.2 External Tutorials

There are several online tutorials available which are geared toward specific subject areas:

- Machine Learning for NeuroImaging in Python
- Machine Learning for Astronomical Data Analysis
2.5.3 Videos

- An introduction to scikit-learn Part I and Part II at Scipy 2013 by Gael Varoquaux, Jake Vanderplas and Olivier Grisel. Notebooks on github.

- Introduction to scikit-learn by Gael Varoquaux at ICML 2010
  A three minute video from a very early stage of the scikit, explaining the basic idea and approach we are following.

- Introduction to statistical learning with scikit-learn by Gael Varoquaux at SciPy 2011
  An extensive tutorial, consisting of four sessions of one hour. The tutorial covers the basics of machine learning, many algorithms and how to apply them using scikit-learn. The material corresponding is now in the scikit-learn documentation section A tutorial on statistical-learning for scientific data processing.

- Statistical Learning for Text Classification with scikit-learn and NLTK (and slides) by Olivier Grisel at PyCon 2011
  Thirty minute introduction to text classification. Explains how to use NLTK and scikit-learn to solve real-world text classification tasks and compares against cloud-based solutions.

- Introduction to Interactive Predictive Analytics in Python with scikit-learn by Olivier Grisel at PyCon 2012
  3-hours long introduction to prediction tasks using scikit-learn.

- scikit-learn - Machine Learning in Python by Jake Vanderplas at the 2012 PyData workshop at Google
  Interactive demonstration of some scikit-learn features. 75 minutes.

- scikit-learn tutorial by Jake Vanderplas at PyData NYC 2012
  Presentation using the online tutorial, 45 minutes.

Note: Doctest Mode

The code-examples in the above tutorials are written in a python-console format. If you wish to easily execute these examples in IPython, use:

```python
%doctest_mode
```

in the IPython-console. You can then simply copy and paste the examples directly into IPython without having to worry about removing the >>>> manually.
3.1 Supervised learning

3.1.1 Generalized Linear Models

The following are a set of methods intended for regression in which the target value is expected to be a linear combination of the input variables. In mathematical notion, if \( \hat{y} \) is the predicted value.

\[
\hat{y}(w, x) = w_0 + w_1 x_1 + \ldots + w_p x_p
\]

Across the module, we designate the vector \( w = (w_1, \ldots, w_p) \) as `coef_` and \( w_0 \) as `intercept_`.

To perform classification with generalized linear models, see `Logistic regression`.

**Ordinary Least Squares**

`LinearRegression` fits a linear model with coefficients \( w = (w_1, \ldots, w_p) \) to minimize the residual sum of squares between the observed responses in the dataset, and the responses predicted by the linear approximation. Mathematically it solves a problem of the form:

\[
\min_w ||Xw - y||^2
\]

`LinearRegression` will take in its `fit` method arrays \( X, y \) and will store the coefficients \( w \) of the linear model in its `coef_` member:
However, coefficient estimates for Ordinary Least Squares rely on the independence of the model terms. When terms are correlated and the columns of the design matrix $X$ have an approximate linear dependence, the design matrix becomes close to singular and as a result, the least-squares estimate becomes highly sensitive to random errors in the observed response, producing a large variance. This situation of *multicollinearity* can arise, for example, when data are collected without an experimental design.

**Examples:**

- *Linear Regression Example*

### Ordinary Least Squares Complexity

This method computes the least squares solution using a singular value decomposition of X. If X is a matrix of size $(n, p)$ this method has a cost of $O(np^2)$, assuming that $n \geq p$.

### Ridge Regression

*Ridge* regression addresses some of the problems of *Ordinary Least Squares* by imposing a penalty on the size of coefficients. The ridge coefficients minimize a penalized residual sum of squares,

$$
\min_w \| Xw - y \|^2 + \alpha \| w \|^2
$$

Here, $\alpha \geq 0$ is a complexity parameter that controls the amount of shrinkage: the larger the value of $\alpha$, the greater the amount of shrinkage and thus the coefficients become more robust to collinearity.

As with other linear models, *Ridge* will take in its `fit` method arrays X, y and will store the coefficients $w$ of the linear model in its `coef_` member:
Examples:

- Plot Ridge coefficients as a function of the regularization
- Classification of text documents using sparse features

Ridge Complexity

This method has the same order of complexity than an Ordinary Least Squares.

Setting the regularization parameter: generalized Cross-Validation

RidgeCV implements ridge regression with built-in cross-validation of the alpha parameter. The object works in the same way as GridSearchCV except that it defaults to Generalized Cross-Validation (GCV), an efficient form of leave-one-out cross-validation:

References

- “Notes on Regularized Least Squares”, Rifkin & Lippert (technical report, course slides).

Lasso

The Lasso is a linear model that estimates sparse coefficients. It is useful in some contexts due to its tendency to prefer solutions with fewer parameter values, effectively reducing the number of variables upon which the given solution is dependent. For this reason, the Lasso and its variants are fundamental to the field of compressed sensing. Under certain conditions, it can recover the exact set of non-zero weights (see Compressive sensing: tomography reconstruction with L1 prior (Lasso)).

Mathematically, it consists of a linear model trained with $\ell_1$ prior as regularizer. The objective function to minimize
is:
\[ \min_w \frac{1}{2n_{\text{samples}}} \| Xw - y \|^2_2 + \alpha \| w \|_1 \]

The lasso estimate thus solves the minimization of the least-squares penalty with \( \alpha \| w \|_1 \) added, where \( \alpha \) is a constant and \( \| w \|_1 \) is the \( \ell_1 \)-norm of the parameter vector.

The implementation in the class \texttt{Lasso} uses coordinate descent as the algorithm to fit the coefficients. See \textit{Least Angle Regression} for another implementation:

```python
>>> from sklearn import linear_model
>>> reg = linear_model.Lasso(alpha = 0.1)
>>> reg.fit([[0, 0], [1, 1]], [0, 1])
Lasso(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000,
   normalize=False, positive=False, precompute=False, random_state=None,
   selection='cyclic', tol=0.0001, warm_start=False)
>>> reg.predict([[1, 1]])
array([ 0.8])
```

Also useful for lower-level tasks is the function \texttt{lasso_path} that computes the coefficients along the full path of possible values.

**Examples:**
- \textit{Lasso and Elastic Net for Sparse Signals}
- \textit{Compressive sensing: tomography reconstruction with L1 prior (Lasso)}

**Note:** \textit{Feature selection with Lasso}

As the Lasso regression yields sparse models, it can thus be used to perform feature selection, as detailed in \textit{L1-based feature selection}.

**Note:** \textit{Randomized sparsity}

For feature selection or sparse recovery, it may be interesting to use \textit{Randomized sparse models}.

**Setting regularization parameter**

The \texttt{alpha} parameter controls the degree of sparsity of the coefficients estimated.

**Using cross-validation**

scikit-learn exposes objects that set the Lasso \texttt{alpha} parameter by cross-validation: \texttt{LassoCV} and \texttt{LassoLarsCV}. \texttt{LassoLarsCV} is based on the \textit{Least Angle Regression} algorithm explained below.

For high-dimensional datasets with many collinear regressors, \texttt{LassoCV} is most often preferable. However, \texttt{LassoLarsCV} has the advantage of exploring more relevant values of \texttt{alpha} parameter, and if the number of samples is very small compared to the number of observations, it is often faster than \texttt{LassoCV}.
Information-criteria based model selection

Alternatively, the estimator `LassoLarsIC` proposes to use the Akaike information criterion (AIC) and the Bayes Information criterion (BIC). It is a computationally cheaper alternative to find the optimal value of alpha as the regularization path is computed only once instead of k+1 times when using k-fold cross-validation. However, such criteria needs a proper estimation of the degrees of freedom of the solution, are derived for large samples (asymptotic results) and assume the model is correct, i.e. that the data are actually generated by this model. They also tend to break when the problem is badly conditioned (more features than samples).

Examples:

- **Lasso model selection: Cross-Validation / AIC / BIC**

Multi-task Lasso

The `MultiTaskLasso` is a linear model that estimates sparse coefficients for multiple regression problems jointly: y is a 2D array, of shape `(n_samples, n_tasks)`. The constraint is that the selected features are the same for all the regression problems, also called tasks.
The following figure compares the location of the non-zeros in W obtained with a simple Lasso or a MultiTaskLasso. The Lasso estimates yields scattered non-zeros while the non-zeros of the MultiTaskLasso are full columns.

Fitting a time-series model, imposing that any active feature be active at all times.

Examples:

- Joint feature selection with multi-task Lasso

Mathematically, it consists of a linear model trained with a mixed $\ell_1 \ell_2$ prior as regularizer. The objective function to minimize is:

$$\min_w \frac{1}{2n_{\text{samples}}} \|Xw - y\|_F^2 + \alpha \|w\|_1$$

where $Fro$ indicates the Frobenius norm:

$$\|A\|_Fro = \sqrt{\sum_{ij} a_{ij}^2}$$

and $\ell_1 \ell_2$ reads:

$$\|A\|_21 = \sum_i \sqrt{\sum_j a_{ij}^2}$$

The implementation in the class `MultiTaskLasso` uses coordinate descent as the algorithm to fit the coefficients.

**Elastic Net**

ElasticNet is a linear regression model trained with L1 and L2 prior as regularizer. This combination allows for learning a sparse model where few of the weights are non-zero like Lasso, while still maintaining the regularization properties of Ridge. We control the convex combination of L1 and L2 using the `l1_ratio` parameter.

Elastic-net is useful when there are multiple features which are correlated with one another. Lasso is likely to pick one of these at random, while elastic-net is likely to pick both.

A practical advantage of trading-off between Lasso and Ridge is it allows Elastic-Net to inherit some of Ridge’s stability under rotation.
The objective function to minimize is in this case

\[
\min_w \frac{1}{2n_{samples}} \|Xw - y\|_2^2 + \alpha \rho \|w\|_1 + \alpha (1 - \rho) \frac{1}{2} \|w\|_2^2
\]

The class `ElasticNetCV` can be used to set the parameters `alpha` (\(\alpha\)) and `l1_ratio` (\(\rho\)) by cross-validation.

Examples:

- *Lasso and Elastic Net for Sparse Signals*
- *Lasso and Elastic Net*

**Multi-task Elastic Net**

The `MultiTaskElasticNet` is an elastic-net model that estimates sparse coefficients for multiple regression problems jointly: \(Y\) is a 2D array, of shape (\(n_{samples}\), \(n_{tasks}\)). The constraint is that the selected features are the same for all the regression problems, also called tasks.

Mathematically, it consists of a linear model trained with a mixed \(\ell_1\) \(\ell_2\) prior and \(\ell_2\) prior as regularizer. The objective function to minimize is:

\[
\min_w \frac{1}{2n_{samples}} \|XW - Y\|_{Fro}^2 + \alpha \rho \|W\|_{21} + \alpha (1 - \rho) \frac{1}{2} \|W\|_{Fro}^2
\]

The implementation in the class `MultiTaskElasticNet` uses coordinate descent as the algorithm to fit the coefficients.

The class `MultiTaskElasticNetCV` can be used to set the parameters `alpha` (\(\alpha\)) and `l1_ratio` (\(\rho\)) by cross-validation.

**Least Angle Regression**

Least-angle regression (LARS) is a regression algorithm for high-dimensional data, developed by Bradley Efron, Trevor Hastie, Iain Johnstone and Robert Tibshirani.

The advantages of LARS are:
• It is numerically efficient in contexts where p >> n (i.e., when the number of dimensions is significantly greater than the number of points)
• It is computationally just as fast as forward selection and has the same order of complexity as an ordinary least squares.
• It produces a full piecewise linear solution path, which is useful in cross-validation or similar attempts to tune the model.
• If two variables are almost equally correlated with the response, then their coefficients should increase at approximately the same rate. The algorithm thus behaves as intuition would expect, and also is more stable.
• It is easily modified to produce solutions for other estimators, like the Lasso.

The disadvantages of the LARS method include:
• Because LARS is based upon an iterative refitting of the residuals, it would appear to be especially sensitive to the effects of noise. This problem is discussed in detail by Weisberg in the discussion section of the Efron et al. (2004) Annals of Statistics article.

The LARS model can be used using estimator `Lars`, or its low-level implementation `lars_path`.

**LARS Lasso**

`LassoLars` is a lasso model implemented using the LARS algorithm, and unlike the implementation based on coordinate descent, this yields the exact solution, which is piecewise linear as a function of the norm of its coefficients.

```python
>>> from sklearn import linear_model

>>> reg = linear_model.LassoLars(alpha=.1)

>>> reg.fit([[0, 0], [1, 1]], [0, 1])

LassoLars(alpha=0.1, copy_X=True, eps=..., fit_intercept=True,
    fit_path=True, max_iter=500, normalize=True, positive=False,
    precompute='auto', verbose=False)

>>> reg.coef_
    array([ 0.717157..., 0. ])```

**Examples:**
**Lasso path using LARS**

The Lars algorithm provides the full path of the coefficients along the regularization parameter almost for free, thus a common operation consist of retrieving the path with function `lars_path`

**Mathematical formulation**

The algorithm is similar to forward stepwise regression, but instead of including variables at each step, the estimated parameters are increased in a direction equiangular to each one’s correlations with the residual.

Instead of giving a vector result, the LARS solution consists of a curve denoting the solution for each value of the L1 norm of the parameter vector. The full coefficients path is stored in the array `coef_path_`, which has size (n_features, max_features+1). The first column is always zero.

**References:**

- Original Algorithm is detailed in the paper *Least Angle Regression* by Hastie et al.

**Orthogonal Matching Pursuit (OMP)**

`OrthogonalMatchingPursuit` and `orthogonal_mp` implements the OMP algorithm for approximating the fit of a linear model with constraints imposed on the number of non-zero coefficients (ie. the L0 pseudo-norm).

Being a forward feature selection method like *Least Angle Regression*, orthogonal matching pursuit can approximate the optimum solution vector with a fixed number of non-zero elements:

$$\arg\min \|y - X\gamma\|_2^2 \text{ subject to } \|\gamma\|_0 \leq n_{\text{nonzero coefs}}$$

Alternatively, orthogonal matching pursuit can target a specific error instead of a specific number of non-zero coefficients. This can be expressed as:

$$\arg\min \|\gamma\|_0 \text{ subject to } \|y - X\gamma\|_2^2 \leq \text{tol}$$

OMP is based on a greedy algorithm that includes at each step the atom most highly correlated with the current residual. It is similar to the simpler matching pursuit (MP) method, but better in that at each iteration, the residual is recomputed using an orthogonal projection on the space of the previously chosen dictionary elements.

**Examples:**

- *Orthogonal Matching Pursuit*

**References:**

- Matching pursuits with time-frequency dictionaries, S. G. Mallat, Z. Zhang,
Bayesian Regression

Bayesian regression techniques can be used to include regularization parameters in the estimation procedure: the regularization parameter is not set in a hard sense but tuned to the data at hand.

This can be done by introducing uninformative priors over the hyper parameters of the model. The $\ell_2$ regularization used in Ridge Regression is equivalent to finding a maximum a-posteriori solution under a Gaussian prior over the parameters $w$ with precision $\lambda^{-1}$. Instead of setting lambda manually, it is possible to treat it as a random variable to be estimated from the data.

To obtain a fully probabilistic model, the output $y$ is assumed to be Gaussian distributed around $Xw$:

$$p(y|X, w, \alpha) = \mathcal{N}(y|Xw, \alpha)$$

Alpha is again treated as a random variable that is to be estimated from the data.

The advantages of Bayesian Regression are:

- It adapts to the data at hand.
- It can be used to include regularization parameters in the estimation procedure.

The disadvantages of Bayesian regression include:

- Inference of the model can be time consuming.

References

- A good introduction to Bayesian methods is given in C. Bishop: Pattern Recognition and Machine Learning
- Original Algorithm is detailed in the book Bayesian learning for neural networks by Radford M. Neal

Bayesian Ridge Regression

BayesianRidge estimates a probabilistic model of the regression problem as described above. The prior for the parameter $w$ is given by a spherical Gaussian:

$$p(w|\lambda) = \mathcal{N}(w|0, \lambda^{-1}I_p)$$

The priors over $\alpha$ and $\lambda$ are chosen to be gamma distributions, the conjugate prior for the precision of the Gaussian.

The resulting model is called Bayesian Ridge Regression, and is similar to the classical Ridge. The parameters $w$, $\alpha$ and $\lambda$ are estimated jointly during the fit of the model. The remaining hyperparameters are the parameters of the gamma priors over $\alpha$ and $\lambda$. These are usually chosen to be non-informative. The parameters are estimated by maximizing the marginal log likelihood.

By default $\alpha_1 = \alpha_2 = \lambda_1 = \lambda_2 = 1.0e^{-6}$.

Bayesian Ridge Regression is used for regression:

```python
>>> from sklearn import linear_model
>>> X = [[0., 0.], [1., 1.], [2., 2.], [3., 3.]]
>>> Y = [0., 1., 2., 3.]
>>> reg = linear_model.BayesianRidge()
>>> reg.fit(X, Y)
BayesianRidge(alpha_1=1e-06, alpha_2=1e-06, compute_score=False, copy_X=True, fit_intercept=True, lambda_1=1e-06, lambda_2=1e-06, n_iter=300, normalize=False, tol=0.001, verbose=False)
```
After being fitted, the model can then be used to predict new values:

```python
>>> reg.predict ([[1, 0.]])
array([ 0.50000013])
```

The weights $w$ of the model can be access:

```python
>>> reg.coef_
array([ 0.49999993, 0.49999993])
```

Due to the Bayesian framework, the weights found are slightly different to the ones found by Ordinary Least Squares. However, Bayesian Ridge Regression is more robust to ill-posed problem.

**Examples:**

- *Bayesian Ridge Regression*

**References**

- More details can be found in the article *Bayesian Interpolation* by MacKay, David J. C.

**Automatic Relevance Determination - ARD**

ARDRegression is very similar to Bayesian Ridge Regression, but can lead to sparser weights $w$.

ARDRegression poses a different prior over $w$, by dropping the assumption of the Gaussian being spherical.

Instead, the distribution over $w$ is assumed to be an axis-parallel, elliptical Gaussian distribution.

This means each weight $w_i$ is drawn from a Gaussian distribution, centered on zero and with a precision $\lambda_i$:

$$p(w|\lambda) = \mathcal{N}(w|0, A^{-1})$$

with $\text{diag}(A) = \lambda = \{\lambda_1, ..., \lambda_p\}$.

---

1 Christopher M. Bishop: Pattern Recognition and Machine Learning, Chapter 7.2.1
2 David Wipf and Srikantan Nagarajan: A new view of automatic relevance determination
In contrast to *Bayesian Ridge Regression*, each coordinate of $w_i$ has its own standard deviation $\lambda_i$. The prior over all $\lambda_i$ is chosen to be the same gamma distribution given by hyperparameters $\lambda_1$ and $\lambda_2$.

ARD is also known in the literature as *Sparse Bayesian Learning* and *Relevance Vector Machine* ³ ⁴.

**Examples:**

- *Automatic Relevance Determination Regression (ARD)*

**References:**

**Logistic regression**

Logistic regression, despite its name, is a linear model for classification rather than regression. Logistic regression is also known in the literature as logit regression, maximum-entropy classification (MaxEnt) or the log-linear classifier. In this model, the probabilities describing the possible outcomes of a single trial are modeled using a logistic function.

The implementation of logistic regression in scikit-learn can be accessed from class *LogisticRegression*. This implementation can fit binary, One-vs- Rest, or multinomial logistic regression with optional L2 or L1 regularization.

As an optimization problem, binary class L2 penalized logistic regression minimizes the following cost function:

$$
\min_{w,c} \frac{1}{2} w^T w + C \sum_{i=1}^{n} \log(\exp(-y_i(X_i^T w + c)) + 1).
$$

Similarly, L1 regularized logistic regression solves the following optimization problem

$$
\min_{w,c} \|w\|_1 + C \sum_{i=1}^{n} \log(\exp(-y_i(X_i^T w + c)) + 1).
$$

The solvers implemented in the class *LogisticRegression* are “liblinear”, “newton-cg”, “lbfgs” and “sag”:

³ Michael E. Tipping: Sparse Bayesian Learning and the Relevance Vector Machine

⁴ Tristan Fletcher: Relevance Vector Machines explained
The solver “liblinear” uses a coordinate descent (CD) algorithm, and relies on the excellent C++ LIBLINEAR library, which is shipped with scikit-learn. However, the CD algorithm implemented in liblinear cannot learn a true multinomial (multiclass) model; instead, the optimization problem is decomposed in a “one-vs-rest” fashion so separate binary classifiers are trained for all classes. This happens under the hood, so LogisticRegression instances using this solver behave as multiclass classifiers. For L1 penalization sklearn.svm.l1_min_c allows to calculate the lower bound for C in order to get a non “null” (all feature weights to zero) model.

The “lbfgs”, “sag” and “newton-cg” solvers only support L2 penalization and are found to converge faster for some high dimensional data. Setting multi_class to “multinomial” with these solvers learns a true multinomial logistic regression model 5, which means that its probability estimates should be better calibrated than the default “one-vs-rest” setting. The “lbfgs”, “sag” and “newton-cg” solvers cannot optimize L1-penalized models, therefore the “multinomial” setting does not learn sparse models.

The solver “sag” uses a Stochastic Average Gradient descent 6. It is faster than other solvers for large datasets, when both the number of samples and the number of features are large.

In a nutshell, one may choose the solver with the following rules:

<table>
<thead>
<tr>
<th>Case</th>
<th>Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small dataset or L1 penalty</td>
<td>“liblinear”</td>
</tr>
<tr>
<td>Multinomial loss or large dataset</td>
<td>“lbfgs”, “sag” or “newton-cg”</td>
</tr>
<tr>
<td>Very Large dataset</td>
<td>“sag”</td>
</tr>
</tbody>
</table>

For large dataset, you may also consider using SGDClassifier with ‘log’ loss.

Examples:

- L1 Penalty and Sparsity in Logistic Regression
- Path with L1 Logistic Regression
- Plot multinomial and One-vs-Rest Logistic Regression

Differences from liblinear:

There might be a difference in the scores obtained between LogisticRegression with solver=liblinear or LinearSVC and the external liblinear library directly, when fit_intercept=False and the fit coef_ (or) the data to be predicted are zeroes. This is because for the sample(s) with decision_function zero, LogisticRegression and LinearSVC predict the negative class, while liblinear predicts the positive class. Note that a model with fit_intercept=False and having many samples with decision_function zero, is likely to be a underfit, bad model and you are advised to set fit_intercept=True and increase the intercept_scaling.

Note: Feature selection with sparse logistic regression

A logistic regression with L1 penalty yields sparse models, and can thus be used to perform feature selection, as detailed in L1-based feature selection.

LogisticRegressionCV implements Logistic Regression with built-in cross-validation to find out the optimal C parameter. “newton-cg”, “sag” and “lbfgs” solvers are found to be faster for high-dimensional dense data, due to warm-starting. For the multiclass case, if multi_class option is set to “ovr”, an optimal C is obtained for each class and if the multi_class option is set to “multinomial”, an optimal C is obtained by minimizing the cross- entropy loss.

---

5 Christopher M. Bishop: Pattern Recognition and Machine Learning. Chapter 4.3.4
6 Mark Schmidt, Nicolas Le Roux, and Francis Bach: Minimizing Finite Sums with the Stochastic Average Gradient.
Stochastic Gradient Descent - SGD

Stochastic gradient descent is a simple yet very efficient approach to fit linear models. It is particularly useful when the number of samples (and the number of features) is very large. The `partial_fit` method allows only/out-of-core learning.

The classes `SGDClassifier` and `SGDRegressor` provide functionality to fit linear models for classification and regression using different (convex) loss functions and different penalties. E.g., with `loss="log"`, `SGDClassifier` fits a logistic regression model, while with `loss="hinge"` it fits a linear support vector machine (SVM).

References

- Stochastic Gradient Descent

Perceptron

The Perceptron is another simple algorithm suitable for large scale learning. By default:

- It does not require a learning rate.
- It is not regularized (penalized).
- It updates its model only on mistakes.

The last characteristic implies that the Perceptron is slightly faster to train than SGD with the hinge loss and that the resulting models are sparser.

Passive Aggressive Algorithms

The passive-aggressive algorithms are a family of algorithms for large-scale learning. They are similar to the Perceptron in that they do not require a learning rate. However, contrary to the Perceptron, they include a regularization parameter $C$.

For classification, `PassiveAggressiveClassifier` can be used with `loss='hinge'` (PA-I) or `loss='squared_hinge'` (PA-II). For regression, `PassiveAggressiveRegressor` can be used with `loss='epsilon_insensitive'` (PA-I) or `loss='squared_epsilon_insensitive'` (PA-II).

References:


Robustness regression: outliers and modeling errors

Robust regression is interested in fitting a regression model in the presence of corrupt data: either outliers, or error in the model.
Different scenario and useful concepts

There are different things to keep in mind when dealing with data corrupted by outliers:

- **Outliers in X or in y?**

<table>
<thead>
<tr>
<th>Outliers in the y direction</th>
<th>Outliers in the X direction</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Corrupt y, Small Deviants" /></td>
<td><img src="image" alt="Corrupt X, Small Deviants" /></td>
</tr>
</tbody>
</table>

- **Fraction of outliers versus amplitude of error**

  The number of outlying points matters, but also how much they are outliers.

<table>
<thead>
<tr>
<th>Small outliers</th>
<th>Large outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Corrupt y, Small Deviants" /></td>
<td><img src="image" alt="Corrupt y, Large Deviants" /></td>
</tr>
</tbody>
</table>

An important notion of robust fitting is that of breakdown point: the fraction of data that can be outlying for the fit to start missing the inlying data.

Note that in general, robust fitting in high-dimensional setting (large $n_{features}$) is very hard. The robust models here will probably not work in these settings.

**Trade-offs: which estimator?**
Scikit-learn provides 3 robust regression estimators: RANSAC, Theil Sen and HuberRegressor

- **HuberRegressor** should be faster than RANSAC and Theil Sen unless the number of samples are very large, i.e \( n_{\text{samples}} \gg n_{\text{features}} \). This is because RANSAC and Theil Sen fit on smaller subsets of the data. However, both Theil Sen and RANSAC are unlikely to be as robust as HuberRegressor for the default parameters.

- **RANSAC** is faster than Theil Sen and scales much better with the number of samples

- **RANSAC** will deal better with large outliers in the y direction (most common situation)

- **Theil Sen** will cope better with medium-size outliers in the X direction, but this property will disappear in large dimensional settings.

When in doubt, use **RANSAC**

**RANSAC: RANdom SAmple Consensus**

RANSAC (RANdom SAmple Consensus) fits a model from random subsets of inliers from the complete data set.

RANSAC is a non-deterministic algorithm producing only a reasonable result with a certain probability, which is dependent on the number of iterations (see `max_trials` parameter). It is typically used for linear and non-linear regression problems and is especially popular in the fields of photogrammetric computer vision.

The algorithm splits the complete input sample data into a set of inliers, which may be subject to noise, and outliers, which are e.g. caused by erroneous measurements or invalid hypotheses about the data. The resulting model is then estimated only from the determined inliers.

![RANSAC Example](image)

**Details of the algorithm**

Each iteration performs the following steps:

1. Select `min_samples` random samples from the original data and check whether the set of data is valid (see `is_data_valid`).

2. Fit a model to the random subset (`base_estimator.fit`) and check whether the estimated model is valid (see `is_model_valid`).
3. Classify all data as inliers or outliers by calculating the residuals to the estimated model
   \((\text{base_estimator.predict}(X) - y)\) - all data samples with absolute residuals smaller than the
   residual_threshold are considered as inliers.

4. Save fitted model as best model if number of inlier samples is maximal. In case the current estimated model has
   the same number of inliers, it is only considered as the best model if it has better score.

These steps are performed either a maximum number of times (max_trials) or until one of the special stop criteria
are met (see stop_n_inliers and stop_score). The final model is estimated using all inlier samples (consensus
set) of the previously determined best model.

The is_data_valid and is_model_valid functions allow to identify and reject degenerate combinations of
random sub-samples. If the estimated model is not needed for identifying degenerate cases, is_data_valid should
be used as it is called prior to fitting the model and thus leading to better computational performance.

**Examples:**

- Robust linear model estimation using RANSAC
- Robust linear estimator fitting

**References:**

- “Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Auto-

**Theil-Sen estimator: generalized-median-based estimator**

The `TheilSenRegressor` estimator uses a generalization of the median in multiple dimensions. It is thus robust
to multivariate outliers. Note however that the robustness of the estimator decreases quickly with the dimensionality of
the problem. It looses its robustness properties and becomes no better than an ordinary least squares in high dimension.

**Examples:**

- Theil-Sen Regression
- Robust linear estimator fitting

**References:**

- https://en.wikipedia.org/wiki/Theil%E2%80%93Sen_estimator

**Theoretical considerations**

`TheilSenRegressor` is comparable to the Ordinary Least Squares (OLS) in terms of asymptotic efficiency and as
an unbiased estimator. In contrast to OLS, Theil-Sen is a non-parametric method which means it makes no assumption.
about the underlying distribution of the data. Since Theil-Sen is a median-based estimator, it is more robust against corrupted data aka outliers. In univariate setting, Theil-Sen has a breakdown point of about 29.3% in case of a simple linear regression which means that it can tolerate arbitrary corrupted data of up to 29.3%.

The implementation of `TheilSenRegressor` in scikit-learn follows a generalization to a multivariate linear regression model using the spatial median which is a generalization of the median to multiple dimensions.

In terms of time and space complexity, Theil-Sen scales according to \(\frac{n_{samples}}{n_{subsamples}}\) which makes it infeasible to be applied exhaustively to problems with a large number of samples and features. Therefore, the magnitude of a subpopulation can be chosen to limit the time and space complexity by considering only a random subset of all possible combinations.

Examples:

- **Theil-Sen Regression**

References:

**Huber Regression**

The `HuberRegressor` is different to `Ridge` because it applies a linear loss to samples that are classified as outliers. A sample is classified as an inlier if the absolute error of that sample is lesser than a certain threshold. It differs from `TheilSenRegressor` and `RANSACRegressor` because it does not ignore the effect of the outliers but gives a lesser weight to them.

The loss function that `HuberRegressor` minimizes is given by

\[
\min_{w, \sigma} \sum_{i=1}^{n} \left( \sigma + H_m \left( \frac{X_iw - y_i}{\sigma} \right) \right) + \alpha |w|_2^2
\]

where

\[ H_m(z) = \begin{cases} 
  z^2, & \text{if } |z| < \epsilon, \\
  2\epsilon|z| - \epsilon^2, & \text{otherwise}
\end{cases} \]

It is advised to set the parameter \( \epsilon \) to 1.35 to achieve 95% statistical efficiency.

Notes

The \texttt{HuberRegressor} differs from using \texttt{SGDRegressor} with loss set to \texttt{huber} in the following ways.

- \texttt{HuberRegressor} is scaling invariant. Once \( \epsilon \) is set, scaling \( X \) and \( y \) down or up by different values would produce the same robustness to outliers as before. as compared to \texttt{SGDRegressor} where \( \epsilon \) has to be set again when \( X \) and \( y \) are scaled.

- \texttt{HuberRegressor} should be more efficient to use on data with small number of samples while \texttt{SGDRegressor} needs a number of passes on the training data to produce the same robustness.

Examples:

- \texttt{HuberRegressor vs Ridge on dataset with strong outliers}

References:

Also, this estimator is different from the R implementation of Robust Regression (http://www.ats.ucla.edu/stat/r/dae/rreg.htm) because the R implementation does a weighted least squares implementation with weights given to each sample on the basis of how much the residual is greater than a certain threshold.

Polynomial regression: extending linear models with basis functions

One common pattern within machine learning is to use linear models trained on nonlinear functions of the data. This approach maintains the generally fast performance of linear methods, while allowing them to fit a much wider range of data.

3.1. Supervised learning
For example, a simple linear regression can be extended by constructing **polynomial features** from the coefficients. In the standard linear regression case, you might have a model that looks like this for two-dimensional data:

\[
\hat{y}(w, x) = w_0 + w_1 x_1 + w_2 x_2
\]

If we want to fit a paraboloid to the data instead of a plane, we can combine the features in second-order polynomials, so that the model looks like this:

\[
\hat{y}(w, x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1 x_2 + w_4 x_1^2 + w_5 x_2^2
\]

The (sometimes surprising) observation is that this is **still a linear model**: to see this, imagine creating a new variable

\[
z = [x_1, x_2, x_1 x_2, x_1^2, x_2^2]
\]

With this re-labeling of the data, our problem can be written

\[
\hat{y}(w, x) = w_0 + w_1 z_1 + w_2 z_2 + w_3 z_3 + w_4 z_4 + w_5 z_5
\]

We see that the resulting **polynomial regression** is in the same class of linear models we’d considered above (i.e. the model is linear in \(w\)) and can be solved by the same techniques. By considering linear fits within a higher-dimensional space built with these basis functions, the model has the flexibility to fit a much broader range of data.

Here is an example of applying this idea to one-dimensional data, using polynomial features of varying degrees:

![Example of polynomial regression](image)

This figure is created using the **PolynomialFeatures** preprocessor. This preprocessor transforms an input data matrix into a new data matrix of a given degree. It can be used as follows:

```python
>>> from sklearn.preprocessing import PolynomialFeatures
>>> import numpy as np

>>> X = np.arange(6).reshape(3, 2)

>>> X
array([[0, 1],
       [2, 3],
       [4, 5]])

>>> poly = PolynomialFeatures(degree=2)

>>> poly.fit_transform(X)
array([[ 1.,  0.,  1.,  0.,  0.,  1.],
       [ 1.,  2.,  3.,  4.,  6.,  9.],
       [ 1.,  4.,  5., 16., 20., 25.]])
```
The features of \( X \) have been transformed from \([x_1, x_2] \) to \([1, x_1, x_2, x_1^2, x_1x_2, x_2^2] \), and can now be used within any linear model.

This sort of preprocessing can be streamlined with the *Pipeline* tools. A single object representing a simple polynomial regression can be created and used as follows:

```python
>>> from sklearn.preprocessing import PolynomialFeatures
>>> from sklearn.linear_model import LinearRegression
>>> from sklearn.pipeline import Pipeline
>>> import numpy as np

>>> model = Pipeline([('poly', PolynomialFeatures(degree=3)),
                    ('linear', LinearRegression(fit_intercept=False))])

>>> # fit to an order-3 polynomial data
>>> x = np.arange(5)
>>> y = 3 - 2 * x + x ** 2 - x ** 3
>>> model = model.fit(x[:, np.newaxis], y)

>>> model.named_steps['linear'].coef_
array([-3., -2., 1., -1.])
```

The linear model trained on polynomial features is able to exactly recover the input polynomial coefficients.

In some cases it’s not necessary to include higher powers of any single feature, but only the so-called *interaction features* that multiply together at most \( d \) distinct features. These can be gotten from *PolynomialFeatures* with the setting `interaction_only=True`.

For example, when dealing with boolean features, \( x_i^n = x_i \) for all \( n \) and is therefore useless; but \( x_ix_j \) represents the conjunction of two booleans. This way, we can solve the XOR problem with a linear classifier:

```python
>>> from sklearn.linear_model import Perceptron
>>> from sklearn.preprocessing import PolynomialFeatures
>>> import numpy as np

>>> X = np.array([[0, 0], [0, 1], [1, 0], [1, 1]])
>>> y = X[:, 0] ^ X[:, 1]

>>> X = PolynomialFeatures(interaction_only=True).fit_transform(X).astype(int)

>>> clf = Perceptron(fit_intercept=False, n_iter=10, shuffle=False).fit(X, y)

And the classifier “predictions” are perfect:

```python
>>> clf.predict(X)
array([0, 1, 1, 0])
>>> clf.score(X, y)
1.0
```

### 3.1.2 Linear and Quadratic Discriminant Analysis

Linear Discriminant Analysis (*discriminant_analysis.LinearDiscriminantAnalysis*) and Quadratic Discriminant Analysis (*discriminant_analysis.QuadraticDiscriminantAnalysis*) are two classic classifiers, with, as their names suggest, a linear and a quadratic decision surface, respectively.

These classifiers are attractive because they have closed-form solutions that can be easily computed, are inherently multiclass, have proven to work well in practice and have no hyperparameters to tune.
The plot shows decision boundaries for Linear Discriminant Analysis and Quadratic Discriminant Analysis. The bottom row demonstrates that Linear Discriminant Analysis can only learn linear boundaries, while Quadratic Discriminant Analysis can learn quadratic boundaries and is therefore more flexible.

**Examples:**

*Linear and Quadratic Discriminant Analysis with confidence ellipsoid:* Comparison of LDA and QDA on synthetic data.

**Dimensionality reduction using Linear Discriminant Analysis**

`discriminant_analysis.LinearDiscriminantAnalysis` can be used to perform supervised dimensionality reduction, by projecting the input data to a linear subspace consisting of the directions which maximize the separation between classes (in a precise sense discussed in the mathematics section below). The dimension of the output is necessarily less than the number of classes, so this is a in general a rather strong dimensionality reduction, and only makes senses in a multiclass setting.

This is implemented in `discriminant_analysis.LinearDiscriminantAnalysis.transform`. The desired dimensionality can be set using the `n_components` constructor parameter. This parameter has no influence on `discriminant_analysis.LinearDiscriminantAnalysis.fit` or `discriminant_analysis.LinearDiscriminantAnalysis.predict`.

**Examples:**

*Comparison of LDA and PCA 2D projection of Iris dataset:* Comparison of LDA and PCA for dimensionality reduction of the Iris dataset
Mathematical formulation of the LDA and QDA classifiers

Both LDA and QDA can be derived from simple probabilistic models which model the class conditional distribution of the data \( P(X|y = k) \) for each class \( k \). Predictions can then be obtained by using Bayes’ rule:

\[
P(y = k|X) = \frac{P(X|y = k)P(y = k)}{P(X)} = \frac{P(X|y = k)P(y = k)}{\sum_l P(X|y = l)P(y = l)}
\]

and we select the class \( k \) which maximizes this conditional probability.

More specifically, for linear and quadratic discriminant analysis, \( P(X|y) \) is modelled as a multivariate Gaussian distribution with density:

\[
p(X|y = k) = \frac{1}{(2\pi)^n|\Sigma_k|^{1/2}} \exp\left(-\frac{1}{2}(X - \mu_k)^\top \Sigma_k^{-1} (X - \mu_k)\right)
\]

To use this model as a classifier, we just need to estimate from the training data the class priors \( P(y = k) \) (by the proportion of instances of class \( k \)), the class means \( \mu_k \) (by the empirical sample class means) and the covariance matrices (either by the empirical sample class covariance matrices, or by a regularized estimator: see the section on shrinkage below).

In the case of LDA, the Gaussians for each class are assumed to share the same covariance matrix: \( \Sigma_k = \Sigma \) for all \( k \). This leads to linear decision surfaces between, as can be seen by comparing the log-probability ratios \( \log[P(y = k|X)/P(y = l|X)] \):

\[
\log \left( \frac{P(y = k|X)}{P(y = l|X)} \right) = 0 \iff (\mu_k - \mu_l)^\top \Sigma^{-1} X = \frac{1}{2}(\mu_k^\top \Sigma^{-1} \mu_k - \mu_l^\top \Sigma^{-1} \mu_l)
\]

In the case of QDA, there are no assumptions on the covariance matrices \( \Sigma_k \) of the Gaussians, leading to quadratic decision surfaces. See \(^3\) for more details.

---

**Note: Relation with Gaussian Naive Bayes**

If in the QDA model one assumes that the covariance matrices are diagonal, then the inputs are assumed to be conditionally independent in each class, and the resulting classifier is equivalent to the Gaussian Naive Bayes classifier `naive_bayes.GaussianNB`.

---

Mathematical formulation of LDA dimensionality reduction

To understand the use of LDA in dimensionality reduction, it is useful to start with a geometric reformulation of the LDA classification rule explained above. We write \( K \) for the total number of target classes. Since in LDA we assume that all classes have the same estimated covariance \( \Sigma \), we can rescale the data so that this covariance is the identity:

\[
X^* = D^{-1/2}U^\top X \text{ with } \Sigma = UDU^\top
\]

Then one can show that to classify a data point after scaling is equivalent to finding the estimated class mean \( \mu_k^* \) which is closest to the data point in the Euclidean distance. But this can be done just as well after projecting on the \( K - 1 \) affine subspace \( H_K \) generated by all the \( \mu_k^* \) for all classes. This shows that, implicit in the LDA classifier, there is a dimensionality reduction by linear projection onto a \( K - 1 \) dimensional space.

We can reduce the dimension even more, to a chosen \( L \), by projecting onto the linear subspace \( H_L \) which maximize the variance of the \( \mu_k^* \) after projection (in effect, we are doing a form of PCA for the transformed class means \( \mu_k^* \)). This \( L \) corresponds to the n_components parameter used in the `discriminant_analysis.LinearDiscriminantAnalysis.transform` method. See \(^3\) for more details.

---

Shrinkage

Shrinkage is a tool to improve estimation of covariance matrices in situations where the number of training samples is small compared to the number of features. In this scenario, the empirical sample covariance is a poor estimator. Shrinkage LDA can be used by setting the shrinkage parameter of the `discriminant_analysis.LinearDiscriminantAnalysis` class to ‘auto’. This automatically determines the optimal shrinkage parameter in an analytic way following the lemma introduced by Ledoit and Wolf 4. Note that currently shrinkage only works when setting the solver parameter to ‘lsqr’ or ‘eigen’.

The shrinkage parameter can also be manually set between 0 and 1. In particular, a value of 0 corresponds to no shrinkage (which means the empirical covariance matrix will be used) and a value of 1 corresponds to complete shrinkage (which means that the diagonal matrix of variances will be used as an estimate for the covariance matrix). Setting this parameter to a value between these two extrema will estimate a shrunk version of the covariance matrix.

Estimation algorithms

The default solver is ‘svd’. It can perform both classification and transform, and it does not rely on the calculation of the covariance matrix. This can be an advantage in situations where the number of features is large. However, the ‘svd’ solver cannot be used with shrinkage.

The ‘lsqr’ solver is an efficient algorithm that only works for classification. It supports shrinkage.

The ‘eigen’ solver is based on the optimization of the between class scatter to within class scatter ratio. It can be used for both classification and transform, and it supports shrinkage. However, the ‘eigen’ solver needs to compute the covariance matrix, so it might not be suitable for situations with a high number of features.

Examples:

Normal and Shrinkage Linear Discriminant Analysis for classification: Comparison of LDA classifiers with and without shrinkage.

References:

3.1.3 Kernel ridge regression

Kernel ridge regression (KRR) [M2012] combines Ridge Regression (linear least squares with l2-norm regularization) with the kernel trick. It thus learns a linear function in the space induced by the respective kernel and the data. For non-linear kernels, this corresponds to a non-linear function in the original space.

The form of the model learned by KernelRidge is identical to support vector regression (SVR). However, different loss functions are used: KRR uses squared error loss while support vector regression uses \( \epsilon \)-insensitive loss, both combined with l2 regularization. In contrast to SVR, fitting KernelRidge can be done in closed-form and is typically faster for medium-sized datasets. On the other hand, the learned model is non-sparse and thus slower than SVR, which learns a sparse model for \( \epsilon > 0 \), at prediction-time.

The following figure compares KernelRidge and SVR on an artificial dataset, which consists of a sinusoidal target function and strong noise added to every fifth datapoint. The learned model of KernelRidge and SVR is plotted, where both complexity/regularization and bandwidth of the RBF kernel have been optimized using grid-search. The learned functions are very similar; however, fitting KernelRidge is approx. seven times faster than fitting SVR (both with grid-search). However, prediction of 100000 target values is more than three times faster with SVR since it has learned a sparse model using only approx. 1/3 of the 100 training datapoints as support vectors.

The next figure compares the time for fitting and prediction of KernelRidge and SVR for different sizes of the training set. Fitting KernelRidge is faster than SVR for medium-sized training sets (less than 1000 samples); however, for larger training sets SVR scales better. With regard to prediction time, SVR is faster than KernelRidge for all sizes of the training set because of the learned sparse solution. Note that the degree of sparsity and thus the prediction time depends on the parameters \( \epsilon \) and \( C \) of the SVR; \( \epsilon = 0 \) would correspond to a dense model.

References:

3.1.4 Support Vector Machines

Support vector machines (SVMs) are a set of supervised learning methods used for classification, regression and outliers detection.

The advantages of support vector machines are:

- Effective in high dimensional spaces.
- Still effective in cases where number of dimensions is greater than the number of samples.
- Uses a subset of training points in the decision function (called support vectors), so it is also memory efficient.
- Versatile: different Kernel functions can be specified for the decision function. Common kernels are provided, but it is also possible to specify custom kernels.

The disadvantages of support vector machines include:

- If the number of features is much greater than the number of samples, the method is likely to give poor performances.
SVR versus Kernel Ridge

- SVR (fit: 0.912s, predict: 0.147s)
- KRR (fit: 0.547s, predict: 0.393s)
- SVR support vectors
- data
SVMs do not directly provide probability estimates, these are calculated using an expensive five-fold cross-validation (see Scores and probabilities, below).

The support vector machines in scikit-learn support both dense (numpy.ndarray and convertible to that by numpy.asarray) and sparse (any scipy.sparse) sample vectors as input. However, to use an SVM to make predictions for sparse data, it must have been fit on such data. For optimal performance, use C-ordered numpy.ndarray (dense) or scipy.sparse.csr_matrix (sparse) with dtype=float64.

Classification

SVC, NuSVC and LinearSVC are classes capable of performing multi-class classification on a dataset.

```
>>> from sklearn import svm

>>> X = [[0, 0], [1, 1]]

>>> y = [0, 1]
```

SVC and NuSVC are similar methods, but accept slightly different sets of parameters and have different mathematical formulations (see section Mathematical formulation). On the other hand, LinearSVC is another implementation of Support Vector Classification for the case of a linear kernel. Note that LinearSVC does not accept keyword kernel, as this is assumed to be linear. It also lacks some of the members of SVC and NuSVC, like support_.

As other classifiers, SVC, NuSVC and LinearSVC take as input two arrays: an array X of size [n_samples, n_features] holding the training samples, and an array y of class labels (strings or integers), size [n_samples]:
**scikit-learn user guide, Release 0.18.2**

```python
>>> clf = svm.SVC()
>>> clf.fit(X, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
    decision_function_shape='ovo', degree=3, gamma='auto', kernel='rbf',
    max_iter=-1, probability=False, random_state=None, shrinking=True,
    tol=0.001, verbose=False)
```

After being fitted, the model can then be used to predict new values:

```python
>>> clf.predict([[2., 2.]])
array([1])
```

SVMs decision function depends on some subset of the training data, called the support vectors. Some properties of these support vectors can be found in members `support_vectors_`, `support_` and `n_support`:

```python
>>> # get support vectors
>>> clf.support_vectors_
array([[ 0., 0.],
       [ 1., 1.]])

>>> # get indices of support vectors
>>> clf.support_
array([0, 1])

>>> # get number of support vectors for each class
>>> clf.n_support_
array([1, 1])
```

### Multi-class classification

*SVC* and *NuSVC* implement the “one-against-one” approach (Knerr et al., 1990) for multi-class classification. If `n_class` is the number of classes, then `n_class * (n_class -1) / 2` classifiers are constructed and each one trains data from two classes. To provide a consistent interface with other classifiers, the `decision_function_shape` option allows to aggregate the results of the “one-against-one” classifiers to a decision function of shape `(n_samples,n_classes)`:

```python
>>> X = [[0], [1], [2], [3]]
>>> Y = [0, 1, 2, 3]
>>> clf = svm.SVC(decision_function_shape='ovo')
>>> clf.fit(X, Y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
    decision_function_shape='ovo', degree=3, gamma='auto', kernel='rbf',
    max_iter=-1, probability=False, random_state=None, shrinking=True,
    tol=0.001, verbose=False)
>>> dec = clf.decision_function([[1]])
>>> dec.shape[1] # 4 classes: 4*3/2 = 6
6
>>> clf.decision_function_shape = "ovr"
>>> dec = clf.decision_function([[1]])
>>> dec.shape[1] # 4 classes
4
```

On the other hand, *LinearSVC* implements “one-vs-the-rest” multi-class strategy, thus training `n_class` models. If there are only two classes, only one model is trained:

```python
>>> lin_clf = svm.LinearSVC()
>>> lin_clf.fit(X, Y)
```
LinearSVC(C=1.0, class_weight=None, dual=True, fit_intercept=True,
intercept_scaling=1, loss='squared_hinge', max_iter=1000,
multi_class='ovr', penalty='l2', random_state=None, tol=0.0001,
verbose=0)

>>> dec = lin_clf.decision_function([[1]])
>>> dec.shape[1]
4

See Mathematical formulation for a complete description of the decision function.

Note that the LinearSVC also implements an alternative multi-class strategy, the so-called multi-class SVM formulated by Crammer and Singer, by using the option multi_class='crammer_singer'. This method is consistent, which is not true for one-vs-rest classification. In practice, one-vs-rest classification is usually preferred, since the results are mostly similar, but the runtime is significantly less.

For “one-vs-rest” LinearSVC the attributes coef_ and intercept_ have the shape [n_class, n_features] and [n_class] respectively. Each row of the coefficients corresponds to one of the n_class many “one-vs-rest” classifiers and similar for the intercepts, in the order of the “one” class.

In the case of “one-vs-one” SVC, the layout of the attributes is a little more involved. In the case of having a linear kernel, The layout of coef_ and intercept_ is similar to the one described for LinearSVC described above, except that the shape of coef_ is [n_class * (n_class -1) / 2, n_features], corresponding to as many binary classifiers. The order for classes 0 to n is “0 vs 1”, “0 vs 2”, ... “0 vs n”, “1 vs 2”, “1 vs 3”, “1 vs n”, ... “n-1 vs n”.

The shape of dual_coef_ is [n_class-1, n_SV] with a somewhat hard to grasp layout. The columns correspond to the support vectors involved in any of the n_class * (n_class -1) / 2 “one-vs-one” classifiers. Each of the support vectors is used in n_class -1 classifiers. The n_class -1 entries in each row correspond to the dual coefficients for these classifiers.

This might be made more clear by an example:

Consider a three class problem with class 0 having three support vectors $v_0^0, v_0^1, v_0^2$ and class 1 and 2 having two support vectors $v_1^0, v_1^1$ and $v_2^0, v_2^1$ respectively. For each support vector $v_i$, there are two dual coefficients. Let’s call the coefficient of support vector $v_i^j$ in the classifier between classes $i$ and $k$ $a_{i,j,k}$. Then dual_coef_ looks like this:

<table>
<thead>
<tr>
<th>$a_{0,1}$</th>
<th>$a_{0,2}$</th>
<th>Coefficients for SVs of class 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{1,0}$</td>
<td>$a_{1,2}$</td>
<td>Coefficients for SVs of class 1</td>
</tr>
<tr>
<td>$a_{2,0}$</td>
<td>$a_{2,1}$</td>
<td>Coefficients for SVs of class 2</td>
</tr>
</tbody>
</table>

Scores and probabilities

The SVC method decision_function gives per-class scores for each sample (or a single score per sample in the binary case). When the constructor option probability is set to True, class membership probability estimates (from the methods predict_proba and predict_log_proba) are enabled. In the binary case, the probabilities are calibrated using Platt scaling: logistic regression on the SVM’s scores, fit by an additional cross-validation on the training data. In the multiclass case, this is extended as per Wu et al. (2004).

Needless to say, the cross-validation involved in Platt scaling is an expensive operation for large datasets. In addition, the probability estimates may be inconsistent with the scores, in the sense that the “argmax” of the scores may not be the argmax of the probabilities. (E.g., in binary classification, a sample may be labeled by predict as belonging
to a class that has probability <½ according to `predict_proba`.) Platt’s method is also known to have theoretical issues. If confidence scores are required, but these do not have to be probabilities, then it is advisable to set `probability=False` and use `decision_function` instead of `predict_proba`.

**References:**


**Unbalanced problems**

In problems where it is desired to give more importance to certain classes or certain individual samples keywords `class_weight` and `sample_weight` can be used.

`SVC` (but not `NuSVC`) implement a keyword `class_weight` in the `fit` method. It’s a dictionary of the form `{class_label : value}`, where value is a floating point number > 0 that sets the parameter C of class `class_label` to `C * value`.

`SVC`, `NuSVC`, `SVR`, `NuSVR` and `OneClassSVM` implement also weights for individual samples in method `fit` through keyword `sample_weight`. Similar to `class_weight`, these set the parameter C for the i-th example to `C * sample_weight[i]`.

**Examples:**

- *Plot different SVM classifiers in the iris dataset*,
- *SVM: Maximum margin separating hyperplane*,

3.1. Supervised learning
The method of Support Vector Classification can be extended to solve regression problems. This method is called Support Vector Regression.

The model produced by support vector classification (as described above) depends only on a subset of the training data, because the cost function for building the model does not care about training points that lie beyond the margin. Analogously, the model produced by Support Vector Regression depends only on a subset of the training data, because the cost function for building the model ignores any training data close to the model prediction.

There are three different implementations of Support Vector Regression: SVR, NuSVR and LinearSVR. LinearSVR provides a faster implementation than SVR but only considers linear kernels, while NuSVR implements a slightly different formulation than SVR and LinearSVR. See Implementation details for further details.

As with classification classes, the fit method will take as argument vectors X, y, only that in this case y is expected to have floating point values instead of integer values:

```python
>>> from sklearn import svm
>>> X = [[0, 0], [2, 2]]
>>> y = [0.5, 2.5]
>>> clf = svm.SVR()
>>> clf.fit(X, y)
SVR(C=1.0, cache_size=200, coef0=0.0, degree=3, epsilon=0.1, gamma='auto',
    kernel='rbf', max_iter=-1, shrinking=True, tol=0.001, verbose=False)
>>> clf.predict([[1, 1]])
array([1.5])
```

Examples:

- Support Vector Regression (SVR) using linear and non-linear kernels
Density estimation, novelty detection

One-class SVM is used for novelty detection, that is, given a set of samples, it will detect the soft boundary of that set so as to classify new points as belonging to that set or not. The class that implements this is called `OneClassSVM`.

In this case, as it is a type of unsupervised learning, the fit method will only take as input an array X, as there are no class labels.

See, section *Novelty and Outlier Detection* for more details on this usage.

![Novelty Detection](image)

#### Examples:

- *One-class SVM with non-linear kernel (RBF)*
- *Species distribution modeling*

#### Complexity

Support Vector Machines are powerful tools, but their compute and storage requirements increase rapidly with the number of training vectors. The core of an SVM is a quadratic programming problem (QP), separating support vectors from the rest of the training data. The QP solver used by this `libsvm`-based implementation scales between $O(n_{features} \times n_{samples}^2)$ and $O(n_{features} \times n_{samples}^3)$ depending on how efficiently the `libsvm` cache is used in practice (dataset dependent). If the data is very sparse $n_{features}$ should be replaced by the average number of non-zero features in a sample vector.

Also note that for the linear case, the algorithm used in `LinearSVC` by the `liblinear` implementation is much more efficient than its `libsvm`-based `SVC` counterpart and can scale almost linearly to millions of samples and/or features.
Tips on Practical Use

- **Avoiding data copy**: For `SVC`, `SVR`, `NuSVC` and `NuSVR`, if the data passed to certain methods is not C-ordered contiguous, and double precision, it will be copied before calling the underlying C implementation. You can check whether a given numpy array is C-contiguous by inspecting its `flags` attribute.

  For `LinearSVC` (and `LogisticRegression`) any input passed as a numpy array will be copied and converted to the liblinear internal sparse data representation (double precision floats and int32 indices of non-zero components). If you want to fit a large-scale linear classifier without copying a dense numpy C-contiguous double precision array as input we suggest to use the `SGDClassifier` class instead. The objective function can be configured to be almost the same as the `LinearSVC` model.

- **Kernel cache size**: For `SVC`, `SVR`, `nuSVC` and `NuSVR`, the size of the kernel cache has a strong impact on run times for larger problems. If you have enough RAM available, it is recommended to set `cache_size` to a higher value than the default of 200(MB), such as 500(MB) or 1000(MB).

- **Setting C**: C is 1 by default and it’s a reasonable default choice. If you have a lot of noisy observations you should decrease it. It corresponds to regularize more the estimation.

- Support Vector Machine algorithms are not scale invariant, so it is highly recommended to scale your data. For variance, scale each attribute on the input vector X to [0,1] or [-1,+1], or standardize it to have mean 0 and variance 1. Note that the same scaling must be applied to the test vector to obtain meaningful results. See section `Preprocessing data` for more details on scaling and normalization.

- Parameter `nu` in `NuSVC/OneClassSVM/NuSVR` approximates the fraction of training errors and support vectors.

- In `SVC`, if data for classification are unbalanced (e.g. many positive and few negative), set `class_weight='balanced'` and/or try different penalty parameters C.

- The underlying `LinearSVC` implementation uses a random number generator to select features when fitting the model. It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a smaller tol parameter.

- Using L1 penalization as provided by `LinearSVC(loss='l2',penalty='l1',dual=False)` yields a sparse solution, i.e. only a subset of feature weights is different from zero and contribute to the decision function. Increasing C yields a more complex model (more feature are selected). The C value that yields a “null” model (all weights equal to zero) can be calculated using `l1_min_c`.

**Kernel functions**

The kernel function can be any of the following:

- linear: $\langle x, x' \rangle$.
- polynomial: $(\gamma \langle x, x' \rangle + r)^d$. $d$ is specified by keyword `degree`, $r$ by `coef0`.
- rbf: $\exp(-\gamma |x - x'|^2)$. $\gamma$ is specified by keyword `gamma`, must be greater than 0.
- sigmoid (tanh($\gamma \langle x, x' \rangle + r$)), where $r$ is specified by `coef0`.

Different kernels are specified by keyword `kernel` at initialization:

```python
>>> linear_svc = svm.SVC(kernel='linear')
>>> linear_svc.kernel
'linear'
>>> rbf_svc = svm.SVC(kernel='rbf')
>>> rbf_svc.kernel
'rbf'
```
Custom Kernels

You can define your own kernels by either giving the kernel as a python function or by precomputing the Gram matrix. Classifiers with custom kernels behave the same way as any other classifiers, except that:

- Field `support_vectors_` is now empty, only indices of support vectors are stored in `support_`
- A reference (and not a copy) of the first argument in the `fit()` method is stored for future reference. If that array changes between the use of `fit()` and `predict()` you will have unexpected results.

Using Python functions as kernels

You can also use your own defined kernels by passing a function to the keyword `kernel` in the constructor.

Your kernel must take as arguments two matrices of shape \((n_{\text{samples}_1}, n_{\text{features}}), (n_{\text{samples}_2}, n_{\text{features}})\) and return a kernel matrix of shape \((n_{\text{samples}_1}, n_{\text{samples}_2})\). The following code defines a linear kernel and creates a classifier instance that will use that kernel:

```python
>>> import numpy as np
>>> from sklearn import svm

>>> def my_kernel(X, Y):
...     return np.dot(X, Y.T)
...

>>> clf = svm.SVC(kernel=my_kernel)
```

Examples:

- *SVM with custom kernel.*

Using the Gram matrix

Set `kernel='precomputed'` and pass the Gram matrix instead of X in the fit method. At the moment, the kernel values between all training vectors and the test vectors must be provided.

```python
>>> import numpy as np
>>> from sklearn import svm

>>> X = np.array([[0, 0], [1, 1]])
>>> y = [0, 1]

>>> clf = svm.SVC(kernel='precomputed')

>>> # linear kernel computation
>>> gram = np.dot(X, X.T)

>>> clf.fit(gram, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
   decision_function_shape=None, degree=3, gamma='auto',
   kernel='precomputed', max_iter=-1, probability=False,
   random_state=None, shrinking=True, tol=0.001, verbose=False)

>>> # predict on training examples
>>> clf.predict(gram)
array([0, 1])
```
Parameters of the RBF Kernel

When training an SVM with the Radial Basis Function (RBF) kernel, two parameters must be considered: $C$ and $\gamma$. The parameter $C$, common to all SVM kernels, trades off misclassification of training examples against simplicity of the decision surface. A low $C$ makes the decision surface smooth, while a high $C$ aims at classifying all training examples correctly. $\gamma$ defines how much influence a single training example has. The larger $\gamma$ is, the closer other examples must be to be affected.

Proper choice of $C$ and $\gamma$ is critical to the SVM’s performance. One is advised to use `sklearn.model_selection.GridSearchCV` with $C$ and $\gamma$ spaced exponentially far apart to choose good values.

**Examples:**

- **RBF SVM parameters**

**Mathematical formulation**

A support vector machine constructs a hyper-plane or set of hyper-planes in a high or infinite dimensional space, which can be used for classification, regression or other tasks. Intuitively, a good separation is achieved by the hyper-plane that has the largest distance to the nearest training data points of any class (so-called functional margin), since in general the larger the margin the lower the generalization error of the classifier.
SVC

Given training vectors $x_i \in \mathbb{R}^p$, $i=1,\ldots, n$, in two classes, and a vector $y \in \{1, -1\}^n$, SVC solves the following primal problem:

$$
\min_{w, b, \zeta} \frac{1}{2} w^T w + C \sum_{i=1}^{n} \zeta_i
$$

subject to $y_i (w^T \phi(x_i) + b) \geq 1 - \zeta_i,$

$\zeta_i \geq 0, i = 1, \ldots, n$

Its dual is

$$
\min_{\alpha} \frac{1}{2} \alpha^T Q \alpha - e^T \alpha
$$

subject to $y^T \alpha = 0$

$0 \leq \alpha_i \leq C, i = 1, \ldots, n$

where $e$ is the vector of all ones, $C > 0$ is the upper bound, $Q$ is an $n$ by $n$ positive semidefinite matrix, $Q_{ij} \equiv y_i y_j K(x_i, x_j)$, where $K(x_i, x_j) = \phi(x_i)^T \phi(x_j)$ is the kernel. Here training vectors are implicitly mapped into a higher (maybe infinite) dimensional space by the function $\phi$.

The decision function is:

$$
\text{sgn}\left(\sum_{i=1}^{n} y_i \alpha_i K(x_i, x) + \rho\right)
$$

Note: While SVM models derived from libsvm and liblinear use $C$ as regularization parameter, most other estimators use $\alpha$. The relation between both is $C = \frac{n_{\text{samples}}}{\alpha}$.

This parameters can be accessed through the members dual_coef_ which holds the product $y_i \alpha_i$, support_vectors_ which holds the support vectors, and intercept_ which holds the independent term $\rho$.

References:


NuSVC

We introduce a new parameter $\nu$ which controls the number of support vectors and training errors. The parameter $\nu \in (0, 1]$ is an upper bound on the fraction of training errors and a lower bound of the fraction of support vectors.

It can be shown that the $\nu$-SVC formulation is a reparametrization of the $C$-SVC and therefore mathematically equivalent.
SVR

Given training vectors \( x_i \in \mathbb{R}^p \), i=1,..., n, and a vector \( y \in \mathbb{R}^n \) \( \varepsilon \)-SVR solves the following primal problem:

\[
\min_{w,b,\zeta,\zeta^*} \frac{1}{2} w^T w + C \sum_{i=1}^{n} (\zeta_i + \zeta_i^*) \\
\text{subject to } y_i - w^T \phi(x_i) - b \leq \varepsilon + \zeta_i, \\
w^T \phi(x_i) + b - y_i \leq \varepsilon + \zeta_i^*, \\
\zeta_i, \zeta_i^* \geq 0, \ i = 1, ..., n
\]

Its dual is

\[
\min_{\alpha,\alpha^*} \frac{1}{2} (\alpha - \alpha^*)^T Q (\alpha - \alpha^*) + \varepsilon e^T (\alpha + \alpha^*) - y^T (\alpha - \alpha^*) \\
\text{subject to } e^T (\alpha - \alpha^*) = 0 \\
0 \leq \alpha_i, \alpha_i^* \leq C, \ i = 1, ..., n
\]

where \( e \) is the vector of all ones, \( C > 0 \) is the upper bound, \( Q \) is an \( n \) by \( n \) positive semidefinite matrix, \( Q_{ij} \equiv K(x_i, x_j) = \phi(x_i)^T \phi(x_j) \) is the kernel. Here training vectors are implicitly mapped into a higher (maybe infinite) dimensional space by the function \( \phi \).

The decision function is:

\[
\sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(x_i, x) + \rho
\]

These parameters can be accessed through the members \texttt{dual_coef\_} which holds the difference \( \alpha_i - \alpha_i^* \), \texttt{support_vectors\_} which holds the support vectors, and \texttt{intercept\_} which holds the independent term \( \rho \)

References:


Implementation details

Internally, we use \texttt{libsvm} and \texttt{liblinear} to handle all computations. These libraries are wrapped using C and Cython.

References:

For a description of the implementation and details of the algorithms used, please refer to

- \texttt{LIBSVM}: A Library for Support Vector Machines.
- \texttt{LIBLINEAR} – A Library for Large Linear Classification.

3.1.5 Stochastic Gradient Descent

\textbf{Stochastic Gradient Descent (SGD)} is a simple yet very efficient approach to discriminative learning of linear classifiers under convex loss functions such as (linear) Support Vector Machines and Logistic Regression. Even though
SGD has been around in the machine learning community for a long time, it has received a considerable amount of attention just recently in the context of large-scale learning.

SGD has been successfully applied to large-scale and sparse machine learning problems often encountered in text classification and natural language processing. Given that the data is sparse, the classifiers in this module easily scale to problems with more than $10^5$ training examples and more than $10^5$ features.

The advantages of Stochastic Gradient Descent are:

- Efficiency.
- Ease of implementation (lots of opportunities for code tuning).

The disadvantages of Stochastic Gradient Descent include:

- SGD requires a number of hyperparameters such as the regularization parameter and the number of iterations.
- SGD is sensitive to feature scaling.

**Classification**

**Warning:** Make sure you permute (shuffle) your training data before fitting the model or use `shuffle=True` to shuffle after each iteration.

The class `SGDClassifier` implements a plain stochastic gradient descent learning routine which supports different loss functions and penalties for classification.

As other classifiers, SGD has to be fitted with two arrays: an array X of size `[n_samples, n_features]` holding the training samples, and an array Y of size `[n_samples]` holding the target values (class labels) for the training samples:
```python
>>> from sklearn.linear_model import SGDClassifier
>>> X = [[0., 0.], [1., 1.]]
>>> y = [0, 1]
>>> clf = SGDClassifier(loss="hinge", penalty="l2")

SGDClassifier(alpha=0.0001, average=False, class_weight=None, epsilon=0.1,
               eta0=0.0, fit_intercept=True, l1_ratio=0.15,
               learning_rate='optimal', loss='hinge', n_iter=5, n_jobs=1,
               penalty='l2', power_t=0.5, random_state=None, shuffle=True,
               verbose=0, warm_start=False)
```

After being fitted, the model can then be used to predict new values:

```python
>>> clf.predict([[2., 2.]])
array([1])
```

SGD fits a linear model to the training data. The member `coef_` holds the model parameters:

```python
>>> clf.coef_
array([[ 9.9..., 9.9...]])
```

Member `intercept_` holds the intercept (aka offset or bias):

```python
>>> clf.intercept_
array([-9.9...])
```

Whether or not the model should use an intercept, i.e. a biased hyperplane, is controlled by the parameter `fit_intercept`.

To get the signed distance to the hyperplane use `SGDClassifier.decision_function`:

```python
>>> clf.decision_function([[2., 2.]])
array([ 29.6...])
```

The concrete loss function can be set via the `loss` parameter. `SGDClassifier` supports the following loss functions:

- `loss="hinge"`: (soft-margin) linear Support Vector Machine,
- `loss="modified_huber"`: smoothed hinge loss,
- `loss="log"`: logistic regression,
- and all regression losses below.

The first two loss functions are lazy, they only update the model parameters if an example violates the margin constraint, which makes training very efficient and may result in sparser models, even when L2 penalty is used.

Using `loss="log"` or `loss="modified_huber"` enables the `predict_proba` method, which gives a vector of probability estimates $P(y|x)$ per sample $x$:

```python
>>> clf = SGDClassifier(loss="log").fit(X, y)
>>> clf.predict_proba([[1., 1.]])
array([[ 0.00..., 0.99...]])
```

The concrete penalty can be set via the `penalty` parameter. SGD supports the following penalties:

- `penalty="l2"`: L2 norm penalty on `coef_`
- `penalty="l1"`: L1 norm penalty on `coef_`
- **penalty="elasticnet"**: Convex combination of L2 and L1; \((1 - \text{l1\_ratio}) \times L2 + \text{l1\_ratio} \times L1\).

The default setting is **penalty="l2"**. The L1 penalty leads to sparse solutions, driving most coefficients to zero. The Elastic Net solves some deficiencies of the L1 penalty in the presence of highly correlated attributes. The parameter **l1\_ratio** controls the convex combination of L1 and L2 penalty.

**SGDClassifier** supports multi-class classification by combining multiple binary classifiers in a “one versus all” (OVA) scheme. For each of the \(K\) classes, a binary classifier is learned that discriminates between that and all other \(K - 1\) classes. At testing time, we compute the confidence score (i.e. the signed distances to the hyperplane) for each classifier and choose the class with the highest confidence. The Figure below illustrates the OVA approach on the iris dataset. The dashed lines represent the three OVA classifiers; the background colors show the decision surface induced by the three classifiers.

In the case of multi-class classification **coef\_** is a two-dimensionally array of shape=[n\_classes, n\_features] and **intercept\_** is a one dimensional array of shape=[n\_classes]. The i-th row of **coef\_** holds the weight vector of the OVA classifier for the i-th class; classes are indexed in ascending order (see attribute **classes\_**). Note that, in principle, since they allow to create a probability model, **loss="log"** and **loss="modified_huber"** are more suitable for one-vs-all classification.

**SGDClassifier** supports both weighted classes and weighted instances via the fit parameters **class\_weight** and **sample\_weight**. See the examples below and the doc string of **SGDClassifier.fit** for further information.

**Examples:**

- **SGD**: Maximum margin separating hyperplane,
- **Plot multi-class SGD on the iris dataset**
- **SGD**: Weighted samples
Comparing various online solvers

SVM: Separating hyperplane for unbalanced classes (See the Note)

SGDClassifier supports averaged SGD (ASGD). Averaging can be enabled by setting `average=True`. ASGD works by averaging the coefficients of the plain SGD over each iteration over a sample. When using ASGD the learning rate can be larger and even constant leading on some datasets to a speed up in training time.

For classification with a logistic loss, another variant of SGD with an averaging strategy is available with Stochastic Average Gradient (SAG) algorithm, available as a solver in LogisticRegression.

Regression

The class SGDRegressor implements a plain stochastic gradient descent learning routine which supports different loss functions and penalties to fit linear regression models. SGDRegressor is well suited for regression problems with a large number of training samples (> 10.000), for other problems we recommend Ridge, Lasso, or ElasticNet.

The concrete loss function can be set via the loss parameter. SGDRegressor supports the following loss functions:

- loss="squared_loss": Ordinary least squares,
- loss="huber": Huber loss for robust regression,
- loss="epsilon_insensitive": linear Support Vector Regression.

The Huber and epsilon-insensitive loss functions can be used for robust regression. The width of the insensitive region has to be specified via the parameter epsilon. This parameter depends on the scale of the target variables.

SGDRegressor supports averaged SGD as SGDClassifier. Averaging can be enabled by setting `average=True`.

For regression with a squared loss and a l2 penalty, another variant of SGD with an averaging strategy is available with Stochastic Average Gradient (SAG) algorithm, available as a solver in Ridge.

Stochastic Gradient Descent for sparse data

Note: The sparse implementation produces slightly different results than the dense implementation due to a shrunk learning rate for the intercept.

There is built-in support for sparse data given in any matrix in a format supported by scipy.sparse. For maximum efficiency, however, use the CSR matrix format as defined in scipy.sparse.csr_matrix.

Examples:

- Classification of text documents using sparse features

Complexity

The major advantage of SGD is its efficiency, which is basically linear in the number of training examples. If X is a matrix of size (n, p) training has a cost of $O(kn\bar{p})$, where k is the number of iterations (epochs) and $\bar{p}$ is the average number of non-zero attributes per sample.
Recent theoretical results, however, show that the runtime to get some desired optimization accuracy does not increase as the training set size increases.

**Tips on Practical Use**

- Stochastic Gradient Descent is sensitive to feature scaling, so it is highly recommended to scale your data. For example, scale each attribute on the input vector $X$ to $[0,1]$ or $[-1,+1]$, or standardize it to have mean 0 and variance 1. Note that the same scaling must be applied to the test vector to obtain meaningful results. This can be easily done using `StandardScaler`:

```python
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler()
scaler.fit(X_train)  # Don't cheat - fit only on training data
X_train = scaler.transform(X_train)
X_test = scaler.transform(X_test)  # apply same transformation to test data
```

If your attributes have an intrinsic scale (e.g. word frequencies or indicator features) scaling is not needed.

- Finding a reasonable regularization term $\alpha$ is best done using `GridSearchCV`, usually in the range $10.0^{-\text{np.arange(1,7)}}$.

- Empirically, we found that SGD converges after observing approx. $10^6$ training samples. Thus, a reasonable first guess for the number of iterations is $n_{\text{iter}} = \text{np.ceil}(10^6 / n)$, where $n$ is the size of the training set.

- If you apply SGD to features extracted using PCA we found that it is often wise to scale the feature values by some constant $c$ such that the average L2 norm of the training data equals one.

- We found that Averaged SGD works best with a larger number of features and a higher $\text{eta0}$

**References:**


**Mathematical formulation**

Given a set of training examples $(x_1, y_1), \ldots, (x_n, y_n)$ where $x_i \in \mathbb{R}^n$ and $y_i \in \{-1, 1\}$, our goal is to learn a linear scoring function $f(x) = w^T x + b$ with model parameters $w \in \mathbb{R}^n$ and intercept $b \in \mathbb{R}$. In order to make predictions, we simply look at the sign of $f(x)$. A common choice to find the model parameters is by minimizing the regularized training error given by

$$E(w, b) = \frac{1}{n} \sum_{i=1}^{n} L(y_i, f(x_i)) + \alpha R(w)$$

where $L$ is a loss function that measures model (mis)fit and $R$ is a regularization term (aka penalty) that penalizes model complexity; $\alpha > 0$ is a non-negative hyperparameter.

Different choices for $L$ entail different classifiers such as

- Hinge: (soft-margin) Support Vector Machines.
- Log: Logistic Regression.
- Least-Squares: Ridge Regression.
- Epsilon-Insensitive: (soft-margin) Support Vector Regression.
All of the above loss functions can be regarded as an upper bound on the misclassification error (Zero-one loss) as shown in the Figure below.

Popular choices for the regularization term $R$ include:

- **L2 norm:** $R(w) := \frac{1}{2} \sum_{i=1}^{n} w_i^2$,
- **L1 norm:** $R(w) := \sum_{i=1}^{n} |w_i|$, which leads to sparse solutions.
- **Elastic Net:** $R(w) := \frac{\rho}{2} \sum_{i=1}^{n} w_i^2 + (1 - \rho) \sum_{i=1}^{n} |w_i|$, a convex combination of L2 and L1, where $\rho$ is given by $1 - \text{l1_ratio}$.

The Figure below shows the contours of the different regularization terms in the parameter space when $R(w) = 1$.

**SGD**

Stochastic gradient descent is an optimization method for unconstrained optimization problems. In contrast to (batch) gradient descent, SGD approximates the true gradient of $E(w, b)$ by considering a single training example at a time.

The class `SGDClassifier` implements a first-order SGD learning routine. The algorithm iterates over the training examples and for each example updates the model parameters according to the update rule given by

$$w \leftarrow w - \eta \left( \alpha \frac{\partial R(w)}{\partial w} + \frac{\partial L(w^T x_i + b, y_i)}{\partial w} \right)$$

where $\eta$ is the learning rate which controls the step-size in the parameter space. The intercept $b$ is updated similarly but without regularization.

The learning rate $\eta$ can be either constant or gradually decaying. For classification, the default learning rate schedule (learning_rate='optimal') is given by

$$\eta(t) = \frac{1}{\alpha(t_0 + t)}$$
where \( t \) is the time step (there are a total of \( n_{\text{samples}} \times n_{\text{iter}} \) time steps), \( t_0 \) is determined based on a heuristic proposed by Léon Bottou such that the expected initial updates are comparable with the expected size of the weights (this assuming that the norm of the training samples is approx. 1). The exact definition can be found in \_init_t in BaseSGD.

For regression the default learning rate schedule is inverse scaling (learning_rate='invscaling'), given by

\[
\eta^{(t)} = \frac{\eta_0}{t^{\text{power}_t}}
\]

where \( \eta_0 \) and \( \text{power}_t \) are hyperparameters chosen by the user via \( \text{eta0} \) and \( \text{power}_t \), resp.

For a constant learning rate use learning_rate='constant' and use \( \text{eta0} \) to specify the learning rate.

The model parameters can be accessed through the members coef_ and intercept_:

- Member coef_ holds the weights \( w \)
- Member intercept_ holds \( b \)

**References:**

- “Towards Optimal One Pass Large Scale Learning with Averaged Stochastic Gradient Descent” Xu, Wei
Implementation details

The implementation of SGD is influenced by the Stochastic Gradient SVM of Léon Bottou. Similar to SvmSGD, the weight vector is represented as the product of a scalar and a vector which allows an efficient weight update in the case of L2 regularization. In the case of sparse feature vectors, the intercept is updated with a smaller learning rate (multiplied by 0.01) to account for the fact that it is updated more frequently. Training examples are picked up sequentially and the learning rate is lowered after each observed example. We adopted the learning rate schedule from Shalev-Shwartz et al. 2007. For multi-class classification, a “one versus all” approach is used. We use the truncated gradient algorithm proposed by Tsuruoka et al. 2009 for L1 regularization (and the Elastic Net). The code is written in Cython.

References:

- “Pegasos: Primal estimated sub-gradient solver for svm” S. Shalev-Shwartz, Y. Singer, N. Srebro - In Proceedings of ICML ‘07.

3.1.6 Nearest Neighbors

sklearn.neighbors provides functionality for unsupervised and supervised neighbors-based learning methods. Unsupervised nearest neighbors is the foundation of many other learning methods, notably manifold learning and spectral clustering. Supervised neighbors-based learning comes in two flavors: classification for data with discrete labels, and regression for data with continuous labels.

The principle behind nearest neighbor methods is to find a predefined number of training samples closest in distance to the new point, and predict the label from these. The number of samples can be a user-defined constant (k-nearest neighbor learning), or vary based on the local density of points (radius-based neighbor learning). The distance can, in general, be any metric measure: standard Euclidean distance is the most common choice. Neighbors-based methods are known as non-generalizing machine learning methods, since they simply “remember” all of its training data (possibly transformed into a fast indexing structure such as a Ball Tree or KD Tree).

Despite its simplicity, nearest neighbors has been successful in a large number of classification and regression problems, including handwritten digits or satellite image scenes. Being a non-parametric method, it is often successful in classification situations where the decision boundary is very irregular.

The classes in sklearn.neighbors can handle either Numpy arrays or scipy.sparse matrices as input. For dense matrices, a large number of possible distance metrics are supported. For sparse matrices, arbitrary Minkowski metrics are supported for searches.

There are many learning routines which rely on nearest neighbors at their core. One example is kernel density estimation, discussed in the density estimation section.

Unsupervised Nearest Neighbors

NearestNeighbors implements unsupervised nearest neighbors learning. It acts as a uniform interface to three different nearest neighbors algorithms: BallTree, KDTree, and a brute-force algorithm based on routines in sklearn.metrics.pairwise. The choice of neighbors search algorithm is controlled through the keyword 'algorithm', which must be one of ['auto', 'ball_tree', 'kd_tree', 'brute']. When the default
value 'auto' is passed, the algorithm attempts to determine the best approach from the training data. For a discussion of the strengths and weaknesses of each option, see *Nearest Neighbor Algorithms*.

**Warning:** Regarding the Nearest Neighbors algorithms, if two neighbors, neighbor \( k+1 \) and \( k \), have identical distances but different labels, the results will depend on the ordering of the training data.

### Finding the Nearest Neighbors

For the simple task of finding the nearest neighbors between two sets of data, the unsupervised algorithms within `sklearn.neighbors` can be used:

```python
>>> from sklearn.neighbors import NearestNeighbors
>>> import numpy as np

>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])

>>> nbrs = NearestNeighbors(n_neighbors=2, algorithm='ball_tree').fit(X)

>>> distances, indices = nbrs.kneighbors(X)
>>> indices
array([[0, 1],
        [1, 0],
        [2, 1],
        [3, 4],
        [4, 3],
        [5, 4]])

>>> distances
array([[ 0. , 1. ],
        [ 0. , 1. ],
        [ 0. , 1.41421356],
        [ 0. , 1. ],
        [ 0. , 1. ],
        [ 0. , 1.41421356]])
```

Because the query set matches the training set, the nearest neighbor of each point is the point itself, at a distance of zero.

It is also possible to efficiently produce a sparse graph showing the connections between neighboring points:

```python
>>> nbrs.kneighbors_graph(X).toarray()
array([[ 1.,  1.,  0.,  0.,  0.,  0.],
        [ 1.,  1.,  0.,  0.,  0.,  0.],
        [ 0.,  1.,  1.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  1.,  1.,  0.],
        [ 0.,  0.,  0.,  1.,  1.,  0.],
        [ 0.,  0.,  0.,  0.,  1.,  1.]]
```

Our dataset is structured such that points nearby in index order are nearby in parameter space, leading to an approximately block-diagonal matrix of K-nearest neighbors. Such a sparse graph is useful in a variety of circumstances which make use of spatial relationships between points for unsupervised learning: in particular, see `sklearn.manifold.Isomap`, `sklearn.manifold.LocallyLinearEmbedding`, and `sklearn.cluster.SpectralClustering`.

### KDTree and BallTree Classes

Alternatively, one can use the `KDTree` or `BallTree` classes directly to find nearest neighbors. This is the functionality wrapped by the `NearestNeighbors` class used above. The Ball Tree and KD Tree have the same interface;
we’ll show an example of using the KD Tree here:

```python
>>> from sklearn.neighbors import KDTree
>>> import numpy as np

>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])

>>> kdt = KDTree(X, leaf_size=30, metric='euclidean')

>>> kdt.query(X, k=2, return_distance=False)
array([[0, 1],
       [1, 0],
       [2, 1],
       [3, 4],
       [4, 3],
       [5, 4]])...
```

Refer to the `KDTree` and `BallTree` class documentation for more information on the options available for neighbors searches, including specification of query strategies, of various distance metrics, etc. For a list of available metrics, see the documentation of the `DistanceMetric` class.

### Nearest Neighbors Classification

Neighbors-based classification is a type of instance-based learning or non-generalizing learning: it does not attempt to construct a general internal model, but simply stores instances of the training data. Classification is computed from a simple majority vote of the nearest neighbors of each point: a query point is assigned the data class which has the most representatives within the nearest neighbors of the point.

scikit-learn implements two different nearest neighbors classifiers: `KNeighborsClassifier` implements learning based on the \( k \) nearest neighbors of each query point, where \( k \) is an integer value specified by the user. `RadiusNeighborsClassifier` implements learning based on the number of neighbors within a fixed radius \( r \) of each training point, where \( r \) is a floating-point value specified by the user.

The \( k \)-neighbors classification in `KNeighborsClassifier` is the more commonly used of the two techniques. The optimal choice of the value \( k \) is highly data-dependent: in general a larger \( k \) suppresses the effects of noise, but makes the classification boundaries less distinct.

In cases where the data is not uniformly sampled, radius-based neighbors classification in `RadiusNeighborsClassifier` can be a better choice. The user specifies a fixed radius \( r \), such that points in sparser neighborhoods use fewer nearest neighbors for the classification. For high-dimensional parameter spaces, this method becomes less effective due to the so-called “curse of dimensionality”.

The basic nearest neighbors classification uses uniform weights: that is, the value assigned to a query point is computed from a simple majority vote of the nearest neighbors. Under some circumstances, it is better to weight the neighbors such that nearer neighbors contribute more to the fit. This can be accomplished through the `weights` keyword. The default value, `weights = 'uniform'`, assigns uniform weights to each neighbor. `weights = 'distance'` assigns weights proportional to the inverse of the distance from the query point. Alternatively, a user-defined function of the distance can be supplied which is used to compute the weights.
Examples:

- **Nearest Neighbors Classification**: an example of classification using nearest neighbors.

**Nearest Neighbors Regression**

Neighbors-based regression can be used in cases where the data labels are continuous rather than discrete variables. The label assigned to a query point is computed based on the mean of the labels of its nearest neighbors.

scikit-learn implements two different neighbors regressors: `KNeighborsRegressor` implements learning based on the \( k \) nearest neighbors of each query point, where \( k \) is an integer value specified by the user. `RadiusNeighborsRegressor` implements learning based on the neighbors within a fixed radius \( r \) of the query point, where \( r \) is a floating-point value specified by the user.

The basic nearest neighbors regression uses uniform weights: that is, each point in the local neighborhood contributes uniformly to the classification of a query point. Under some circumstances, it can be advantageous to weight points such that nearby points contribute more to the regression than faraway points. This can be accomplished through the `weights` keyword. The default value, `weights = 'uniform'`, assigns equal weights to all points. `weights = 'distance'` assigns weights proportional to the inverse of the distance from the query point. Alternatively, a user-defined function of the distance can be supplied, which will be used to compute the weights.

The use of multi-output nearest neighbors for regression is demonstrated in *Face completion with a multi-output estimators*. In this example, the inputs \( X \) are the pixels of the upper half of faces and the outputs \( Y \) are the pixels of
the lower half of those faces.

Examples:

- **Nearest Neighbors regression**: an example of regression using nearest neighbors.
- **Face completion with a multi-output estimators**: an example of multi-output regression using nearest neighbors.

### Nearest Neighbor Algorithms

#### Brute Force

Fast computation of nearest neighbors is an active area of research in machine learning. The most naive neighbor search implementation involves the brute-force computation of distances between all pairs of points in the dataset: for \( N \) samples in \( D \) dimensions, this approach scales as \( O(DN^2) \). Efficient brute-force neighbors searches can be very competitive for small data samples. However, as the number of samples \( N \) grows, the brute-force approach quickly becomes infeasible. In the classes within `sklearn.neighbors`, brute-force neighbors searches are specified using the keyword `algorithm = 'brute'`, and are computed using the routines available in `sklearn.metrics.pairwise`.

#### K-D Tree

To address the computational inefficiencies of the brute-force approach, a variety of tree-based data structures have been invented. In general, these structures attempt to reduce the required number of distance calculations by efficiently encoding aggregate distance information for the sample. The basic idea is that if point \( A \) is very distant from point \( B \), and point \( B \) is very close to point \( C \), then we know that points \( A \) and \( C \) are very distant, without having to
Face completion with multi-output estimators

true faces | Extra trees | K-nn | Linear regression | Ridge

---

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explicitly calculate their distance. In this way, the computational cost of a nearest neighbors search can be reduced to $O(DN \log(N))$ or better. This is a significant improvement over brute-force for large $N$.

An early approach to taking advantage of this aggregate information was the KD tree data structure (short for $K$-dimensional tree), which generalizes two-dimensional Quad-trees and 3-dimensional Oct-trees to an arbitrary number of dimensions. The KD tree is a binary tree structure which recursively partitions the parameter space along the data axes, dividing it into nested orthotopic regions into which data points are filed. The construction of a KD tree is very fast: because partitioning is performed only along the data axes, no $D$-dimensional distances need to be computed. Once constructed, the nearest neighbor of a query point can be determined with only $O(\log(N))$ distance computations. Though the KD tree approach is very fast for low-dimensional ($D < 20$) neighbors searches, it becomes inefficient as $D$ grows very large: this is one manifestation of the so-called “curse of dimensionality”. In scikit-learn, KD tree neighbors searches are specified using the keyword $algorithm = 'kd_tree'$, and are computed using the class $KDTree$.

Ball Tree

To address the inefficiencies of KD Trees in higher dimensions, the ball tree data structure was developed. Where KD trees partition data along Cartesian axes, ball trees partition data in a series of nesting hyper-spheres. This makes tree construction more costly than that of the KD tree, but results in a data structure which can be very efficient on highly-structured data, even in very high dimensions.

A ball tree recursively divides the data into nodes defined by a centroid $C$ and radius $r$, such that each point in the node lies within the hyper-sphere defined by $r$ and $C$. The number of candidate points for a neighbor search is reduced through use of the triangle inequality:

$$|x + y| \leq |x| + |y|$$

With this setup, a single distance calculation between a test point and the centroid is sufficient to determine a lower and upper bound on the distance to all points within the node. Because of the spherical geometry of the ball tree nodes, it can out-perform a KD-tree in high dimensions, though the actual performance is highly dependent on the structure of the training data. In scikit-learn, ball-tree-based neighbors searches are specified using the keyword $algorithm = 'ball_tree'$, and are computed using the class $sklearn.neighbors.BallTree$. Alternatively, the user can work with the $BallTree$ class directly.

Choice of Nearest Neighbors Algorithm

The optimal algorithm for a given dataset is a complicated choice, and depends on a number of factors:

- number of samples $N$ (i.e. $n_samples$) and dimensionality $D$ (i.e. $n_features$).
  - Brute force query time grows as $O(DN)$
Ball tree query time grows as approximately $O[D \log(N)]$

KD tree query time changes with $D$ in a way that is difficult to precisely characterise. For small $D$ (less than 20 or so) the cost is approximately $O[D \log(N)]$, and the KD tree query can be very efficient. For larger $D$, the cost increases to nearly $O[DN]$, and the overhead due to the tree structure can lead to queries which are slower than brute force.

For small data sets ($N$ less than 30 or so), $\log(N)$ is comparable to $N$, and brute force algorithms can be more efficient than a tree-based approach. Both KDTree and BallTree address this through providing a leaf size parameter: this controls the number of samples at which a query switches to brute-force. This allows both algorithms to approach the efficiency of a brute-force computation for small $N$.

- data structure: intrinsic dimensionality of the data and/or sparsity of the data. Intrinsic dimensionality refers to the dimension $d \leq D$ of a manifold on which the data lies, which can be linearly or non-linearly embedded in the parameter space. Sparsity refers to the degree to which the data fills the parameter space (this is to be distinguished from the concept as used in “sparse” matrices. The data matrix may have no zero entries, but the structure can still be “sparse” in this sense).

- Brute force query time is unchanged by data structure.

- Ball tree and KD tree query times can be greatly influenced by data structure. In general, sparser data with a smaller intrinsic dimensionality leads to faster query times. Because the KD tree internal representation is aligned with the parameter axes, it will not generally show as much improvement as ball tree for arbitrarily structured data.

Datasets used in machine learning tend to be very structured, and are very well-suited for tree-based queries.

- number of neighbors $k$ requested for a query point.

  - Brute force query time is largely unaffected by the value of $k$

  - Ball tree and KD tree query time will become slower as $k$ increases. This is due to two effects: first, a larger $k$ leads to the necessity to search a larger portion of the parameter space. Second, using $k > 1$ requires internal queueing of results as the tree is traversed.

As $k$ becomes large compared to $N$, the ability to prune branches in a tree-based query is reduced. In this situation, Brute force queries can be more efficient.

- number of query points. Both the ball tree and the KD Tree require a construction phase. The cost of this construction becomes negligible when amortized over many queries. If only a small number of queries will be performed, however, the construction can make up a significant fraction of the total cost. If very few query points will be required, brute force is better than a tree-based method.

Currently, algorithm = 'auto' selects 'kd_tree' if $k < N/2$ and the 'effective_metric_' is in the 'VALID_METRICS' list of 'kd_tree'. It selects 'ball_tree' if $k < N/2$ and the 'effective_metric_' is not in the 'VALID_METRICS' list of 'kd_tree'. It selects 'brute' if $k \geq N/2$. This choice is based on the assumption that the number of query points is at least the same order as the number of training points, and that leaf_size is close to its default value of 30.

**Effect of leaf_size**

As noted above, for small sample sizes a brute force search can be more efficient than a tree-based query. This fact is accounted for in the ball tree and KD tree by internally switching to brute force searches within leaf nodes. The level of this switch can be specified with the parameter leaf_size. This parameter choice has many effects:

**construction time** A larger leaf_size leads to a faster tree construction time, because fewer nodes need to be created
query time  Both a large or small leaf_size can lead to suboptimal query cost. For leaf_size approaching 1, the overhead involved in traversing nodes can significantly slow query times. For leaf_size approaching the size of the training set, queries become essentially brute force. A good compromise between these is leaf_size = 30, the default value of the parameter.

memory  As leaf_size increases, the memory required to store a tree structure decreases. This is especially important in the case of ball tree, which stores a $D$-dimensional centroid for each node. The required storage space for BallTree is approximately $1 / \text{leaf_size}$ times the size of the training set.

leaf_size is not referenced for brute force queries.

Nearest Centroid Classifier

The NearestCentroid classifier is a simple algorithm that represents each class by the centroid of its members. In effect, this makes it similar to the label updating phase of the sklearn.KMeans algorithm. It also has no parameters to choose, making it a good baseline classifier. It does, however, suffer on non-convex classes, as well as when classes have drastically different variances, as equal variance in all dimensions is assumed. See Linear Discriminant Analysis (sklearn.discriminant_analysis.LinearDiscriminantAnalysis) and Quadratic Discriminant Analysis (sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis) for more complex methods that do not make this assumption. Usage of the default NearestCentroid is simple:

```python
>>> from sklearn.neighbors.nearest_centroid import NearestCentroid
>>> import numpy as np

>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = NearestCentroid()
>>> clf.fit(X, y)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

Nearest Shrunken Centroid

The NearestCentroid classifier has a shrink_threshold parameter, which implements the nearest shrunken centroid classifier. In effect, the value of each feature for each centroid is divided by the within-class variance of that feature. The feature values are then reduced by shrink_threshold. Most notably, if a particular feature value crosses zero, it is set to zero. In effect, this removes the feature from affecting the classification. This is useful, for example, for removing noisy features.

In the example below, using a small shrink threshold increases the accuracy of the model from 0.81 to 0.82.
Examples:

- **Nearest Centroid Classification**: an example of classification using nearest centroid with different shrink thresholds.

### Approximate Nearest Neighbors

There are many efficient exact nearest neighbor search algorithms for low dimensions $d$ (approximately 50). However, these algorithms perform poorly with respect to space and query time when $d$ increases. These algorithms are not any better than comparing query point to each point from the database in a high dimension (see Brute Force). This is a well-known consequence of the phenomenon called “The Curse of Dimensionality”.

There are certain applications where we do not need the exact nearest neighbors but having a “good guess” would suffice. When answers do not have to be exact, the `LSHForest` class implements an approximate nearest neighbor search. Approximate nearest neighbor search methods have been designed to try to speedup query time with high dimensional data. These techniques are useful when the aim is to characterize the neighborhood rather than identifying the exact neighbors themselves (eg: k-nearest neighbors classification and regression). Some of the most popular approximate nearest neighbor search techniques are locality sensitive hashing, best bin fit and balanced box-decomposition tree based search.
Locality Sensitive Hashing Forest

The vanilla implementation of locality sensitive hashing has a hyper-parameter that is hard to tune in practice, therefore scikit-learn implements a variant called `LSHForest` that has more reasonable hyperparameters. Both methods use internally random hyperplanes to index the samples into buckets and actual cosine similarities are only computed for samples that collide with the query hence achieving sublinear scaling. (see *Mathematical description of Locality Sensitive Hashing*).

`LSHForest` has two main hyper-parameters: `n_estimators` and `n_candidates`. The accuracy of queries can be controlled using these parameters as demonstrated in the following plots:

As a rule of thumb, a user can set `n_estimators` to a large enough value (e.g. between 10 and 50) and then adjust `n_candidates` to trade off accuracy for query time.

For small data sets, the brute force method for exact nearest neighbor search can be faster than LSH Forest. However LSH Forest has a sub-linear query time scalability with the index size. The exact break even point where LSH Forest queries become faster than brute force depends on the dimensionality, structure of the dataset, required level of precision, characteristics of the runtime environment such as availability of BLAS optimizations, number of CPU cores and size of the CPU caches. Following graphs depict scalability of LSHForest queries with index size.

For fixed `LSHForest` parameters, the accuracy of queries tends to slowly decrease with larger datasets. The error bars on the previous plots represent standard deviation across different queries.
Examples:

- **Hyper-parameters of Approximate Nearest Neighbors**: an example of the behavior of hyperparameters of approximate nearest neighbor search using LSH Forest.
- **Scalability of Approximate Nearest Neighbors**: an example of scalability of approximate nearest neighbor search using LSH Forest.

**Mathematical description of Locality Sensitive Hashing**

Locality sensitive hashing (LSH) techniques have been used in many areas where nearest neighbor search is performed in high dimensions. The main concept behind LSH is to hash each data point in the database using multiple (often simple) hash functions to form a digest (also called a hash) in high dimensions. The main concept behind LSH is to hash each data point in the database using multiple (often simple) hash functions to form a digest (also called a hash). At this point the probability of collision - where two objects have similar digests - is much higher for the points which are close to each other than that of the distant points. We describe the requirements for a hash function family to be locality sensitive as follows.

A family $H$ of functions from a domain $S$ to a range $U$ is called $(r, e, p_1, p_2)$-sensitive, with $r, e > 0$, $p_1 > p_2 > 0$, if for any $p, q \in S$, the following conditions hold ($D$ is the distance function):

- If $D(p, q) \leq r$ then $P_H[h(p) = h(q)] \geq p_1$.
- If $D(p, q) > r(1 + e)$ then $P_H[h(p) = h(q)] \leq p_2$.

As defined, nearby points within a distance of $r$ to each other are likely to collide with probability $p_1$. In contrast, distant points which are located with the distance more than $r(1 + e)$ have a small probability of $p_2$ of collision. Suppose there is a family of LSH function $H$. An LSH index is built as follows:

1. Choose $k$ functions $h_1, h_2, \ldots, h_k$ uniformly at random (with replacement) from $H$. For any $p \in S$, place $p$ in the bucket with label $g(p) = (h_1(p), h_2(p), \ldots, h_k(p))$. Observe that if each $h_i$ outputs one “digit”, each bucket has a $k$-digit label.

2. Independently perform step 1 $l$ times to construct $l$ separate estimators, with hash functions $g_1, g_2, \ldots, g_l$.

The reason to concatenate hash functions in the step 1 is to decrease the probability of the collision of distant points as much as possible. The probability drops from $p_2$ to $p_2^k$ which is negligibly small for large $k$. The choice of $k$ is strongly dependent on the data set size and structure and is therefore hard to tune in practice. There is a side effect of having a large $k$; it has the potential of decreasing the chance of nearby points getting collided. To address this issue, multiple estimators are constructed in step 2.

The requirement to tune $k$ for a given dataset makes classical LSH cumbersome to use in practice. The LSH Forest variant has been designed to alleviate this requirement by automatically adjusting the number of digits used to hash the samples.

LSH Forest is formulated with prefix trees with each leaf of a tree corresponding to an actual data point in the database. There are $l$ such trees which compose the forest and they are constructed using independently drawn random sequence of hash functions from $H$. In this implementation, “Random Projections” is being used as the LSH technique which is an approximation for the cosine distance. The length of the sequence of hash functions is kept fixed at 32. Moreover, a prefix tree is implemented using sorted arrays and binary search.

There are two phases of tree traversals used in order to answer a query to find the $m$ nearest neighbors of a point $q$. First, a top-down traversal is performed using a binary search to identify the leaf having the longest prefix match (maximum depth) with $q$’s label after subjecting $q$ to the same hash functions. $M >> m$ points (total candidates) are extracted from the forest, moving up from the previously found maximum depth towards the root synchronously across all trees in the bottom-up traversal. $M$ is set to $cM$ where $c$, the number of candidates extracted from each tree, is a constant. Finally, the similarity of each of these $M$ points against point $q$ is calculated and the top $m$ points are returned as the nearest neighbors of $q$. Since most of the time in these queries is spent calculating the distances to
candidates, the speedup compared to brute force search is approximately $N/M$, where $N$ is the number of points in database.

References:


3.1.7 Gaussian Processes

Gaussian Processes (GP) are a generic supervised learning method designed to solve regression and probabilistic classification problems.

The advantages of Gaussian processes are:

- The prediction interpolates the observations (at least for regular kernels).
- The prediction is probabilistic (Gaussian) so that one can compute empirical confidence intervals and decide based on those if one should refit (online fitting, adaptive fitting) the prediction in some region of interest.
- Versatile: different kernels can be specified. Common kernels are provided, but it is also possible to specify custom kernels.

The disadvantages of Gaussian processes include:

- They are not sparse, i.e., they use the whole samples/features information to perform the prediction.
- They lose efficiency in high dimensional spaces – namely when the number of features exceeds a few dozens.

Gaussian Process Regression (GPR)

The GaussianProcessRegressor implements Gaussian processes (GP) for regression purposes. For this, the prior of the GP needs to be specified. The prior mean is assumed to be constant and zero (for normalize_y=False) or the training data’s mean (for normalize_y=True). The prior’s covariance is specified by a passing a kernel object. The hyperparameters of the kernel are optimized during fitting of GaussianProcessRegressor by maximizing the log-marginal-likelihood (LML) based on the passed optimizer. As the LML may have multiple local optima, the optimizer can be started repeatedly by specifying n_restarts_optimizer. The first run is always conducted starting from the initial hyperparameter values of the kernel; subsequent runs are conducted from hyperparameter values that have been chosen randomly from the range of allowed values. If the initial hyperparameters should be kept fixed, None can be passed as optimizer.

The noise level in the targets can be specified by passing it via the parameter alpha, either globally as a scalar or per datapoint. Note that a moderate noise level can also be helpful for dealing with numeric issues during fitting as it is effectively implemented as Tikhonov regularization, i.e., by adding it to the diagonal of the kernel matrix. An alternative to specifying the noise level explicitly is to include a WhiteKernel component into the kernel, which can estimate the global noise level from the data (see example below).

The implementation is based on Algorithm 2.1 of [RW2006]. In addition to the API of standard scikit-learn estimators, GaussianProcessRegressor:

- allows prediction without prior fitting (based on the GP prior)
- provides an additional method sample_y(X), which evaluates samples drawn from the GPR (prior or posterior) at given inputs

3.1. Supervised learning
- exposes a method `log_marginal_likelihood(theta)`, which can be used externally for other ways of selecting hyperparameters, e.g., via Markov chain Monte Carlo.

GPR examples

GPR with noise-level estimation

This example illustrates that GPR with a sum-kernel including a WhiteKernel can estimate the noise level of data. An illustration of the log-marginal-likelihood (LML) landscape shows that there exist two local maxima of LML.

\[
	ext{Initial: } 1^{\ast} 2 \ast \text{RBF}(\text{length\_scale}=100) + \text{WhiteKernel}(\text{noise\_level}=1)\\
\text{Optimum: } 0.00316^{\ast} 2 \ast \text{RBF}(\text{length\_scale}=109) + \text{WhiteKernel}(\text{noise\_level}=0.63)\\
\text{Log-Marginal-Likelihood: } -23.872337362
\]

The first corresponds to a model with a high noise level and a large length scale, which explains all variations in the data by noise.

The second one has a smaller noise level and shorter length scale, which explains most of the variation by the noise-free functional relationship. The second model has a higher likelihood; however, depending on the initial value for the hyperparameters, the gradient-based optimization might also converge to the high-noise solution. It is thus important to repeat the optimization several times for different initializations.

Comparison of GPR and Kernel Ridge Regression

Both kernel ridge regression (KRR) and GPR learn a target function by employing internally the “kernel trick”. KRR learns a linear function in the space induced by the respective kernel which corresponds to a non-linear function in
Initial: $1^2 \cdot \text{RBF}(\text{length_scale}=1) + \text{WhiteKernel}(\text{noise_level}=1e-05)$
Optimum: $0.64^2 \cdot \text{RBF}(\text{length_scale}=0.365) + \text{WhiteKernel}(\text{noise_level}=0.294)$
Log-Marginal-Likelihood: -21.8050908902
the original space. The linear function in the kernel space is chosen based on the mean-squared error loss with ridge regularization. GPR uses the kernel to define the covariance of a prior distribution over the target functions and uses the observed training data to define a likelihood function. Based on Bayes theorem, a (Gaussian) posterior distribution over target functions is defined, whose mean is used for prediction. A major difference is that GPR can choose the kernel’s hyperparameters based on gradient-ascent on the marginal likelihood function while KRR needs to perform a grid search on a cross-validated loss function (mean-squared error loss). A further difference is that GPR learns a generative, probabilistic model of the target function and can thus provide meaningful confidence intervals and posterior samples along with the predictions while KRR only provides predictions. The following figure illustrates both methods on an artificial dataset, which consists of a sinusoidal target function and strong noise. The figure compares the learned model of KRR and GPR based on a ExpSineSquared kernel, which is suited for learning periodic functions. The kernel’s hyperparameters control the smoothness (length_scale) and periodicity of the kernel (periodicity). Moreover, the noise level of the data is learned explicitly by GPR by an additional WhiteKernel component in the kernel and by the regularization parameter alpha of KRR.

The figure shows that both methods learn reasonable models of the target function. GPR correctly identifies the periodicity of the function to be roughly $2 \times \pi$ (6.28), while KRR chooses the doubled periodicity $4 \times \pi$. Besides that, GPR provides reasonable confidence bounds on the prediction which are not available for KRR. A major difference between the two methods is the time required for fitting and predicting: while fitting KRR is fast in principle, the grid-search for hyperparameter optimization scales exponentially with the number of hyperparameters (“curse of dimensionality”). The gradient-based optimization of the parameters in GPR does not suffer from this exponential scaling and is thus considerably faster on this example with 3-dimensional hyperparameter space. The time for predicting is similar; however, generating the variance of the predictive distribution of GPR takes considerably longer than just predicting the mean.

**GPR on Mauna Loa CO2 data**

This example is based on Section 5.4.3 of [RW2006]. It illustrates an example of complex kernel engineering and hyperparameter optimization using gradient ascent on the log-marginal-likelihood. The data consists of the monthly average atmospheric CO2 concentrations (in parts per million by volume (ppmv)) collected at the Mauna Loa Observatory in Hawaii, between 1958 and 1997. The objective is to model the CO2 concentration as a function of the time...
t.

The kernel is composed of several terms that are responsible for explaining different properties of the signal:

- a long term, smooth rising trend is to be explained by an RBF kernel. The RBF kernel with a large length-scale enforces this component to be smooth; it is not enforced that the trend is rising which leaves this choice to the GP. The specific length-scale and the amplitude are free hyperparameters.

- a seasonal component, which is to be explained by the periodic ExpSineSquared kernel with a fixed periodicity of 1 year. The length-scale of this periodic component, controlling its smoothness, is a free parameter. In order to allow decaying away from exact periodicity, the product with an RBF kernel is taken. The length-scale of this RBF component controls the decay time and is a further free parameter.

- smaller, medium term irregularities are to be explained by a RationalQuadratic kernel component, whose length-scale and alpha parameter, which determines the diffuseness of the length-scales, are to be determined. According to [RW2006], these irregularities can better be explained by a RationalQuadratic than an RBF kernel component, probably because it can accommodate several length-scales.

- a “noise” term, consisting of an RBF kernel contribution, which shall explain the correlated noise components such as local weather phenomena, and a WhiteKernel contribution for the white noise. The relative amplitudes and the RBF’s length scale are further free parameters.

Maximizing the log-marginal-likelihood after subtracting the target’s mean yields the following kernel with an LML of -83.214:

\[
34.4^{**2} \times \text{RBF}(\text{length\_scale}=41.8) \\
+ 3.27^{**2} \times \text{RBF}(\text{length\_scale}=180) \times \text{ExpSineSquared}(\text{length\_scale}=1.44, \text{periodicity}=1) \\
+ 0.446^{**2} \times \text{RationalQuadratic}(\text{alpha}=17.7, \text{length\_scale}=0.957) \\
+ 0.197^{**2} \times \text{RBF}(\text{length\_scale}=0.138) + \text{WhiteKernel}(\text{noise\_level}=0.0336)
\]

Thus, most of the target signal (34.4ppm) is explained by a long-term rising trend (length-scale 41.8 years). The periodic component has an amplitude of 3.27ppm, a decay time of 180 years and a length-scale of 1.44. The long decay time indicates that we have a locally very close to periodic seasonal component. The correlated noise has an amplitude of 0.197ppm with a length scale of 0.138 years and a white-noise contribution of 0.197ppm. Thus, the overall noise level is very small, indicating that the data can be very well explained by the model. The figure shows also that the model makes very confident predictions until around 2015.

**Gaussian Process Classification (GPC)**

The `GaussianProcessClassifier` implements Gaussian processes (GP) for classification purposes, more specifically for probabilistic classification, where test predictions take the form of class probabilities. `GaussianProcessClassifier` places a GP prior on a latent function \( f \), which is then squashed through a link function to obtain the probabilistic classification. The latent function \( f \) is a so-called nuisance function, whose values are not observed and are not relevant by themselves. Its purpose is to allow a convenient formulation of the model, and \( f \) is removed (integrated out) during prediction. `GaussianProcessClassifier` implements the logistic link function, for which the integral cannot be computed analytically but is easily approximated in the binary case.

In contrast to the regression setting, the posterior of the latent function \( f \) is not Gaussian even for a GP prior since a Gaussian likelihood is inappropriate for discrete class labels. Rather, a non-Gaussian likelihood corresponding to the logistic link function (logit) is used. `GaussianProcessClassifier` approximates the non-Gaussian posterior with a Gaussian based on the Laplace approximation. More details can be found in Chapter 3 of [RW2006].

The GP prior mean is assumed to be zero. The prior’s covariance is specified by a passing a `kernel` object. The hyperparameters of the kernel are optimized during fitting of `GaussianProcessRegressor` by maximizing the log-marginal-likelihood (LML) based on the passed optimizer. As the LML may have multiple local optima, the optimizer can be started repeatedly by specifying `n_restarts_optimizer`. The first run is always conducted starting from the initial hyperparameter values of the kernel; subsequent runs are conducted from hyperparameter values that have been
Atmospheric CO$_2$ concentration at Mauna Loa

Year

CO$_2$ in ppm
chosen randomly from the range of allowed values. If the initial hyperparameters should be kept fixed, None can be passed as optimizer.

`GaussianProcessClassifier` supports multi-class classification by performing either one-versus-rest or one-versus-one based training and prediction. In one-versus-rest, one binary Gaussian process classifier is fitted for each class, which is trained to separate this class from the rest. In “one_vs_one”, one binary Gaussian process classifier is fitted for each pair of classes, which is trained to separate these two classes. The predictions of these binary predictors are combined into multi-class predictions. See the section on multi-class classification for more details.

In the case of Gaussian process classification, “one_vs_one” might be computationally cheaper since it has to solve many problems involving only a subset of the whole training set rather than fewer problems on the whole dataset. Since Gaussian process classification scales cubically with the size of the dataset, this might be considerably faster. However, note that “one_vs_one” does not support predicting probability estimates but only plain predictions. Moreover, note that `GaussianProcessClassifier` does not (yet) implement a true multi-class Laplace approximation internally, but as discussed above is based on solving several binary classification tasks internally, which are combined using one-versus-rest or one-versus-one.

**GPC examples**

**Probabilistic predictions with GPC**

This example illustrates the predicted probability of GPC for an RBF kernel with different choices of the hyperparameters. The first figure shows the predicted probability of GPC with arbitrarily chosen hyperparameters and with the hyperparameters corresponding to the maximum log-marginal-likelihood (LML).

While the hyperparameters chosen by optimizing LML have a considerable larger LML, they perform slightly worse according to the log-loss on test data. The figure shows that this is because they exhibit a steep change of the class probabilities at the class boundaries (which is good) but have predicted probabilities close to 0.5 far away from the class boundaries (which is bad). This undesirable effect is caused by the Laplace approximation used internally by GPC.

The second figure shows the log-marginal-likelihood for different choices of the kernel’s hyperparameters, highlighting the two choices of the hyperparameters used in the first figure by black dots.

**Illustration of GPC on the XOR dataset**

This example illustrates GPC on XOR data. Compared are a stationary, isotropic kernel (RBF) and a non-stationary kernel (DotProduct). On this particular dataset, the DotProduct kernel obtains considerably better results because the class-boundaries are linear and coincide with the coordinate axes. In practice, however, stationary kernels such as RBF often obtain better results.

**Gaussian process classification (GPC) on iris dataset**

This example illustrates the predicted probability of GPC for an isotropic and anisotropic RBF kernel on a two-dimensional version for the iris-dataset. This illustrates the applicability of GPC to non-binary classification. The anisotropic RBF kernel obtains slightly higher log-marginal-likelihood by assigning different length-scales to the two feature dimensions.

**Kernels for Gaussian Processes**

Kernels (also called “covariance functions” in the context of GPs) are a crucial ingredient of GPs which determine the shape of prior and posterior of the GP. They encode the assumptions on the function being learned by defining the
“similarity” of two datapoints combined with the assumption that similar datapoints should have similar target values. Two categories of kernels can be distinguished: stationary kernels depend only on the distance of two datapoints and not on their absolute values \( k(x_i, x_j) = k(d(x_i, x_j)) \) and are thus invariant to translations in the input space, while non-stationary kernels depend also on the specific values of the datapoints. Stationary kernels can further be subdivided into isotropic and anisotropic kernels, where isotropic kernels are also invariant to rotations in the input space. For more details, we refer to Chapter 4 of [RW2006].

**Gaussian Process Kernel API**

The main usage of a `Kernel` is to compute the GP’s covariance between datapoints. For this, the method `__call__` of the kernel can be called. This method can either be used to compute the “auto-covariance” of all pairs of datapoints in a 2d array \( X \), or the “cross-covariance” of all combinations of datapoints of a 2d array \( X \) with datapoints in a 2d array \( Y \). The following identity holds true for all kernels \( k \) (except for the `WhiteKernel`): \( k(X) == K(X, Y=X) \)

If only the diagonal of the auto-covariance is being used, the method `diag()` of a kernel can be called, which is more computationally efficient than the equivalent call to `__call__`: \( \text{np.diag}(k(X,X)) == k.diag(X) \)

Kernels are parameterized by a vector \( \theta \) of hyperparameters. These hyperparameters can for instance control length-scales or periodicity of a kernel (see below). All kernels support computing analytic gradients of of the kernel’s auto-covariance with respect to \( \theta \) via setting `eval_gradient=True` in the `__call__` method. This gradient is used by the Gaussian process (both regressor and classifier) in computing the gradient of the log-marginal-likelihood, which in turn is used to determine the value of \( \theta \), which maximizes the log-marginal-likelihood, via gradient ascent. For each hyperparameter, the initial value and the bounds need to be specified when creating an instance of the kernel. The current value of \( \theta \) can be get and set via the property `theta` of the kernel object. Moreover, the bounds of the hyperparameters can be accessed by the property `bounds` of the kernel. Note that both properties (theta and bounds) return log-transformed values of the internally used values since those are typically more amenable to gradient-based optimization. The specification of each hyperparameter is stored in the form of an instance of `Hyperparameter` in the respective kernel. Note that a kernel using a hyperparameter with name “\( x \)” must have the attributes self.x and self.x_bounds.

The abstract base class for all kernels is `Kernel`. Kernel implements a similar interface as `Estimator`, providing the methods `get_params()`, `set_params()`, and `clone()`. This allows setting kernel values also via meta-estimators such as `Pipeline` or `GridSearch`. Note that due to the nested structure of kernels (by applying kernel
operators, see below), the names of kernel parameters might become relatively complicated. In general, for a binary
kernel operator, parameters of the left operand are prefixed with \texttt{k1\_} and parameters of the right operand with \texttt{k2\_}.
An additional convenience method is \texttt{clone_with_theta(\theta)}, which returns a cloned version of the kernel
but with the hyperparameters set to \theta. An illustrative example:

```python
>>> from sklearn.gaussian_process.kernels import ConstantKernel, RBF
>>> kernel = ConstantKernel(constant_value=1.0, constant_value_bounds=(0.0, 10.0)) * RBF(length_scale=0.5, length_scale_bounds=(0.0, 10.0)) + RBF(length_scale=2.0, length_scale_bounds=(0.0, 10.0))
>>> for hyperparameter in kernel.hyperparameters: print(hyperparameter)
Hyperparameter(name='k1\_k1\_constant_value', value_type='numeric', bounds=array([[ 0., 10.]]), n_elements=1, fixed=False)
Hyperparameter(name='k1\_k2\_length_scale', value_type='numeric', bounds=array([[ 0., 10.]]), n_elements=1, fixed=False)
Hyperparameter(name='k2\_length_scale', value_type='numeric', bounds=array([[ 0., 10.]]), n_elements=1, fixed=False)
>>> params = kernel.get_params()
>>> for key in sorted(params): print("%s: %s" % (key, params[key]))
k1 : 1**2 * RBF(length_scale=0.5)
k1\_k1 : 1**2
k1\_k1\_constant_value : 1.0
k1\_k1\_constant_value_bounds : (0.0, 10.0)
k1\_k2 : RBF(length_scale=0.5)
k1\_k2\_length_scale : 0.5
k1\_k2\_length_scale_bounds : (0.0, 10.0)
k2 : RBF(length_scale=2)
k2\_length_scale : 2.0
k2\_length_scale_bounds : (0.0, 10.0)
>>> print(kernel.theta) # Note: log-transformed
[ 0. -0.69314718 0.69314718]
>>> print(kernel.bounds) # Note: log-transformed
[[ -inf 2.30258509 0.69314718]
 [ -inf 2.30258509]
 [ -inf 2.30258509]]
```

All Gaussian process kernels are interoperable with \texttt{sklearn.metrics.pairwise} and vice versa: instances
of subclasses of \texttt{Kernel} can be passed as \texttt{metric} to \texttt{pairwise.kernels\_} from \texttt{sklearn.metrics.pairwise}.
Moreover, kernel functions from pairwise can be used as GP kernels by using the wrapper class \texttt{PairwiseKernel}.
The only caveat is that the gradient of the hyperparameters is not analytic but numeric and all those kernels support
only isotropic distances. The parameter \texttt{gamma} is considered to be a hyperparameter and may be optimized. The other
kernel parameters are set directly at initialization and are kept fixed.

### Basic kernels

The \texttt{ConstantKernel} kernel can be used as part of a \texttt{Product} kernel where it scales the magnitude of the other
factor (kernel) or as part of a \texttt{Sum} kernel, where it modifies the mean of the Gaussian process. It depends on a
parameter \texttt{constant\_value}. It is defined as:

\[
k(x_i, x_j) = constant\_value \quad \forall \ x_1, x_2
\]

The main use-case of the \texttt{WhiteKernel} kernel is as part of a sum-kernel where it explains the noise-component of
the signal. Tuning its parameter \texttt{noise\_level} corresponds to estimating the noise-level. It is defined as:

\[
k(x_i, x_j) = noise\_level \text{ if } x_i == x_j \text{ else } 0
\]
Kernel operators

Kernel operators take one or two base kernels and combine them into a new kernel. The Sum kernel takes two kernels \( k_1 \) and \( k_2 \) and combines them via \( k_{\text{sum}}(X, Y) = k_1(X, Y) + k_2(X, Y) \). The Product kernel takes two kernels \( k_1 \) and \( k_2 \) and combines them via \( k_{\text{product}}(X, Y) = k_1(X, Y) \times k_2(X, Y) \). The Exponentiation kernel takes one base kernel and a scalar parameter \( \text{exponent} \) and combines them via \( k_{\text{exp}}(X, Y) = k(X, Y)^{\text{exponent}} \).

Radial-basis function (RBF) kernel

The RBF kernel is a stationary kernel. It is also known as the “squared exponential” kernel. It is parameterized by a length-scale parameter \( l > 0 \), which can either be a scalar (isotropic variant of the kernel) or a vector with the same number of dimensions as the inputs \( x \) (anisotropic variant of the kernel). The kernel is given by:

\[
k(x_i, x_j) = \exp\left(-\frac{1}{2} \frac{d(x_i, x_j)}{l}^2\right)
\]

This kernel is infinitely differentiable, which implies that GPs with this kernel as covariance function have mean square derivatives of all orders, and are thus very smooth. The prior and posterior of a GP resulting from an RBF kernel are shown in the following figure:

Matérn kernel

The Matérn kernel is a stationary kernel and a generalization of the RBF kernel. It has an additional parameter \( \nu \) which controls the smoothness of the resulting function. It is parameterized by a length-scale parameter \( l > 0 \), which can either be a scalar (isotropic variant of the kernel) or a vector with the same number of dimensions as the inputs \( x \) (anisotropic variant of the kernel). The kernel is given by:

\[
k(x_i, x_j) = \sigma^2 \frac{1}{\Gamma(\nu)} \left(\frac{\gamma \sqrt{2 \nu} d(x_i, x_j)}{l}\right)^\nu K_\nu\left(\frac{\gamma \sqrt{2 \nu} d(x_i, x_j)}{l}\right),
\]

As \( \nu \to \infty \), the Matérn kernel converges to the RBF kernel. When \( \nu = 1/2 \), the Matérn kernel becomes identical to the absolute exponential kernel, i.e.,

\[
k(x_i, x_j) = \sigma^2 \exp\left(-\gamma d(x_i, x_j)\right) \quad \nu = \frac{1}{2}
\]

In particular, \( \nu = 3/2 \):

\[
k(x_i, x_j) = \sigma^2 \left(1 + \gamma \sqrt{3} d(x_i, x_j)\right) \exp\left(-\gamma \sqrt{3} d(x_i, x_j)\right) \quad \nu = \frac{3}{2}
\]

and \( \nu = 5/2 \):

\[
k(x_i, x_j) = \sigma^2 \left(1 + \gamma \sqrt{5} d(x_i, x_j) + \frac{5}{3} \gamma^2 d(x_i, x_j)^2\right) \exp\left(-\gamma \sqrt{5} d(x_i, x_j)\right) \quad \nu = \frac{5}{2}
\]

are popular choices for learning functions that are not infinitely differentiable (as assumed by the RBF kernel) but at least once (\( \nu = 3/2 \)) or twice differentiable (\( \nu = 5/2 \)).

The flexibility of controlling the smoothness of the learned function via \( \nu \) allows adapting to the properties of the true underlying functional relation. The prior and posterior of a GP resulting from a Matérn kernel are shown in the following figure:

See [RW2006], pp84 for further details regarding the different variants of the Matérn kernel.
Prior (kernel: $1**2 \times \text{RBF}(\text{length\_scale}=1))$

Posterior (kernel: $0.594**2 \times \text{RBF}(\text{length\_scale}=0.279))$

Log-Likelihood: -0.067
**Rational quadratic kernel**

The `RationalQuadratic` kernel can be seen as a scale mixture (an infinite sum) of RBF kernels with different characteristic length-scales. It is parameterized by a length-scale parameter $l > 0$ and a scale mixture parameter $\alpha > 0$. Only the isotropic variant where $l$ is a scalar is supported at the moment. The kernel is given by:

$$k(x_i, x_j) = \left(1 + \frac{d(x_i, x_j)^2}{2\alpha l^2}\right)^\alpha$$

The prior and posterior of a GP resulting from an RBF kernel are shown in the following figure:
Exp-Sine-Squared kernel

The `ExpSineSquared` kernel allows modeling periodic functions. It is parameterized by a length-scale parameter $l > 0$ and a periodicity parameter $p > 0$. Only the isotropic variant where $l$ is a scalar is supported at the moment. The kernel is given by:

$$k(x_i, x_j) = \exp\left(-2\sin\left(\frac{\pi}{p} \cdot d(x_i, x_j)\right)/l\right)^2$$

The prior and posterior of a GP resulting from an ExpSineSquared kernel are shown in the following figure:
Dot-Product kernel

The DotProduct kernel is non-stationary and can be obtained from linear regression by putting $N(0, 1)$ priors on the coefficients of $x_d (d = 1, ..., D)$ and a prior of $N(0, \sigma^2_0)$ on the bias. The DotProduct kernel is invariant to a rotation of the coordinates about the origin, but not translations. It is parameterized by a parameter $\sigma^2_0$. For $\sigma^2_0 = 0$, the kernel is called the homogeneous linear kernel, otherwise it is inhomogeneous. The kernel is given by

$$k(x_i, x_j) = \sigma^2_0 + x_i \cdot x_j$$

The DotProduct kernel is commonly combined with exponentiation. An example with exponent 2 is shown in the following figure:
References


Legacy Gaussian Processes

In this section, the implementation of Gaussian processes used in scikit-learn until release 0.16.1 is described. Note that this implementation is deprecated and will be removed in version 0.18.

An introductory regression example

Say we want to surrogate the function \( g(x) = x \sin(x) \). To do so, the function is evaluated onto a design of experiments. Then, we define a GaussianProcess model whose regression and correlation models might be specified using additional kwargs, and ask for the model to be fitted to the data. Depending on the number of parameters provided at instantiation, the fitting procedure may recourse to maximum likelihood estimation for the parameters or alternatively it uses the given parameters.

```python
>>> import numpy as np
>>> from sklearn import gaussian_process
>>> def f(x):
...     return x * np.sin(x)
>>> X = np.atleast_2d([1., 3., 5., 6., 7., 8.]).T
>>> y = f(X).ravel()
>>> x = np.atleast_2d(np.linspace(0, 10, 1000)).T
>>> gp = gaussian_process.GaussianProcess(theta0=1e-2, thetaL=1e-4, thetaU=1e-1)
>>> gp.fit(X, y)
GaussianProcess(beta0=None, corr=<function squared_exponential at 0x...>,
normalize=True, nugget=array(2.22...-15),
optimizer='fmin_cobyla', random_start=1, random_state=...
regr=<function constant at 0x...>, storage_mode='full',
theta0=array([[ 0.01]]), thetaL=array([[ 0.0001]]),
thetaU=array([[ 0.1]]), verbose=False)
>>> y_pred, sigma2_pred = gp.predict(x, eval_MSE=True)
```

Fitting Noisy Data

When the data to be fit includes noise, the Gaussian process model can be used by specifying the variance of the noise for each point. `GaussianProcess` takes a parameter `nugget` which is added to the diagonal of the correlation matrix between training points: in general this is a type of Tikhonov regularization. In the special case of a squared-exponential correlation function, this normalization is equivalent to specifying a fractional variance in the input. That is

\[
\text{nugget}_i = \left( \frac{\sigma_i}{y_i} \right)^2
\]

With `nugget` and `corr` properly set, Gaussian Processes can be used to robustly recover an underlying function from noisy data.
Mathematical formulation

The initial assumption

Suppose one wants to model the output of a computer experiment, say a mathematical function:

\[ g : \mathbb{R}^{n_{\text{features}}} \rightarrow \mathbb{R} \]
\[ X \mapsto y = g(X) \]

GPML starts with the assumption that this function is a conditional sample path of a Gaussian process \( G \) which is additionally assumed to read as follows:

\[ G(X) = f(X)^T \beta + Z(X) \]

where \( f(X)^T \beta \) is a linear regression model and \( Z(X) \) is a zero-mean Gaussian process with a fully stationary covariance function:

\[ C(X, X') = \sigma^2 R(|X - X'|) \]

\( \sigma^2 \) being its variance and \( R \) being the correlation function which solely depends on the absolute relative distance between each sample, possibly featurewise (this is the stationarity assumption).

From this basic formulation, note that GPML is nothing but an extension of a basic least squares linear regression problem:

\[ g(X) \approx f(X)^T \beta \]

Except we additionally assume some spatial coherence (correlation) between the samples dictated by the correlation function. Indeed, ordinary least squares assumes the correlation model \( R(|X - X'|) \) is one when \( X = X' \) and zero otherwise: a \textit{dirac} correlation model – sometimes referred to as a \textit{nugget} correlation model in the kriging literature.

The best linear unbiased prediction (BLUP)

We now derive the best linear unbiased prediction of the sample path \( g \) conditioned on the observations:

\[ \hat{G}(X) = G(X | y_1 = g(X_1), \ldots, y_{n_{\text{samples}}} = g(X_{n_{\text{samples}}})) \]

It is derived from its \textit{given properties}:

- It is linear (a linear combination of the observations)
  \[ \hat{G}(X) \equiv a(X)^T y \]
- It is unbiased
  \[ \mathbb{E}[G(X) - \hat{G}(X)] = 0 \]
- It is the best (in the Mean Squared Error sense)
  \[ \hat{G}(X)^* = \arg \min_{\hat{G}(X)} \mathbb{E}[(G(X) - \hat{G}(X))^2] \]

So that the optimal weight vector \( a(X) \) is solution of the following equality constrained optimization problem:

\[ a(X)^* = \arg \min_{a(X)} \mathbb{E}[(G(X) - a(X)^T y)^2] \]
\[ \text{s.t. } \mathbb{E}[G(X) - a(X)^T y] = 0 \]
Rewriting this constrained optimization problem in the form of a Lagrangian and looking further for the first order optimality conditions to be satisfied, one ends up with a closed form expression for the sought predictor – see references for the complete proof.

In the end, the BLUP is shown to be a Gaussian random variate with mean:

$$\mu_{Y}(X) = f(X)^T \hat{\beta} + r(X)^T \gamma$$

and variance:

$$\sigma_{Y}^2(X) = \sigma_Y^2 (1 - r(X)^T R^{-1} r(X) + u(X)^T (F^T R^{-1} F)^{-1} u(X))$$

where we have introduced:

- the correlation matrix whose terms are defined wrt the autocorrelation function and its built-in parameters $\theta$:
  $$R_{ij} = R(|X_i - X_j|, \theta), \ i, j = 1, ..., m$$

- the vector of cross-correlations between the point where the prediction is made and the points in the DOE:
  $$r_i = R(|X - X_i|, \theta), \ i = 1, ..., m$$

- the regression matrix (eg the Vandermonde matrix if $f$ is a polynomial basis):
  $$F_{ij} = f_i(x_j), \ i = 1, ..., p, j = 1, ..., m$$

- the generalized least square regression weights:
  $$\hat{\beta} = (F^T R^{-1} F)^{-1} F^T R^{-1} Y$$

- and the vectors:
  $$\gamma = R^{-1} (Y - F \hat{\beta})$$
  $$u(X) = F^T R^{-1} r(X) - f(X)$$

It is important to notice that the probabilistic response of a Gaussian Process predictor is fully analytic and mostly relies on basic linear algebra operations. More precisely the mean prediction is the sum of two simple linear combinations (dot products), and the variance requires two matrix inversions, but the correlation matrix can be decomposed only once using a Cholesky decomposition algorithm.

**The empirical best linear unbiased predictor (EBLUP)**

Until now, both the autocorrelation and regression models were assumed given. In practice however they are never known in advance so that one has to make (motivated) empirical choices for these models *Correlation Models*.

Provided these choices are made, one should estimate the remaining unknown parameters involved in the BLUP. To do so, one uses the set of provided observations in conjunction with some inference technique. The present implementation, which is based on the DACE’s Matlab toolbox uses the *maximum likelihood estimation* technique – see DACE manual in references for the complete equations. This maximum likelihood estimation problem is turned into a global optimization problem onto the autocorrelation parameters. In the present implementation, this global optimization is solved by means of the fmin_cobyla optimization function from scipy.optimize. In the case of anisotropy however, we provide an implementation of Welch’s componentwise optimization algorithm – see references.
Correlation Models

Common correlation models match some famous SVM’s kernels because they are mostly built on equivalent assumptions. They must fulfill Mercer’s conditions and should additionally remain stationary. Note however, that the choice of the correlation model should be made in agreement with the known properties of the original experiment from which the observations come. For instance:

- If the original experiment is known to be infinitely differentiable (smooth), then one should use the squared-exponential correlation model.
- If it’s not, then one should rather use the exponential correlation model.
- Note also that there exists a correlation model that takes the degree of derivability as input: this is the Matern correlation model, but it’s not implemented here (TODO).

For a more detailed discussion on the selection of appropriate correlation models, see the book by Rasmussen & Williams in references.

Regression Models

Common linear regression models involve zero- (constant), first- and second-order polynomials. But one may specify its own in the form of a Python function that takes the features X as input and that returns a vector containing the values of the functional set. The only constraint is that the number of functions must not exceed the number of available observations so that the underlying regression problem is not underdetermined.

Implementation details

The implementation is based on a translation of the DACE Matlab toolbox.

References:

- DACE, A Matlab Kriging Toolbox S Lophaven, HB Nielsen, J Sondergaard 2002,

3.1.8 Cross decomposition

The cross decomposition module contains two main families of algorithms: the partial least squares (PLS) and the canonical correlation analysis (CCA).

These families of algorithms are useful to find linear relations between two multivariate datasets: the X and Y arguments of the fit method are 2D arrays.

Cross decomposition algorithms find the fundamental relations between two matrices (X and Y). They are latent variable approaches to modeling the covariance structures in these two spaces. They will try to find the multidimensional direction in the X space that explains the maximum multidimensional variance direction in the Y space. PLS-regression is particularly suited when the matrix of predictors has more variables than observations, and when there is multicollinearity among X values. By contrast, standard regression will fail in these cases.

Classes included in this module are PLSRegression, PLSCanonical, CCA and PLSSVD
Naive Bayes methods are a set of supervised learning algorithms based on applying Bayes’ theorem with the “naive” assumption of independence between every pair of features. Given a class variable $y$ and a dependent feature vector $x_1$ through $x_n$, Bayes’ theorem states the following relationship:

$$P(y \mid x_1, \ldots, x_n) = \frac{P(y)P(x_1, \ldots, x_n \mid y)}{P(x_1, \ldots, x_n)}$$

Using the naive independence assumption that

$$P(x_i \mid y, x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) = P(x_i \mid y),$$

for all $i$, this relationship is simplified to

$$P(y \mid x_1, \ldots, x_n) = \frac{P(y) \prod_{i=1}^{n} P(x_i \mid y)}{P(x_1, \ldots, x_n)}$$

Reference:

- JA Wegelin *A survey of Partial Least Squares (PLS) methods, with emphasis on the two-block case*
Since $P(x_1, \ldots, x_n)$ is constant given the input, we can use the following classification rule:

$$P(y \mid x_1, \ldots, x_n) \propto P(y) \prod_{i=1}^{n} P(x_i \mid y)$$

$$\hat{y} = \arg \max_y P(y) \prod_{i=1}^{n} P(x_i \mid y),$$

and we can use Maximum A Posteriori (MAP) estimation to estimate $P(y)$ and $P(x_i \mid y)$; the former is then the relative frequency of class $y$ in the training set.

The different naive Bayes classifiers differ mainly by the assumptions they make regarding the distribution of $P(x_i \mid y)$.

In spite of their apparently over-simplified assumptions, naive Bayes classifiers have worked quite well in many real-world situations, famously document classification and spam filtering. They require a small amount of training data to estimate the necessary parameters. (For theoretical reasons why naive Bayes works well, and on which types of data it does, see the references below.)

Naive Bayes learners and classifiers can be extremely fast compared to more sophisticated methods. The decoupling of the class conditional feature distributions means that each distribution can be independently estimated as a one dimensional distribution. This in turn helps to alleviate problems stemming from the curse of dimensionality.

On the flip side, although naive Bayes is known as a decent classifier, it is known to be a bad estimator, so the probability outputs from `predict_proba` are not to be taken too seriously.

References:


Gaussian Naive Bayes

`GaussianNB` implements the Gaussian Naive Bayes algorithm for classification. The likelihood of the features is assumed to be Gaussian:

$$P(x_i \mid y) = \frac{1}{\sqrt{2\pi\sigma^2_y}} \exp \left( -\frac{(x_i - \mu_y)^2}{2\sigma^2_y} \right)$$

The parameters $\sigma_y$ and $\mu_y$ are estimated using maximum likelihood.

```python
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> from sklearn.naive_bayes import GaussianNB
>>> gnb = GaussianNB()
>>> y_pred = gnb.fit(iris.data, iris.target).predict(iris.data)
>>> print("Number of mislabeled points out of a total %d points : %d" % (iris.data.shape[0],(iris.target != y_pred).sum()))
```

Number of mislabeled points out of a total 150 points : 6

Multinomial Naive Bayes

`MultinomialNB` implements the naive Bayes algorithm for multinomially distributed data, and is one of the two classic naive Bayes variants used in text classification (where the data are typically represented as word vector counts,
although tf-idf vectors are also known to work well in practice). The distribution is parametrized by vectors $\theta_y = (\theta_{y1}, \ldots, \theta_{yn})$ for each class $y$, where $n$ is the number of features (in text classification, the size of the vocabulary) and $\theta_{yi}$ is the probability $P(x_i \mid y)$ of feature $i$ appearing in a sample belonging to class $y$.

The parameters $\theta_y$ is estimated by a smoothed version of maximum likelihood, i.e. relative frequency counting:

$$\hat{\theta}_{yi} = \frac{N_{yi} + \alpha}{N_y + \alpha n}$$

where $N_{yi} = \sum_{x \in T} x_i$ is the number of times feature $i$ appears in a sample of class $y$ in the training set $T$, and $N_y = \sum_{i=1}^{\mid T \mid} N_{yi}$ is the total count of all features for class $y$.

The smoothing priors $\alpha \geq 0$ accounts for features not present in the learning samples and prevents zero probabilities in further computations. Setting $\alpha = 1$ is called Laplace smoothing, while $\alpha < 1$ is called Lidstone smoothing.

**Bernoulli Naive Bayes**

`BernoulliNB` implements the naive Bayes training and classification algorithms for data that is distributed according to multivariate Bernoulli distributions; i.e., there may be multiple features but each one is assumed to be a binary-valued (Bernoulli, boolean) variable. Therefore, this class requires samples to be represented as binary-valued feature vectors; if handed any other kind of data, a `BernoulliNB` instance may binarize its input (depending on the `binarize` parameter).

The decision rule for Bernoulli naive Bayes is based on

$$P(x_i \mid y) = P(i \mid y)x_i + (1 - P(i \mid y))(1 - x_i)$$

which differs from multinomial NB’s rule in that it explicitly penalizes the non-occurrence of a feature $i$ that is an indicator for class $y$, where the multinomial variant would simply ignore a non-occurring feature.

In the case of text classification, word occurrence vectors (rather than word count vectors) may be used to train and use this classifier. `BernoulliNB` might perform better on some datasets, especially those with shorter documents. It is advisable to evaluate both models, if time permits.

**References:**


**Out-of-core naive Bayes model fitting**

Naive Bayes models can be used to tackle large scale classification problems for which the full training set might not fit in memory. To handle this case, `MultinomialNB`, `BernoulliNB`, and `GaussianNB` expose a `partial_fit` method that can be used incrementally as done with other classifiers as demonstrated in Out-of-core classification of text documents. All naive Bayes classifiers support sample weighting.

Contrary to the `fit` method, the first call to `partial_fit` needs to be passed the list of all the expected class labels. For an overview of available strategies in scikit-learn, see also the out-of-core learning documentation.
Note: The `partial_fit` method call of naive Bayes models introduces some computational overhead. It is recommended to use data chunk sizes that are as large as possible, that is as the available RAM allows.

### 3.1.10 Decision Trees

**Decision Trees (DTs)** are a non-parametric supervised learning method used for *classification* and *regression*. The goal is to create a model that predicts the value of a target variable by learning simple decision rules inferred from the data features.

For instance, in the example below, decision trees learn from data to approximate a sine curve with a set of if-then-else decision rules. The deeper the tree, the more complex the decision rules and the fitter the model.

Some advantages of decision trees are:

- Simple to understand and to interpret. Trees can be visualised.
- Requires little data preparation. Other techniques often require data normalisation, dummy variables need to be created and blank values to be removed. Note however that this module does not support missing values.
- The cost of using the tree (i.e., predicting data) is logarithmic in the number of data points used to train the tree.
- Able to handle both numerical and categorical data. Other techniques are usually specialised in analysing datasets that have only one type of variable. See *algorithms* for more information.
- Able to handle multi-output problems.
- Uses a white box model. If a given situation is observable in a model, the explanation for the condition is easily explained by boolean logic. By contrast, in a black box model (e.g., in an artificial neural network), results may be more difficult to interpret.
- Possible to validate a model using statistical tests. That makes it possible to account for the reliability of the model.
• Performs well even if its assumptions are somewhat violated by the true model from which the data were generated.

The disadvantages of decision trees include:

• Decision-tree learners can create over-complex trees that do not generalise the data well. This is called overfitting. Mechanisms such as pruning (not currently supported), setting the minimum number of samples required at a leaf node or setting the maximum depth of the tree are necessary to avoid this problem.

• Decision trees can be unstable because small variations in the data might result in a completely different tree being generated. This problem is mitigated by using decision trees within an ensemble.

• The problem of learning an optimal decision tree is known to be NP-complete under several aspects of optimality and even for simple concepts. Consequently, practical decision-tree learning algorithms are based on heuristic algorithms such as the greedy algorithm where locally optimal decisions are made at each node. Such algorithms cannot guarantee to return the globally optimal decision tree. This can be mitigated by training multiple trees in an ensemble learner, where the features and samples are randomly sampled with replacement.

• There are concepts that are hard to learn because decision trees do not express them easily, such as XOR, parity or multiplexer problems.

• Decision tree learners create biased trees if some classes dominate. It is therefore recommended to balance the dataset prior to fitting with the decision tree.

Classification

DecisionTreeClassifier is a class capable of performing multi-class classification on a dataset.

As with other classifiers, DecisionTreeClassifier takes as input two arrays: an array X, sparse or dense, of size [n_samples, n_features] holding the training samples, and an array Y of integer values, size [n_samples], holding the class labels for the training samples:

```python
>>> from sklearn import tree
>>> X = [[0, 0], [1, 1]]
>>> Y = [0, 1]
>>> clf = tree.DecisionTreeClassifier()
>>> clf = clf.fit(X, Y)
```

After being fitted, the model can then be used to predict the class of samples:

```python
>>> clf.predict([[2., 2.]])
array([1])
```

Alternatively, the probability of each class can be predicted, which is the fraction of training samples of the same class in a leaf:

```python
>>> clf.predict_proba([[2., 2.]])
array([[0., 1.]])
```

DecisionTreeClassifier is capable of both binary (where the labels are [-1, 1]) classification and multiclass (where the labels are [0, ..., K-1]) classification.

Using the Iris dataset, we can construct a tree as follows:

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn import tree
>>> iris = load_iris()
>>> clf = tree.DecisionTreeClassifier()
>>> clf = clf.fit(iris.data, iris.target)
```
Once trained, we can export the tree in Graphviz format using the `export_graphviz` exporter. Below is an example export of a tree trained on the entire iris dataset:

```python
>>> with open("iris.dot", "w") as f:
...     f = tree.export_graphviz(clf, out_file=f)
```

Then we can use Graphviz's `dot` tool to create a PDF file (or any other supported file type): `dot -Tpdf iris.dot -o iris.pdf`.

```python
>>> import os
>>> os.unlink('iris.dot')
```

Alternatively, if we have Python module `pydotplus` installed, we can generate a PDF file (or any other supported file type) directly in Python:

```python
>>> import pydotplus
>>> dot_data = tree.export_graphviz(clf, out_file=None)
>>> graph = pydotplus.graph_from_dot_data(dot_data)
>>> graph.write_pdf("iris.pdf")
```

The `export_graphviz` exporter also supports a variety of aesthetic options, including coloring nodes by their class (or value for regression) and using explicit variable and class names if desired. IPython notebooks can also render these plots inline using the `Image()` function:

```python
>>> from IPython.display import Image
>>> dot_data = tree.export_graphviz(clf, out_file=None,
...     feature_names=iris.feature_names,
...     class_names=iris.target_names,
...     filled=True, rounded=True,
...     special_characters=True)
>>> graph = pydotplus.graph_from_dot_data(dot_data)
>>> Image(graph.create_png())
```

After being fitted, the model can then be used to predict the class of samples:

```python
>>> clf.predict(iris.data[:1, :])
array([0])
```

Alternatively, the probability of each class can be predicted, which is the fraction of training samples of the same class in a leaf:

```python
>>> clf.predict_proba(iris.data[:1, :])
array([[ 1., 0., 0.]])
```

**Examples:**

- Plot the decision surface of a decision tree on the iris dataset

**Regression**

Decision trees can also be applied to regression problems, using the `DecisionTreeRegressor` class.

As in the classification setting, the fit method will take as argument arrays X and y, only that in this case y is expected to have floating point values instead of integer values:
3.1. Supervised learning
>>> from sklearn import tree
>>> X = [[0, 0], [2, 2]]
>>> y = [0.5, 2.5]
>>> clf = tree.DecisionTreeRegressor()
>>> clf = clf.fit(X, y)
>>> clf.predict([[1, 1]])
asarray([0.5])

Examples:

• Decision Tree Regression

Multi-output problems

A multi-output problem is a supervised learning problem with several outputs to predict, that is when Y is a 2d array of size [n_samples, n_outputs].

When there is no correlation between the outputs, a very simple way to solve this kind of problem is to build n independent models, i.e. one for each output, and then to use those models to independently predict each one of the n outputs. However, because it is likely that the output values related to the same input are themselves correlated, an often better way is to build a single model capable of predicting simultaneously all n outputs. First, it requires lower training time since only a single estimator is built. Second, the generalization accuracy of the resulting estimator may often be increased.

With regard to decision trees, this strategy can readily be used to support multi-output problems. This requires the following changes:

• Store n output values in leaves, instead of 1;
• Use splitting criteria that compute the average reduction across all n outputs.
This module offers support for multi-output problems by implementing this strategy in both `DecisionTreeClassifier` and `DecisionTreeRegressor`. If a decision tree is fit on an output array $Y$ of size $[n_{\text{samples}}, n_{\text{outputs}}]$ then the resulting estimator will:

- Output $n_{\text{output}}$ values upon `predict`;
- Output a list of $n_{\text{output}}$ arrays of class probabilities upon `predict_proba`.

The use of multi-output trees for regression is demonstrated in *Multi-output Decision Tree Regression*. In this example, the input $X$ is a single real value and the outputs $Y$ are the sine and cosine of $X$.

![Multi-output Decision Tree Regression](image.png)

The use of multi-output trees for classification is demonstrated in *Face completion with a multi-output estimators*. In this example, the inputs $X$ are the pixels of the upper half of faces and the outputs $Y$ are the pixels of the lower half of those faces.

**Examples:**

- *Multi-output Decision Tree Regression*
- *Face completion with a multi-output estimators*

**References:**

Face completion with multi-output estimators

true faces  Extra trees  K-nn  Linear regression  Ridge
Complexity

In general, the run time cost to construct a balanced binary tree is $O(n_{samples}n_{features}\log(n_{samples}))$ and query time $O(\log(n_{samples}))$. Although the tree construction algorithm attempts to generate balanced trees, they will not always be balanced. Assuming that the subtrees remain approximately balanced, the cost at each node consists of searching through $O(n_{features})$ to find the feature that offers the largest reduction in entropy. This has a cost of $O(n_{features}n_{samples}\log(n_{samples}))$ at each node, leading to a total cost over the entire trees (by summing the cost at each node) of $O(n_{features}n_{samples}^2\log(n_{samples}))$.

Scikit-learn offers a more efficient implementation for the construction of decision trees. A naive implementation (as above) would recompute the class label histograms (for classification) or the means (for regression) at for each new split point along a given feature. Presorting the feature over all relevant samples, and retaining a running label count, will reduce the complexity at each node to $O(n_{features}\log(n_{samples}))$, which results in a total cost of $O(n_{features}n_{samples}\log(n_{samples}))$. This is an option for all tree based algorithms. By default it is turned on for gradient boosting, where in general it makes training faster, but turned off for all other algorithms as it tends to slow down training when training deep trees.

Tips on practical use

- Decision trees tend to overfit on data with a large number of features. Getting the right ratio of samples to number of features is important, since a tree with few samples in high dimensional space is very likely to overfit.

- Consider performing dimensionality reduction (PCA, ICA, or Feature selection) beforehand to give your tree a better chance of finding features that are discriminative.

- Visualise your tree as you are training by using the export function. Use max_depth=3 as an initial tree depth to get a feel for how the tree is fitting to your data, and then increase the depth.

- Remember that the number of samples required to populate the tree doubles for each additional level the tree grows to. Use max_depth to control the size of the tree to prevent overfitting.

- Use min_samples_split or min_samples_leaf to control the number of samples at a leaf node. A very small number will usually mean the tree will overfit, whereas a large number will prevent the tree from learning the data. Try min_samples_leaf=5 as an initial value. If the sample size varies greatly, a float number can be used as percentage in these two parameters. The main difference between the two is that min_samples_leaf guarantees a minimum number of samples in a leaf, while min_samples_split can create arbitrary small leaves, though min_samples_split is more common in the literature.

- Balance your dataset before training to prevent the tree from being biased toward the classes that are dominant. Class balancing can be done by sampling an equal number of samples from each class, or preferably by normalizing the sum of the sample weights (sample_weight) for each class to the same value. Also note that weight-based pre-pruning criteria, such as min_weight_fraction_leaf, will then be less biased toward dominant classes than criteria that are not aware of the sample weights, like min_samples_leaf.

- If the samples are weighted, it will be easier to optimize the tree structure using weight-based pre-pruning criterion such as min_weight_fraction_leaf, which ensure that leaf nodes contain at least a fraction of the overall sum of the sample weights.

- All decision trees use np.float32 arrays internally. If training data is not in this format, a copy of the dataset will be made.

- If the input matrix X is very sparse, it is recommended to convert to sparse csc_matrix before calling fit and sparse csr_matrix before calling predict. Training time can be orders of magnitude faster for a sparse matrix input compared to a dense matrix when features have zero values in most of the samples.
Tree algorithms: ID3, C4.5, C5.0 and CART

What are all the various decision tree algorithms and how do they differ from each other? Which one is implemented in scikit-learn?

ID3 (Iterative Dichotomiser 3) was developed in 1986 by Ross Quinlan. The algorithm creates a multiway tree, finding for each node (i.e. in a greedy manner) the categorical feature that will yield the largest information gain for categorical targets. Trees are grown to their maximum size and then a pruning step is usually applied to improve the ability of the tree to generalise to unseen data.

C4.5 is the successor to ID3 and removed the restriction that features must be categorical by dynamically defining a discrete attribute (based on numerical variables) that partitions the continuous attribute value into a discrete set of intervals. C4.5 converts the trained trees (i.e. the output of the ID3 algorithm) into sets of if-then rules. These accuracy of each rule is then evaluated to determine the order in which they should be applied. Pruning is done by removing a rule’s precondition if the accuracy of the rule improves without it.

C5.0 is Quinlan’s latest version release under a proprietary license. It uses less memory and builds smaller rule sets than C4.5 while being more accurate.

CART (Classification and Regression Trees) is very similar to C4.5, but it differs in that it supports numerical target variables (regression) and does not compute rule sets. CART constructs binary trees using the feature and threshold that yield the largest information gain at each node.

scikit-learn uses an optimised version of the CART algorithm.

Mathematical formulation

Given training vectors $x_i \in \mathbb{R}^n$, $i=1,\ldots, l$ and a label vector $y \in \mathbb{R}^l$, a decision tree recursively partitions the space such that the samples with the same labels are grouped together.

Let the data at node $m$ be represented by $Q$. For each candidate split $\theta = (j, t_m)$ consisting of a feature $j$ and threshold $t_m$, partition the data into $Q_{left}(\theta)$ and $Q_{right}(\theta)$ subsets

$$Q_{left}(\theta) = \{(x, y)|x_j \leq t_m\}$$
$$Q_{right}(\theta) = Q \setminus Q_{left}(\theta)$$

The impurity at $m$ is computed using an impurity function $H(\cdot)$, the choice of which depends on the task being solved (classification or regression)

$$G(Q, \theta) = \frac{n_{left}}{N_m} H(Q_{left}(\theta)) + \frac{n_{right}}{N_m} H(Q_{right}(\theta))$$

Select the parameters that minimises the impurity

$$\theta^* = \arg\min_\theta G(Q, \theta)$$

Recurse for subsets $Q_{left}(\theta^*)$ and $Q_{right}(\theta^*)$ until the maximum allowable depth is reached, $N_m < \min_{\text{samples}}$ or $N_m = 1$.

Classification criteria

If a target is a classification outcome taking on values 0,1,...,K-1, for node $m$, representing a region $R_m$ with $N_m$ observations, let

$$p_{mk} = 1/N_m \sum_{x_i \in R_m} I(y_i = k)$$
be the proportion of class k observations in node m

Common measures of impurity are Gini

\[ H(X_m) = \sum_k p_{mk}(1 - p_{mk}) \]

Cross-Entropy

\[ H(X_m) = -\sum_k p_{mk} \log(p_{mk}) \]

and Misclassification

\[ H(X_m) = 1 - \max(p_{mk}) \]

**Regression criteria**

If the target is a continuous value, then for node m, representing a region \( R_m \) with \( N_m \) observations, a common criterion to minimise is the Mean Squared Error

\[
c_m = \frac{1}{N_m} \sum_{i \in N_m} y_i
\]

\[
H(X_m) = \frac{1}{N_m} \sum_{i \in N_m} (y_i - c_m)^2
\]

**References:**

- https://en.wikipedia.org/wiki/Predictive_analytics

### 3.1.11 Ensemble methods

The goal of **ensemble methods** is to combine the predictions of several base estimators built with a given learning algorithm in order to improve generalizability / robustness over a single estimator.

Two families of ensemble methods are usually distinguished:

- **In averaging methods**, the driving principle is to build several estimators independently and then to average their predictions. On average, the combined estimator is usually better than any of the single base estimator because its variance is reduced.

  **Examples:** Bagging methods, Forests of randomized trees, ...

- By contrast, in **boosting methods**, base estimators are built sequentially and one tries to reduce the bias of the combined estimator. The motivation is to combine several weak models to produce a powerful ensemble.

  **Examples:** AdaBoost, Gradient Tree Boosting, ...
Bagging meta-estimator

In ensemble algorithms, bagging methods form a class of algorithms which build several instances of a black-box estimator on random subsets of the original training set and then aggregate their individual predictions to form a final prediction. These methods are used as a way to reduce the variance of a base estimator (e.g., a decision tree), by introducing randomization into its construction procedure and then making an ensemble out of it. In many cases, bagging methods constitute a very simple way to improve with respect to a single model, without making it necessary to adapt the underlying base algorithm. As they provide a way to reduce overfitting, bagging methods work best with strong and complex models (e.g., fully developed decision trees), in contrast with boosting methods which usually work best with weak models (e.g., shallow decision trees).

Bagging methods come in many flavours but mostly differ from each other by the way they draw random subsets of the training set:

- When random subsets of the dataset are drawn as random subsets of the samples, then this algorithm is known as Pasting [B1999].
- When samples are drawn with replacement, then the method is known as Bagging [B1996].
- When random subsets of the dataset are drawn as random subsets of the features, then the method is known as Random Subspaces [H1998].
- Finally, when base estimators are built on subsets of both samples and features, then the method is known as Random Patches [LG2012].

In scikit-learn, bagging methods are offered as a unified `BaggingClassifier` meta-estimator (resp. `BaggingRegressor`), taking as input a user-specified base estimator along with parameters specifying the strategy to draw random subsets. In particular, `max_samples` and `max_features` control the size of the subsets (in terms of samples and features), while `bootstrap` and `bootstrap_features` control whether samples and features are drawn with or without replacement. When using a subset of the available samples the generalization accuracy can be estimated with the out-of-bag samples by setting `oob_score=True`. As an example, the snippet below illustrates how to instantiate a bagging ensemble of `KNeighborsClassifier` base estimators, each built on random subsets of 50% of the samples and 50% of the features.

```python
>>> from sklearn.ensemble import BaggingClassifier
>>> from sklearn.neighbors import KNeighborsClassifier
>>> bagging = BaggingClassifier(KNeighborsClassifier(),
...                              max_samples=0.5, max_features=0.5)
```

Examples:

- Single estimator versus bagging: bias-variance decomposition

References

Forests of randomized trees

The `sklearn.ensemble` module includes two averaging algorithms based on randomized decision trees: the RandomForest algorithm and the Extra-Trees method. Both algorithms are perturb-and-combine techniques [B1998] specifically designed for trees. This means a diverse set of classifiers is created by introducing randomness in the classifier construction. The prediction of the ensemble is given as the averaged prediction of the individual classifiers.
As other classifiers, forest classifiers have to be fitted with two arrays: a sparse or dense array \(X\) of size \([n_{\text{samples}}, n_{\text{features}}]\) holding the training samples, and an array \(Y\) of size \([n_{\text{samples}}]\) holding the target values (class labels) for the training samples:

```python
>>> from sklearn.ensemble import RandomForestClassifier
>>> X = [[0, 0], [1, 1]]
>>> Y = [0, 1]
>>> clf = RandomForestClassifier(n_estimators=10)
>>> clf = clf.fit(X, Y)
```

Like decision trees, forests of trees also extend to multi-output problems (if \(Y\) is an array of size \([n_{\text{samples}}, n_{\text{outputs}}]\)).

**Random Forests**

In random forests (see `RandomForestClassifier` and `RandomForestRegressor` classes), each tree in the ensemble is built from a sample drawn with replacement (i.e., a bootstrap sample) from the training set. In addition, when splitting a node during the construction of the tree, the split that is chosen is no longer the best split among all features. Instead, the split that is picked is the best split among a random subset of the features. As a result of this randomness, the bias of the forest usually slightly increases (with respect to the bias of a single non-random tree) but, due to averaging, its variance also decreases, usually more than compensating for the increase in bias, hence yielding an overall better model.

In contrast to the original publication [B2001], the scikit-learn implementation combines classifiers by averaging their probabilistic prediction, instead of letting each classifier vote for a single class.

**Extremely Randomized Trees**

In extremely randomized trees (see `ExtraTreesClassifier` and `ExtraTreesRegressor` classes), randomness goes one step further in the way splits are computed. As in random forests, a random subset of candidate features is used, but instead of looking for the most discriminative thresholds, thresholds are drawn at random for each candidate feature and the best of these randomly-generated thresholds is picked as the splitting rule. This usually allows to reduce the variance of the model a bit more, at the expense of a slightly greater increase in bias:
```python
>>> clf = ExtraTreesClassifier(n_estimators=10, max_depth=None,
... min_samples_split=2, random_state=0)
>>> scores = cross_val_score(clf, X, y)
>>> scores.mean() > 0.999
True
```

### Parameters

The main parameters to adjust when using these methods is `n_estimators` and `max_features`. The former is the number of trees in the forest. The larger the better, but also the longer it will take to compute. In addition, note that results will stop getting significantly better beyond a critical number of trees. The latter is the size of the random subsets of features to consider when splitting a node. The lower the greater the reduction of variance, but also the greater the increase in bias. Empirical good default values are `max_features=n_features` for regression problems, and `max_features=sqrt(n_features)` for classification tasks (where `n_features` is the number of features in the data). Good results are often achieved when setting `max_depth=None` in combination with `min_samples_split=1` (i.e., when fully developing the trees). Bear in mind though that these values are usually not optimal, and might result in models that consume a lot of RAM. The best parameter values should always be cross-validated. In addition, note that in random forests, bootstrap samples are used by default (`bootstrap=True`) while the default strategy for extra-trees is to use the whole dataset (`bootstrap=False`). When using bootstrap sampling the generalization accuracy can be estimated on the left out or out-of-bag samples. This can be enabled by setting `oob_score=True`.

### Parallelization

Finally, this module also features the parallel construction of the trees and the parallel computation of the predictions through the `n_jobs` parameter. If `n_jobs=k` then computations are partitioned into `k` jobs, and run on `k` cores of the machine. If `n_jobs=-1` then all cores available on the machine are used. Note that because of inter-process communication overhead, the speedup might not be linear (i.e., using `k` jobs will unfortunately not be `k` times as fast).
Significant speedup can still be achieved though when building a large number of trees, or when building a single tree requires a fair amount of time (e.g., on large datasets).

Examples:

- Plot the decision surfaces of ensembles of trees on the iris dataset
- Pixel importances with a parallel forest of trees
- Face completion with a multi-output estimators

References

Feature importance evaluation

The relative rank (i.e. depth) of a feature used as a decision node in a tree can be used to assess the relative importance of that feature with respect to the predictability of the target variable. Features used at the top of the tree contribute to the final prediction decision of a larger fraction of the input samples. The expected fraction of the samples they contribute to can thus be used as an estimate of the relative importance of the features.

By averaging those expected activity rates over several randomized trees one can reduce the variance of such an estimate and use it for feature selection.

The following example shows a color-coded representation of the relative importances of each individual pixel for a face recognition task using an ExtraTreesClassifier model.

In practice those estimates are stored as an attribute named feature_importances_ on the fitted model. This is an array with shape (n_features,) whose values are positive and sum to 1.0. The higher the value, the more important is the contribution of the matching feature to the prediction function.

Examples:

- Pixel importances with a parallel forest of trees
- Feature importances with forests of trees

Totally Random Trees Embedding

RandomTreesEmbedding implements an unsupervised transformation of the data. Using a forest of completely random trees, RandomTreesEmbedding encodes the data by the indices of the leaves a data point ends up in. This index is then encoded in a one-of-K manner, leading to a high dimensional, sparse binary coding. This coding can be computed very efficiently and can then be used as a basis for other learning tasks. The size and sparsity of the code can be influenced by choosing the number of trees and the maximum depth per tree. For each tree in the ensemble, the coding contains one entry of one. The size of the coding is at most n_estimators * 2 ** max_depth, the maximum number of leaves in the forest.

As neighboring data points are more likely to lie within the same leaf of a tree, the transformation performs an implicit, non-parametric density estimation.
Examples:

- Hashing feature transformation using Totally Random Trees
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap... compares non-linear dimensionality reduction techniques on handwritten digits.
- Feature transformations with ensembles of trees compares supervised and unsupervised tree based feature transformations.

See also:

Manifold learning techniques can also be useful to derive non-linear representations of feature space, also these approaches focus also on dimensionality reduction.

AdaBoost

The module `sklearn.ensemble` includes the popular boosting algorithm AdaBoost, introduced in 1995 by Freund and Schapire [FS1995].

The core principle of AdaBoost is to fit a sequence of weak learners (i.e., models that are only slightly better than random guessing, such as small decision trees) on repeatedly modified versions of the data. The predictions from all of them are then combined through a weighted majority vote (or sum) to produce the final prediction. The data modifications at each so-called boosting iteration consist of applying weights $w_1, w_2, ..., w_N$ to each of the training samples. Initially, those weights are all set to $w_i = 1/N$, so that the first step simply trains a weak learner on the original data. For each successive iteration, the sample weights are individually modified and the learning algorithm is
reapplied to the reweighted data. At a given step, those training examples that were incorrectly predicted by the boosted model induced at the previous step have their weights increased, whereas the weights are decreased for those that were predicted correctly. As iterations proceed, examples that are difficult to predict receive ever-increasing influence. Each subsequent weak learner is thereby forced to concentrate on the examples that are missed by the previous ones in the sequence [HTF].

AdaBoost can be used both for classification and regression problems:

- For multi-class classification, `AdaBoostClassifier` implements AdaBoost-SAMME and AdaBoost-SAMME.R [ZZRH2009].
- For regression, `AdaBoostRegressor` implements AdaBoost.R2 [D1997].

**Usage**

The following example shows how to fit an AdaBoost classifier with 100 weak learners:

```python
>>> from sklearn.model_selection import cross_val_score
>>> from sklearn.datasets import load_iris
>>> from sklearn.ensemble import AdaBoostClassifier

>>> iris = load_iris()
>>> clf = AdaBoostClassifier(n_estimators=100)
>>> scores = cross_val_score(clf, iris.data, iris.target)
>>> scores.mean()
0.9...
```

The number of weak learners is controlled by the parameter `n_estimators`. The `learning_rate` parameter controls the contribution of the weak learners in the final combination. By default, weak learners are decision stumps. Different weak learners can be specified through the `base_estimator` parameter. The main parameters to tune to obtain good results are `n_estimators` and the complexity of the base estimators (e.g., its depth `max_depth` or minimum required number of samples at a leaf `min_samples_leaf` in case of decision trees).

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Examples:

- **Discrete versus Real AdaBoost** compares the classification error of a decision stump, decision tree, and a boosted decision stump using AdaBoost-SAMME and AdaBoost-SAMME.R.
- **Two-class AdaBoost** shows the decision boundary and decision function values for a non-linearly separable two-class problem using AdaBoost-SAMME.
- **Decision Tree Regression with AdaBoost** demonstrates regression with the AdaBoost.R2 algorithm.

References

**Gradient Tree Boosting**

Gradient Tree Boosting or Gradient Boosted Regression Trees (GBRT) is a generalization of boosting to arbitrary differentiable loss functions. GBRT is an accurate and effective off-the-shelf procedure that can be used for both regression and classification problems. Gradient Tree Boosting models are used in a variety of areas including Web search ranking and ecology.

The advantages of GBRT are:

- Natural handling of data of mixed type (= heterogeneous features)
- Predictive power
- Robustness to outliers in output space (via robust loss functions)

The disadvantages of GBRT are:

- Scalability, due to the sequential nature of boosting it can hardly be parallelized.

The module `sklearn.ensemble` provides methods for both classification and regression via gradient boosted regression trees.

**Classification**

`GradientBoostingClassifier` supports both binary and multi-class classification. The following example shows how to fit a gradient boosting classifier with 100 decision stumps as weak learners:

```python
>>> from sklearn.datasets import make_hastie_10_2
>>> from sklearn.ensemble import GradientBoostingClassifier

>>> X, y = make_hastie_10_2(random_state=0)
>>> X_train, X_test = X[:2000], X[2000:]
>>> y_train, y_test = y[:2000], y[2000:]

>>> clf = GradientBoostingClassifier(n_estimators=100, learning_rate=1.0, ...
... max_depth=1, random_state=0).fit(X_train, y_train)
>>> clf.score(X_test, y_test)
0.913...
```
The number of weak learners (i.e. regression trees) is controlled by the parameter `n_estimators`; the size of each tree can be controlled either by setting the tree depth via `max_depth` or by setting the number of leaf nodes via `max_leaf_nodes`. The learning rate is a hyper-parameter in the range (0.0, 1.0] that controls overfitting via shrinkage.

**Note:** Classification with more than 2 classes requires the induction of `n_classes` regression trees at each iteration, thus, the total number of induced trees equals `n_classes * n_estimators`. For datasets with a large number of classes we strongly recommend to use `RandomForestClassifier` as an alternative to `GradientBoostingClassifier`.

### Regression

`GradientBoostingRegressor` supports a number of different loss functions for regression which can be specified via the argument `loss`; the default loss function for regression is least squares (`'ls'`).

```python
>>> import numpy as np
>>> from sklearn.metrics import mean_squared_error
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.ensemble import GradientBoostingRegressor

>>> X, y = make_friedman1(n_samples=1200, random_state=0, noise=1.0)
>>> X_train, X_test = X[:200], X[200:]
>>> y_train, y_test = y[:200], y[200:]
>>> est = GradientBoostingRegressor(n_estimators=100, learning_rate=0.1,
... max_depth=1, random_state=0, loss='ls').fit(X_train, y_train)
>>> mean_squared_error(y_test, est.predict(X_test))
5.00...
```

The figure below shows the results of applying `GradientBoostingRegressor` with least squares loss and 500 base learners to the Boston house price dataset (`sklearn.datasets.load_boston`). The plot on the left shows the train and test error at each iteration. The train error at each iteration is stored in the `train_score_` attribute of the gradient boosting model. The test error at each iterations can be obtained via the `staged_predict` method which returns a generator that yields the predictions at each stage. Plots like these can be used to determine the optimal number of trees (i.e. `n_estimators`) by early stopping. The plot on the right shows the feature importances which can be obtained via the `feature_importances_` property.
Examples:

- Gradient Boosting regression
- Gradient Boosting Out-of-Bag estimates

Fitting additional weak-learners

Both GradientBoostingRegressor and GradientBoostingClassifier support warm_start=True which allows you to add more estimators to an already fitted model.

```python
>>> _ = est.set_params(n_estimators=200, warm_start=True)  # set warm_start and new nr of trees
>>> _ = est.fit(X_train, y_train)  # fit additional 100 trees to est
>>> mean_squared_error(y_test, est.predict(X_test))
3.84...
```

Controlling the tree size

The size of the regression tree base learners defines the level of variable interactions that can be captured by the gradient boosting model. In general, a tree of depth $h$ can capture interactions of order $h$. There are two ways in which the size of the individual regression trees can be controlled.

If you specify max_depth=$h$ then complete binary trees of depth $h$ will be grown. Such trees will have (at most) $2^h$ leaf nodes and $2^h - 1$ split nodes.

Alternatively, you can control the tree size by specifying the number of leaf nodes via the parameter max_leaf_nodes. In this case, trees will be grown using best-first search where nodes with the highest improvement in impurity will be expanded first. A tree with max_leaf_nodes=$k$ has $k - 1$ split nodes and thus can model interactions of up to order max_leaf_nodes -1.

We found that max_leaf_nodes=$k$ gives comparable results to max_depth=$k-1$ but is significantly faster to train at the expense of a slightly higher training error. The parameter max_leaf_nodes corresponds to the variable $J$ in the chapter on gradient boosting in [F2001] and is related to the parameter interaction.depth in R’s gbm package where max_leaf_nodes == interaction.depth + 1.

Mathematical formulation

GBRT considers additive models of the following form:

$$ F(x) = \sum_{m=1}^{M} \gamma_m h_m(x) $$

where $h_m(x)$ are the basis functions which are usually called weak learners in the context of boosting. Gradient Tree Boosting uses decision trees of fixed size as weak learners. Decision trees have a number of abilities that make them valuable for boosting, namely the ability to handle data of mixed type and the ability to model complex functions.

Similar to other boosting algorithms GBRT builds the additive model in a forward stagewise fashion:

$$ F_m(x) = F_{m-1}(x) + \gamma_m h_m(x) $$
At each stage the decision tree $h_m(x)$ is chosen to minimize the loss function $L$ given the current model $F_{m-1}$ and its fit $F_{m-1}(x_i)$

$$F_m(x) = F_{m-1}(x) + \arg \min_h \sum_{i=1}^{n} L(y_i, F_{m-1}(x_i) - h(x))$$

The initial model $F_0$ is problem specific, for least-squares regression one usually chooses the mean of the target values.

**Note:** The initial model can also be specified via the `init` argument. The passed object has to implement `fit` and `predict`.

Gradient Boosting attempts to solve this minimization problem numerically via steepest descent: The steepest descent direction is the negative gradient of the loss function evaluated at the current model $F_{m-1}$ which can be calculated for any differentiable loss function:

$$F_m(x) = F_{m-1}(x) + \sum_{i=1}^{n} \gamma_m \nabla_F L(y_i, F_{m-1}(x_i))$$

Where the step length $\gamma_m$ is chosen using line search:

$$\gamma_m = \arg \min_\gamma \sum_{i=1}^{n} L(y_i, F_{m-1}(x_i)) - \gamma \frac{\partial L(y_i, F_{m-1}(x_i))}{\partial F_{m-1}(x_i)}$$

The algorithms for regression and classification only differ in the concrete loss function used.

**Loss Functions**

The following loss functions are supported and can be specified using the parameter `loss`:

- **Regression**
  - Least squares (`'ls'`): The natural choice for regression due to its superior computational properties. The initial model is given by the mean of the target values.
  - Least absolute deviation (`'lad'`): A robust loss function for regression. The initial model is given by the median of the target values.
  - Huber (`'huber'`): Another robust loss function that combines least squares and least absolute deviation; use `alpha` to control the sensitivity with regards to outliers (see [F2001] for more details).
  - Quantile (`'quantile'`): A loss function for quantile regression. Use $0 < \alpha < 1$ to specify the quantile. This loss function can be used to create prediction intervals (see *Prediction Intervals for Gradient Boosting Regression*).

- **Classification**
  - Binomial deviance (`'deviance'`): The negative binomial log-likelihood loss function for binary classification (provides probability estimates). The initial model is given by the log odds-ratio.
Multinomial deviance ('deviance'): The negative multinomial log-likelihood loss function for multi-class classification with n_classes mutually exclusive classes. It provides probability estimates. The initial model is given by the prior probability of each class. At each iteration n_classes regression trees have to be constructed which makes GBRT rather inefficient for data sets with a large number of classes.

Exponential loss ('exponential'): The same loss function as AdaBoostClassifier. Less robust to mislabeled examples than 'deviance'; can only be used for binary classification.

Regularization

Shrinkage

[F2001] proposed a simple regularization strategy that scales the contribution of each weak learner by a factor $\nu$:

$$F_m(x) = F_{m-1}(x) + \nu \gamma_m h_m(x)$$

The parameter $\nu$ is also called the learning rate because it scales the step length the gradient descent procedure; it can be set via the learning_rate parameter.

The parameter learning_rate strongly interacts with the parameter n_estimators, the number of weak learners to fit. Smaller values of learning_rate require larger numbers of weak learners to maintain a constant training error. Empirical evidence suggests that small values of learning_rate favor better test error. [HTF2009] recommend to set the learning rate to a small constant (e.g. learning_rate <= 0.1) and choose n_estimators by early stopping. For a more detailed discussion of the interaction between learning_rate and n_estimators see [R2007].

Subsampling

[F1999] proposed stochastic gradient boosting, which combines gradient boosting with bootstrap averaging (bagging). At each iteration the base classifier is trained on a fraction subsample of the available training data. The subsample is drawn without replacement. A typical value of subsample is 0.5.

The figure below illustrates the effect of shrinkage and subsampling on the goodness-of-fit of the model. We can clearly see that shrinkage outperforms no-shrinkage. Subsampling with shrinkage can further increase the accuracy of the model. Subsampling without shrinkage, on the other hand, does poorly.

Another strategy to reduce the variance is by subsampling the features analogous to the random splits in RandomForestClassifier. The number of subsampled features can be controlled via the max_features parameter.

Note: Using a small max_features value can significantly decrease the runtime.

Stochastic gradient boosting allows to compute out-of-bag estimates of the test deviance by computing the improvement in deviance on the examples that are not included in the bootstrap sample (i.e. the out-of-bag examples). The improvements are stored in the attribute oob_improvement_. oob_improvement_[i] holds the improvement in terms of the loss on the OOB samples if you add the i-th stage to the current predictions. Out-of-bag estimates can be used for model selection, for example to determine the optimal number of iterations. OOB estimates are usually very pessimistic thus we recommend to use cross-validation instead and only use OOB if cross-validation is too time consuming.

Examples:
Interpretation

Individual decision trees can be interpreted easily by simply visualizing the tree structure. Gradient boosting models, however, comprise hundreds of regression trees thus they cannot be easily interpreted by visual inspection of the individual trees. Fortunately, a number of techniques have been proposed to summarize and interpret gradient boosting models.

Feature importance

Often features do not contribute equally to predict the target response; in many situations the majority of the features are in fact irrelevant. When interpreting a model, the first question usually is: what are those important features and how do they contributing in predicting the target response?

Individual decision trees intrinsically perform feature selection by selecting appropriate split points. This information can be used to measure the importance of each feature; the basic idea is: the more often a feature is used in the split points of a tree the more important that feature is. This notion of importance can be extended to decision tree ensembles by simply averaging the feature importance of each tree (see Feature importance evaluation for more details).

The feature importance scores of a fit gradient boosting model can be accessed via the feature_importances_ property:

```python
>>> from sklearn.datasets import make_hastie_10_2
>>> from sklearn.ensemble import GradientBoostingClassifier
```
>>> X, y = make_hastie_10_2(random_state=0)
>>> clf = GradientBoostingClassifier(n_estimators=100, learning_rate=1.0,
... max_depth=1, random_state=0).fit(X, y)
>>> clf.feature_importances_
array([ 0.11, 0.1 , 0.11, ...

Examples:

- **Gradient Boosting regression**

Partial dependence

Partial dependence plots (PDP) show the dependence between the target response and a set of ‘target’ features, marginalizing over the values of all other features (the ‘complement’ features). Intuitively, we can interpret the partial dependence as the expected target response \(^1\) as a function of the ‘target’ features \(^2\).

Due to the limits of human perception the size of the target feature set must be small (usually, one or two) thus the target features are usually chosen among the most important features.

The Figure below shows four one-way and one two-way partial dependence plots for the California housing dataset:

One-way PDPs tell us about the interaction between the target response and the target feature (e.g. linear, non-linear). The upper left plot in the above Figure shows the effect of the median income in a district on the median house price; we can clearly see a linear relationship among them.

PDPs with two target features show the interactions among the two features. For example, the two-variable PDP in the above Figure shows the dependence of median house price on joint values of house age and avg. occupants per

---

\(^1\) For classification with \texttt{loss=\textquotesingle deviance\textquotesingle} the target response is \texttt{logit(p)}.
\(^2\) More precisely its the expectation of the target response after accounting for the initial model; partial dependence plots do not include the init model.
household. We can clearly see an interaction between the two features: For an avg. occupancy greater than two, the house price is nearly independent of the house age, whereas for values less than two there is a strong dependence on age.

The module partial_dependence provides a convenience function plot_partial_dependence to create one-way and two-way partial dependence plots. In the below example we show how to create a grid of partial dependence plots: two one-way PDPs for the features 0 and 1 and a two-way PDP between the two features:

```python
>>> from sklearn.datasets import make_hastie_10_2
>>> from sklearn.ensemble import GradientBoostingClassifier
>>> from sklearn.ensemble.partial_dependence import plot_partial_dependence

>>> X, y = make_hastie_10_2(random_state=0)
>>> clf = GradientBoostingClassifier(n_estimators=100, learning_rate=1.0,
... max_depth=1, random_state=0).fit(X, y)
>>> features = [0, 1, (0, 1)]
>>> fig, axs = plot_partial_dependence(clf, X, features)
```

For multi-class models, you need to set the class label for which the PDPs should be created via the label argument:

```python
>>> from sklearn.datasets import load_iris
>>> iris = load_iris()
>>> mc_clf = GradientBoostingClassifier(n_estimators=10,
... max_depth=1).fit(iris.data, iris.target)
>>> features = [3, 2, (3, 2)]
>>> fig, axs = plot_partial_dependence(mc_clf, X, features, label=0)
```

If you need the raw values of the partial dependence function rather than the plots you can use the partial_dependence function:

```python
>>> from sklearn.ensemble.partial_dependence import partial_dependence

>>> pdp, axes = partial_dependence(clf, [0], X=X)
>>> pdp
array([[ 2.46643157, 2.46643157, ...]])
>>> axes
[array([-1.62497054, -1.59201391, ...])]
```

The function requires either the argument grid which specifies the values of the target features on which the partial dependence function should be evaluated or the argument X which is a convenience mode for automatically creating grid from the training data. If X is given, the axes value returned by the function gives the axis for each target feature.

For each value of the ‘target’ features in the grid the partial dependence function need to marginalize the predictions of a tree over all possible values of the ‘complement’ features. In decision trees this function can be evaluated efficiently without reference to the training data. For each grid point a weighted tree traversal is performed: if a split node involves a ‘target’ feature, the corresponding left or right branch is followed, otherwise both branches are followed, each branch is weighted by the fraction of training samples that entered that branch. Finally, the partial dependence is given by a weighted average of all visited leaves. For tree ensembles the results of each individual tree are again averaged.

**Examples:**

- Partial Dependence Plots
VotingClassifier

The idea behind the voting classifier implementation is to combine conceptually different machine learning classifiers and use a majority vote or the average predicted probabilities (soft vote) to predict the class labels. Such a classifier can be useful for a set of equally well performing model in order to balance out their individual weaknesses.

Majority Class Labels (Majority/Hard Voting)

In majority voting, the predicted class label for a particular sample is the class label that represents the majority (mode) of the class labels predicted by each individual classifier.

E.g., if the prediction for a given sample is

- classifier 1 -> class 1
- classifier 2 -> class 1
- classifier 3 -> class 2

the VotingClassifier (with `voting='hard'`) would classify the sample as “class 1” based on the majority class label.

In the cases of a tie, the `VotingClassifier` will select the class based on the ascending sort order. E.g., in the following scenario

- classifier 1 -> class 2
- classifier 2 -> class 1

the class label 1 will be assigned to the sample.

Usage

The following example shows how to fit the majority rule classifier:

```python
>>> from sklearn import datasets
>>> from sklearn.model_selection import cross_val_score
>>> from sklearn.linear_model import LogisticRegression
>>> from sklearn.naive_bayes import GaussianNB
>>> from sklearn.ensemble import RandomForestClassifier,
    VotingClassifier

>>> iris = datasets.load_iris()
>>> X, y = iris.data[:, 1:3], iris.target

>>> clf1 = LogisticRegression(random_state=1)
>>> clf2 = RandomForestClassifier(random_state=1)
>>> clf3 = GaussianNB()

>>> eclf = VotingClassifier(estimators=[('lr', clf1), ('rf', clf2), ('gnb', clf3)],
    voting='hard')

>>> for clf, label in zip([clf1, clf2, clf3, eclf],
    ['Logistic Regression', 'Random Forest', 'naive Bayes', 'Ensemble']):
    ...     scores = cross_val_score(clf, X, y, cv=5, scoring='accuracy')
```
... print("Accuracy: %0.2f (+/- %0.2f) [%s]" % (scores.mean(), scores.std(), -label))
Accuracy: 0.90 (+/- 0.05) [Logistic Regression]
Accuracy: 0.93 (+/- 0.05) [Random Forest]
Accuracy: 0.91 (+/- 0.04) [naive Bayes]
Accuracy: 0.95 (+/- 0.05) [Ensemble]

**Weighted Average Probabilities (Soft Voting)**

In contrast to majority voting (hard voting), soft voting returns the class label as argmax of the sum of predicted probabilities.

Specific weights can be assigned to each classifier via the *weights* parameter. When weights are provided, the predicted class probabilities for each classifier are collected, multiplied by the classifier weight, and averaged. The final class label is then derived from the class label with the highest average probability.

To illustrate this with a simple example, let's assume we have 3 classifiers and a 3-class classification problems where we assign equal weights to all classifiers: \(w_1=1, w_2=1, w_3=1\).

The weighted average probabilities for a sample would then be calculated as follows:

<table>
<thead>
<tr>
<th>classifier</th>
<th>class 1</th>
<th>class 2</th>
<th>class 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>classifier 1</td>
<td>(w_1 \times 0.2)</td>
<td>(w_1 \times 0.5)</td>
<td>(w_1 \times 0.3)</td>
</tr>
<tr>
<td>classifier 2</td>
<td>(w_2 \times 0.6)</td>
<td>(w_2 \times 0.3)</td>
<td>(w_2 \times 0.1)</td>
</tr>
<tr>
<td>classifier 3</td>
<td>(w_3 \times 0.3)</td>
<td>(w_3 \times 0.4)</td>
<td>(w_3 \times 0.3)</td>
</tr>
<tr>
<td>weighted average</td>
<td>0.37</td>
<td>0.4</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Here, the predicted class label is 2, since it has the highest average probability.

The following example illustrates how the decision regions may change when a soft *VotingClassifier* is used based on an linear Support Vector Machine, a Decision Tree, and a K-nearest neighbor classifier:

```python
>>> from sklearn import datasets
>>> from sklearn.tree import DecisionTreeClassifier
>>> from sklearn.neighbors import KNeighborsClassifier
>>> from sklearn.svm import SVC
>>> from itertools import product
>>> from sklearn.ensemble import VotingClassifier

>>> # Loading some example data
>>> iris = datasets.load_iris()
>>> X = iris.data[:, [0,2]]
>>> y = iris.target

>>> # Training classifiers
>>> clf1 = DecisionTreeClassifier(max_depth=4)
>>> clf2 = KNeighborsClassifier(n_neighbors=7)
>>> clf3 = SVC(kernel='rbf', probability=True)
>>> eclf = VotingClassifier(estimators=[('dt', clf1), ('knn', clf2), ('svc', clf3)], voting='soft', weights=[2,1,2])

>>> clf1 = clf1.fit(X,y)
>>> clf2 = clf2.fit(X,y)
>>> clf3 = clf3.fit(X,y)
>>> eclf = eclf.fit(X,y)
```

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Using the VotingClassifier with GridSearch

The VotingClassifier can also be used together with GridSearch in order to tune the hyperparameters of the individual estimators:

```python
>>> from sklearn.model_selection import GridSearchCV
>>> clf1 = LogisticRegression(random_state=1)
>>> clf2 = RandomForestClassifier(random_state=1)
>>> clf3 = GaussianNB()
>>> eclf = VotingClassifier(estimators=[('lr', clf1), ('rf', clf2), ('gnb', clf3)], voting='soft')
>>> params = {'lr__C': [1.0, 100.0], 'rf__n_estimators': [20, 200],}
>>> grid = GridSearchCV(estimator=eclf, param_grid=params, cv=5)
>>> grid = grid.fit(iris.data, iris.target)
```

Usage

In order to predict the class labels based on the predicted class-probabilities (scikit-learn estimators in the VotingClassifier must support predict_proba method):

```python
>>> eclf = VotingClassifier(estimators=[('lr', clf1), ('rf', clf2), ('gnb', clf3)], voting='soft')
```

Optionally, weights can be provided for the individual classifiers:

```python
>>> eclf = VotingClassifier(estimators=[('lr', clf1), ('rf', clf2), ('gnb', clf3)], voting='soft', weights=[2,5,1])
```
Warning: All classifiers in scikit-learn do multiclass classification out-of-the-box. You don’t need to use the sklearn.multiclass module unless you want to experiment with different multiclass strategies.

The sklearn.multiclass module implements meta-estimators to solve multiclass and multilabel classification problems by decomposing such problems into binary classification problems. Multitarget regression is also supported.

- **Multiclass classification** means a classification task with more than two classes; e.g., classify a set of images of fruits which may be oranges, apples, or pears. Multiclass classification makes the assumption that each sample is assigned to one and only one label: a fruit can be either an apple or a pear but not both at the same time.

- **Multilabel classification** assigns to each sample a set of target labels. This can be thought as predicting properties of a data-point that are not mutually exclusive, such as topics that are relevant for a document. A text might be about any of religion, politics, finance or education at the same time or none of these.

- **Multioutput regression** assigns each sample a set of target values. This can be thought of as predicting several properties for each data-point, such as wind direction and magnitude at a certain location.

- **Multioutput-multiclass classification** and **multi-task classification** means that a single estimator has to handle several joint classification tasks. This is both a generalization of the multi-label classification task, which only considers binary classification, as well as a generalization of the multi-class classification task. **The output format is a 2d numpy array or sparse matrix.**

  The set of labels can be different for each output variable. For instance, a sample could be assigned “pear” for an output variable that takes possible values in a finite set of species such as “pear”, “apple”; and “blue” or “green” for a second output variable that takes possible values in a finite set of colors such as “green”, “red”, “blue”, “yellow”...

  This means that any classifiers handling multi-output multiclass or multi-task classification tasks, support the multi-label classification task as a special case. Multi-task classification is similar to the multi-output classification task with different model formulations. For more information, see the relevant estimator documentation.

All scikit-learn classifiers are capable of multiclass classification, but the meta-estimators offered by sklearn.multiclass permit changing the way they handle more than two classes because this may have an effect on classifier performance (either in terms of generalization error or required computational resources).

Below is a summary of the classifiers supported by scikit-learn grouped by strategy; you don’t need the meta-estimators in this class if you’re using one of these, unless you want custom multiclass behavior:

- **Inherently multiclass**: Naive Bayes, LDA and QDA, Decision Trees, Random Forests, Nearest Neighbors, setting multi_class='multinomial' in sklearn.linear_model.LogisticRegression.

- **Support multilabel**: Decision Trees, Random Forests, Nearest Neighbors.

- **One-Vs-One**: sklearn.svm.SVC.

- **One-Vs-All**: all linear models except sklearn.svm.SVC.

Some estimators also support multioutput-multiclass classification tasks Decision Trees, Random Forests, Nearest Neighbors.
Warning: At present, no metric in sklearn.metrics supports the multioutput-multiclass classification task.

Multilabel classification format

In multilabel learning, the joint set of binary classification tasks is expressed with label binary indicator array: each sample is one row of a 2d array of shape (n_samples, n_classes) with binary values: the one, i.e. the non zero elements, corresponds to the subset of labels. An array such as np.array([[1, 0, 0], [0, 1, 1], [0, 0, 0]]) represents label 0 in the first sample, labels 1 and 2 in the second sample, and no labels in the third sample.

Producing multilabel data as a list of sets of labels may be more intuitive. The MultiLabelBinarizer transformer can be used to convert between a collection of collections of labels and the indicator format.

```python
>>> from sklearn.preprocessing import MultiLabelBinarizer
>>> y = [[2, 3, 4], [2], [0, 1, 3], [0, 1, 2, 3, 4], [0, 1, 2]]
>>> MultiLabelBinarizer().fit_transform(y)
array([[0, 0, 1, 1, 1],
       [0, 0, 1, 0, 0],
       [1, 1, 0, 1, 0],
       [1, 1, 1, 1, 1],
       [1, 1, 1, 0, 0]])
```

One-Vs-The-Rest

This strategy, also known as one-vs-all, is implemented in OneVsRestClassifier. The strategy consists in fitting one classifier per class. For each classifier, the class is fitted against all the other classes. In addition to its computational efficiency (only n_classes classifiers are needed), one advantage of this approach is its interpretability. Since each class is represented by one and only one classifier, it is possible to gain knowledge about the class by inspecting its corresponding classifier. This is the most commonly used strategy and is a fair default choice.

Multiclass learning

Below is an example of multiclass learning using OvR:

```python
>>> from sklearn import datasets
>>> from sklearn.multiclass import OneVsRestClassifier
>>> from sklearn.svm import LinearSVC

>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target
>>> OneVsRestClassifier(LinearSVC(random_state=0)).fit(X, y).predict(X)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Multilabel learning

OneVsRestClassifier also supports multilabel classification. To use this feature, feed the classifier an indicator matrix, in which cell [i, j] indicates the presence of label j in sample i.
Examples:

- Multilabel classification

One-Vs-One

OneVsOneClassifier constructs one classifier per pair of classes. At prediction time, the class which received the most votes is selected. In the event of a tie (among two classes with an equal number of votes), it selects the class with the highest aggregate classification confidence by summing over the pair-wise classification confidence levels computed by the underlying binary classifiers.

Since it requires to fit \( n_{\text{classes}} \times (n_{\text{classes}} - 1) / 2 \) classifiers, this method is usually slower than one-vs-the-rest, due to its \( O(n_{\text{classes}}^2) \) complexity. However, this method may be advantageous for algorithms such as kernel algorithms which don’t scale well with \( n_{\text{samples}} \). This is because each individual learning problem only involves a small subset of the data whereas, with one-vs-the-rest, the complete dataset is used \( n_{\text{classes}} \) times.

Multiclass learning

Below is an example of multiclass learning using OvO:

```python
>>> from sklearn import datasets
>>> from sklearn.multiclass import OneVsOneClassifier
>>> from sklearn.svm import LinearSVC

>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target
>>> OneVsOneClassifier(LinearSVC(random_state=0)).fit(X, y).predict(X)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```
Error-Correcting Output-Codes

Output-code based strategies are fairly different from one-vs-the-rest and one-vs-one. With these strategies, each class is represented in a Euclidean space, where each dimension can only be 0 or 1. Another way to put it is that each class is represented by a binary code (an array of 0 and 1). The matrix which keeps track of the location/code of each class is called the code book. The code size is the dimensionality of the aforementioned space. Intuitively, each class should be represented by a code as unique as possible and a good code book should be designed to optimize classification accuracy. In this implementation, we simply use a randomly-generated code book as advocated in 3 although more elaborate methods may be added in the future.

At fitting time, one binary classifier per bit in the code book is fitted. At prediction time, the classifiers are used to project new points in the class space and the class closest to the points is chosen.

In `OutputCodeClassifier`, the `code_size` attribute allows the user to control the number of classifiers which will be used. It is a percentage of the total number of classes.

A number between 0 and 1 will require fewer classifiers than one-vs-the-rest. In theory, \( \log_2(n_{\text{classes}}) / n_{\text{classes}} \) is sufficient to represent each class unambiguously. However, in practice, it may not lead to good accuracy since \( \log_2(n_{\text{classes}}) \) is much smaller than \( n_{\text{classes}} \).

A number greater than 1 will require more classifiers than one-vs-the-rest. In this case, some classifiers will in theory correct for the mistakes made by other classifiers, hence the name “error-correcting”. In practice, however, this may not happen as classifier mistakes will typically be correlated. The error-correcting output codes have a similar effect to bagging.

Multiclass learning

Below is an example of multiclass learning using Output-Codes:

```python
>>> from sklearn import datasets
>>> from sklearn.multiclass import OutputCodeClassifier
>>> from sklearn.svm import LinearSVC

iris = datasets.load_iris()
X, y = iris.data, iris.target
clf = OutputCodeClassifier(LinearSVC(random_state=0),
                          code_size=2, random_state=0)

>>> clf.fit(X, y).predict(X)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Multioutput regression

Multioutput regression support can be added to any regressor with `MultiOutputRegressor`. This strategy consists of fitting one regressor per target. Since each target is represented by exactly one regressor it is possible to gain knowledge about the target by inspecting its corresponding regressor. As `MultiOutputRegressor` fits one regressor per target it can not take advantage of correlations between targets.

Below is an example of multioutput regression:

```python
>>> from sklearn.datasets import make_regression
>>> from sklearn.multioutput import MultiOutputRegressor
>>> from sklearn.ensemble import GradientBoostingRegressor

>>> X, y = make_regression(n_samples=10, n_targets=3, random_state=1)

>>> MultiOutputRegressor(GradientBoostingRegressor(random_state=0)).fit(X, y).predict(X)
array([[-154.75474165, -147.03498585, -50.03812219],
       [  7.12165031,  5.12914884, -81.46081961],
       [-187.8948621 , -100.44373091, 13.88978285],
       [-141.62745778,  95.02891072, -191.48204257],
       [  97.03260883, 165.34867495, 139.52003279],
       [ 123.92529176,  21.25719016, -7.84253  ],
       [-122.25193977, -85.16443186, -107.12274212],
       [-30.170388 , -94.80956739, 12.16979946],
       [ 140.72667194, 176.50941682, -17.50447799],
       [ 149.37967282, -81.15699552, -5.72850319]])
```

Multioutput classification

Multioutput classification support can be added to any classifier with `MultiOutputClassifier`. This strategy consists of fitting one classifier per target. This allows multiple target variable classifications. The purpose of this class is to extend estimators to be able to estimate a series of target functions (f1,f2,f3,...,fn) that are trained on a single X predictor matrix to predict a series of responses (y1,y2,y3,...,yn).

Below is an example of multioutput classification:

```python
>>> from sklearn.datasets import make_classification
>>> from sklearn.multioutput import MultiOutputClassifier
>>> from sklearn.ensemble import RandomForestClassifier
>>> from sklearn.utils import shuffle
>>> import numpy as np

>>> X, y1 = make_classification(n_samples=10, n_features=100, n_informative=30, n_classes=3, random_state=1)

>>> y2 = shuffle(y1, random_state=1)

>>> y3 = shuffle(y1, random_state=2)

>>> Y = np.vstack((y1, y2, y3)).T

>>> n_samples, n_features = X.shape # 10,100
>>> n_outputs = Y.shape[1] # 3
>>> n_classes = 3

>>> forest = RandomForestClassifier(n_estimators=100, random_state=1)

>>> multi_target_forest = MultiOutputClassifier(forest, n_jobs=-1)
```
>>> multi_target_forest.fit(X, Y).predict(X)
array([[2, 2, 0],
[1, 2, 1],
[2, 1, 0],
[0, 0, 2],
[0, 2, 1],
[0, 0, 2],
[1, 1, 0],
[1, 1, 1],
[0, 0, 2],
[2, 0, 0]])

3.1.13 Feature selection

The classes in the `sklearn.feature_selection` module can be used for feature selection/dimensionality reduction on sample sets, either to improve estimators' accuracy scores or to boost their performance on very high-dimensional datasets.

Removing features with low variance

`VarianceThreshold` is a simple baseline approach to feature selection. It removes all features whose variance doesn't meet some threshold. By default, it removes all zero-variance features, i.e. features that have the same value in all samples.

As an example, suppose that we have a dataset with boolean features, and we want to remove all features that are either one or zero (on or off) in more than 80% of the samples. Boolean features are Bernoulli random variables, and the variance of such variables is given by

$$\text{Var}[X] = p(1-p)$$

so we can select using the threshold \(0.8 \times (1 - 0.8)\):

```python
>>> from sklearn.feature_selection import VarianceThreshold
>>> X = [[0, 0, 1], [0, 1, 0], [1, 0, 0], [0, 1, 1], [0, 1, 0], [0, 1, 1]]
>>> sel = VarianceThreshold(threshold=(.8 * (1 - .8)))
>>> sel.fit_transform(X)
array([[0, 1],
[1, 0],
[0, 0],
[1, 1],
[1, 0],
[1, 1]])
```

As expected, `VarianceThreshold` has removed the first column, which has a probability \(p = 5/6 > 0.8\) of containing a zero.

Univariate feature selection

Univariate feature selection works by selecting the best features based on univariate statistical tests. It can be seen as a preprocessing step to an estimator. Scikit-learn exposes feature selection routines as objects that implement the `transform` method:

- `SelectKBest` removes all but the \(k\) highest scoring features
SelectPercentile removes all but a user-specified highest scoring percentage of features

using common univariate statistical tests for each feature: false positive rate SelectFpr, false discovery rate SelectFdr, or family wise error SelectFwe.

GenericUnivariateSelect allows to perform univariate feature selection with a configurable strategy.
This allows to select the best univariate selection strategy with hyper-parameter search estimator.

For instance, we can perform a $\chi^2$ test to the samples to retrieve only the two best features as follows:

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn.feature_selection import SelectKBest
>>> from sklearn.feature_selection import chi2
>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> X.shape
(150, 4)
>>> X_new = SelectKBest(chi2, k=2).fit_transform(X, y)
>>> X_new.shape
(150, 2)
```

These objects take as input a scoring function that returns univariate scores and p-values (or only scores for SelectKBest and SelectPercentile):

- For regression: f_regression, mutual_info_regression
- For classification: chi2, f_classif, mutual_info_classif

The methods based on F-test estimate the degree of linear dependency between two random variables. On the other hand, mutual information methods can capture any kind of statistical dependency, but being nonparametric, they require more samples for accurate estimation.

**Feature selection with sparse data**

If you use sparse data (i.e. data represented as sparse matrices), chi2, mutual_info_regression, mutual_info_classif will deal with the data without making it dense.

**Warning:** Beware not to use a regression scoring function with a classification problem, you will get useless results.

**Examples:**

- Univariate Feature Selection
- Comparison of F-test and mutual information

**Recursive feature elimination**

Given an external estimator that assigns weights to features (e.g., the coefficients of a linear model), recursive feature elimination (RFE) is to select features by recursively considering smaller and smaller sets of features. First, the estimator is trained on the initial set of features and weights are assigned to each one of them. Then, features whose absolute weights are the smallest are pruned from the current set features. That procedure is recursively repeated on the pruned set until the desired number of features to select is eventually reached.

3.1. Supervised learning
RFECV performs RFE in a cross-validation loop to find the optimal number of features.

Examples:

- **Recursive feature elimination**: A recursive feature elimination example showing the relevance of pixels in a digit classification task.
- **Recursive feature elimination with cross-validation**: A recursive feature elimination example with automatic tuning of the number of features selected with cross-validation.

**Feature selection using SelectFromModel**

`SelectFromModel` is a meta-transformer that can be used along with any estimator that has a `coef_` or `feature_importances_` attribute after fitting. The features are considered unimportant and removed, if the corresponding `coef_` or `feature_importances_` values are below the provided threshold parameter. Apart from specifying the threshold numerically, there are built-in heuristics for finding a threshold using a string argument. Available heuristics are “mean”, “median” and float multiples of these like “0.1*mean”.

For examples on how it is to be used refer to the sections below.

Examples

- **Feature selection using SelectFromModel and LassoCV**: Selecting the two most important features from the Boston dataset without knowing the threshold beforehand.

**L1-based feature selection**

Linear models penalized with the L1 norm have sparse solutions: many of their estimated coefficients are zero. When the goal is to reduce the dimensionality of the data to use with another classifier, they can be used along with `feature_selection.SelectFromModel` to select the non-zero coefficients. In particular, sparse estimators useful for this purpose are the `linear_model.Lasso` for regression, and of `linear_model.LogisticRegression` and `svm.LinearSVC` for classification:

```python
>>> from sklearn.svm import LinearSVC
>>> from sklearn.datasets import load_iris
>>> from sklearn.feature_selection import SelectFromModel

>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> X.shape
(150, 4)
>>> lsvc = LinearSVC(C=0.01, penalty="l1", dual=False).fit(X, y)
>>> model = SelectFromModel(lsvc, prefit=True)
>>> X_new = model.transform(X)
>>> X_new.shape
(150, 3)
```

With SVMs and logistic-regression, the parameter C controls the sparsity: the smaller C the fewer features selected. With Lasso, the higher the alpha parameter, the fewer features selected.
Examples:

- **Classification of text documents using sparse features**: Comparison of different algorithms for document classification including L1-based feature selection.

L1-recovery and compressive sensing

For a good choice of alpha, the Lasso can fully recover the exact set of non-zero variables using only few observations, provided certain specific conditions are met. In particular, the number of samples should be “sufficiently large”, or L1 models will perform at random, where “sufficiently large” depends on the number of non-zero coefficients, the logarithm of the number of features, the amount of noise, the smallest absolute value of non-zero coefficients, and the structure of the design matrix X. In addition, the design matrix must display certain specific properties, such as not being too correlated.

There is no general rule to select an alpha parameter for recovery of non-zero coefficients. It can be set by cross-validation (LassoCV or LassoLarsCV), though this may lead to under-penalized models: including a small number of non-relevant variables is not detrimental to prediction score. BIC (LassoLarsIC) tends, on the opposite, to set high values of alpha.


Randomized sparse models

In terms of feature selection, there are some well-known limitations of L1-penalized models for regression and classification. For example, it is known that the Lasso will tend to select an individual variable out of a group of highly correlated features. Furthermore, even when the correlation between features is not too high, the conditions under which L1-penalized methods consistently select “good” features can be restrictive in general.

To mitigate this problem, it is possible to use randomization techniques such as those presented in [B2009] and [M2010]. The latter technique, known as stability selection, is implemented in the module sklearn.linear_model. In the stability selection method, a subsample of the data is fit to a L1-penalized model where the penalty of a random subset of coefficients has been scaled. Specifically, given a subsample of the data \((x_i, y_i), i \in I\), where \(I \subset \{1, 2, \ldots, n\}\) is a random subset of the data of size \(n_I\), the following modified Lasso fit is obtained:

\[
\hat{w}_I = \arg \min_w \frac{1}{2n_I} \sum_{i \in I} (y_i - x_i^T w)^2 + \alpha \sum_{j=1}^p \frac{|w_j|}{s_j},
\]

where \(s_j \in \{s, 1\}\) are independent trials of a fair Bernoulli random variable, and \(0 < s < 1\) is the scaling factor. By repeating this procedure across different random subsamples and Bernoulli trials, one can count the fraction of times the randomized procedure selected each feature, and used these fractions as scores for feature selection.

RandomizedLasso implements this strategy for regression settings, using the Lasso, while RandomizedLogisticRegression uses the logistic regression and is suitable for classification tasks. To get a full path of stability scores you can use lasso_stability_path.

Note that for randomized sparse models to be more powerful than standard F statistics at detecting non-zero features, the ground truth model should be sparse, in other words, there should be only a small fraction of features non zero.
Sparse recovery: feature selection for sparse linear models: An example comparing different feature selection approaches and discussing in which situation each approach is to be favored.

References:

Tree-based feature selection

Tree-based estimators (see the `sklearn.tree` module and forest of trees in the `sklearn.ensemble` module) can be used to compute feature importances, which in turn can be used to discard irrelevant features (when coupled with the `sklearn.feature_selection.SelectFromModel` meta-transformer):

```
>>> from sklearn.ensemble import ExtraTreesClassifier
>>> from sklearn.datasets import load_iris
>>> from sklearn.feature_selection import SelectFromModel
>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> X.shape
(150, 4)
>>> clf = ExtraTreesClassifier()
>>> clf = clf.fit(X, y)
>>> clf.feature_importances_
array([ 0.04..., 0.05..., 0.4..., 0.4...])
>>> model = SelectFromModel(clf, prefit=True)
>>> X_new = model.transform(X)
>>> X_new.shape
(150, 2)
```

Examples:

Feature importances with forests of trees: example on synthetic data showing the recovery of the actually meaningful features.
Feature selection as part of a pipeline

Feature selection is usually used as a pre-processing step before doing the actual learning. The recommended way to do this in scikit-learn is to use a `sklearn.pipeline.Pipeline`:

```python
clf = Pipeline(
    [{'feature_selection', SelectFromModel(LinearSVC(penalty="l1"))},
     {'classification', RandomForestClassifier()}
])
clf.fit(X, y)
```

In this snippet we make use of a `sklearn.svm.LinearSVC` coupled with `sklearn.feature_selection.SelectFromModel` to evaluate feature importances and select the most relevant features. Then, a `sklearn.ensemble.RandomForestClassifier` is trained on the transformed output, i.e. using only relevant features. You can perform similar operations with the other feature selection methods and also classifiers that provide a way to evaluate feature importances of course. See the `sklearn.pipeline.Pipeline` examples for more details.

3.1.14 Semi-Supervised

Semi-supervised learning is a situation in which in your training data some of the samples are not labeled. The semi-supervised estimators in `sklearn.semi_supervised` are able to make use of this additional unlabeled data to better capture the shape of the underlying data distribution and generalize better to new samples. These algorithms can perform well when we have a very small amount of labeled points and a large amount of unlabeled points.

Unlabeled entries in y

It is important to assign an identifier to unlabeled points along with the labeled data when training the model with the `fit` method. The identifier that this implementation uses is the integer value $-1$.

Label Propagation

Label propagation denotes a few variations of semi-supervised graph inference algorithms.

A few features available in this model:

- Can be used for classification and regression tasks
- Kernel methods to project data into alternate dimensional spaces

`scikit-learn` provides two label propagation models: `LabelPropagation` and `LabelSpreading`. Both work by constructing a similarity graph over all items in the input dataset.

`LabelPropagation` and `LabelSpreading` differ in modifications to the similarity matrix that graph and the clamping effect on the label distributions. Clamping allows the algorithm to change the weight of the true ground labeled data to some degree. The `LabelPropagation` algorithm performs hard clamping of input labels, which means $\alpha = 1$. This clamping factor can be relaxed, to say $\alpha = 0.8$, which means that we will always retain 80 percent of our original label distribution, but the algorithm gets to change its confidence of the distribution within 20 percent.

`LabelPropagation` uses the raw similarity matrix constructed from the data with no modifications. In contrast, `LabelSpreading` minimizes a loss function that has regularization properties, as such it is often more robust to
Fig. 3.1: An illustration of label-propagation: the structure of unlabeled observations is consistent with the class structure, and thus the class label can be propagated to the unlabeled observations of the training set.

noise. The algorithm iterates on a modified version of the original graph and normalizes the edge weights by computing the normalized graph Laplacian matrix. This procedure is also used in Spectral clustering.

Label propagation models have two built-in kernel methods. Choice of kernel effects both scalability and performance of the algorithms. The following are available:

- rbf \((\exp(-\gamma |x - y|^2), \gamma > 0)\). \(\gamma\) is specified by keyword gamma.
- knn \((1|x' \in kNN(x)|)\). \(k\) is specified by keyword n_neighbors.

The RBF kernel will produce a fully connected graph which is represented in memory by a dense matrix. This matrix may be very large and combined with the cost of performing a full matrix multiplication calculation for each iteration of the algorithm can lead to prohibitively long running times. On the other hand, the KNN kernel will produce a much more memory-friendly sparse matrix which can drastically reduce running times.

Examples

- Decision boundary of label propagation versus SVM on the Iris dataset
- Label Propagation learning a complex structure
- Label Propagation digits active learning

References


3.1.15 Isotonic regression

The class IsotonicRegression fits a non-decreasing function to data. It solves the following problem:

\[
\begin{align*}
\text{minimize} & \quad \sum_i w_i (y_i - \hat{y}_i)^2 \\
\text{subject to} & \quad \hat{y}_{\min} = \hat{y}_1 \leq \hat{y}_2 \ldots \leq \hat{y}_n = \hat{y}_{\max}
\end{align*}
\]
where each $w_i$ is strictly positive and each $y_i$ is an arbitrary real number. It yields the vector which is composed of non-decreasing elements the closest in terms of mean squared error. In practice this list of elements forms a function that is piecewise linear.

3.1.16 Probability calibration

When performing classification you often want not only to predict the class label, but also obtain a probability of the respective label. This probability gives you some kind of confidence on the prediction. Some models can give you poor estimates of the class probabilities and some even do not support probability prediction. The calibration module allows you to better calibrate the probabilities of a given model, or to add support for probability prediction.

Well calibrated classifiers are probabilistic classifiers for which the output of the predict_proba method can be directly interpreted as a confidence level. For instance, a well calibrated (binary) classifier should classify the samples such that among the samples to which it gave a predict_proba value close to 0.8, approximately 80% actually belong to the positive class. The following plot compares how well the probabilistic predictions of different classifiers are calibrated:

LogisticRegression returns well calibrated predictions by default as it directly optimizes log-loss. In contrast, the other methods return biased probabilities; with different biases per method:

- GaussianNB tends to push probabilities to 0 or 1 (note the counts in the histograms). This is mainly because it makes the assumption that features are conditionally independent given the class, which is not the case in this dataset which contains 2 redundant features.
**RandomForestClassifier** shows the opposite behavior: the histograms show peaks at approximately 0.2 and 0.9 probability, while probabilities close to 0 or 1 are very rare. An explanation for this is given by Niculescu-Mizil and Caruana [4]: “Methods such as bagging and random forests that average predictions from a base set of models can have difficulty making predictions near 0 and 1 because variance in the underlying base models will bias predictions that should be near zero or one away from these values. Because predictions are restricted to the interval [0,1], errors caused by variance tend to be one-sided near zero and one. For example, if a model should predict \( p = 0 \) for a case, the only way bagging can achieve this is if all bagged trees predict zero. If we add noise to the trees that bagging is averaging over, this noise will cause some trees to predict values larger than 0 for this case, thus moving the average prediction of the bagged ensemble away from 0. We observe this effect most strongly with random forests because the base-level trees trained with random forests have relatively high variance due to feature subseting.” As a result, the calibration curve shows a characteristic sigmoid shape, indicating that the classifier could trust its “intuition” more and return probabilities closer to 0 or 1 typically.

**Linear Support Vector Classification (LinearSVC)** shows an even more sigmoid curve as the RandomForestClassifier, which is typical for maximum-margin methods (compare Niculescu-Mizil and Caruana [4]), which focus on hard samples that are close to the decision boundary (the support vectors).

Two approaches for performing calibration of probabilistic predictions are provided: a parametric approach based on Platt’s sigmoid model and a non-parametric approach based on isotonic regression (sklearn.isotonic). Probability calibration should be done on new data not used for model fitting. The class CalibratedClassifierCV uses a cross-validation generator and estimates for each split the model parameter on the train samples and the calibration of the test samples. The probabilities predicted for the folds are then averaged. Already fitted classifiers can be calibrated by CalibratedClassifierCV via the parameter cv="prefit". In this case, the user has to take care manually that data for model fitting and calibration are disjoint.

The following images demonstrate the benefit of probability calibration. The first image presents a dataset with 2 classes and 3 blobs of data. The blob in the middle contains random samples of each class. The probability for the samples in this blob should be 0.5.

The following image shows on the data above the estimated probability using a Gaussian naive Bayes classifier without calibration, with a sigmoid calibration and with a non-parametric isotonic calibration. One can observe that the non-parametric model provides the most accurate probability estimates for samples in the middle, i.e., 0.5.

The following experiment is performed on an artificial dataset for binary classification with 100,000 samples (1,000 of them are used for model fitting) with 20 features. Of the 20 features, only 2 are informative and 10 are redundant. The figure shows the estimated probabilities obtained with logistic regression, a linear support-vector classifier (SVC), and linear SVC with both isotonic calibration and sigmoid calibration. The calibration performance is evaluated with Brier score brier_score_loss, reported in the legend (the smaller the better).

One can observe here that logistic regression is well calibrated as its curve is nearly diagonal. Linear SVC’s calibration curve has a sigmoid curve, which is typical for an under-confident classifier. In the case of LinearSVC, this is caused by the margin property of the hinge loss, which lets the model focus on hard samples that are close to the decision boundary (the support vectors). Both kinds of calibration can fix this issue and yield nearly identical results. The next figure shows the calibration curve of Gaussian naive Bayes on the same data, with both kinds of calibration and also without calibration.

One can see that Gaussian naive Bayes performs very badly but does so in an other way than linear SVC: While linear SVC exhibited a sigmoid calibration curve, Gaussian naive Bayes’ calibration curve has a transposed-sigmoid shape. This is typical for an over-confident classifier. In this case, the classifier’s overconfidence is caused by the redundant features which violate the naive Bayes assumption of feature-independence.

Calibration of the probabilities of Gaussian naive Bayes with isotonic regression can fix this issue as can be seen from the nearly diagonal calibration curve. Sigmoid calibration also improves the brier score slightly, albeit not as strongly as the non-parametric isotonic calibration. This is an intrinsic limitation of sigmoid calibration, whose parametric form assumes a sigmoid rather than a transposed-sigmoid curve. The non-parametric isotonic calibration model, however, makes no such strong assumptions and can deal with either shape, provided that there is sufficient calibration data. In general, sigmoid calibration is preferable in cases where the calibration curve is sigmoid and where there is limited
calibration data, while isotonic calibration is preferable for non-sigmoid calibration curves and in situations where large amounts of data are available for calibration.

`CalibratedClassifierCV` can also deal with classification tasks that involve more than two classes if the base estimator can do so. In this case, the classifier is calibrated first for each class separately in an one-vs-rest fashion. When predicting probabilities for unseen data, the calibrated probabilities for each class are predicted separately. As those probabilities do not necessarily sum to one, a postprocessing is performed to normalize them.

The next image illustrates how sigmoid calibration changes predicted probabilities for a 3-class classification problem. Illustrated is the standard 2-simplex, where the three corners correspond to the three classes. Arrows point from the probability vectors predicted by an uncalibrated classifier to the probability vectors predicted by the same classifier after sigmoid calibration on a hold-out validation set. Colors indicate the true class of an instance (red: class 1, green: class 2, blue: class 3).

The base classifier is a random forest classifier with 25 base estimators (trees). If this classifier is trained on all 800 training datapoints, it is overly confident in its predictions and thus incurs a large log-loss. Calibrating an identical classifier, which was trained on 600 datapoints, with method='sigmoid' on the remaining 200 datapoints reduces the confidence of the predictions, i.e., moves the probability vectors from the edges of the simplex towards the center:

This calibration results in a lower log-loss. Note that an alternative would have been to increase the number of base estimators which would have resulted in a similar decrease in log-loss.
Illustration of sigmoid calibrator

- Probability class 1
- Probability class 2
3.1.17 Neural network models (supervised)

**Warning:** This implementation is not intended for large-scale applications. In particular, scikit-learn offers no GPU support. For much faster, GPU-based implementations, as well as frameworks offering much more flexibility to build deep learning architectures, see Related Projects.

**Multi-layer Perceptron**

Multi-layer Perceptron (MLP) is a supervised learning algorithm that learns a function \( f(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^o \) by training on a dataset, where \( m \) is the number of dimensions for input and \( o \) is the number of dimensions for output. Given a set of features \( X = x_1, x_2, ..., x_m \) and a target \( y \), it can learn a non-linear function approximator for either classification or regression. It is different from logistic regression, in that between the input and the output layer, there can be one or more non-linear layers, called hidden layers. Figure 1 shows a one hidden layer MLP with scalar output.

![One hidden layer MLP](image)

**Fig. 3.2: Figure 1 : One hidden layer MLP.**

The leftmost layer, known as the input layer, consists of a set of neurons \( \{ x_i | x_1, x_2, ..., x_m \} \) representing the input features. Each neuron in the hidden layer transforms the values from the previous layer with a weighted linear sum \( w_1x_1 + w_2x_2 + ... + w_mx_m \), followed by a non-linear activation function \( g(\cdot): \mathbb{R} \rightarrow \mathbb{R} \) - like the hyperbolic tan function. The output layer receives the values from the last hidden layer and transforms them into output values.

The module contains the public attributes `coefs_` and `intercepts_`. `coefs_` is a list of weight matrices, where weight matrix at index \( i \) represents the weights between layer \( i \) and layer \( i+1 \). `intercepts_` is a list of bias vectors, where the vector at index \( i \) represents the bias values added to layer \( i+1 \).

The advantages of Multi-layer Perceptron are:
- Capability to learn non-linear models.
- Capability to learn models in real-time (on-line learning) using `partial_fit`.

The disadvantages of Multi-layer Perceptron (MLP) include:
• MLP with hidden layers have a non-convex loss function where there exists more than one local minimum. Therefore different random weight initializations can lead to different validation accuracy.
• MLP requires tuning a number of hyperparameters such as the number of hidden neurons, layers, and iterations.
• MLP is sensitive to feature scaling.

Please see *Tips on Practical Use* section that addresses some of these disadvantages.

### Classification

Class `MLPClassifier` implements a multi-layer perceptron (MLP) algorithm that trains using Backpropagation.

MLP trains on two arrays: array X of size \((n\text{\_samples}, n\text{\_features})\), which holds the training samples represented as floating point feature vectors; and array y of size \((n\text{\_samples})\), which holds the target values (class labels) for the training samples:

```python
>>> from sklearn.neural_network import MLPClassifier
>>> X = [[0., 0.], [1., 1.]]
>>> y = [0, 1]
>>> clf = MLPClassifier(solver='lbfgs', alpha=1e-5,
...                     hidden_layer_sizes=(5, 2), random_state=1)
... >>> clf.fit(X, y)
```

```python
MLPClassifier(activation='relu', alpha=1e-05, batch_size='auto',
               beta_1=0.9, beta_2=0.999, early_stopping=False,
               epsilon=1e-08, hidden_layer_sizes=(5, 2), learning_rate='constant',
               learning_rate_init=0.001, max_iter=200, momentum=0.9,
               nesterovs_momentum=True, power_t=0.5, random_state=1, shuffle=True,
               solver='lbfgs', tol=0.0001, validation_fraction=0.1, verbose=False,
               warm_start=False)
```

After fitting (training), the model can predict labels for new samples:

```python
>>> clf.predict([[2., 2.], [-1., -2.]])
array([1, 0])
```

MLP can fit a non-linear model to the training data. `clf.coefs_` contains the weight matrices that constitute the model parameters:

```python
>>> [coef.shape for coef in clf.coefs_]
[(2, 5), (5, 2), (2, 1)]
```

Currently, `MLPClassifier` supports only the Cross-Entropy loss function, which allows probability estimates by running the `predict_proba` method.

MLP trains using Backpropagation. More precisely, it trains using some form of gradient descent and the gradients are calculated using Backpropagation. For classification, it minimizes the Cross-Entropy loss function, giving a vector of probability estimates \(P(y|x)\) per sample \(x\):

```python
>>> clf.predict_proba([[2., 2.], [1., 2.]])
array([[ 1.967...e-04, 9.998...-01],
       [ 1.967...e-04, 9.998...-01]])
```

`MLPClassifier` supports multi-class classification by applying `Softmax` as the output function.

Further, the model supports *multi-label classification* in which a sample can belong to more than one class. For each class, the raw output passes through the logistic function. Values larger or equal to 0.5 are rounded to 1, otherwise to 0. For a predicted output of a sample, the indices where the value is 1 represents the assigned classes of that sample:
>>> X = [[0., 0.], [1., 1.]]
>>> y = [[0, 1], [1, 1]]
>>> clf = MLPClassifier(solver='lbfgs', alpha=1e-5,
...                     hidden_layer_sizes=(15,), random_state=1)
... clf.fit(X, y)
MLPClassifier(activation='relu', alpha=1e-05, batch_size='auto',
               beta_1=0.9, beta_2=0.999, early_stopping=False,
               epsilon=1e-08, hidden_layer_sizes=(15,), learning_rate='constant',
               learning_rate_init=0.001, max_iter=200, momentum=0.9,
               nesterovs_momentum=True, power_t=0.5, random_state=1, shuffle=True,
               solver='lbfgs', tol=0.0001, validation_fraction=0.1, verbose=False,
               warm_start=False)
>>> clf.predict([[1., 2.]])
array([[1, 1]])
>>> clf.predict([[0., 0.]])
array([[0, 1]])

See the examples below and the doc string of **MLPClassifier.fit** for further information.

**Examples:**

- Compare Stochastic learning strategies for **MLPClassifier**
- Visualization of MLP weights on MNIST

**Regression**

Class **MLPRegressor** implements a multi-layer perceptron (MLP) that trains using backpropagation with no activation function in the output layer, which can also be seen as using the identity function as activation function. Therefore, it uses the square error as the loss function, and the output is a set of continuous values.

**MLPRegressor** also supports multi-output regression, in which a sample can have more than one target.

**Regularization**

Both **MLPRegressor** and class **MLPClassifier** use parameter alpha for regularization (L2 regularization) term which helps in avoiding overfitting by penalizing weights with large magnitudes. Following plot displays varying decision function with value of alpha.

See the examples below for further information.

**Examples:**

- Varying regularization in Multi-layer Perceptron
Algorithms

MLP trains using Stochastic Gradient Descent, Adam, or L-BFGS. Stochastic Gradient Descent (SGD) updates parameters using the gradient of the loss function with respect to a parameter that needs adaptation, i.e.

\[ w \leftarrow w - \eta \left( \partial R(w) / \partial w + \partial \text{Loss} / \partial w \right) \]

where \( \eta \) is the learning rate which controls the step-size in the parameter space search. \text{Loss} is the loss function used for the network.

More details can be found in the documentation of SGD

Adam is similar to SGD in a sense that it is a stochastic optimizer, but it can automatically adjust the amount to update parameters based on adaptive estimates of lower-order moments.

With SGD or Adam, training supports online and mini-batch learning.

L-BFGS is a solver that approximates the Hessian matrix which represents the second-order partial derivative of a function. Further it approximates the inverse of the Hessian matrix to perform parameter updates. The implementation uses the Scipy version of L-BFGS.

If the selected solver is ‘L-BFGS’, training does not support online nor mini-batch learning.

Complexity

Suppose there are \( n \) training samples, \( m \) features, \( k \) hidden layers, each containing \( h \) neurons - for simplicity, and \( o \) output neurons. The time complexity of backpropagation is \( O(n \cdot m \cdot h^k \cdot o \cdot i) \), where \( i \) is the number of iterations. Since backpropagation has a high time complexity, it is advisable to start with smaller number of hidden neurons and few hidden layers for training.

Mathematical formulation

Given a set of training examples \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) where \( x_i \in \mathbb{R}^n \) and \( y_i \in \{0, 1\} \), a one hidden layer one hidden neuron MLP learns the function \( f(x) = W_2g(W_1^T x + b_1) + b_2 \) where \( W_1 \in \mathbb{R}^{m} \) and \( W_2, b_1, b_2 \in \mathbb{R} \) are model parameters. \( W_1, W_2 \) represent the weights of the input layer and hidden layer, respectively; and \( b_1, b_2 \) represent the bias added to the hidden layer and the output layer, respectively. \( g(\cdot) : R \rightarrow R \) is the activation function, set by
default as the hyperbolic tan. It is given as,

\[ g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}} \]

For binary classification, \( f(x) \) passes through the logistic function \( g(z) = \frac{1}{1+e^{-z}} \) to obtain output values between zero and one. A threshold, set to 0.5, would assign samples of outputs larger or equal 0.5 to the positive class, and the rest to the negative class.

If there are more than two classes, \( f(x) \) itself would be a vector of size \((n_{\text{classes}},)\). Instead of passing through logistic function, it passes through the softmax function, which is written as,

\[
\text{softmax}(z)_i = \frac{\exp(z_i)}{\sum_{k=1}^{K} \exp(z_k)}
\]

where \( z_i \) represents the \( i \) th element of the input to softmax, which corresponds to class \( i \), and \( K \) is the number of classes. The result is a vector containing the probabilities that sample \( x \) belong to each class. The output is the class with the highest probability.

In regression, the output remains as \( f(x) \); therefore, output activation function is just the identity function.

MLP uses different loss functions depending on the problem type. The loss function for classification is Cross-Entropy, which in binary case is given as,

\[
\text{Loss}(\hat{y}, y, W) = -y \ln \hat{y} - (1 - y) \ln (1 - \hat{y}) + \alpha ||W||_2^2
\]

where \( \alpha ||W||_2^2 \) is an L2-regularization term (aka penalty) that penalizes complex models; and \( \alpha > 0 \) is a non-negative hyperparameter that controls the magnitude of the penalty.

For regression, MLP uses the Square Error loss function; written as,

\[
\text{Loss}(\hat{y}, y, W) = \frac{1}{2} ||\hat{y} - y||_2^2 + \alpha ||W||_2^2
\]

Starting from initial random weights, multi-layer perceptron (MLP) minimizes the loss function by repeatedly updating these weights. After computing the loss, a backward pass propagates it from the output layer to the previous layers, providing each weight parameter with an update value meant to decrease the loss.

In gradient descent, the gradient \( \nabla \text{Loss}_W \) of the loss with respect to the weights is computed and deducted from \( W \). More formally, this is expressed as,

\[
W^{i+1} = W^i - \epsilon \nabla \text{Loss}_W^i
\]

where \( i \) is the iteration step, and \( \epsilon \) is the learning rate with a value larger than 0.

The algorithm stops when it reaches a preset maximum number of iterations; or when the improvement in loss is below a certain, small number.

**Tips on Practical Use**

- Multi-layer Perceptron is sensitive to feature scaling, so it is highly recommended to scale your data. For example, scale each attribute on the input vector \( X \) to \([0, 1]\) or \([-1, +1]\), or standardize it to have mean 0 and variance 1. Note that you must apply the *same* scaling to the test set for meaningful results. You can use `StandardScaler` for standardization.

```python
>>> from sklearn.preprocessing import StandardScaler
>>> scaler = StandardScaler()
>>> # Don't cheat - fit only on training data
>>> scaler.fit(X_train)
>>> X_train = scaler.transform(X_train)
>>> # apply same transformation to test data
>>> X_test = scaler.transform(X_test)
```
An alternative and recommended approach is to use `StandardScaler` in a `Pipeline`:

- Finding a reasonable regularization parameter \( \alpha \) is best done using `GridSearchCV`, usually in the range \( 10.0 ** -\text{np.arange}(1,7) \).
- Empirically, we observed that `L-BFGS` converges faster and with better solutions on small datasets. For relatively large datasets, however, `Adam` is very robust. It usually converges quickly and gives pretty good performance. `SGD` with momentum or nesterov’s momentum, on the other hand, can perform better than those two algorithms if learning rate is correctly tuned.

### More control with warm_start

If you want more control over stopping criteria or learning rate in SGD, or want to do additional monitoring, using `warm_start=True` and `max_iter=1` and iterating yourself can be helpful:

```python
>>> X = [[0., 0.], [1., 1.]]
>>> y = [0, 1]
>>> clf = MLPClassifier(hidden_layer_sizes=(15,), random_state=1, max_iter=1, warm_start=True)
>>> for i in range(10):
...     clf.fit(X, y)
...     # additional monitoring / inspection
```

### References:

- “Backpropagation” Andrew Ng, Jiquan Ngiam, Chuan Yu Foo, Yifan Mai, Caroline Suen - Website, 2011.

## 3.2 Unsupervised learning

### 3.2.1 Gaussian mixture models

`sklearn.mixture` is a package which enables one to learn Gaussian Mixture Models (diagonal, spherical, tied and full covariance matrices supported), sample them, and estimate them from data. Facilities to help determine the appropriate number of components are also provided.

A Gaussian mixture model is a probabilistic model that assumes all the data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters. One can think of mixture models as generalizing k-means clustering to incorporate information about the covariance structure of the data as well as the centers of the latent Gaussians.

Scikit-learn implements different classes to estimate Gaussian mixture models, that correspond to different estimation strategies, detailed below.
Gaussian Mixture

The `GaussianMixture` object implements the expectation-maximization (EM) algorithm for fitting mixture-of-Gaussian models. It can also draw confidence ellipsoids for multivariate models, and compute the Bayesian Information Criterion to assess the number of clusters in the data. A `GaussianMixture.fit` method is provided that learns a Gaussian Mixture Model from train data. Given test data, it can assign to each sample the Gaussian it mostly probably belong to using the `GaussianMixture.predict` method.

The `GaussianMixture` comes with different options to constrain the covariance of the difference classes estimated: spherical, diagonal, tied or full covariance.

**Examples:**

- See `GMM covariances` for an example of using the Gaussian mixture as clustering on the iris dataset.
- See `Density Estimation for a Gaussian mixture` for an example on plotting the density estimation.

**Pros and cons of class GaussianMixture**

**Pros**

- **Speed** It is the fastest algorithm for learning mixture models
- **Agnostic** As this algorithm maximizes only the likelihood, it will not bias the means towards zero, or bias the cluster sizes to have specific structures that might or might not apply.

**Cons**

- **Singularities** When one has insufficiently many points per mixture, estimating the covariance matrices becomes difficult, and the algorithm is known to diverge and find solutions with infinite likelihood unless one regularizes the covariances artificially.
**Number of components**  This algorithm will always use all the components it has access to, needing held-out data or information theoretical criteria to decide how many components to use in the absence of external cues.

**Selecting the number of components in a classical Gaussian Mixture Model**

The BIC criterion can be used to select the number of components in a Gaussian Mixture in an efficient way. In theory, it recovers the true number of components only in the asymptotic regime (i.e. if much data is available and assuming that the data was actually generated i.i.d. from a mixture of Gaussian distribution). Note that using a Variational Bayesian Gaussian mixture avoids the specification of the number of components for a Gaussian mixture model.

**Examples:**

- See *Gaussian Mixture Model Selection* for an example of model selection performed with classical Gaussian mixture.

**Estimation algorithm Expectation-maximization**

The main difficulty in learning Gaussian mixture models from unlabeled data is that it is one usually doesn’t know which points came from which latent component (if one has access to this information it gets very easy to fit a separate Gaussian distribution to each set of points). Expectation-maximization is a well-founded statistical algorithm to get around this problem by an iterative process. First one assumes random components (randomly centered on data points,
learned from k-means, or even just normally distributed around the origin) and computes for each point a probability of being generated by each component of the model. Then, one tweaks the parameters to maximize the likelihood of the data given those assignments. Repeating this process is guaranteed to always converge to a local optimum.

**Variational Bayesian Gaussian Mixture**

The `BayesianGaussianMixture` object implements a variant of the Gaussian mixture model with variational inference algorithms. The API is similar as the one defined by `GaussianMixture`.

**Estimation algorithm: variational inference**

Variational inference is an extension of expectation-maximization that maximizes a lower bound on model evidence (including priors) instead of data likelihood. The principle behind variational methods is the same as expectation-maximization (that is both are iterative algorithms that alternate between finding the probabilities for each point to be generated by each mixture and fitting the mixture to these assigned points), but variational methods add regularization by integrating information from prior distributions. This avoids the singularities often found in expectation-maximization solutions but introduces some subtle biases to the model. Inference is often notably slower, but not usually as much so as to render usage unpractical.

Due to its Bayesian nature, the variational algorithm needs more hyper-parameters than expectation-maximization, the most important of these being the concentration parameter `weight_concentration_prior`. Specifying a low value for the concentration prior will make the model put most of the weight on few components set the remaining components weights very close to zero. High values of the concentration prior will allow a larger number of components to be active in the mixture.

The parameters implementation of the `BayesianGaussianMixture` class proposes two types of prior for the weights distribution: a finite mixture model with Dirichlet distribution and an infinite mixture model with the Dirichlet Process. In practice Dirichlet Process inference algorithm is approximated and uses a truncated distribution with a fixed maximum number of components (called the Stick-breaking representation). The number of components actually used almost always depends on the data.

The next figure compares the results obtained for the different type of the weight concentration prior (parameter `weight_concentration_prior_type`) for different values of `weight_concentration_prior`. Here, we can see the the value of the `weight_concentration_prior` parameter has a strong impact on the effective number of active components obtained. We can also notice that large values for the concentration weight prior lead to more uniform weights when the type of prior is `dirichlet_distribution` while this is not necessarily the case for the `dirichlet_process` type (used by default).
The examples below compare Gaussian mixture models with a fixed number of components, to the variational Gaussian mixture models with a Dirichlet process prior. Here, a classical Gaussian mixture is fitted with 5 components on a dataset composed of 2 clusters. We can see that the variational Gaussian mixture with a Dirichlet process prior is able to limit itself to only 2 components whereas the Gaussian mixture fits the data with a fixed number of components that has to be set a priori by the user. In this case the user has selected `n_components=5` which does not match the true generative distribution of this toy dataset. Note that with very little observations, the variational Gaussian mixture models with a Dirichlet process prior can take a conservative stand, and fit only one component.

On the following figure we are fitting a dataset not well-depicted by a Gaussian mixture. Adjusting the `weight_concentration_prior` parameter of the class: `BayesianGaussianMixture` controls the number of components used to fit this data. We also present on the last two plots a random sampling generated from the two resulting mixtures.

**Examples:**

- See *Gaussian Mixture Model Ellipsoids* for an example on plotting the confidence ellipsoids for both `GaussianMixture` and `BayesianGaussianMixture`.

- *Gaussian Mixture Model Sine Curve* shows using `GaussianMixture` and `BayesianGaussianMixture` to fit a sine wave.
Pros and cons of variational inference with `BayesianGaussianMixture`

**Pros**

**Automatic selection** when `weight_concentration_prior` is small enough and `n_components` is larger than what is found necessary by the model, the Variational Bayesian mixture model has a natural tendency to set some mixture weights values close to zero. This makes it possible to let the model choose a suitable number of effective components automatically. Only an upper bound of this number needs to be provided. Note however that the “ideal” number of active components is very application specific and is typically ill-defined in a data exploration setting.

**Less sensitivity to the number of parameters** unlike finite models, which will almost always use all components as much as they can, and hence will produce wildly different solutions for different numbers of components, the variational inference with a Dirichlet process prior (`weight_concentration_prior_type='dirichlet_process'`) won’t change much with changes to the parameters, leading to more stability and less tuning.

**Regularization** due to the incorporation of prior information, variational solutions have less pathological special cases than expectation-maximization solutions.
Cons

**Speed** the extra parametrization necessary for variational inference make inference slower, although not by much.

**Hyperparameters** this algorithm needs an extra hyperparameter that might need experimental tuning via cross-validation.

**Bias** there are many implicit biases in the inference algorithms (and also in the Dirichlet process if used), and whenever there is a mismatch between these biases and the data it might be possible to fit better models using a finite mixture.

The Dirichlet Process

Here we describe variational inference algorithms on Dirichlet process mixture. The Dirichlet process is a prior probability distribution on *clusterings with an infinite, unbounded, number of partitions*. Variational techniques let us incorporate this prior structure on Gaussian mixture models at almost no penalty in inference time, comparing with a finite Gaussian mixture model.

An important question is how can the Dirichlet process use an infinite, unbounded number of clusters and still be consistent. While a full explanation doesn’t fit this manual, one can think of its *stick breaking process* analogy to help understanding it. The stick breaking process is a generative story for the Dirichlet process. We start with a unit-length stick and in each step we break off a portion of the remaining stick. Each time, we associate the length of the piece of the stick to the proportion of points that falls into a group of the mixture. At the end, to represent the infinite mixture, we associate the last remaining piece of the stick to the proportion of points that don’t fall into all the other groups. The length of each piece is random variable with probability proportional to the concentration parameter. Smaller value of the concentration will divide the unit-length into larger pieces of the stick (defining more concentrated distribution). Larger concentration values will create smaller pieces of the stick (increasing the number of components with non zero weights).

Variational inference techniques for the Dirichlet process still work with a finite approximation to this infinite mixture model, but instead of having to specify a priori how many components one wants to use, one just specifies the concentration parameter and an upper bound on the number of mixture components (this upper bound, assuming it is higher than the “true” number of components, affects only algorithmic complexity, not the actual number of components used).

3.2.2 Manifold learning

Look for the bare necessities
The simple bare necessities
Forget about your worries and your strife
I mean the bare necessities
Old Mother Nature’s recipes
That bring the bare necessities of life

– Baloo’s song [The Jungle Book]

Manifold learning is an approach to non-linear dimensionality reduction. Algorithms for this task are based on the idea that the dimensionality of many data sets is only artificially high.
Introduction

High-dimensional datasets can be very difficult to visualize. While data in two or three dimensions can be plotted to show the inherent structure of the data, equivalent high-dimensional plots are much less intuitive. To aid visualization of the structure of a dataset, the dimension must be reduced in some way.

The simplest way to accomplish this dimensionality reduction is by taking a random projection of the data. Though this allows some degree of visualization of the data structure, the randomness of the choice leaves much to be desired. In a random projection, it is likely that the more interesting structure within the data will be lost.
To address this concern, a number of supervised and unsupervised linear dimensionality reduction frameworks have been designed, such as Principal Component Analysis (PCA), Independent Component Analysis, Linear Discriminant Analysis, and others. These algorithms define specific rubrics to choose an “interesting” linear projection of the data. These methods can be powerful, but often miss important non-linear structure in the data.

Manifold Learning can be thought of as an attempt to generalize linear frameworks like PCA to be sensitive to non-linear structure in data. Though supervised variants exist, the typical manifold learning problem is unsupervised: it learns the high-dimensional structure of the data from the data itself, without the use of predetermined classifications.

**Examples:**

- See *Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...* for an example of dimensionality reduction on handwritten digits.
- See *Comparison of Manifold Learning methods* for an example of dimensionality reduction on a toy “S-curve” dataset.

The manifold learning implementations available in scikit-learn are summarized below.

### Isomap

One of the earliest approaches to manifold learning is the Isomap algorithm, short for Isometric Mapping. Isomap can be viewed as an extension of Multi-dimensional Scaling (MDS) or Kernel PCA. Isomap seeks a lower-dimensional
embedding which maintains geodesic distances between all points. Isomap can be performed with the object `Isomap`.

**Complexity**

The Isomap algorithm comprises three stages:

1. **Nearest neighbor search.** Isomap uses `sklearn.neighbors.BallTree` for efficient neighbor search. The cost is approximately $O[D \log(k)N \log(N)]$, for $k$ nearest neighbors of $N$ points in $D$ dimensions.

2. **Shortest-path graph search.** The most efficient known algorithms for this are Dijkstra’s Algorithm, which is approximately $O[N^2(k + \log(N))]$, or the Floyd-Warshall algorithm, which is $O[N^3]$. The algorithm can be selected by the user with the `path_method` keyword of `Isomap`. If unspecified, the code attempts to choose the best algorithm for the input data.

3. **Partial eigenvalue decomposition.** The embedding is encoded in the eigenvectors corresponding to the $d$ largest eigenvalues of the $N \times N$ isomap kernel. For a dense solver, the cost is approximately $O[dN^2]$. This cost can often be improved using the ARPACK solver. The eigensolver can be specified by the user with the `path_method` keyword of `Isomap`. If unspecified, the code attempts to choose the best algorithm for the input data.

The overall complexity of Isomap is $O[D \log(k)N \log(N)] + O[N^2(k + \log(N))] + O[dN^2]$.

- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

**References:**

- “A global geometric framework for nonlinear dimensionality reduction” Tenenbaum, J.B.; De Silva, V.; & Langford, J.C. Science 290 (5500)
Locally Linear Embedding

Locally linear embedding (LLE) seeks a lower-dimensional projection of the data which preserves distances within local neighborhoods. It can be thought of as a series of local Principal Component Analyses which are globally compared to find the best non-linear embedding.

Locally linear embedding can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`.

Complexity

The standard LLE algorithm comprises three stages:

1. **Nearest Neighbors Search.** See discussion under Isomap above.
2. **Weight Matrix Construction.** $O[DNk^3]$. The construction of the LLE weight matrix involves the solution of a $k \times k$ linear equation for each of the $N$ local neighborhoods
3. **Partial Eigenvalue Decomposition.** See discussion under Isomap above.

The overall complexity of standard LLE is $O[D \log(k)N \log(N)] + O[DNk^3] + O[dN^2]$.

- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

References:


Modified Locally Linear Embedding

One well-known issue with LLE is the regularization problem. When the number of neighbors is greater than the number of input dimensions, the matrix defining each local neighborhood is rank-deficient. To address this, standard
LLE applies an arbitrary regularization parameter $r$, which is chosen relative to the trace of the local weight matrix. Though it can be shown formally that as $r \to 0$, the solution converges to the desired embedding, there is no guarantee that the optimal solution will be found for $r > 0$. This problem manifests itself in embeddings which distort the underlying geometry of the manifold.

One method to address the regularization problem is to use multiple weight vectors in each neighborhood. This is the essence of modified locally linear embedding (MLLE). MLLE can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`, with the keyword `method = 'modified'`. It requires $n_{neighbors} > n_{components}$.

Complexity

The MLLE algorithm comprises three stages:

1. **Nearest Neighbors Search.** Same as standard LLE

2. **Weight Matrix Construction.** Approximately $O[DNk^3] + O[N(k - D)k^2]$. The first term is exactly equivalent to that of standard LLE. The second term has to do with constructing the weight matrix from multiple weights. In practice, the added cost of constructing the MLLE weight matrix is relatively small compared to the cost of steps 1 and 3.

3. **Partial Eigenvalue Decomposition.** Same as standard LLE

The overall complexity of MLLE is $O[D \log(k) N \log(N)] + O[DNk^3] + O[N(k - D)k^2] + O[dN^2]$.

- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

References:

- “MLLE: Modified Locally Linear Embedding Using Multiple Weights” Zhang, Z. & Wang, J.
Hessian Eigenmapping

Hessian Eigenmapping (also known as Hessian-based LLE: HLLE) is another method of solving the regularization problem of LLE. It revolves around a hessian-based quadratic form at each neighborhood which is used to recover the locally linear structure. Though other implementations note its poor scaling with data size, sklearn implements some algorithmic improvements which make its cost comparable to that of other LLE variants for small output dimension. HLLE can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`, with the keyword `method = 'hessian'`. It requires `n_neighbors > n_components + (n_components + 3) / 2`.

Complexity

The HLLE algorithm comprises three stages:

1. **Nearest Neighbors Search.** Same as standard LLE
2. **Weight Matrix Construction.** Approximately $O[DNk^3] + O[Nd^6]$. The first term reflects a similar cost to that of standard LLE. The second term comes from a QR decomposition of the local hessian estimator.
3. **Partial Eigenvalue Decomposition.** Same as standard LLE


- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

References:

Spectral Embedding

Spectral Embedding (also known as Laplacian Eigenmaps) is one method to calculate non-linear embedding. It finds a low dimensional representation of the data using a spectral decomposition of the graph Laplacian. The graph generated can be considered as a discrete approximation of the low dimensional manifold in the high dimensional space. Minimization of a cost function based on the graph ensures that points close to each other on the manifold are mapped close to each other in the low dimensional space, preserving local distances. Spectral embedding can be performed with the function `spectral_embedding` or its object-oriented counterpart `SpectralEmbedding`.

Complexity

The Spectral Embedding algorithm comprises three stages:

1. **Weighted Graph Construction.** Transform the raw input data into graph representation using affinity (adjacency) matrix representation.
2. **Graph Laplacian Construction.** Unnormalized graph Laplacian is constructed as \( L = D - A \) and normalized one as \( L = D^{-\frac{1}{2}}(D - A)D^{-\frac{1}{2}} \).
3. **Partial Eigenvalue Decomposition.** Eigenvalue decomposition is done on graph Laplacian.

The overall complexity of spectral embedding is \( O[D \log(k)N \log(N)] + O[DNk^3] + O[dN^2] \).

- \( N \): number of training data points
- \( D \): input dimension
- \( k \): number of nearest neighbors
- \( d \): output dimension

References:

- “Laplacian Eigenmaps for Dimensionality Reduction and Data Representation” M. Belkin, P. Niyogi, Neural Computation, June 2003; 15 (6):1373-1396

Local Tangent Space Alignment

Though not technically a variant of LLE, Local tangent space alignment (LTSA) is algorithmically similar enough to LLE that it can be put in this category. Rather than focusing on preserving neighborhood distances as in LLE, LTSA seeks to characterize the local geometry at each neighborhood via its tangent space, and performs a global optimization to align these local tangent spaces to learn the embedding. LTSA can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`, with the keyword `method = 'ltsa'`.

Complexity

The LTSA algorithm comprises three stages:

1. **Nearest Neighbors Search.** Same as standard LLE.
2. **Weight Matrix Construction.** Approximately \( O[DNk^3] + O[k^2d] \). The first term reflects a similar cost to that of standard LLE.
3. **Partial Eigenvalue Decomposition.** Same as standard LLE.
The overall complexity of standard LTSA is \( O[D \log(k)N \log(N)] + O[DNk^3] + O[k^2d] + O[dN^2] \).

- \( N \): number of training data points
- \( D \): input dimension
- \( k \): number of nearest neighbors
- \( d \): output dimension

**References:**


### Multi-dimensional Scaling (MDS)

Multidimensional scaling (MDS) seeks a low-dimensional representation of the data in which the distances respect well the distances in the original high-dimensional space.

In general, is a technique used for analyzing similarity or dissimilarity data. MDS attempts to model similarity or dissimilarity data as distances in a geometric spaces. The data can be ratings of similarity between objects, interaction frequencies of molecules, or trade indices between countries.

There exists two types of MDS algorithm: metric and non metric. In the scikit-learn, the class MDS implements both. In Metric MDS, the input similarity matrix arises from a metric (and thus respects the triangular inequality), the distances between output two points are then set to be as close as possible to the similarity or dissimilarity data. In the non-metric version, the algorithms will try to preserve the order of the distances, and hence seek for a monotonic relationship between the distances in the embedded space and the similarities/dissimilarities.

Let \( S \) be the similarity matrix, and \( X \) the coordinates of the \( n \) input points. Disparities \( \hat{d}_{ij} \) are transformation of the similarities chosen in some optimal ways. The objective, called the stress, is then defined by \( \text{sum}_{i<j}d_{ij}(X) - \hat{d}_{ij}(X) \)

#### Metric MDS

The simplest metric MDS model, called absolute MDS, disparities are defined by \( \hat{d}_{ij} = S_{ij} \). With absolute MDS, the value \( S_{ij} \) should then correspond exactly to the distance between point \( i \) and \( j \) in the embedding point.
Most commonly, disparities are set to $\hat{d}_{ij} = bS_{ij}$.

**Nonmetric MDS**

Non metric MDS focuses on the ordination of the data. If $S_{ij} < S_{kl}$, then the embedding should enforce $d_{ij} < d_{jk}$. A simple algorithm to enforce that is to use a monotonic regression of $d_{ij}$ on $S_{ij}$, yielding disparities $\hat{d}_{ij}$ in the same order as $S_{ij}$.

A trivial solution to this problem is to set all the points on the origin. In order to avoid that, the disparities $\hat{d}_{ij}$ are normalized.

References:

- “Nonmetric multidimensional scaling: a numerical method” Kruskal, J. Psychometrika, 29 (1964)
t-distributed Stochastic Neighbor Embedding (t-SNE)

t-SNE (TSNE) converts affinities of data points to probabilities. The affinities in the original space are represented by Gaussian joint probabilities and the affinities in the embedded space are represented by Student’s t-distributions. This allows t-SNE to be particularly sensitive to local structure and has a few other advantages over existing techniques:

- Revealing the structure at many scales on a single map
- Revealing data that lie in multiple, different, manifolds or clusters
- Reducing the tendency to crowd points together at the center

While Isomap, LLE and variants are best suited to unfold a single continuous low dimensional manifold, t-SNE will focus on the local structure of the data and will tend to extract clustered local groups of samples as highlighted on the S-curve example. This ability to group samples based on the local structure might be beneficial to visually disentangle a dataset that comprises several manifolds at once as is the case in the digits dataset.

The Kullback-Leibler (KL) divergence of the joint probabilities in the original space and the embedded space will be minimized by gradient descent. Note that the KL divergence is not convex, i.e. multiple restarts with different initializations will end up in local minima of the KL divergence. Hence, it is sometimes useful to try different seeds and select the embedding with the least KL divergence.

The disadvantages to using t-SNE are roughly:

- t-SNE is computationally expensive, and can take several hours on million-sample datasets where PCA will finish in seconds or minutes
- The Barnes-Hut t-SNE method is limited to two or three dimensional embeddings.
- The algorithm is stochastic and multiple restarts with different seeds can yield different embeddings. However, it is perfectly legitimate to pick the embedding with the least error.
- Global structure is not explicitly preserved. This is problem is mitigated by initializing points with PCA (using init='pca').
Optimizing t-SNE

The main purpose of t-SNE is visualization of high-dimensional data. Hence, it works best when the data will be embedded on two or three dimensions.

Optimizing the KL divergence can be a little bit tricky sometimes. There are five parameters that control the optimization of t-SNE and therefore possibly the quality of the resulting embedding:

- perplexity
- early exaggeration factor
- learning rate
- maximum number of iterations
- angle (not used in the exact method)

The perplexity is defined as \( k = 2^S \) where \( S \) is the Shannon entropy of the conditional probability distribution. The perplexity of a \( k \)-sided die is \( k \), so that \( k \) is effectively the number of nearest neighbors t-SNE considers when generating the conditional probabilities. Larger perplexities lead to more nearest neighbors and less sensitive to small structure. Larger datasets tend to require larger perplexities. The maximum number of iterations is usually high enough and does not need any tuning. The optimization consists of two phases: the early exaggeration phase and the final optimization. During early exaggeration the joint probabilities in the original space will be artificially increased by multiplication with a given factor. Larger factors result in larger gaps between natural clusters in the data. If the factor is too high, the KL divergence could increase during this phase. Usually it does not have to be tuned. A critical parameter is the learning rate. If it is too low gradient descent will get stuck in a bad local minimum. If it is too high the KL divergence will increase during optimization. More tips can be found in Laurens van der Maaten’s FAQ (see references). The last parameter, angle, is a tradeoff between performance and accuracy. Larger angles imply that we can approximate larger regions by a single point, leading to better speed but less accurate results.

Barnes-Hut t-SNE

The Barnes-Hut t-SNE that has been implemented here is usually much slower than other manifold learning algorithms. The optimization is quite difficult and the computation of the gradient is \( O[dN\log(N)] \), where \( d \) is the number of output dimensions and \( N \) is the number of samples. The Barnes-Hut method improves on the exact method where t-SNE complexity is \( O[dN^2] \), but has several other notable differences:

- The Barnes-Hut implementation only works when the target dimensionality is 3 or less. The 2D case is typical when building visualizations.
- Barnes-Hut only works with dense input data. Sparse data matrices can only be embedded with the exact method or can be approximated by a dense low rank projection for instance using `sklearn.decomposition.TruncatedSVD`
- Barnes-Hut is an approximation of the exact method. The approximation is parameterized with the angle parameter, therefore the angle parameter is unused when method=”exact”
- Barnes-Hut is significantly more scalable. Barnes-Hut can be used to embed hundred of thousands of data points while the exact method can handle thousands of samples before becoming computationally intractable

For visualization purpose (which is the main use case of t-SNE), using the Barnes-Hut method is strongly recommended. The exact t-SNE method is useful for checking the theoretically properties of the embedding possibly in higher dimensional space but limit to small datasets due to computational constraints.

Also note that the digits labels roughly match the natural grouping found by t-SNE while the linear 2D projection of the PCA model yields a representation where label regions largely overlap. This is a strong clue that this data can be well separated by non linear methods that focus on the local structure (e.g. an SVM with a Gaussian RBF kernel). However, failing to visualize well separated homogeneously labeled groups with t-SNE in 2D does not necessarily
implie that the data cannot be correctly classified by a supervised model. It might be the case that 2 dimensions are not enough low to accurately represents the internal structure of the data.

References:

- “t-Distributed Stochastic Neighbor Embedding” van der Maaten, L.J.P.

Tips on practical use

- Make sure the same scale is used over all features. Because manifold learning methods are based on a nearest-neighbor search, the algorithm may perform poorly otherwise. See StandardScaler for convenient ways of scaling heterogeneous data.
- The reconstruction error computed by each routine can be used to choose the optimal output dimension. For a \( d \)-dimensional manifold embedded in a \( D \)-dimensional parameter space, the reconstruction error will decrease as \( n\_components \) is increased until \( n\_components == d \).
- Note that noisy data can “short-circuit” the manifold, in essence acting as a bridge between parts of the manifold that would otherwise be well-separated. Manifold learning on noisy and/or incomplete data is an active area of research.
- Certain input configurations can lead to singular weight matrices, for example when more than two points in the dataset are identical, or when the data is split into disjointed groups. In this case, solver='arpack' will fail to find the null space. The easiest way to address this is to use solver='dense' which will work on a singular matrix, though it may be very slow depending on the number of input points. Alternatively, one can attempt to understand the source of the singularity: if it is due to disjoint sets, increasing \( n\_neighbors \) may help. If it is due to identical points in the dataset, removing these points may help.

See also:

Totally Random Trees Embedding can also be useful to derive non-linear representations of feature space, also it does not perform dimensionality reduction.

3.2.3 Clustering

Clustering of unlabeled data can be performed with the module sklearn.cluster.

Each clustering algorithm comes in two variants: a class, that implements the fit method to learn the clusters on train data, and a function, that, given train data, returns an array of integer labels corresponding to the different clusters. For the class, the labels over the training data can be found in the labels_ attribute.

Input data

One important thing to note is that the algorithms implemented in this module can take different kinds of matrix as input. All the methods accept standard data matrices of shape \([n\_samples, n\_features]\). These can be obtained from the classes in the sklearn.feature_extraction module. For
AffinityPropagation, SpectralClustering and DBSCAN one can also input similarity matrices of shape \([n\_samples, n\_samples]\). These can be obtained from the functions in the 
\texttt{sklearn.metrics.pairwise} module.

### Overview of clustering methods

![Fig. 3.4: A comparison of the clustering algorithms in scikit-learn](image)

<table>
<thead>
<tr>
<th>Method name</th>
<th>Parameters</th>
<th>Scalability</th>
<th>Usecase</th>
<th>Geometry (metric used)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-Means</td>
<td>number of clusters</td>
<td>Very large (n_samples), medium (n_clusters) with \textit{MiniBatch code}</td>
<td>General-purpose, even cluster size, flat geometry, not too many clusters</td>
<td>Distances between points</td>
</tr>
<tr>
<td>Affinity propagation</td>
<td>damping, sample preference</td>
<td>Not scalable with (n_samples)</td>
<td>Many clusters, uneven cluster size, non-flat geometry</td>
<td>Graph distance (e.g. nearest-neighbor graph)</td>
</tr>
<tr>
<td>Mean-shift</td>
<td>bandwidth</td>
<td>Not scalable with (n_samples)</td>
<td>Many clusters, uneven cluster size, non-flat geometry</td>
<td>Distances between points</td>
</tr>
<tr>
<td>Spectral clustering</td>
<td>number of clusters</td>
<td>Medium (n_samples), small (n_clusters)</td>
<td>Few clusters, even cluster size, non-flat geometry</td>
<td>Graph distance (e.g. nearest-neighbor graph)</td>
</tr>
<tr>
<td>Ward hierarchical clustering</td>
<td>number of clusters</td>
<td>Large (n_samples) and (n_clusters)</td>
<td>Many clusters, possibly connectivity constraints</td>
<td>Distances between points</td>
</tr>
<tr>
<td>Agglomerative clustering</td>
<td>number of clusters, linkage type, distance</td>
<td>Large (n_samples) and (n_clusters)</td>
<td>Many clusters, possibly connectivity constraints, non Euclidean distances</td>
<td>Any pairwise distance</td>
</tr>
<tr>
<td>DBSCAN</td>
<td>neighborhood size</td>
<td>Very large (n_samples), medium (n_clusters)</td>
<td>Non-flat geometry, uneven cluster sizes</td>
<td>Distances between nearest points</td>
</tr>
<tr>
<td>Gaussian mixtures</td>
<td>many</td>
<td>Not scalable</td>
<td>Flat geometry, good for density estimation</td>
<td>Mahalanobis distances to centers</td>
</tr>
<tr>
<td>Birch</td>
<td>branching factor, threshold, optional global clusterer</td>
<td>Large (n_clusters) and (n_samples)</td>
<td>Large dataset, outlier removal, data reduction.</td>
<td>Euclidean distance between points</td>
</tr>
</tbody>
</table>

Non-flat geometry clustering is useful when the clusters have a specific shape, i.e. a non-flat manifold, and the standard
euclidean distance is not the right metric. This case arises in the two top rows of the figure above.

Gaussian mixture models, useful for clustering, are described in another chapter of the documentation dedicated to mixture models. KMeans can be seen as a special case of Gaussian mixture model with equal covariance per component.

**K-means**

The KMeans algorithm clusters data by trying to separate samples in n groups of equal variance, minimizing a criterion known as the inertia or within-cluster sum-of-squares. This algorithm requires the number of clusters to be specified. It scales well to large number of samples and has been used across a large range of application areas in many different fields.

The k-means algorithm divides a set of N samples X into K disjoint clusters C, each described by the mean \( \mu_j \) of the samples in the cluster. The means are commonly called the cluster “centroids”; note that they are not, in general, points from X, although they live in the same space. The K-means algorithm aims to choose centroids that minimise the **inertia**, or within-cluster sum of squared criterion:

\[
\sum_{i=0}^{K} \min_{\mu_j \in C} (||x_j - \mu_i||^2)
\]

Inertia, or the within-cluster sum of squares criterion, can be recognized as a measure of how internally coherent clusters are. It suffers from various drawbacks:

- Inertia makes the assumption that clusters are convex and isotropic, which is not always the case. It responds poorly to elongated clusters, or manifolds with irregular shapes.

- Inertia is not a normalized metric: we just know that lower values are better and zero is optimal. But in very high-dimensional spaces, Euclidean distances tend to become inflated (this is an instance of the so-called “curse of dimensionality”). Running a dimensionality reduction algorithm such as PCA prior to k-means clustering can alleviate this problem and speed up the computations.

K-means is often referred to as Lloyd’s algorithm. In basic terms, the algorithm has three steps. The first step chooses the initial centroids, with the most basic method being to choose \( k \) samples from the dataset X. After initialization, K-means consists of looping between the two other steps. The first step assigns each sample to its nearest centroid. The second step creates new centroids by taking the mean value of all of the samples assigned to each previous centroid. The difference between the old and the new centroids
are computed and the algorithm repeats these last two steps until this value is less than a threshold. In other words, it
repeats until the centroids do not move significantly.

K-means is equivalent to the expectation-maximization algorithm with a small, all-equal, diagonal covariance matrix.

The algorithm can also be understood through the concept of Voronoi diagrams. First the Voronoi diagram of the points is calculated using the current centroids. Each segment in the Voronoi diagram becomes a separate cluster. Secondly, the centroids are updated to the mean of each segment. The algorithm then repeats this until a stopping criterion is fulfilled. Usually, the algorithm stops when the relative decrease in the objective function between iterations is less than the given tolerance value. This is not the case in this implementation: iteration stops when centroids move less than the tolerance.

Given enough time, K-means will always converge, however this may be to a local minimum. This is highly dependent on the initialization of the centroids. As a result, the computation is often done several times, with different initializations of the centroids. One method to help address this issue is the k-means++ initialization scheme, which has been implemented in scikit-learn (use the init='kmeans++' parameter). This initializes the centroids to be (generally) distant from each other, leading to provably better results than random initialization, as shown in the reference.

A parameter can be given to allow K-means to be run in parallel, called n_jobs. Giving this parameter a positive value uses that many processors (default: 1). A value of -1 uses all available processors, with -2 using one less, and so on. Parallelization generally speeds up computation at the cost of memory (in this case, multiple copies of centroids need to be stored, one for each job).

**Warning:** The parallel version of K-Means is broken on OS X when numpy uses the Accelerate Framework. This is expected behavior: Accelerate can be called after a fork but you need to execv the subprocess with the Python binary (which multiprocessing does not do under posix).

K-means can be used for vector quantization. This is achieved using the transform method of a trained model of KMeans.

**Examples:**

- *Demonstration of k-means assumptions:* Demonstrating when k-means performs intuitively and when it does not
- *A demo of K-Means clustering on the handwritten digits data:* Clustering handwritten digits

**References:**

Mini Batch K-Means

The MiniBatchKMeans is a variant of the KMeans algorithm which uses mini-batches to reduce the computation time, while still attempting to optimise the same objective function. Mini-batches are subsets of the input data, randomly sampled in each training iteration. These mini-batches drastically reduce the amount of computation required to converge to a local solution. In contrast to other algorithms that reduce the convergence time of k-means, mini-batch k-means produces results that are generally only slightly worse than the standard algorithm.

The algorithm iterates between two major steps, similar to vanilla k-means. In the first step, \( b \) samples are drawn randomly from the dataset, to form a mini-batch. These are then assigned to the nearest centroid. In the second step, the centroids are updated. In contrast to k-means, this is done on a per-sample basis. For each sample in the mini-batch, the assigned centroid is updated by taking the streaming average of the sample and all previous samples assigned to that centroid. This has the effect of decreasing the rate of change for a centroid over time. These steps are performed until convergence or a predetermined number of iterations is reached.

MiniBatchKMeans converges faster than KMeans, but the quality of the results is reduced. In practice this difference in quality can be quite small, as shown in the example and cited reference.

Examples:
- Comparison of the K-Means and MiniBatchKMeans clustering algorithms: Comparison of KMeans and MiniBatchKMeans
- Clustering text documents using k-means: Document clustering using sparse MiniBatchKMeans
- Online learning of a dictionary of parts of faces

References:

Affinity Propagation

AffinityPropagation creates clusters by sending messages between pairs of samples until convergence. A dataset is then described using a small number of exemplars, which are identified as those most representative of other
samples. The messages sent between pairs represent the suitability for one sample to be the exemplar of the other, which is updated in response to the values from other pairs. This updating happens iteratively until convergence, at which point the final exemplars are chosen, and hence the final clustering is given.

Affinity Propagation can be interesting as it chooses the number of clusters based on the data provided. For this purpose, the two important parameters are the preference, which controls how many exemplars are used, and the damping factor.

The main drawback of Affinity Propagation is its complexity. The algorithm has a time complexity of the order $O(N^2 T)$, where $N$ is the number of samples and $T$ is the number of iterations until convergence. Further, the memory complexity is of the order $O(N^2)$ if a dense similarity matrix is used, but reducible if a sparse similarity matrix is used. This makes Affinity Propagation most appropriate for small to medium sized datasets.

**Examples:**

- *Demo of affinity propagation clustering algorithm*: Affinity Propagation on a synthetic 2D datasets with 3 classes.
- *Visualizing the stock market structure* Affinity Propagation on Financial time series to find groups of companies

**Algorithm description:** The messages sent between points belong to one of two categories. The first is the responsibility $r(i, k)$, which is the accumulated evidence that sample $k$ should be the exemplar for sample $i$. The second is the availability $a(i, k)$ which is the accumulated evidence that sample $i$ should choose sample $k$ to be its exemplar, and considers the values for all other samples that $k$ should be an exemplar. In this way, exemplars are chosen by samples if they are (1) similar enough to many samples and (2) chosen by many samples to be representative of themselves.

More formally, the responsibility of a sample $k$ to be the exemplar of sample $i$ is given by:

$$r(i, k) \leftarrow s(i, k) - \max[a(i, k) + s(i, \hat{k}) \forall \hat{k} \neq k]$$

Where $s(i, k)$ is the similarity between samples $i$ and $k$. The availability of sample $k$ to be the exemplar of sample $i$ is given by:

$$a(i, k) \leftarrow \min[0, r(k, k) + \sum_{i \text{ s.t. } i \notin \{i, k\}} r(i, k)]$$

To begin with, all values for $r$ and $a$ are set to zero, and the calculation of each iterates until convergence.
Mean Shift

MeanShift clustering aims to discover blobs in a smooth density of samples. It is a centroid based algorithm, which works by updating candidates for centroids to be the mean of the points within a given region. These candidates are then filtered in a post-processing stage to eliminate near-duplicates to form the final set of centroids.

Given a candidate centroid \(x_i\) for iteration \(t\), the candidate is updated according to the following equation:

\[
x_{i}^{t+1} = x_{i}^{t} + m(x_{i}^{t})
\]

Where \(N(x_i)\) is the neighborhood of samples within a given distance around \(x_i\) and \(m\) is the mean shift vector that is computed for each centroid that points towards a region of the maximum increase in the density of points. This is computed using the following equation, effectively updating a centroid to be the mean of the samples within its neighborhood:

\[
m(x_i) = \frac{\sum_{x_j \in N(x_i)} K(x_j - x_i)x_j}{\sum_{x_j \in N(x_i)} K(x_j - x_i)}
\]

The algorithm automatically sets the number of clusters, instead of relying on a parameter bandwidth, which dictates the size of the region to search through. This parameter can be set manually, but can be estimated using the provided estimate_bandwidth function, which is called if the bandwidth is not set.

The algorithm is not highly scalable, as it requires multiple nearest neighbor searches during the execution of the algorithm. The algorithm is guaranteed to converge, however the algorithm will stop iterating when the change in centroids is small.

Labelling a new sample is performed by finding the nearest centroid for a given sample.

Examples:

- **A demo of the mean-shift clustering algorithm**: Mean Shift clustering on a synthetic 2D datasets with 3 classes.

References:

Spectral clustering

*SpectralClustering* does a low-dimension embedding of the affinity matrix between samples, followed by a
KMeans in the low dimensional space. It is especially efficient if the affinity matrix is sparse and the *pyamg* module
is installed. *SpectralClustering* requires the number of clusters to be specified. It works well for a small number of
clusters but is not advised when using many clusters.

For two clusters, it solves a convex relaxation of the *normalised cuts* problem on the similarity graph: cutting the
graph in two so that the weight of the edges cut is small compared to the weights of the edges inside each cluster. This
criteria is especially interesting when working on images: graph vertices are pixels, and edges of the similarity graph
are a function of the gradient of the image.

**Warning:** Transforming distance to well-behaved similarities

Note that if the values of your similarity matrix are not well distributed, e.g. with negative values or with a distance
matrix rather than a similarity, the spectral problem will be singular and the problem not solvable. In which case
it is advised to apply a transformation to the entries of the matrix. For instance, in the case of a signed distance
matrix, is common to apply a heat kernel:

```
similarity = np.exp(-beta * distance / distance.std())
```

See the examples for such an application.

**Examples:**

- *Spectral clustering for image segmentation:* Segmenting objects from a noisy background using spectral
  clustering.
- *Segmenting the picture of a raccoon face in regions:* Spectral clustering to split the image of the raccoon face
  in regions.
Different label assignment strategies

Different label assignment strategies can be used, corresponding to the assign_labels parameter of SpectralClustering. The "kmeans" strategy can match finer details of the data, but it can be more unstable. In particular, unless you control the random_state, it may not be reproducible from run-to-run, as it depends on a random initialization. On the other hand, the "discretize" strategy is 100% reproducible, but it tends to create parcels of fairly even and geometrical shape.

References:

- “Normalized cuts and image segmentation” Jianbo Shi, Jitendra Malik, 2000
- “On Spectral Clustering: Analysis and an algorithm” Andrew Y. Ng, Michael I. Jordan, Yair Weiss, 2001

Hierarchical clustering

Hierarchical clustering is a general family of clustering algorithms that build nested clusters by merging or splitting them successively. This hierarchy of clusters is represented as a tree (or dendrogram). The root of the tree is the unique cluster that gathers all the samples, the leaves being the clusters with only one sample. See the Wikipedia page for more details.

The AgglomerativeClustering object performs a hierarchical clustering using a bottom up approach: each observation starts in its own cluster, and clusters are successively merged together. The linkage criteria determines the metric used for the merge strategy:

- **Ward** minimizes the sum of squared differences within all clusters. It is a variance-minimizing approach and in this sense is similar to the k-means objective function but tackled with an agglomerative hierarchical approach.
- **Maximum** or **complete linkage** minimizes the maximum distance between observations of pairs of clusters.
- **Average linkage** minimizes the average of the distances between all observations of pairs of clusters.

AgglomerativeClustering can also scale to large number of samples when it is used jointly with a connectivity matrix, but is computationally expensive when no connectivity constraints are added between samples: it considers at each step all the possible merges.
FeatureAgglomeration

The FeatureAgglomeration uses agglomerative clustering to group together features that look very similar, thus decreasing the number of features. It is a dimensionality reduction tool, see Unsupervised dimensionality reduction.

Different linkage type: Ward, complete and average linkage

AgglomerativeClustering supports Ward, average, and complete linkage strategies.

Agglomerative cluster has a “rich get richer” behavior that leads to uneven cluster sizes. In this regard, complete linkage is the worst strategy, and Ward gives the most regular sizes. However, the affinity (or distance used in clustering) cannot be varied with Ward, thus for non Euclidean metrics, average linkage is a good alternative.

Examples:

- Various Agglomerative Clustering on a 2D embedding of digits: exploration of the different linkage strategies in a real dataset.

Adding connectivity constraints

An interesting aspect of AgglomerativeClustering is that connectivity constraints can be added to this algorithm (only adjacent clusters can be merged together), through a connectivity matrix that defines for each sample the neighboring samples following a given structure of the data. For instance, in the swiss-roll example below, the connectivity constraints forbid the merging of points that are not adjacent on the swiss roll, and thus avoid forming clusters that extend across overlapping folds of the roll.
These constraint are useful to impose a certain local structure, but they also make the algorithm faster, especially when the number of the samples is high.

The connectivity constraints are imposed via an connectivity matrix: a scipy sparse matrix that has elements only at the intersection of a row and a column with indices of the dataset that should be connected. This matrix can be constructed from a-priori information: for instance, you may wish to cluster web pages by only merging pages with a link pointing from one to another. It can also be learned from the data, for instance using `sklearn.neighbors.kneighbors_graph` to restrict merging to nearest neighbors as in this example, or using `sklearn.feature_extraction.image.grid_to_graph` to enable only merging of neighboring pixels on an image, as in the raccoon face example.

**Examples:**

- **A demo of structured Ward hierarchical clustering on a raccoon face image:** Ward clustering to split the image of a raccoon face in regions.
- **Hierarchical clustering: structured vs unstructured ward:** Example of Ward algorithm on a swiss-roll, comparison of structured approaches versus unstructured approaches.
- **Feature agglomeration vs. univariate selection:** Example of dimensionality reduction with feature agglomeration based on Ward hierarchical clustering.
- **Agglomerative clustering with and without structure**

**Warning:** Connectivity constraints with average and complete linkage

Connectivity constraints and complete or average linkage can enhance the ‘rich getting richer’ aspect of agglomerative clustering, particularly so if they are built with `sklearn.neighbors.kneighbors_graph`. In the limit of a small number of clusters, they tend to give a few macroscopically occupied clusters and almost empty ones. (see the discussion in Agglomerative clustering with and without structure).
Varying the metric

Average and complete linkage can be used with a variety of distances (or affinities), in particular Euclidean distance ($l2$), Manhattan distance (or Cityblock, or $l1$), cosine distance, or any precomputed affinity matrix.

- $l1$ distance is often good for sparse features, or sparse noise: ie many of the features are zero, as in text mining using occurrences of rare words.

- cosine distance is interesting because it is invariant to global scalings of the signal.

The guidelines for choosing a metric is to use one that maximizes the distance between samples in different classes, and minimizes that within each class.

Examples:

- Agglomerative clustering with different metrics

DBSCAN

The DBSCAN algorithm views clusters as areas of high density separated by areas of low density. Due to this rather generic view, clusters found by DBSCAN can be any shape, as opposed to k-means which assumes that clusters are convex shaped. The central component to the DBSCAN is the concept of core samples, which are samples that are in areas of high density. A cluster is therefore a set of core samples, each close to each other (measured by some distance measure) and a set of non-core samples that are close to a core sample (but are not themselves core samples). There are two parameters to the algorithm, min_samples and eps, which define formally what we mean when we say dense. Higher min_samples or lower eps indicate higher density necessary to form a cluster.

More formally, we define a core sample as being a sample in the dataset such that there exist min_samples other samples within a distance of eps, which are defined as neighbors of the core sample. This tells us that the core sample is in a dense area of the vector space. A cluster is a set of core samples that can be built by recursively taking a core sample, finding all of its neighbors that are core samples, finding all of their neighbors that are core samples, and so on. A cluster also has a set of non-core samples, which are samples that are neighbors of a core sample in the cluster but are not themselves core samples. Intuitively, these samples are on the fringes of a cluster.

Any core sample is part of a cluster, by definition. Any sample that is not a core sample, and is at least eps in distance from any core sample, is considered an outlier by the algorithm.

In the figure below, the color indicates cluster membership, with large circles indicating core samples found by the algorithm. Smaller circles are non-core samples that are still part of a cluster. Moreover, the outliers are indicated by black points below.
Examples:

- Demo of DBSCAN clustering algorithm

Implementation

The algorithm is non-deterministic, but the core samples will always belong to the same clusters (although the labels may be different). The non-determinism comes from deciding to which cluster a non-core sample belongs. A non-core sample can have a distance lower than \( \text{eps} \) to two core samples in different clusters. By the triangular inequality, those two core samples must be more distant than \( \text{eps} \) from each other, or they would be in the same cluster. The non-core sample is assigned to whichever cluster is generated first, where the order is determined randomly. Other than the ordering of the dataset, the algorithm is deterministic, making the results relatively stable between runs on the same data.

The current implementation uses ball trees and kd-trees to determine the neighborhood of points, which avoids calculating the full distance matrix (as was done in scikit-learn versions before 0.14). The possibility to use custom metrics is retained; for details, see NearestNeighbors.

Memory consumption for large sample sizes

This implementation is by default not memory efficient because it constructs a full pairwise similarity matrix in the case where kd-trees or ball-trees cannot be used (e.g. with sparse matrices). This matrix will consume \( n^2 \) floats. A couple of mechanisms for getting around this are:

- A sparse radius neighborhood graph (where missing entries are presumed to be out of \( \text{eps} \)) can be precomputed in a memory-efficient way and dbscan can be run over this with metric='precomputed'.

- The dataset can be compressed, either by removing exact duplicates if these occur in your data, or by using BIRCH. Then you only have a relatively small number of representatives for a large number of points. You can then provide a sample_weight when fitting DBSCAN.

References:

- “A Density-Based Algorithm for Discovering Clusters in Large Spatial Databases with Noise” Ester, M., H. P.
Birch

The Birch builds a tree called the Characteristic Feature Tree (CFT) for the given data. The data is essentially lossy compressed to a set of Characteristic Feature nodes (CF Nodes). The CF Nodes have a number of subclusters called Characteristic Feature subclusters (CF Subclusters) and these CF Subclusters located in the non-terminal CF Nodes can have CF Nodes as children.

The CF Subclusters hold the necessary information for clustering which prevents the need to hold the entire input data in memory. This information includes:

- Number of samples in a subcluster.
- Linear Sum - A n-dimensional vector holding the sum of all samples
- Squared Sum - Sum of the squared L2 norm of all samples.
- Centroids - To avoid recalculation linear sum / n_samples.
- Squared norm of the centroids.

The Birch algorithm has two parameters, the threshold and the branching factor. The branching factor limits the number of subclusters in a node and the threshold limits the distance between the entering sample and the existing subclusters.

This algorithm can be viewed as an instance or data reduction method, since it reduces the input data to a set of subclusters which are obtained directly from the leaves of the CFT. This reduced data can be further processed by feeding it into a global clusterer. This global clusterer can be set by n_clusters. If n_clusters is set to None, the subclusters from the leaves are directly read off, otherwise a global clustering step labels these subclusters into global clusters (labels) and the samples are mapped to the global label of the nearest subcluster.

Algorithm description:

- A new sample is inserted into the root of the CF Tree which is a CF Node. It is then merged with the subcluster of the root, that has the smallest radius after merging, constrained by the threshold and branching factor conditions. If the subcluster has any child node, then this is done repeatedly till it reaches a leaf. After finding the nearest subcluster in the leaf, the properties of this subcluster and the parent subclusters are recursively updated.
- If the radius of the subcluster obtained by merging the new sample and the nearest subcluster is greater than the square of the threshold and if the number of subclusters is greater than the branching factor, then a space is temporarily allocated to this new sample. The two farthest subclusters are taken and the subclusters are divided into two groups on the basis of the distance between these subclusters.
- If this split node has a parent subcluster and there is room for a new subcluster, then the parent is split into two. If there is no room, then this node is again split into two and the process is continued recursively, till it reaches the root.

Birch or MiniBatchKMeans?

- Birch does not scale very well to high dimensional data. As a rule of thumb if n_features is greater than twenty, it is generally better to use MiniBatchKMeans.
- If the number of instances of data needs to be reduced, or if one wants a large number of subclusters either as a preprocessing step or otherwise, Birch is more useful than MiniBatchKMeans.

How to use partial_fit?

To avoid the computation of global clustering, for every call of partial_fit the user is advised
1. To set `n_clusters=None` initially
2. Train all data by multiple calls to `partial_fit`.
3. Set `n_clusters` to a required value using `brc.set_params(n_clusters=n_clusters)`.
4. Call `partial_fit` finally with no arguments, i.e `brc.partial_fit()` which performs the global clustering.

![Birch without global clustering](image1)
![Birch with global clustering](image2)
![MiniBatchKMeans](image3)

**References:**

- Roberto Perdisci JBiCh - Java implementation of BIRCH clustering algorithm https://code.google.com/archive/p/jbirch

**Clustering performance evaluation**

Evaluating the performance of a clustering algorithm is not as trivial as counting the number of errors or the precision and recall of a supervised classification algorithm. In particular any evaluation metric should not take the absolute values of the cluster labels into account but rather if this clustering define separations of the data similar to some ground truth set of classes or satisfying some assumption such that members belong to the same class are more similar that members of different classes according to some similarity metric.

**Adjusted Rand index**

Given the knowledge of the ground truth class assignments `labels_true` and our clustering algorithm assignments of the same samples `labels_pred`, the *adjusted Rand index* is a function that measures the *similarity* of the two assignments, ignoring permutations and *with chance normalization*:

```python
>>> from sklearn import metrics
>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]
>>> metrics.adjusted_rand_score(labels_true, labels_pred)
0.24...
```

One can permute 0 and 1 in the predicted labels, rename 2 to 3, and get the same score:
Furthermore, `adjusted_rand_score` is symmetric: swapping the argument does not change the score. It can thus be used as a consensus measure:

```python
>>> metrics.adjusted_rand_score(labels_pred, labels_true)
0.24...
```

Perfect labeling is scored 1.0:

```python
>>> labels_pred = labels_true[:]
>>> metrics.adjusted_rand_score(labels_true, labels_pred)
1.0
```

Bad (e.g. independent labelings) have negative or close to 0.0 scores:

```python
>>> labels_true = [0, 1, 2, 0, 3, 4, 5, 1]
>>> labels_pred = [1, 1, 0, 0, 2, 2, 2, 2]
>>> metrics.adjusted_rand_score(labels_true, labels_pred)
-0.12...
```

**Advantages**

- Random (uniform) label assignments have a ARI score close to 0.0 for any value of `n_clusters` and `n_samples` (which is not the case for raw Rand index or the V-measure for instance).
- Bounded range [-1, 1]: negative values are bad (independent labelings), similar clusterings have a positive ARI, 1.0 is the perfect match score.
- No assumption is made on the cluster structure: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

**Drawbacks**

- Contrary to inertia, **ARI requires knowledge of the ground truth classes** while is almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

However ARI can also be useful in a purely unsupervised setting as a building block for a Consensus Index that can be used for clustering model selection (TODO).

**Examples:**

- Adjustment for chance in clustering performance evaluation: Analysis of the impact of the dataset size on the value of clustering measures for random assignments.

**Mathematical formulation**

If $C$ is a ground truth class assignment and $K$ the clustering, let us define $a$ and $b$ as:
• $a$, the number of pairs of elements that are in the same set in $C$ and in the same set in $K$
• $b$, the number of pairs of elements that are in different sets in $C$ and in different sets in $K$

The raw (unadjusted) Rand index is then given by:

$$RI = \frac{a + b}{\binom{N_{\text{samples}}}{2}}$$

Where $\binom{N_{\text{samples}}}{2}$ is the total number of possible pairs in the dataset (without ordering).

However the RI score does not guarantee that random label assignments will get a value close to zero (esp. if the number of clusters is in the same order of magnitude as the number of samples).

To counter this effect we can discount the expected RI $E[RI]$ of random labelings by defining the adjusted Rand index as follows:

$$ARI = \frac{RI - E[RI]}{\max(RI) - E[RI]}$$

**References**

• Comparing Partitions L. Hubert and P. Arabie, Journal of Classification 1985
• Wikipedia entry for the adjusted Rand index

### Mutual Information based scores

Given the knowledge of the ground truth class assignments `labels_true` and our clustering algorithm assignments of the same samples `labels_pred`, the **Mutual Information** is a function that measures the agreement of the two assignments, ignoring permutations. Two different normalized versions of this measure are available, **Normalized Mutual Information(NMI)** and **Adjusted Mutual Information(AMI)**. NMI is often used in the literature while AMI was proposed more recently and is normalized against chance:

```python
>>> from sklearn import metrics
>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]

>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
0.22504...
```

One can permute 0 and 1 in the predicted labels, rename 2 to 3 and get the same score:

```python
>>> labels_pred = [1, 1, 0, 0, 3, 3]

>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
0.22504...
```

All, `mutual_info_score`, `adjusted_mutual_info_score` and `normalized_mutual_info_score` are symmetric: swapping the argument does not change the score. Thus they can be used as a consensus measure:

```python
>>> metrics.adjusted_mutual_info_score(labels_pred, labels_true)
0.22504...
```

Perfect labeling is scored 1.0:
This is not true for `mutual_info_score`, which is therefore harder to judge:

```
>>> metrics.mutual_info_score(labels_true, labels_pred)
0.69...
```

Bad (e.g. independent labelings) have non-positive scores:

```
>>> labels_true = [0, 1, 2, 0, 3, 4, 5, 1]
>>> labels_pred = [1, 1, 0, 0, 2, 2, 2, 2]
>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
-0.10526...
```

**Advantages**

- Random (uniform) label assignments have a AMI score close to 0.0 for any value of `n_clusters` and `n_samples` (which is not the case for raw Mutual Information or the V-measure for instance).

- **Bounded range [0, 1]**: Values close to zero indicate two label assignments that are largely independent, while values close to one indicate significant agreement. Further, values of exactly 0 indicate purely independent label assignments and a AMI of exactly 1 indicates that the two label assignments are equal (with or without permutation).

- **No assumption is made on the cluster structure**: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

**Drawbacks**

- Contrary to inertia, MI-based measures require the knowledge of the ground truth classes while almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

  However MI-based measures can also be useful in purely unsupervised setting as a building block for a Consensus Index that can be used for clustering model selection.

- NMI and MI are not adjusted against chance.

**Examples:**

- *Adjustment for chance in clustering performance evaluation*: Analysis of the impact of the dataset size on the value of clustering measures for random assignments. This example also includes the Adjusted Rand Index.
Mathematical formulation

Assume two label assignments (of the same N objects), $U$ and $V$. Their entropy is the amount of uncertainty for a partition set, defined by:

$$H(U) = \sum_{i=1}^{\vert U \vert} P(i) \log(P(i))$$

where $P(i) = \vert U_i \vert / N$ is the probability that an object picked at random from $U$ falls into class $U_i$. Likewise for $V$:

$$H(V) = \sum_{j=1}^{\vert V \vert} P'(j) \log(P'(j))$$

With $P'(j) = \vert V_j \vert / N$. The mutual information (MI) between $U$ and $V$ is calculated by:

$$\text{MI}(U, V) = \sum_{i=1}^{\vert U \vert} \sum_{j=1}^{\vert V \vert} P(i,j) \log \left( \frac{P(i,j)}{P(i)P'(j)} \right)$$

where $P(i,j) = \vert U_i \cap V_j \vert / N$ is the probability that an object picked at random falls into both classes $U_i$ and $V_j$.

The normalized mutual information is defined as

$$\text{NMI}(U, V) = \frac{\text{MI}(U, V)}{\sqrt{H(U)H(V)}}$$

This value of the mutual information and also the normalized variant is not adjusted for chance and will tend to increase as the number of different labels (clusters) increases, regardless of the actual amount of “mutual information” between the label assignments.

The expected value for the mutual information can be calculated using the following equation, from Vinh, Epps, and Bailey, (2009). In this equation, $a_i = \vert U_i \vert$ (the number of elements in $U_i$) and $b_j = \vert V_j \vert$ (the number of elements in $V_j$).

$$E[\text{MI}(U, V)] = \sum_{i=1}^{\vert U \vert} \sum_{j=1}^{\vert V \vert} \min(a_i, b_j) \frac{n_{ij}}{N} \log \left( \frac{Nn_{ij}}{a_ib_j} \right) \frac{a_i!b_j!(N-a_i)!(N-b_j)!}{N!n_{ij}!(a_i-n_{ij})!(b_j-n_{ij})!(N-a_i-b_j+n_{ij})!}$$

Using the expected value, the adjusted mutual information can then be calculated using a similar form to that of the adjusted Rand index:

$$\text{AMI} = \frac{\text{MI} - E[\text{MI}]}{\max(H(U), H(V)) - E[\text{MI}]}$$

References

Homogeneity, completeness and V-measure

Given the knowledge of the ground truth class assignments of the samples, it is possible to define some intuitive metric using conditional entropy analysis.

In particular Rosenberg and Hirschberg (2007) define the following two desirable objectives for any cluster assignment:

- **homogeneity**: each cluster contains only members of a single class.
- **completeness**: all members of a given class are assigned to the same cluster.

We can turn those concept as scores `homogeneity_score` and `completeness_score`. Both are bounded below by 0.0 and above by 1.0 (higher is better):

```python
>>> from sklearn import metrics

>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]

>>> metrics.homogeneity_score(labels_true, labels_pred)
0.66...

>>> metrics.completeness_score(labels_true, labels_pred)
0.42...
```

Their harmonic mean called **V-measure** is computed by `v_measure_score`:

```python
>>> metrics.v_measure_score(labels_true, labels_pred)
0.51...
```

The V-measure is actually equivalent to the mutual information (NMI) discussed above normalized by the sum of the label entropies [B2011].

Homogeneity, completeness and V-measure can be computed at once using `homogeneity_completeness_v_measure` as follows:

```python
>>> metrics.homogeneity_completeness_v_measure(labels_true, labels_pred)
...
(0.66..., 0.42..., 0.51...)
```

The following clustering assignment is slightly better, since it is homogeneous but not complete:

```python
>>> labels_pred = [0, 0, 0, 1, 2, 2]

>>> metrics.homogeneity_completeness_v_measure(labels_true, labels_pred)
...
(1.0, 0.68..., 0.81...)
```

**Note**: `v_measure_score` is **symmetric**: it can be used to evaluate the **agreement** of two independent assignments on the same dataset.

This is not the case for `completeness_score` and `homogeneity_score`: both are bound by the relationship:
homogeneity_score(a, b) == completeness_score(b, a)

Advantages

- **Bounded scores**: 0.0 is as bad as it can be, 1.0 is a perfect score.
- Intuitive interpretation: clustering with bad V-measure can be **qualitatively analyzed in terms of homogeneity and completeness** to better feel what ‘kind’ of mistakes is done by the assignment.
- **No assumption is made on the cluster structure**: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

Drawbacks

- The previously introduced metrics are **not normalized with regards to random labeling**: this means that depending on the number of samples, clusters and ground truth classes, a completely random labeling will not always yield the same values for homogeneity, completeness and hence v-measure. In particular random labeling won’t yield zero scores especially when the number of clusters is large. This problem can safely be ignored when the number of samples is more than a thousand and the number of clusters is less than 10. **For smaller sample sizes or larger number of clusters it is safer to use an adjusted index such as the Adjusted Rand Index (ARI).**
- These metrics **require the knowledge of the ground truth classes** while almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

Examples:

- *Adjustment for chance in clustering performance evaluation*: Analysis of the impact of the dataset size on the value of clustering measures for random assignments.

Mathematical formulation

Homogeneity and completeness scores are formally given by:

\[ h = 1 - \frac{H(C|K)}{H(C)} \]

\[ c = 1 - \frac{H(K|C)}{H(K)} \]

where \( H(C|K) \) is the **conditional entropy of the classes given the cluster assignments** and is given by:

\[ H(C|K) = - \sum_{c=1}^{\lvert C \rvert} \sum_{k=1}^{\lvert K \rvert} \frac{n_{c,k}}{n} \cdot \log \left( \frac{n_{c,k}}{n_k} \right) \]

and \( H(C) \) is the **entropy of the classes** and is given by:

\[ H(C) = - \sum_{c=1}^{\lvert C \rvert} \frac{n_c}{n} \cdot \log \left( \frac{n_c}{n} \right) \]
Clustering measures for 2 random uniform labelings with equal number of clusters

![Graph showing clustering measures for 2 random uniform labelings with equal number of clusters.](image)
with \( n \) the total number of samples, \( n_c \) and \( n_k \) the number of samples respectively belonging to class \( c \) and cluster \( k \), and finally \( n_{c,k} \) the number of samples from class \( c \) assigned to cluster \( k \).

The **conditional entropy of clusters given class** \( H(K|C) \) and the **entropy of clusters** \( H(K) \) are defined in a symmetric manner.

Rosenberg and Hirschberg further define **V-measure** as the harmonic mean of **homogeneity** and **completeness**:

\[
v = 2 \cdot \frac{h \cdot c}{h + c}
\]

### References

### Fowlkes-Mallows scores

The Fowlkes-Mallows index (`sklearn.metrics.fowlkes_mallows_score`) can be used when the ground truth class assignments of the samples is known. The Fowlkes-Mallows score FMI is defined as the geometric mean of the pairwise precision and recall:

\[
\text{FMI} = \frac{\text{TP}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})}}
\]

Where TP is the number of **True Positive** (i.e. the number of pair of points that belong to the same clusters in both the true labels and the predicted labels), FP is the number of **False Positive** (i.e. the number of pair of points that belong to the same clusters in the true labels and not in the predicted labels) and FN is the number of **False Negative** (i.e. the number of pair of points that belongs in the same clusters in the predicted labels and not in the true labels).

The score ranges from 0 to 1. A high value indicates a good similarity between two clusters.

```python
>>> from sklearn import metrics
>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]

>>> metrics.fowlkes_mallows_score(labels_true, labels_pred)
0.47140...
```

One can permute 0 and 1 in the predicted labels, rename 2 to 3 and get the same score:

```python
>>> labels_pred = [1, 1, 0, 0, 3, 3]

>>> metrics.fowlkes_mallows_score(labels_true, labels_pred)
0.47140...
```

Perfect labeling is scored 1.0:

```python
>>> labels_pred = labels_true[:]

>>> metrics.fowlkes_mallows_score(labels_true, labels_pred)
1.0
```

Bad (e.g. independent labelings) have zero scores:

```python
>>> labels_true = [0, 1, 2, 0, 3, 4, 5, 1]

>>> labels_pred = [1, 1, 0, 0, 2, 2, 2, 2]

>>> metrics.fowlkes_mallows_score(labels_true, labels_pred)
0.0
```
Advantages

- Random (uniform) label assignments have a FMI score close to 0.0 for any value of n_clusters and n_samples (which is not the case for raw Mutual Information or the V-measure for instance).
- Bounded range \([0, 1]\): Values close to zero indicate two label assignments that are largely independent, while values close to one indicate significant agreement. Further, values of exactly 0 indicate purely independent label assignments and a AMI of exactly 1 indicates that the two label assignments are equal (with or without permutation).
- No assumption is made on the cluster structure: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

Drawbacks

- Contrary to inertia, FMI-based measures require the knowledge of the ground truth classes while almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

References

- Wikipedia entry for the Fowlkes-Mallows Index

Silhouette Coefficient

If the ground truth labels are not known, evaluation must be performed using the model itself. The Silhouette Coefficient (sklearn.metrics.silhouette_score) is an example of such an evaluation, where a higher Silhouette Coefficient score relates to a model with better defined clusters. The Silhouette Coefficient is defined for each sample and is composed of two scores:

- \(a\): The mean distance between a sample and all other points in the same class.
- \(b\): The mean distance between a sample and all other points in the next nearest cluster.

The Silhouette Coefficient \(s\) for a single sample is then given as:

\[
s = \frac{b - a}{\max(a, b)}
\]

The Silhouette Coefficient for a set of samples is given as the mean of the Silhouette Coefficient for each sample.

```python
>>> from sklearn import metrics
>>> from sklearn.metrics import pairwise_distances
>>> from sklearn import datasets
>>> dataset = datasets.load_iris()
>>> X = dataset.data
>>> y = dataset.target
```

In normal usage, the Silhouette Coefficient is applied to the results of a cluster analysis.
>>> import numpy as np
>>> from sklearn.cluster import KMeans
>>> kmeans_model = KMeans(n_clusters=3, random_state=1).fit(X)
>>> labels = kmeans_model.labels_
>>> metrics.silhouette_score(X, labels, metric='euclidean')
... 0.55...

References


Advantages

- The score is bounded between -1 for incorrect clustering and +1 for highly dense clustering. Scores around zero indicate overlapping clusters.
- The score is higher when clusters are dense and well separated, which relates to a standard concept of a cluster.

Drawbacks

- The Silhouette Coefficient is generally higher for convex clusters than other concepts of clusters, such as density based clusters like those obtained through DBSCAN.

Examples:

- Selecting the number of clusters with silhouette analysis on KMeans clustering: In this example the silhouette analysis is used to choose an optimal value for n_clusters.

Calinski-Harabaz Index

If the ground truth labels are not known, the Calinski-Harabaz index (sklearn.metrics.calinski_harabaz_score) can be used to evaluate the model, where a higher Calinski-Harabaz score relates to a model with better defined clusters.

For k clusters, the Calinski-Harabaz score $s$ is given as the ratio of the between-clusters dispersion mean and the within-cluster dispersion:

$$ s(k) = \frac{\text{Tr}(B_k)}{\text{Tr}(W_k)} \times \frac{N - k}{k - 1} $$

where $B_k$ is the between group dispersion matrix and $W_k$ is the within-cluster dispersion matrix defined by:

$$ W_k = \sum_{q=1}^{k} \sum_{x \in C_q} (x - c_q)(x - c_q)^T $$

$$ B_k = \sum_{q} n_q (c_q - c)(c_q - c)^T $$

3.2. Unsupervised learning
with $N$ be the number of points in our data, $C_q$ be the set of points in cluster $q$, $c_q$ be the center of cluster $q$, $c$ be the center of $E$, $n_q$ be the number of points in cluster $q$.

```python
>>> from sklearn import metrics
>>> from sklearn.metrics import pairwise_distances
>>> from sklearn import datasets
>>> dataset = datasets.load_iris()
>>> X = dataset.data
>>> y = dataset.target

In normal usage, the Calinski-Harabaz index is applied to the results of a cluster analysis.

```python
>>> import numpy as np
>>> from sklearn.cluster import KMeans
>>> kmeans_model = KMeans(n_clusters=3, random_state=1).fit(X)
>>> labels = kmeans_model.labels_
>>> metrics.calinski_harabaz_score(X, labels)
560.39...

Advantages

- The score is higher when clusters are dense and well separated, which relates to a standard concept of a cluster.
- The score is fast to compute

Drawbacks

- The Calinski-Harabaz index is generally higher for convex clusters than other concepts of clusters, such as density based clusters like those obtained through DBSCAN.

References


3.2.4 Biclustering

Biclustering can be performed with the module `sklearn.cluster.bicluster`. Biclustering algorithms simultaneously cluster rows and columns of a data matrix. These clusters of rows and columns are known as biclusters. Each determines a submatrix of the original data matrix with some desired properties.

For instance, given a matrix of shape $(10, 10)$, one possible bicluster with three rows and two columns induces a submatrix of shape $(3, 2)$:

```python
>>> import numpy as np
>>> data = np.arange(100).reshape(10, 10)
>>> rows = np.array([0, 2, 3])[:, np.newaxis]
>>> columns = np.array([1, 2])
>>> data[rows, columns]
array([[ 1,  2],
       [21, 22],
       [31, 32]])
```
For visualization purposes, given a bicluster, the rows and columns of the data matrix may be rearranged to make the bicluster contiguous.

Algorithms differ in how they define biclusters. Some of the common types include:

- constant values, constant rows, or constant columns
- unusually high or low values
- submatrices with low variance
- correlated rows or columns

Algorithms also differ in how rows and columns may be assigned to biclusters, which leads to different bicluster structures. Block diagonal or checkerboard structures occur when rows and columns are divided into partitions.

If each row and each column belongs to exactly one bicluster, then rearranging the rows and columns of the data matrix reveals the biclusters on the diagonal. Here is an example of this structure where biclusters have higher average values than the other rows and columns:

![Figure 3.5: An example of biclusters formed by partitioning rows and columns.](image)

In the checkerboard case, each row belongs to all column clusters, and each column belongs to all row clusters. Here is an example of this structure where the variance of the values within each bicluster is small:

After fitting a model, row and column cluster membership can be found in the `rows_` and `columns_` attributes. `rows_[i]` is a binary vector with nonzero entries corresponding to rows that belong to bicluster `i`. Similarly, `columns_[i]` indicates which columns belong to bicluster `i`.

Some models also have `row_labels_` and `column_labels_` attributes. These models partition the rows and columns, such as in the block diagonal and checkerboard bicluster structures.

**Note:** Biclustering has many other names in different fields including co-clustering, two-mode clustering, two-way clustering, block clustering, coupled two-way clustering, etc. The names of some algorithms, such as the Spectral Co-Clustering algorithm, reflect these alternate names.
The **SpectralCoclustering** algorithm finds biclusters with values higher than those in the corresponding other rows and columns. Each row and each column belongs to exactly one bicluster, so rearranging the rows and columns to make partitions contiguous reveals these high values along the diagonal:

**Note:** The algorithm treats the input data matrix as a bipartite graph: the rows and columns of the matrix correspond to the two sets of vertices, and each entry corresponds to an edge between a row and a column. The algorithm approximates the normalized cut of this graph to find heavy subgraphs.

**Mathematical formulation**

An approximate solution to the optimal normalized cut may be found via the generalized eigenvalue decomposition of the Laplacian of the graph. Usually this would mean working directly with the Laplacian matrix. If the original data matrix \( A \) has shape \( m \times n \), the Laplacian matrix for the corresponding bipartite graph has shape \( (m+n) \times (m+n) \). However, in this case it is possible to work directly with \( A \), which is smaller and more efficient.

The input matrix \( A \) is preprocessed as follows:

\[
A_n = R^{-1/2} AC^{-1/2}
\]

Where \( R \) is the diagonal matrix with entry \( i \) equal to \( \sum_j A_{ij} \) and \( C \) is the diagonal matrix with entry \( j \) equal to \( \sum_i A_{ij} \).

The singular value decomposition, \( A_n = U \Sigma V^T \), provides the partitions of the rows and columns of \( A \). A subset of the left singular vectors gives the row partitions, and a subset of the right singular vectors gives the column partitions.

The \( \ell = \lceil \log_2 k \rceil \) singular vectors, starting from the second, provide the desired partitioning information. They are used to form the matrix \( Z \):

\[
Z = \begin{bmatrix} R^{-1/2}U \\ C^{-1/2}V \end{bmatrix}
\]
where the columns of $U$ are $u_2, \ldots, u_{\ell+1}$, and similarly for $V$.

Then the rows of $Z$ are clustered using $k$-means. The first $n\_rows$ labels provide the row partitioning, and the remaining $n\_columns$ labels provide the column partitioning.

Examples:

- *A demo of the Spectral Co-Clustering algorithm:* A simple example showing how to generate a data matrix with biclusters and apply this method to it.

- *Biclustering documents with the Spectral Co-clustering algorithm:* An example of finding biclusters in the twenty newsgroup dataset.

References:


**Spectral Biclustering**

The *SpectralBiclustering* algorithm assumes that the input data matrix has a hidden checkerboard structure. The rows and columns of a matrix with this structure may be partitioned so that the entries of any bicluster in the Cartesian product of row clusters and column clusters are approximately constant. For instance, if there are two row partitions and three column partitions, each row will belong to three biclusters, and each column will belong to two biclusters.

The algorithm partitions the rows and columns of a matrix so that a corresponding blockwise-constant checkerboard matrix provides a good approximation to the original matrix.

**Mathematical formulation**

The input matrix $A$ is first normalized to make the checkerboard pattern more obvious. There are three possible methods:

1. *Independent row and column normalization*, as in Spectral Co-Clustering. This method makes the rows sum to a constant and the columns sum to a different constant.

2. *Bistochastization*: repeated row and column normalization until convergence. This method makes both rows and columns sum to the same constant.

3. *Log normalization*: the log of the data matrix is computed: $L = \log A$. Then the column mean $\overline{L}_i$, row mean $\overline{L}_j$, and overall mean $\overline{L}$ of $L$ are computed. The final matrix is computed according to the formula

$$K_{ij} = L_{ij} - \overline{L}_i - \overline{L}_j + \overline{L}.$$

After normalizing, the first few singular vectors are computed, just as in the Spectral Co-Clustering algorithm.

If log normalization was used, all the singular vectors are meaningful. However, if independent normalization or bistochastization were used, the first singular vectors, $u_1$ and $v_1$, are discarded. From now on, the “first” singular vectors refers to $u_2 \ldots u_{p+1}$ and $v_2 \ldots v_{p+1}$ except in the case of log normalization.

Given these singular vectors, they are ranked according to which can be best approximated by a piecewise-constant vector. The approximations for each vector are found using one-dimensional k-means and scored using the Euclidean distance. Some subset of the best left and right singular vector are selected. Next, the data is projected to this best subset of singular vectors and clustered.
For instance, if $p$ singular vectors were calculated, the $q$ best are found as described, where $q < p$. Let $U$ be the matrix with columns the $q$ best left singular vectors, and similarly $V$ for the right. To partition the rows, the rows of $A$ are projected to a $q$ dimensional space: $A * V$. Treating the $m$ rows of this $m \times q$ matrix as samples and clustering using k-means yields the row labels. Similarly, projecting the columns to $A^T * U$ and clustering this $n \times q$ matrix yields the column labels.

**Examples:**

- *A demo of the Spectral Biclustering algorithm:* a simple example showing how to generate a checkerboard matrix and bicluster it.

**References:**

- Kluger, Yuval, et. al., 2003. Spectral biclustering of microarray data: coclustering genes and conditions.

**Biclustering evaluation**

There are two ways of evaluating a biclustering result: internal and external. Internal measures, such as cluster stability, rely only on the data and the result themselves. Currently there are no internal bicluster measures in scikit-learn. External measures refer to an external source of information, such as the true solution. When working with real data the true solution is usually unknown, but biclustering artificial data may be useful for evaluating algorithms precisely because the true solution is known.

To compare a set of found biclusters to the set of true biclusters, two similarity measures are needed: a similarity measure for individual biclusters, and a way to combine these individual similarities into an overall score.

To compare individual biclusters, several measures have been used. For now, only the Jaccard index is implemented:

$$J(A, B) = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}$$

where $A$ and $B$ are biclusters, $|A \cap B|$ is the number of elements in their intersection. The Jaccard index achieves its minimum of 0 when the biclusters to not overlap at all and its maximum of 1 when they are identical.

Several methods have been developed to compare two sets of biclusters. For now, only `consensus_score` (Hochreiter et. al., 2010) is available:

1. Compute bicluster similarities for pairs of biclusters, one in each set, using the Jaccard index or a similar measure.
2. Assign biclusters from one set to another in a one-to-one fashion to maximize the sum of their similarities. This step is performed using the Hungarian algorithm.
3. The final sum of similarities is divided by the size of the larger set.

The minimum consensus score, 0, occurs when all pairs of biclusters are totally dissimilar. The maximum score, 1, occurs when both sets are identical.

**References:**

- Hochreiter, Bodenhofer, et. al., 2010. FABIA: factor analysis for bicluster acquisition.
3.2.5 Decomposing signals in components (matrix factorization problems)

Principal component analysis (PCA)

Exact PCA and probabilistic interpretation

PCA is used to decompose a multivariate dataset in a set of successive orthogonal components that explain a maximum amount of the variance. In scikit-learn, PCA is implemented as a transformer object that learns $n$ components in its fit method, and can be used on new data to project it on these components.

The optional parameter whiten=True makes it possible to project the data onto the singular space while scaling each component to unit variance. This is often useful if the models down-stream make strong assumptions on the isotropy of the signal: this is for example the case for Support Vector Machines with the RBF kernel and the K-Means clustering algorithm.

Below is an example of the iris dataset, which is comprised of 4 features, projected on the 2 dimensions that explain most variance:

![PCA of IRIS dataset](image)

The PCA object also provides a probabilistic interpretation of the PCA that can give a likelihood of data based on the amount of variance it explains. As such it implements a score method that can be used in cross-validation:

Examples:

- Comparison of LDA and PCA 2D projection of Iris dataset
- Model selection with Probabilistic PCA and Factor Analysis (FA)
Incremental PCA

The PCA object is very useful, but has certain limitations for large datasets. The biggest limitation is that PCA only supports batch processing, which means all of the data to be processed must fit in main memory. The IncrementalPCA object uses a different form of processing and allows for partial computations which almost exactly match the results of PCA while processing the data in a minibatch fashion. IncrementalPCA makes it possible to implement out-of-core Principal Component Analysis either by:

- Using its `partial_fit` method on chunks of data fetched sequentially from the local hard drive or a network database.
- Calling its `fit` method on a memory mapped file using `numpy.memmap`.

IncrementalPCA only stores estimates of component and noise variances, in order update `explained_variance_ratio_` incrementally. This is why memory usage depends on the number of samples per batch, rather than the number of samples to be processed in the dataset.

Examples:

- Incremental PCA

PCA using randomized SVD

It is often interesting to project data to a lower-dimensional space that preserves most of the variance, by dropping the singular vector of components associated with lower singular values.

For instance, if we work with 64x64 pixel gray-level pictures for face recognition, the dimensionality of the data is 4096 and it is slow to train an RBF support vector machine on such wide data. Furthermore we know that the intrinsic dimensionality of the data is much lower than 4096 since all pictures of human faces look somewhat alike. The samples lie on a manifold of much lower dimension (say around 200 for instance). The PCA algorithm can be used to
Incremental PCA of iris dataset
Mean absolute unsigned error 0.004077

- setosa
- versicolor
- virginica
linearly transform the data while both reducing the dimensionality and preserve most of the explained variance at the same time.

The class \texttt{PCA} used with the optional parameter \texttt{svd_solver='randomized'} is very useful in that case: since we are going to drop most of the singular vectors it is much more efficient to limit the computation to an approximated estimate of the singular vectors we will keep to actually perform the transform.

For instance, the following shows 16 sample portraits (centered around 0.0) from the Olivetti dataset. On the right hand side are the first 16 singular vectors reshaped as portraits. Since we only require the top 16 singular vectors of a dataset with size $n_{\text{samples}} = 400$ and $n_{\text{features}} = 64 \times 64 = 4096$, the computation time is less than 1s:

Note: with the optional parameter \texttt{svd_solver='randomized'}, we also need to give \texttt{PCA} the size of the lower-dimensional space \texttt{n\_components} as a mandatory input parameter.

If we note $n_{\text{max}} = \max(n_{\text{samples}}, n_{\text{features}})$ and $n_{\text{min}} = \min(n_{\text{samples}}, n_{\text{features}})$, the time complexity of the randomized \texttt{PCA} is $O(n_{\text{max}}^2 \cdot n_{\text{components}})$ instead of $O(n_{\text{max}}^2 \cdot n_{\text{min}})$ for the exact method implemented in \texttt{PCA}.

The memory footprint of randomized \texttt{PCA} is also proportional to $2 \cdot n_{\text{max}} \cdot n_{\text{components}}$ instead of $n_{\text{max}} \cdot n_{\text{min}}$ for the exact method.

Note: the implementation of \texttt{inverse\_transform} in \texttt{PCA} with \texttt{svd\_solver='randomized'} is not the exact inverse transform of \texttt{transform} even when \texttt{whiten=False} (default).
Kernel PCA

*KernelPCA* is an extension of PCA which achieves non-linear dimensionality reduction through the use of kernels (see *Pairwise metrics, Affinities and Kernels*). It has many applications including denoising, compression and structured prediction (kernel dependency estimation). *KernelPCA* supports both *transform* and *inverse_transform*.

Examples:

- *Kernel PCA*

Sparse principal components analysis (*SparsePCA* and *MiniBatchSparsePCA*)

*SparsePCA* is a variant of PCA, with the goal of extracting the set of sparse components that best reconstruct the
Mini-batch sparse PCA (`MiniBatchSparsePCA`) is a variant of `SparsePCA` that is faster but less accurate. The increased speed is reached by iterating over small chunks of the set of features, for a given number of iterations.

Principal component analysis (`PCA`) has the disadvantage that the components extracted by this method have exclusively dense expressions, i.e. they have non-zero coefficients when expressed as linear combinations of the original variables. This can make interpretation difficult. In many cases, the real underlying components can be more naturally imagined as sparse vectors; for example in face recognition, components might naturally map to parts of faces.

Sparse principal components yields a more parsimonious, interpretable representation, clearly emphasizing which of the original features contribute to the differences between samples.

The following example illustrates 16 components extracted using sparse PCA from the Olivetti faces dataset. It can be seen how the regularization term induces many zeros. Furthermore, the natural structure of the data causes the non-zero coefficients to be vertically adjacent. The model does not enforce this mathematically: each component is a vector $h \in \mathbb{R}^{4096}$, and there is no notion of vertical adjacency except during the human-friendly visualization as 64x64 pixel images. The fact that the components shown below appear local is the effect of the inherent structure of the data, which makes such local patterns minimize reconstruction error. There exist sparsity-inducing norms that take into account adjacency and different kinds of structure; see [Jen09] for a review of such methods. For more details on how to use Sparse PCA, see the Examples section, below.

Note that there are many different formulations for the Sparse PCA problem. The one implemented here is based...
on [Mrl09]. The optimization problem solved is a PCA problem (dictionary learning) with an $\ell_1$ penalty on the components:

$$(U^*, V^*) = \arg \min_{U, V} \frac{1}{2} \| X - UV \|_2^2 + \alpha \| V \|_1$$

subject to $\| U_k \|_2 = 1$ for all $0 \leq k < n_{\text{components}}$

The sparsity-inducing $\ell_1$ norm also prevents learning components from noise when few training samples are available. The degree of penalization (and thus sparsity) can be adjusted through the hyperparameter $\alpha$. Small values lead to a gently regularized factorization, while larger values shrink many coefficients to zero.

**Note:** While in the spirit of an online algorithm, the class `MiniBatchSparsePCA` does not implement `partial_fit` because the algorithm is online along the features direction, not the samples direction.

**Examples:**

- *Faces dataset decompositions*

**References:**

**Truncated singular value decomposition and latent semantic analysis**

`TruncatedSVD` implements a variant of singular value decomposition (SVD) that only computes the $k$ largest singular values, where $k$ is a user-specified parameter.

When truncated SVD is applied to term-document matrices (as returned by `CountVectorizer` or `TfidfVectorizer`), this transformation is known as latent semantic analysis (LSA), because it transforms such matrices to a “semantic” space of low dimensionality. In particular, LSA is known to combat the effects of synonymy and polysemy (both of which roughly mean there are multiple meanings per word), which cause term-document matrices to be overly sparse and exhibit poor similarity under measures such as cosine similarity.

**Note:** LSA is also known as latent semantic indexing, LSI, though strictly that refers to its use in persistent indexes for information retrieval purposes.

Mathematically, truncated SVD applied to training samples $X$ produces a low-rank approximation $X$:

$$X \approx X_k = U_k \Sigma_k V_k^T$$

After this operation, $U_k \Sigma_k V_k^T$ is the transformed training set with $k$ features (called `n_components` in the API).

To also transform a test set $X$, we multiply it with $V_k$:

$$X' = XV_k$$

**Note:** Most treatments of LSA in the natural language processing (NLP) and information retrieval (IR) literature swap the axes of the matrix $X$ so that it has shape $n_{\text{features}} \times n_{\text{samples}}$. We present LSA in a different way that matches the scikit-learn API better, but the singular values found are the same.
TruncatedSVD is very similar to PCA, but differs in that it works on sample matrices $X$ directly instead of their covariance matrices. When the columnwise (per-feature) means of $X$ are subtracted from the feature values, truncated SVD on the resulting matrix is equivalent to PCA. In practical terms, this means that the TruncatedSVD transformer accepts scipy.sparse matrices without the need to densify them, as densifying may fill up memory even for medium-sized document collections.

While the TruncatedSVD transformer works with any (sparse) feature matrix, using it on tf–idf matrices is recommended over raw frequency counts in an LSA/document processing setting. In particular, sublinear scaling and inverse document frequency should be turned on (sublinear_tf=True, use_idf=True) to bring the feature values closer to a Gaussian distribution, compensating for LSA’s erroneous assumptions about textual data.

**Examples:**

- Clustering text documents using k-means

**References:**

- Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze (2008), Introduction to Information Retrieval, Cambridge University Press, chapter 18: Matrix decompositions & latent semantic indexing

**Dictionary Learning**

**Sparse coding with a precomputed dictionary**

The SparseCoder object is an estimator that can be used to transform signals into sparse linear combination of atoms from a fixed, precomputed dictionary such as a discrete wavelet basis. This object therefore does not implement a fit method. The transformation amounts to a sparse coding problem: finding a representation of the data as a linear combination of as few dictionary atoms as possible. All variations of dictionary learning implement the following transform methods, controllable via the transform_method initialization parameter:

- Orthogonal matching pursuit (Orthogonal Matching Pursuit (OMP))
- Least-angle regression (Least Angle Regression)
- Lasso computed by least-angle regression
- Lasso using coordinate descent (Lasso)
- Thresholding

Thresholding is very fast but it does not yield accurate reconstructions. They have been shown useful in literature for classification tasks. For image reconstruction tasks, orthogonal matching pursuit yields the most accurate, unbiased reconstruction.

The dictionary learning objects offer, via the split_code parameter, the possibility to separate the positive and negative values in the results of sparse coding. This is useful when dictionary learning is used for extracting features that will be used for supervised learning, because it allows the learning algorithm to assign different weights to negative loadings of a particular atom, from to the corresponding positive loading.

The split code for a single sample has length $2 \times n_{components}$ and is constructed using the following rule: First, the regular code of length $n_{components}$ is computed. Then, the first $n_{components}$ entries of the split_code are filled with the positive part of the regular code vector. The second half of the split code is filled with the negative part of the code vector, only with a positive sign. Therefore, the split_code is non-negative.
Examples:

- Sparse coding with a precomputed dictionary

Generic dictionary learning

Dictionary learning (DictionaryLearning) is a matrix factorization problem that amounts to finding a (usually overcomplete) dictionary that will perform good at sparsely encoding the fitted data.

Representing data as sparse combinations of atoms from an overcomplete dictionary is suggested to be the way the mammal primary visual cortex works. Consequently, dictionary learning applied on image patches has been shown to give good results in image processing tasks such as image completion, inpainting and denoising, as well as for supervised recognition tasks.

Dictionary learning is an optimization problem solved by alternatively updating the sparse code, as a solution to multiple Lasso problems, considering the dictionary fixed, and then updating the dictionary to best fit the sparse code.

\[
(U^*, V^*) = \arg \min_{U,V} \frac{1}{2} ||X - UV||_2^2 + \alpha ||U||_1
\]

subject to \( ||V_k||_2 = 1 \) for all \( 0 \leq k < n_{atoms} \)

genfaces - PCA using randomized SVD - Train time 0.:

MiniBatchDictionaryLearning - Train time 1.0s
After using such a procedure to fit the dictionary, the transform is simply a sparse coding step that shares the same implementation with all dictionary learning objects (see Sparse coding with a precomputed dictionary). The following image shows how a dictionary learned from 4x4 pixel image patches extracted from part of the image of a raccoon face looks like.

Examples:

- Image denoising using dictionary learning

References:


Mini-batch dictionary learning

MiniBatchDictionaryLearning implements a faster, but less accurate version of the dictionary learning algorithm that is better suited for large datasets.

By default, MiniBatchDictionaryLearning divides the data into mini-batches and optimizes in an online manner by cycling over the mini-batches for the specified number of iterations. However, at the moment it does not implement a stopping condition.

The estimator also implements partial_fit, which updates the dictionary by iterating only once over a mini-batch. This can be used for online learning when the data is not readily available from the start, or for when the data does not fit into the memory.
Clustering for dictionary learning

Note that when using dictionary learning to extract a representation (e.g. for sparse coding) clustering can be a good proxy to learn the dictionary. For instance the MiniBatchKMeans estimator is computationally efficient and implements on-line learning with a partial_fit method.

Example: Online learning of a dictionary of parts of faces

Factor Analysis

In unsupervised learning we only have a dataset \( X = \{x_1, x_2, \ldots, x_n\} \). How can this dataset be described mathematically? A very simple continuous latent variable model for \( X \) is

\[
x_i = Wh_i + \mu + \epsilon
\]

The vector \( h_i \) is called “latent” because it is unobserved. \( \epsilon \) is considered a noise term distributed according to a Gaussian with mean 0 and covariance \( \Psi \) (i.e. \( \epsilon \sim \mathcal{N}(0, \Psi) \)). \( \mu \) is some arbitrary offset vector. Such a model is called “generative” as it describes how \( x_i \) is generated from \( h_i \). If we use all the \( x_i \)'s as columns to form a matrix \( X \) and all the \( h_i \)'s as columns of a matrix \( H \) then we can write (with suitably defined \( M \) and \( E \)):

\[
X = WH + M + E
\]

In other words, we decomposed matrix \( X \).

If \( h_i \) is given, the above equation automatically implies the following probabilistic interpretation:

\[
p(x_i | h_i) = \mathcal{N}(Wh_i + \mu, \Psi)
\]

For a complete probabilistic model we also need a prior distribution for the latent variable \( h \). The most straightforward assumption (based on the nice properties of the Gaussian distribution) is \( h \sim \mathcal{N}(0, I) \). This yields a Gaussian as the marginal distribution of \( x \):

\[
p(x) = \mathcal{N}(\mu, WW^T + \Psi)
\]

Now, without any further assumptions the idea of having a latent variable \( h \) would be superfluous – \( x \) can be completely modelled with a mean and a covariance. We need to impose some more specific structure on one of these two parameters. A simple additional assumption regards the structure of the error covariance \( \Psi \):

- \( \Psi = \sigma^2 I \): This assumption leads to the probabilistic model of PCA.
- \( \Psi = \text{diag}(\psi_1, \psi_2, \ldots, \psi_n) \): This model is called FactorAnalysis, a classical statistical model. The matrix \( W \) is sometimes called the “factor loading matrix”.

Both models essentially estimate a Gaussian with a low-rank covariance matrix. Because both models are probabilistic they can be integrated in more complex models, e.g. Mixture of Factor Analysers. One gets very different models (e.g. FastICA) if non-Gaussian priors on the latent variables are assumed.

Factor analysis can produce similar components (the columns of its loading matrix) to PCA. However, one can not make any general statements about these components (e.g. whether they are orthogonal):
The main advantage for Factor Analysis (over PCA) is that it can model the variance in every direction of the input space independently (heteroscedastic noise):

This allows better model selection than probabilistic PCA in the presence of heteroscedastic noise:

Examples:

- Model selection with Probabilistic PCA and Factor Analysis (FA)
Independent component analysis (ICA)

Independent component analysis separates a multivariate signal into additive subcomponents that are maximally independent. It is implemented in scikit-learn using the Fast ICA algorithm. Typically, ICA is not used for reducing dimensionality but for separating superimposed signals. Since the ICA model does not include a noise term, for the model to be correct, whitening must be applied. This can be done internally using the whiten argument or manually using one of the PCA variants.

It is classically used to separate mixed signals (a problem known as blind source separation), as in the example below:

ICA can also be used as yet another non linear decomposition that finds components with some sparsity:
Examples:

- Blind source separation using FastICA
- FastICA on 2D point clouds
- Faces dataset decompositions

Non-negative matrix factorization (NMF or NNMF)

NMF is an alternative approach to decomposition that assumes that the data and the components are non-negative. NMF can be plugged in instead of PCA or its variants, in the cases where the data matrix does not contain negative values. It finds a decomposition of samples $X$ into two matrices $W$ and $H$ of non-negative elements, by optimizing for the squared Frobenius norm:

$$\arg \min_{W,H} \frac{1}{2} \|X - WH\|_F^2 = \frac{1}{2} \sum_{i,j} (X_{ij} - WH_{ij})^2$$

This norm is an obvious extension of the Euclidean norm to matrices. (Other optimization objectives have been suggested in the NMF literature, in particular Kullback-Leibler divergence, but these are not currently implemented.)
Unlike PCA, the representation of a vector is obtained in an additive fashion, by superimposing the components, without subtracting. Such additive models are efficient for representing images and text.

It has been observed in [Hoyer, 04] that, when carefully constrained, NMF can produce a parts-based representation of the dataset, resulting in interpretable models. The following example displays 16 sparse components found by NMF from the images in the Olivetti faces dataset, in comparison with the PCA eigenfaces.

The `init` attribute determines the initialization method applied, which has a great impact on the performance of the method. NMF implements the method Nonnegative Double Singular Value Decomposition. NNDSVD is based on two SVD processes, one approximating the data matrix, the other approximating positive sections of the resulting partial SVD factors utilizing an algebraic property of unit rank matrices. The basic NNDSVD algorithm is better fit for sparse factorization. Its variants NNDSVDa (in which all zeros are set equal to the mean of all elements of the data), and NNDSVDar (in which the zeros are set to random perturbations less than the mean of the data divided by 100) are recommended in the dense case.

NMF can also be initialized with correctly scaled random non-negative matrices by setting `init="random"`. An integer seed or a `RandomState` can also be passed to `random_state` to control reproducibility.

In NMF, L1 and L2 priors can be added to the loss function in order to regularize the model. The L2 prior uses the Frobenius norm, while the L1 prior uses an elementwise L1 norm. As in ElasticNet, we control the combination of L1 and L2 with the `l1_ratio (ρ)` parameter, and the intensity of the regularization with the `alpha (α)` parameter.
Then the priors terms are:

\[ \alpha \rho ||W||_1 + \alpha \rho ||H||_1 + \frac{\alpha(1 - \rho)}{2} ||W||_{Fro}^2 + \frac{\alpha(1 - \rho)}{2} ||H||_{Fro}^2 \]

and the regularized objective function is:

\[ \frac{1}{2} ||X - WH||_{Fro}^2 + \alpha \rho ||W||_1 + \alpha \rho ||H||_1 + \frac{\alpha(1 - \rho)}{2} ||W||_{Fro}^2 + \frac{\alpha(1 - \rho)}{2} ||H||_{Fro}^2 \]

NMF regularizes both W and H. The public function `non_negative_factorization` allows a finer control through the `regularization` attribute, and may regularize only W, only H, or both.

**Examples:**

- *Faces dataset decompositions*
- *Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation*

**References:**

- “Learning the parts of objects by non-negative matrix factorization” D. Lee, S. Seung, 1999
- “Non-negative Matrix Factorization with Sparseness Constraints” P. Hoyer, 2004
- “SVD based initialization: A head start for nonnegative matrix factorization” C. Boutsidis, E. Gallopoulos, 2008
- “Fast local algorithms for large scale nonnegative matrix and tensor factorizations.” A. Cichocki, P. Anh-Huy, 2009

**Latent Dirichlet Allocation (LDA)**

Latent Dirichlet Allocation is a generative probabilistic model for collections of discrete dataset such as text corpora. It is also a topic model that is used for discovering abstract topics from a collection of documents.

The graphical model of LDA is a three-level Bayesian model:
When modeling text corpora, the model assumes the following generative process for a corpus with $D$ documents and $K$ topics:

1. For each topic $k$, draw $\beta_k \sim \text{Dirichlet} (\eta)$, $k = 1...K$
2. For each document $d$, draw $\theta_d \sim \text{Dirichlet} (\alpha)$, $d = 1...D$
3. For each word $i$ in document $d$:
   1. Draw a topic index $z_{di} \sim \text{Multinomial} (\theta_d)$
   2. Draw the observed word $w_{ij} \sim \text{Multinomial} (\beta_{z_{di}})$

For parameter estimation, the posterior distribution is:

$$p(z, \theta, \beta | w, \alpha, \eta) = \frac{p(z, \theta, \beta | \alpha, \eta)}{p(w | \alpha, \eta)}$$

Since the posterior is intractable, variational Bayesian method uses a simpler distribution $q(z, \theta, \beta | \lambda, \phi, \gamma)$ to approximate it, and those variational parameters $\lambda, \phi, \gamma$ are optimized to maximize the Evidence Lower Bound (ELBO):

$$\log P(w | \alpha, \eta) \geq E_q[\log p(w, z, \theta, \beta | \alpha, \eta)] - E_q[\log q(z, \theta, \beta)]$$

Maximizing ELBO is equivalent to minimizing the Kullback-Leibler (KL) divergence between $q(z, \theta, \beta)$ and the true posterior $p(z, \theta, \beta | w, \alpha, \eta)$.

**LatentDirichletAllocation** implements online variational Bayes algorithm and supports both online and batch update method. While batch method updates variational variables after each full pass through the data, online method updates variational variables from mini-batch data points.

**Note:** Although online method is guaranteed to converge to a local optimum point, the quality of the optimum point and the speed of convergence may depend on mini-batch size and attributes related to learning rate setting.

When **LatentDirichletAllocation** is applied on a “document-term” matrix, the matrix will be decomposed into a “topic-term” matrix and a “document-topic” matrix. While “topic-term” matrix is stored as components in the model, “document-topic” matrix can be calculated from transform method.

**LatentDirichletAllocation** also implements partial_fit method. This is used when data can be fetched sequentially.
3.2.6 Covariance estimation

Many statistical problems require at some point the estimation of a population’s covariance matrix, which can be seen as an estimation of data set scatter plot shape. Most of the time, such an estimation has to be done on a sample whose properties (size, structure, homogeneity) has a large influence on the estimation’s quality. The sklearn.covariance package aims at providing tools affording an accurate estimation of a population’s covariance matrix under various settings.

We assume that the observations are independent and identically distributed (i.i.d.).

Empirical covariance

The covariance matrix of a data set is known to be well approximated with the classical maximum likelihood estimator (or “empirical covariance”), provided the number of observations is large enough compared to the number of features (the variables describing the observations). More precisely, the Maximum Likelihood Estimator of a sample is an unbiased estimator of the corresponding population covariance matrix.

The empirical covariance matrix of a sample can be computed using the empirical_covariance function of the package, or by fitting an EmpiricalCovariance object to the data sample with the EmpiricalCovariance.fit method. Be careful that depending whether the data are centered or not, the result will be different, so one may want to use the assume_centered parameter accurately. More precisely if one uses assume_centered=False, then the test set is supposed to have the same mean vector as the training set. If not so, both should be centered by the user, and assume_centered=True should be used.

Examples:

- See Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood for an example on how to fit an EmpiricalCovariance object to data.

Shrunk Covariance

Basic shrinkage

Despite being an unbiased estimator of the covariance matrix, the Maximum Likelihood Estimator is not a good estimator of the eigenvalues of the covariance matrix, so the precision matrix obtained from its inversion is not accurate.
Sometimes, it even occurs that the empirical covariance matrix cannot be inverted for numerical reasons. To avoid such an inversion problem, a transformation of the empirical covariance matrix has been introduced: the shrinkage.

In the scikit-learn, this transformation (with a user-defined shrinkage coefficient) can be directly applied to a pre-computed covariance with the `shrinked_covariance` method. Also, a shrunk estimator of the covariance can be fitted to data with a `ShrunkCovariance` object and its `ShrunkCovariance.fit` method. Again, depending whether the data are centered or not, the result will be different, so one may want to use the `assume_centered` parameter accurately.

Mathematically, this shrinkage consists in reducing the ratio between the smallest and the largest eigenvalue of the empirical covariance matrix. It can be done by simply shifting every eigenvalue according to a given offset, which is equivalent of finding the l2-penalized Maximum Likelihood Estimator of the covariance matrix. In practice, shrinkage boils down to a simple a convex transformation:

\[ \Sigma_{\text{shrunk}} = (1 - \alpha) \hat{\Sigma} + \alpha \frac{Tr \hat{\Sigma}}{p} \text{Id}. \]

Choosing the amount of shrinkage, \( \alpha \) amounts to setting a bias/variance trade-off, and is discussed below.

**Examples:**

- See [Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood](https://scikit-learn.org/stable/modules/covariance.html#shrinkage) for an example on how to fit a `ShrunkCovariance` object to data.

---

**Ledoit-Wolf shrinkage**

In their 2004 paper [1], O. Ledoit and M. Wolf propose a formula so as to compute the optimal shrinkage coefficient \( \alpha \) that minimizes the Mean Squared Error between the estimated and the real covariance matrix.

The Ledoit-Wolf estimator of the covariance matrix can be computed on a sample with the `ledoit_wolf` function of the `sklearn.covariance` package, or it can be otherwise obtained by fitting a `LedoitWolf` object to the same sample.

**Examples:**

- See [Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood](https://scikit-learn.org/stable/modules/covariance.html#shrinkage) for an example on how to fit a `LedoitWolf` object to data and for visualizing the performances of the Ledoit-Wolf estimator in terms of likelihood.


---

**Oracle Approximating Shrinkage**

Under the assumption that the data are Gaussian distributed, Chen et al. [2] derived a formula aimed at choosing a shrinkage coefficient that yields a smaller Mean Squared Error than the one given by Ledoit and Wolf’s formula. The resulting estimator is known as the Oracle Shrinkage Approximating estimator of the covariance.

The OAS estimator of the covariance matrix can be computed on a sample with the `oas` function of the `sklearn.covariance` package, or it can be otherwise obtained by fitting an `OAS` object to the same sample.

Sparse inverse covariance

The matrix inverse of the covariance matrix, often called the precision matrix, is proportional to the partial correlation matrix. It gives the partial independence relationship. In other words, if two features are independent conditionally on the others, the corresponding coefficient in the precision matrix will be zero. This is why it makes sense to estimate a sparse precision matrix: by learning independence relations from the data, the estimation of the covariance matrix is better conditioned. This is known as covariance selection.

In the small-samples situation, in which \( n_{\text{samples}} \) is on the order of \( n_{\text{features}} \) or smaller, sparse inverse covariance estimators tend to work better than shrunk covariance estimators. However, in the opposite situation, or for very correlated data, they can be numerically unstable. In addition, unlike shrinkage estimators, sparse estimators are able to recover off-diagonal structure.

The \texttt{GraphLasso} estimator uses an \texttt{L1} penalty to enforce sparsity on the precision matrix: the higher its \texttt{alpha} parameter, the more sparse the precision matrix. The corresponding \texttt{GraphLassoCV} object uses cross-validation to automatically set the \texttt{alpha} parameter.

\textbf{Note: Structure recovery}

Recovering a graphical structure from correlations in the data is a challenging thing. If you are interested in such recovery keep in mind that:
Fig. 3.8: A comparison of maximum likelihood, shrinkage and sparse estimates of the covariance and precision matrix in the very small samples settings.
• Recovery is easier from a correlation matrix than a covariance matrix: standardize your observations before running GraphLasso

• If the underlying graph has nodes with much more connections than the average node, the algorithm will miss some of these connections.

• If your number of observations is not large compared to the number of edges in your underlying graph, you will not recover it.

• Even if you are in favorable recovery conditions, the alpha parameter chosen by cross-validation (e.g. using the GraphLassoCV object) will lead to selecting too many edges. However, the relevant edges will have heavier weights than the irrelevant ones.

The mathematical formulation is the following:

\[ \hat{K} = \arg\min_K \left( \text{tr} \, S \hat{K} - \log \det \hat{K} + \alpha \|\hat{K}\|_1 \right) \]

Where \( \hat{K} \) is the precision matrix to be estimated, and \( S \) is the sample covariance matrix. \( \|\hat{K}\|_1 \) is the sum of the absolute values of off-diagonal coefficients of \( \hat{K} \). The algorithm employed to solve this problem is the GLasso algorithm, from the Friedman 2008 Biostatistics paper. It is the same algorithm as in the R glasso package.

Examples:

• Sparse inverse covariance estimation: example on synthetic data showing some recovery of a structure, and comparing to other covariance estimators.

• Visualizing the stock market structure: example on real stock market data, finding which symbols are most linked.

References:

• Friedman et al, “Sparse inverse covariance estimation with the graphical lasso”, Biostatistics 9, pp 432, 2008

Robust Covariance Estimation

Real data set are often subjects to measurement or recording errors. Regular but uncommon observations may also appear for a variety of reason. Every observation which is very uncommon is called an outlier. The empirical covariance estimator and the shrunk covariance estimators presented above are very sensitive to the presence of outlying observations in the data. Therefore, one should use robust covariance estimators to estimate the covariance of its real data sets. Alternatively, robust covariance estimators can be used to perform outlier detection and discard/downweight some observations according to further processing of the data.

The sklearn.covariance package implements a robust estimator of covariance, the Minimum Covariance Determinant [3].

Minimum Covariance Determinant

The Minimum Covariance Determinant estimator is a robust estimator of a data set’s covariance introduced by P.J. Rousseuw in [3]. The idea is to find a given proportion (h) of “good” observations which are not outliers and compute their empirical covariance matrix. This empirical covariance matrix is then rescaled to compensate the performed selection of observations (“consistency step”). Having computed the Minimum Covariance Determinant estimator,
one can give weights to observations according to their Mahalanobis distance, leading to a reweighted estimate of the covariance matrix of the data set (“reweighting step”).

Rousseeuw and Van Driessen [4] developed the FastMCD algorithm in order to compute the Minimum Covariance Determinant. This algorithm is used in scikit-learn when fitting an MCD object to data. The FastMCD algorithm also computes a robust estimate of the data set location at the same time.

Raw estimates can be accessed as `raw_location_` and `raw_covariance_` attributes of a `MinCovDet` robust covariance estimator object.


Examples:

- See `Robust vs Empirical covariance estimate` for an example on how to fit a `MinCovDet` object to data and see how the estimate remains accurate despite the presence of outliers.
- See `Robust covariace estimation and Mahalanobis distances relevance` to visualize the difference between `EmpiricalCovariance` and `MinCovDet` covariance estimators in terms of Mahalanobis distance (so we get a better estimate of the precision matrix too).

### 3.2.7 Novelty and Outlier Detection

Many applications require being able to decide whether a new observation belongs to the same distribution as existing observations (it is an *inlier*), or should be considered as different (it is an outlier). Often, this ability is used to clean real data sets. Two important distinction must be made:

**novelty detection** The training data is not polluted by outliers, and we are interested in detecting anomalies in new observations.

**outlier detection** The training data contains outliers, and we need to fit the central mode of the training data, ignoring the deviant observations.

The scikit-learn project provides a set of machine learning tools that can be used both for novelty or outliers detection. This strategy is implemented with objects learning in an unsupervised way from the data:

```
estimator.fit(X_train)
```

new observations can then be sorted as inliers or outliers with a `predict` method:
**Novelty Detection**

Consider a data set of $n$ observations from the same distribution described by $p$ features. Consider now that we add one more observation to that data set. Is the new observation so different from the others that we can doubt it is regular? (i.e. does it come from the same distribution?) Or on the contrary, is it so similar to the other that we cannot distinguish it from the original observations? This is the question addressed by the novelty detection tools and methods.

In general, it is about to learn a rough, close frontier delimiting the contour of the initial observations distribution, plotted in embedding $p$-dimensional space. Then, if further observations lay within the frontier-delimited subspace, they are considered as coming from the same population than the initial observations. Otherwise, if they lay outside the frontier, we can say that they are abnormal with a given confidence in our assessment.

The One-Class SVM has been introduced by Schölkopf et al. for that purpose and implemented in the `Support Vector Machines` module in the `svm.OneClassSVM` object. It requires the choice of a kernel and a scalar parameter to define a frontier. The RBF kernel is usually chosen although there exists no exact formula or algorithm to set its bandwidth parameter. This is the default in the scikit-learn implementation. The $\nu$ parameter, also known as the margin of the One-Class SVM, corresponds to the probability of finding a new, but regular, observation outside the frontier.

**References:**


**Examples:**

- See One-class SVM with non-linear kernel (RBF) for visualizing the frontier learned around some data by a `svm.OneClassSVM` object.

**Outlier Detection**

Outlier detection is similar to novelty detection in the sense that the goal is to separate a core of regular observations from some polluting ones, called “outliers”. Yet, in the case of outlier detection, we don’t have a clean data set representing the population of regular observations that can be used to train any tool.

**Fitting an elliptic envelope**

One common way of performing outlier detection is to assume that the regular data come from a known distribution (e.g. data are Gaussian distributed). From this assumption, we generally try to define the “shape” of the data, and can define outlying observations as observations which stand far enough from the fit shape.

The scikit-learn provides an object `covariance.EllipticEnvelope` that fits a robust covariance estimate to the data, and thus fits an ellipse to the central data points, ignoring points outside the central mode.
For instance, assuming that the inlier data are Gaussian distributed, it will estimate the inlier location and covariance in a robust way (i.e. without being influenced by outliers). The Mahalanobis distances obtained from this estimate is used to derive a measure of outlyingness. This strategy is illustrated below.

Examples:

- See *Robust covariance estimation and Mahalanobis distances relevance* for an illustration of the difference between using a standard (covariance.EmpiricalCovariance) or a robust estimate (covariance.MinCovDet) of location and covariance to assess the degree of outlyingness of an observation.

References:

**Isolation Forest**

One efficient way of performing outlier detection in high-dimensional datasets is to use random forests. The *ensemble.IsolationForest* “isolates” observations by randomly selecting a feature and then randomly selecting a split value between the maximum and minimum values of the selected feature.

Since recursive partitioning can be represented by a tree structure, the number of splittings required to isolate a sample is equivalent to the path length from the root node to the terminating node.

This path length, averaged over a forest of such random trees, is a measure of normality and our decision function.

Random partitioning produces noticeably shorter paths for anomalies. Hence, when a forest of random trees collectively produce shorter path lengths for particular samples, they are highly likely to be anomalies.

This strategy is illustrated below.
3.2. Unsupervised learning

1. from non-robust estimates
(Maximum Likelihood)

2. from robust estimates
(Minimum Covariance Determinant)

IsolationForest

- training observations
- new regular observations
- new abnormal observations
Examples:

- See *IsolationForest example* for an illustration of the use of IsolationForest.
- See *Outlier detection with several methods.* for a comparison of `ensemble.IsolationForest` with `svm.OneClassSVM` (tuned to perform like an outlier detection method) and a covariance-based outlier detection with `covariance.MinCovDet`.

References:

**One-class SVM versus Elliptic Envelope versus Isolation Forest**

Strictly-speaking, the One-class SVM is not an outlier-detection method, but a novelty-detection method: its training set should not be contaminated by outliers as it may fit them. That said, outlier detection in high-dimension, or without any assumptions on the distribution of the inlying data is very challenging, and a One-class SVM gives useful results in these situations.

The examples below illustrate how the performance of the `covariance.EllipticEnvelope` degrades as the data is less and less unimodal. The `svm.OneClassSVM` works better on data with multiple modes and `ensemble.IsolationForest` performs well in every cases.

**Table 3.1: Comparing One-class SVM approach, and elliptic envelope**

<table>
<thead>
<tr>
<th>Inlier Mode</th>
<th>SVM OneClassSVM</th>
<th>Elliptic Envelope</th>
<th>Isolation Forest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well-centered and elliptic</td>
<td>Cannot benefit from rotational symmetry</td>
<td>ELLIPSE FIT</td>
<td>Performs well</td>
</tr>
<tr>
<td>Bimodal</td>
<td>Difficulties to detect the two modes</td>
<td>Overfits</td>
<td>Performs well</td>
</tr>
<tr>
<td>Strongly non-Gaussian</td>
<td>Reaches a reasonable approximation</td>
<td>Completely fails</td>
<td>Performs well</td>
</tr>
</tbody>
</table>

Examples:

- See *Outlier detection with several methods.* for a comparison of the `svm.OneClassSVM` (tuned to perform like an outlier detection method), the `ensemble.IsolationForest` and a covariance-based outlier detection with `covariance.MinCovDet`. 
3.2.8 Density Estimation

Density estimation walks the line between unsupervised learning, feature engineering, and data modeling. Some of the most popular and useful density estimation techniques are mixture models such as Gaussian Mixtures (sklearn.mixture.GaussianMixture), and neighbor-based approaches such as the kernel density estimate (sklearn.neighbors.KernelDensity). Gaussian Mixtures are discussed more fully in the context of clustering, because the technique is also useful as an unsupervised clustering scheme.

Density estimation is a very simple concept, and most people are already familiar with one common density estimation technique: the histogram.

Density Estimation: Histograms

A histogram is a simple visualization of data where bins are defined, and the number of data points within each bin is tallied. An example of a histogram can be seen in the upper-left panel of the following figure:

A major problem with histograms, however, is that the choice of binning can have a disproportionate effect on the resulting visualization. Consider the upper-right panel of the above figure. It shows a histogram over the same data, with the bins shifted right. The results of the two visualizations look entirely different, and might lead to different interpretations of the data.

Intuitively, one can also think of a histogram as a stack of blocks, one block per point. By stacking the blocks in the appropriate grid space, we recover the histogram. But what if, instead of stacking the blocks on a regular grid, we center each block on the point it represents, and sum the total height at each location? This idea leads to the lower-left visualization. It is perhaps not as clean as a histogram, but the fact that the data drive the block locations mean that it is a much better representation of the underlying data.

This visualization is an example of a kernel density estimation, in this case with a top-hat kernel (i.e. a square block at each point). We can recover a smoother distribution by using a smoother kernel. The bottom-right plot shows a Gaussian kernel density estimate, in which each point contributes a Gaussian curve to the total. The result is a smooth
Kernel Density Estimation

Kernel density estimation in scikit-learn is implemented in the `sklearn.neighbors.KernelDensity` estimator, which uses the Ball Tree or KD Tree for efficient queries (see Nearest Neighbors for a discussion of these). Though the above example uses a 1D data set for simplicity, kernel density estimation can be performed in any number of dimensions, though in practice the curse of dimensionality causes its performance to degrade in high dimensions.

In the following figure, 100 points are drawn from a bimodal distribution, and the kernel density estimates are shown for three choices of kernels:

It’s clear how the kernel shape affects the smoothness of the resulting distribution. The scikit-learn kernel density estimator can be used as follows:

```python
>>> from sklearn.neighbors.kde import KernelDensity
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> kde = KernelDensity(kernel='gaussian', bandwidth=0.2).fit(X)
>>> kde.score_samples(X)
array([-0.41075698, -0.41075698, -0.41076071, -0.41075698, -0.41075698,
       -0.41076071])
```

Here we have used `kernel='gaussian'`, as seen above. Mathematically, a kernel is a positive function $K(x; h)$ which is controlled by the bandwidth parameter $h$. Given this kernel form, the density estimate at a point $y$ within a group of points $x_i; i = 1 \cdots N$ is given by:

$$\rho_K(y) = \sum_{i=1}^{N} K((y - x_i)/h)$$
The bandwidth here acts as a smoothing parameter, controlling the tradeoff between bias and variance in the result. A large bandwidth leads to a very smooth (i.e. high-bias) density distribution. A small bandwidth leads to an unsmooth (i.e. high-variance) density distribution.

`sklearn.neighbors.KernelDensity` implements several common kernel forms, which are shown in the following figure:

The form of these kernels is as follows:

- **Gaussian kernel** (kernel = 'gaussian')
  \[ K(x; h) \propto \exp\left(-\frac{x^2}{2h^2}\right) \]

- **Tophat kernel** (kernel = 'tophat')
  \[ K(x; h) \propto 1 \text{ if } x < h \]

- **Epanechnikov kernel** (kernel = 'epanechnikov')
  \[ K(x; h) \propto 1 - \frac{x^2}{h^2} \]

- **Exponential kernel** (kernel = 'exponential')
  \[ K(x; h) \propto \exp\left(-\frac{x}{h}\right) \]

- **Linear kernel** (kernel = 'linear')
  \[ K(x; h) \propto 1 - x/h \text{ if } x < h \]

- **Cosine kernel** (kernel = 'cosine')
  \[ K(x; h) \propto \cos\left(\frac{\pi x}{2h}\right) \text{ if } x < h \]

The kernel density estimator can be used with any of the valid distance metrics (see `sklearn.neighbors.DistanceMetric` for a list of available metrics), though the results are properly normalized only for the Euclidean metric. One particularly useful metric is the **Haversine distance** which measures the angular distance between points on a sphere. Here is an example of using a kernel density estimate for a visualization.
of geospatial data, in this case the distribution of observations of two different species on the South American continent:

![Maps of Bradypus Variegatus and Microromyzomys Minutus distributions on South America](image)

One other useful application of kernel density estimation is to learn a non-parametric generative model of a dataset in order to efficiently draw new samples from this generative model. Here is an example of using this process to create a new set of hand-written digits, using a Gaussian kernel learned on a PCA projection of the data:

<table>
<thead>
<tr>
<th>Selection from the input data</th>
</tr>
</thead>
<tbody>
<tr>
<td>01234567890</td>
</tr>
<tr>
<td>111345678101</td>
</tr>
<tr>
<td>2745678025</td>
</tr>
<tr>
<td>5650898417</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>&quot;New&quot; digits drawn from the kernel density model</th>
</tr>
</thead>
<tbody>
<tr>
<td>87103548414</td>
</tr>
<tr>
<td>42691563559</td>
</tr>
<tr>
<td>73461901245</td>
</tr>
<tr>
<td>638481757345</td>
</tr>
</tbody>
</table>
The “new” data consists of linear combinations of the input data, with weights probabilistically drawn given the KDE model.

Examples:

- **Simple 1D Kernel Density Estimation**: computation of simple kernel density estimates in one dimension.
- **Kernel Density Estimation**: an example of using Kernel Density estimation to learn a generative model of the hand-written digits data, and drawing new samples from this model.
- **Kernel Density Estimate of Species Distributions**: an example of Kernel Density estimation using the Haversine distance metric to visualize geospatial data

### 3.2.9 Neural network models (unsupervised)

**Restricted Boltzmann machines**

Restricted Boltzmann machines (RBM) are unsupervised nonlinear feature learners based on a probabilistic model. The features extracted by an RBM or a hierarchy of RBMs often give good results when fed into a linear classifier such as a linear SVM or a perceptron.

The model makes assumptions regarding the distribution of inputs. At the moment, scikit-learn only provides `BernoulliRBM`, which assumes the inputs are either binary values or values between 0 and 1, each encoding the probability that the specific feature would be turned on.

The RBM tries to maximize the likelihood of the data using a particular graphical model. The parameter learning algorithm used (**Stochastic Maximum Likelihood**) prevents the representations from straying far from the input data, which makes them capture interesting regularities, but makes the model less useful for small datasets, and usually not useful for density estimation.

The method gained popularity for initializing deep neural networks with the weights of independent RBMs. This method is known as unsupervised pre-training.

Examples:

- **Restricted Boltzmann Machine features for digit classification**

**Graphical model and parametrization**

The graphical model of an RBM is a fully-connected bipartite graph.
The nodes are random variables whose states depend on the state of the other nodes they are connected to. The model is therefore parameterized by the weights of the connections, as well as one intercept (bias) term for each visible and hidden unit, ommitted from the image for simplicity.

The energy function measures the quality of a joint assignment:

$$E(v,h) = \sum_i \sum_j w_{ij} v_i h_j + \sum_i b_i v_i + \sum_j c_j h_j$$

In the formula above, \(b\) and \(c\) are the intercept vectors for the visible and hidden layers, respectively. The joint
probability of the model is defined in terms of the energy:

\[ P(v, h) = \frac{e^{-E(v, h)}}{Z} \]

The word restricted refers to the bipartite structure of the model, which prohibits direct interaction between hidden units, or between visible units. This means that the following conditional independencies are assumed:

\[ h_i \perp h_j | v \]
\[ v_i \perp v_j | h \]

The bipartite structure allows for the use of efficient block Gibbs sampling for inference.

**Bernoulli Restricted Boltzmann machines**

In the BernoulliRBM, all units are binary stochastic units. This means that the input data should either be binary, or real-valued between 0 and 1 signifying the probability that the visible unit would turn on or off. This is a good model for character recognition, where the interest is on which pixels are active and which aren’t. For images of natural scenes it no longer fits because of background, depth and the tendency of neighbouring pixels to take the same values.

The conditional probability distribution of each unit is given by the logistic sigmoid activation function of the input it receives:

\[ P(v_i = 1 | h) = \sigma \left( \sum_j w_{ij} h_j + b_i \right) \]
\[ P(h_i = 1 | v) = \sigma \left( \sum_i w_{ij} v_i + c_j \right) \]

where \( \sigma \) is the logistic sigmoid function:

\[ \sigma(x) = \frac{1}{1 + e^{-x}} \]

**Stochastic Maximum Likelihood learning**

The training algorithm implemented in BernoulliRBM is known as Stochastic Maximum Likelihood (SML) or Persistent Contrastive Divergence (PCD). Optimizing maximum likelihood directly is infeasible because of the form of the data likelihood:

\[ \log P(v) = \log \sum_h e^{-E(v, h)} - \log \sum_{x, y} e^{-E(x, y)} \]

For simplicity the equation above is written for a single training example. The gradient with respect to the weights is formed of two terms corresponding to the ones above. They are usually known as the positive gradient and the negative gradient, because of their respective signs. In this implementation, the gradients are estimated over mini-batches of samples.

In maximizing the log-likelihood, the positive gradient makes the model prefer hidden states that are compatible with the observed training data. Because of the bipartite structure of RBMs, it can be computed efficiently. The negative gradient, however, is intractable. Its goal is to lower the energy of joint states that the model prefers, therefore making it stay true to the data. It can be approximated by Markov chain Monte Carlo using block Gibbs sampling by iteratively sampling each of \( v \) and \( h \) given the other, until the chain mixes. Samples generated in this way are sometimes referred as fantasy particles. This is inefficient and it is difficult to determine whether the Markov chain mixes.

The Contrastive Divergence method suggests to stop the chain after a small number of iterations, \( k \), usually even 1. This method is fast and has low variance, but the samples are far from the model distribution.
Persistent Contrastive Divergence addresses this. Instead of starting a new chain each time the gradient is needed, and performing only one Gibbs sampling step, in PCD we keep a number of chains (fantasy particles) that are updated \( k \) Gibbs steps after each weight update. This allows the particles to explore the space more thoroughly.

**References:**

- “Training Restricted Boltzmann Machines using Approximations to the Likelihood Gradient” T. Tieleman, 2008

### 3.3 Model selection and evaluation

#### 3.3.1 Cross-validation: evaluating estimator performance

Learning the parameters of a prediction function and testing it on the same data is a methodological mistake: a model that would just repeat the labels of the samples that it has just seen would have a perfect score but would fail to predict anything useful on yet-unseen data. This situation is called **overfitting**. To avoid it, it is common practice when performing a (supervised) machine learning experiment to hold out part of the available data as a test set \( X_{\text{test}}, y_{\text{test}} \). Note that the word “experiment” is not intended to denote academic use only, because even in commercial settings machine learning usually starts out experimentally.

In scikit-learn a random split into training and test sets can be quickly computed with the `train_test_split` helper function. Let’s load the iris data set to fit a linear support vector machine on it:

```python
>>> import numpy as np
>>> from sklearn.model_selection import train_test_split
>>> from sklearn import datasets
>>> from sklearn import svm

>>> iris = datasets.load_iris()
>>> iris.data.shape, iris.target.shape
((150, 4), (150,))

We can now quickly sample a training set while holding out 40% of the data for testing (evaluating) our classifier:

```python
>>> X_train, X_test, y_train, y_test = train_test_split(
...     iris.data, iris.target, test_size=0.4, random_state=0)

>>> X_train.shape, y_train.shape
((90, 4), (90,))
>>> X_test.shape, y_test.shape
((60, 4), (60,))

>>> clf = svm.SVC(kernel='linear', C=1).fit(X_train, y_train)
>>> clf.score(X_test, y_test)
0.96...
```

When evaluating different settings (“hyperparameters”) for estimators, such as the \( C \) setting that must be manually set for an SVM, there is still a risk of overfitting on the test set because the parameters can be tweaked until the estimator performs optimally. This way, knowledge about the test set can “leak” into the model and evaluation metrics no longer report on generalization performance. To solve this problem, yet another part of the dataset can be held out as a so-called “validation set”: training proceeds on the training set, after which evaluation is done on the validation set, and when the experiment seems to be successful, final evaluation can be done on the test set.
However, by partitioning the available data into three sets, we drastically reduce the number of samples which can be used for learning the model, and the results can depend on a particular random choice for the pair of (train, validation) sets.

A solution to this problem is a procedure called cross-validation (CV for short). A test set should still be held out for final evaluation, but the validation set is no longer needed when doing CV. In the basic approach, called \( k \)-fold CV, the training set is split into \( k \) smaller sets (other approaches are described below, but generally follow the same principles). The following procedure is followed for each of the \( k \) “folds”:

- A model is trained using \( k - 1 \) of the folds as training data;
- the resulting model is validated on the remaining part of the data (i.e., it is used as a test set to compute a performance measure such as accuracy).

The performance measure reported by \( k \)-fold cross-validation is then the average of the values computed in the loop. This approach can be computationally expensive, but does not waste too much data (as it is the case when fixing an arbitrary test set), which is a major advantage in problem such as inverse inference where the number of samples is very small.

### Computing cross-validated metrics

The simplest way to use cross-validation is to call the `cross_val_score` helper function on the estimator and the dataset.

The following example demonstrates how to estimate the accuracy of a linear kernel support vector machine on the iris dataset by splitting the data, fitting a model and computing the score 5 consecutive times (with different splits each time):

```python
>>> from sklearn.model_selection import cross_val_score
>>> clf = svm.SVC(kernel='linear', C=1)
>>> scores = cross_val_score(clf, iris.data, iris.target, cv=5)
>>> scores
array([ 0.96..., 1. ..., 0.96..., 0.96..., 1. ])
```

The mean score and the 95% confidence interval of the score estimate are hence given by:

```python
>>> print("Accuracy: %0.2f (+/- %0.2f)" % (scores.mean(), scores.std() * 2))
Accuracy: 0.98 (+/- 0.03)
```

By default, the score computed at each CV iteration is the `score` method of the estimator. It is possible to change this by using the scoring parameter:

```python
>>> from sklearn import metrics
>>> scores = cross_val_score(... clf, iris.data, iris.target, cv=5, scoring='f1_macro')
>>> scores
array([ 0.96..., 1. ..., 0.96..., 0.96..., 1. ])
```

See [The scoring parameter: defining model evaluation rules](#) for details. In the case of the Iris dataset, the samples are balanced across target classes hence the accuracy and the F1-score are almost equal.

When the `cv` argument is an integer, `cross_val_score` uses the `KFold` or `StratifiedKFold` strategies by default, the latter being used if the estimator derives from `ClassifierMixin`.

It is also possible to use other cross validation strategies by passing a cross validation iterator instead, for instance:

```python
>>> from sklearn.model_selection import ShuffleSplit
>>> n_samples = iris.data.shape[0]
>>> cv = ShuffleSplit(n_splits=3, test_size=0.3, random_state=0)
```
>>> cross_val_score(clf, iris.data, iris.target, cv=cv)
...  
array([ 0.97...,  0.97...,  1.   ])  

Data transformation with held-out data

Just as it is important to test a predictor on data held-out from training, preprocessing (such as standardization, feature selection, etc.) and similar data transformations similarly should be learnt from a training set and applied to held-out data for prediction:

```python
>>> from sklearn import preprocessing
>>> X_train, X_test, y_train, y_test = train_test_split(  
...     iris.data, iris.target, test_size=0.4, random_state=0)
>>> scaler = preprocessing.StandardScaler().fit(X_train)
>>> X_train_transformed = scaler.transform(X_train)
>>> clf = svm.SVC(C=1).fit(X_train_transformed, y_train)
>>> X_test_transformed = scaler.transform(X_test)
>>> clf.score(X_test_transformed, y_test)
0.9333...
```

A Pipeline makes it easier to compose estimators, providing this behavior under cross-validation:

```python
>>> from sklearn.pipeline import make_pipeline
>>> clf = make_pipeline(preprocessing.StandardScaler(), svm.SVC(C=1))
>>> cross_val_score(clf, iris.data, iris.target, cv=cv)
...  
array([ 0.97...,  0.93...,  0.95...,  0.97...,  0.96...,  0.93...,  0.96...,  0.95...,  0.96...,  0.96...])  
```

See Pipeline and FeatureUnion: combining estimators.

Obtaining predictions by cross-validation

The function `cross_val_predict` has a similar interface to `cross_val_score`, but returns, for each element in the input, the prediction that was obtained for that element when it was in the test set. Only cross-validation strategies that assign all elements to a test set exactly once can be used (otherwise, an exception is raised).

These prediction can then be used to evaluate the classifier:

```python
>>> from sklearn.model_selection import cross_val_predict
>>> predicted = cross_val_predict(clf, iris.data, iris.target, cv=10)
>>> metrics.accuracy_score(iris.target, predicted)
0.966...
```

Note that the result of this computation may be slightly different from those obtained using `cross_val_score` as the elements are grouped in different ways.

The available cross validation iterators are introduced in the following section.

Examples

- Receiver Operating Characteristic (ROC) with cross validation,
- Recursive feature elimination with cross-validation,
- Parameter estimation using grid search with cross-validation,
Cross validation iterators

The following sections list utilities to generate indices that can be used to generate dataset splits according to different cross validation strategies.

Cross-validation iterators for i.i.d. data

Assuming that some data is Independent Identically Distributed (i.i.d.) is making the assumption that all samples stem from the same generative process and that the generative process is assumed to have no memory of past generated samples.

The following cross-validators can be used in such cases.

NOTE

While i.i.d. data is a common assumption in machine learning theory, it rarely holds in practice. If one knows that the samples have been generated using a time-dependent process, it’s safer to use a time-series aware cross-validation scheme <time_series_cv> Similarly if we know that the generative process has a group structure (samples from collected from different subjects, experiments, measurement devices) it safer to use group-wise cross-validation <group_cv>.

K-fold

K-fold divides all the samples in \( k \) groups of samples, called folds (if \( k = n \), this is equivalent to the Leave One Out strategy), of equal sizes (if possible). The prediction function is learned using \( k - 1 \) folds, and the fold left out is used for test.

Example of 2-fold cross-validation on a dataset with 4 samples:

```python
>>> import numpy as np
>>> from sklearn.model_selection import KFold

>>> X = ["a", "b", "c", "d"]
>>> kf = KFold(n_splits=2)
>>> for train, test in kf.split(X):
...     print("%s %s" % (train, test))
[2 3] [0 1]
[0 1] [2 3]
```

Each fold is constituted by two arrays: the first one is related to the training set, and the second one to the test set. Thus, one can create the training/test sets using numpy indexing:

```python
>>> X = np.array([[0., 0.], [1., 1.], [-1., -1.], [2., 2.]])
>>> y = np.array([0, 1, 0, 1])
>>> X_train, X_test, y_train, y_test = X[train], X[test], y[train], y[test]
```
Leave One Out (LOO)

LeaveOneOut (or LOO) is a simple cross-validation. Each learning set is created by taking all the samples except one, the test set being the sample left out. Thus, for \( n \) samples, we have \( n \) different training sets and \( n \) different tests set. This cross-validation procedure does not waste much data as only one sample is removed from the training set:

```python
>>> from sklearn.model_selection import LeaveOneOut

>>> X = [1, 2, 3, 4]
>>> loo = LeaveOneOut()
>>> for train, test in loo.split(X):
...    print("%s %s" % (train, test))
[1 2 3] [0]
[0 2 3] [1]
[0 1 3] [2]
[0 1 2] [3]
```

Potential users of LOO for model selection should weigh a few known caveats. When compared with \( k \)-fold cross validation, one builds \( n \) models from \( n \) samples instead of \( k \) models, where \( n > k \). Moreover, each is trained on \( n - 1 \) samples rather than \( (k - 1)n/k \). In both ways, assuming \( k \) is not too large and \( k < n \), LOO is more computationally expensive than \( k \)-fold cross validation.

In terms of accuracy, LOO often results in high variance as an estimator for the test error. Intuitively, since \( n - 1 \) of the \( n \) samples are used to build each model, models constructed from folds are virtually identical to each other and to the model built from the entire training set.

However, if the learning curve is steep for the training size in question, then 5- or 10- fold cross validation can overestimate the generalization error.

As a general rule, most authors, and empirical evidence, suggest that 5- or 10- fold cross validation should be preferred to LOO.

References:


Leave P Out (LPO)

LeavePOut is very similar to LeaveOneOut as it creates all the possible training/test sets by removing \( p \) samples from the complete set. For \( n \) samples, this produces \( \binom{n}{p} \) train-test pairs. Unlike LeaveOneOut and KFold, the test sets will overlap for \( p > 1 \).

Example of Leave-2-Out on a dataset with 4 samples:
Random permutations cross-validation a.k.a. Shuffle & Split

**ShuffleSplit**

The **ShuffleSplit** iterator will generate a user defined number of independent train / test dataset splits. Samples are first shuffled and then split into a pair of train and test sets.

It is possible to control the randomness for reproducibility of the results by explicitly seeding the `random_state` pseudo random number generator.

Here is a usage example:

```python
>>> from sklearn.model_selection import ShuffleSplit
>>> X = np.arange(5)
>>> ss = ShuffleSplit(n_splits=3, test_size=0.25, random_state=0)
>>> for train_index, test_index in ss.split(X):
...     print("%s %s") % (train_index, test_index))
...     [1 3 4] [2 0]
[1 4 3] [0 2]
[4 0 2] [1 3]
```

**ShuffleSplit** is thus a good alternative to **KFold** cross validation that allows a finer control on the number of iterations and the proportion of samples on each side of the train / test split.

### Cross-validation iterators with stratification based on class labels.

Some classification problems can exhibit a large imbalance in the distribution of the target classes: for instance there could be several times more negative samples than positive samples. In such cases it is recommended to use stratified sampling as implemented in **StratifiedKFold** and **StratifiedShuffleSplit** to ensure that relative class frequencies is approximately preserved in each train and validation fold.

**Stratified k-fold**

**StratifiedKFold** is a variation of **k-fold** which returns *stratified* folds: each set contains approximately the same percentage of samples of each target class as the complete set.

Example of stratified 3-fold cross-validation on a dataset with 10 samples from two slightly unbalanced classes:
>>> from sklearn.model_selection import StratifiedKFold

>>> X = np.ones(10)
>>> y = [0, 0, 0, 0, 1, 1, 1, 1, 1, 1]
>>> skf = StratifiedKFold(n_splits=3)
>>> for train, test in skf.split(X, y):
...    print("%s %s" % (train, test))
[2 3 6 7 8 9] [0 1 4 5]
[0 1 3 4 5 8 9] [2 6 7]
[0 1 2 4 5 6 7] [3 8 9]

Stratified Shuffle Split

StratifiedShuffleSplit is a variation of ShuffleSplit, which returns stratified splits, i.e which creates splits by preserving the same percentage for each target class as in the complete set.

Cross-validation iterators for grouped data.

The i.i.d. assumption is broken if the underlying generative process yield groups of dependent samples.

Such a grouping of data is domain specific. An example would be when there is medical data collected from multiple patients, with multiple samples taken from each patient. And such data is likely to be dependent on the individual group. In our example, the patient id for each sample will be its group identifier.

In this case we would like to know if a model trained on a particular set of groups generalizes well to the unseen groups. To measure this, we need to ensure that all the samples in the validation fold come from groups that are not represented at all in the paired training fold.

The following cross-validation splitters can be used to do that. The grouping identifier for the samples is specified via the groups parameter.

Group k-fold

class:GroupKFold is a variation of k-fold which ensures that the same group is not represented in both testing and training sets. For example if the data is obtained from different subjects with several samples per-subject and if the model is flexible enough to learn from highly person specific features it could fail to generalize to new subjects. class:GroupKFold makes it possible to detect this kind of overfitting situations.

Imagine you have three subjects, each with an associated number from 1 to 3:

>>> from sklearn.model_selection import GroupKFold

>>> X = [0.1, 0.2, 2.2, 2.4, 2.3, 4.55, 5.8, 8.8, 9, 10]
>>> y = ["a", "b", "b", "b", "c", "c", "c", "d", "d", "d"]
>>> groups = [1, 1, 1, 2, 2, 2, 3, 3, 3, 3]

>>> gkf = GroupKFold(n_splits=3)
>>> for train, test in gkf.split(X, y, groups=groups):
...    print("%s %s" % (train, test))
[0 1 2 3 4 5] [6 7 8 9]
[0 1 2 6 7 8 9] [3 4 5]
[3 4 5 6 7 8 9] [0 1 2]
Each subject is in a different testing fold, and the same subject is never in both testing and training. Notice that the folds do not have exactly the same size due to the imbalance in the data.

**Leave One Group Out**

*LeaveOneGroupOut* is a cross-validation scheme which holds out the samples according to a third-party provided array of integer groups. This group information can be used to encode arbitrary domain specific pre-defined cross-validation folds.

Each training set is thus constituted by all the samples except the ones related to a specific group.

For example, in the cases of multiple experiments, *LeaveOneGroupOut* can be used to create a cross-validation based on the different experiments: we create a training set using the samples of all the experiments except one:

```python
>>> from sklearn.model_selection import LeaveOneGroupOut

>>> X = [1, 5, 10, 50, 60, 70, 80]
>>> y = [0, 1, 1, 2, 2, 2, 2]
>>> groups = [1, 1, 2, 2, 3, 3, 3]
>>> logo = LeaveOneGroupOut()
>>> for train, test in logo.split(X, y, groups=groups):
...     print("%s %s") % (train, test))
[2 3 4 5 6] [0 1]
[0 1 4 5 6] [2 3]
[0 1 2 3] [4 5 6]
```

Another common application is to use time information: for instance the groups could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

**Leave P Groups Out**

*LeavePGroupsOut* is similar as *LeaveOneGroupOut*, but removes samples related to \( P \) groups for each training/test set.

Example of Leave-2-Group Out:

```python
>>> from sklearn.model_selection import LeavePGroupsOut

>>> X = np.arange(6)
>>> y = [1, 1, 1, 2, 2, 2]
>>> groups = [1, 1, 2, 2, 3, 3]
>>> lpgo = LeavePGroupsOut(n_groups=2)
>>> for train, test in lpgo.split(X, y, groups=groups):
...     print("%s %s") % (train, test))
[4 5] [0 1 2 3]
[2 3] [0 1 4 5]
[0 1] [2 3 4 5]
```

**Group Shuffle Split**

*GroupShuffleSplit* iterator behaves as a combination of *ShuffleSplit* and *LeavePGroupsOut*, and generates a sequence of randomized partitions in which a subset of groups are held out for each split.
Here is a usage example:

```python
>>> from sklearn.model_selection import GroupShuffleSplit

>>> X = [0.1, 0.2, 2.2, 2.4, 2.3, 4.55, 5.8, 0.001]
>>> y = ['a', 'b', 'b', 'b', 'c', 'c', 'c', 'a']
>>> groups = [1, 1, 2, 2, 3, 3, 4, 4]
>>> gss = GroupShuffleSplit(n_splits=4, test_size=0.5, random_state=0)
>>> for train, test in gss.split(X, y, groups=groups):
...     print("%s %s" % (train, test))
...
[0 1 2 3] [4 5 6 7]
[2 3 6 7] [0 1 4 5]
[2 3 4 5] [0 1 6 7]
[4 5 6 7] [0 1 2 3]
```

This class is useful when the behavior of `LeavePGroupsOut` is desired, but the number of groups is large enough that generating all possible partitions with $P$ groups withheld would be prohibitively expensive. In such a scenario, `GroupShuffleSplit` provides a random sample (with replacement) of the train / test splits generated by `LeavePGroupsOut`.

**Predefined Fold-Splits / Validation-Sets**

For some datasets, a pre-defined split of the data into training- and validation fold or into several cross-validation folds already exists. Using `PredefinedSplit` it is possible to use these folds e.g. when searching for hyperparameters.

For example, when using a validation set, set the `test_fold` to 0 for all samples that are part of the validation set, and to -1 for all other samples.

**Cross validation of time series data**

Time series data is characterised by the correlation between observations that are near in time (autocorrelation). However, classical cross-validation techniques such as `KFold` and `ShuffleSplit` assume the samples are independent and identically distributed, and would result in unreasonable correlation between training and testing instances (yielding poor estimates of generalisation error) on time series data. Therefore, it is very important to evaluate our model for time series data on the “future” observations least like those that are used to train the model. To achieve this, one solution is provided by `TimeSeriesSplit`.

**Time Series Split**

`TimeSeriesSplit` is a variation of $k$-fold which returns first $k$ folds as train set and the $(k + 1)$ th fold as test set. Note that unlike standard cross-validation methods, successive training sets are supersets of those that come before them. Also, it adds all surplus data to the first training partition, which is always used to train the model.

This class can be used to cross-validate time series data samples that are observed at fixed time intervals.

Example of 3-split time series cross-validation on a dataset with 6 samples:

```python
>>> from sklearn.model_selection import TimeSeriesSplit

>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([1, 2, 3, 4, 5, 6])
>>> tscv = TimeSeriesSplit(n_splits=3)
>>> print(tscv)
TimeSeriesSplit(n_splits=3)
```
A note on shuffling

If the data ordering is not arbitrary (e.g. samples with the same class label are contiguous), shuffling it first may be essential to get a meaningful cross-validation result. However, the opposite may be true if the samples are not independently and identically distributed. For example, if samples correspond to news articles, and are ordered by their time of publication, then shuffling the data will likely lead to a model that is overfit and an inflated validation score: it will be tested on samples that are artificially similar (close in time) to training samples.

Some cross validation iterators, such as KFold, have an inbuilt option to shuffle the data indices before splitting them. Note that:

- This consumes less memory than shuffling the data directly.
- By default no shuffling occurs, including for the (stratified) K-fold cross-validation performed by specifying cv=some_integer to cross_val_score, grid search, etc. Keep in mind that train_test_split still returns a random split.
- The random_state parameter defaults to None, meaning that the shuffling will be different every time KFold(..., shuffle=True) is iterated. However, GridSearchCV will use the same shuffling for each set of parameters validated by a single call to its fit method.
- To ensure results are repeatable (on the same platform), use a fixed value for random_state.

Cross validation and model selection

Cross validation iterators can also be used to directly perform model selection using Grid Search for the optimal hyperparameters of the model. This is the topic of the next section: Tuning the hyper-parameters of an estimator.

3.3.2 Tuning the hyper-parameters of an estimator

Hyper-parameters are parameters that are not directly learnt within estimators. In scikit-learn they are passed as arguments to the constructor of the estimator classes. Typical examples include C, kernel and gamma for Support Vector Classifier, alpha for Lasso, etc.

It is possible and recommended to search the hyper-parameter space for the best Cross-validation: evaluating estimator performance score.

Any parameter provided when constructing an estimator may be optimized in this manner. Specifically, to find the names and current values for all parameters for a given estimator, use:

```python
estimator.get_params()
```

A search consists of:

- an estimator (regressor or classifier such as sklearn.svm.SVC());
- a parameter space;
- a method for searching or sampling candidates;
- a cross-validation scheme; and
Some models allow for specialized, efficient parameter search strategies, outlined below. Two generic approaches to sampling search candidates are provided in scikit-learn: for given values, GridSearchCV exhaustively considers all parameter combinations, while RandomizedSearchCV can sample a given number of candidates from a parameter space with a specified distribution. After describing these tools we detail best practice applicable to both approaches.

Note that it is common that a small subset of those parameters can have a large impact on the predictive or computation performance of the model while others can be left to their default values. It is recommend to read the docstring of the estimator class to get a finer understanding of their expected behavior, possibly by reading the enclosed reference to the literature.

### Exhaustive Grid Search

The grid search provided by GridSearchCV exhaustively generates candidates from a grid of parameter values specified with the param_grid parameter. For instance, the following param_grid:

```python
param_grid = [
    {'C': [1, 10, 100, 1000], 'kernel': ['linear']},
    {'C': [1, 10, 100, 1000], 'gamma': [0.001, 0.0001], 'kernel': ['rbf']},
]
```

specifies that two grids should be explored: one with a linear kernel and C values in [1, 10, 100, 1000], and the second one with an RBF kernel, and the cross-product of C values ranging in [1, 10, 100, 1000] and gamma values in [0.001, 0.0001].

The GridSearchCV instance implements the usual estimator API: when “fitting” it on a dataset all the possible combinations of parameter values are evaluated and the best combination is retained.

### Randomized Parameter Optimization

While using a grid of parameter settings is currently the most widely used method for parameter optimization, other search methods have more favourable properties. RandomizedSearchCV implements a randomized search over parameters, where each setting is sampled from a distribution over possible parameter values. This has two main benefits over an exhaustive search:

- A budget can be chosen independent of the number of parameters and possible values.
- Adding parameters that do not influence the performance does not decrease efficiency.

Specifying how parameters should be sampled is done using a dictionary, very similar to specifying parameters for GridSearchCV. Additionally, a computation budget, being the number of sampled candidates or sampling itera-
tions, is specified using the n_iter parameter. For each parameter, either a distribution over possible values or a list of discrete choices (which will be sampled uniformly) can be specified:

```python
{'C': scipy.stats.expon(scale=100), 'gamma': scipy.stats.expon(scale=.1),
 'kernel': ['rbf'], 'class_weight': ['balanced', None]}
```

This example uses the scipy.stats module, which contains many useful distributions for sampling parameters, such as expon, gamma, uniform or randint. In principle, any function can be passed that provides a rvs (random variate sample) method to sample a value. A call to the rvs function should provide independent random samples from possible parameter values on consecutive calls.

**Warning:** The distributions in scipy.stats prior to version scipy 0.16 do not allow specifying a random state. Instead, they use the global numpy random state, that can be seeded via np.random.seed or set using np.random.set_state. However, beginning scikit-learn 0.18, the sklearn.model_selection module sets the random state provided by the user if scipy >= 0.16 is also available.

For continuous parameters, such as C above, it is important to specify a continuous distribution to take full advantage of the randomization. This way, increasing n_iter will always lead to a finer search.

**Examples:**

- Comparing randomized search and grid search for hyperparameter estimation compares the usage and efficiency of randomized search and grid search.

**References:**


**Tips for parameter search**

**Specifying an objective metric**

By default, parameter search uses the score function of the estimator to evaluate a parameter setting. These are the sklearn.metrics.accuracy_score for classification and sklearn.metrics.r2_score for regression. For some applications, other scoring functions are better suited (for example in unbalanced classification, the accuracy score is often uninformative). An alternative scoring function can be specified via the scoring parameter to GridSearchCV, RandomizedSearchCV and many of the specialized cross-validation tools described below. See The scoring parameter: defining model evaluation rules for more details.

**Composite estimators and parameter spaces**

*Pipeline: chaining estimators* describes building composite estimators whose parameter space can be searched with these tools.
Model selection: development and evaluation

Model selection by evaluating various parameter settings can be seen as a way to use the labeled data to “train” the parameters of the grid.

When evaluating the resulting model it is important to do it on held-out samples that were not seen during the grid search process: it is recommended to split the data into a development set (to be fed to the GridSearchCV instance) and an evaluation set to compute performance metrics.

This can be done by using the `train_test_split` utility function.

Parallelism

`GridSearchCV` and `RandomizedSearchCV` evaluate each parameter setting independently. Computations can be run in parallel if your OS supports it, by using the keyword `n_jobs=-1`. See function signature for more details.

Robustness to failure

Some parameter settings may result in a failure to fit one or more folds of the data. By default, this will cause the entire search to fail, even if some parameter settings could be fully evaluated. Setting `error_score=0` (or `=np.NaN`) will make the procedure robust to such failure, issuing a warning and setting the score for that fold to 0 (or `NaN`), but completing the search.

Alternatives to brute force parameter search

Model specific cross-validation

Some models can fit data for a range of values of some parameter almost as efficiently as fitting the estimator for a single value of the parameter. This feature can be leveraged to perform a more efficient cross-validation used for model selection of this parameter.

The most common parameter amenable to this strategy is the parameter encoding the strength of the regularizer. In this case we say that we compute the regularization path of the estimator.

Here is the list of such models:

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>linear_model.ElasticNetCV([l1_ratio, eps, ...])</code></td>
<td>Elastic Net model with iterative fitting along a regularization path</td>
</tr>
<tr>
<td><code>linear_model.LarsCV([fit_intercept, ...])</code></td>
<td>Cross-validated Least Angle Regression model</td>
</tr>
<tr>
<td><code>linear_model.LassoCV([eps, n_alphas, ...])</code></td>
<td>Lasso linear model with iterative fitting along a regularization path</td>
</tr>
<tr>
<td><code>linear_model.LassoLarsCV([fit_intercept, ...])</code></td>
<td>Cross-validated Lasso, using the LARS algorithm</td>
</tr>
<tr>
<td><code>linear_model.LogisticRegressionCV([Cs, ...])</code></td>
<td>Logistic Regression CV (aka logit, MaxEnt) classifier.</td>
</tr>
<tr>
<td><code>linear_model.MultiTaskLassoCV([eps, ...])</code></td>
<td>Multi-task L1/L2 Lasso with built-in cross-validation.</td>
</tr>
<tr>
<td><code>linear_model.OrthogonalMatchingPursuitCV([...])</code></td>
<td>Cross-validated Orthogonal Matching Pursuit model (OMP)</td>
</tr>
<tr>
<td><code>linear_model.RidgeCV([alphas, ...])</code></td>
<td>Ridge regression with built-in cross-validation.</td>
</tr>
<tr>
<td><code>linear_model.RidgeClassifierCV([alphas, ...])</code></td>
<td>Ridge classifier with built-in cross-validation.</td>
</tr>
</tbody>
</table>
Elastic Net model with iterative fitting along a regularization path

The best model is selected by cross-validation.

Read more in the User Guide.

**Parameters**

- **l1_ratio**: float or array of floats, optional
  
  Float between 0 and 1 passed to ElasticNet (scaling between l1 and l2 penalties). For l1_ratio = 0 the penalty is an L2 penalty. For l1_ratio = 1 it is an L1 penalty. For 0 < l1_ratio < 1, the penalty is a combination of L1 and L2 This parameter can be a list, in which case the different values are tested by cross-validation and the one giving the best prediction score is used. Note that a good choice of list of values for l1_ratio is often to put more values close to 1 (i.e. Lasso) and less close to 0 (i.e. Ridge), as in [.1,.5,.7,.9,.95,.99,1]

- **eps**: float, optional
  
  Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

- **n_alphas**: int, optional
  
  Number of alphas along the regularization path, used for each l1_ratio.

- **alphas**: numpy array, optional
  
  List of alphas where to compute the models. If None alphas are set automatically

- **precompute**: True | False | ‘auto’ | array-like
  
  Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

- **max_iter**: int, optional
  
  The maximum number of iterations

- **tol**: float, optional
  
  The tolerance for the optimization: if the updates are smaller than tol, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

- **cv**: int, cross-validation generator or an iterable, optional
  
  Determines the cross-validation splitting strategy. Possible inputs for cv are:
  
  • None, to use the default 3-fold cross-validation,
  
  • integer, to specify the number of folds.
  
  • An object to be used as a cross-validation generator.
  
  • An iterable yielding train/test splits.
  
  For integer/None inputs, KFold is used.
  
  Refer User Guide for the various cross-validation strategies that can be used here.

- **verbose**: bool or integer
Amount of verbosity.

**n_jobs** : integer, optional

Number of CPUs to use during the cross validation. If -1, use all the CPUs.

**positive** : bool, optional

When set to True, forces the coefficients to be positive.

**selection** : str, default 'cyclic'

If set to 'random', a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to 'random') often leads to significantly faster convergence especially when tol is higher than 1e-4.

**random_state** : int, RandomState instance, or None (default)

The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to 'random'.

**fit_intercept** : boolean

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**normalize** : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**Attributes**

**alpha_** : float

The amount of penalization chosen by cross validation

**l1_ratio_** : float

The compromise between l1 and l2 penalization chosen by cross validation

**coef_** : array, shape (n_features,) | (n_targets, n_features)

Parameter vector (w in the cost function formula),

**intercept_** : float | array, shape (n_targets, n_features)

Independent term in the decision function.

**mse_path_** : array, shape (n_l1_ratio, n_alpha, n_folds)

Mean square error for the test set on each fold, varying l1_ratio and alpha.

**alphas_** : numpy array, shape (n_alphas,) or (n_l1_ratio, n_alphas)

The grid of alphas used for fitting, for each l1_ratio.

**n_iter_** : int

number of iterations run by the coordinate descent solver to reach the specified tolerance for the optimal alpha.
See also:

enet_path, ElasticNet

Notes

See examples/linear_model/plot_lasso_model_selection.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

The parameter l1_ratio corresponds to alpha in the glmnet R package while alpha corresponds to the lambda parameter in glmnet. More specifically, the optimization objective is:

\[
\frac{1}{2 \times n\_samples} \times ||y - Xw||^2_2 \\
+ \alpha \times l1\_ratio \times ||w||_1 \\
+ 0.5 \times \alpha \times (1 - l1\_ratio) \times ||w||^2_2
\]

If you are interested in controlling the L1 and L2 penalty separately, keep in mind that this is equivalent to:

\[
a \times L1 + b \times L2
\]

for:

\[
\alpha = a + b \quad \text{and} \quad l1\_ratio = a / (a + b).
\]

Methods

decision_function(*args, **kwargs) DEPRECATED: and will be removed in 0.19.
fit(X, y) Fit linear model with coordinate descent
get_params([deep]) Get parameters for this estimator.
path(X, y[, l1_ratio, eps, n_alphas, ...]) Compute elastic net path with coordinate descent
predict(X) Predict using the linear model
score(X, y[, sample_weight]) Returns the coefficient of determination R^2 of the prediction.
set_params(**params) Set the parameters of this estimator.

__init__(l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, cv=None, copy_X=True, verbose=0, n_jobs=1, positive=False, random_state=None, selection='cyclic')
decision_function(*args, **kwargs) DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returns:

C : array, shape = (n_samples,)

Returns predicted values.

fit(X, y)
Fit linear model with coordinate descent
Fit is on grid of alphas and best alpha estimated by cross-validation.

**Parameters**

- **X**: {array-like}, shape (n_samples, n_features)
  
  Training data. Pass directly as float64, Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output, X can be sparse.

- **y**: array-like, shape (n_samples,) or (n_samples, n_targets)
  
  Target values

**get_params** *(deep=True)*

Get parameters for this estimator.

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**

  **params**: mapping of string to any

  Parameter names mapped to their values.

**static path** *(X, y, l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, check_input=True, **params)*

Compute elastic net path with coordinate descent

The elastic net optimization function varies for mono and multi-outputs.

For mono-output tasks it is:

\[
1 / (2 * n_samples) * ||y - Xw||^2_2 \\
+ \alpha * l1_ratio * ||w||_1 \\
+ 0.5 * \alpha * (1 - l1_ratio) * ||w||^2_2
\]

For multi-output tasks it is:

\[
\frac{1}{(2 * n_samples)} * ||Y - XW||_{Fro}^2 \\
+ \alpha * l1_ratio * ||W||_{21} \\
+ 0.5 * \alpha * (1 - l1_ratio) * ||W||_{Fro}^2
\]

Where:

\[
||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the **User Guide**.

- **Parameters**

  - **X**: {array-like}, shape (n_samples, n_features)
    
    Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output then X can be sparse.

  - **y**: ndarray, shape (n_samples,) or (n_samples, n_outputs)
    
    Target values

  - **l1_ratio**: float, optional
    
    float between 0 and 1 passed to elastic net (scaling between l1 and l2 penalties). l1_ratio=1 corresponds to the Lasso

  - **eps**: float
Length of the path. \( \text{eps} = 1e^{-3} \) means that \( \alpha_{\text{min}} / \alpha_{\text{max}} = 1e^{-3} \)

**n_alphas**: int, optional

Number of alphas along the regularization path

**alphas**: ndarray, optional

List of alphas where to compute the models. If None alphas are set automatically

**precompute**: True | False | 'auto' | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto' let us decide. The Gram matrix can also be passed as argument.

**Xy**: array-like, optional

\( Xy = \text{np.dot}(X.T, y) \) that can be precomputed. It is useful only when the Gram matrix is precomputed.

**copy_X**: boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**coef_init**: array, shape (n_features,) | None

The initial values of the coefficients.

**verbose**: bool or integer

Amount of verbosity.

**params**: kwargs

Keyword arguments passed to the coordinate descent solver.

**return_n_iter**: bool

Whether to return the number of iterations or not.

**positive**: bool, default False

If set to True, forces coefficients to be positive.

**check_input**: bool, default True

Skip input validation checks, including the Gram matrix when provided assuming there are handled by the caller when check_input=False.

**Returns**

**alphas**: array, shape (n_alphas,)

The alphas along the path where models are computed.

**coefs**: array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)

Coefficients along the path.

**dual_gaps**: array, shape (n_alphas,)

The dual gaps at the end of the optimization for each alpha.

**n_iters**: array-like, shape (n_alphas,)

The number of iterations taken by the coordinate descent optimizer to reach the specified tolerance for each alpha. (Is returned when return_n_iter is set to True).

See also:


3.3. Model selection and evaluation
Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

**predict** *(X)*

Predict using the linear model

**Parameters**

<table>
<thead>
<tr>
<th>X</th>
<th>{array-like, sparse matrix}, shape = (n_samples, n_features) Samples.</th>
</tr>
</thead>
</table>

**Returns**

| C | array, shape = (n_samples,) Returns predicted values. |

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ((y_true - y_pred) ** 2).sum() and $v$ is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

<table>
<thead>
<tr>
<th>X</th>
<th>array-like, shape = (n_samples, n_features) Test samples.</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>array-like, shape = (n_samples) or (n_samples, n_outputs) True values for X.</td>
</tr>
<tr>
<td>sample_weight</td>
<td>array-like, shape = [n_samples], optional Sample weights.</td>
</tr>
</tbody>
</table>

**Returns**

| score | float $R^2$ of self.predict(X) wrt. y. |

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

| self | |

**sklearn.linear_model.LarsCV**

Class **sklearn.linear_model.LarsCV**(fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True, positive=False)

Cross-validated Least Angle Regression model

Read more in the **User Guide**.

**Parameters**

| fit_intercept | boolean whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered). |
| positive | boolean (default=False) |
Restrict coefficients to be $\geq 0$. Be aware that you might want to remove fit_intercept which is set True by default.

**verbose** : boolean or integer, optional
Sets the verbosity amount

**normalize** : boolean, optional, default False
If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

**copy_X** : boolean, optional, default True
If True, X will be copied; else, it may be overwritten.

**precompute** : True | False | ‘auto’ | array-like
Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto', let us decide. The Gram matrix can also be passed as argument.

**max_iter** : integer, optional
Maximum number of iterations to perform.

**cv** : int, cross-validation generator or an iterable, optional
Determines the cross-validation splitting strategy. Possible inputs for cv are:
• None, to use the default 3-fold cross-validation,
• integer, to specify the number of folds.
• An object to be used as a cross-validation generator.
• An iterable yielding train/test splits.
For integer/None inputs, KFold is used.
Refer User Guide for the various cross-validation strategies that can be used here.

**max_n_alphas** : integer, optional
The maximum number of points on the path used to compute the residuals in the cross-validation

**n_jobs** : integer, optional
Number of CPUs to use during the cross validation. If -1, use all the CPUs

**eps** : float, optional
The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**Attributes**

**coef_** : array, shape (n_features,)
parameter vector (w in the formulation formula)

**intercept_** : float
independent term in decision function

**coef_path_** : array, shape (n_features, n_alphas)
the varying values of the coefficients along the path

**alpha_**: float

the estimated regularization parameter alpha

**alphas_**: array, shape (n_alphas,)

the different values of alpha along the path

**cv_alphas_**: array, shape (n_cv_alphas,)

all the values of alpha along the path for the different folds

**cv_mse_path_**: array, shape (n_folds, n_cv_alphas)

the mean square error on left-out for each fold along the path (alpha values given by cv_alphas)

**n_iter_**: array-like or int

the number of iterations run by Lars with the optimal alpha.

See also:

*lars_path*, *LassoLars*, *LassoLarsCV*

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(*args, **kwargs)</td>
<td>DEPRECATED: and will be removed in 0.19.</td>
</tr>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(*kwargs)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__*(fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute=’auto’, cv=None, max_n_alphas=1000, n_jobs=1, eps=2.220446049250313e-16, copy_X=True, positive=False)*

**decision_function**(*args, **kwargs)*

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  
  Samples.

- **y**: array-like, shape = (n_samples,)
  
  Returns predicted values.

**fit**(X, y)

Fit the model using X, y as training data.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  
  Training data.

- **y**: array-like, shape (n_samples,)
  
  Target values.
Target values.

**Returns**
- `self`: object
  - returns an instance of self.

**get_params** *(deep=True)*
Get parameters for this estimator.

**Parameters**
- `deep` : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- `params` : mapping of string to any
  - Parameter names mapped to their values.

**predict** *(X)*
Predict using the linear model

**Parameters**
- `X` : {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**
- `C` : array, shape = (n_samples,)
  - Returns predicted values.

**score** *(X, y, sample_weight=None)*
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true.mean()})^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**
- `X` : array-like, shape = (n_samples, n_features)
  - Test samples.
- `y` : array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True values for X.

- `sample_weight` : array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**
- `score` : float
  - $R^2$ of self.predict(X) wrt. y.

**set_params** *(**params)*
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**
- `self` :
Lasso linear model with iterative fitting along a regularization path

The best model is selected by cross-validation.

The optimization objective for Lasso is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||y - Xw||_2^2 + \alpha \times ||w||_1
\]

Read more in the User Guide.

**Parameters**

- `eps` : float, optional
  Length of the path. `eps=1e-3` means that `alpha_min / alpha_max = 1e-3`.

- `n_alphas` : int, optional
  Number of alphas along the regularization path

- `alphas` : numpy array, optional
  List of alphas where to compute the models. If `None` alphas are set automatically

- `precompute` : True | False | ‘auto’ | array-like
  Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’, let us decide. The Gram matrix can also be passed as argument.

- `max_iter` : int, optional
  The maximum number of iterations

- `tol` : float, optional
  The tolerance for the optimization: if the updates are smaller than `tol`, the optimization code checks the dual gap for optimality and continues until it is smaller than `tol`.

- `cv` : int, cross-validation generator or an iterable, optional
  Determines the cross-validation splitting strategy. Possible inputs for `cv` are:
  - None, to use the default 3-fold cross-validation,
  - integer, to specify the number of folds.
  - An object to be used as a cross-validation generator.
  - An iterable yielding train/test splits.
  For integer/None inputs, `KFold` is used.
  Refer User Guide for the various cross-validation strategies that can be used here.

- `verbose` : bool or integer
  Amount of verbosity.

- `n_jobs` : integer, optional
  Number of CPUs to use during the cross validation. If -1, use all the CPUs.

- `positive` : bool, optional
If positive, restrict regression coefficients to be positive

**selection** : str, default ‘cyclic’

If set to ‘random’, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to ‘random’) often leads to significantly faster convergence especially when tol is higher than 1e-4.

**random_state** : int, RandomState instance, or None (default)

The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to ‘random’.

**fit_intercept** : boolean, default True

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**normalize** : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**Attributes**

- **alpha_** : float
  The amount of penalization chosen by cross validation
- **coef_** : array, shape (n_features,) | (n_targets, n_features)
  parameter vector (w in the cost function formula)
- **intercept_** : float | array, shape (n_targets,)
  independent term in decision function.
- **mse_path_** : array, shape (n_alphas, n_folds)
  mean square error for the test set on each fold, varying alpha
- **alphas_** : numpy array, shape (n_alphas,)
  The grid of alphas used for fitting
- **dual_gap_** : ndarray, shape ()
  The dual gap at the end of the optimization for the optimal alpha (alpha_).
- **n_iter_** : int
  number of iterations run by the coordinate descent solver to reach the specified tolerance for the optimal alpha.

See also:

- lars_path
- lasso_path
- LassoLars
- Lasso
- LassoLarsCV

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Notes

See examples/linear_model/plot_lasso_model_selection.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

Methods

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<td><code>decision_function</code></td>
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<td><code>fit(X, y)</code></td>
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<td><code>path(X, y[, eps, n_alphas, alphas, ...])</code></td>
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</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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**__init__**( **eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, copy_X=True, cv=None, verbose=False, n_jobs=1, positive=False, random_state=None, selection='cyclic')

**decision_function** (*args, **kwargs)

Decision function of the linear model.

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.
- **Returns** C : array, shape = (n_samples,)
  - Returns predicted values.

**fit** (X, y)

Fit linear model with coordinate descent

Fit is on grid of alphas and best alpha estimated by cross-validation.

Parameters:
- **X**: {array-like}, shape (n_samples, n_features)
  - Training data. Pass directly as float64, Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output, X can be sparse.
- **y**: array-like, shape (n_samples,) or (n_samples, n_targets)
  - Target values

**get_params** (deep=True)

Get parameters for this estimator.

Parameters:
- **deep** : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params** : mapping of string to any
  - Parameter names mapped to their values.
**path** `(X, y, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, **params)`

Compute Lasso path with coordinate descent

The Lasso optimization function varies for mono and multi-outputs.

For mono-output tasks it is:

$$\frac{1}{(2 \times n\_samples)} \times ||y - Xw||^2_2 + \alpha \times ||w||_1$$

For multi-output tasks it is:

$$\frac{1}{(2 \times n\_samples)} \times \sum_{i=1}^{n\_samples} \sum_{j=1}^{n\_outputs} ||y_{i} - Xw_{i}||^2 \text{Fro} + \alpha \times \sum_{i=1}^{n\_samples} \sum_{j=1}^{n\_outputs} ||w_{i}||_2$$

Where:

$$||w||_2 = \sqrt{\sum_{i} \sum_{j} w_{ij}^2}$$

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

- `X` : {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If `y` is mono-output then `X` can be sparse.
- `y` : ndarray, shape (n_samples,), or (n_samples, n_outputs)
  - Target values
- `eps` : float, optional
  - Length of the path. `eps=1e-3` means that `alpha_min / alpha_max = 1e-3`
- `n_alphas` : int, optional
  - Number of alphas along the regularization path
- `alphas` : ndarray, optional
  - List of alphas where to compute the models. If `None` alphas are set automatically
- `precompute` : True | False | `’auto’` | array-like
  - Whether to use a precomputed Gram matrix to speed up calculations. If set to `’auto’` let us decide. The Gram matrix can also be passed as argument.
- `Xy` : array-like, optional
  - `Xy = np.dot(X.T, y)` that can be precomputed. It is useful only when the Gram matrix is precomputed.
- `copy_X` : boolean, optional, default True
  - If True, `X` will be copied; else, it may be overwritten.
- `coef_init` : array, shape (n_features,) | None
  - The initial values of the coefficients.
- `verbose` : bool or integer
  - Amount of verbosity.
- `params` : kwargs

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keyword arguments passed to the coordinate descent solver.

**positive** : bool, default False
If set to True, forces coefficients to be positive.

**return_n_iter** : bool
whether to return the number of iterations or not.

Returns:

**alphas** : array, shape (n_alphas,)
The alphas along the path where models are computed.

**coefs** : array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
Coefficients along the path.

**dual_gaps** : array, shape (n_alphas,)
The dual gaps at the end of the optimization for each alpha.

**n_iters** : array-like, shape (n_alphas,)
The number of iterations taken by the coordinate descent optimizer to reach the specified
tolerance for each alpha.

See also:

lars_path, Lasso, LassoLars, LassoCV, LassoLarsCV, sklearn.decomposition.sparse_encode

Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a
Fortran-contiguous numpy array.

Note that in certain cases, the Lars solver may be significantly faster to implement this functionality. In
particular, linear interpolation can be used to retrieve model coefficients between the values output by
lars_path

Examples

Comparing lasso_path and lars_path with interpolation:

```python
>>> X = np.array([[1, 2, 3.1], [2.3, 5.4, 4.3]]).T
>>> y = np.array([1, 2, 3.1])
>>> # Use lasso_path to compute a coefficient path
>>> _, coef_path, _ = lasso_path(X, y, alphas=[5., 1., .5])
>>> print(coef_path)
[[ 0.  0.  0.46874778]
 [ 0.2159048  0.4425765  0.23689075]]

>>> # Now use lars_path and 1D linear interpolation to compute the
>>> # same path
>>> from sklearn.linear_model import lars_path
>>> from scipy import interpolate
>>> alphas, active, coef_path_lars = lars_path(X, y, method='lasso')
>>> coef_path_continuous = interpolate.interp1d(alphas[::-1],
```
... 

```python
>> print(coef_path_continuous([5., 1., 0.5]))
[['0
0.46915237
[0.2159048 0.4425765 0.23668876]]
```

**predict** *(X)*

Predict using the linear model

**Parameters**

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  
  Samples.

**Returns**

- **C**: array, shape = (n_samples,)
  
  Returns predicted values.

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares \((y_{true} - y_{pred})^2\).sum() and $v$ is the residual sum of squares \((y_{true} - y_{true}.mean())^2\).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  
  Test samples.

- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  
  True values for X.

- **sample_weight**: array-like, shape = [n_samples], optional
  
  Sample weights.

**Returns**

- **score**: float
  
  $R^2$ of self.predict(X) wrt. $y$.

**set_params** (**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**Examples using** *sklearn.linear_model.LassoCV*

- Cross-validation on diabetes Dataset Exercise
- Feature selection using SelectFromModel and LassoCV
- Lasso model selection: Cross-Validation / AIC / BIC
class sklearn.linear_model.LassoLarsCV (fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.220446049250313e-16, copy_X=True, positive=False)

Cross-validated Lasso, using the LARS algorithm

The optimization objective for Lasso is:

\[ \frac{1}{2n} ||y - Xw||^2_2 + \alpha ||w||_1 \]

Read more in the User Guide.

Parameters

fit_intercept : boolean

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

positive : boolean (default=False)

Restrict coefficients to be >= 0. Be aware that you might want to remove fit_intercept which is set True by default. Under the positive restriction the model coefficients do not converge to the ordinary-least-squares solution for small values of alpha. Only coefficients up to the smallest alpha value (alphas_[alphas_ > 0.].min() when fit_path=True) reached by the stepwise Lars-Lasso algorithm are typically in congruence with the solution of the coordinate descent Lasso estimator. As a consequence using LassoLarsCV only makes sense for problems where a sparse solution is expected and/or reached.

verbose : boolean or integer, optional

Sets the verbosity amount

normalize : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

precompute : True | False | 'auto' | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto', let us decide. The Gram matrix can also be passed as argument.

max_iter : integer, optional

Maximum number of iterations to perform.

cv : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the default 3-fold cross-validation,
- integer, to specify the number of folds.
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.
For integer/None inputs, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**max_n_alphas**: integer, optional

The maximum number of points on the path used to compute the residuals in the cross-validation

**n_jobs**: integer, optional

Number of CPUs to use during the cross validation. If -1, use all the CPUs

**eps**: float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**copy_X**: boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**Attributes**

**coef_**: array, shape (n_features,)

parameter vector (w in the formulation formula)

**intercept_**: float

independent term in decision function.

**coef_path_**: array, shape (n_features, n_alphas)

the varying values of the coefficients along the path

**alpha_**: float

the estimated regularization parameter alpha

**alphas_**: array, shape (n_alphas,)

the different values of alpha along the path

**cv_alphas_**: array, shape (n_cv_alphas,)

all the values of alpha along the path for the different folds

**cv_mse_path_**: array, shape (n_folds, n_cv_alphas)

the mean square error on left-out for each fold along the path (alpha values given by cv_alphas)

**n_iter_**: array-like or int

the number of iterations run by Lars with the optimal alpha.

See also:

lars_path, LassoLars, LarsCV, LassoCV

Notes

The object solves the same problem as the LassoCV object. However, unlike the LassoCV, it find the relevant alphas values by itself. In general, because of this property, it will be more stable. However, it is more fragile to heavily multicollinear datasets.

It is more efficient than the LassoCV if only a small number of features are selected compared to the total number, for instance if there are very few samples compared to the number of features.
Methods

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<tr>
<td><code>fit(X, y)</code></td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td><code>get_params()</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict using the linear model.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__(fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True, positive=False)

`decision_function(*args, **kwargs)`  
DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.
- **Y**: array-like, shape = (n_samples,)
  Returns predicted values.

`fit(X, y)`  
Fit the model using X, y as training data.

Parameters:
- **X**: array-like, shape (n_samples, n_features)
  Training data.
- **y**: array-like, shape (n_samples,)
  Target values.

Returns:
- **self**: object
  returns an instance of self.

`get_params(deep=True)`  
Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params**: mapping of string to any
  Parameter names mapped to their values.

`predict(X)`  
Predict using the linear model.

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

Returns:
- **C**: array, shape = (n_samples,)
  Returns predicted values.
**score** (*X, y, sample_weight=None*)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true}.mean())^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **score**: float
  $R^2$ of self.predict(X) wrt. $y$.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

---

**Examples using sklearn.linear_model.LassoLarsCV**

- **Lasso model selection**: Cross-Validation / AIC / BIC
- **Sparse recovery**: feature selection for sparse linear models

**sklearn.linear_model.LogisticRegressionCV**

```python
class sklearn.linear_model.LogisticRegressionCV(Cs=10, fit_intercept=True, cv=None, dual=False, penalty='l2', scoring=None, solver='lbfgs', tol=0.0001, max_iter=100, class_weight=None, n_jobs=1, verbose=0, refit=True, intercept_scaling=1.0, multi_class='ovr', random_state=None)
```

Logistic Regression CV (aka logit, MaxEnt) classifier.

This class implements logistic regression using liblinear, newton-cg, sag of lbfgs optimizer. The newton-cg, sag and lbfgs solvers support only L2 regularization with primal formulation. The liblinear solver supports both L1 and L2 regularization, with a dual formulation only for the L2 penalty.

For the grid of Cs values (that are set by default to be ten values in a logarithmic scale between 1e-4 and 1e4), the best hyperparameter is selected by the cross-validator StratifiedKFold, but it can be changed using the cv parameter. In the case of newton-cg and lbfgs solvers, we warm start along the path i.e guess the initial coefficients of the present fit to be the coefficients got after convergence in the previous fit, so it is supposed to be faster for high-dimensional dense data.
For a multiclass problem, the hyperparameters for each class are computed using the best scores got by doing a one-vs-rest in parallel across all folds and classes. Hence this is not the true multinomial loss.

Read more in the User Guide.

**Parameters**

- **Cs**: list of floats | int
  
  Each of the values in Cs describes the inverse of regularization strength. If Cs is as an int, then a grid of Cs values are chosen in a logarithmic scale between 1e-4 and 1e4. Like in support vector machines, smaller values specify stronger regularization.

- **fit_intercept**: bool, default: True
  
  Specifies if a constant (a.k.a. bias or intercept) should be added to the decision function.

- **class_weight**: dict or ‘balanced’, optional
  
  Weights associated with classes in the form `{class_label: weight}`. If not given, all classes are supposed to have weight one.
  
  The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as `n_samples / (n_classes * np.bincount(y))`.
  
  Note that these weights will be multiplied with sample_weight (passed through the fit method) if sample_weight is specified.

  New in version 0.17: class_weight == ‘balanced’

- **cv**: integer or cross-validation generator
  
  The default cross-validation generator used is Stratified K-Folds. If an integer is provided, then it is the number of folds used. See the module `sklearn.model_selection` module for the list of possible cross-validation objects.

- **penalty**: str, ‘l1’ or ‘l2’
  
  Used to specify the norm used in the penalization. The ‘newton-cg’, ‘sag’ and ‘lbfgs’ solvers support only l2 penalties.

- **dual**: bool
  
  Dual or primal formulation. Dual formulation is only implemented for l2 penalty with liblinear solver. Prefer dual=False when n_samples > n_features.

- **scoring**: callable
  
  Scoring function to use as cross-validation criteria. For a list of scoring functions that can be used, look at `sklearn.metrics`. The default scoring option used is accuracy_score.

- **solver**: ['newton-cg', 'lbfgs', 'liblinear', 'sag']
  
  Algorithm to use in the optimization problem.

  - For small datasets, ‘liblinear’ is a good choice, whereas ‘sag’ is faster for large ones.
  
  - For multiclass problems, only ‘newton-cg’, ‘sag’ and ‘lbfgs’ handle multinomial loss; ‘liblinear’ is limited to one-versus-rest schemes.
  
  - ‘newton-cg’, ‘lbfgs’ and ‘sag’ only handle L2 penalty.
  
  - ‘liblinear’ might be slower in LogisticRegressionCV because it does not handle warm-starting.
Note that ‘sag’ fast convergence is only guaranteed on features with approximately the same scale. You can preprocess the data with a scaler from sklearn.preprocessing.

New in version 0.17: Stochastic Average Gradient descent solver.

tol : float, optional
Tolerance for stopping criteria.

max_iter : int, optional
Maximum number of iterations of the optimization algorithm.

n_jobs : int, optional
Number of CPU cores used during the cross-validation loop. If given a value of -1, all cores are used.

verbose : int
For the ‘liblinear’, ‘sag’ and ‘lbfgs’ solvers set verbose to any positive number for verbosity.

refit : bool
If set to True, the scores are averaged across all folds, and the coefs and the C that corresponds to the best score is taken, and a final refit is done using these parameters. Otherwise the coefs, intercepts and C that correspond to the best scores across folds are averaged.

multi_class : str, {'ovr', 'multinomial'}
Multiclass option can be either ‘ovr’ or ‘multinomial’. If the option chosen is ‘ovr’, then a binary problem is fit for each label. Else the loss minimised is the multinomial loss fit across the entire probability distribution. Works only for the ‘newton-cg’, ‘sag’ and ‘lbfgs’ solver.

New in version 0.18: Stochastic Average Gradient descent solver for ‘multinomial’ case.

intercept_scaling : float, default 1.
Useful only when the solver ‘liblinear’ is used and self.fit_intercept is set to True. In this case, x becomes [x, self.intercept_scaling], i.e. a “synthetic” feature with constant value equal to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic_feature_weight.

Note! the synthetic feature weight is subject to l1/l2 regularization as all other features. To lessen the effect of regularization on synthetic feature weight (and therefore on the intercept) intercept_scaling has to be increased.

random_state : int seed, RandomState instance, or None (default)
The seed of the pseudo random number generator to use when shuffling the data.

Attributes

coef_ : array, shape (1, n_features) or (n_classes, n_features)
Coefficient of the features in the decision function.

coeff_ is of shape (1, n_features) when the given problem is binary. coef_ is readonly property derived from raw_coef_ that follows the internal memory layout of liblinear.

intercept_ : array, shape (1,) or (n_classes,)
Intercept (a.k.a. bias) added to the decision function. It is available only when parameter intercept is set to True and is of shape(1,) when the problem is binary.
Cs_ : array

Array of C i.e. inverse of regularization parameter values used for cross-validation.

coeffs_paths_ : array, shape (n_folds, len(Cs_), n_features) or (n_folds, len(Cs_), n_features + 1)

dict with classes as the keys, and the path of coefficients obtained during cross-validating across each fold and then across each Cs after doing an OvR for the corresponding class as values. If the ‘multi_class’ option is set to ‘multinomial’, then the coefs_paths are the coefficients corresponding to each class. Each dict value has shape (n_folds, len(Cs_), n_features) or (n_folds, len(Cs_), n_features + 1) depending on whether the intercept is fit or not.

scores_ : dict

dict with classes as the keys, and the values as the grid of scores obtained during cross-validating each fold, after doing an OvR for the corresponding class. If the ‘multi_class’ option given is ‘multinomial’ then the same scores are repeated across all classes, since this is the multinomial class. Each dict value has shape (n_folds, len(Cs))

C_ : array, shape (n_classes,) or (n_classes - 1,)

Array of C that maps to the best scores across every class. If refit is set to False, then for each class, the best C is the average of the C’s that correspond to the best scores for each fold.

n_iter_ : array, shape (n_classes, n_folds, n_cs) or (1, n_folds, n_cs)

Actual number of iterations for all classes, folds and Cs. In the binary or multinomial cases, the first dimension is equal to 1.

See also:

LogisticRegression

Methods

decision_function(X) Predict confidence scores for samples.
densify() Convert coefficient matrix to dense array format.
fit(X, y[, sample_weight]) Fit the model according to the given training data.
fit_transform(X[, y]) Fit to data, then transform it.
get_params([deep]) Get parameters for this estimator.
predict(X) Predict class labels for samples in X.
predict_log_proba(X) Log of probability estimates.
predict_proba(X) Probability estimates.
score(X, y[, sample_weight]) Returns the mean accuracy on the given test data and labels.
set_params(**params) Set the parameters of this estimator.
sparseify() Convert coefficient matrix to sparse format.
transform(*args, **kwargs) DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.

__init__ (Cs=10, fit_intercept=True, cv=None, dual=False, penalty='l2', scoring=None, solver='lbfgs', tol=0.0001, max_iter=100, class_weight=None, n_jobs=1, verbose=0, refit=True, intercept_scaling=1.0, multi_class='ovr', random_state=None)
**decision_function** *(X)*

Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**

- array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes)
  - Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

**densify** ()

Convert coefficient matrix to dense array format.

Converts the `coef_` member (back) to a numpy.ndarray. This is the default format of `coef_` and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

**Returns**

- `self`: estimator

**fit** *(X, y, sample_weight=None)*

Fit the model according to the given training data.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training vector, where n_samples is the number of samples and n_features is the number of features.
- **y**: array-like, shape (n_samples,)
  - Target vector relative to X.
- **sample_weight**: array-like, shape (n_samples,) optional
  - Array of weights that are assigned to individual samples. If not provided, then each sample is given unit weight.

**Returns**

- `self`: object
  - Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- `X_new`: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.
**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

**predict** $(X)$

Predict class labels for samples in $X$.

- **Parameters**
  - $X$: array-like, sparse matrix, shape = [n_samples, n_features]
    - Samples.
  - **Returns**
    - $C$: array, shape = [n_samples]
      - Predicted class label per sample.

**predict_log_proba** $(X)$

Log of probability estimates.

The returned estimates for all classes are ordered by the label of classes.

- **Parameters**
  - $X$: array-like, shape = [n_samples, n_features]
  - **Returns**
    - $T$: array-like, shape = [n_samples, n_classes]
      - Returns the log-probability of the sample for each class in the model, where classes are ordered as they are in `self.classes_`.

**predict_proba** $(X)$

Probability estimates.

The returned estimates for all classes are ordered by the label of classes.

For a multi_class problem, if `multi_class` is set to be “multinomial” the softmax function is used to find the predicted probability of each class. Else use a one-vs-rest approach, i.e calculate the probability of each class assuming it to be positive using the logistic function, and normalize these values across all the classes.

- **Parameters**
  - $X$: array-like, shape = [n_samples, n_features]
  - **Returns**
    - $T$: array-like, shape = [n_samples, n_classes]
      - Returns the probability of the sample for each class in the model, where classes are ordered as they are in `self.classes_`.

**score** $(X, y, sample_weight=None)$

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

- **Parameters**
  - $X$: array-like, shape = (n_samples, n_features)
    - Test samples.
  - $y$: array-like, shape = (n_samples) or (n_samples, n_outputs)
    - True labels for $X$.
  - `sample_weight`: array-like, shape = [n_samples], optional
    - Sample weights.
  - **Returns**
    - `score`: float
      - Mean accuracy of `self.predict(X)` wrt. $y$. 
**set_params** (**params**)  
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns** self:

**sparsify** ()  
Convert coefficient matrix to sparse format.

Converts the `coef_` member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The `intercept_` member is not converted.

**Returns** self:

**Notes**

For non-sparse models, i.e. when there are not many zeros in `coef_`, this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with `(coef_ == 0).sum()`, must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

**transform** (*args, **kwargs*)  
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]
  
The input samples.

- **threshold**: string, float or None, optional (default=None)  
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  
The input samples with only the selected features.
Multi-task L1/L2 ElasticNet with built-in cross-validation.

The optimization objective for MultiTaskElasticNet is:

\[
\frac{1}{(2 \cdot n_{\text{samples}})} \cdot ||Y - XW||_{\text{Fro}}^2 + \alpha \cdot l1_{\text{ratio}} \cdot ||W||_{2,1} + 0.5 \cdot \alpha \cdot (1 - l1_{\text{ratio}}) \cdot ||W||_{\text{Fro}}^2
\]

Where:

\[
||W||_{2,1} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

- **eps** : float, optional
  Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

- **alphas** : array-like, optional
  List of alphas where to compute the models. If not provided, set automatically.

- **n_alphas** : int, optional
  Number of alphas along the regularization path

- **l1_ratio** : float or array of floats
  The ElasticNet mixing parameter, with 0 < l1_ratio <= 1. For l1_ratio = 0 the penalty is an L1/L2 penalty. For l1_ratio = 1 it is an L1 penalty. For 0 < l1_ratio < 1, the penalty is a combination of L1/L2 and L2. This parameter can be a list, in which case the different values are tested by cross-validation and the one giving the best prediction score is used. Note that a good choice of list of values for l1_ratio is often to put more values close to 1 (i.e. Lasso) and less close to 0 (i.e. Ridge), as in [.1,.5,.7,.9,.95,.99,1]

- **fit_intercept** : boolean
  whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **normalize** : boolean, optional, default False
  If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

- **copy_X** : boolean, optional, default True
If True, X will be copied; else, it may be overwritten.

**max_iter** : int, optional

The maximum number of iterations

**tol** : float, optional

The tolerance for the optimization: if the updates are smaller than tol, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

**cv** : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

• None, to use the default 3-fold cross-validation,

• integer, to specify the number of folds.

• An object to be used as a cross-validation generator.

• An iterable yielding train/test splits.

For integer/None inputs, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**verbose** : bool or integer

Amount of verbosity.

**n_jobs** : integer, optional

Number of CPUs to use during the cross validation. If -1, use all the CPUs. Note that this is used only if multiple values for l1_ratio are given.

**selection** : str, default ‘cyclic’

If set to ‘random’, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to ‘random’) often leads to significantly faster convergence especially when tol is higher than 1e-4.

**random_state** : int, RandomState instance, or None (default)

The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to ‘random’.

**Attributes**

**intercept_** : array, shape (n_tasks,)

Independent term in decision function.

**coef_** : array, shape (n_tasks, n_features)

Parameter vector (W in the cost function formula).

**alpha_** : float

The amount of penalization chosen by cross validation

**mse_path_** : array, shape (n_alphas, n_folds) or (n_l1_ratio, n_alphas, n_folds)

mean square error for the test set on each fold, varying alpha

**alphas_** : numpy array, shape (n_alphas,) or (n_l1_ratio, n_alphas)

The grid of alphas used for fitting, for each l1_ratio

**l1_ratio_** : float

best l1_ratio obtained by cross-validation.
n_iter_ : int

number of iterations run by the coordinate descent solver to reach the specified tolerance
for the optimal alpha.

See also:
MultiTaskElasticNet, ElasticNetCV, MultiTaskLassoCV

Notes

The algorithm used to fit the model is coordinate descent.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a
Fortran-contiguous numpy array.

Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.MultiTaskElasticNetCV()
>>> clf.fit([[0, 0], [1, 1], [2, 2]],
...         [[0, 0], [1, 1], [2, 2]])
... MultiTaskElasticNetCV(alphas=None, copy_X=True, cv=None, eps=0.001,
  fit_intercept=True, l1_ratio=0.5, max_iter=1000, n_alphas=100,
  n_jobs=1, normalize=False, random_state=None, selection='cyclic',
  tol=0.0001, verbose=0)
>>> print(clf.coef_)
[[ 0.52875032  0.46958558]
 [ 0.52875032  0.46958558]]
>>> print(clf.intercept_)
[ 0.00166409  0.00166409]
```

Methods

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<td>set_params(**params)</td>
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__init__ (l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, max_iter=1000, tol=0.0001, cv=None, copy_X=True, verbose=0, n_jobs=1, random_state=None, selection='cyclic')

decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.
Parameters

**X**: {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

**Returns**

**C**: array, shape = (n_samples,)

Returns predicted values.

**fit**(X, y)

Fit linear model with coordinate descent

Fit is on grid of alphas and best alpha estimated by cross-validation.

**Parameters**

**X**: {array-like}, shape (n_samples, n_features)

Training data. Pass directly as float64, Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output, X can be sparse.

**y**: array-like, shape (n_samples,) or (n_samples, n_targets)

Target values

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

**deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

**params** : mapping of string to any

Parameter names mapped to their values.

**static path**(X, y, l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, check_input=True, **params)

Compute elastic net path with coordinate descent

The elastic net optimization function varies for mono and multi-outputs.

For mono-output tasks it is:

\[
\frac{1}{(2 \times \text{n_samples})} \times \|y - Xw\|^2_2 + 
\alpha \times \text{l1_ratio} \times \|w\|_1 + 0.5 \times \alpha \times (1 - \text{l1_ratio}) \times \|w\|^2_2
\]

For multi-output tasks it is:

\[
\left(\frac{1}{(2 \times \text{n_samples})}\right) \times \|Y - XW\|^\text{Fro}_2 + 
\alpha \times \text{l1_ratio} \times \|W\|_21 + 0.5 \times \alpha \times (1 - \text{l1_ratio}) \times \|W\|_\text{Fro}^2
\]

Where:

\[
\|W\|_21 = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the *User Guide*.

**Parameters**

**X**: {array-like}, shape (n_samples, n_features)

Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output then X can be sparse.
y : ndarray, shape (n_samples,) or (n_samples, n_outputs)
    Target values

l1_ratio : float, optional
    float between 0 and 1 passed to elastic net (scaling between l1 and l2 penalties).
    l1_ratio=1 corresponds to the Lasso

eps : float
    Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3

n_alphas : int, optional
    Number of alphas along the regularization path

alphas : ndarray, optional
    List of alphas where to compute the models. If None alphas are set automatically

precompute : True | False | ‘auto’ | array-like
    Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto'
    let us decide. The Gram matrix can also be passed as argument.

Xy : array-like, optional
    Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is
    precomputed.

copy_X : boolean, optional, default True
    If True, X will be copied; else, it may be overwritten.

coef_init : array, shape (n_features,) | None
    The initial values of the coefficients.

verbose : bool or integer
    Amount of verbosity.

params : kwargs
    Keyword arguments passed to the coordinate descent solver.

return_n_iter : bool
    whether to return the number of iterations or not.

positive : bool, default False
    If set to True, forces coefficients to be positive.

check_input : bool, default True
    Skip input validation checks, including the Gram matrix when provided assuming there
    are handled by the caller when check_input=False.

Returns:
alphas : array, shape (n_alphas,)
    The alphas along the path where models are computed.

coops : array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
    Coefficients along the path.

dual_gaps : array, shape (n_alphas,)
    The dual gaps along the path.
The dual gaps at the end of the optimization for each alpha.

**n_iters** : array-like, shape (n_alphas,)

The number of iterations taken by the coordinate descent optimizer to reach the specified tolerance for each alpha. (Is returned when return_n_iter is set to True).

**See also:**

*MutiTaskElasticNet, MultiTaskElasticNetCV, ElasticNet, ElasticNetCV*

**Notes**

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

**predict** *(X)*

Predict using the linear model

**Parameters**

- **X** : {array-like, sparse matrix}, shape = (n_samples, n_features)
  
  Samples.

**Returns**

- **C** : array, shape = (n_samples,)
  
  Returns predicted values.

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true.mean})^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X** : array-like, shape = (n_samples, n_features)
  
  Test samples.

- **y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
  
  True values for $X$.

- **sample_weight** : array-like, shape = [n_samples], optional
  
  Sample weights.

**Returns**

- **score** : float
  
  $R^2$ of self.predict(X) wrt. $y$.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :
**sklearn.linear_model.MultiTaskLassoCV**

**class sklearn.linear_model.MultiTaskLassoCV (eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, max_iter=1000, tol=0.0001, copy_X=True, cv=None, verbose=False, n_jobs=1, random_state=None, selection='cyclic')**

Multi-task L1/L2 Lasso with built-in cross-validation.

The optimization objective for MultiTaskLasso is:

\[
\frac{1}{(2 \times n\_samples)} \times ||Y - XW||_{\text{Fro}}^2 + \alpha \times ||W||_{21}
\]

Where:

\[
||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.


**Parameters**

- **eps**: float, optional
  Length of the path. \( \text{eps}=1e-3 \) means that \( \alpha_{\text{min}} / \alpha_{\text{max}} = 1e-3 \).

- **alphas**: array-like, optional
  List of alphas where to compute the models. If not provided, set automatically.

- **n_alphas**: int, optional
  Number of alphas along the regularization path

- **fit_intercept**: boolean
  Whether to calculate the intercept for this model. If set to False, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **normalize**: boolean, optional, default False
  If True, the regressors \( X \) will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

- **copy_X**: boolean, optional, default True
  If True, \( X \) will be copied; else, it may be overwritten.

- **max_iter**: int, optional
  The maximum number of iterations.

- **tol**: float, optional
  The tolerance for the optimization: if the updates are smaller than \( \text{tol} \), the optimization code checks the dual gap for optimality and continues until it is smaller than \( \text{tol} \).

- **cv**: int, cross-validation generator or an iterable, optional
  Determines the cross-validation splitting strategy. Possible inputs for cv are:
  - None, to use the default 3-fold cross-validation,
• integer, to specify the number of folds.
• An object to be used as a cross-validation generator.
• An iterable yielding train/test splits.

For integer/None inputs, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**verbose** : bool or integer
Amount of verbosity.

**n_jobs** : integer, optional
Number of CPUs to use during the cross validation. If -1, use all the CPUs. Note that this is used only if multiple values for l1_ratio are given.

**selection** : str, default ‘cyclic’
If set to ‘random’, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to ‘random’) often leads to significantly faster convergence especially when tol is higher than 1e-4.

**random_state** : int, RandomState instance, or None (default)
The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to ‘random’.

**Attributes**
- **intercept_** : array, shape (n_tasks,)
  Independent term in decision function.
- **coef_** : array, shape (n_tasks, n_features)
  Parameter vector (W in the cost function formula).
- **alpha_** : float
  The amount of penalization chosen by cross validation
- **mse_path_** : array, shape (n_alphas, n_folds)
  mean square error for the test set on each fold, varying alpha
- **alphas_** : numpy array, shape (n_alphas,)
  The grid of alphas used for fitting.
- **n_iter_** : int
  number of iterations run by the coordinate descent solver to reach the specified tolerance for the optimal alpha.

**See also:**
MultiTaskElasticNet, ElasticNetCV, MultiTaskElasticNetCV

**Notes**
The algorithm used to fit the model is coordinate descent.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.
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<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
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<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (eps=0.001, n_alphas=100, alphas= None, fit_intercept=True, normalize=False, max_iter=1000, tol=0.0001, copy_X=True, cv=None, verbose=False, n_jobs=1, random_state=None, selection='cyclic')

decision_function (*args, **kwargs)
DEPRECATED: and will be removed in 0.19.
Decision function of the linear model.

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.
- **y**: array-like, shape (n_samples,)
  - Returns predicted values.

fit (X, y)
Fit linear model with coordinate descent
Fit is on grid of alphas and best alpha estimated by cross-validation.

Parameters:
- **X**: {array-like}, shape (n_samples, n_features)
  - Training data. Pass directly as float64, Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output, X can be sparse.
- **y**: array-like, shape (n_samples,) or (n_samples, n_targets)
  - Target values

get_params (deep=True)
Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params**: mapping of string to any
  - Parameter names mapped to their values.

static path (X, y, eps=0.001, n_alphas=100, alphas= None, precompute='auto', Xy=None, copy_X=True, coef_init= None, verbose=False, return_n_iter=False, positive=False, **params)
Compute Lasso path with coordinate descent
The Lasso optimization function varies for mono and multi-outputs.
For mono-output tasks it is:
For multi-output tasks it is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||Y - XW||^2_{\text{Fro}} + \alpha \times ||W||_2^1
\]

Where:

\[||W||_2^1 = \sum_i \sqrt{\sum_j w_{ij}^2}\]

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

- **X**: array-like, sparse matrix, shape (n_samples, n_features)
  - Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output then X can be sparse.

- **y**: ndarray, shape (n_samples,), or (n_samples, n_outputs)
  - Target values

- **eps**: float, optional
  - Length of the path. eps=1e-3 means that \(\alpha_{\text{min}} / \alpha_{\text{max}} = 1e-3\)

- **n_alphas**: int, optional
  - Number of alphas along the regularization path

- **alphas**: ndarray, optional
  - List of alphas where to compute the models. If None alphas are set automatically

- **precompute**: True | False | 'auto' | array-like
  - Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto' let us decide. The Gram matrix can also be passed as argument.

- **Xy**: array-like, optional
  - \(Xy = \text{np.dot}(X.T, y)\) that can be precomputed. It is useful only when the Gram matrix is precomputed.

- **copy_X**: boolean, optional, default True
  - If True, X will be copied; else, it may be overwritten.

- **coeff_init**: array, shape (n_features, ) | None
  - The initial values of the coefficients.

- **verbose**: bool or integer
  - Amount of verbosity.

- **params**: kwargs
  - Keyword arguments passed to the coordinate descent solver.

- **positive**: bool, default False
  - If set to True, forces coefficients to be positive.

- **return_n_iter**: bool
whether to return the number of iterations or not.

**Returns**

- **alphas** : array, shape (n_alphas,)
  
The alphas along the path where models are computed.

- **coefs** : array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
  
  Coefficients along the path.

- **dual_gaps** : array, shape (n_alphas,)
  
The dual gaps at the end of the optimization for each alpha.

- **n_iters** : array-like, shape (n_alphas,)
  
The number of iterations taken by the coordinate descent optimizer to reach the specified
tolerance for each alpha.

**See also:**

- `lars_path`
- `Lasso`
- `LassoLars`
- `LassoCV`
- `LassoLarsCV`
- `sklearn.decomposition.sparse_encode`

**Notes**

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a
Fortran-contiguous numpy array.

Note that in certain cases, the Lars solver may be significantly faster to implement this functionality. In
particular, linear interpolation can be used to retrieve model coefficients between the values output by
`lars_path`

**Examples**

Comparing lasso_path and lars_path with interpolation:

```python
>>> X = np.array([[1, 2, 3.1], [2.3, 5.4, 4.3]]).T
>>> y = np.array([1, 2, 3.1])
>>> # Use lasso_path to compute a coefficient path
>>> _, coef_path, _ = lasso_path(X, y, alphas=[5., 1., .5])
>>> print(coef_path)
[[ 0.          0.          0.46874778]
 [ 0.2159048   0.4425765  0.23689075]]
```

```python
>>> # Now use lars_path and 1D linear interpolation to compute the
>>> # same path
>>> from sklearn.linear_model import lars_path
>>> alphas, active, coef_path_lars = lars_path(X, y, method='lasso')
>>> from scipy import interpolate
>>> coef_path_continuous = interpolate.interp1d(alphas[::-1],
...                                              coef_path_lars[:, ::-1])
>>> print(coef_path_continuous([5., 1., .5]))
[[ 0.          0.          0.46915237]
 [ 0.2159048   0.4425765  0.23668876]]
```
predict \((X)\)
Predict using the linear model

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

Returns:
- **C**: array, shape = (n_samples,)
  - Returns predicted values.

score \((X, y, \text{sample_weight=None})\)
Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \(1 - u/v\), where \(u\) is the regression sum of squares ((\(y_{\text{true}} - y_{\text{pred}}\) ** 2).sum() and \(v\) is the residual sum of squares ((\(y_{\text{true}} - y_{\text{true.mean()}}\) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \(y\), disregarding the input features, would get a \(R^2\) score of 0.0.

Parameters:
- **X**: array-like, shape = (n_samples, n_features)
  - Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True values for \(X\).
- **sample_weight**: array-like, shape = [n_samples], optional
  - Sample weights.

Returns:
- **score**: float
  - \(R^2\) of self.predict(X) wrt. \(y\).

set_params (**params)**
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
- **self**:

```python
from sklearn.linear_model import OrthogonalMatchingPursuitCV

class sklearn.linear_model.OrthogonalMatchingPursuitCV(
copy=True, fit_intercept=True,
normalize=True,
max_iter=None, cv=None,
n_jobs=1, verbose=False)
```

Cross-validated Orthogonal Matching Pursuit model (OMP)

Parameters:
- **copy**: bool, optional
  - Whether the design matrix \(X\) must be copied by the algorithm. A false value is only helpful if \(X\) is already Fortran-ordered, otherwise a copy is made anyway.
- **fit_intercept**: boolean, optional
  - Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).
- **normalize**: boolean, optional, default False
  - Whether the design matrix \(X\) is normalized.

3.3. Model selection and evaluation
If True, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to `False`. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

**max_iter** : integer, optional

Maximum numbers of iterations to perform, therefore maximum features to include. 10% of n_features but at least 5 if available.

**cv** : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:
- None, to use the default 3-fold cross-validation,
- integer, to specify the number of folds.
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.

For integer/None inputs, `KFold` is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**n_jobs** : integer, optional

Number of CPUs to use during the cross validation. If -1, use all the CPUs

**verbose** : boolean or integer, optional

Sets the verbosity amount

Read more in the :ref:`User Guide <omp>`:

**Attributes**

**intercept_** : float or array, shape (n_targets,)

Independent term in decision function.

**coef_** : array, shape (n_features,) or (n_features, n_targets)

Parameter vector (w in the problem formulation).

**n_nonzero_coefs_** : int

Estimated number of non-zero coefficients giving the best mean squared error over the cross-validation folds.

**n_iter_** : int or array-like

Number of active features across every target for the model refit with the best hyperparameters got by cross-validating across all folds.

See also:

- `orthogonal_mp`
- `orthogonal_mp_gram`
- `lars_path`
- `Lars`
- `LassoLars`
- `OrthogonalMatchingPursuit`
- `LarsCV`
- `LassoLarsCV`
- `decomposition.sparse_encode`

**Methods**

`decision_function`(*args, **kwargs) DEPRECATED: and will be removed in 0.19.
Continued on next page
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<td>Fit the model using X, y as training data.</td>
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<td><code>get_params</code></td>
<td>Get parameters for this estimator.</td>
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<td><code>predict(X)</code></td>
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<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
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</table>

#### __init__

```python
(copy=True, fit_intercept=True, normalize=True, max_iter=None, cv=None, n_jobs=1, verbose=False)
```

#### decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

**Parameters**

- `X`: array-like, sparse matrix, shape = (n_samples, n_features)
  - Samples.

**Returns**

- `C`: array, shape = (n_samples,)
  - Returns predicted values.

#### fit (X, y)

Fit the model using X, y as training data.

**Parameters**

- `X`: array-like, shape [n_samples, n_features]
  - Training data.
- `y`: array-like, shape [n_samples]
  - Target values.

**Returns**

- `self`: object
  - returns an instance of self.

#### get_params (deep=True)

Get parameters for this estimator.

**Parameters**

- `deep`: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- `params`: mapping of string to any
  - Parameter names mapped to their values.

#### predict (X)

Predict using the linear model.

**Parameters**

- `X`: array-like, sparse matrix, shape = (n_samples, n_features)
  - Samples.

**Returns**

- `C`: array, shape = (n_samples,)
  - Returns predicted values.

#### score (X, y, sample_weight=None)

Returns the coefficient of determination $R^2$ of the prediction.

### 3.3. Model selection and evaluation

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The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ($(y_{true} - y_{pred})^2$).sum() and $v$ is the residual sum of squares ($(y_{true} - y_{true}.mean())^2$).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **score**: float
  $R^2$ of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**:

**Examples using sklearn.linear_model.OrthogonalMatchingPursuitCV**

- Orthogonal Matching Pursuit

**sklearn.linear_model.RidgeCV**

class sklearn.linear_model.RidgeCV(alphas=(0.1, 1.0, 10.0), fit_intercept=True, normalize=False, scoring=None, cv=None, gcv_mode=None, store_cv_values=False)

Ridge regression with built-in cross-validation.

By default, it performs Generalized Cross-Validation, which is a form of efficient Leave-One-Out cross-validation.

Read more in the **User Guide**.

**Parameters**

- **alphas**: numpy array of shape [n_alphas]
  Array of alpha values to try. Regularization strength; must be a positive float. Regularization improves the conditioning of the problem and reduces the variance of the estimates. Larger values specify stronger regularization. Alpha corresponds to $C^{-1}$ in other linear models such as LogisticRegression or LinearSVC.

- **fit_intercept**: boolean
  Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **normalize**: boolean, optional, default False
If True, the regressors $X$ will be normalized before regression. This parameter is ignored when $\text{fit\_intercept}$ is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use $\text{preprocessing.StandardScaler}$ before calling $\text{fit}$ on an estimator with $\text{normalize=False}$.

**scoring**: string, callable or None, optional, default: None

A string (see model evaluation documentation) or a scorer callable object / function with signature $\text{scorer}(\text{estimator},X,y)$.

**cv**: int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the efficient Leave-One-Out cross-validation
- integer, to specify the number of folds.
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.

For integer/None inputs, if $y$ is binary or multiclass, $\text{sklearn.model_selection.StratifiedKFold}$ is used, else, $\text{sklearn.model_selection.KFold}$ is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**gcv_mode**: {None, ‘auto’, ‘svd’, ‘eigen’}, optional

Flag indicating which strategy to use when performing Generalized Cross-Validation. Options are:

- ‘auto’: use svd if $n\_samples > n\_features$ or when $X$ is a sparse matrix, otherwise use eigen
- ‘svd’: force computation via singular value decomposition of $X$ (does not work for sparse matrices)
- ‘eigen’: force computation via eigendecomposition of $X^T X$

The ‘auto’ mode is the default and is intended to pick the cheaper option of the two depending upon the shape and format of the training data.

**store_cv_values**: boolean, default=False

Flag indicating if the cross-validation values corresponding to each alpha should be stored in the $\text{cv\_values}$ attribute (see below). This flag is only compatible with $cv=\text{None}$ (i.e. using Generalized Cross-Validation).

**cv_values**: array, shape = [n_samples, n_alphas] or shape = [n_samples, n_targets, n_alphas], optional

Cross-validation values for each alpha (if $\text{store\_cv\_values=\text{True}}$ and $\text{cv=\text{None}}$). After $\text{fit()}$ has been called, this attribute will contain the mean squared errors (by default) or the values of the $\text{(loss,\_score)}\_\text{func}$ function (if provided in the constructor).

**coef_**: array, shape = [n_features] or [n_targets, n_features]

Weight vector(s).

**intercept_**: float | array, shape = (n_targets,)

Independent term in decision function. Set to 0.0 if $\text{fit\_intercept = False}$.
alpha_ : float

Estimated regularization parameter.

See also:

Ridge         Ridge regression
RidgeClassifier Ridge classifier
RidgeClassifierCV Ridge classifier with built-in cross validation

Methods

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<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit Ridge regression model</td>
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<td>get_params([deep])</td>
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<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(*&quot;n&quot;params)</td>
<td>Set the parameters of this estimator.</td>
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__init__(alphas=(0.1, 1.0, 10.0), fit_intercept=True, normalize=False, scoring=None, cv=None, gcv_mode=None, store_cv_values=False)

decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:

**X**: {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returns:

**C**: array, shape = (n_samples,)

Returns predicted values.

fit (X, y, sample_weight=None)

Fit Ridge regression model

Parameters:

**X**: array-like, shape = [n_samples, n_features]

Training data

**y**: array-like, shape = [n_samples] or [n_samples, n_targets]

Target values

**sample_weight**: float or array-like of shape [n_samples]

Sample weight

Returns:

**self**: Returns self.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

**deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.
Returns: mapping of string to any
Parameter names mapped to their values.

**predict** *(X)*
Predict using the linear model

Parameters:

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

Returns:

- **C**: array, shape = (n_samples,)
  Returns predicted values.

**score** *(X, y, sample_weight=None)*
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $\sum((y_{true} - y_{pred})^2)$ and $v$ is the residual sum of squares $\sum((y_{true} - y_{true.mean})^2)$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters:

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns:

- **score**: float
  $R^2$ of self.predict(X) wrt. y.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

**Examples using sklearn.linear_model.RidgeCV**

- Face completion with a multi-output estimators

**sklearn.linear_model.RidgeClassifierCV**

class sklearn.linear_model.RidgeClassifierCV (alphas=(0.1, 1.0, 10.0), fit_intercept=True, normalize=False, scoring=None, cv=None, class_weight=None)

Ridge classifier with built-in cross-validation.

By default, it performs Generalized Cross-Validation, which is a form of efficient Leave-One-Out cross-validation. Currently, only the n_features > n_samples case is handled efficiently.

Read more in the User Guide.
Parameters:

**alphas**: numpy array of shape [n_alphas]

Array of alpha values to try. Regularization strength; must be a positive float. Regularization improves the conditioning of the problem and reduces the variance of the estimates. Larger values specify stronger regularization. Alpha corresponds to $C^{-1}$ in other linear models such as LogisticRegression or LinearSVC.

**fit_intercept**: boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**normalize**: boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

**scoring**: string, callable or None, optional, default: None

A string (see model evaluation documentation) or a scorer callable object / function with signature `scorer(estimator,X,y)`.

**cv**: int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the efficient Leave-One-Out cross-validation
- integer, to specify the number of folds.
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.

Refer *User Guide* for the various cross-validation strategies that can be used here.

**class_weight**: dict or ‘balanced’, optional

Weights associated with classes in the form `{class_label: weight}`. If not given, all classes are supposed to have weight one.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as `n_samples / (n_classes * np.bincount(y))`.

Attributes:

**cv_values_**: array, shape = [n_samples, n_alphas] or shape = [n_samples, n_responses, n_alphas], optional

Cross-validation values for each alpha (if `store_cv_values=True` and `cv=None`). After `fit()` has been called, this attribute will contain the mean squared errors (by default) or the values of the `{loss, score}_func` function (if provided in the constructor).

**coef_**: array, shape = [n_features] or [n_targets, n_features]

Weight vector(s).

**intercept_**: float | array, shape = (n_targets,)

Independent term in decision function. Set to 0.0 if `fit_intercept = False`. 

---

*scikit-learn user guide, Release 0.18.2*
alpha_ : float

Estimated regularization parameter

See also:

RidgeRegression
Ridge regression

RidgeClassifier
Ridge classifier

RidgeCV
Ridge regression with built-in cross validation

Notes

For multi-class classification, n_class classifiers are trained in a one-versus-all approach. Concretely, this is implemented by taking advantage of the multi-variate response support in Ridge.

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<td>Predict confidence scores for samples.</td>
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<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit the ridge classifier.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict class labels for samples in X.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (alphas=(0.1, 1.0, 10.0), fit_intercept=True, normalize=False, scoring=None, cv=None, class_weight=None)

decision_function (X)

Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

Parameters:

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returns:

array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes) :

Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

fit (X, y, sample_weight=None)

Fit the ridge classifier.

Parameters:

X : array-like, shape (n_samples, n_features)

Training vectors, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape (n_samples,)

Target values.

sample_weight : float or numpy array of shape (n_samples,)

Sample weight.
Returns self:

Returns self.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

deep : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns:

params : mapping of string to any

Parameter names mapped to their values.

predict (X)

Predict class labels for samples in X.

Parameters:

X : array-like, sparse matrix, shape = [n_samples, n_features]

Samples.

Returns:

C : array, shape = [n_samples]

Predicted class label per sample.

score (X, y, sample_weight=None)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each
sample that each label set be correctly predicted.

Parameters:

X : array-like, shape = (n_samples, n_features)

Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)

True labels for X.

sample_weight : array-like, shape = [n_samples], optional

Sample weights.

Return:

score : float

Mean accuracy of self.predict(X) wrt. y.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns:

self:

Information Criterion

Some models can offer an information-theoretic closed-form formula of the optimal estimate of the regularization
parameter by computing a single regularization path (instead of several when using cross-validation).

Here is the list of models benefitting from the Akaike Information Criterion (AIC) or the Bayesian Information Crite-
rion (BIC) for automated model selection:
Lasso model fit with Lars using BIC or AIC for model selection

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||y - Xw||^2_2 + \alpha \times ||w||_1
\]

AIC is the Akaike information criterion and BIC is the Bayes Information criterion. Such criteria are useful to select the value of the regularization parameter by making a trade-off between the goodness of fit and the complexity of the model. A good model should explain well the data while being simple.

Read more in the User Guide.

Parameters:

- **criterion**: 'bic' | 'aic'
  
  The type of criterion to use.

- **fit_intercept**: boolean
  
  whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **positive**: boolean (default=False)
  
  Restrict coefficients to be \( \geq 0 \). Be aware that you might want to remove fit_intercept which is set True by default. Under the positive restriction the model coefficients do not converge to the ordinary-least-squares solution for small values of alpha. Only coefficients up to the smallest alpha value (alphas_[alphas_ > 0.].min() when fit_path=True) reached by the stepwise Lars-Lasso algorithm are typically in congruence with the solution of the coordinate descent Lasso estimator. As a consequence using LassoLarsIC only makes sense for problems where a sparse solution is expected and/or reached.

- **verbose**: boolean or integer, optional
  
  Sets the verbosity amount

- **normalize**: boolean, optional, default False
  
  If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

- **copy_X**: boolean, optional, default True
  
  If True, X will be copied; else, it may be overwritten.

- **precompute**: True | False | ‘auto’ | array-like
  
  Additional options for the coordinate descent solver. See coordinate_descent.py for more information.
Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto' let us decide. The Gram matrix can also be passed as argument.

max_iter : integer, optional

Maximum number of iterations to perform. Can be used for early stopping.

eps : float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the tol parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

Attributes

coef_ : array, shape (n_features,)

parameter vector (w in the formulation formula)

intercept_ : float

independent term in decision function.

alpha_ : float

the alpha parameter chosen by the information criterion

n_iter_ : int

number of iterations run by lars_path to find the grid of alphas.

criterion_ : array, shape (n_alphas,)

The value of the information criteria ('aic', 'bic') across all alphas. The alpha which has the smallest information criteria is chosen.

See also:

lars_path, LassoLars, LassoLarsCV

Notes

The estimation of the number of degrees of freedom is given by:


Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.LassoLarsIC(criterion='bic')
>>> clf.fit([[-1, 1], [0, 0], [1, 1]], [-1.1111, 0, -1.1111])
...
LassoLarsIC(copy_X=True, criterion='bic', eps=..., fit_intercept=True,
max_iter=500, normalize=True, positive=False, precompute='auto',
verbose=False)
>>> print(clf.coef_)
[ 0.  -1.11...]
```
Methods

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__init__(criterion='aic', fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, copy_X=True, positive=False)

decision_function(*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

Returns:

- **C**: array, shape = (n_samples,)
  - Returns predicted values.

fit(X, y, copy_X=True)

Fit the model using X, y as training data.

Parameters:

- **X**: array-like, shape (n_samples, n_features)
  - training data.
- **y**: array-like, shape (n_samples,)
  - target values.
- **copy_X**: boolean, optional, default True
  - If True, X will be copied; else, it may be overwritten.

Returns:

- **self**: object
  - returns an instance of self.

get_params(deep=True)

Get parameters for this estimator.

Parameters:

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

- **params**: mapping of string to any
  - Parameter names mapped to their values.

predict(X)

Predict using the linear model.

Parameters:

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.
Returns:

C : array, shape = (n_samples,)

Returns predicted values.

score (X, y, sample_weight=None)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

Parameters:

X : array-like, shape = (n_samples, n_features)

Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for X.

sample_weight : array-like, shape = [n_samples], optional

Sample weights.

Returns:

score : float

R^2 of self.predict(X) wrt. y.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:

self :

Examples using sklearn.linear_model.LassoLarsIC

- Lasso model selection: Cross-Validation / AIC / BIC

Out of Bag Estimates

When using ensemble methods base upon bagging, i.e. generating new training sets using sampling with replacement, part of the training set remains unused. For each classifier in the ensemble, a different part of the training set is left out.

This left out portion can be used to estimate the generalization error without having to rely on a separate validation set. This estimate comes “for free” as no additional data is needed and can be used for model selection.

This is currently implemented in the following classes:

- ensemble.RandomForestClassifier([...]) A random forest classifier.
- ensemble.RandomForestRegressor([...]) A random forest regressor.
- ensemble.GradientBoostingClassifier([loss, ...]) Gradient Boosting for classification.
A random forest classifier.

A random forest is a meta estimator that fits a number of decision tree classifiers on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting. The sub-sample size is always the same as the original input sample size but the samples are drawn with replacement if `bootstrap=True` (default).

Read more in the User Guide.

**Parameters**

- `n_estimators` : integer, optional (default=10)
  The number of trees in the forest.

- `criterion` : string, optional (default=”gini”)
  The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain. Note: this parameter is tree-specific.

- `max_features` : int, float, string or None, optional (default=”auto”)
  The number of features to consider when looking for the best split:
  - If int, then consider `max_features` features at each split.
  - If float, then `max_features` is a percentage and `int(max_features * n_features)` features are considered at each split.
  - If “auto”, then `max_features=sqrt(n_features)`.
  - If “sqrt”, then `max_features=sqrt(n_features)` (same as “auto”).
  - If “log2”, then `max_features=log2(n_features)`.
  - If None, then `max_features=n_features`.
  Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than `max_features` features.

- `max_depth` : integer or None, optional (default=None)
  The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than `min_samples_split` samples.

- `min_samples_split` : int, float, optional (default=2)
The minimum number of samples required to split an internal node:

- If int, then consider `min_samples_split` as the minimum number.
- If float, then `min_samples_split` is a percentage and `ceil(min_samples_split * n_samples)` are the minimum number of samples for each split.

Changed in version 0.18: Added float values for percentages.

**min_samples_leaf**: int, float, optional (default=1)

The minimum number of samples required to be at a leaf node:

- If int, then consider `min_samples_leaf` as the minimum number.
- If float, then `min_samples_leaf` is a percentage and `ceil(min_samples_leaf * n_samples)` are the minimum number of samples for each node.

Changed in version 0.18: Added float values for percentages.

**min_weight_fraction_leaf**: float, optional (default=0.)

The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when `sample_weight` is not provided.

**max_leaf_nodes**: int or None, optional (default=None)

Grow trees with `max_leaf_nodes` in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

**min_impurity_split**: float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.

New in version 0.18.

**bootstrap**: boolean, optional (default=True)

Whether bootstrap samples are used when building trees.

**oob_score**: bool (default=False)

Whether to use out-of-bag samples to estimate the generalization accuracy.

**n_jobs**: integer, optional (default=1)

The number of jobs to run in parallel for both `fit` and `predict`. If -1, then the number of jobs is set to the number of cores.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**verbose**: int, optional (default=0)

Controls the verbosity of the tree building process.

**warm_start**: bool, optional (default=False)

When set to `True`, reuse the solution of the previous call to fit and add more estimators to the ensemble, otherwise, just fit a whole new forest.

**class_weight**: dict, list of dicts, “balanced”,
“balanced_subsample” or None, optional (default=None) Weights associated with classes in the form \{class_label: weight\}. If not given, all classes are supposed to have weight one. For multi-output problems, a list of dicts can be provided in the same order as the columns of y.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as \(\frac{n\text{\_samples}}{(n\text{\_classes} * \text{np.bincount}(y))}\)

The “balanced_subsample” mode is the same as “balanced” except that weights are computed based on the bootstrap sample for every tree grown.

For multi-output, the weights of each column of y will be multiplied.

Note that these weights will be multiplied with sample_weight (passed through the fit method) if sample_weight is specified.

**Attributes**

- **estimators_**: list of DecisionTreeClassifier
  
The collection of fitted sub-estimators.

- **classes_**: array of shape = [n\_classes] or a list of such arrays
  
The classes labels (single output problem), or a list of arrays of class labels (multi-output problem).

- **n\_classes_**: int or list
  
The number of classes (single output problem), or a list containing the number of classes for each output (multi-output problem).

- **n\_features_**: int
  
The number of features when fit is performed.

- **n\_outputs_**: int
  
The number of outputs when fit is performed.

- **feature\_importances_**: array of shape = [n\_features]
  
The feature importances (the higher, the more important the feature).

- **oob\_score_**: float
  
Score of the training dataset obtained using an out-of-bag estimate.

- **oob\_decision\_function_**: array of shape = [n\_samples, n\_classes]
  
Decision function computed with out-of-bag estimate on the training set. If n\_estimators is small it might be possible that a data point was never left out during the bootstrap. In this case, oob\_decision\_function_ might contain NaN.

See also:

- DecisionTreeClassifier, ExtraTreesClassifier

References

[R23]

Methods
apply(X)  
Apply trees in the forest to X, return leaf indices.

decision_path(X)  
Return the decision path in the forest

fit(X, y[, sample_weight])  
Build a forest of trees from the training set (X, y).

fit_transform(X[, y])  
Fit to data, then transform it.

get_params([deep])  
Get parameters for this estimator.

predict(X)  
Predict class for X.

predict_log_proba(X)  
Predict class log-probabilities for X.

predict_proba(X)  
Predict class probabilities for X.

score(X, y[, sample_weight])  
Returns the mean accuracy on the given test data and labels.

set_params(**params)  
Set the parameters of this estimator.

transform(*args, **kwargs)  
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.

__init__(n_estimators=10, criterion='gini', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, bootstrap=True, oob_score=False, n_jobs=1, random_state=None, verbose=0, warm_start=False, class_weight=None)

apply(X)  
Apply trees in the forest to X, return leaf indices.

Parameters:
X : array-like or sparse matrix, shape = [n_samples, n_features]
    The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns:
X_leaves : array-like, shape = [n_samples, n_estimators]
    For each datapoint x in X and for each tree in the forest, return the index of the leaf x ends up in.

decision_path(X)  
Return the decision path in the forest

New in version 0.18.

Parameters:
X : array-like or sparse matrix, shape = [n_samples, n_features]
    The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns:
indicator : sparse csr array, shape = [n_samples, n_nodes]
    Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

n_nodes_ptr : array of size (n_estimators + 1, )
    The columns from indicator[n_nodes_ptr[i]:n_nodes_ptr[i+1]] gives the indicator value for the i-th estimator.

feature_importances_

Return the feature importances (the higher, the more important the feature).

Returns:
feature_importances_ : array, shape = [n_features]

fit(X, y, sample_weight=None)
Build a forest of trees from the training set (X, y).
Parameters:

- **X**: array-like or sparse matrix of shape = [n_samples, n_features]
  - The training input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csc_matrix.

- **y**: array-like, shape = [n_samples] or [n_samples, n_outputs]
  - The target values (class labels in classification, real numbers in regression).

- **sample_weight**: array-like, shape = [n_samples] or None
  - Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node. In the case of classification, splits are also ignored if they would result in any single class carrying a negative weight in either child node.

Returns:

- **self**: object
  - Returns self.

**fit_transform**(X, y=None, **fit_params)

- Fit to data, then transform it.

  - Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

- **Parameters**:
  - **X**: numpy array of shape [n_samples, n_features]
    - Training set.
  - **y**: numpy array of shape [n_samples]
    - Target values.

- **Returns**:
  - **X_new**: numpy array of shape [n_samples, n_features_new]
    - Transformed array.

**get_params**(deep=True)

- Get parameters for this estimator.

- **Parameters**:
  - **deep**: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**:
  - **params**: mapping of string to any
    - Parameter names mapped to their values.

**predict**(X)

- Predict class for X.

  - The predicted class of an input sample is a vote by the trees in the forest, weighted by their probability estimates. That is, the predicted class is the one with highest mean probability estimate across the trees.

- **Parameters**:
  - **X**: array-like or sparse matrix of shape = [n_samples, n_features]
    - The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

- **Returns**:
  - **y**: array of shape = [n_samples] or [n_samples, n_outputs]
    - The predicted classes.
**predict_log_proba** *(X)*

Predict class log-probabilities for X.

The predicted class log-probabilities of an input sample is computed as the log of the mean predicted class probabilities of the trees in the forest.

**Parameters**

- **X**: array-like or sparse matrix of shape = [n_samples, n_features]
  
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

**Returns**

- **p**: array of shape = [n_samples, n_classes], or a list of n_outputs
  
such arrays if n_outputs > 1. The class probabilities of the input samples. The order of the classes corresponds to that in the attribute classes_.

**predict_proba** *(X)*

Predict class probabilities for X.

The predicted class probabilities of an input sample are computed as the mean predicted class probabilities of the trees in the forest. The class probability of a single tree is the fraction of samples of the same class in a leaf.

**Parameters**

- **X**: array-like or sparse matrix of shape = [n_samples, n_features]
  
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

**Returns**

- **p**: array of shape = [n_samples, n_classes], or a list of n_outputs
  
such arrays if n_outputs > 1. The class probabilities of the input samples. The order of the classes corresponds to that in the attribute classes_.

**score** *(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  
Test samples.

- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  
True labels for X.

- **sample_weight**: array-like, shape = [n_samples], optional
  
Sample weights.

**Returns**

- **score**: float
  
Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>___<parameter> so that it’s possible to update each component of a nested object.

**Returns**

- **self**: object
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses coef_ or feature_importances_ to determine the most important features. For models with a coef_ for each class, the absolute sum over the classes is used.

**Parameters**

- X : array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.

- threshold : string, float or None, optional (default=None)
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

- X_r : array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

**Examples using sklearn.ensemble.RandomForestClassifier**

- Probability Calibration for 3-class classification
- Comparison of Calibration of Classifiers
- Classifier comparison
- OOB Errors for Random Forests
- Feature transformations with ensembles of trees
- Plot the decision surfaces of ensembles of trees on the iris dataset
- Plot class probabilities calculated by the VotingClassifier
- Comparing randomized search and grid search for hyperparameter estimation
- Classification of text documents using sparse features

**sklearn.ensemble.RandomForestRegressor**

```
class sklearn.ensemble.RandomForestRegressor(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, bootstrap=True, oob_score=False, n_jobs=1, random_state=None, verbose=0, warm_start=False)
```

A random forest regressor.

A random forest is a meta estimator that fits a number of classifying decision trees on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting. The sub-sample size is
always the same as the original input sample size but the samples are drawn with replacement if bootstrap=True (default).

Read more in the User Guide.

**Parameters**

- **n_estimators**: integer, optional (default=10)
  
  The number of trees in the forest.

- **criterion**: string, optional (default="mse")
  
  The function to measure the quality of a split. Supported criteria are “mse” for the mean squared error, which is equal to variance reduction as feature selection criterion, and “mae” for the mean absolute error.

  New in version 0.18: Mean Absolute Error (MAE) criterion.

- **max_features**: int, float, string or None, optional (default="auto")
  
  The number of features to consider when looking for the best split:

  - If int, then consider max_features features at each split.
  - If float, then max_features is a percentage and int(max_features * n_features) features are considered at each split.
  - If “auto”, then max_features=n_features.
  - If “sqrt”, then max_features=sqrt(n_features).
  - If “log2”, then max_features=log2(n_features).
  - If None, then max_features=n_features.

  Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than max_features features.

- **max_depth**: integer or None, optional (default=None)
  
  The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples.

- **min_samples_split**: int, float, optional (default=2)
  
  The minimum number of samples required to split an internal node:

  - If int, then consider min_samples_split as the minimum number.
  - If float, then min_samples_split is a percentage and ceil(min_samples_split * n_samples) are the minimum number of samples for each split.

  Changed in version 0.18: Added float values for percentages.

- **min_samples_leaf**: int, float, optional (default=1)
  
  The minimum number of samples required to be at a leaf node:

  - If int, then consider min_samples_leaf as the minimum number.
  - If float, then min_samples_leaf is a percentage and ceil(min_samples_leaf * n_samples) are the minimum number of samples for each node.

  Changed in version 0.18: Added float values for percentages.

- **min_weight_fraction_leaf**: float, optional (default=0.)
The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.

**max_leaf_nodes** : int or None, optional (default=None)

Grow trees with max_leaf_nodes in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

**min_impurity_split** : float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.

New in version 0.18.

**bootstrap** : boolean, optional (default=True)

Whether bootstrap samples are used when building trees.

**oob_score** : bool, optional (default=False)

whether to use out-of-bag samples to estimate the R^2 on unseen data.

**n_jobs** : integer, optional (default=1)

The number of jobs to run in parallel for both fit and predict. If -1, then the number of jobs is set to the number of cores.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**verbose** : int, optional (default=0)

Controls the verbosity of the tree building process.

**warm_start** : bool, optional (default=False)

When set to True, reuse the solution of the previous call to fit and add more estimators to the ensemble, otherwise, just fit a whole new forest.

**Attributes**

**estimators_** : list of DecisionTreeRegressor

The collection of fitted sub-estimators.

**feature_importances_** : array of shape = [n_features]

The feature importances (the higher, the more important the feature).

**n_features_** : int

The number of features when fit is performed.

**n_outputs_** : int

The number of outputs when fit is performed.

**oob_score_** : float

Score of the training dataset obtained using an out-of-bag estimate.

**oob_prediction_** : array of shape = [n_samples]

Prediction computed with out-of-bag estimate on the training set.
See also:

DecisionTreeRegressor, ExtraTreesRegressor

References

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Methods

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<th>Method</th>
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<td>apply(X)</td>
<td>Apply trees in the forest to X, return leaf indices.</td>
</tr>
<tr>
<td>decision_path(X)</td>
<td>Return the decision path in the forest</td>
</tr>
<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Build a forest of trees from the training set (X, y).</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params({deep})</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict regression target for X.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(*args, **kwargs)</td>
<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
</tr>
</tbody>
</table>

__init__(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, bootstrap=True, oob_score=False, n_jobs=1, random_state=None, verbose=0, warm_start=False)

apply (X)

Apply trees in the forest to X, return leaf indices.

Parameters

X : array-like or sparse matrix, shape = [n_samples, n_features]

The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns

X_leaves : array_like, shape = [n_samples, n_estimators]

For each datapoint x in X and for each tree in the forest, return the index of the leaf x ends up in.

decision_path (X)

Return the decision path in the forest

New in version 0.18.

Parameters

X : array-like or sparse matrix, shape = [n_samples, n_features]

The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns

indicator : sparse csr array, shape = [n_samples, n_nodes]

Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

n_nodes_ptr : array of size (n_estimators + 1,)
feature_importances_

Return the feature importances (the higher, the more important the feature).

Returns feature_importances_ : array, shape = [n_features]

fit (X, y, sample_weight=None)

Build a forest of trees from the training set (X, y).

Parameters X : array-like or sparse matrix of shape = [n_samples, n_features]
The training input samples. Internally, its dtype will be converted to
dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse
csc_matrix.

y : array-like, shape = [n_samples] or [n_samples, n_outputs]
The target values (class labels in classification, real numbers in regression).

sample_weight : array-like, shape = [n_samples] or None
Sample weights. If None, then samples are equally weighted. Splits that would create
child nodes with net zero or negative weight are ignored while searching for a split in
each node. In the case of classification, splits are also ignored if they would result in
any single class carrying a negative weight in either child node.

Returns self : object
Returns self.

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

get_params (deep=True)
Get parameters for this estimator.

Parameters deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns params : mapping of string to any
Parameter names mapped to their values.

predict (X)
Predict regression target for X.
The predicted regression target of an input sample is computed as the mean predicted regression targets of the trees in the forest.

**Parameters**

- \( X \) : array-like or sparse matrix of shape \([n_{\text{samples}}, n_{\text{features}}]\)
  
  The input samples. Internally, its dtype will be converted to \( \text{dtype=\text{np.float32}} \). If a sparse matrix is provided, it will be converted into a sparse \( \text{csr_matrix} \).

- \( y \) : array of shape \([n_{\text{samples}}]\) or \([n_{\text{samples}}, n_{\text{outputs}}]\)
  
  The predicted values.

**score** \((X, y, \text{sample_weight=\text{None}})\)

Returns the coefficient of determination \( R^2 \) of the prediction.

The coefficient \( R^2 \) is defined as \((1 - u/v)\), where \( u \) is the regression sum of squares \(((y_{\text{true}} - y_{\text{pred}}) ** 2).sum()\) and \( v \) is the residual sum of squares \(((y_{\text{true}} - y_{\text{true}}.mean()) ** 2).sum()\). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \( y \), disregarding the input features, would get a \( R^2 \) score of 0.0.

**Parameters**

- \( X \) : array-like, shape \( = (n_{\text{samples}}, n_{\text{features}}) \)
  
  Test samples.

- \( y \) : array-like, shape \( = (n_{\text{samples}}) \) or \( (n_{\text{samples}}, n_{\text{outputs}}) \)
  
  True values for \( X \).

- \( \text{sample_weight} \) : array-like, shape \( = [n_{\text{samples}}] \), optional
  
  Sample weights.

**Returns**

- \( \text{score} \) : float
  
  \( R^2 \) of self.predict(X) wrt. \( y \).

**set_params** \((**params)\)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \( \text{<component>__<parameter>} \) so that it’s possible to update each component of a nested object.

**Returns**

- \( \text{self} \)

**transform** \((*args, **kwargs)\)

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce \( X \) to its most important features.

Uses \( \text{coef} \) or \( \text{feature_importances} \) to determine the most important features. For models with a \( \text{coef} \) for each class, the absolute sum over the classes is used.

**Parameters**

- \( X \) : array or scipy sparse matrix of shape \([n_{\text{samples}}, n_{\text{features}}]\)
  
  The input samples.

- \( \text{threshold} \) [string, float or None, optional (default=\text{None})]
  
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*\text{mean}”) may also be used. If None and if available, the object attribute \( \text{threshold} \) is used. Otherwise, “mean” is used by default.
Returns $X_r$ : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

Examples using `sklearn.ensemble.RandomForestRegressor`

- **Imputing missing values before building an estimator**
- Prediction Latency
- Comparing random forests and the multi-output meta estimator

### `sklearn.ensemble.ExtraTreesClassifier`

```python
class sklearn.ensemble.ExtraTreesClassifier(n_estimators=10, criterion='gini',
min_samples_split=2, min_samples_leaf=1,
min_weight_fraction_leaf=0.0,
max_features='auto', max_leaf_nodes=None,
min_impurity_split=1e-07, bootstrap=False, oob_score=False, n_jobs=1,
random_state=None, verbose=0,
warm_start=False, class_weight=None)
```

An extra-trees classifier.

This class implements a meta estimator that fits a number of randomized decision trees (a.k.a. extra-trees) on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.


**Parameters**

- `n_estimators` : integer, optional (default=10)
  
The number of trees in the forest.

- `criterion` : string, optional (default="gini")
  
The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain.

- `max_features` : int, float, string or None, optional (default="auto")
  
The number of features to consider when looking for the best split:
  
  - If int, then consider `max_features` features at each split.
  - If float, then `max_features` is a percentage and `int(max_features * n_features)` features are considered at each split.
  - If “auto”, then `max_features=sqrt(n_features)`.
  - If “sqrt”, then `max_features=sqrt(n_features)`.
  - If “log2”, then `max_features=log2(n_features)`.
  - If None, then `max_features=n_features`.

  Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than `max_features` features.

- `max_depth` : integer or None, optional (default=None)
The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples.

**min_samples_split** : int, float, optional (default=2)

The minimum number of samples required to split an internal node:

- If int, then consider min_samples_split as the minimum number.
- If float, then min_samples_split is a percentage and \( \text{ceil}(\text{min_samples_split} \times n\_samples) \) are the minimum number of samples for each split.

Changed in version 0.18: Added float values for percentages.

**min_samples_leaf** : int, float, optional (default=1)

The minimum number of samples required to be at a leaf node:

- If int, then consider min_samples_leaf as the minimum number.
- If float, then min_samples_leaf is a percentage and \( \text{ceil}(\text{min_samples_leaf} \times n\_samples) \) are the minimum number of samples for each node.

Changed in version 0.18: Added float values for percentages.

**min_weight_fraction_leaf** : float, optional (default=0.)

The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.

**max_leaf_nodes** : int or None, optional (default=None)

Grow trees with max_leaf_nodes in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

**min_impurity_split** : float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.

New in version 0.18.

**bootstrap** : boolean, optional (default=False)

Whether bootstrap samples are used when building trees.

**oob_score** : bool, optional (default=False)

Whether to use out-of-bag samples to estimate the generalization accuracy.

**n_jobs** : integer, optional (default=1)

The number of jobs to run in parallel for both fit and predict. If -1, then the number of jobs is set to the number of cores.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**verbose** : int, optional (default=0)

Controls the verbosity of the tree building process.

**warm_start** : bool, optional (default=False)
When set to True, reuse the solution of the previous call to fit and add more estimators to the ensemble, otherwise, just fit a whole new forest.

**class_weight** : dict, list of dict, “balanced”, “balanced_subsample” or None, optional (default=None)

Weights associated with classes in the form `{class_label: weight}`. If not given, all classes are supposed to have weight one. For multi-output problems, a list of dict can be provided in the same order as the columns of y.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as \( n_{\text{samples}} / (n_{\text{classes}} \times \text{np.bincount}(y)) \)

The “balanced_subsample” mode is the same as “balanced” except that weights are computed based on the bootstrap sample for every tree grown.

For multi-output, the weights of each column of y will be multiplied.

Note that these weights will be multiplied with sample_weight (passed through the fit method) if sample_weight is specified.

**Attributes**

- **estimators_** : list of DecisionTreeClassifier

  The collection of fitted sub-estimators.

- **classes_** : array of shape = [n_classes] or a list of such arrays

  The classes labels (single output problem), or a list of arrays of class labels (multi-output problem).

- **n_classes_** : int or list

  The number of classes (single output problem), or a list containing the number of classes for each output (multi-output problem).

- **feature_importances_** : array of shape = [n_features]

  The feature importances (the higher, the more important the feature).

- **n_features_** : int

  The number of features when fit is performed.

- **n_outputs_** : int

  The number of outputs when fit is performed.

- **oob_score_** : float

  Score of the training dataset obtained using an out-of-bag estimate.

- **oob_decision_function_** : array of shape = [n_samples, n_classes]

  Decision function computed with out-of-bag estimate on the training set. If n_estimators is small it might be possible that a data point was never left out during the bootstrap. In this case, oob_decision_function_ might contain NaN.

See also:

- **sklearn.tree.ExtraTreeClassifier**: Base classifier for this ensemble.

- **RandomForestClassifier**: Ensemble Classifier based on trees with optimal splits.
References

[R19]

Methods

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__init__(n_estimators=10, criterion='gini', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, bootstrap=False, oob_score=False, n_jobs=1, random_state=None, verbose=0, warm_start=False, class_weight=None)

apply(X)
Apply trees in the forest to X, return leaf indices.

Parameters

X : array-like or sparse matrix, shape = [n_samples, n_features]
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns

X_leaves : array_like, shape = [n_samples, n_estimators]
For each datapoint x in X and for each tree in the forest, return the index of the leaf x ends up in.

decision_path(X)
Return the decision path in the forest

New in version 0.18.

Parameters

X : array-like or sparse matrix, shape = [n_samples, n_features]
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns

indicator : sparse csr array, shape = [n_samples, n_nodes]
Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

n_nodes_ptr : array of size (n_estimators + 1,)
The columns from indicator[n_nodes_ptr[i]:n_nodes_ptr[i+1]] gives the indicator value for the i-th estimator.
feature_importances_

Return the feature importances (the higher, the more important the feature).

Returns feature_importances_ : array, shape = [n_features]

fit (X, y, sample_weight=None)

Build a forest of trees from the training set (X, y).

Parameters

X : array-like or sparse matrix of shape = [n_samples, n_features]

The training input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csc_matrix.

y : array-like, shape = [n_samples] or [n_samples, n_outputs]

The target values (class labels in classification, real numbers in regression).

sample_weight : array-like, shape = [n_samples] or None

Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node. In the case of classification, splits are also ignored if they would result in any single class carrying a negative weight in either child node.

Returns self : object

Returns self.

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

X : numpy array of shape [n_samples, n_features]

Training set.

y : numpy array of shape [n_samples]

Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]

Transformed array.

get_params (deep=True)

Get parameters for this estimator.

Parameters

deep : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns params : mapping of string to any

Parameter names mapped to their values.

predict (X)

Predict class for X.

The predicted class of an input sample is a vote by the trees in the forest, weighted by their probability estimates. That is, the predicted class is the one with highest mean probability estimate across the trees.

Parameters

X : array-like or sparse matrix of shape = [n_samples, n_features]
The input samples. Internally, its dtype will be converted to `dtype=np.float32`. If a sparse matrix is provided, it will be converted into a sparse `csr_matrix`.

Returns

- `y`: array of shape = `[n_samples]` or `[n_samples, n_outputs]`
  - The predicted classes.

**predict_log_proba** (*X*)

Predict class log-probabilities for *X*.

The predicted class log-probabilities of an input sample is computed as the log of the mean predicted class probabilities of the trees in the forest.

Parameters

- `X`: array-like or sparse matrix of shape = `[n_samples, n_features]`
  - The input samples. Internally, its dtype will be converted to `dtype=np.float32`. If a sparse matrix is provided, it will be converted into a sparse `csr_matrix`.

Returns

- `p`: array of shape = `[n_samples, n_classes]`, or a list of `n_outputs` such arrays if `n_outputs > 1. The class probabilities of the input samples. The order of the classes corresponds to that in the attribute `classes_`.

**predict_proba** (*X*)

Predict class probabilities for *X*.

The predicted class probabilities of an input sample are computed as the mean predicted class probabilities of the trees in the forest. The class probability of a single tree is the fraction of samples of the same class in a leaf.

Parameters

- `X`: array-like or sparse matrix of shape = `[n_samples, n_features]`
  - The input samples. Internally, its dtype will be converted to `dtype=np.float32`. If a sparse matrix is provided, it will be converted into a sparse `csr_matrix`.

Returns

- `p`: array of shape = `[n_samples, n_classes]`, or a list of `n_outputs` such arrays if `n_outputs > 1. The class probabilities of the input samples. The order of the classes corresponds to that in the attribute `classes_`.

**score** (*X, y, sample_weight=None*)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters

- `X`: array-like, shape = (n_samples, n_features)
  - Test samples.
- `y`: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True labels for *X*.
- `sample_weight`: array-like, shape = [n_samples], optional
  - Sample weights.

Returns

- `score`: float
  - Mean accuracy of `self.predict(X)` wrt. *y*.

**set_params** (**params**)

Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns:

self:

`transform(*args, **kwargs)`

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

Parameters:

`X` : array or scipy sparse matrix of shape `[n_samples, n_features]`

The input samples.

`threshold` [string, float or None, optional (default=None)] The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

Returns:

`X_r` : array of shape `[n_samples, n_selected_features]`

The input samples with only the selected features.

Examples using `sklearn.ensemble.ExtraTreesClassifier`

- Feature importances with forests of trees
- Pixel importances with a parallel forest of trees
- Plot the decision surfaces of ensembles of trees on the iris dataset
- Hashing feature transformation using Totally Random Trees

`sklearn.ensemble.ExtraTreesRegressor`

class `sklearn.ensemble.ExtraTreesRegressor` (`n_estimators=10, criterion='mse', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, bootstrap=False, oob_score=False, n_jobs=1, random_state=None, verbose=0, warm_start=False)`

An extra-trees regressor.

This class implements a meta estimator that fits a number of randomized decision trees (a.k.a. extra-trees) on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

Read more in the User Guide.
Parameters:

- **n_estimators**: integer, optional (default=10)
  The number of trees in the forest.

- **criterion**: string, optional (default="mse")
  The function to measure the quality of a split. Supported criteria are “mse” for the mean squared error, which is equal to variance reduction as feature selection criterion, and “mae” for the mean absolute error.
  New in version 0.18: Mean Absolute Error (MAE) criterion.

- **max_features**: int, float, string or None, optional (default="auto")
  The number of features to consider when looking for the best split:
  - If int, then consider max_features features at each split.
  - If float, then max_features is a percentage and int(max_features * n_features) features are considered at each split.
  - If “auto”, then max_features=n_features.
  - If “sqrt”, then max_features=sqrt(n_features).
  - If “log2”, then max_features=log2(n_features).
  - If None, then max_features=n_features.
  Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than max_features features.

- **max_depth**: integer or None, optional (default=None)
  The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples.

- **min_samples_split**: int, float, optional (default=2)
  The minimum number of samples required to split an internal node:
  - If int, then consider min_samples_split as the minimum number.
  - If float, then min_samples_split is a percentage and ceil(min_samples_split * n_samples) are the minimum number of samples for each split.
  Changed in version 0.18: Added float values for percentages.

- **min_samples_leaf**: int, float, optional (default=1)
  The minimum number of samples required to be at a leaf node:
  - If int, then consider min_samples_leaf as the minimum number.
  - If float, then min_samples_leaf is a percentage and ceil(min_samples_leaf * n_samples) are the minimum number of samples for each node.
  Changed in version 0.18: Added float values for percentages.

- **min_weight_fraction_leaf**: float, optional (default=0.)
  The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.

- **max_leaf_nodes**: int or None, optional (default=None)
Grow trees with `max_leaf_nodes` in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

**min_impurity_split** : float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.

New in version 0.18.

**bootstrap** : boolean, optional (default=False)

Whether bootstrap samples are used when building trees.

**oob_score** : bool, optional (default=False)

Whether to use out-of-bag samples to estimate the $R^2$ on unseen data.

**n_jobs** : integer, optional (default=1)

The number of jobs to run in parallel for both `fit` and `predict`. If -1, then the number of jobs is set to the number of cores.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**verbose** : int, optional (default=0)

Controls the verbosity of the tree building process.

**warm_start** : bool, optional (default=False)

When set to `True`, reuse the solution of the previous call to `fit` and add more estimators to the ensemble, otherwise, just fit a whole new forest.

**Attributes**

- `estimators_` : list of DecisionTreeRegressor
  
  The collection of fitted sub-estimators.

- `feature_importances_` : array of shape = [n_features]
  
  The feature importances (the higher, the more important the feature).

- `n_features_` : int
  
  The number of features.

- `n_outputs_` : int
  
  The number of outputs.

- `oob_score_` : float
  
  Score of the training dataset obtained using an out-of-bag estimate.

- `oob_prediction_` : array of shape = [n_samples]
  
  Prediction computed with out-of-bag estimate on the training set.

**See also:**

- `sklearn.tree.ExtraTreeRegressor` Base estimator for this ensemble.

- `RandomForestRegressor` Ensemble regressor using trees with optimal splits.
References

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<td>Get parameters for this estimator.</td>
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<tr>
<td>predict(X)</td>
<td>Predict regression target for X.</td>
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<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
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<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td>transform(**args, **kwargs)</td>
<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
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__init__(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, bootstrap=False, oob_score=False, n_jobs=1, random_state=None, verbose=0, warm_start=False)

apply(X)

Apply trees in the forest to X, return leaf indices.

Parameters

X : array-like or sparse matrix, shape = [n_samples, n_features]

The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns

X_leaves : array_like, shape = [n_samples, n_estimators]

For each datapoint x in X and for each tree in the forest, return the index of the leaf x ends up in.

decision_path (X)

Return the decision path in the forest

New in version 0.18.

Parameters

X : array-like or sparse matrix, shape = [n_samples, n_features]

The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns

indicator : sparse csr array, shape = [n_samples, n_nodes]

Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

n_nodes_ptr : array of size (n_estimators + 1,)

The columns from indicator[n_nodes_ptr[i]:n_nodes_ptr[i+1]] gives the indicator value for the i-th estimator.

feature_importances_

Return the feature importances (the higher, the more important the feature).
Returns feature importances : array, shape = [n_features]

**fit** *(X, y, sample_weight=None)*

Build a forest of trees from the training set (X, y).

**Parameters**

- **X** : array-like or sparse matrix of shape = [n_samples, n_features]
  
The training input samples. Internally, its dtypes will be converted to
dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse
csc_matrix.

- **y** : array-like, shape = [n_samples] or [n_samples, n_outputs]
  
The target values (class labels in classification, real numbers in regression).

- **sample_weight** : array-like, shape = [n_samples] or None
  
  Sample weights. If None, then samples are equally weighted. Splits that would create
child nodes with net zero or negative weight are ignored while searching for a split in
each node. In the case of classification, splits are also ignored if they would result in
any single class carrying a negative weight in either child node.

Returns **self** : object

Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X** : numpy array of shape [n_samples, n_features]
  
  Training set.

- **y** : numpy array of shape [n_samples]
  
  Target values.

Returns **X_new** : numpy array of shape [n_samples, n_features_new]

Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns **params** : mapping of string to any

Parameter names mapped to their values.

**predict** *(X)*

Predict regression target for X.

The predicted regression target of an input sample is computed as the mean predicted regression targets of
the trees in the forest.

**Parameters**

- **X** : array-like or sparse matrix of shape = [n_samples, n_features]
  
  The input samples. Internally, its dtypes will be converted to dtype=np.float32. If
a sparse matrix is provided, it will be converted into a sparse csr_matrix.

Returns **y** : array of shape = [n_samples] or [n_samples, n_outputs]
The predicted values.

**score** (*X*, *y*, *sample_weight=None*)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ($(y_{true} - y_{pred})^2$).sum() and $v$ is the residual sum of squares ($(y_{true} - y_{true.mean})^2$).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **score**: float
  $R^2$ of self.predict(X) wrt. $y$.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: object

**transform** (*args, **kwargs*)

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.

- **threshold**: string, float or None, optional (default=None)
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.
Examples using `sklearn.ensemble.ExtraTreesRegressor`

- Face completion with a multi-output estimators
- Sparse recovery: feature selection for sparse linear models

`sklearn.ensemble.GradientBoostingClassifier`

class `sklearn.ensemble.GradientBoostingClassifier` (loss='deviance', learning_rate=0.1, n_estimators=100, sample=1.0, criterion='friedman_mse', min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_depth=3, min_impurity_split=1e-07, init=None, random_state=None, max_features=None, verbose=0, max_leaf_nodes=None, warm_start=False, presort='auto')

Gradient Boosting for classification.

GB builds an additive model in a forward stage-wise fashion; it allows for the optimization of arbitrary differentiable loss functions. In each stage \( n_{\text{classes}} \) regression trees are fit on the negative gradient of the binomial or multinomial deviance loss function. Binary classification is a special case where only a single regression tree is induced.

Read more in the User Guide.

Parameters

- **loss**: {'deviance', 'exponential'}, optional (default='deviance')
  
  loss function to be optimized. 'deviance' refers to deviance (= logistic regression) for classification with probabilistic outputs. For loss 'exponential' gradient boosting recovers the AdaBoost algorithm.

- **learning_rate**: float, optional (default=0.1)
  
  learning rate shrinks the contribution of each tree by learning_rate. There is a trade-off between learning_rate and n_estimators.

- **n_estimators**: int (default=100)
  
  The number of boosting stages to perform. Gradient boosting is fairly robust to overfitting so a large number usually results in better performance.

- **max_depth**: integer, optional (default=3)
  
  maximum depth of the individual regression estimators. The maximum depth limits the number of nodes in the tree. Tune this parameter for best performance; the best value depends on the interaction of the input variables.

- **criterion**: string, optional (default="friedman_mse")
  
  The function to measure the quality of a split. Supported criteria are “friedman_mse” for the mean squared error with improvement score by Friedman, “mse” for mean squared error, and “mae” for the mean absolute error. The default value of “friedman_mse” is generally the best as it can provide a better approximation in some cases.

  New in version 0.18.

- **min_samples_split**: int, float, optional (default=2)
The minimum number of samples required to split an internal node:

• If int, then consider \textit{min\_samples\_split} as the minimum number.

• If float, then \textit{min\_samples\_split} is a percentage and \textit{ceil(min\_samples\_split * n\_samples)} are the minimum number of samples for each split.

Changed in version 0.18: Added float values for percentages.

\textbf{min\_samples\_leaf} : int, float, optional (default=1)

The minimum number of samples required to be at a leaf node:

• If int, then consider \textit{min\_samples\_leaf} as the minimum number.

• If float, then \textit{min\_samples\_leaf} is a percentage and \textit{ceil(min\_samples\_leaf * n\_samples)} are the minimum number of samples for each node.

Changed in version 0.18: Added float values for percentages.

\textbf{min\_weight\_fraction\_leaf} : float, optional (default=0.)

The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.

\textbf{subsample} : float, optional (default=1.0)

The fraction of samples to be used for fitting the individual base learners. If smaller than 1.0 this results in Stochastic Gradient Boosting. \textit{subsample} interacts with the parameter \textit{n\_estimators}. Choosing \textit{subsample < 1.0} leads to a reduction of variance and an increase in bias.

\textbf{max\_features} : int, float, string or None, optional (default=None)

The number of features to consider when looking for the best split:

• If int, then consider \textit{max\_features} features at each split.

• If float, then \textit{max\_features} is a percentage and \textit{int(max\_features * n\_features)} features are considered at each split.

• If “auto”, then \textit{max\_features=sqrt(n\_features)}.

• If “sqrt”, then \textit{max\_features=sqrt(n\_features)}.

• If “log2”, then \textit{max\_features=log2(n\_features)}.

• If None, then \textit{max\_features=n\_features}.

Choosing \textit{max\_features < n\_features} leads to a reduction of variance and an increase in bias.

Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than \textit{max\_features} features.

\textbf{max\_leaf\_nodes} : int or None, optional (default=None)

Grow trees with \textit{max\_leaf\_nodes} in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

\textbf{min\_impurity\_split} : float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.
New in version 0.18.

**init**: BaseEstimator, None, optional (default=None)

An estimator object that is used to compute the initial predictions. init has to provide fit and predict. If None it uses loss.init_estimator.

**verbose**: int, default: 0

Enable verbose output. If 1 then it prints progress and performance once in a while (the more trees the lower the frequency). If greater than 1 then it prints progress and performance for every tree.

**warm_start**: bool, default: False

When set to True, reuse the solution of the previous call to fit and add more estimators to the ensemble, otherwise, just erase the previous solution.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**presort**: bool or ‘auto’, optional (default='auto')

Whether to presort the data to speed up the finding of best splits in fitting. Auto mode by default will use presorting on dense data and default to normal sorting on sparse data. Setting presort to true on sparse data will raise an error.

New in version 0.17: presort parameter.

**Attributes**

- **feature_importances_**: array, shape = [n_features]
  
  The feature importances (the higher, the more important the feature).

- **oob_improvement_**: array, shape = [n_estimators]
  
  The improvement in loss (= deviance) on the out-of-bag samples relative to the previous iteration. oob_improvement_[0] is the improvement in loss of the first stage over the init estimator.

- **train_score_**: array, shape = [n_estimators]
  
  The i-th score train_score_[i] is the deviance (= loss) of the model at iteration i on the in-bag sample. If subsample == 1 this is the deviance on the training data.

- **loss_**: LossFunction
  
  The concrete LossFunction object.

- **init**: BaseEstimator
  
  The estimator that provides the initial predictions. Set via the init argument or loss.init_estimator.

- **estimators_**: ndarray of DecisionTreeRegressor, shape = [n_estimators, loss_.K]
  
  The collection of fitted sub-estimators. loss_.K is 1 for binary classification, otherwise n_classes.

See also:

- sklearn.tree.DecisionTreeClassifier
- RandomForestClassifier
- AdaBoostClassifier

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## References


Friedman, Stochastic Gradient Boosting, 1999


## Methods

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### apply(X)

Apply trees in the ensemble to X, return leaf indices.

**New in version 0.17.**

**Parameters**

- `X`: array-like or sparse matrix, shape = [n_samples, n_features]
  - The input samples. Internally, its dtype will be converted to `dtype=np.float32`. If a sparse matrix is provided, it will be converted to a sparse `csr_matrix`.

**Returns**

- `X_leaves`: array_like, shape = [n_samples, n_estimators, n_classes]
  - For each datapoint x in X and for each tree in the ensemble, return the index of the leaf x ends up in each estimator. In the case of binary classification n_classes is 1.

### decision_function(X)

Compute the decision function of X.

**Parameters**

- `X`: array-like of shape = [n_samples, n_features]
  - The input samples.

**Returns**

- `score`: array, shape = [n_samples, n_classes] or [n_samples]
The decision function of the input samples. The order of the classes corresponds to that in the attribute classes_. Regression and binary classification produce an array of shape [n_samples].

feature_importances_

Return the feature importances (the higher, the more important the feature).

Returns feature_importances_ : array, shape = [n_features]

fit (X, y, sample_weight=None, monitor=None)

Fit the gradient boosting model.

Parameters:
X : array-like, shape = [n_samples, n_features]
    Training vectors, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]
    Target values (integers in classification, real numbers in regression) For classification, labels must correspond to classes.

sample_weight : array-like, shape = [n_samples] or None
    Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node. In the case of classification, splits are also ignored if they would result in any single class carrying a negative weight in either child node.

monitor : callable, optional
    The monitor is called after each iteration with the current iteration, a reference to the estimator and the local variables of _fit_stages as keyword arguments callable(i, self, locals()). If the callable returns True the fitting procedure is stopped. The monitor can be used for various things such as computing held-out estimates, early stopping, model introspect, and snapshoting.

Returns:
self : object
    Returns self.

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters:
X : numpy array of shape [n_samples, n_features]
    Training set.

y : numpy array of shape [n_samples]
    Target values.

Returns:
X_new : numpy array of shape [n_samples, n_features_new]
    Transformed array.

get_params (deep=True)

Get parameters for this estimator.

Parameters:
deep : boolean, optional

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If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns: params : mapping of string to any
Parameter names mapped to their values.

**predict**(X)
Predict class for X.

Parameters:
X : array-like of shape = [n_samples, n_features]
The input samples.

Returns:
y : array of shape = ["n_samples"]
The predicted values.

**predict_log_proba**(X)
Predict class log-probabilities for X.

Parameters:
X : array-like of shape = [n_samples, n_features]
The input samples.

Returns:
p : array of shape = [n_samples]
The class log-probabilities of the input samples. The order of the classes corresponds to that in the attribute `classes_`.

Raises: AttributeError
If the `loss` does not support probabilities.

**predict_proba**(X)
Predict class probabilities for X.

Parameters:
X : array-like of shape = [n_samples, n_features]
The input samples.

Returns:
p : array of shape = [n_samples]
The class probabilities of the input samples. The order of the classes corresponds to that in the attribute `classes_`.

Raises: AttributeError
If the `loss` does not support probabilities.

**score**(X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters:
X : array-like, shape = (n_samples, n_features)
Test samples.
y : array-like, shape = (n_samples) or (n_samples, n_outputs)
True labels for X.
sample_weight : array-like, shape = [n_samples], optional
Sample weights.

Returns:
score : float
Mean accuracy of self.predict(X) wrt. y.

**set_params(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

**staged_decision_function**(X)

Compute decision function of X for each iteration.

This method allows monitoring (i.e. determine error on testing set) after each stage.

Parameters:

- **X**: array-like of shape = [n_samples, n_features]

  The input samples.

Returns:

- **score**: generator of array, shape = [n_samples, k]

  The decision function of the input samples. The order of the classes corresponds to that in the attribute `classes_`. Regression and binary classification are special cases with k == 1, otherwise k==n_classes.

**staged_predict**(X)

Predict class at each stage for X.

This method allows monitoring (i.e. determine error on testing set) after each stage.

Parameters:

- **X**: array-like of shape = [n_samples, n_features]

  The input samples.

Returns:

- **y**: generator of array of shape = [n_samples]

  The predicted value of the input samples.

**staged_predict_proba**(X)

Predict class probabilities at each stage for X.

This method allows monitoring (i.e. determine error on testing set) after each stage.

Parameters:

- **X**: array-like of shape = [n_samples, n_features]

  The input samples.

Returns:

- **y**: generator of array of shape = [n_samples]

  The predicted value of the input samples.

**transform**(**args**, **kwargs**)

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses coef_ or feature_importances_ to determine the most important features. For models with a coef_ for each class, the absolute sum over the classes is used.

Parameters:

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]

  The input samples.
**scikit-learn user guide, Release 0.18.2**

**threshold**: [string, float or None, optional (default=None)] The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

**Examples using sklearn.ensemble.GradientBoostingClassifier**

- Feature transformations with ensembles of trees
- Gradient Boosting Out-of-Bag estimates
- Gradient Boosting regularization

**sklearn.ensemble.GradientBoostingRegressor**

```python
class sklearn.ensemble.GradientBoostingRegressor(loss='ls', learning_rate=0.1, n_estimators=100, subsample=1.0, criterion='friedman_mse', min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_depth=3, min_impurity_split=1e-07, init=None, random_state=None, max_features=None, alpha=0.9, verbose=0, max_leaf_nodes=None, warm_start=False, presort='auto')
```

Gradient Boosting for regression.

GB builds an additive model in a forward stage-wise fashion; it allows for the optimization of arbitrary differentiable loss functions. In each stage a regression tree is fit on the negative gradient of the given loss function.

Read more in the User Guide.

**Parameters**

- **loss**: {'ls', 'lad', 'huber', 'quantile'}, optional (default='ls')
  loss function to be optimized. ‘ls’ refers to least squares regression. ‘lad’ (least absolute deviation) is a highly robust loss function solely based on order information of the input variables. ‘huber’ is a combination of the two. ‘quantile’ allows quantile regression (use alpha to specify the quantile).

- **learning_rate**: float, optional (default=0.1)
  learning rate shrinks the contribution of each tree by learning_rate. There is a trade-off between learning_rate and n_estimators.

- **n_estimators**: int (default=100)
  The number of boosting stages to perform. Gradient boosting is fairly robust to overfitting so a large number usually results in better performance.

- **max_depth**: integer, optional (default=3)
maximum depth of the individual regression estimators. The maximum depth limits the
number of nodes in the tree. Tune this parameter for best performance; the best value
depends on the interaction of the input variables.

criterion : string, optional (default="friedman_mse")
The function to measure the quality of a split. Supported criteria are “friedman_mse” for
the mean squared error with improvement score by Friedman, “mse” for mean squared
error, and “mae” for the mean absolute error. The default value of “friedman_mse” is
generally the best as it can provide a better approximation in some cases.

New in version 0.18.

min_samples_split : int, float, optional (default=2)
The minimum number of samples required to split an internal node:
• If int, then consider min_samples_split as the minimum number.
• If float, then min_samples_split is a percentage and ceil(min_samples_split * n_samples) are the minimum number of samples for each split.

Changed in version 0.18: Added float values for percentages.

min_samples_leaf : int, float, optional (default=1)
The minimum number of samples required to be at a leaf node:
• If int, then consider min_samples_leaf as the minimum number.
• If float, then min_samples_leaf is a percentage and ceil(min_samples_leaf * n_samples) are the minimum number of samples for each node.

Changed in version 0.18: Added float values for percentages.

min_weight_fraction_leaf : float, optional (default=0.0)
The minimum weighted fraction of the sum total of weights (of all the input samples)
required to be at a leaf node. Samples have equal weight when sample_weight is not
provided.

subsample : float, optional (default=1.0)
The fraction of samples to be used for fitting the individual base learners. If smaller
than 1.0 this results in Stochastic Gradient Boosting. subsample interacts with the pa-
rameter n_estimators. Choosing subsample < 1.0 leads to a reduction of variance and
an increase in bias.

max_features : int, float, string or None, optional (default=None)
The number of features to consider when looking for the best split:
• If int, then consider max_features features at each split.
• If float, then max_features is a percentage and int(max_features * n_features) features
are considered at each split.
• If “auto”, then max_features=n_features.
• If “sqrt”, then max_features=sqrt(n_features).
• If “log2”, then max_features=log2(n_features).
• If None, then max_features=n_features.
Choosing \texttt{max\_features < n\_features} leads to a reduction of variance and an increase in bias.

Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than \texttt{max\_features} features.

**max\_leaf\_nodes**: int or None, optional (default=None)

Grow trees with \texttt{max\_leaf\_nodes} in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

**min\_impurity\_split**: float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.

New in version 0.18.

**alpha**: float (default=0.9)

The alpha-quantile of the huber loss function and the quantile loss function. Only if \texttt{loss='huber'} or \texttt{loss='quantile'}.

**init**: BaseEstimator, None, optional (default=None)

An estimator object that is used to compute the initial predictions. \texttt{init} has to provide \texttt{fit} and \texttt{predict}. If None it uses \texttt{loss.init_estimator}.

**verbose**: int, default: 0

Enable verbose output. If 1 then it prints progress and performance once in a while (the more trees the lower the frequency). If greater than 1 then it prints progress and performance for every tree.

**warm\_start**: bool, default: False

When set to True, reuse the solution of the previous call to fit and add more estimators to the ensemble, otherwise, just erase the previous solution.

**random\_state**: int, RandomState instance or None, optional (default=None)

If int, random\_state is the seed used by the random number generator; If RandomState instance, random\_state is the random number generator; If None, the random number generator is the RandomState instance used by \texttt{np.random}.

**presort**: bool or ‘auto’, optional (default=’auto’)

Whether to presort the data to speed up the finding of best splits in fitting. Auto mode by default will use presorting on dense data and default to normal sorting on sparse data. Setting presort to true on sparse data will raise an error.

New in version 0.17: optional parameter \texttt{presort}.

**Attributes**

\texttt{feature\_importances\_: array, shape = [n\_features]}

The feature importances (the higher, the more important the feature).

\texttt{oob\_improvement\_: array, shape = [n\_estimators]}

The improvement in loss (= deviance) on the out-of-bag samples relative to the previous iteration. \texttt{oob\_improvement\_[0]} is the improvement in loss of the first stage over the \texttt{init} estimator.

\texttt{train\_score\_: array, shape = [n\_estimators]}

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The i-th score \texttt{train_score}[i] is the deviance (= loss) of the model at iteration i on the in-bag sample. If \texttt{subsample == 1} this is the deviance on the training data.

\textbf{loss_} : LossFunction

The concrete LossFunction object.

\textbf{‘init’} : BaseEstimator

The estimator that provides the initial predictions. Set via the \texttt{init} argument or \texttt{loss.init_estimator}.

\textbf{estimators_} : ndarray of DecisionTreeRegressor, shape = [n_estimators, 1]

The collection of fitted sub-estimators.

See also:

DecisionTreeRegressor, RandomForestRegressor

References


10. Friedman, Stochastic Gradient Boosting, 1999


Methods

\texttt{apply(X)}

Apply trees in the ensemble to X, return leaf indices.

\texttt{decision_function(X, y)}

DEPRECATED: and will be removed in 0.19

\texttt{fit(X, y)}

Fit the gradient boosting model.

\texttt{fit_transform(X, y)}

Fit to data, then transform it.

\texttt{get_params([deep])}

Get parameters for this estimator.

\texttt{predict(X)}

Predict regression target for X.

\texttt{score(X, y)}

Returns the coefficient of determination \(R^2\) of the prediction.

\texttt{set_params(**params)}

Set the parameters of this estimator.

\texttt{staged_decision_function(X, y)}

DEPRECATED: and will be removed in 0.19

\texttt{staged_predict(X)}

Predict regression target at each stage for X.

\texttt{transform(X, y)}

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.

__init__(loss='ls', learning_rate=0.1, n_estimators=100, subsample=1.0, criterion='friedman_mse', min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_depth=3, min_impurity_split=1e-07, init=None, random_state=None, max_features=None, alpha=0.9, verbose=0, max_leaf_nodes=None, warm_start=False, presort='auto')

\texttt{apply(X)}

Apply trees in the ensemble to X, return leaf indices.

New in version 0.17.
Parameters:

- **X**: array-like or sparse matrix, shape = [n_samples, n_features]
  
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted to a sparse csr_matrix.

Returns:

- **X_leaves**: array_like, shape = [n_samples, n_estimators]
  
  For each datapoint x in X and for each tree in the ensemble, return the index of the leaf x ends up in each estimator.

**decision_function**(*args, **kwargs)**

DEPRECATED: and will be removed in 0.19

Compute the decision function of X.

Parameters:

- **X**: array-like of shape = [n_samples, n_features]
  
  The input samples.

Returns:

- **score**: array, shape = [n_samples, n_classes] or [n_samples]
  
  The decision function of the input samples. The order of the classes corresponds to that in the attribute classes_. Regression and binary classification produce an array of shape [n_samples].

**feature_importances_**

Return the feature importances (the higher, the more important the feature).

Returns:

- **feature_importances_**: array, shape = [n_features]

**fit**(*X, y, sample_weight=None, monitor=None*)

Fit the gradient boosting model.

Parameters:

- **X**: array-like, shape = [n_samples, n_features]
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

- **y**: array-like, shape = [n_samples]
  
  Target values (integers in classification, real numbers in regression) For classification, labels must correspond to classes.

- **sample_weight**: array-like, shape = [n_samples] or None
  
  Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node. In the case of classification, splits are also ignored if they would result in any single class carrying a negative weight in either child node.

- **monitor**: callable, optional
  
  The monitor is called after each iteration with the current iteration, a reference to the estimator and the local variables of _fit_stages as keyword arguments callable(i, self, locals()). If the callable returns True the fitting procedure is stopped. The monitor can be used for various things such as computing held-out estimates, early stopping, model introspect, and snapshots.

Returns:

- **self**: object
  
  Returns self.
fit_transform \((X, y=None, **fit_params)\)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

\- **X**: numpy array of shape \([n_{samples}, n_{features}]\)
  - Training set.

\- **y**: numpy array of shape \([n_{samples}]\)
  - Target values.

Returns

\- **X_new**: numpy array of shape \([n_{samples}, n_{features_new}]\)
  - Transformed array.

get_params \((deep=True)\)

Get parameters for this estimator.

Parameters

\- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

\- **params**: mapping of string to any
  - Parameter names mapped to their values.

predict \((X)\)

Predict regression target for X.

Parameters

\- **X**: array-like of shape \([n_{samples}, n_{features}]\)
  - The input samples.

Returns

\- **y**: array of shape \([n_{samples}]\)
  - The predicted values.

score \((X, y, sample_weight=None)\)

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y_{true} - y_{pred})^2).sum()\) and \(v\) is the residual sum of squares \(((y_{true} - y_{true}.mean())^2).sum()\). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \(y\), disregarding the input features, would get a \(R^2\) score of 0.0.

Parameters

\- **X**: array-like, shape \((n_{samples}, n_{features})\)
  - Test samples.

\- **y**: array-like, shape \((n_{samples})\) or \((n_{samples}, n_{outputs})\)
  - True values for X.

\- **sample_weight**: array-like, shape \([n_{samples}]\), optional
  - Sample weights.

Returns

\- **score**: float
  - \(R^2\) of self.predict(X) wrt. \(y\).

set_params \((**params)\)

Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

**Return** self:

**staged_decision_function**(*args, **kwargs*)

DEPRECATED: and will be removed in 0.19

Compute decision function of X for each iteration.

This method allows monitoring (i.e. determine error on testing set) after each stage.

**Parameters**

X : array-like of shape = [n_samples, n_features]

The input samples.

**Returns**

cscore : generator of array, shape = [n_samples, k]

The decision function of the input samples. The order of the classes corresponds to that in the attribute classes_. Regression and binary classification are special cases with k == 1, otherwise k==n_classes.

**staged_predict**(X)

Predict regression target at each stage for X.

This method allows monitoring (i.e. determine error on testing set) after each stage.

**Parameters**

X : array-like of shape = [n_samples, n_features]

The input samples.

**Returns**

c : generator of array of shape = [n_samples]

The predicted value of the input samples.

**transform**(*args, **kwargs*)

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses coef_ or feature_importances_ to determine the most important features. For models with a coef_ for each class, the absolute sum over the classes is used.

**Parameters**

X : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold[string, float or None, optional (default=None)] The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

X_r : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.
Examples using `sklearn.ensemble.GradientBoostingRegressor`

- Model Complexity Influence
- Prediction Intervals for Gradient Boosting Regression
- Gradient Boosting regression
- Partial Dependence Plots

### 3.3.3 Model evaluation: quantifying the quality of predictions

There are 3 different approaches to evaluate the quality of predictions of a model:

- **Estimator score method**: Estimators have a `score` method providing a default evaluation criterion for the problem they are designed to solve. This is not discussed on this page, but in each estimator’s documentation.

- **Scoring parameter**: Model-evaluation tools using `cross-validation` (such as `model_selection.cross_val_score` and `model_selection.GridSearchCV`) rely on an internal `scoring` strategy. This is discussed in the section *The scoring parameter: defining model evaluation rules*.

- **Metric functions**: The `metrics` module implements functions assessing prediction error for specific purposes. These metrics are detailed in sections on *Classification metrics*, *Multilabel ranking metrics*, *Regression metrics* and *Clustering metrics*.

Finally, *Dummy estimators* are useful to get a baseline value of those metrics for random predictions.

See also:

For “pairwise” metrics, between samples and not estimators or predictions, see the *Pairwise metrics, Affinities and Kernels* section.

### The scoring parameter: defining model evaluation rules

Model selection and evaluation using tools, such as `model_selection.GridSearchCV` and `model_selection.cross_val_score`, take a `scoring` parameter that controls what metric they apply to the estimators evaluated.

### Common cases: predefined values

For the most common use cases, you can designate a scorer object with the `scoring` parameter; the table below shows all possible values. All scorer objects follow the convention that **higher return values are better than lower return values**. Thus metrics which measure the distance between the model and the data, like `metrics.mean_squared_error`, are available as `neg_mean_squared_error` which return the negated value of the metric.
## Scoring

<table>
<thead>
<tr>
<th>Scoring</th>
<th>Function</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Classification</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>‘accuracy’</td>
<td>metrics.accuracy_score</td>
<td></td>
</tr>
<tr>
<td>‘average_precision’</td>
<td>metrics.average_precision_score</td>
<td></td>
</tr>
<tr>
<td>‘f1’</td>
<td>metrics.f1_score</td>
<td>for binary targets</td>
</tr>
<tr>
<td>‘f1_micro’</td>
<td>metrics.f1_score</td>
<td>micro-averaged</td>
</tr>
<tr>
<td>‘f1_macro’</td>
<td>metrics.f1_score</td>
<td>macro-averaged</td>
</tr>
<tr>
<td>‘f1_weighted’</td>
<td>metrics.f1_score</td>
<td>weighted average</td>
</tr>
<tr>
<td>‘f1_samples’</td>
<td>metrics.f1_score</td>
<td>by multilabel sample</td>
</tr>
<tr>
<td>‘neg_log_loss’</td>
<td>metrics.log_loss</td>
<td>requires predict_proba support</td>
</tr>
<tr>
<td>‘precision’ etc.</td>
<td>metrics.precision_score</td>
<td>suffixes apply as with ‘f1’</td>
</tr>
<tr>
<td>‘recall’ etc.</td>
<td>metrics.recall_score</td>
<td>suffixes apply as with ‘f1’</td>
</tr>
<tr>
<td>‘roc_auc’</td>
<td>metrics.roc_auc_score</td>
<td></td>
</tr>
<tr>
<td><strong>Clustering</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>‘adjusted_rand_score’</td>
<td>metrics.adjusted_rand_score</td>
<td></td>
</tr>
<tr>
<td><strong>Regression</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>‘neg_mean_absolute_error’</td>
<td>metrics.mean_absolute_error</td>
<td></td>
</tr>
<tr>
<td>‘neg_mean_squared_error’</td>
<td>metrics.mean_squared_error</td>
<td></td>
</tr>
<tr>
<td>‘neg_median_absolute_error’</td>
<td>metrics.median_absolute_error</td>
<td></td>
</tr>
<tr>
<td>‘r2’</td>
<td>metrics.r2_score</td>
<td></td>
</tr>
</tbody>
</table>

### Usage examples:

```python
>>> from sklearn import svm, datasets
>>> from sklearn.model_selection import cross_val_score
>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target
>>> clf = svm.SVC(probability=True, random_state=0)
>>> cross_val_score(clf, X, y, scoring='neg_log_loss')
array([-0.07..., -0.16..., -0.06...])
>>> model = svm.SVC()
>>> cross_val_score(model, X, y, scoring='wrong_choice')
ValueError: 'wrong_choice' is not a valid scoring value. Valid options are ['accuracy', 'adjusted_rand_score', 'average_precision', 'f1', 'f1_macro', 'f1_micro', 'f1_samples', 'f1_weighted', 'neg_log_loss', 'neg_mean_absolute_error', 'neg_mean_squared_error', 'neg_median_absolute_error', 'precision', 'precision_macro', 'precision_micro', 'precision_samples', 'precision_weighted', 'r2', 'recall', 'recall_macro', 'recall_micro', 'recall_samples', 'recall_weighted', 'roc_auc']
```

### Note:

The values listed by the ValueError exception correspond to the functions measuring prediction accuracy described in the following sections. The scorer objects for those functions are stored in the dictionary `sklearn.metrics.SCORERS`.

### Defining your scoring strategy from metric functions

The module `sklearn.metric` also exposes a set of simple functions measuring a prediction error given ground truth and prediction:

- functions ending with `_score` return a value to maximize, the higher the better.
- functions ending with `_error` or `_loss` return a value to minimize, the lower the better. When converting into
a scorer object using `make_scorer`, set the `greater_is_better` parameter to False (True by default; see the parameter description below).

Metrics available for various machine learning tasks are detailed in sections below.

Many metrics are not given names to be used as `scoring` values, sometimes because they require additional parameters, such as `fbeta_score`. In such cases, you need to generate an appropriate scoring object. The simplest way to generate a callable object for scoring is by using `make_scorer`. That function converts metrics into callables that can be used for model evaluation.

One typical use case is to wrap an existing metric function from the library with non-default values for its parameters, such as the `beta` parameter for the `fbeta_score` function:

```python
>>> from sklearn.metrics import fbeta_score, make_scorer
>>> ftwo_scorer = make_scorer(fbeta_score, beta=2)
>>> from sklearn.model_selection import GridSearchCV
>>> from sklearn.svm import LinearSVC
>>> grid = GridSearchCV(LinearSVC(), param_grid={'C': [1, 10]}, scoring=ftwo_scorer)
```

The second use case is to build a completely custom scorer object from a simple python function using `make_scorer`, which can take several parameters:

- the python function you want to use (`my_custom_loss_func` in the example below)
- whether the python function returns a score (`greater_is_better=True`, the default) or a loss (`greater_is_better=False`). If a loss, the output of the python function is negated by the scorer object, conforming to the cross validation convention that scorers return higher values for better models.
- for classification metrics only: whether the python function you provided requires continuous decision certainties (`needs_threshold=True`). The default value is False.
- any additional parameters, such as `beta` or `labels` in `f1_score`.

Here is an example of building custom scorers, and of using the `greater_is_better` parameter:

```python
>>> import numpy as np
>>> def my_custom_loss_func(ground_truth, predictions):
...     diff = np.abs(ground_truth - predictions).max()
...     return np.log(1 + diff)
...
>>> # loss_func will negate the return value of my_custom_loss_func,
>>> # which will be np.log(2), 0.693, given the values for ground_truth
>>> # and predictions defined below.
>>> loss = make_scorer(my_custom_loss_func, greater_is_better=False)
>>> score = make_scorer(my_custom_loss_func, greater_is_better=True)
>>> ground_truth = [[1, 1]]
>>> predictions = [0, 1]
>>> from sklearn.dummy import DummyClassifier
>>> clf = DummyClassifier(strategy='most_frequent', random_state=0)
>>> clf = clf.fit(ground_truth, predictions)
>>> loss(clf, ground_truth, predictions)
-0.69...
>>> score(clf, ground_truth, predictions)
0.69...
```

### Implementing your own scoring object

You can generate even more flexible model scorers by constructing your own scoring object from scratch, without using the `make_scorer` factory. For a callable to be a scorer, it needs to meet the protocol specified by the following two
rules:

- It can be called with parameters `(estimator, X, y)`, where `estimator` is the model that should be evaluated, `X` is validation data, and `y` is the ground truth target for `X` (in the supervised case) or `None` (in the unsupervised case).
- It returns a floating point number that quantifies the `estimator` prediction quality on `X`, with reference to `y`. Again, by convention higher numbers are better, so if your scorer returns loss, that value should be negated.

### Classification metrics

The `sklearn.metrics` module implements several loss, score, and utility functions to measure classification performance. Some metrics might require probability estimates of the positive class, confidence values, or binary decisions values. Most implementations allow each sample to provide a weighted contribution to the overall score, through the `sample_weight` parameter.

Some of these are restricted to the binary classification case:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>matthews_corrcoef</code></td>
<td>Compute the Matthews correlation coefficient (MCC) for binary classes</td>
</tr>
<tr>
<td><code>precision_recall_curve</code></td>
<td>Compute precision-recall pairs for different probability thresholds</td>
</tr>
<tr>
<td><code>roc_curve</code></td>
<td>Compute Receiver operating characteristic (ROC)</td>
</tr>
</tbody>
</table>

Others also work in the multiclass case:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>cohen_kappa_score</code></td>
<td>Cohen’s kappa: a statistic that measures inter-annotator agreement.</td>
</tr>
<tr>
<td><code>confusion_matrix</code></td>
<td>Compute confusion matrix to evaluate the accuracy of a classification</td>
</tr>
<tr>
<td><code>hinge_loss</code></td>
<td>Average hinge loss (non-regularized)</td>
</tr>
</tbody>
</table>

Some also work in the multilabel case:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>accuracy_score</code></td>
<td>Accuracy classification score.</td>
</tr>
<tr>
<td><code>classification_report</code></td>
<td>Build a text report showing the main classification metrics</td>
</tr>
<tr>
<td><code>f1_score</code></td>
<td>Compute the F1 score, also known as balanced F-score or F-measure</td>
</tr>
<tr>
<td><code>fbeta_score</code></td>
<td>Compute the F-beta score</td>
</tr>
<tr>
<td><code>hamming_loss</code></td>
<td>Compute the average Hamming loss.</td>
</tr>
<tr>
<td><code>jaccard_similarity_score</code></td>
<td>Jaccard similarity coefficient score</td>
</tr>
<tr>
<td><code>log_loss</code></td>
<td>Log loss, aka logistic loss or cross-entropy loss.</td>
</tr>
<tr>
<td><code>precision_recall_fscore_support</code></td>
<td>Compute precision, recall, F-measure and support for each class</td>
</tr>
<tr>
<td><code>precision_score</code></td>
<td>Compute the precision</td>
</tr>
<tr>
<td><code>recall_score</code></td>
<td>Compute the recall</td>
</tr>
<tr>
<td><code>zero_one_loss</code></td>
<td>Zero-one classification loss.</td>
</tr>
</tbody>
</table>

And some work with binary and multilabel (but not multiclass) problems:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>average_precision_score</code></td>
<td>Compute average precision (AP) from prediction scores</td>
</tr>
</tbody>
</table>
In the following sub-sections, we will describe each of those functions, preceded by some notes on common API and metric definition.

From binary to multiclass and multilabel

Some metrics are essentially defined for binary classification tasks (e.g. f1_score, roc_auc_score). In these cases, by default only the positive label is evaluated, assuming by default that the positive class is labelled 1 (though this may be configurable through the pos_label parameter). In extending a binary metric to multiclass or multilabel problems, the data is treated as a collection of binary problems, one for each class. There are then a number of ways to average binary metric calculations across the set of classes, each of which may be useful in some scenario. Where available, you should select among these using the average parameter.

- "macro" simply calculates the mean of the binary metrics, giving equal weight to each class. In problems where infrequent classes are nonetheless important, macro-averaging may be a means of highlighting their performance. On the other hand, the assumption that all classes are equally important is often untrue, such that macro-averaging will over-emphasize the typically low performance on an infrequent class.

- "weighted" accounts for class imbalance by computing the average of binary metrics in which each class’s score is weighted by its presence in the true data sample.

- "micro" gives each sample-class pair an equal contribution to the overall metric (except as a result of sample-weight). Rather than summing the metric per class, this sums the dividends and divisors that make up the per-class metrics to calculate an overall quotient. Micro-averaging may be preferred in multilabel settings, including multiclass classification where a majority class is to be ignored.

- "samples" applies only to multilabel problems. It does not calculate a per-class measure, instead calculating the metric over the true and predicted classes for each sample in the evaluation data, and returning their (sample_weight-weighted) average.

- Selecting average=None will return an array with the score for each class.

While multiclass data is provided to the metric, like binary targets, as an array of class labels, multilabel data is specified as an indicator matrix, in which cell [i, j] has value 1 if sample i has label j and value 0 otherwise.

Accuracy score

The accuracy_score function computes the accuracy, either the fraction (default) or the count (normalize=False) of correct predictions.

In multilabel classification, the function returns the subset accuracy. If the entire set of predicted labels for a sample strictly match with the true set of labels, then the subset accuracy is 1.0; otherwise it is 0.0.

If \( \hat{y}_i \) is the predicted value of the \( i \)-th sample and \( y_i \) is the corresponding true value, then the fraction of correct predictions over \( n_{\text{samples}} \) is defined as

\[
\text{accuracy}(y, \hat{y}) = \frac{1}{n_{\text{samples}}} \sum_{i=0}^{n_{\text{samples}}-1} 1(\hat{y}_i = y_i)
\]

where \( 1(x) \) is the indicator function.
```python
>>> import numpy as np
data
>>> from sklearn.metrics import accuracy_score
data
>>> y_pred = [0, 2, 1, 3]
data
>>> y_true = [0, 1, 2, 3]
data
>>> accuracy_score(y_true, y_pred)
data
0.5
data
>>> accuracy_score(y_true, y_pred, normalize=False)
data
2
```
Here is a visual representation of such a confusion matrix (this figure comes from the *Confusion matrix* example):

![Confusion matrix image](image)

get counts of true negatives, false positives, false negatives and true positives as follows:

```python
>>> y_true = [0, 0, 0, 1, 1, 1, 1, 1]
>>> y_pred = [0, 1, 0, 1, 0, 1, 0, 1]
>>> tn, fp, fn, tp = confusion_matrix(y_true, y_pred).ravel()
>>> tn, fp, fn, tp
(2, 1, 2, 3)
```

Example:

- See *Confusion matrix* for an example of using a confusion matrix to evaluate classifier output quality.
- See *Recognizing hand-written digits* for an example of using a confusion matrix to classify hand-written digits.
- See *Classification of text documents using sparse features* for an example of using a confusion matrix to classify text documents.

Classification report

The `classification_report` function builds a text report showing the main classification metrics. Here is a small example with custom `target_names` and inferred labels:

```python
>>> from sklearn.metrics import classification_report
>>> y_true = [0, 1, 2, 2, 0]
>>> y_pred = [0, 0, 2, 1, 0]
>>> target_names = ['class 0', 'class 1', 'class 2']
>>> print(classification_report(y_true, y_pred, target_names=target_names))
    precision    recall  f1-score   support

    class 0       1.00      1.00    1.00         2
    class 1       0.80      0.80    0.80         2
    class 2       1.00      1.00    1.00         3

    accuracy                           1.00         7
    macro avg       1.00      1.00    1.00         7
    weighted avg       1.00      1.00    1.00         7
```

3.3. Model selection and evaluation
Hamming loss

The `hamming_loss` computes the average Hamming loss or Hamming distance between two sets of samples. If $\hat{y}_j$ is the predicted value for the $j$-th label of a given sample, $y_j$ is the corresponding true value, and $n_{\text{labels}}$ is the number of classes or labels, then the Hamming loss $L_{\text{Hamming}}$ between two samples is defined as:

$$L_{\text{Hamming}}(y, \hat{y}) = \frac{1}{n_{\text{labels}}} \sum_{j=0}^{n_{\text{labels}}-1} 1(\hat{y}_j \neq y_j)$$

where $1(x)$ is the indicator function.

```python
>>> from sklearn.metrics import hamming_loss
>>> y_pred = [1, 2, 3, 4]
>>> y_true = [2, 2, 3, 4]
>>> hamming_loss(y_true, y_pred)
0.25
```

In the multilabel case with binary label indicators:

```python
>>> hamming_loss(np.array([[0, 1], [1, 1]]), np.zeros((2, 2)))
0.75
```

**Note:** In multiclass classification, the Hamming loss corresponds to the Hamming distance between $y_{\text{true}}$ and $y_{\text{pred}}$ which is similar to the Zero one loss function. However, while zero-one loss penalizes prediction sets that do not strictly match true sets, the Hamming loss penalizes individual labels. Thus the Hamming loss, upper bounded by the zero-one loss, is always between zero and one, inclusive; and predicting a proper subset or superset of the true labels will give a Hamming loss between zero and one, exclusive.

Jaccard similarity coefficient score

The `jaccard_similarity_score` function computes the average (default) or sum of Jaccard similarity coefficients, also called the Jaccard index, between pairs of label sets.
The Jaccard similarity coefficient of the $i$-th samples, with a ground truth label set $y_i$ and predicted label set $\hat{y}_i$, is defined as

$$J(y_i, \hat{y}_i) = \frac{|y_i \cap \hat{y}_i|}{|y_i \cup \hat{y}_i|}.$$ 

In binary and multiclass classification, the Jaccard similarity coefficient score is equal to the classification accuracy.

```python
>>> import numpy as np
>>> from sklearn.metrics import jaccard_similarity_score
>>> y_pred = [0, 2, 1, 3]
>>> y_true = [0, 1, 2, 3]
>>> jaccard_similarity_score(y_true, y_pred)
0.5
>>> jaccard_similarity_score(y_true, y_pred, normalize=False)
2
```

In the multilabel case with binary label indicators:

```python
>>> jaccard_similarity_score(np.array([[0, 1], [1, 1]]), np.ones((2, 2)))
0.75
```

### Precision, recall and F-measures

Intuitively, precision is the ability of the classifier not to label as positive a sample that is negative, and recall is the ability of the classifier to find all the positive samples.

The F-measure ($F_\beta$ and $F_1$ measures) can be interpreted as a weighted harmonic mean of the precision and recall. A $F_\beta$ measure reaches its best value at 1 and its worst score at 0. With $\beta = 1$, $F_\beta$ and $F_1$ are equivalent, and the recall and the precision are equally important.

The `precision_recall_curve` function computes a precision-recall curve from the ground truth label and a score given by the classifier by varying a decision threshold.

The `average_precision_score` function computes the average precision (AP) from prediction scores. This score corresponds to the area under the precision-recall curve. The value is between 0 and 1 and higher is better. With random predictions, the AP is the fraction of positive samples.

Several functions allow you to analyze the precision, recall and F-measures score:

- `average_precision_score(y_true, y_score[, ...])`: Compute average precision (AP) from prediction scores
- `f1_score(y_true, y_pred[, labels, ...])`: Compute the F1 score, also known as balanced F-score or F-measure
- `fbeta_score(y_true, y_pred, beta[, labels, ...])`: Compute the F-beta score
- `precision_recall_curve(y_true, probas_pred)`: Compute precision-recall pairs for different probability thresholds
- `precision_recall_fscore_support(y_true, y_pred)`: Compute precision, recall, F-measure and support for each class
- `precision_score(y_true, y_pred[, labels, ...])`: Compute the precision
- `recall_score(y_true, y_pred[, labels, ...])`: Compute the recall

Note that the `precision_recall_curve` function is restricted to the binary case. The `average_precision_score` function works only in binary classification and multilabel indicator format.
Examples:

- See Classification of text documents using sparse features for an example of \texttt{f1_score} usage to classify text documents.
- See Parameter estimation using grid search with cross-validation for an example of \texttt{precision_score} and \texttt{recall_score} usage to estimate parameters using grid search with nested cross-validation.
- See Precision-Recall for an example of \texttt{precision_recall_curve} usage to evaluate classifier output quality.
- See Sparse recovery: feature selection for sparse linear models for an example of \texttt{precision_recall_curve} usage to select features for sparse linear models.

Binary classification

In a binary classification task, the terms “positive” and “negative” refer to the classifier’s prediction, and the terms “true” and “false” refer to whether that prediction corresponds to the external judgment (sometimes known as the “observation”). Given these definitions, we can formulate the following table:

<table>
<thead>
<tr>
<th>Actual class (observation)</th>
<th>Predicted class (expectation)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tp (true positive) Correct result</td>
</tr>
<tr>
<td></td>
<td>fn (false negative) Missing result</td>
</tr>
</tbody>
</table>

In this context, we can define the notions of precision, recall and F-measure:

\[
\text{precision} = \frac{tp}{tp + fp},
\]

\[
\text{recall} = \frac{tp}{tp + fn},
\]

\[
F_\beta = (1 + \beta^2) \frac{\text{precision} \times \text{recall}}{\beta^2 \text{precision} + \text{recall}}.
\]

Here are some small examples in binary classification:

```python
>>> from sklearn import metrics
>>> y_pred = [0, 1, 0, 0]
>>> y_true = [0, 1, 0, 1]
>>> metrics.precision_score(y_true, y_pred)
1.0
>>> metrics.recall_score(y_true, y_pred)
0.5
>>> metrics.f1_score(y_true, y_pred)
0.66...
>>> metrics.fbeta_score(y_true, y_pred, beta=0.5)
0.83...
>>> metrics.fbeta_score(y_true, y_pred, beta=1)
0.66...
>>> metrics.fbeta_score(y_true, y_pred, beta=2)
0.55...
>>> metrics.precision_recall_fscore_support(y_true, y_pred, beta=0.5)
(array([ 0.66...,  1.]), array([ 1. ,  0.5]), array([ 0.71...,  0.83...]), array([2, 2]))
```

```python
>>> import numpy as np
```
In multiclass and multilabel classification task, the notions of precision, recall, and F-measures can be applied to each label independently. There are a few ways to combine results across labels, specified by the `average` argument to the `average_precision_score` (multilabel only), `f1_score`, `fbeta_score`, `precision_recall_fscore_support`, `precision_score`, and `recall_score` functions, as described above. Note that for “micro”-averaging in a multiclass setting with all labels included will produce equal precision, recall and $F$, while “weighted” averaging may produce an F-score that is not between precision and recall.

To make this more explicit, consider the following notation:

- $y$ the set of predicted (sample, label) pairs
- $\hat{y}$ the set of true (sample, label) pairs
- $L$ the set of labels
- $S$ the set of samples
- $y_s$ the subset of $y$ with sample $s$, i.e. $y_s := \{(s',l) \in y | s' = s\}$
- $y_l$ the subset of $y$ with label $l$
- similarly, $\hat{y}_s$ and $\hat{y}_l$ are subsets of $\hat{y}$
- $P(A,B) := \frac{|A \cap B|}{|A|}$
- $R(A,B) := \frac{|A \cap B|}{|B|}$ (Conventions vary on handling $B = \emptyset$; this implementation uses $R(A, B) := 0$, and similar for $P$.)
- $F_\beta(A,B) := (1 + \beta^2) \frac{P(A,B) \times R(A,B)}{\beta^2 P(A,B) + R(A,B)}$

Then the metrics are defined as:

<table>
<thead>
<tr>
<th>average</th>
<th>Precision</th>
<th>Recall</th>
<th>$F$-beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;micro&quot;</td>
<td>$P(y, \hat{y})$</td>
<td>$R(y, \hat{y})$</td>
<td>$F_\beta(y, \hat{y})$</td>
</tr>
<tr>
<td>&quot;samples&quot;</td>
<td>$\frac{1}{</td>
<td>S</td>
<td>} \sum_{s \in S} P(y_s, \hat{y}_s)$</td>
</tr>
<tr>
<td>&quot;macro&quot;</td>
<td>$\frac{1}{</td>
<td>L</td>
<td>} \sum_{l \in L} P(y_l, \hat{y}_l)$</td>
</tr>
<tr>
<td>&quot;weighted&quot;</td>
<td>$\frac{1}{</td>
<td>L</td>
<td>} \sum_{l \in L}</td>
</tr>
<tr>
<td>None</td>
<td>$P(y_l, \hat{y}_l)</td>
<td>l \in L }$</td>
<td>$R(y_l, \hat{y}_l)</td>
</tr>
</tbody>
</table>

```python
>>> from sklearn.metrics import precision_recall_curve
>>> from sklearn.metrics import average_precision_score
>>> y_true = np.array([0, 0, 1, 1])
>>> y_scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> precision, recall, threshold = precision_recall_curve(y_true, y_scores)
>>> precision
array([ 0.66..., 0.5 , 1. , 1. ])
>>> recall
array([ 1. , 0.5, 0.5, 0. ])
>>> threshold
array([ 0.35, 0.4 , 0.8 ])
>>> average_precision_score(y_true, y_scores)
0.79...
```
0.22...
>>> metrics.recall_score(y_true, y_pred, average='micro')
...
0.33...
>>> metrics.f1_score(y_true, y_pred, average='weighted')
0.26...
>>> metrics.fbeta_score(y_true, y_pred, average='macro', beta=0.5)
0.23...
>>> metrics.precision_recall_fscore_support(y_true, y_pred, beta=0.5, average=None)
...
(array([ 0.66..., 0. , 0. ]), array([ 1. , 0. , 0. ]), array([ 0.71..., 0. , 0. ]), array([2, 2, 2]))

For multiclass classification with a “negative class”, it is possible to exclude some labels:

>>> metrics.recall_score(y_true, y_pred, labels=[1, 2], average='micro')
...  # excluding 0, no labels were correctly recalled
0.0

Similarly, labels not present in the data sample may be accounted for in macro-averaging.

>>> metrics.precision_score(y_true, y_pred, labels=[0, 1, 2, 3], average='macro')
...
0.166...

**Hinge loss**

The `hinge_loss` function computes the average distance between the model and the data using hinge loss, a one-sided metric that considers only prediction errors. (Hinge loss is used in maximal margin classifiers such as support vector machines.)

If the labels are encoded with +1 and -1, \( y \) is the true value, and \( w \) is the predicted decisions as output by `decision_function`, then the hinge loss is defined as:

\[
    L_{\text{Hinge}}(y, w) = \max\{1 - wy, 0\} = |1 - wy|_+
\]

If there are more than two labels, `hinge_loss` uses a multiclass variant due to Crammer & Singer. Here is the paper describing it.

If \( y_w \) is the predicted decision for true label and \( y_t \) is the maximum of the predicted decisions for all other labels, where predicted decisions are output by decision function, then multiclass hinge loss is defined by:

\[
    L_{\text{Hinge}}(y_w, y_t) = \max\{1 + y_t - y_w, 0\}
\]

Here a small example demonstrating the use of the `hinge_loss` function with a svm classifier in a binary class problem:

```python
>>> from sklearn import svm
>>> from sklearn.metrics import hinge_loss
>>> X = [[0], [1]]
>>> y = [-1, 1]
>>> est = svm.LinearSVC(random_state=0)
>>> est.fit(X, y)
```

LinearSVC(C=1.0, class_weight=None, dual=True, fit_intercept=True, intercept_scaling=1, loss='squared_hinge', max_iter=1000, multi_class='ovr', penalty='l2', random_state=0, tol=0.0001,
Here is an example demonstrating the use of the `hinge_loss` function with a SVM classifier in a multiclass problem:

```python
>>> X = np.array([[0], [1], [2], [3]])
>>> Y = np.array([0, 1, 2, 3])
>>> labels = np.array([0, 1, 2, 3])
>>> est = svm.LinearSVC()
>>> est.fit(X, Y)
LinearSVC(C=1.0, class_weight=None, dual=True, fit_intercept=True,
        intercept_scaling=1, loss='squared_hinge', max_iter=1000,
        multi_class='ovr', penalty='l2', random_state=None, tol=0.0001,
        verbose=0)
>>> pred_decision = est.decision_function([[[-1], [2], [3]]])
>>> y_true = [0, 2, 3]
>>> hinge_loss(y_true, pred_decision, labels)
0.56...
```

**Log loss**

Log loss, also called logistic regression loss or cross-entropy loss, is defined on probability estimates. It is commonly used in (multinomial) logistic regression and neural networks, as well as in some variants of expectation-maximization, and can be used to evaluate the probability outputs (`predict_proba`) of a classifier instead of its discrete predictions.

For binary classification with a true label \( y \in \{0, 1\} \) and a probability estimate \( p = \Pr(y = 1) \), the log loss per sample is the negative log-likelihood of the classifier given the true label:

\[
L_{\text{log}}(y, p) = - \log \Pr(y|p) = -(y \log(p) + (1 - y) \log(1 - p))
\]

This extends to the multiclass case as follows. Let the true labels for a set of samples be encoded as a 1-of-K binary indicator matrix \( Y \), i.e., \( y_{i,k} = 1 \) if sample \( i \) has label \( k \) taken from a set of \( K \) labels. Let \( P \) be a matrix of probability estimates, with \( p_{i,k} = \Pr(t_{i,k} = 1) \). Then the log loss of the whole set is

\[
L_{\text{log}}(Y, P) = - \log \Pr(Y|P) = -\frac{1}{N} \sum_{i=0}^{N-1} \sum_{k=0}^{K-1} y_{i,k} \log p_{i,k}
\]

To see how this generalizes the binary log loss given above, note that in the binary case, \( p_{i,0} = 1 - p_{i,1} \) and \( y_{i,0} = 1 - y_{i,1} \), so expanding the inner sum over \( y_{i,k} \in \{0, 1\} \) gives the binary log loss.

The `log_loss` function computes log loss given a list of ground-truth labels and a probability matrix, as returned by an estimator’s `predict_proba` method.

```python
>>> from sklearn.metrics import log_loss
>>> y_true = [0, 0, 1, 1]
>>> y_pred = [[.9, .1], [.8, .2], [.3, .7], [.01, .99]]
>>> log_loss(y_true, y_pred)
0.1738...
```

The first [.9, .1] in `y_pred` denotes 90% probability that the first sample has label 0. The log loss is non-negative.
Matthews correlation coefficient

The \texttt{matthews_corrcoef} function computes the Matthew's correlation coefficient (MCC) for binary classes. Quoting Wikipedia:

“The Matthews correlation coefficient is used in machine learning as a measure of the quality of binary (two-class) classifications. It takes into account true and false positives and negatives and is generally regarded as a balanced measure which can be used even if the classes are of very different sizes. The MCC is in essence a correlation coefficient value between -1 and +1. A coefficient of +1 represents a perfect prediction, 0 an average random prediction and -1 an inverse prediction. The statistic is also known as the phi coefficient.”

If $tp$, $tn$, $fp$ and $fn$ are respectively the number of true positives, true negatives, false positives and false negatives, the MCC coefficient is defined as

$$ MCC = \frac{tp \times tn - fp \times fn}{\sqrt{(tp + fp)(tp + fn)(tn + fp)(tn + fn)}}. $$

Here is a small example illustrating the usage of the \texttt{matthews_corrcoef} function:

```python
>>> from sklearn.metrics import matthews_corrcoef
>>> y_true = [+1, +1, +1, -1]
>>> y_pred = [+1, -1, +1, +1]
>>> matthews_corrcoef(y_true, y_pred)
-0.33...
```

Receiver operating characteristic (ROC)

The function \texttt{roc_curve} computes the receiver operating characteristic curve, or ROC curve. Quoting Wikipedia:

“A receiver operating characteristic (ROC), or simply ROC curve, is a graphical plot which illustrates the performance of a binary classifier system as its discrimination threshold is varied. It is created by plotting the fraction of true positives out of the positives (TPR = true positive rate) vs. the fraction of false positives out of the negatives (FPR = false positive rate), at various threshold settings. TPR is also known as sensitivity, and FPR is one minus the specificity or true negative rate.”

This function requires the true binary value and the target scores, which can either be probability estimates of the positive class, confidence values, or binary decisions. Here is a small example of how to use the \texttt{roc_curve} function:

```python
>>> import numpy as np
>>> from sklearn.metrics import roc_curve
>>> y = np.array([1, 1, 2, 2])
>>> scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> fpr, tpr, thresholds = roc_curve(y, scores, pos_label=2)
>>> fpr
array([ 0. , 0.5, 0.5, 1. ])
>>> tpr
array([ 0.5, 0.5, 1. , 1. ])
>>> thresholds
array([ 0.8 , 0.4 , 0.35, 0.1 ])```
This figure shows an example of such an ROC curve:
The \texttt{roc_auc_score} function computes the area under the receiver operating characteristic (ROC) curve, which is also denoted by AUC or AUROC. By computing the area under the roc curve, the curve information is summarized in one number. For more information see the Wikipedia article on AUC.

```python
>>> import numpy as np
>>> from sklearn.metrics import roc_auc_score

>>> y_true = np.array([0, 0, 1, 1])
>>> y_scores = np.array([0.1, 0.4, 0.35, 0.8])

>>> roc_auc_score(y_true, y_scores)
0.75
```

In multi-label classification, the \texttt{roc_auc_score} function is extended by averaging over the labels as above.

Compared to metrics such as the subset accuracy, the Hamming loss, or the F1 score, ROC doesn’t require optimizing a threshold for each label. The \texttt{roc_auc_score} function can also be used in multi-class classification, if the predicted
outputs have been binarized.

Examples:

- See Receiver Operating Characteristic (ROC) for an example of using ROC to evaluate the quality of the output of a classifier.
- See Receiver Operating Characteristic (ROC) with cross validation for an example of using ROC to evaluate classifier output quality, using cross-validation.
- See Species distribution modeling for an example of using ROC to model species distribution.

Zero one loss

The zero_one_loss function computes the sum or the average of the 0-1 classification loss ($L_{0-1}$) over $n_{\text{samples}}$. By default, the function normalizes over the sample. To get the sum of the $L_{0-1}$, set normalize to False.

In multilabel classification, the zero_one_loss scores a subset as one if its labels strictly match the predictions, and as a zero if there are any errors. By default, the function returns the percentage of imperfectly predicted subsets. To get the count of such subsets instead, set normalize to False.

If $\hat{y}_i$ is the predicted value of the $i$-th sample and $y_i$ is the corresponding true value, then the 0-1 loss $L_{0-1}$ is defined as:

$$L_{0-1}(y_i, \hat{y}_i) = 1(\hat{y}_i \neq y_i)$$

where $1(x)$ is the indicator function.

```python
gt from sklearn.metrics import zero_one_loss
gt y_pred = [1, 2, 3, 4]  
gt y_true = [2, 2, 3, 4]  
gt zero_one_loss(y_true, y_pred)  
0.25```
zero_one_loss(y_true, y_pred, normalize=False)

1

In the multilabel case with binary label indicators, where the first label set [0,1] has an error:

zero_one_loss(np.array([[0, 1], [1, 1]]), np.ones((2, 2)))
0.5

zero_one_loss(np.array([[0, 1], [1, 1]]), np.ones((2, 2)), normalize=False)
1

Example:

- See Recursive feature elimination with cross-validation for an example of zero one loss usage to perform recursive feature elimination with cross-validation.

Brier score loss

The brier_score_loss function computes the Brier score for binary classes. Quoting Wikipedia:

“The Brier score is a proper score function that measures the accuracy of probabilistic predictions. It is applicable to tasks in which predictions must assign probabilities to a set of mutually exclusive discrete outcomes.”

This function returns a score of the mean square difference between the actual outcome and the predicted probability of the possible outcome. The actual outcome has to be 1 or 0 (true or false), while the predicted probability of the actual outcome can be a value between 0 and 1.

The brier score loss is also between 0 to 1 and the lower the score (the mean square difference is smaller), the more accurate the prediction is. It can be thought of as a measure of the “calibration” of a set of probabilistic predictions.

\[
BS = \frac{1}{N} \sum_{t=1}^{N} (f_t - o_t)^2
\]

where : $N$ is the total number of predictions, $f_t$ is the predicted probability of the actual outcome $o_t$.

Here is a small example of usage of this function:

```python
>>> import numpy as np
>>> from sklearn.metrics import brier_score_loss
>>> y_true = np.array([0, 1, 1, 0])
>>> y_true_categorical = np.array(["spam", "ham", "ham", "spam"])
>>> y_prob = np.array([0.1, 0.9, 0.8, 0.4])
>>> y_pred = np.array([0, 1, 1, 0])
>>> brier_score_loss(y_true, y_prob)
0.055
>>> brier_score_loss(y_true, 1-y_prob, pos_label=0)
0.055
>>> brier_score_loss(y_true_categorical, y_prob, pos_label="ham")
0.055
>>> brier_score_loss(y_true, y_prob > 0.5)
0.0
```
Example:

- See *Probability calibration of classifiers* for an example of Brier score loss usage to perform probability calibration of classifiers.

References:

- G. Brier, *Verification of forecasts expressed in terms of probability*, Monthly weather review 78.1 (1950)

Multilabel ranking metrics

In multilabel learning, each sample can have any number of ground truth labels associated with it. The goal is to give high scores and better rank to the ground truth labels.

Coverage error

The `coverage_error` function computes the average number of labels that have to be included in the final prediction such that all true labels are predicted. This is useful if you want to know how many top-scored-labels you have to predict in average without missing any true one. The best value of this metrics is thus the average number of true labels.

Formally, given a binary indicator matrix of the ground truth labels \( y \in \{0, 1\}^{n_{\text{samples}} \times n_{\text{labels}}} \) and the score associated with each label \( \hat{f} \in \mathbb{R}^{n_{\text{samples}} \times n_{\text{labels}}} \), the coverage is defined as

\[
\text{coverage}(y, \hat{f}) = \frac{1}{n_{\text{samples}}} \sum_{i=0}^{n_{\text{samples}}-1} \max_{j : y_{ij} = 1} \text{rank}_{ij}
\]

with \( \text{rank}_{ij} = \left| \left\{ k : \hat{f}_{ik} \geq \hat{f}_{ij} \right\} \right| \). Given the rank definition, ties in \( y_{\text{scores}} \) are broken by giving the maximal rank that would have been assigned to all tied values.

Here is a small example of usage of this function:

```python
>>> import numpy as np
>>> from sklearn.metrics import coverage_error
>>> y_true = np.array([[1, 0, 0], [0, 0, 1]])
>>> y_score = np.array([[0.75, 0.5, 1], [1, 0.2, 0.1]])
>>> coverage_error(y_true, y_score)
2.5
```

Label ranking average precision

The `label_ranking_average_precision_score` function implements label ranking average precision (LRAP). This metric is linked to the `average_precision_score` function, but is based on the notion of label ranking instead of precision and recall.

Label ranking average precision (LRAP) is the average over each ground truth label assigned to each sample, of the ratio of true vs. total labels with lower score. This metric will yield better scores if you are able to give better rank to the labels associated with each sample. The obtained score is always strictly greater than 0, and the best value is 1. If there is exactly one relevant label per sample, label ranking average precision is equivalent to the mean reciprocal rank.
Formally, given a binary indicator matrix of the ground truth labels \( y \in \mathcal{R}^{n_{\text{samples}} \times n_{\text{labels}}} \) and the score associated with each label \( f \in \mathcal{R}^{n_{\text{samples}} \times n_{\text{labels}}} \), the average precision is defined as

\[
LRAP(y, f) = \frac{1}{n_{\text{samples}}} \sum_{i=0}^{n_{\text{samples}}-1} \frac{1}{|y_i|} \sum_{j : y_{ij} = 1} |\mathcal{L}_{ij}| \cdot \text{rank}_{ij}
\]

with \( \mathcal{L}_{ij} = \{k : y_{ik} = 1, f_{ik} \geq f_{ij}\} \), \( \text{rank}_{ij} = |\{k : f_{ik} \geq f_{ij}\}| \) and \(|\cdot|\) is the \( l_0 \) norm or the cardinality of the set.

Here is a small example of usage of this function:

```python
code_snippet
```

**Regression metrics**

The `sklearn.metrics` module implements several loss, score, and utility functions to measure regression performance. Some of those have been enhanced to handle the multioutput case: `mean_squared_error`, `mean_absolute_error`, `explained_variance_score` and `r2_score`.

These functions have an `multioutput` keyword argument which specifies the way the scores or losses for each individual target should be averaged. The default is `'uniform_average'`, which specifies a uniformly weighted mean over outputs. If an `ndarray` of shape `(n_outputs,)` is passed, then its entries are interpreted as weights.
and an according weighted average is returned. If `multioutput='raw_values'` is specified, then all unaltered individual scores or losses will be returned in an array of shape \((n\_outputs,)\).

The `r2_score` and `explained_variance_score` accept an additional value 'variance_weighted' for the `multioutput` parameter. This option leads to a weighting of each individual score by the variance of the corresponding target variable. This setting quantifies the globally captured unscaled variance. If the target variables are of different scale, then this score puts more importance on well explaining the higher variance variables. `multioutput='variance_weighted'` is the default value for `r2_score` for backward compatibility. This will be changed to `uniform_average` in the future.

**Explained variance score**

The `explained_variance_score` computes the explained variance regression score.

If \(\hat{y}\) is the estimated target output, \(y\) the corresponding (correct) target output, and \(\text{Var}\) is Variance, the square of the standard deviation, then the explained variance is estimated as follow:

\[
\text{explained\_variance}(y, \hat{y}) = 1 - \frac{\text{Var}\{y - \hat{y}\}}{\text{Var}\{y\}}
\]

The best possible score is 1.0, lower values are worse.

Here is a small example of usage of the `explained_variance_score` function:

```python
>>> from sklearn.metrics import explained_variance_score
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> explained_variance_score(y_true, y_pred)
0.957...
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
>>> explained_variance_score(y_true, y_pred, multioutput='raw_values')
...
array([ 0.967..., 1. ...])
>>> explained_variance_score(y_true, y_pred, multioutput=[0.3, 0.7])
...
0.990...
```

**Mean absolute error**

The `mean_absolute_error` function computes mean absolute error, a risk metric corresponding to the expected value of the absolute error loss or \(l1\)-norm loss.

If \(\hat{y}_i\) is the predicted value of the \(i\)-th sample, and \(y_i\) is the corresponding true value, then the mean absolute error (MAE) estimated over \(n\_samples\) is defined as

\[
\text{MAE}(y, \hat{y}) = \frac{1}{n\_samples} \sum_{i=0}^{n\_samples-1} |y_i - \hat{y}_i|.
\]

Here is a small example of usage of the `mean_absolute_error` function:

```python
>>> from sklearn.metrics import mean_absolute_error
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> mean_absolute_error(y_true, y_pred)
0.5
```
```python
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
>>> mean_absolute_error(y_true, y_pred)
0.75
>>> mean_absolute_error(y_true, y_pred, multioutput='raw_values')
array([ 0.5, 1. ])
>>> mean_absolute_error(y_true, y_pred, multioutput=[0.3, 0.7])
...
0.849...
```

### Mean squared error

The `mean_squared_error` function computes mean square error, a risk metric corresponding to the expected value of the squared (quadratic) error loss or loss.

If \( \hat{y}_i \) is the predicted value of the \( i \)-th sample, and \( y_i \) is the corresponding true value, then the mean squared error (MSE) estimated over \( n_{\text{samples}} \) samples is defined as

\[
\text{MSE}(y, \hat{y}) = \frac{1}{n_{\text{samples}}} \sum_{i=0}^{n_{\text{samples}}-1} (y_i - \hat{y}_i)^2.
\]

Here is a small example of usage of the `mean_squared_error` function:

```python
>>> from sklearn.metrics import mean_squared_error
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, -0.5, 2, 8]
>>> mean_squared_error(y_true, y_pred)
0.375
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
>>> mean_squared_error(y_true, y_pred)
0.7083...
```

**Examples:**

- See *Gradient Boosting regression* for an example of mean squared error usage to evaluate gradient boosting regression.

### Median absolute error

The `median_absolute_error` is particularly interesting because it is robust to outliers. The loss is calculated by taking the median of all absolute differences between the target and the prediction.

If \( \hat{y}_i \) is the predicted value of the \( i \)-th sample and \( y_i \) is the corresponding true value, then the median absolute error (MedAE) estimated over \( n_{\text{samples}} \) samples is defined as

\[
\text{MedAE}(y, \hat{y}) = \text{median}(| y_1 - \hat{y}_1 |, \ldots, | y_n - \hat{y}_n |).
\]

The `median_absolute_error` does not support multioutput.

Here is a small example of usage of the `median_absolute_error` function:
R² score, the coefficient of determination

The \texttt{r2_score} function computes R², the coefficient of determination. It provides a measure of how well future samples are likely to be predicted by the model. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R² score of 0.0.

If \( \hat{y}_i \) is the predicted value of the \( i \)-th sample and \( y_i \) is the corresponding true value, then the score R² estimated over \( n_{\text{samples}} \) samples is defined as

\[
R^2(y, \hat{y}) = 1 - \frac{\sum_{i=0}^{n_{\text{samples}}-1}(y_i - \hat{y}_i)^2}{\sum_{i=0}^{n_{\text{samples}}-1}(y_i - \bar{y})^2}
\]

where \( \bar{y} = \frac{1}{n_{\text{samples}}} \sum_{i=0}^{n_{\text{samples}}-1} y_i \).

Here is a small example of usage of the \texttt{r2_score} function:

```python
>>> from sklearn.metrics import r2_score
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> r2_score(y_true, y_pred)
0.948...
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
>>> r2_score(y_true, y_pred, multioutput='variance_weighted')
... 0.938...
>>> r2_score(y_true, y_pred, multioutput='uniform_average')
... 0.936...
>>> r2_score(y_true, y_pred, multioutput='raw_values')
... array([ 0.965..., 0.908...])
>>> r2_score(y_true, y_pred, multioutput=[0.3, 0.7])
... 0.925...
```

Example:

- See \textit{Lasso and Elastic Net for Sparse Signals} for an example of R² score usage to evaluate Lasso and Elastic Net on sparse signals.
Clustering metrics

The `sklearn.metrics` module implements several loss, score, and utility functions. For more information see the `Clustering performance evaluation` section for instance clustering, and `Biclustering evaluation` for biclustering.

Dummy estimators

When doing supervised learning, a simple sanity check consists of comparing one’s estimator against simple rules of thumb. `DummyClassifier` implements several such simple strategies for classification:

- **stratified** generates random predictions by respecting the training set class distribution.
- **most_frequent** always predicts the most frequent label in the training set.
- **prior** always predicts the class that maximizes the class prior (like `most_frequent`) and `predict_proba` returns the class prior.
- **uniform** generates predictions uniformly at random.
- **constant** always predicts a constant label that is provided by the user. A major motivation of this method is F1-scoring, when the positive class is in the minority.

Note that with all these strategies, the `predict` method completely ignores the input data!

To illustrate `DummyClassifier`, first let’s create an imbalanced dataset:

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn.model_selection import train_test_split

>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> y[y != 1] = -1
>>> X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)
```

Next, let’s compare the accuracy of `SVC` and `most_frequent`:

```python
>>> from sklearn.dummy import DummyClassifier
>>> from sklearn.svm import SVC

>>> clf = SVC(kernel='linear', C=1).fit(X_train, y_train)
>>> clf.score(X_test, y_test)
0.63...

>>> clf = DummyClassifier(strategy='most_frequent',random_state=0)
>>> clf.fit(X_train, y_train)
DummyClassifier(constant=None, random_state=0, strategy='most_frequent')
>>> clf.score(X_test, y_test)
0.57...
```

We see that `SVC` doesn’t do much better than a dummy classifier. Now, let’s change the kernel:

```python
>>> clf = SVC(kernel='rbf', C=1).fit(X_train, y_train)
>>> clf.score(X_test, y_test)
0.97...
```

We see that the accuracy was boosted to almost 100%. A cross validation strategy is recommended for a better estimate of the accuracy, if it is not too CPU costly. For more information see the `Cross-validation: evaluating estimator performance` section. Moreover if you want to optimize over the parameter space, it is highly recommended to use an appropriate methodology; see the `Tuning the hyper-parameters of an estimator` section for details.

More generally, when the accuracy of a classifier is too close to random, it probably means that something went wrong: features are not helpful, a hyperparameter is not correctly tuned, the classifier is suffering from class imbalance, etc...

3.3. Model selection and evaluation
DummyRegressor also implements four simple rules of thumb for regression:

- **mean** always predicts the mean of the training targets.
- **median** always predicts the median of the training targets.
- **quantile** always predicts a user provided quantile of the training targets.
- **constant** always predicts a constant value that is provided by the user.

In all these strategies, the predict method completely ignores the input data.

### 3.3.4 Model persistence

After training a scikit-learn model, it is desirable to have a way to persist the model for future use without having to retrain. The following section gives you an example of how to persist a model with pickle. We’ll also review a few security and maintainability issues when working with pickle serialization.

**Persistence example**

It is possible to save a model in the scikit by using Python’s built-in persistence model, namely pickle:

```python
>>> from sklearn import svm
>>> from sklearn import datasets

>>> clf = svm.SVC()
>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target

>>> clf.fit(X, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
   decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
   max_iter=-1, probability=False, random_state=None, shrinking=True,
   tol=0.001, verbose=False)

>>> import pickle

>>> s = pickle.dumps(clf)
>>> clf2 = pickle.loads(s)

>>> clf2.predict(X[0:1])
array([0])

>>> y[0]
0
```

In the specific case of the scikit, it may be more interesting to use joblib’s replacement of pickle (`joblib.dump` & `joblib.load`), which is more efficient on objects that carry large numpy arrays internally as is often the case for fitted scikit-learn estimators, but can only pickle to the disk and not to a string:

```python
>>> from sklearn.externals import joblib

>>> joblib.dump(clf, 'filename.pkl')

Later you can load back the pickled model (possibly in another Python process) with:

```python
>>> clf = joblib.load('filename.pkl')
```

**Note:** joblib.dump and joblib.load functions also accept file-like object instead of filenames. More information on data persistence with Joblib is available here.
Security & maintainability limitations

pickle (and joblib by extension), has some issues regarding maintainability and security. Because of this,

- Never unpickle untrusted data as it could lead to malicious code being executed upon loading.
- While models saved using one version of scikit-learn might load in other versions, this is entirely unsupported
  and inadvisable. It should also be kept in mind that operations performed on such data could give different and
  unexpected results.

In order to rebuild a similar model with future versions of scikit-learn, additional metadata should be saved along the
pickled model:

- The training data, e.g. a reference to a immutable snapshot
- The python source code used to generate the model
- The versions of scikit-learn and its dependencies
- The cross validation score obtained on the training data

This should make it possible to check that the cross-validation score is in the same range as before.

If you want to know more about these issues and explore other possible serialization methods, please refer to this talk
by Alex Gaynor.

3.3.5 Validation curves: plotting scores to evaluate models

Every estimator has its advantages and drawbacks. Its generalization error can be decomposed in terms of bias,
variance and noise. The bias of an estimator is its average error for different training sets. The variance of an
estimator indicates how sensitive it is to varying training sets. Noise is a property of the data.

In the following plot, we see a function $f(x) = \cos\left(\frac{3}{2}\pi x\right)$ and some noisy samples from that function. We use three
different estimators to fit the function: linear regression with polynomial features of degree 1, 4 and 15. We see that
the first estimator can at best provide only a poor fit to the samples and the true function because it is too simple
(high bias), the second estimator approximates it almost perfectly and the last estimator approximates the training data
perfectly but does not fit the true function very well, i.e. it is very sensitive to varying training data (high variance).

Bias and variance are inherent properties of estimators and we usually have to select learning algorithms and hyper-
parameters so that both bias and variance are as low as possible (see Bias-variance dilemma). Another way to reduce
the variance of a model is to use more training data. However, you should only collect more training data if the true
function is too complex to be approximated by an estimator with a lower variance.

In the simple one-dimensional problem that we have seen in the example it is easy to see whether the estimator suffers
from bias or variance. However, in high-dimensional spaces, models can become very difficult to visualize. For this
reason, it is often helpful to use the tools described below.

Examples:
Validation curve

To validate a model we need a scoring function (see *Model evaluation: quantifying the quality of predictions*), for example accuracy for classifiers. The proper way of choosing multiple hyperparameters of an estimator are of course grid search or similar methods (see *Tuning the hyper-parameters of an estimator*) that select the hyperparameter with the maximum score on a validation set or multiple validation sets. Note that if we optimized the hyperparameters based on a validation score the validation score is biased and not a good estimate of the generalization any longer. To get a proper estimate of the generalization we have to compute the score on another test set.

However, it is sometimes helpful to plot the influence of a single hyperparameter on the training score and the validation score to find out whether the estimator is overfitting or underfitting for some hyperparameter values.

The function `validation_curve` can help in this case:

```python
>>> import numpy as np
>>> from sklearn.model_selection import validation_curve
>>> from sklearn.datasets import load_iris
>>> from sklearn.linear_model import Ridge

>>> np.random.seed(0)
>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> indices = np.arange(y.shape[0])
>>> np.random.shuffle(indices)
>>> X, y = X[indices], y[indices]

>>> train_scores, valid_scores = validation_curve(Ridge(), X, y, "alpha", ...
              np.logspace(-7, 3, 3))

>>> train_scores
array([[ 0.94..., 0.92..., 0.92...],
        [ 0.94..., 0.92..., 0.92...],
        [ 0.47..., 0.45..., 0.42...]])

>>> valid_scores
array([[ 0.90..., 0.92..., 0.94...],
        [ 0.90..., 0.92..., 0.94...],
        [ 0.44..., 0.39..., 0.45...]])
```

If the training score and the validation score are both low, the estimator will be underfitting. If the training score is high and the validation score is low, the estimator is overfitting and otherwise it is working very well. A low training score and a high validation score is usually not possible. All three cases can be found in the plot below where we vary the parameter $\gamma$ of an SVM on the digits dataset.

Learning curve

A learning curve shows the validation and training score of an estimator for varying numbers of training samples. It is a tool to find out how much we benefit from adding more training data and whether the estimator suffers more from a variance error or a bias error. If both the validation score and the training score converge to a value that is too low with increasing size of the training set, we will not benefit much from more training data. In the following plot you can see an example: naive Bayes roughly converges to a low score.
We will probably have to use an estimator or a parametrization of the current estimator that can learn more complex concepts (i.e. has a lower bias). If the training score is much greater than the validation score for the maximum number of training samples, adding more training samples will most likely increase generalization. In the following plot you can see that the SVM could benefit from more training examples.

![Learning Curve Plot](image)

We can use the function `learning_curve` to generate the values that are required to plot such a learning curve (number of samples that have been used, the average scores on the training sets and the average scores on the validation sets):

```python
>>> from sklearn.model_selection import learning_curve
>>> from sklearn.svm import SVC

>>> train_sizes, train_scores, valid_scores = learning_curve(... SVC(kernel='linear'), X, y, train_sizes=[50, 80, 110], cv=5)

>>> train_sizes
array([ 50, 80, 110])

>>> train_scores
array([[ 0.98..., 0.98 , 0.98..., 0.98..., 0.98...],
       [ 0.98..., 1. , 0.98..., 0.98..., 0.98...],
       [ 0.98..., 1. , 0.98..., 0.98..., 0.99...]])

>>> valid_scores
array([[ 1. , 0.93..., 1. , 1. , 0.96...],
       [ 1. , 0.96..., 1. , 1. , 0.96...],
       [ 1. , 0.96..., 1. , 1. , 0.96...]])
```

### 3.4 Dataset Transformations

scikit-learn provides a library of transformers, which may clean (see Preprocessing data), reduce (see Unsupervised dimensionality reduction), expand (see Kernel Approximation) or generate (see Feature extraction) feature representations.

Like other estimators, these are represented by classes with a `fit` method, which learns model parameters (e.g. mean and standard deviation for normalization) from a training set, and a `transform` method which applies this transformation model to unseen data. `fit_transform` may be more convenient and efficient for modelling and transforming the training data simultaneously.

Combining such transformers, either in parallel or series is covered in Pipeline and FeatureUnion: combining estimators. Pairwise metrics, Affinities and Kernels covers transforming feature spaces into affinity matrices, while...
Transforming the prediction target \((y)\) considers transformations of the target space (e.g. categorical labels) for use in scikit-learn.

### 3.4.1 Pipeline and FeatureUnion: combining estimators

**Pipeline: chaining estimators**

*Pipeline* can be used to chain multiple estimators into one. This is useful as there is often a fixed sequence of steps in processing the data, for example feature selection, normalization and classification. *Pipeline* serves two purposes here:

**Convenience**: You only have to call *fit* and *predict* once on your data to fit a whole sequence of estimators.

**Joint parameter selection**: You can *grid search* over parameters of all estimators in the pipeline at once.

All estimators in a pipeline, except the last one, must be transformers (i.e. must have a *transform* method). The last estimator may be any type (transformer, classifier, etc.).

**Usage**

The *Pipeline* is built using a list of *(key, value)* pairs, where the *key* is a string containing the name you want to give this step and *value* is an estimator object:

```python
>>> from sklearn.pipeline import Pipeline
>>> from sklearn.svm import SVC
>>> from sklearn.decomposition import PCA

>>> estimators = [('reduce_dim', PCA()), ('clf', SVC())]
>>> pipe = Pipeline(estimators)
>>> pipe
Pipeline(steps=[('reduce_dim', PCA(copy=True, iterated_power='auto', n_components=None, random_state=None, svd_solver='auto', tol=0.0, whiten=False)), ('clf', SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, decision_function_shape=None, degree=3, gamma='auto', kernel='rbf', max_iter=-1, probability=False, random_state=None, shrinking=True, tol=0.001, verbose=False))])
```

The utility function *make_pipeline* is a shorthand for constructing pipelines; it takes a variable number of estimators and returns a pipeline, filling in the names automatically:

```python
>>> from sklearn.pipeline import make_pipeline
>>> from sklearn.naive_bayes import MultinomialNB
>>> from sklearn.preprocessing import Binarizer

>>> make_pipeline(Binarizer(), MultinomialNB())
Pipeline(steps=[('binarizer', Binarizer(copy=True, threshold=0.0)),
               ('multinomialnb', MultinomialNB(alpha=1.0,
                                           class_prior=None,
                                           fit_prior=True))])
```

The estimators of a pipeline are stored as a list in the *steps* attribute:

```python
>>> pipe.steps[0]
('reduce_dim', PCA(copy=True, iterated_power='auto', n_components=None, random_˓→state=None, svd_solver='auto', tol=0.0, whiten=False))
```
and as a dict in named_steps:

```python
>>> pipe.named_steps['reduce_dim']
PCA(copy=True, iterated_power='auto', n_components=None, random_state=None,
   svd_solver='auto', tol=0.0, whiten=False)
```

Parameters of the estimators in the pipeline can be accessed using the `<estimator>__<parameter>` syntax:

```python
>>> pipe.set_params(clf__C=10)
Pipeline(steps=[('reduce_dim', PCA(copy=True, iterated_power='auto',
    n_components=None, random_state=None, svd_solver='auto', tol=0.0,
    whiten=False)), ('clf', SVC(C=10, cache_size=200, class_weight=None,
    coef0=0.0, decision_function_shape=None, degree=3, gamma='auto',
    kernel='rbf', max_iter=-1, probability=False, random_state=None,
    shrinking=True, tol=0.001, verbose=False))])
```

This is particularly important for doing grid searches:

```python
>>> from sklearn.model_selection import GridSearchCV
>>> params = dict(reduce_dim__n_components=[2, 5, 10],
                 ...       clf__C=[0.1, 10, 100])
>>> grid_search = GridSearchCV(pipe, param_grid=params)
```

Individual steps may also be replaced as parameters, and non-final steps may be ignored by setting them to `None`:

```python
>>> from sklearn.linear_model import LogisticRegression
>>> params = dict(reduce_dim=[None, PCA(5), PCA(10)],
                 ...       clf=[SVC(), LogisticRegression()],
                 ...       clf__C=[0.1, 10, 100])
>>> grid_search = GridSearchCV(pipe, param_grid=params)
```

Examples:

- Pipeline Anova SVM
- Sample pipeline for text feature extraction and evaluation
- Pipelining: chaining a PCA and a logistic regression
- Explicit feature map approximation for RBF kernels
- SVM-Anova: SVM with univariate feature selection

See also:

- Tuning the hyper-parameters of an estimator

Notes

Calling `fit` on the pipeline is the same as calling `fit` on each estimator in turn, `transform` the input and pass it on to the next step. The pipeline has all the methods that the last estimator in the pipeline has, i.e. if the last estimator is a classifier, the `Pipeline` can be used as a classifier. If the last estimator is a transformer, again, so is the pipeline.
FeatureUnion: composite feature spaces

FeatureUnion combines several transformer objects into a new transformer that combines their output. A FeatureUnion takes a list of transformer objects. During fitting, each of these is fit to the data independently. For transforming data, the transformers are applied in parallel, and the sample vectors they output are concatenated end-to-end into larger vectors.

FeatureUnion serves the same purposes as Pipeline - convenience and joint parameter estimation and validation.

FeatureUnion and Pipeline can be combined to create complex models.

(A FeatureUnion has no way of checking whether two transformers might produce identical features. It only produces a union when the feature sets are disjoint, and making sure they are the caller’s responsibility.)

Usage

A FeatureUnion is built using a list of (key, value) pairs, where the key is the name you want to give to a given transformation (an arbitrary string; it only serves as an identifier) and value is an estimator object:

```python
>>> from sklearn.pipeline import FeatureUnion
>>> from sklearn.decomposition import PCA
>>> from sklearn.decomposition import KernelPCA

>>> estimators = [('linear_pca', PCA()), ('kernel_pca', KernelPCA())]
>>> combined = FeatureUnion(estimators)
```

Like pipelines, feature unions have a shorthand constructor called `make_union` that does not require explicit naming of the components.

Like Pipeline, individual steps may be replaced using `set_params`, and ignored by setting to None:

```python
>>> combined.set_params(kernel_pca=None)
```

Examples:

- Concatenating multiple feature extraction methods
- Feature Union with Heterogeneous Data Sources
3.4.2 Feature extraction

The `sklearn.feature_extraction` module can be used to extract features in a format supported by machine learning algorithms from datasets consisting of formats such as text and image.

**Note:** Feature extraction is very different from Feature selection: the former consists in transforming arbitrary data, such as text or images, into numerical features usable for machine learning. The latter is a machine learning technique applied on these features.

### Loading features from dicts

The class `DictVectorizer` can be used to convert feature arrays represented as lists of standard Python `dict` objects to the NumPy/SciPy representation used by scikit-learn estimators.

While not particularly fast to process, Python’s `dict` has the advantages of being convenient to use, being sparse (absent features need not be stored) and storing feature names in addition to values.

`DictVectorizer` implements what is called one-of-K or “one-hot” coding for categorical (aka nominal, discrete) features. Categorical features are “attribute-value” pairs where the value is restricted to a list of discrete possibilities without ordering (e.g. topic identifiers, types of objects, tags, names...).

In the following, “city” is a categorical attribute while “temperature” is a traditional numerical feature:

```python
>>> measurements = [
...     {'city': 'Dubai', 'temperature': 33.},
...     {'city': 'London', 'temperature': 12.},
...     {'city': 'San Francisco', 'temperature': 18.},
...]

>>> from sklearn.feature_extraction import DictVectorizer

>>> vec = DictVectorizer()

>>> vec.fit_transform(measurements).toarray()
array([[ 1.,  0.,  0.,  33.],
       [ 0.,  1.,  0.,  12.],
       [ 0.,  0.,  1.,  18.]]

>>> vec.get_feature_names()
['city=Dubai', 'city=London', 'city=San Francisco', 'temperature']
```

`DictVectorizer` is also a useful representation transformation for training sequence classifiers in Natural Language Processing models that typically work by extracting feature windows around a particular word of interest.

For example, suppose that we have a first algorithm that extracts Part of Speech (PoS) tags that we want to use as complementary tags for training a sequence classifier (e.g. a chunker). The following dict could be such a window of features extracted around the word ‘sat’ in the sentence ‘The cat sat on the mat’:

```python
>>> pos_window = [
...     {'word-2': 'the',
...      'pos-2': 'DT',
...      'word-1': 'cat',
...      'pos-1': 'NN',
...      'word+1': 'on',
...      'pos+1': 'PP'},
... ]
```
This description can be vectorized into a sparse two-dimensional matrix suitable for feeding into a classifier (maybe after being piped into a `text.TfidfTransformer` for normalization):

```
>>> vec = DictVectorizer()
>>> pos_vectorized = vec.fit_transform(pos_window)
>>> pos_vectorized
<1x6 sparse matrix of type '<... 'numpy.float64'>'
   with 6 stored elements in Compressed Sparse ... format>
>>> pos_vectorized.toarray()
array([[ 1.,  1.,  1.,  1.,  1.,  1.]])
>>> vec.get_feature_names()
['pos+1=PP', 'pos-1=NN', 'pos-2=DT', 'word+1=on', 'word-1=cat', 'word-2=the']
```

As you can imagine, if one extracts such a context around each individual word of a corpus of documents the resulting matrix will be very wide (many one-hot-features) with most of them being valued to zero most of the time. So as to make the resulting data structure able to fit in memory the `DictVectorizer` class uses a `scipy.sparse` matrix by default instead of a `numpy.ndarray`.

**Feature hashing**

The class `FeatureHasher` is a high-speed, low-memory vectorizer that uses a technique known as feature hashing, or the “hashing trick”. Instead of building a hash table of the features encountered in training, as the vectorizers do, instances of `FeatureHasher` apply a hash function to the features to determine their column index in sample matrices directly. The result is increased speed and reduced memory usage, at the expense of inspectability; the hasher does not remember what the input features looked like and has no `inverse_transform` method.

Since the hash function might cause collisions between (unrelated) features, a signed hash function is used and the sign of the hash value determines the sign of the value stored in the output matrix for a feature. This way, collisions are likely to cancel out rather than accumulate error, and the expected mean of any output feature’s value is zero.

If `non_negative=True` is passed to the constructor, the absolute value is taken. This undoes some of the collision handling, but allows the output to be passed to estimators like `sklearn.naive_bayes.MultinomialNB` or `sklearn.feature_selection.chi2` feature selectors that expect non-negative inputs.

Feature hashing can be employed in document classification, but unlike `text.CountVectorizer`, `FeatureHasher` does not do word splitting or any other preprocessing except Unicode-to-UTF-8 encoding; see *Vectorizing a large text corpus with the hashing trick*, below, for a combined tokenizer/hasher.

As an example, consider a word-level natural language processing task that needs features extracted from `(token, part_of_speech)` pairs. One could use a Python generator function to extract features:

```
def token_features(token, part_of_speech):
    if token.isdigit():
        yield "numeric"
    else:
        yield "token={}".format(token.lower())
```
yield "token,pos=\[,\]/\].format(token, part_of_speech)
if token[0].isupper():
    yield "uppercase_initial"
if token.isupper():
    yield "all_uppercase"
yield "pos=/\].format(part_of_speech)

Then, the raw_{X} to be fed to FeatureHasher.transform can be constructed using:

raw_{X} = (token_features(tok, pos_tagger(tok)) for tok in corpus)

and fed to a hasher with:

hasher = FeatureHasher(input_type='string')
{X} = hasher.transform(raw_{X})

to get a scipy.sparse matrix {X}.

Note the use of a generator comprehension, which introduces laziness into the feature extraction: tokens are only processed on demand from the hasher.

Implementation details

FeatureHasher uses the signed 32-bit variant of MurmurHash3. As a result (and because of limitations in scipy.sparse), the maximum number of features supported is currently \(2^{31} - 1\).

The original formulation of the hashing trick by Weinberger et al. used two separate hash functions \(h\) and \(\xi\) to determine the column index and sign of a feature, respectively. The present implementation works under the assumption that the sign bit of MurmurHash3 is independent of its other bits.

Since a simple modulo is used to transform the hash function to a column index, it is advisable to use a power of two as the n_features parameter; otherwise the features will not be mapped evenly to the columns.

References:

- MurmurHash3.

Text feature extraction

The Bag of Words representation

Text Analysis is a major application field for machine learning algorithms. However the raw data, a sequence of symbols cannot be fed directly to the algorithms themselves as most of them expect numerical feature vectors with a fixed size rather than the raw text documents with variable length.

In order to address this, scikit-learn provides utilities for the most common ways to extract numerical features from text content, namely:

- **tokenizing** strings and giving an integer id for each possible token, for instance by using white-spaces and punctuation as token separators.
- **counting** the occurrences of tokens in each document.
normalizing and weighting with diminishing importance tokens that occur in the majority of samples / documents.

In this scheme, features and samples are defined as follows:

- each individual token occurrence frequency (normalized or not) is treated as a feature.
- the vector of all the token frequencies for a given document is considered a multivariate sample.

A corpus of documents can thus be represented by a matrix with one row per document and one column per token (e.g. word) occurring in the corpus.

We call vectorization the general process of turning a collection of text documents into numerical feature vectors. This specific strategy (tokenization, counting and normalization) is called the Bag of Words or “Bag of n-grams” representation. Documents are described by word occurrences while completely ignoring the relative position information of the words in the document.

**Sparsity**

As most documents will typically use a very small subset of the words used in the corpus, the resulting matrix will have many feature values that are zeros (typically more than 99% of them).

For instance a collection of 10,000 short text documents (such as emails) will use a vocabulary with a size in the order of 100,000 unique words in total while each document will use 100 to 1000 unique words individually.

In order to be able to store such a matrix in memory but also to speed up algebraic operations matrix / vector, implementations will typically use a sparse representation such as the implementations available in the scipy.sparse package.

**Common Vectorizer usage**

`CountVectorizer` implements both tokenization and occurrence counting in a single class:

```python
>>> from sklearn.feature_extraction.text import CountVectorizer
```

This model has many parameters, however the default values are quite reasonable (please see the reference documentation for the details):

```python
>>> vectorizer = CountVectorizer(min_df=1)
>>> vectorizer
CountVectorizer(analyzer='word', binary=False, decode_error='strict',
    dtype=<... 'numpy.int64'>, encoding='utf-8', input='content',
    lowercase=True, max_df=1.0, max_features=None, min_df=1,
    ngram_range=(1, 1), preprocessor=None, stop_words=None,
    strip_accents=None, token_pattern='(?u)\b\w\w+\b',
    tokenizer=None, vocabulary=None)
```

Let’s use it to tokenize and count the word occurrences of a minimalistic corpus of text documents:

```python
>>> corpus = ['This is the first document.',
... 'This is the second second document.',
... 'And the third one.',
... 'Is this the first document?']
>>> X = vectorizer.fit_transform(corpus)
>>> X
```

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The default configuration tokenizes the string by extracting words of at least 2 letters. The specific function that does this step can be requested explicitly:

```python
>>> analyze = vectorizer.build_analyzer()
>>> analyze("This is a text document to analyze.") == (...
... ['this', 'is', 'text', 'document', 'to', 'analyze'])
True
```

Each term found by the analyzer during the fit is assigned a unique integer index corresponding to a column in the resulting matrix. This interpretation of the columns can be retrieved as follows:

```python
>>> vectorizer.get_feature_names() == (...
... ['and', 'document', 'first', 'is', 'one',
... 'second', 'the', 'third', 'this'])
True
```

```python
>>> X.toarray()
array([[0, 1, 1, 1, 0, 1, 0, 1, 1],
       [0, 1, 0, 1, 0, 2, 1, 0, 1],
       [1, 0, 0, 1, 0, 1, 1, 0, 0],
       [0, 1, 1, 0, 0, 1, 0, 1, 1]])...
```

The converse mapping from feature name to column index is stored in the `vocabulary_` attribute of the vectorizer:

```python
>>> vectorizer.vocabulary_.get('document')
1
```

Hence words that were not seen in the training corpus will be completely ignored in future calls to the transform method:

```python
>>> vectorizer.transform(['Something completely new.']).toarray()
... array([[0, 0, 0, 0, 0, 0, 0, 0, 0]])...
```

Note that in the previous corpus, the first and the last documents have exactly the same words hence are encoded in equal vectors. In particular we lose the information that the last document is an interrogative form. To preserve some of the local ordering information we can extract 2-grams of words in addition to the 1-grams (individual words):

```python
>>> bigram_vectorizer = CountVectorizer(ngram_range=(1, 2),
... token_pattern=r'\b\w+\b', min_df=1)
>>> analyze = bigram_vectorizer.build_analyzer()
>>> analyze('Bi-grams are cool!') == (...
... ['bi', 'grams', 'are', 'cool', 'bi grams', 'grams are', 'are cool'])
True
```

The vocabulary extracted by this vectorizer is hence much bigger and can now resolve ambiguities encoded in local positioning patterns:

```python
>>> X_2 = bigram_vectorizer.fit_transform(corpus).toarray()
>>> X_2
... array([[0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0],
        [0, 1, 1, 0, 1, 0, 1, 1, 0, 2, 1, 1, 0, 1, 0, 0, 1, 1, 0],
        [0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0],
        ... [1, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1],
        [0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0],
        [0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0],
        [0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0]]...)
```
In particular the interrogative form “Is this” is only present in the last document:

```
>>> feature_index = bigram_vectorizer.vocabulary_.get('is this')
>>> X_2[:, feature_index]
array([0, 0, 0, 1]...)
```

### Tf–idf term weighting

In a large text corpus, some words will be very present (e.g. “the”, “a”, “is” in English) hence carrying very little meaningful information about the actual contents of the document. If we were to feed the direct count data directly to a classifier those very frequent terms would shadow the frequencies of rarer yet more interesting terms.

In order to re-weight the count features into floating point values suitable for usage by a classifier it is very common to use the tf–idf transform.

Tf means **term-frequency** while tf–idf means term-frequency times **inverse document-frequency**: $\text{tf-idf}(t,d) = \text{tf}(t,d) \times \text{idf}(t)$.

Using the TfidfTransformer’s default settings, TfidfTransformer(norm='l2', use_idf=True, smooth_idf=True, sublinear_tf=False), the term frequency, the number of times a term occurs in a given document, is multiplied with idf component, which is computed as

$$\text{idf}(t) = \log \frac{1+n_d}{1+\text{df}(d,t)} + 1,$$

where $n_d$ is the total number of documents, and $\text{df}(d,t)$ is the number of documents that contain term $t$. The resulting tf-idf vectors are then normalized by the Euclidean norm:

$$\|v\|_2 = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}.$$

This was originally a term weighting scheme developed for information retrieval (as a ranking function for search engines results) that has also found good use in document classification and clustering.

The following sections contain further explanations and examples that illustrate how the tf-idfs are computed exactly and how the tf-ids computed in scikit-learn’s TfidfTransformer and TfidfVectorizer differ slightly from the standard textbook notation that defines the idf as

$$\text{idf}(t) = \log \frac{n_d}{1+\text{df}(d,t)}.$$

In the TfidfTransformer and TfidfVectorizer with smooth_idf=False, the “1” count is added to the idf instead of the idf’s denominator:

$$\text{idf}(t) = \log \frac{n_d + 1}{1+\text{df}(d,t)} + 1$$

This normalization is implemented by the TfidfTransformer class:

```
>>> from sklearn.feature_extraction.text import TfidfTransformer
>>> transformer = TfidfTransformer(smooth_idf=False)
>>> transformer
TfidfTransformer(norm='l2', use_idf=False, sublinear_tf=False, smooth_idf=True)
```

Again please see the reference documentation for the details on all the parameters.

Let’s take an example with the following counts. The first term is present 100% of the time hence not very interesting. The two other features only in less than 50% of the time hence probably more representative of the content of the documents:
Each row is normalized to have unit Euclidean norm:

\[ v_{\text{norm}} = \frac{v}{\|v\|_2} = \frac{v_1^2 + v_2^2 + \cdots + v_n^2}{\sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}} \]

For example, we can compute the tf-idf of the first term in the first document in the `counts` array as follows:

\[ n_{d,\text{term1}} = 6 \]
\[ df(d, t)_{\text{term1}} = 6 \]
\[ \text{idf}(d, t)_{\text{term1}} = \log \frac{n_d}{\text{df}(d, t)_{\text{term1}}} + 1 = \log(6) + 1 = 1 \]
\[ \text{tf-idf}_{\text{term1}} = \text{tf} \times \text{idf} = 3 \times 1 = 3 \]

Now, if we repeat this computation for the remaining 2 terms in the document, we get

\[ \text{tf-idf}_{\text{term2}} = 0 \times \log(6/1) + 1 = 0 \]
\[ \text{tf-idf}_{\text{term3}} = 1 \times \log(6/2) + 1 \approx 2.0986 \]

and the vector of raw tf-ids:

\[ \text{tf-idf}_{aw} = [3, 0, 2.0986] \]

Then, applying the Euclidean (L2) norm, we obtain the following tf-ids for document 1:

\[ \sqrt{\frac{3^2 + 0^2 + 2.0986^2}{3^2 + 0^2 + 2.0986^2}} = [0.819, 0, 0.573] \]

Furthermore, the default parameter `smooth_idf=True` adds “1” to the numerator and denominator as if an extra document was seen containing every term in the collection exactly once, which prevents zero divisions:

\[ \text{idf}(t) = \log \frac{1 + n_d}{1 + \text{df}(d, t)} + 1 \]

Using this modification, the tf-idf of the third term in document 1 changes to 1.8473:

\[ \text{tf-idf}_{\text{term3}} = 1 \times \log(7/3) + 1 \approx 1.8473 \]

And the L2-normalized tf-idf changes to

\[ \sqrt{\frac{3^2 + 0^2 + 1.8473^2}{3^2 + 0^2 + 1.8473^2}} = [0.8515, 0, 0.5243] \]
The weights of each feature computed by the `fit` method call are stored in a model attribute:

```python
>>> transformer.idf_
array([ 1. ..., 2.25..., 1.84...])
```

As tf–idf is very often used for text features, there is also another class called `TfidfVectorizer` that combines all the options of `CountVectorizer` and `TfidfTransformer` in a single model:

```python
>>> from sklearn.feature_extraction.text import TfidfVectorizer
>>> vectorizer = TfidfVectorizer(min_df=1)
>>> vectorizer.fit_transform(corpus)
...<4x9 sparse matrix of type '<... 'numpy.float64'>'
   with 19 stored elements in Compressed Sparse ... format>
```

While the tf–idf normalization is often very useful, there might be cases where the binary occurrence markers might offer better features. This can be achieved by using the `binary` parameter of `CountVectorizer`. In particular, some estimators such as Bernoulli Naive Bayes explicitly model discrete boolean random variables. Also, very short texts are likely to have noisy tf–idf values while the binary occurrence info is more stable.

As usual the best way to adjust the feature extraction parameters is to use a cross-validated grid search, for instance by pipelining the feature extractor with a classifier:

- **Sample pipeline for text feature extraction and evaluation**

### Decoding text files

Text is made of characters, but files are made of bytes. These bytes represent characters according to some *encoding*. To work with text files in Python, their bytes must be *decoded* to a character set called Unicode. Common encodings are ASCII, Latin-1 (Western Europe), KOI8-R (Russian) and the universal encodings UTF-8 and UTF-16. Many others exist.

**Note:** An encoding can also be called a `character set`, but this term is less accurate: several encodings can exist for a single character set.

The text feature extractors in scikit-learn know how to decode text files, but only if you tell them what encoding the files are in. The `CountVectorizer` takes an `encoding` parameter for this purpose. For modern text files, the correct encoding is probably UTF-8, which is therefore the default (`encoding="utf-8"`).

If the text you are loading is not actually encoded with UTF-8, however, you will get a `UnicodeDecodeError`. The vectorizers can be told to be silent about decoding errors by setting the `decode_error` parameter to either `"ignore"` or `"replace"`. See the documentation for the Python function `bytes.decode` for more details (type `help(bytes.decode)` at the Python prompt).

If you are having trouble decoding text, here are some things to try:
Find out what the actual encoding of the text is. The file might come with a header or README that tells you the encoding, or there might be some standard encoding you can assume based on where the text comes from.

You may be able to find out what kind of encoding it is in general using the UNIX command `file`. The Python `chardet` module comes with a script called `chardetect.py` that will guess the specific encoding, though you cannot rely on its guess being correct.

You could try UTF-8 and disregard the errors. You can decode byte strings with `bytes.decode(errors='replace')` to replace all decoding errors with a meaningless character, or set `decode_error='replace'` in the vectorizer. This may damage the usefulness of your features.

Real text may come from a variety of sources that may have used different encodings, or even be sloppily decoded in a different encoding than the one it was encoded with. This is common in text retrieved from the Web. The Python package `ftfy` can automatically sort out some classes of decoding errors, so you could try decoding the unknown text as `latin-1` and then using `ftfy` to fix errors.

If the text is in a mish-mash of encodings that is simply too hard to sort out (which is the case for the 20 Newsgroups dataset), you can fall back on a simple single-byte encoding such as `latin-1`. Some text may display incorrectly, but at least the same sequence of bytes will always represent the same feature.

For example, the following snippet uses `chardet` (not shipped with scikit-learn, must be installed separately) to figure out the encoding of three texts. It then vectorizes the texts and prints the learned vocabulary. The output is not shown here.

```python
>>> import chardet
>>> text1 = b"Sei mir gegr".decode(chardet.detect(x)['encoding'])
... for x in [text1, text2, text3]:
...     decoded = [x.decode(chardet.detect(x)['encoding'])]
...     v = CountVectorizer().fit(decoded).vocabulary_
...     for term in v: print(v)
```

For an introduction to Unicode and character encodings in general, see Joel Spolsky’s Absolute Minimum Every Software Developer Must Know About Unicode.

**Applications and examples**

The bag of words representation is quite simplistic but surprisingly useful in practice.

In particular in a supervised setting it can be successfully combined with fast and scalable linear models to train document classifiers, for instance:

- Classification of text documents using sparse features

In an unsupervised setting it can be used to group similar documents together by applying clustering algorithms such as K-means:

- Clustering text documents using k-means

Finally it is possible to discover the main topics of a corpus by relaxing the hard assignment constraint of clustering, for instance by using Non-negative matrix factorization (NMF or NNMF):

- Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation
Limitations of the Bag of Words representation

A collection of unigrams (what bag of words is) cannot capture phrases and multi-word expressions, effectively disregarding any word order dependence. Additionally, the bag of words model doesn’t account for potential misspellings or word derivations.

N-grams to the rescue! Instead of building a simple collection of unigrams (n=1), one might prefer a collection of bigrams (n=2), where occurrences of pairs of consecutive words are counted.

One might alternatively consider a collection of character n-grams, a representation resilient against misspellings and derivations.

For example, let’s say we’re dealing with a corpus of two documents: ['words', 'wprds']. The second document contains a misspelling of the word ‘words’. A simple bag of words representation would consider these two as very distinct documents, differing in both of the two possible features. A character 2-gram representation, however, would find the documents matching in 4 out of 8 features, which may help the preferred classifier decide better:

```python
>>> ngram_vectorizer = CountVectorizer(analyzer='char_wb', ngram_range=(2, 2), min_df=1)
>>> counts = ngram_vectorizer.fit_transform(['words', 'wprds'])
>>> ngram_vectorizer.get_feature_names() == ('w ', 'ds', 'or', 'pr', 'rd', 's ', 'wo', 'wp'))
True
>>> counts.toarray().astype(int)
array([[1, 1, 1, 0, 1, 1, 1, 0],
       [1, 1, 0, 1, 1, 1, 0, 1]])
```

In the above example, 'char_wb analyzer is used, which creates n-grams only from characters inside word boundaries (padded with space on each side). The 'char' analyzer, alternatively, creates n-grams that span across words:

```python
>>> ngram_vectorizer = CountVectorizer(analyzer='char_wb', ngram_range=(5, 5), min_df=1)
>>> ngram_vectorizer.fit_transform(['jumpy fox'])
...<1x4 sparse matrix of type '<... 'numpy.int64'>'
  with 4 stored elements in Compressed Sparse ... format>
>>> ngram_vectorizer.get_feature_names() == ('fox ', 'jump', 'jumpy', 'umpy ')
True

>>> ngram_vectorizer = CountVectorizer(analyzer='char', ngram_range=(5, 5), min_df=1)
>>> ngram_vectorizer.fit_transform(['jumpy fox'])
...<1x5 sparse matrix of type '<... 'numpy.int64'>'
  with 5 stored elements in Compressed Sparse ... format>
>>> ngram_vectorizer.get_feature_names() == ('jumpy', 'mpy f', 'py fo', 'umpy ', 'y fox')
True
```

The word boundaries-aware variant char_wb is especially interesting for languages that use white-spaces for word separation as it generates significantly less noisy features than the raw char variant in that case. For such languages it can increase both the predictive accuracy and convergence speed of classifiers trained using such features while retaining the robustness with regards to misspellings and word derivations.

While some local positioning information can be preserved by extracting n-grams instead of individual words, bag of words and bag of n-grams destroy most of the inner structure of the document and hence most of the meaning carried by that internal structure.

In order to address the wider task of Natural Language Understanding, the local structure of sentences and paragraphs

3.4. Dataset transformations
should thus be taken into account. Many such models will thus be casted as “Structured output” problems which are currently outside of the scope of scikit-learn.

### Vectorizing a large text corpus with the hashing trick

The above vectorization scheme is simple but the fact that it holds an in-memory mapping from the string tokens to the integer feature indices (the `vocabulary_` attribute) causes several problems when dealing with large datasets:

- the larger the corpus, the larger the vocabulary will grow and hence the memory use too,
- fitting requires the allocation of intermediate data structures of size proportional to that of the original dataset.
- building the word-mapping requires a full pass over the dataset hence it is not possible to fit text classifiers in a strictly online manner.
- pickling and un-pickling vectorizers with a large `vocabulary_` can be very slow (typically much slower than pickling / un-pickling flat data structures such as a NumPy array of the same size),
- it is not easily possible to split the vectorization work into concurrent sub tasks as the `vocabulary_` attribute would have to be a shared state with a fine grained synchronization barrier: the mapping from token string to feature index is dependent on ordering of the first occurrence of each token hence would have to be shared, potentially harming the concurrent workers’ performance to the point of making them slower than the sequential variant.

It is possible to overcome those limitations by combining the “hashing trick” ([Feature hashing](#)) implemented by the `sklearn.feature_extraction.FeatureHasher` class and the text preprocessing and tokenization features of the `CountVectorizer`.

This combination is implementing in `HashingVectorizer`, a transformer class that is mostly API compatible with `CountVectorizer`. `HashingVectorizer` is stateless, meaning that you don’t have to call `fit` on it:

```python
>>> from sklearn.feature_extraction.text import HashingVectorizer
>>> hv = HashingVectorizer(n_features=10)
>>> hv.transform(corpus)
...<4x10 sparse matrix of type '<... 'numpy.float64'>'
    with 16 stored elements in Compressed Sparse ... format>
```

You can see that 16 non-zero feature tokens were extracted in the vector output: this is less than the 19 non-zeros extracted previously by the `CountVectorizer` on the same toy corpus. The discrepancy comes from hash function collisions because of the low value of the `n_features` parameter.

In a real world setting, the `n_features` parameter can be left to its default value of $2^{20}$ (roughly one million possible features). If memory or downstream models size is an issue selecting a lower value such as $2^{18}$ might help without introducing too many additional collisions on typical text classification tasks.

Note that the dimensionality does not affect the CPU training time of algorithms which operate on CSR matrices (`LinearSVC(dual=True)`, `Perceptron`, `SGDClassifier`, `PassiveAggressive`) but it does for algorithms that work with CSC matrices (`LinearSVC(dual=False)`, `Lasso`, etc).

Let’s try again with the default setting:

```python
>>> hv = HashingVectorizer()
>>> hv.transform(corpus)
...<4x1048576 sparse matrix of type '<... 'numpy.float64'>'
    with 19 stored elements in Compressed Sparse ... format>
```
We no longer get the collisions, but this comes at the expense of a much larger dimensionality of the output space. Of course, other terms than the 19 used here might still collide with each other.

The *HashingVectorizer* also comes with the following limitations:

- it is not possible to invert the model (no `inverse_transform` method), nor to access the original string representation of the features, because of the one-way nature of the hash function that performs the mapping.
- it does not provide IDF weighting as that would introduce statefulness in the model. A *TfidfTransformer* can be appended to it in a pipeline if required.

### Performing out-of-core scaling with HashingVectorizer

An interesting development of using a *HashingVectorizer* is the ability to perform out-of-core scaling. This means that we can learn from data that does not fit into the computer’s main memory.

A strategy to implement out-of-core scaling is to stream data to the estimator in mini-batches. Each mini-batch is vectorized using *HashingVectorizer* so as to guarantee that the input space of the estimator has always the same dimensionality. The amount of memory used at any time is thus bounded by the size of a mini-batch. Although there is no limit to the amount of data that can be ingested using such an approach, from a practical point of view the learning time is often limited by the CPU time one wants to spend on the task.

For a full-fledged example of out-of-core scaling in a text classification task see *Out-of-core classification of text documents*.

### Customizing the vectorizer classes

It is possible to customize the behavior by passing a callable to the vectorizer constructor:

```python
>>> def my_tokenizer(s):
...    return s.split()
...>>> vectorizer = CountVectorizer(tokenizer=my_tokenizer)
>>> vectorizer.build_analyzer()(u"Some... punctuation!") == 
...    ['some...', 'punctuation!'])
True
```

In particular we name:

- **preprocessor**: a callable that takes an entire document as input (as a single string), and returns a possibly transformed version of the document, still as an entire string. This can be used to remove HTML tags, lowercase the entire document, etc.

- **tokenizer**: a callable that takes the output from the preprocessor and splits it into tokens, then returns a list of these.

- **analyzer**: a callable that replaces the preprocessor and tokenizer. The default analyzers all call the preprocessor and tokenizer, but custom analyzers will skip this. N-gram extraction and stop word filtering take place at the analyzer level, so a custom analyzer may have to reproduce these steps.

(Lucene users might recognize these names, but be aware that scikit-learn concepts may not map one-to-one onto Lucene concepts.)

To make the preprocessor, tokenizer and analyzers aware of the model parameters it is possible to derive from the class and override the `build_preprocessor`, `build_tokenizer` and `build_analyzer` factory methods instead of passing custom functions.

Some tips and tricks:
• If documents are pre-tokenized by an external package, then store them in files (or strings) with the tokens separated by whitespace and pass `analyzer=str.split`

• Fancy token-level analysis such as stemming, lemmatizing, compound splitting, filtering based on part-of-speech, etc. are not included in the scikit-learn codebase, but can be added by customizing either the tokenizer or the analyzer. Here’s a `CountVectorizer` with a tokenizer and lemmatizer using NLTK:

```
>>> from nltk import word_tokenize
>>> from nltk.stem import WordNetLemmatizer

>>> class LemmaTokenizer(object):
...     def __init__(self):
...         self.wnl = WordNetLemmatizer()
...     def __call__(self, doc):
...         return [self.wnl.lemmatize(t) for t in word_tokenize(doc)]
...

>>> vect = CountVectorizer(tokenizer=LemmaTokenizer())
```

(Note that this will not filter out punctuation.)

Customizing the vectorizer can also be useful when handling Asian languages that do not use an explicit word separator such as whitespace.

**Image feature extraction**

**Patch extraction**

The `extract_patches_2d` function extracts patches from an image stored as a two-dimensional array, or three-dimensional with color information along the third axis. For rebuilding an image from all its patches, use `reconstruct_from_patches_2d`. For example let use generate a 4x4 pixel picture with 3 color channels (e.g. in RGB format):

```
>>> import numpy as np
>>> from sklearn.feature_extraction import image

>>> one_image = np.arange(4 * 4 * 3).reshape((4, 4, 3))
>>> one_image[:, :, 0]  # R channel of a fake RGB picture
array([[ 0,  3,  6,  9],
       [12, 15, 18, 21],
       [24, 27, 30, 33],
       [36, 39, 42, 45]])

>>> patches = image.extract_patches_2d(one_image, (2, 2), max_patches=2,
...                                      random_state=0)
>>> patches.shape
(2, 2, 2, 3)
>>> patches[:, :, :, 0]
array([[[ 0,  3],
       [12, 15]],
       [[15, 18],
       [27, 30]])]

>>> patches = image.extract_patches_2d(one_image, (2, 2))
>>> patches.shape
(9, 2, 2, 3)
>>> patches[4, :, :, 0]
array([[15, 18],
       [27, 30]])
```
Let us now try to reconstruct the original image from the patches by averaging on overlapping areas:

```python
>>> reconstructed = image.reconstruct_from_patches_2d(patches, (4, 4, 3))
>>> np.testing.assert_array_equal(one_image, reconstructed)
```

The `PatchExtractor` class works in the same way as `extract_patches_2d`, only it supports multiple images as input. It is implemented as an estimator, so it can be used in pipelines. See:

```python
>>> five_images = np.arange(5 * 4 * 4 * 3).reshape(5, 4, 4, 3)
>>> patches = image.PatchExtractor((2, 2)).transform(five_images)
>>> patches.shape
(45, 2, 2, 3)
```

### Connectivity graph of an image

Several estimators in the scikit-learn can use connectivity information between features or samples. For instance Ward clustering ([Hierarchical clustering](#)) can cluster together only neighboring pixels of an image, thus forming contiguous patches:

For this purpose, the estimators use a ‘connectivity’ matrix, giving which samples are connected.

The function `img_to_graph` returns such a matrix from a 2D or 3D image. Similarly, `grid_to_graph` build a connectivity matrix for images given the shape of these image.

These matrices can be used to impose connectivity in estimators that use connectivity information, such as Ward clustering ([Hierarchical clustering](#)), but also to build precomputed kernels, or similarity matrices.

**Note:** Examples

- [A demo of structured Ward hierarchical clustering on a raccoon face image](#)
- [Spectral clustering for image segmentation](#)
- [Feature agglomeration vs. univariate selection](#)

### 3.4.3 Preprocessing data

The `sklearn.preprocessing` package provides several common utility functions and transformer classes to change raw feature vectors into a representation that is more suitable for the downstream estimators.

3.4. Dataset transformations
Standardization, or mean removal and variance scaling

Standardization of datasets is a common requirement for many machine learning estimators implemented in scikit-learn; they might behave badly if the individual features do not more or less look like standard normally distributed data: Gaussian with zero mean and unit variance.

In practice we often ignore the shape of the distribution and just transform the data to center it by removing the mean value of each feature, then scale it by dividing non-constant features by their standard deviation.

For instance, many elements used in the objective function of a learning algorithm (such as the RBF kernel of Support Vector Machines or the l1 and l2 regularizers of linear models) assume that all features are centered around zero and have variance in the same order. If a feature has a variance that is orders of magnitude larger than others, it might dominate the objective function and make the estimator unable to learn from other features correctly as expected.

The function `scale` provides a quick and easy way to perform this operation on a single array-like dataset:

```python
>>> from sklearn import preprocessing
>>> import numpy as np

>>> X = np.array([[ 1., -1., 2.], ...
                   [ 2., 0., 0.], ...
                   [ 0., 1., -1.]])
>>> X_scaled = preprocessing.scale(X)

>>> X_scaled
array([[ 0. ..., -1.22..., 1.33...],
       [ 1.22..., 0. ..., -0.26...],
       [-1.22..., 1.22..., -1.06...]])
```

Scaled data has zero mean and unit variance:

```python
>>> X_scaled.mean(axis=0)
array([ 0., 0., 0.])

>>> X_scaled.std(axis=0)
array([ 1., 1., 1.])
```

The `preprocessing` module further provides a utility class `StandardScaler` that implements the Transformer API to compute the mean and standard deviation on a training set so as to be able to later reapply the same transformation on the testing set. This class is hence suitable for use in the early steps of a `sklearn.pipeline.Pipeline`:

```python
>>> scaler = preprocessing.StandardScaler().fit(X)
>>> scaler
StandardScaler(copy=True, with_mean=True, with_std=True)

>>> scaler.mean_
array([ 1. ..., 0. ..., 0.33...])

>>> scaler.scale_
array([ 0.81..., 0.81..., 1.24...])

>>> scaler.transform(X)
array([[ 0. ..., -1.22..., 1.33...],
       [ 1.22..., 0. ..., -0.26...],
       [-1.22..., 1.22..., -1.06...]])
```

The scaler instance can then be used on new data to transform it the same way it did on the training set:
It is possible to disable either centering or scaling by either passing `with_mean=False` or `with_std=False` to the constructor of `StandardScaler`.

### Scaling features to a range

An alternative standardization is scaling features to lie between a given minimum and maximum value, often between zero and one, or so that the maximum absolute value of each feature is scaled to unit size. This can be achieved using `MinMaxScaler` or `MaxAbsScaler`, respectively.

The motivation to use this scaling include robustness to very small standard deviations of features and preserving zero entries in sparse data.

Here is an example to scale a toy data matrix to the `[0,1]` range:

```python
>>> X_train = np.array([[ 1., -1., 2.],
...                      [ 2., 0., 0.],
...                      [ 0., 1., -1.]])
...
>>> min_max_scaler = preprocessing.MinMaxScaler()
>>> X_train_minmax = min_max_scaler.fit_transform(X_train)
>>> X_train_minmax
array([[ 0.5,  0. ,  1. ],
       [ 1. ,  0.5,  0.33333333],
       [ 0. ,  1. ,  0. ]])
```

The same instance of the transformer can then be applied to some new test data unseen during the fit call: the same scaling and shifting operations will be applied to be consistent with the transformation performed on the train data:

```python
>>> X_test = np.array([[-3., -1., 4.]])
>>> X_test_minmax = min_max_scaler.transform(X_test)
>>> X_test_minmax
array([[-1.5,  0. ,  1.66666667]])
```

It is possible to introspect the scaler attributes to find about the exact nature of the transformation learned on the training data:

```python
>>> min_max_scaler.scale_
array([ 0.5,  0.5,  0.33...])

>>> min_max_scaler.min_
array([ 0. ,  0.5,  0.33...])
```

If `MinMaxScaler` is given an explicit `feature_range=(min,max)` the full formula is:

\[
X_{std} = \frac{(X - X.min(\text{axis}=0))}{(X.max(\text{axis}=0) - X.min(\text{axis}=0))}
\]

\[
X_{scaled} = X_{std} / (\text{max} - \text{min}) + \text{min}
\]

`MaxAbsScaler` works in a very similar fashion, but scales in a way that the training data lies within the range `[-1,1]` by dividing through the largest maximum value in each feature. It is meant for data that is already centered at zero or sparse data.

Here is how to use the toy data from the previous example with this scaler:
>>> X_train = np.array([[ 1., -1., 2.],
...                     [ 2.,  0.,  0.],
...                     [ 0.,  1., -1.]])
...  
>>> max_abs_scaler = preprocessing.MaxAbsScaler()
>>> X_train_maxabs = max_abs_scaler.fit_transform(X_train)

# doctest +NORMALIZE_WHITESPACE

array([[ 0.5, -1. , 1. ],
        [ 1. ,  0. ,  0. ],
        [ 0. ,  1. , -0.5]])

>>> X_test = np.array([[ -3., -1., 4.]])
>>> X_test_maxabs = max_abs_scaler.transform(X_test)
>>> X_test_maxabs
array([[ -1.5, -1. , 2. ]])

As with `scale`, the module further provides convenience functions `minmax_scale` and `maxabs_scale` if you don’t want to create an object.

### Scaling sparse data

Centering sparse data would destroy the sparseness structure in the data, and thus rarely is a sensible thing to do. However, it can make sense to scale sparse inputs, especially if features are on different scales.

`MaxAbsScaler` and `maxabs_scale` were specifically designed for scaling sparse data, and are the recommended way to go about this. However, `scale` and `StandardScaler` can accept `scipy.sparse` matrices as input, as long as `with_mean=False` is explicitly passed to the constructor. Otherwise a `ValueError` will be raised as silently centering would break the sparsity and would often crash the execution by allocating excessive amounts of memory unintentionally. `RobustScaler` cannot be fitted to sparse inputs, but you can use the `transform` method on sparse inputs.

Note that the scalers accept both Compressed Sparse Rows and Compressed Sparse Columns format (see `scipy.sparse.csr_matrix` and `scipy.sparse.csc_matrix`). Any other sparse input will be converted to the Compressed Sparse Rows representation. To avoid unnecessary memory copies, it is recommended to choose the CSR or CSC representation upstream.

Finally, if the centered data is expected to be small enough, explicitly converting the input to an array using the `toarray` method of sparse matrices is another option.

### Scaling data with outliers

If your data contains many outliers, scaling using the mean and variance of the data is likely to not work very well. In these cases, you can use `robust_scale` and `RobustScaler` as drop-in replacements instead. They use more robust estimates for the center and range of your data.

### References:

Further discussion on the importance of centering and scaling data is available on this FAQ: Should I normalize/standardize/rescale the data?
Scaling vs Whitening

It is sometimes not enough to center and scale the features independently, since a downstream model can further make some assumption on the linear independence of the features.

To address this issue you can use `sklearn.decomposition.PCA` or `sklearn.decomposition.RandomizedPCA` with `whiten=True` to further remove the linear correlation across features.

Scaling target variables in regression

`spl` and `StandardScaler` work out-of-the-box with 1d arrays. This is very useful for scaling the target / response variables used for regression.

Centering kernel matrices

If you have a kernel matrix of a kernel $K$ that computes a dot product in a feature space defined by function $phi$, a `KernelCenterer` can transform the kernel matrix so that it contains inner products in the feature space defined by $phi$ followed by removal of the mean in that space.

Normalization

Normalization is the process of scaling individual samples to have unit norm. This process can be useful if you plan to use a quadratic form such as the dot-product or any other kernel to quantify the similarity of any pair of samples.

This assumption is the base of the Vector Space Model often used in text classification and clustering contexts.

The function `normalize` provides a quick and easy way to perform this operation on a single array-like dataset, either using the l1 or l2 norms:

```python
>>> X = [[ 1., -1., 2.],
      ... [ 2., 0., 0.],
      ... [ 0., 1., -1.]]
>>> X_normalized = preprocessing.normalize(X, norm='l2')
>>> X_normalized
array([[ 0.40..., -0.40..., 0.81...],
       [ 1. ..., 0. ..., 0. ...],
       [ 0. ..., 0.70..., -0.70...]])
```

The `preprocessing` module further provides a utility class `Normalizer` that implements the same operation using the Transformer API (even though the `fit` method is useless in this case: the class is stateless as this operation treats samples independently).

This class is hence suitable for use in the early steps of a `sklearn.pipeline.Pipeline`:

```python
>>> normalizer = preprocessing.Normalizer().fit(X)  # fit does nothing
>>> normalizer
Normalizer(copy=True, norm='l2')
```

The normalizer instance can then be used on sample vectors as any transformer:
>>> normalizer.transform(X)
array([[ 0.40..., -0.40..., 0.81...],
       [ 1. ..., 0. ..., 0. ...],
       [ 0. ..., 0.70..., -0.70...]])

>>> normalizer.transform([[1., 1., 0.]])
array([[0.70..., 0.70..., 0. ...]])

Sparse input

normalize and Normalizer accept both dense array-like and sparse matrices from scipy.sparse as input.
For sparse input the data is converted to the Compressed Sparse Rows representation (see scipy.sparse.csr_matrix) before being fed to efficient Cython routines. To avoid unnecessary memory copies, it is recommended to choose the CSR representation upstream.

Binarization

Feature binarization

Feature binarization is the process of thresholding numerical features to get boolean values. This can be useful for downstream probabilistic estimators that make assumption that the input data is distributed according to a multi-variate Bernoulli distribution. For instance, this is the case for the sklearn.neural_network.BernoulliRBM.

It is also common among the text processing community to use binary feature values (probably to simplify the probabilistic reasoning) even if normalized counts (a.k.a. term frequencies) or TF-IDF valued features often perform slightly better in practice.

As for the Normalizer, the utility class Binarizer is meant to be used in the early stages of sklearn.pipeline.Pipeline. The fit method does nothing as each sample is treated independently of others:

>>> X = [[ 1., -1., 2.],
      [ 2., 0., 0.],
      [ 0., 1., -1.]]

>>> binarizer = preprocessing.Binarizer().fit(X)  # fit does nothing
>>> binarizer
Binarizer(copy=True, threshold=0.0)

>>> binarizer.transform(X)
array([[ 1., 0., 1.],
      [ 1., 0., 0.],
      [ 0., 1., 0.]])

It is possible to adjust the threshold of the binarizer:

>>> binarizer = preprocessing.Binarizer(threshold=1.1)

>>> binarizer.transform(X)
array([[ 0., 0., 1.],
      [ 1., 0., 0.],
      [ 0., 0., 0.]])
As for the `StandardScaler` and `Normalizer` classes, the preprocessing module provides a companion function `binarize` to be used when the transformer API is not necessary.

**Sparse input**

`binarize` and `Binarizer` accept both dense array-like and sparse matrices from scipy.sparse as input. For sparse input the data is converted to the Compressed Sparse Rows representation (see `scipy.sparse.csr_matrix`). To avoid unnecessary memory copies, it is recommended to choose the CSR representation upstream.

### Encoding categorical features

Often features are not given as continuous values but categorical. For example a person could have features `["male", "female"], ["from Europe", "from US", "from Asia"], ["uses Firefox", "uses Chrome", "uses Safari", "uses Internet Explorer"]`. Such features can be efficiently coded as integers, for instance `["male", "from US", "uses Internet Explorer"]` could be expressed as `[0, 1, 3]` while `["female", "from Asia", "uses Chrome"]` would be `[1, 2, 1]`.

Such integer representation can not be used directly with scikit-learn estimators, as these expect continuous input, and would interpret the categories as being ordered, which is often not desired (i.e. the set of browsers was ordered arbitrarily).

One possibility to convert categorical features to features that can be used with scikit-learn estimators is to use a one-of-K or one-hot encoding, which is implemented in `OneHotEncoder`. This estimator transforms each categorical feature with `m` possible values into `m` binary features, with only one active.

Continuing the example above:

```python
>>> enc = preprocessing.OneHotEncoder()
>>> enc.fit([[0, 0, 3], [1, 1, 0], [0, 2, 1], [1, 0, 2]])
OneHotEncoder(categorical_features='all', dtype=<... 'numpy.float64'>,
   handle_unknown='error', n_values='auto', sparse=True)
>>> enc.transform([[0, 1, 3]]).toarray()
array([[ 1., 0., 0., 0., 1.]])
```

By default, how many values each feature can take is inferred automatically from the dataset. It is possible to specify this explicitly using the parameter `n_values`. There are two genders, three possible continents and four web browsers in our dataset. Then we fit the estimator, and transform a data point. In the result, the first two numbers encode the gender, the next set of three numbers the continent and the last four the web browser.

Note that, if there is a possibility that the training data might have missing categorical features, one has to explicitly set `n_values`. For example,

```python
>>> enc = preprocessing.OneHotEncoder(n_values=[2, 3, 4])
>>> # Note that there are missing categorical values for the 2nd and 3rd
categorical features
>>> enc.fit([[1, 2, 3], [0, 2, 0]])
OneHotEncoder(categorical_features='all', dtype=<... 'numpy.float64'>,
   handle_unknown='error', n_values=[2, 3, 4], sparse=True)
>>> enc.transform([[1, 0, 0]]).toarray()
array([[ 0., 1., 1., 0., 0., 0., 0.]])
```

See `Loading features from dicts` for categorical features that are represented as a dict, not as integers.
Imputation of missing values

For various reasons, many real world datasets contain missing values, often encoded as blanks, NaNs or other placeholders. Such datasets however are incompatible with scikit-learn estimators which assume that all values in an array are numerical, and that all have and hold meaning. A basic strategy to use incomplete datasets is to discard entire rows and/or columns containing missing values. However, this comes at the price of losing data which may be valuable (even though incomplete). A better strategy is to impute the missing values, i.e., to infer them from the known part of the data.

The `Imputer` class provides basic strategies for imputing missing values, either using the mean, the median or the most frequent value of the row or column in which the missing values are located. This class also allows for different missing values encodings.

The following snippet demonstrates how to replace missing values, encoded as `np.nan`, using the mean value of the columns (axis 0) that contain the missing values:

```python
>>> import numpy as np
>>> from sklearn.preprocessing import Imputer
>>> imp = Imputer(missing_values='NaN', strategy='mean', axis=0)
>>> imp.fit([[1, 2], [np.nan, 3], [7, 6]])
Imputer(axis=0, copy=True, missing_values='NaN', strategy='mean', verbose=0)
>>> X = [[np.nan, 2], [6, np.nan], [7, 6]]
>>> print(imp.transform(X))
[[ 4.  2.]
 [ 6. 3.666...]
 [ 7.  6.]]
```

The `Imputer` class also supports sparse matrices:

```python
>>> import scipy.sparse as sp
>>> X = sp.csc_matrix([[1, 2], [0, 3], [7, 6]])
>>> imp = Imputer(missing_values=0, strategy='mean', axis=0)
>>> imp.fit(X)
Imputer(axis=0, copy=True, missing_values=0, strategy='mean', verbose=0)
>>> X_test = sp.csc_matrix([[0, 2], [6, 0], [7, 6]])
>>> print(imp.transform(X_test))
[[ 4.  2.]
 [ 6. 3.666...]
 [ 7.  6.]]
```

Note that, here, missing values are encoded by 0 and are thus implicitly stored in the matrix. This format is thus suitable when there are many more missing values than observed values.

`Imputer` can be used in a Pipeline as a way to build a composite estimator that supports imputation. See Imputing missing values before building an estimator

Generating polynomial features

Often it’s useful to add complexity to the model by considering nonlinear features of the input data. A simple and common method to use is polynomial features, which can get features’ high-order and interaction terms. It is implemented in `PolynomialFeatures`:

```python
>>> import numpy as np
>>> from sklearn.preprocessing import PolynomialFeatures
>>> X = np.arange(6).reshape(3, 2)
>>> print(X)
[[ 0.  1.]
 [ 2.  3.]
 [ 4.  5.]]
```
The features of X have been transformed from $\mathbf{X} = (X_1, X_2)$ to $\mathbf{X} = (1, X_1, X_2, X_1^2, X_1X_2, X_2^2)$.

In some cases, only interaction terms among features are required, and it can be gotten with the setting `interaction_only=True`:

```python
>>> X = np.arange(9).reshape(3, 3)
>>> X
array([[0, 1, 2],
       [3, 4, 5],
       [6, 7, 8]])
>>> poly = PolynomialFeatures(degree=3, interaction_only=True)
>>> poly.fit_transform(X)
array([[ 1., 0., 1., 2., 0., 0., 2., 0.],
       [ 1., 3., 4., 5., 12., 15., 20., 60.],
       [ 1., 6., 7., 8., 42., 48., 56., 336.]])
```

The features of X have been transformed from $\mathbf{X} = (X_1, X_2, X_3)$ to $\mathbf{X} = (1, X_1, X_2, X_3, X_1X_2, X_1X_3, X_2X_3, X_1X_2X_3)$.

Note that polynomial features are used implicitly in kernel methods (e.g., `sklearn.svm.SVC`, `sklearn.decomposition.KernelPCA`) when using polynomial kernel functions.

See Polynomial interpolation for Ridge regression using created polynomial features.

### Custom transformers

Often, you will want to convert an existing Python function into a transformer to assist in data cleaning or processing. You can implement a transformer from an arbitrary function with `FunctionTransformer`. For example, to build a transformer that applies a log transformation in a pipeline, do:

```python
>>> import numpy as np
>>> from sklearn.preprocessing import FunctionTransformer
>>> transformer = FunctionTransformer(np.log1p)
>>> X = np.array([[0. , 1.09861229, 1.38629436]])
>>> transformer.transform(X)
array([[ 0. , 0.69314718, 0.69314718],
       [ 1.09861229, 1.38629436]])
```

For a full code example that demonstrates using a `FunctionTransformer` to do custom feature selection, see Using `FunctionTransformer` to select columns.

### 3.4.4 Unsupervised dimensionality reduction

If your number of features is high, it may be useful to reduce it with an unsupervised step prior to supervised steps. Many of the Unsupervised learning methods implement a `transform` method that can be used to reduce the dimensionality. Below we discuss two specific example of this pattern that are heavily used.
Pipelining

The unsupervised data reduction and the supervised estimator can be chained in one step. See *Pipeline: chaining estimators*.

PCA: principal component analysis

decomposition.PCA looks for a combination of features that capture well the variance of the original features. See *Decomposing signals in components (matrix factorization problems)*.

Examples

- *Faces recognition example using eigenfaces and SVMs*

Random projections

The module: random_projection provides several tools for data reduction by random projections. See the relevant section of the documentation: *Random Projection*.

Examples

- *The Johnson-Lindenstrauss bound for embedding with random projections*

Feature agglomeration

cluster.FeatureAgglomeration applies *Hierarchical clustering* to group together features that behave similarly.

Examples

- *Feature agglomeration vs. univariate selection*
- *Feature agglomeration*

Feature scaling

Note that if features have very different scaling or statistical properties, cluster.FeatureAgglomeration may not be able to capture the links between related features. Using a preprocessing.StandardScaler can be useful in these settings.

3.4.5 Random Projection

The sklearn.random_projection module implements a simple and computationally efficient way to reduce the dimensionality of the data by trading a controlled amount of accuracy (as additional variance) for faster processing
times and smaller model sizes. This module implements two types of unstructured random matrix: Gaussian random matrix and sparse random matrix.

The dimensions and distribution of random projections matrices are controlled so as to preserve the pairwise distances between any two samples of the dataset. Thus random projection is a suitable approximation technique for distance based method.

References:


The Johnson-Lindenstrauss lemma

The main theoretical result behind the efficiency of random projection is the Johnson-Lindenstrauss lemma (quoting Wikipedia):

In mathematics, the Johnson-Lindenstrauss lemma is a result concerning low-distortion embeddings of points from high-dimensional into low-dimensional Euclidean space. The lemma states that a small set of points in a high-dimensional space can be embedded into a space of much lower dimension in such a way that distances between the points are nearly preserved. The map used for the embedding is at least Lipschitz, and can even be taken to be an orthogonal projection.

Knowing only the number of samples, the `sklearn.random_projection.johnson_lindenstrauss_min_dim` estimates conservatively the minimal size of the random subspace to guarantee a bounded distortion introduced by the random projection:

```python
>>> from sklearn.random_projection import johnson_lindenstrauss_min_dim
>>> johnson_lindenstrauss_min_dim(n_samples=1e6, eps=0.5)
663
>>> johnson_lindenstrauss_min_dim(n_samples=1e6, eps=[0.5, 0.1, 0.01])
array([ 663, 11841, 1112658])
>>> johnson_lindenstrauss_min_dim(n_samples=[1e4, 1e5, 1e6], eps=0.1)
array([ 7894,  9868, 11841])
```

Example:

- See *The Johnson-Lindenstrauss bound for embedding with random projections* for a theoretical explication on the Johnson-Lindenstrauss lemma and an empirical validation using sparse random matrices.

References:

Gaussian random projection

The `sklearn.random_projection.GaussianRandomProjection` reduces the dimensionality by projecting the original input space on a randomly generated matrix where components are drawn from the following distribution $N(0, \frac{1}{n_{\text{components}}})$.

Here a small excerpt which illustrates how to use the Gaussian random projection transformer:

```python
>>> import numpy as np
>>> from sklearn import random_projection
>>> X = np.random.rand(100, 10000)
>>> transformer = random_projection.GaussianRandomProjection()
>>> X_new = transformer.fit_transform(X)
>>> X_new.shape
(100, 3947)
```

Sparse random projection

The `sklearn.random_projection.SparseRandomProjection` reduces the dimensionality by projecting the original input space using a sparse random matrix.

Sparse random matrices are an alternative to dense Gaussian random projection matrix that guarantees similar embedding quality while being much more memory efficient and allowing faster computation of the projected data.

If we define $s = \frac{1}{\text{density}}$, the elements of the random matrix are drawn from

\[
\begin{cases}
-\sqrt{s} & 1/2s \\
0 & 1 - 1/s \\
+\sqrt{s} & 1/2s \\
\end{cases}
\]

where $n_{\text{components}}$ is the size of the projected subspace. By default the density of non zero elements is set to the minimum density as recommended by Ping Li et al.: $1/\sqrt{n_{\text{features}}}$.

Here a small excerpt which illustrates how to use the sparse random projection transformer:

```python
>>> import numpy as np
>>> from sklearn import random_projection
>>> X = np.random.rand(100,10000)
>>> transformer = random_projection.SparseRandomProjection()
>>> X_new = transformer.fit_transform(X)
>>> X_new.shape
(100, 3947)
```

References:

3.4.6 Kernel Approximation

This submodule contains functions that approximate the feature mappings that correspond to certain kernels, as they are used for example in support vector machines (see Support Vector Machines). The following feature functions perform non-linear transformations of the input, which can serve as a basis for linear classification or other algorithms.

The advantage of using approximate explicit feature maps compared to the kernel trick, which makes use of feature maps implicitly, is that explicit mappings can be better suited for online learning and can significantly reduce the cost of learning with very large datasets. Standard kernelized SVMs do not scale well to large datasets, but using an approximate kernel map it is possible to use much more efficient linear SVMs. In particular, the combination of kernel map approximations with SGDClassifier can make non-linear learning on large datasets possible.

Since there has not been much empirical work using approximate embeddings, it is advisable to compare results against exact kernel methods when possible.

See also:
Polynomial regression: extending linear models with basis functions for an exact polynomial transformation.

Nystroem Method for Kernel Approximation

The Nystroem method, as implemented in Nystroem is a general method for low-rank approximations of kernels. It achieves this by essentially subsampling the data on which the kernel is evaluated. By default Nystroem uses the rbf kernel, but it can use any kernel function or a precomputed kernel matrix. The number of samples used - which is also the dimensionality of the features computed - is given by the parameter n_components.

Radial Basis Function Kernel

The RBFSampler constructs an approximate mapping for the radial basis function kernel, also known as Random Kitchen Sinks [RR2007]. This transformation can be used to explicitly model a kernel map, prior to applying a linear algorithm, for example a linear SVM:

```python
>>> from sklearn.kernel_approximation import RBFSampler
>>> from sklearn.linear_model import SGDClassifier

>>> X = [[0, 0], [1, 1], [1, 0], [0, 1]]
>>> y = [0, 0, 1, 1]
>>> rbf_feature = RBFSampler(gamma=1, random_state=1)
>>> X_features = rbf_feature.fit_transform(X)
>>> clf = SGDClassifier()
>>> clf.fit(X_features, y)
SGDClassifier(alpha=0.0001, average=False, class_weight=None, epsilon=0.1,
  eta0=0.0, fit_intercept=True, l1_ratio=0.15,
  learning_rate='optimal', loss='hinge', n_iter=5, n_jobs=1,
  penalty='l2', power_t=0.5, random_state=None, shuffle=True,
  verbose=0, warm_start=False)

>>> clf.score(X_features, y)
1.0
```

The mapping relies on a Monte Carlo approximation to the kernel values. The fit function performs the Monte Carlo sampling, whereas the transform method performs the mapping of the data. Because of the inherent randomness of the process, results may vary between different calls to the fit function.

The fit function takes two arguments: n_components, which is the target dimensionality of the feature transform, and gamma, the parameter of the RBF-kernel. A higher n_components will result in a better approximation of the kernel and will yield results more similar to those produced by a kernel SVM. Note that “fitting” the feature function does not actually depend on the data given to the fit function. Only the dimensionality of the data is used. Details on the method can be found in [RR2007].
For a given value of `n_components` `RBFSampler` is often less accurate as `Nystroem`. `RBFSampler` is cheaper to compute, though, making use of larger feature spaces more efficient.

![Fig. 3.9: Comparing an exact RBF kernel (left) with the approximation (right)](image)

**Examples:**

- **Explicit feature map approximation for RBF kernels**

### Additive Chi Squared Kernel

The additive chi squared kernel is a kernel on histograms, often used in computer vision.

The additive chi squared kernel as used here is given by

$$ k(x, y) = \sum_i \frac{2x_i y_i}{x_i + y_i} $$

This is not exactly the same as `sklearn.metrics.additive_chi2_kernel`. The authors of [VZ2010] prefer the version above as it is always positive definite. Since the kernel is additive, it is possible to treat all components $x_i$ separately for embedding. This makes it possible to sample the Fourier transform in regular intervals, instead of approximating using Monte Carlo sampling.

The class `AdditiveChi2Sampler` implements this component wise deterministic sampling. Each component is sampled $n$ times, yielding $2n + 1$ dimensions per input dimension (the multiple of two stems from the real and complex part of the Fourier transform). In the literature, $n$ is usually chosen to be 1 or 2, transforming the dataset to size $\text{n_samples} \times 5 \times \text{n_features}$ (in the case of $n = 2$).

The approximate feature map provided by `AdditiveChi2Sampler` can be combined with the approximate feature map provided by `RBFSampler` to yield an approximate feature map for the exponentiated chi squared kernel. See the [VZ2010] for details and [VVZ2010] for combination with the `RBFSampler`.

### Skewed Chi Squared Kernel

The skewed chi squared kernel is given by:

$$ k(x, y) = \prod_i \frac{2\sqrt{x_i + c\sqrt{y_i + c}}}{x_i + y_i + 2c} $$

It has properties that are similar to the exponentiated chi squared kernel often used in computer vision, but allows for a simple Monte Carlo approximation of the feature map.

The usage of the `SkewedChi2Sampler` is the same as the usage described above for the `RBFSampler`. The only difference is in the free parameter, that is called $c$. For a motivation for this mapping and the mathematical details see [LS2010].
Mathematical Details

Kernel methods like support vector machines or kernelized PCA rely on a property of reproducing kernel Hilbert spaces. For any positive definite kernel function \( k \) (a so called Mercer kernel), it is guaranteed that there exists a mapping \( \phi \) into a Hilbert space \( \mathcal{H} \), such that

\[
k(x, y) = \langle \phi(x), \phi(y) \rangle
\]

Where \( \langle \cdot, \cdot \rangle \) denotes the inner product in the Hilbert space.

If an algorithm, such as a linear support vector machine or PCA, relies only on the scalar product of data points \( x_i \), one may use the value of \( k(x_i, x_j) \), which corresponds to applying the algorithm to the mapped data points \( \phi(x_i) \). The advantage of using \( k \) is that the mapping \( \phi \) never has to be calculated explicitly, allowing for arbitrary large features (even infinite).

One drawback of kernel methods is, that it might be necessary to store many kernel values \( k(x_i, x_j) \) during optimization. If a kernelized classifier is applied to new data \( y_j \), \( k(x_i, y_j) \) needs to be computed to make predictions, possibly for many different \( x_i \) in the training set.

The classes in this submodule allow to approximate the embedding \( \phi \), thereby working explicitly with the representations \( \phi(x_i) \), which obviates the need to apply the kernel or store training examples.

References:

3.4.7 Pairwise metrics, Affinities and Kernels

The `sklearn.metrics.pairwise` submodule implements utilities to evaluate pairwise distances or affinity of sets of samples.

This module contains both distance metrics and kernels. A brief summary is given on the two here.

Distance metrics are functions \( d(a, b) \) such that \( d(a, b) < d(a, c) \) if objects \( a \) and \( b \) are considered “more similar” than objects \( a \) and \( c \). Two objects exactly alike would have a distance of zero. One of the most popular examples is Euclidean distance. To be a ‘true’ metric, it must obey the following four conditions:

1. \( d(a, b) \geq 0 \), for all \( a \) and \( b \)
2. \( d(a, b) = 0 \), if and only if \( a = b \), positive definiteness
3. \( d(a, b) = d(b, a) \), symmetry
4. \( d(a, c) \leq d(a, b) + d(b, c) \), the triangle inequality

Kernels are measures of similarity, i.e. \( s(a, b) > s(a, c) \) if objects \( a \) and \( b \) are considered “more similar” than objects \( a \) and \( c \). A kernel must also be positive semi-definite.

There are a number of ways to convert between a distance metric and a similarity measure, such as a kernel. Let \( D \) be the distance, and \( S \) be the kernel:

1. \( S = \exp(-D \times \text{gamma}) \), where one heuristic for choosing \( \text{gamma} \) is \( 1 / \text{num_features} \)
2. \( S = 1 / (D / \text{np.max}(D)) \)

Cosine similarity

`cosine_similarity` computes the L2-normalized dot product of vectors. That is, if \( x \) and \( y \) are row vectors, their cosine similarity \( k \) is defined as:

\[
k(x, y) = \frac{xy^\top}{\|x\|\|y\|}
\]
This is called cosine similarity, because Euclidean (L2) normalization projects the vectors onto the unit sphere, and their dot product is then the cosine of the angle between the points denoted by the vectors.

This kernel is a popular choice for computing the similarity of documents represented as tf-idf vectors. \texttt{cosine_similarity} accepts \texttt{scipy.sparse} matrices. (Note that the tf-idf functionality in \texttt{sklearn.feature_extraction.text} can produce normalized vectors, in which case \texttt{cosine_similarity} is equivalent to \texttt{linear_kernel}, only slower.)

\begin{quote}
\textbf{References:}
\begin{itemize}
\end{itemize}
\end{quote}

\section*{Linear kernel}

The function \texttt{linear_kernel} computes the linear kernel, that is, a special case of \texttt{polynomial_kernel} with degree=1 and coef0=0 (homogeneous). If \(x\) and \(y\) are column vectors, their linear kernel is:

\[ k(x, y) = x^\top y \]

\section*{Polynomial kernel}

The function \texttt{polynomial_kernel} computes the degree-d polynomial kernel between two vectors. The polynomial kernel represents the similarity between two vectors. Conceptually, the polynomial kernels considers not only the similarity between vectors under the same dimension, but also across dimensions. When used in machine learning algorithms, this allows to account for feature interaction.

The polynomial kernel is defined as:

\[ k(x, y) = (\gamma x^\top y + c_0)^d \]

where:

\begin{itemize}
  \item \(x, y\) are the input vectors
  \item \(d\) is the kernel degree
\end{itemize}

If \(c_0 = 0\) the kernel is said to be homogeneous.

\section*{Sigmoid kernel}

The function \texttt{sigmoid_kernel} computes the sigmoid kernel between two vectors. The sigmoid kernel is also known as hyperbolic tangent, or Multilayer Perceptron (because, in the neural network field, it is often used as neuron activation function). It is defined as:

\[ k(x, y) = \tanh(\gamma x^\top y + c_0) \]

where:

\begin{itemize}
  \item \(x, y\) are the input vectors
  \item \(\gamma\) is known as slope
  \item \(c_0\) is known as intercept
\end{itemize}
RBF kernel

The function `rbf_kernel` computes the radial basis function (RBF) kernel between two vectors. This kernel is defined as:

\[ k(x, y) = \exp(-\gamma ||x - y||^2) \]

where \( x \) and \( y \) are the input vectors. If \( \gamma = \sigma^{-2} \) the kernel is known as the Gaussian kernel of variance \( \sigma^2 \).

Laplacian kernel

The function `laplacian_kernel` is a variant on the radial basis function kernel defined as:

\[ k(x, y) = \exp(-\gamma ||x - y||_1) \]

where \( x \) and \( y \) are the input vectors and \( ||x - y||_1 \) is the Manhattan distance between the input vectors.

It has proven useful in ML applied to noiseless data. See e.g. Machine learning for quantum mechanics in a nutshell.

Chi-squared kernel

The chi-squared kernel is a very popular choice for training non-linear SVMs in computer vision applications. It can be computed using `chi2_kernel` and then passed to an `sklearn.svm.SVC` with `kernel="precomputed"`:

```python
>>> from sklearn.svm import SVC
>>> from sklearn.metrics.pairwise import chi2_kernel
>>> X = [[0, 1], [1, 0], [.2, .8], [.7, .3]]
>>> y = [0, 1, 0, 1]
>>> K = chi2_kernel(X, gamma=.5)
>>> K
array([[ 1.        , 0.36..., 0.89..., 0.58...],
       [ 0.36..., 1.        , 0.51..., 0.83...],
       [ 0.89..., 0.51..., 1.        , 0.77... ],
       [ 0.58..., 0.83..., 0.77..., 1.        ]])

>>> svm = SVC(kernel='precomputed').fit(K, y)
>>> svm.predict(K)
array([0, 1, 0, 1])
```

It can also be directly used as the `kernel` argument:

```python
>>> svm = SVC(kernel=chi2_kernel).fit(X, y)
>>> svm.predict(X)
array([0, 1, 0, 1])
```

The chi-squared kernel is given by

\[ k(x, y) = \exp\left(-\gamma \sum_i \frac{(x[i] - y[i])^2}{x[i] + y[i]}\right) \]

The data is assumed to be non-negative, and is often normalized to have an L1-norm of one. The normalization is rationalized with the connection to the chi-squared distance, which is a distance between discrete probability distributions.

The chi squared kernel is most commonly used on histograms (bags) of visual words.
References:


3.4.8 Transforming the prediction target (y)

Label binarization

_LabelBinarizer_ is a utility class to help create a label indicator matrix from a list of multi-class labels:

```python
>>> from sklearn import preprocessing
>>> lb = preprocessing.LabelBinarizer()
>>> lb.fit([1, 2, 6, 4, 2])
LabelBinarizer(neg_label=0, pos_label=1, sparse_output=False)
>>> lb.classes_
array([1, 2, 4, 6])
>>> lb.transform([1, 6])
array([[1, 0, 0, 0],
       [0, 0, 0, 1]])
```

For multiple labels per instance, use _MultiLabelBinarizer_:

```python
>>> lb = preprocessing.MultiLabelBinarizer()
>>> lb.fit_transform([(1, 2), (3,)])
array([[1, 1, 0],
       [0, 0, 1]])
>>> lb.classes_
array([1, 2, 3])
```

Label encoding

_LabelEncoder_ is a utility class to help normalize labels such that they contain only values between 0 and n_classes-1. This is sometimes useful for writing efficient Cython routines. _LabelEncoder_ can be used as follows:

```python
>>> from sklearn import preprocessing
>>> le = preprocessing.LabelEncoder()
>>> le.fit([1, 2, 2, 6])
LabelEncoder()  
>>> le.classes_
array([1, 2, 6])
>>> le.transform([1, 1, 2, 6])
array([0, 0, 1, 2])
>>> le.inverse_transform([0, 0, 1, 2])
array([1, 1, 2, 6])
```

It can also be used to transform non-numerical labels (as long as they are hashable and comparable) to numerical labels:

```python
>>> le = preprocessing.LabelEncoder()
>>> le.fit(['paris', 'paris', 'tokyo', 'amsterdam'])
LabelEncoder()
```

3.4. Dataset transformations
3.5 Dataset loading utilities

The `sklearn.datasets` package embeds some small toy datasets as introduced in the *Getting Started* section.

To evaluate the impact of the scale of the dataset (\(n_{\text{samples}}\) and \(n_{\text{features}}\)) while controlling the statistical properties of the data (typically the correlation and informativeness of the features), it is also possible to generate synthetic data.

This package also features helpers to fetch larger datasets commonly used by the machine learning community to benchmark algorithm on data that comes from the 'real world'.

3.5.1 General dataset API

There are three distinct kinds of dataset interfaces for different types of datasets. The simplest one is the interface for sample images, which is described below in the *Sample images* section.

The dataset generation functions and the `svmlight` loader share a simplistic interface, returning a tuple \((X, y)\) consisting of a \(n_{\text{samples}} \times n_{\text{features}}\) numpy array \(X\) and an array of length \(n_{\text{samples}}\) containing the targets \(y\).

The toy datasets as well as the 'real world' datasets and the datasets fetched from mldata.org have more sophisticated structure. These functions return a dictionary-like object holding at least two items: an array of shape \(n_{\text{samples}} \times n_{\text{features}}\) with key `data` (except for `20newsgroups`) and a numpy array of length \(n_{\text{samples}}\), containing the target values, with key `target`.

The datasets also contain a description in `DESCR` and some contain `feature_names` and `target_names`. See the dataset descriptions below for details.

3.5.2 Toy datasets

scikit-learn comes with a few small standard datasets that do not require to download any file from some external website.

<table>
<thead>
<tr>
<th>Function Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>load_boston([return_X_y])</code></td>
<td>Load and return the boston house-prices dataset (regression).</td>
</tr>
<tr>
<td><code>load_iris([return_X_y])</code></td>
<td>Load and return the iris dataset (classification).</td>
</tr>
<tr>
<td><code>load_diabetes([return_X_y])</code></td>
<td>Load and return the diabetes dataset (regression).</td>
</tr>
<tr>
<td><code>load_digits([n_class, return_X_y])</code></td>
<td>Load and return the digits dataset (classification).</td>
</tr>
<tr>
<td><code>load_linnerud([return_X_y])</code></td>
<td>Load and return the linnerud dataset (multivariate regression).</td>
</tr>
</tbody>
</table>

These datasets are useful to quickly illustrate the behavior of the various algorithms implemented in the scikit. They are however often too small to be representative of real world machine learning tasks.
3.5.3 Sample images

The scikit also embed a couple of sample JPEG images published under Creative Commons license by their authors. Those image can be useful to test algorithms and pipeline on 2D data.

```
load_sample_images()  Load sample images for image manipulation.
load_sample_image(image_name)  Load the numpy array of a single sample image
```

**Warning:** The default coding of images is based on the uint8 dtype to spare memory. Often machine learning algorithms work best if the input is converted to a floating point representation first. Also, if you plan to use `matplotlib.pyplot.imshow` don’t forget to scale to the range 0 - 1 as done in the following example.

**Examples:**

- [Color Quantization using K-Means](#)

3.5.4 Sample generators

In addition, scikit-learn includes various random sample generators that can be used to build artificial datasets of controlled size and complexity.

**Generators for classification and clustering**

These generators produce a matrix of features and corresponding discrete targets.

**Single label**

Both `make_blobs` and `make_classification` create multiclass datasets by allocating each class one or more normally-distributed clusters of points. `make_blobs` provides greater control regarding the centers and standard deviations of each cluster, and is used to demonstrate clustering. `make_classification` specialises in introducing noise by way of: correlated, redundant and uninformative features; multiple Gaussian clusters per class; and linear transformations of the feature space.

`make_gaussian_quantiles` divides a single Gaussian cluster into near-equal-size classes separated by concentric hyperspheres. `make_hastie_10_2` generates a similar binary, 10-dimensional problem.
make_circles and make_moons generate 2d binary classification datasets that are challenging to certain algorithms (e.g. centroid-based clustering or linear classification), including optional Gaussian noise. They are useful for visualisation. produces Gaussian data with a spherical decision boundary for binary classification.

Multilabel

make_multilabel_classification generates random samples with multiple labels, reflecting a bag of words drawn from a mixture of topics. The number of topics for each document is drawn from a Poisson distribution, and the topics themselves are drawn from a fixed random distribution. Similarly, the number of words is drawn from Poisson, with words drawn from a multinomial, where each topic defines a probability distribution over words. Simplifications with respect to true bag-of-words mixtures include:

- Per-topic word distributions are independently drawn, where in reality all would be affected by a sparse base distribution, and would be correlated.
- For a document generated from multiple topics, all topics are weighted equally in generating its bag of words.
- Documents without labels words at random, rather than from a base distribution.

Biclustering
### Generators for regression

**make_regression** produces regression targets as an optionally-sparse random linear combination of random features, with noise. Its informative features may be uncorrelated, or low rank (few features account for most of the variance).

Other regression generators generate functions deterministically from randomized features. **make_sparse_uncorrelated** produces a target as a linear combination of four features with fixed coefficients. Others encode explicitly non-linear relations: **make_friedman1** is related by polynomial and sine transforms; **make_friedman2** includes feature multiplication and reciprocation; and **make_friedman3** is similar with an arctan transformation on the target.

### Generators for manifold learning

**make_s_curve** generates an S curve dataset. 
**make_swiss_roll** generates a swiss roll dataset.

### Generators for decomposition

**make_low_rank_matrix** generates a mostly low rank matrix with bell-shaped singular values. 
**make_sparse_coded_signal** generates a signal as a sparse combination of dictionary elements. 
**make_spd_matrix** generates a random symmetric, positive-definite matrix. 
**make_sparse_spd_matrix** generates a sparse symmetric definite positive matrix.

### 3.5.5 Datasets in svmlight / libsvm format

scikit-learn includes utility functions for loading datasets in the svmlight / libsvm format. In this format, each line takes the form `<label> <feature-id>:<feature-value> <feature-id>:<feature-value> ...`. This format is especially suitable for sparse datasets. In this module, scipy sparse CSR matrices are used for X and numpy arrays are used for y.

You may load a dataset like as follows:

```python
>>> from sklearn.datasets import load_svmlight_file
>>> X_train, y_train = load_svmlight_file("/path/to/train_dataset.txt")
```

You may also load two (or more) datasets at once:

```python
>>> X_train, y_train, X_test, y_test = load_svmlight_files(
...     ("/path/to/train_dataset.txt", "/path/to/test_dataset.txt"))
```
In this case, \(X_{\text{train}}\) and \(X_{\text{test}}\) are guaranteed to have the same number of features. Another way to achieve the same result is to fix the number of features:

```python
>>> X_test, y_test = load_svmlight_file(
    ...   "/path/to/test_dataset.txt", n_features=X_train.shape[1])
```

### Related links:

- Faster API-compatible implementation: [https://github.com/mblondel/svmlight-loader](https://github.com/mblondel/svmlight-loader)

### 3.5.6 Loading from external datasets

`scikit-learn` works on any numeric data stored as numpy arrays or scipy sparse matrices. Other types that are convertible to numeric arrays such as pandas DataFrame are also acceptable.

Here are some recommended ways to load standard columnar data into a format usable by `scikit-learn`:

- `pandas.io` provides tools to read data from common formats including CSV, Excel, JSON and SQL. DataFrames may also be constructed from lists of tuples or dicts. Pandas handles heterogeneous data smoothly and provides tools for manipulation and conversion into a numeric array suitable for `scikit-learn`.
- `scipy.io` specializes in binary formats often used in scientific computing context such as `.mat` and `.arff`
- `numpy/routines.io` for standard loading of columnar data into numpy arrays
- `scikit-learn`’s `datasets.load_svmlight_file` for the svmlight or libSVM sparse format
- `scikit-learn`’s `datasets.load_files` for directories of text files where the name of each directory is the name of each category and each file inside of each directory corresponds to one sample from that category

For some miscellaneous data such as images, videos, and audio, you may wish to refer to:

- `skimage.io` or `Imageio` for loading images and videos to numpy arrays
- `scipy.misc.imread` (requires the `Pillow` package) to load pixel intensities data from various image file formats
- `scipy.io.wavfile.read` for reading WAV files into a numpy array

Categorical (or nominal) features stored as strings (common in pandas DataFrames) will need converting to integers, and integer categorical variables may be best exploited when encoded as one-hot variables (`sklearn.preprocessing.OneHotEncoder`) or similar. See `Preprocessing data`.

Note: if you manage your own numerical data it is recommended to use an optimized file format such as HDF5 to reduce data load times. Various libraries such as H5Py, PyTables and pandas provides a Python interface for reading and writing data in that format.

### The Olivetti faces dataset

This dataset contains a set of face images taken between April 1992 and April 1994 at AT&T Laboratories Cambridge. The `sklearn.datasets.fetch_olivetti_faces` function is the data fetching / caching function that downloads the data archive from AT&T.

As described on the original website:
There are ten different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement).

The image is quantized to 256 grey levels and stored as unsigned 8-bit integers; the loader will convert these to floating point values on the interval [0, 1], which are easier to work with for many algorithms.

The “target” for this database is an integer from 0 to 39 indicating the identity of the person pictured; however, with only 10 examples per class, this relatively small dataset is more interesting from an unsupervised or semi-supervised perspective.

The original dataset consisted of 92 x 112, while the version available here consists of 64x64 images.

When using these images, please give credit to AT&T Laboratories Cambridge.

The 20 newsgroups text dataset

The 20 newsgroups dataset comprises around 18000 newsgroups posts on 20 topics split in two subsets: one for training (or development) and the other one for testing (or for performance evaluation). The split between the train and test set is based upon a messages posted before and after a specific date.

This module contains two loaders. The first one, sklearn.datasets.fetch_20newsgroups, returns a list of the raw texts that can be fed to text feature extractors such as sklearn.feature_extraction.text.CountVectorizer with custom parameters so as to extract feature vectors. The second one, sklearn.datasets.fetch_20newsgroups_vectorized, returns ready-to-use features, i.e., it is not necessary to use a feature extractor.

Usage

The sklearn.datasets.fetch_20newsgroups function is a data fetching / caching functions that downloads the data archive from the original 20 newsgroups website, extracts the archive contents in the ~/scikit_learn_data/20news_home folder and calls the sklearn.datasets.load_files on either the training or testing set folder, or both of them:

```python
>>> from sklearn.datasets import fetch_20newsgroups
>>> newsgroups_train = fetch_20newsgroups(subset='train')

>>> from pprint import pprint
>>> pprint(list(newsgroups_train.target_names))
['alt.atheism',
 'comp.graphics',
 'comp.os.ms-windows.misc',
 'comp.sys.ibm.pc.hardware',
 'comp.sys.mac.hardware',
 'comp.windows.x',
 'misc.forsale',
 'rec.autos',
 'rec.motorcycles',
 'rec.sport.baseball',
 'rec.sport.hockey',
 'sci.crypt',
 'sci.electronics',
 'sci.med',
 'sci.space',
 'soc.religion.christian',
 'talk.politics.guns',
]
```
The real data lies in the filenames and target attributes. The target attribute is the integer index of the category:

```
>>> newsgroups_train.filenames.shape
(11314,)
>>> newsgroups_train.target.shape
(11314,)
>>> newsgroups_train.target[:10]
array([12,  6,  9,  8,  6,  7,  9,  2, 13, 19])
```

It is possible to load only a sub-selection of the categories by passing the list of the categories to load to the `sklearn.datasets.fetch_20newsgroups` function:

```
>>> cats = ['alt.atheism', 'sci.space']
>>> newsgroups_train = fetch_20newsgroups(subset='train', categories=cats)

```

Converting text to vectors

In order to feed predictive or clustering models with the text data, one first need to turn the text into vectors of numerical values suitable for statistical analysis. This can be achieved with the utilities of the `sklearn.feature_extraction.text` as demonstrated in the following example that extract TF-IDF vectors of unigram tokens from a subset of 20news:

```
>>> from sklearn.feature_extraction.text import TfidfVectorizer

```

The extracted TF-IDF vectors are very sparse, with an average of 159 non-zero components by sample in a more than 30000-dimensional space (less than .5% non-zero features):

```
>>> vectors.nnz / float(vectors.shape[0])
159.01327433628319
```

`sklearn.datasets.fetch_20newsgroups_vectorized` is a function which returns ready-to-use tfidf features instead of file names.
Filtering text for more realistic training

It is easy for a classifier to overfit on particular things that appear in the 20 Newsgroups data, such as newsgroup headers. Many classifiers achieve very high F-scores, but their results would not generalize to other documents that aren’t from this window of time.

For example, let’s look at the results of a multinomial Naive Bayes classifier, which is fast to train and achieves a decent F-score:

```python
>>> from sklearn.naive_bayes import MultinomialNB
>>> from sklearn import metrics

>>> newsgroups_test = fetch_20newsgroups(subset='test', categories=categories)
>>> vectors_test = vectorizer.transform(newsgroups_test.data)
>>> clf = MultinomialNB(alpha=.01)
>>> clf.fit(vectors, newsgroups_train.target)
>>> pred = clf.predict(vectors_test)
>>> metrics.f1_score(newsgroups_test.target, pred, average='macro')
0.88213592402729568
```

(The example Classification of text documents using sparse features shuffles the training and test data, instead of segmenting by time, and in that case multinomial Naive Bayes gets a much higher F-score of 0.88. Are you suspicious yet of what’s going on inside this classifier?)

Let’s take a look at what the most informative features are:

```python
>>> import numpy as np
>>> def show_top10(classifier, vectorizer, categories):
...     feature_names = np.asarray(vectorizer.get_feature_names())
...     for i, category in enumerate(categories):
...         top10 = np.argsort(classifier.coef_[i])[-10:]
...         print("%s: %s" % (category, " ".join(feature_names[top10])))

>>> show_top10(clf, vectorizer, newsgroups_train.target_names)
alt.atheism: sgi livesey atheists writes people caltech com god keith edu
comp.graphics: organization thanks files subject com image lines university edu...
sci.space: toronto moon gov com alaska access henry nasa edu space
talk.religion.misc: article writes kent people christian jesus sandvik edu com god
```

You can now see many things that these features have overfit to:

- Almost every group is distinguished by whether headers such as NNTP-Posting-Host: and Distribution: appear more or less often.
- Another significant feature involves whether the sender is affiliated with a university, as indicated either by their headers or their signature.
- The word “article” is a significant feature, based on how often people quote previous posts like this: “In article [article ID], [name] <[e-mail address]> wrote:”
- Other features match the names and e-mail addresses of particular people who were posting at the time.

With such an abundance of clues that distinguish newsgroups, the classifiers barely have to identify topics from text at all, and they all perform at the same high level.

For this reason, the functions that load 20 Newsgroups data provide a parameter called remove, telling it what kinds of information to strip out of each file. remove should be a tuple containing any subset of ('headers', 'footers', 'quotes'), telling it to remove headers, signature blocks, and quotation blocks respectively.
This classifier lost over a lot of its F-score, just because we removed metadata that has little to do with topic classification. It loses even more if we also strip this metadata from the training data:

```python
>>> newsgroups_train = fetch_20newsgroups(subset='train',
    remove=('headers', 'footers', 'quotes'),
    categories=categories)
>>> vectors = vectorizer.fit_transform(newsgroups_train.data)
>>> clf = MultinomialNB(alpha=.01)
>>> clf.fit(vectors, newsgroups_train.target)
>>> vectors_test = vectorizer.transform(newsgroups_test.data)
>>> pred = clf.predict(vectors_test)
>>> metrics.f1_score(newsgroups_test.target, pred, average='macro')
0.76995175184521725
```

Some other classifiers cope better with this harder version of the task. Try running `Sample pipeline for text feature extraction and evaluation` with and without the `--filter` option to compare the results.

**Recommendation**

When evaluating text classifiers on the 20 Newsgroups data, you should strip newsgroup-related metadata. In scikit-learn, you can do this by setting `remove=('headers','footers','quotes')`. The F-score will be lower because it is more realistic.

**Examples**

- `Sample pipeline for text feature extraction and evaluation`
- `Classification of text documents using sparse features`

**Downloading datasets from the mldata.org repository**

mldata.org is a public repository for machine learning data, supported by the PASCAL network.

The `sklearn.datasets` package is able to directly download data sets from the repository using the function `sklearn.datasets.fetch_mldata`.

For example, to download the MNIST digit recognition database:

```python
>>> from sklearn.datasets import fetch_mldata
>>> mnist = fetch_mldata('MNIST original', data_home=custom_data_home)
```

The MNIST database contains a total of 70000 examples of handwritten digits of size 28x28 pixels, labeled from 0 to 9:
After the first download, the dataset is cached locally in the path specified by the `data_home` keyword argument, which defaults to `~/scikit_learn_data/`:

```python
>>> os.listdir(os.path.join(custom_data_home, 'mldata'))
['mnist-original.mat']
```

Data sets in mldata.org do not adhere to a strict naming or formatting convention. `sklearn.datasets.fetch_mldata` is able to make sense of the most common cases, but allows to tailor the defaults to individual datasets:

- The data arrays in mldata.org are most often shaped as `(n_features,n_samples)`. This is the opposite of the scikit-learn convention, so `sklearn.datasets.fetch_mldata` transposes the matrix by default. The `transpose_data` keyword controls this behavior:

```python
>>> iris = fetch_mldata('iris', data_home=custom_data_home)
>>> iris.data.shape
(150, 4)
>>> iris = fetch_mldata('iris', transpose_data=False,
...                     data_home=custom_data_home)
>>> iris.data.shape
(4, 150)
```

- For datasets with multiple columns, `sklearn.datasets.fetch_mldata` tries to identify the target and data columns and rename them to `target` and `data`. This is done by looking for arrays named `label` and `data` in the dataset, and failing that by choosing the first array to be `target` and the second to be `data`. This behavior can be changed with the `target_name` and `data_name` keywords, setting them to a specific name or index number (the name and order of the columns in the datasets can be found at its mldata.org under the tab “Data”):

```python
>>> iris2 = fetch_mldata('datasets-UCI iris', target_name=1, data_name=0,
...                       data_home=custom_data_home)
>>> iris3 = fetch_mldata('datasets-UCI iris', target_name='class',
...                       data_name='double0', data_home=custom_data_home)
```

### The Labeled Faces in the Wild face recognition dataset

This dataset is a collection of JPEG pictures of famous people collected over the internet, all details are available on the official website:


Each picture is centered on a single face. The typical task is called Face Verification: given a pair of two pictures, a binary classifier must predict whether the two images are from the same person.

An alternative task, Face Recognition or Face Identification is: given the picture of the face of an unknown person, identify the name of the person by referring to a gallery of previously seen pictures of identified persons.

Both Face Verification and Face Recognition are tasks that are typically performed on the output of a model trained to perform Face Detection. The most popular model for Face Detection is called Viola-Jones and is implemented in the OpenCV library. The LFW faces were extracted by this face detector from various online websites.
Usage

scikit-learn provides two loaders that will automatically download, cache, parse the metadata files, decode the jpeg and convert the interesting slices into memmaped numpy arrays. This dataset size is more than 200 MB. The first load typically takes more than a couple of minutes to fully decode the relevant part of the JPEG files into numpy arrays. If the dataset has been loaded once, the following times the loading times less than 200ms by using a memmaped version memoized on the disk in the ~scikit_learn_data/lfw_home/ folder using joblib.

The first loader is used for the Face Identification task: a multi-class classification task (hence supervised learning):

```python
>>> from sklearn.datasets import fetch_lfw_people
>>> lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)
>>> for name in lfw_people.target_names:
    ...     print(name)
    ...
Ariel Sharon
Colin Powell
Donald Rumsfeld
George W Bush
Gerhard Schroeder
Hugo Chavez
Tony Blair
```

The default slice is a rectangular shape around the face, removing most of the background:

```python
>>> lfw_people.data.dtype
dtype('float32')

>>> lfw_people.data.shape
(1288, 1850)

>>> lfw_people.images.shape
(1288, 50, 37)
```

Each of the 1140 faces is assigned to a single person id in the target array:

```python
>>> lfw_people.target.shape
(1288,)

>>> list(lfw_people.target[:10])
[5, 6, 3, 1, 0, 1, 3, 4, 3, 0]
```

The second loader is typically used for the face verification task: each sample is a pair of two picture belonging or not to the same person:

```python
>>> from sklearn.datasets import fetch_lfw_pairs
>>> lfw_pairs_train = fetch_lfw_pairs(subset='train')

>>> list(lfw_pairs_train.target_names)
['Different persons', 'Same person']

>>> lfw_pairs_train.pairs.shape
(2200, 2, 62, 47)

>>> lfw_pairs_train.data.shape
(2200, 5828)
```
Both for the `sklearn.datasets.fetch_lfw_people` and `sklearn.datasets.fetch_lfw_pairs` function it is possible to get an additional dimension with the RGB color channels by passing `color=True`, in that case the shape will be `(2200, 2, 62, 47, 3)`.

The `sklearn.datasets.fetch_lfw_pairs` datasets is subdivided into 3 subsets: the development `train` set, the development `test` set and an evaluation `10_folds` set meant to compute performance metrics using a 10-folds cross validation scheme.

### References


### Examples

#### Faces recognition example using eigenfaces and SVMs

#### Forest covetypes

The samples in this dataset correspond to 30×30m patches of forest in the US, collected for the task of predicting each patch’s cover type, i.e. the dominant species of tree. There are seven covetypes, making this a multiclass classification problem. Each sample has 54 features, described on the dataset's homepage. Some of the features are boolean indicators, while others are discrete or continuous measurements.

`sklearn.datasets.fetch_covtype` will load the covtype dataset; it returns a dictionary-like object with the feature matrix in the `data` member and the target values in `target`. The dataset will be downloaded from the web if necessary.

#### RCV1 dataset

Reuters Corpus Volume I (RCV1) is an archive of over 800,000 manually categorized newswire stories made available by Reuters, Ltd. for research purposes. The dataset is extensively described in \(^1\). `sklearn.datasets.fetch_rcv1` will load the following version: RCV1-v2, vectors, full sets, topics multilabels:

```python
>>> from sklearn.datasets import fetch_rcv1
>>>
rcv1 = fetch_rcv1()
```

It returns a dictionary-like object, with the following attributes:

- `data`: The feature matrix is a scipy CSR sparse matrix, with 804414 samples and 47236 features. Non-zero values contains cosine-normalized, log TF-IDF vectors. A nearly chronological split is proposed in \(^1\): The first 23149 samples are the training set. The last 781265 samples are the testing set. This follows the official LYRL2004 chronological split. The array has 0.16% of non zero values:

---

target: The target values are stored in a scipy CSR sparse matrix, with 804414 samples and 103 categories. Each sample has a value of 1 in its categories, and 0 in others. The array has 3.15% of non zero values:

```python
>>> rcv1.target.shape
(804414, 103)
```

sample_id: Each sample can be identified by its ID, ranging (with gaps) from 2286 to 810596:

```python
>>> rcv1.sample_id[:3]
array([2286, 2287, 2288], dtype=uint32)
```

target_names: The target values are the topics of each sample. Each sample belongs to at least one topic, and to up to 17 topics. There are 103 topics, each represented by a string. Their corpus frequencies span five orders of magnitude, from 5 occurrences for ‘GMIL’, to 381327 for ‘CCAT’:

```python
>>> rcv1.target_names[:3].tolist()
['E11', 'ECAT', 'M11']
```

The dataset will be downloaded from the rcv1 homepage if necessary. The compressed size is about 656 MB.

### References

#### 3.5.7 The Olivetti faces dataset

This dataset contains a set of face images taken between April 1992 and April 1994 at AT&T Laboratories Cambridge. The `sklearn.datasets.fetch_olivetti_faces` function is the data fetching / caching function that downloads the data archive from AT&T.

As described on the original website:

> There are ten different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement).

The image is quantized to 256 grey levels and stored as unsigned 8-bit integers; the loader will convert these to floating point values on the interval [0, 1], which are easier to work with for many algorithms.

The “target” for this database is an integer from 0 to 39 indicating the identity of the person pictured; however, with only 10 examples per class, this relatively small dataset is more interesting from an unsupervised or semi-supervised perspective.

The original dataset consisted of 92 x 112, while the version available here consists of 64x64 images.

When using these images, please give credit to AT&T Laboratories Cambridge.

#### 3.5.8 The 20 newsgroups text dataset

The 20 newsgroups dataset comprises around 18000 newsgroups posts on 20 topics split in two subsets: one for training (or development) and the other one for testing (or for performance evaluation). The split between the train and test set is based upon a messages posted before and after a specific date.
This module contains two loaders. The first one, `sklearn.datasets.fetch_20newsgroups`, returns a list of the raw texts that can be fed to text feature extractors such as `sklearn.feature_extraction.text.CountVectorizer` with custom parameters so as to extract feature vectors. The second one, `sklearn.datasets.fetch_20newsgroups_vectorized`, returns ready-to-use features, i.e., it is not necessary to use a feature extractor.

**Usage**

The `sklearn.datasets.fetch_20newsgroups` function is a data fetching / caching functions that downloads the data archive from the original 20 newsgroups website, extracts the archive contents in the `~/scikit_learn_data/20news_home` folder and calls the `sklearn.datasets.load_files` on either the training or testing set folder, or both of them:

```python
>>> from sklearn.datasets import fetch_20newsgroups
>>> newsgroups_train = fetch_20newsgroups(subset='train')

>>> from pprint import pprint
>>> pprint(list(newsgroups_train.target_names))
```

The real data lies in the `filenames` and `target` attributes. The target attribute is the integer index of the category:

```python
>>> newsgroups_train.filenames.shape
(11314,)

>>> newsgroups_train.target.shape
(11314,)

>>> newsgroups_train.target[:10]
array([12, 6, 9, 8, 6, 7, 9, 2, 13, 19])
```

It is possible to load only a sub-selection of the categories by passing the list of the categories to load to the `sklearn.datasets.fetch_20newsgroups` function:

```python
>>> cats = ['alt.atheism', 'sci.space']

>>> newsgroups_train = fetch_20newsgroups(subset='train', categories=cats)

>>> list(newsgroups_train.target_names)
['alt.atheism', 'sci.space']

>>> newsgroups_train.filenames.shape
```

3.5. Dataset loading utilities
Converting text to vectors

In order to feed predictive or clustering models with the text data, one first need to turn the text into vectors of numerical values suitable for statistical analysis. This can be achieved with the utilities of the `sklearn.feature_extraction.text` as demonstrated in the following example that extract TF-IDF vectors of unigram tokens from a subset of 20news:

```python
>>> from sklearn.feature_extraction.text import TfidfVectorizer
... categories = ['alt.atheism', 'talk.religion.misc',
... 'comp.graphics', 'sci.space']
... newsgroups_train = fetch_20newsgroups(subset='train',
... categories=categories)
... vectorizer = TfidfVectorizer()
... vectors = vectorizer.fit_transform(newsgroups_train.data)
... vectors.shape
(2034, 34118)
```

The extracted TF-IDF vectors are very sparse, with an average of 159 non-zero components by sample in a more than 30000-dimensional space (less than .5% non-zero features):

```python
>>> vectors.nnz / float(vectors.shape[0])
159.01327433628319
```

`sklearn.datasets.fetch_20newsgroups_vectorized` is a function which returns ready-to-use tfidf features instead of file names.

Filtering text for more realistic training

It is easy for a classifier to overfit on particular things that appear in the 20 Newsgroups data, such as newsgroup headers. Many classifiers achieve very high F-scores, but their results would not generalize to other documents that aren’t from this window of time.

For example, let’s look at the results of a multinomial Naive Bayes classifier, which is fast to train and achieves a decent F-score:

```python
>>> from sklearn.naive_bayes import MultinomialNB
>>> from sklearn import metrics
... newsgroups_test = fetch_20newsgroups(subset='test',
... categories=categories)
... vectors_test = vectorizer.transform(newsgroups_test.data)
... clf = MultinomialNB(alpha=.01)
... clf.fit(vectors, newsgroups_train.target)
... pred = clf.predict(vectors_test)
... metrics.f1_score(newsgroups_test.target, pred, average='macro')
0.88213592402729568
```

(The example Classification of text documents using sparse features shuffles the training and test data, instead of segmenting by time, and in that case multinomial Naive Bayes gets a much higher F-score of 0.88. Are you suspicious yet of what’s going on inside this classifier?)
Let’s take a look at what the most informative features are:

```python
>>> import numpy as np
>>> def show_top10(classifier, vectorizer, categories):
...     feature_names = np.asarray(vectorizer.get_feature_names())
...     for i, category in enumerate(categories):
...         top10 = np.argsort(classifier.coef_[i])[-10:]
...         print("%s: %s" % (category, " ".join(feature_names[top10])))
...
>>> show_top10(clf, vectorizer, newsgroups_train.target_names)
alt.atheism: sgi livesey atheists writes people caltech com god keith edu
comp.graphics: organization thanks files subject com image lines university edu...
sci.space: toronto moon gov com alaska access henry nasa edu space
talk.religion.misc: article writes kent people christian jesus sandvik edu com god
```

You can now see many things that these features have overfit to:

- Almost every group is distinguished by whether headers such as NNTP-Posting-Host: and Distribution: appear more or less often.
- Another significant feature involves whether the sender is affiliated with a university, as indicated either by their headers or their signature.
- The word “article” is a significant feature, based on how often people quote previous posts like this: “In article [article ID], [name] <[e-mail address]> wrote:”
- Other features match the names and e-mail addresses of particular people who were posting at the time.

With such an abundance of clues that distinguish newsgroups, the classifiers barely have to identify topics from text at all, and they all perform at the same high level.

For this reason, the functions that load 20 Newsgroups data provide a parameter called `remove`, telling it what kinds of information to strip out of each file. `remove` should be a tuple containing any subset of ('headers', 'footers', 'quotes'), telling it to remove headers, signature blocks, and quotation blocks respectively.

```python
>>> newsgroups_test = fetch_20newsgroups(subset='test',
...                                        remove=('headers', 'footers', 'quotes'),
...                                        categories=categories)
>>> vectors_test = vectorizer.transform(newsgroups_test.data)
>>> pred = clf.predict(vectors_test)
>>> metrics.f1_score(pred, newsgroups_test.target, average='macro')
0.77310350681274775
```

This classifier lost over a lot of its F-score, just because we removed metadata that has little to do with topic classification. It loses even more if we also strip this metadata from the training data:

```python
>>> newsgroups_train = fetch_20newsgroups(subset='train',
...                                        remove=('headers', 'footers', 'quotes'),
...                                        categories=categories)
>>> vectors = vectorizer.fit_transform(newsgroups_train.data)
>>> clf = MultinomialNB(alpha=.01)
>>> clf.fit(vectors, newsgroups_train.target)
>>> vectors_test = vectorizer.transform(newsgroups_test.data)
>>> pred = clf.predict(vectors_test)
>>> metrics.f1_score(newsgroups_test.target, pred, average='macro')
0.76995175184521725
```
Some other classifiers cope better with this harder version of the task. Try running Sample pipeline for text feature extraction and evaluation with and without the --filter option to compare the results.

**Recommendation**

When evaluating text classifiers on the 20 Newsgroups data, you should strip newsgroup-related metadata. In scikit-learn, you can do this by setting remove=('headers','footers','quotes'). The F-score will be lower because it is more realistic.

**Examples**

- Sample pipeline for text feature extraction and evaluation
- Classification of text documents using sparse features

### 3.5.9 Downloading datasets from the mldata.org repository

mldata.org is a public repository for machine learning data, supported by the PASCAL network.

The sklearn.datasets package is able to directly download data sets from the repository using the function sklearn.datasets.fetch_mldata.

For example, to download the MNIST digit recognition database:

```python
>>> from sklearn.datasets import fetch_mldata

>>> mnist = fetch_mldata('MNIST original', data_home=custom_data_home)
```

The MNIST database contains a total of 70000 examples of handwritten digits of size 28x28 pixels, labeled from 0 to 9:

```python
>>> mnist.data.shape
(70000, 784)

>>> mnist.target.shape
(70000,)

>>> np.unique(mnist.target)
array([ 0., 1., 2., 3., 4., 5., 6., 7., 8., 9.])
```

After the first download, the dataset is cached locally in the path specified by the data_home keyword argument, which defaults to ~/scikit_learn_data/:

```python
>>> os.listdir(os.path.join(custom_data_home, 'mldata'))
['mnist-original.mat']
```

Data sets in mldata.org do not adhere to a strict naming or formatting convention. sklearn.datasets.fetch_mldata is able to make sense of the most common cases, but allows to tailor the defaults to individual datasets:

- The data arrays in mldata.org are most often shaped as (n_features,n_samples). This is the opposite of the scikit-learn convention, so sklearn.datasets.fetch_mldata transposes the matrix by default. The transpose_data keyword controls this behavior:

```python
>>> iris = fetch_mldata('iris', data_home=custom_data_home)

>>> iris.data.shape
(150, 4)
```
For datasets with multiple columns, `sklearn.datasets.fetch_mldata` tries to identify the target and data columns and rename them to target and data. This is done by looking for arrays named label and data in the dataset, and failing that by choosing the first array to be target and the second to be data. This behavior can be changed with the target_name and data_name keywords, setting them to a specific name or index number (the name and order of the columns in the datasets can be found at its mldata.org under the tab “Data”):

```python
>>> iris2 = fetch_mldata('datasets-UCI iris', target_name=1, data_name=0,
... data_home=custom_data_home)
>>> iris3 = fetch_mldata('datasets-UCI iris', target_name='class',
... data_name='double0', data_home=custom_data_home)
```

### 3.5.10 The Labeled Faces in the Wild face recognition dataset

This dataset is a collection of JPEG pictures of famous people collected over the internet, all details are available on the official website:

http://vis-www.cs.umass.edu/lfw/

Each picture is centered on a single face. The typical task is called Face Verification: given a pair of two pictures, a binary classifier must predict whether the two images are from the same person.

An alternative task, Face Recognition or Face Identification is: given the picture of the face of an unknown person, identify the name of the person by referring to a gallery of previously seen pictures of identified persons.

Both Face Verification and Face Recognition are tasks that are typically performed on the output of a model trained to perform Face Detection. The most popular model for Face Detection is called Viola-Jones and is implemented in the OpenCV library. The LFW faces were extracted by this face detector from various online websites.

**Usage**

`scikit-learn` provides two loaders that will automatically download, cache, parse the metadata files, decode the jpeg and convert the interesting slices into memmapped numpy arrays. This dataset size is more than 200 MB. The first load typically takes more than a couple of minutes to fully decode the relevant part of the JPEG files into numpy arrays. If the dataset has been loaded once, the following times the loading times less than 200ms by using a memmapped version memoized on the disk in the ~/scikit_learn_data/lfw_home/ folder using `joblib`.

The first loader is used for the Face Identification task: a multi-class classification task (hence supervised learning):

```python
>>> from sklearn.datasets import fetch_lfw_people
>>> lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)

>>> for name in lfw_people.target_names:
...     print(name)
...     ...
Ariel Sharon
Colin Powell
Donald Rumsfeld
George W Bush
Gerhard Schroeder
```
scikit-learn user guide, Release 0.18.2

Hugo Chavez
Tony Blair

The default slice is a rectangular shape around the face, removing most of the background:

```python
>>> lfw_people.data.dtype
dtype('float32')
```

```python
>>> lfw_people.data.shape
(1288, 1850)
```

```python
>>> lfw_people.images.shape
(1288, 50, 37)
```

Each of the 1140 faces is assigned to a single person id in the target array:

```python
>>> lfw_people.target.shape
(1288,)
```

```python
>>> list(lfw_people.target[:10])
[5, 6, 3, 1, 0, 1, 3, 4, 3, 0]
```

The second loader is typically used for the face verification task: each sample is a pair of two picture belonging or not to the same person:

```python
>>> from sklearn.datasets import fetch_lfw_pairs
>>> lfw_pairs_train = fetch_lfw_pairs(subset='train')
```

```python
>>> list(lfw_pairs_train.target_names)
['Different persons', 'Same person']
```

```python
>>> lfw_pairs_train.pairs.shape
(2200, 2, 62, 47)
```

```python
>>> lfw_pairs_train.data.shape
(2200, 5828)
```

```python
>>> lfw_pairs_train.target.shape
(2200,)
```

Both for the `sklearn.datasets.fetch_lfw_people` and `sklearn.datasets.fetch_lfw_pairs` function it is possible to get an additional dimension with the RGB color channels by passing `color=True`, in that case the shape will be `(2200,2,62,47,3)`.

The `sklearn.datasets.fetch_lfw_pairs` datasets is subdivided into 3 subsets: the development train set, the development test set and an evaluation 10_folds set meant to compute performance metrics using a 10-folds cross validation scheme.

References:

Examples

Faces recognition example using eigenfaces and SVMs

3.5.11 Forest covetypes

The samples in this dataset correspond to 30x30m patches of forest in the US, collected for the task of predicting each patch’s cover type, i.e. the dominant species of tree. There are seven covertypes, making this a multiclass classification problem. Each sample has 54 features, described on the dataset’s homepage. Some of the features are boolean indicators, while others are discrete or continuous measurements.

sklearn.datasets.fetch_covtype will load the covertype dataset; it returns a dictionary-like object with the feature matrix in the data member and the target values in target. The dataset will be downloaded from the web if necessary.

3.5.12 RCV1 dataset

Reuters Corpus Volume I (RCV1) is an archive of over 800,000 manually categorized newswire stories made available by Reuters, Ltd. for research purposes. The dataset is extensively described in 1.

sklearn.datasets.fetch_rcv1 will load the following version: RCV1-v2, vectors, full sets, topics multilabels:

```python
>>> from sklearn.datasets import fetch_rcv1
>>> rcv1 = fetch_rcv1()
```

It returns a dictionary-like object, with the following attributes:

data: The feature matrix is a scipy CSR sparse matrix, with 804414 samples and 47236 features. Non-zero values contains cosine-normalized, log TF-IDF vectors. A nearly chronological split is proposed in 1: The first 23149 samples are the training set. The last 781265 samples are the testing set. This follows the official LYRL2004 chronological split. The array has 0.16% of non zero values:

```python
>>> rcv1.data.shape
(804414, 47236)
```

target: The target values are stored in a scipy CSR sparse matrix, with 804414 samples and 103 categories. Each sample has a value of 1 in its categories, and 0 in others. The array has 3.15% of non zero values:

```python
>>> rcv1.target.shape
(804414, 103)
```
sample_id: Each sample can be identified by its ID, ranging (with gaps) from 2286 to 810596:

```python
>>> rcv1.sample_id[:3]
array([2286, 2287, 2288], dtype=uint32)
```
target_names: The target values are the topics of each sample. Each sample belongs to at least one topic, and to up to 17 topics. There are 103 topics, each represented by a string. Their corpus frequencies span five orders of magnitude, from 5 occurrences for ‘GMIL’, to 381327 for ‘CCAT’:

```python
>>> rcv1.target_names[:3].tolist()
['E11', 'ECAT', 'M11']
```

The dataset will be downloaded from the rcv1 homepage if necessary. The compressed size is about 656 MB.

References

3.5.13 Boston House Prices dataset

Notes

Data Set Characteristics:

  Number of Instances  506
  Number of Attributes  13 numeric/categorical predictive

:Median Value (attribute 14) is usually the target

Attribute Information (in order)

  • CRIM per capita crime rate by town
  • ZN proportion of residential land zoned for lots over 25,000 sq.ft.
  • INDUS proportion of non-retail business acres per town
  • CHAS Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
  • NOX nitric oxides concentration (parts per 10 million)
  • RM average number of rooms per dwelling
  • AGE proportion of owner-occupied units built prior to 1940
  • DIS weighted distances to five Boston employment centres
  • RAD index of accessibility to radial highways
  • TAX full-value property-tax rate per $10,000
  • PTRATIO pupil-teacher ratio by town
  • B 1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
  • LSTAT % lower status of the population
  • MEDV Median value of owner-occupied homes in $1000’s

Missing Attribute Values  None

Creator  Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset, http://archive.ics.uci.edu/ml/datasets/Housing

This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.


The Boston house-price data has been used in many machine learning papers that address regression problems.

References


• many more! (see http://archive.ics.uci.edu/ml/datasets/Housing)

3.5.14 Breast Cancer Wisconsin (Diagnostic) Database

Notes

Data Set Characteristics:

- **Number of Instances**: 569
- **Number of Attributes**: 30 numeric, predictive attributes and the class

**Attribute Information**

- radius (mean of distances from center to points on the perimeter)
- texture (standard deviation of gray-scale values)
- perimeter
- area
- smoothness (local variation in radius lengths)
- compactness (perimeter^2 / area - 1.0)
- concavity (severity of concave portions of the contour)
- concave points (number of concave portions of the contour)
- symmetry
- fractal dimension (“coastline approximation” - 1)

The mean, standard error, and “worst” or largest (mean of the three largest values) of these features were computed for each image, resulting in 30 features. For instance, field 3 is Mean Radius, field 13 is Radius SE, field 23 is Worst Radius.

- **class**:
  - WDBC-Malignant
  - WDBC-Benign

**Summary Statistics**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Mean</th>
<th>Standard Error</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius (mean):</td>
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Table 3.32 – continued from previous page

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</table>

Missing Attribute Values  None

Class Distribution  212 - Malignant, 357 - Benign

Creator  Dr. William H. Wolberg, W. Nick Street, Olvi L. Mangasarian

Donor  Nick Street

Date  November, 1995

This is a copy of UCI ML Breast Cancer Wisconsin (Diagnostic) datasets. https://goo.gl/U2Uwz2

Features are computed from a digitized image of a fine needle aspirate (FNA) of a breast mass. They describe characteristics of the cell nuclei present in the image.

Separating plane described above was obtained using Multisurface Method-Tree (MSM-T) [K. P. Bennett, “Decision Tree Construction Via Linear Programming.” Proceedings of the 4th Midwest Artificial Intelligence and Cognitive Science Society, pp. 97-101, 1992], a classification method which uses linear programming to construct a decision tree. Relevant features were selected using an exhaustive search in the space of 1-4 features and 1-3 separating planes.

The actual linear program used to obtain the separating plane in the 3-dimensional space is that described in: [K. P. Bennett and O. L. Mangasarian: “Robust Linear Programming Discrimination of Two Linearly Inseparable Sets”, Optimization Methods and Software 1, 1992, 23-34].

This database is also available through the UW CS ftp server:


References


### 3.5.15 Diabetes dataset

#### Notes

Ten baseline variables, age, sex, body mass index, average blood pressure, and six blood serum measurements were obtained for each of $n = 442$ diabetes patients, as well as the response of interest, a quantitative measure of disease progression one year after baseline.

Data Set Characteristics:

- **Number of Instances** 442
- **Number of Attributes** First 10 columns are numeric predictive values
- **Target** Column 11 is a quantitative measure of disease progression one year after baseline
- **Attributes**
  - Age
  - Sex
  - Body mass index
  - Average blood pressure
  - S1
  - S2
  - S3
  - S4
  - S5
  - S6

Note: Each of these 10 feature variables have been mean centered and scaled by the standard deviation times $n_{samples}$ (i.e. the sum of squares of each column totals 1).

Source URL: [http://www4.stat.ncsu.edu/~boos/var.select/diabetes.html](http://www4.stat.ncsu.edu/~boos/var.select/diabetes.html)


### 3.5.16 Optical Recognition of Handwritten Digits Data Set

#### Notes

Data Set Characteristics:

- **Number of Instances** 5620
- **Number of Attributes** 64
- **Attribute Information** 8x8 image of integer pixels in the range 0..16.
- **Missing Attribute Values** None
This is a copy of the test set of the UCI ML hand-written digits datasets http://archive.ics.uci.edu/ml/datasets/Optical+Recognition+of+Handwritten+Digits

The data set contains images of hand-written digits: 10 classes where each class refers to a digit.

Preprocessing programs made available by NIST were used to extract normalized bitmaps of handwritten digits from a preprinted form. From a total of 43 people, 30 contributed to the training set and different 13 to the test set. 32x32 bitmaps are divided into nonoverlapping blocks of 4x4 and the number of on pixels are counted in each block. This generates an input matrix of 8x8 where each element is an integer in the range 0..16. This reduces dimensionality and gives invariance to small distortions.


References

• Ken Tang and Ponnuthurai N. Suganthan and Xi Yao and A. Kai Qin. Linear dimensionality reduction using relevance weighted LDA. School of Electrical and Electronic Engineering Nanyang Technological University. 2005.

### 3.5.17 Iris Plants Database

**Notes**

**Data Set Characteristics:**

- **Number of Instances**: 150 (50 in each of three classes)
- **Number of Attributes**: 4 numeric, predictive attributes and the class

**Attribute Information**

- sepal length in cm
- sepal width in cm
- petal length in cm
- petal width in cm
- **class**:  
  - Iris-Setosa
  - Iris-Versicolour
  - Iris-Virginica

**Summary Statistics**
The famous Iris database, first used by Sir R.A Fisher

This is perhaps the best known database to be found in the pattern recognition literature. Fisher’s paper is a classic in the field and is referenced frequently to this day. (See Duda & Hart, for example.) The data set contains 3 classes of 50 instances each, where each class refers to a type of iris plant. One class is linearly separable from the other 2; the latter are NOT linearly separable from each other.

References

- See also: 1988 MLC Proceedings, 54-64. Cheeseman et al’s AUTOCLASS II conceptual clustering system finds 3 classes in the data.
- Many, many more ...

3.5.18 Linnerud dataset

Notes

Data Set Characteristics:

- **Number of Instances**: 20
- **Number of Attributes**: 3
- **Missing Attribute Values**: None

The Linnerud dataset contains two small dataset:

- **exercise**: A list containing the following components: exercise data with 20 observations on 3 exercise variables: Weight, Waist and Pulse.
- physiological: Data frame with 20 observations on 3 physiological variables: Chins, Situps and Jumps.

References


3.6 Strategies to scale computationally: bigger data

For some applications the amount of examples, features (or both) and/or the speed at which they need to be processed are challenging for traditional approaches. In these cases scikit-learn has a number of options you can consider to make your system scale.

3.6.1 Scaling with instances using out-of-core learning

Out-of-core (or “external memory”) learning is a technique used to learn from data that cannot fit in a computer’s main memory (RAM).

Here is sketch of a system designed to achieve this goal:

1. a way to stream instances
2. a way to extract features from instances
3. an incremental algorithm

Streaming instances

Basically, 1. may be a reader that yields instances from files on a hard drive, a database, from a network stream etc. However, details on how to achieve this are beyond the scope of this documentation.

Extracting features

2. could be any relevant way to extract features among the different feature extraction methods supported by scikit-learn. However, when working with data that needs vectorization and where the set of features or values is not known in advance one should take explicit care. A good example is text classification where unknown terms are likely to be found during training. It is possible to use a statefull vectorizer if making multiple passes over the data is reasonable from an application point of view. Otherwise, one can turn up the difficulty by using a stateless feature extractor. Currently the preferred way to do this is to use the so-called hashing trick as implemented by sklearn.feature_extraction.FeatureHasher for datasets with categorical variables represented as list of Python dicts or sklearn.feature_extraction.text.HashingVectorizer for text documents.

Incremental learning

Finally, for 3. we have a number of options inside scikit-learn. Although all algorithms cannot learn incrementally (i.e. without seeing all the instances at once), all estimators implementing the partial_fit API are candidates. Actually, the ability to learn incrementally from a mini-batch of instances (sometimes called “online learning”) is key to out-of-core learning as it guarantees that at any given time there will be only a small amount of instances in the
main memory. Choosing a good size for the mini-batch that balances relevancy and memory footprint could involve some tuning¹.

Here is a list of incremental estimators for different tasks:

- **Classification**
  - sklearn.naive_bayes.MultinomialNB
  - sklearn.naive_bayes.BernoulliNB
  - sklearn.linear_model.Perceptron
  - sklearn.linear_model.SGDClassifier
  - sklearn.linear_model.PassiveAggressiveClassifier

- **Regression**
  - sklearn.linear_model.SGDRegressor
  - sklearn.linear_model.PassiveAggressiveRegressor

- **Clustering**
  - sklearn.cluster.MiniBatchKMeans

- **Decomposition / feature Extraction**
  - sklearn.decomposition.MiniBatchDictionaryLearning
  - sklearn.decomposition.IncrementalPCA
  - sklearn.decomposition.LatentDirichletAllocation
  - sklearn.cluster.MiniBatchKMeans

For classification, a somewhat important thing to note is that although a stateless feature extraction routine may be able to cope with new/unseen attributes, the incremental learner itself may be unable to cope with new/unseen targets classes. In this case you have to pass all the possible classes to the first `partial_fit` call using the `classes=` parameter.

Another aspect to consider when choosing a proper algorithm is that all of them don’t put the same importance on each example over time. Namely, the Perceptron is still sensitive to badly labeled examples even after many examples whereas the SGD* and PassiveAggressive* families are more robust to this kind of artifacts. Conversely, the later also tend to give less importance to remarkably different, yet properly labeled examples when they come late in the stream as their learning rate decreases over time.

**Examples**

Finally, we have a full-fledged example of *Out-of-core classification of text documents*. It is aimed at providing a starting point for people wanting to build out-of-core learning systems and demonstrates most of the notions discussed above.

Furthermore, it also shows the evolution of the performance of different algorithms with the number of processed examples.

¹ Depending on the algorithm the mini-batch size can influence results or not. SGD*, PassiveAggressive*, and discrete NaiveBayes are truly online and are not affected by batch size. Conversely, MiniBatchKMeans convergence rate is affected by the batch size. Also, its memory footprint can vary dramatically with batch size.
Now looking at the computation time of the different parts, we see that the vectorization is much more expensive than learning itself. From the different algorithms, MultinomialNB is the most expensive, but its overhead can be mitigated by increasing the size of the mini-batches (exercise: change `minibatch_size` to 100 and 10000 in the program and compare).
Notes

3.7 Computational Performance

For some applications the performance (mainly latency and throughput at prediction time) of estimators is crucial. It may also be of interest to consider the training throughput but this is often less important in a production setup (where it often takes place offline).

We will review here the orders of magnitude you can expect from a number of scikit-learn estimators in different contexts and provide some tips and tricks for overcoming performance bottlenecks.

Prediction latency is measured as the elapsed time necessary to make a prediction (e.g. in micro-seconds). Latency is often viewed as a distribution and operations engineers often focus on the latency at a given percentile of this distribution (e.g. the 90 percentile).

Prediction throughput is defined as the number of predictions the software can deliver in a given amount of time (e.g. in predictions per second).

An important aspect of performance optimization is also that it can hurt prediction accuracy. Indeed, simpler models (e.g. linear instead of non-linear, or with fewer parameters) often run faster but are not always able to take into account the same exact properties of the data as more complex ones.

3.7.1 Prediction Latency

One of the most straight-forward concerns one may have when using/choosing a machine learning toolkit is the latency at which predictions can be made in a production environment.

The main factors that influence the prediction latency are

1. Number of features
2. Input data representation and sparsity
3. Model complexity
4. Feature extraction

A last major parameter is also the possibility to do predictions in bulk or one-at-a-time mode.

Bulk versus Atomic mode

In general doing predictions in bulk (many instances at the same time) is more efficient for a number of reasons (branching predictability, CPU cache, linear algebra libraries optimizations etc.). Here we see on a setting with few features that independently of estimator choice the bulk mode is always faster, and for some of them by 1 to 2 orders of magnitude:
To benchmark different estimators for your case you can simply change the `n_features` parameter in this example: Prediction Latency. This should give you an estimate of the order of magnitude of the prediction latency.

**Influence of the Number of Features**

Obviously when the number of features increases so does the memory consumption of each example. Indeed, for a matrix of $M$ instances with $N$ features, the space complexity is in $O(NM)$. From a computing perspective it also means that the number of basic operations (e.g., multiplications for vector-matrix products in linear models) increases too. Here is a graph of the evolution of the prediction latency with the number of features:
Overall you can expect the prediction time to increase at least linearly with the number of features (non-linear cases can happen depending on the global memory footprint and estimator).

### Influence of the Input Data Representation

Scipy provides sparse matrix datastructures which are optimized for storing sparse data. The main feature of sparse formats is that you don’t store zeros so if your data is sparse then you use much less memory. A non-zero value in a sparse (CSR or CSC) representation will only take on average one 32bit integer position + the 64 bit floating point value + an additional 32bit per row or column in the matrix. Using sparse input on a dense (or sparse) linear model can speedup prediction by quite a bit as only the non zero valued features impact the dot product and thus the model predictions. Hence if you have 100 non zeros in 1e6 dimensional space, you only need 100 multiply and add operation instead of 1e6.

Calculation over a dense representation, however, may leverage highly optimised vector operations and multithreading in BLAS, and tends to result in fewer CPU cache misses. So the sparsity should typically be quite high (10% non-zeros max, to be checked depending on the hardware) for the sparse input representation to be faster than the dense input representation on a machine with many CPUs and an optimized BLAS implementation.

Here is sample code to test the sparsity of your input:

```python
def sparsity_ratio(X):
    return 1.0 - np.count_nonzero(X) / float(X.shape[0] * X.shape[1])
print("input sparsity ratio: ", sparsity_ratio(X))
```

As a rule of thumb you can consider that if the sparsity ratio is greater than 90% you can probably benefit from sparse formats. Check Scipy’s sparse matrix formats [documentation](https://docs.scipy.org/doc/scipy/reference/sparse.html) for more information on how to build (or convert your data to) sparse matrix formats. Most of the time the CSR and CSC formats work best.

### Influence of the Model Complexity

Generally speaking, when model complexity increases, predictive power and latency are supposed to increase. Increasing predictive power is usually interesting, but for many applications we would better not increase prediction latency too much. We will now review this idea for different families of supervised models.
For `sklearn.linear_model` (e.g. Lasso, ElasticNet, SGDClassifier/Regressor, Ridge & RidgeClassifier, PassiveAgressiveClassifier/Regressor, LinearSVC, LogisticRegression...) the decision function that is applied at prediction time is the same (a dot product), so latency should be equivalent.

Here is an example using `sklearn.linear_model.stochastic_gradient.SGDClassifier` with the elasticnet penalty. The regularization strength is globally controlled by the alpha parameter. With a sufficiently high alpha, one can then increase the l1_ratio parameter of elasticnet to enforce various levels of sparsity in the model coefficients. Higher sparsity here is interpreted as less model complexity as we need fewer coefficients to describe it fully. Of course sparsity influences in turn the prediction time as the sparse dot-product takes time roughly proportional to the number of non-zero coefficients.

![Influence of Model Complexity - SGDC](image1)

For the `sklearn.svm` family of algorithms with a non-linear kernel, the latency is tied to the number of support vectors (the fewer the faster). Latency and throughput should (asymptotically) grow linearly with the number of support vectors in a SVC or SVR model. The kernel will also influence the latency as it is used to compute the projection of the input vector once per support vector. In the following graph the nu parameter of `sklearn.svm.classes.NuSVR` was used to influence the number of support vectors.

![Influence of Model Complexity - NuSV](image2)

For `sklearn.ensemble` of trees (e.g. RandomForest, GBT, ExtraTrees etc) the number of trees and their depth play the most important role. Latency and throughput should scale linearly with the number of trees. In this case we used directly the n_estimators parameter of `sklearn.ensemble.gradient_boosting.GradientBoostingRegressor`. 

![Influence of Model Complexity - Ensemble](image3)
In any case be warned that decreasing model complexity can hurt accuracy as mentioned above. For instance a non-linearly separable problem can be handled with a speedy linear model but prediction power will very likely suffer in the process.

**Feature Extraction Latency**

Most scikit-learn models are usually pretty fast as they are implemented either with compiled Cython extensions or optimized computing libraries. On the other hand, in many real world applications the feature extraction process (i.e. turning raw data like database rows or network packets into numpy arrays) governs the overall prediction time. For example on the Reuters text classification task the whole preparation (reading and parsing SGML files, tokenizing the text and hashing it into a common vector space) is taking 100 to 500 times more time than the actual prediction code, depending on the chosen model.
In many cases it is thus recommended to carefully time and profile your feature extraction code as it may be a good place to start optimizing when your overall latency is too slow for your application.

3.7.2 Prediction Throughput

Another important metric to care about when sizing production systems is the throughput i.e. the number of predictions you can make in a given amount of time. Here is a benchmark from the Prediction Latency example that measures this quantity for a number of estimators on synthetic data:
These throughputs are achieved on a single process. An obvious way to increase the throughput of your application is to spawn additional instances (usually processes in Python because of the GIL) that share the same model. One might also add machines to spread the load. A detailed explanation on how to achieve this is beyond the scope of this documentation though.

### 3.7.3 Tips and Tricks

#### Linear algebra libraries

As scikit-learn relies heavily on Numpy/Scipy and linear algebra in general it makes sense to take explicit care of the versions of these libraries. Basically, you ought to make sure that Numpy is built using an optimized BLAS / LAPACK library.

Not all models benefit from optimized BLAS and Lapack implementations. For instance models based on (randomized) decision trees typically do not rely on BLAS calls in their inner loops, nor do kernel SVMs (SVC, SVR, NuSVC, NuSVR). On the other hand a linear model implemented with a BLAS Dgemm call (via numpy.dot) will typically benefit hugely from a tuned BLAS implementation and lead to orders of magnitude speedup over a non-optimized BLAS.

You can display the BLAS / LAPACK implementation used by your NumPy / SciPy / scikit-learn install with the following commands:

```
from numpy.distutils.system_info import get_info
print(get_info('blas_opt'))
print(get_info('lapack_opt'))
```

Optimized BLAS / LAPACK implementations include:

- Atlas (need hardware specific tuning by rebuilding on the target machine)
- OpenBLAS
- MKL
- Apple Accelerate and vecLib frameworks (OSX only)

More information can be found on the [Scipy install page](http://docs.scipy.org/doc/scipy/reference/install.html) and in this [blog post](http://blog.datadog.com/optimize-scikit-learn-performance-with-openblas/) from Daniel Nouri which has some nice step by step install instructions for Debian / Ubuntu.

#### Warning:

Multithreaded BLAS libraries sometimes conflict with Python's `multiprocessing` module, which is used by e.g. GridSearchCV and most other estimators that take an `n_jobs` argument (with the exception of SGDClassifier, SGDRegressor, Perceptron, PassiveAggressiveClassifier and tree-based methods such as random forests). This is true of Apple’s Accelerate and OpenBLAS when built with OpenMP support.

Besides scikit-learn, NumPy and SciPy also use BLAS internally, as explained earlier.

If you experience hanging subprocesses with `n_jobs>1` or `n_jobs=-1`, make sure you have a single-threaded BLAS library, or set `n_jobs=1`, or upgrade to Python 3.4 which has a new version of multiprocessing that should be immune to this problem.

#### Model Compression

Model compression in scikit-learn only concerns linear models for the moment. In this context it means that we want to control the model sparsity (i.e. the number of non-zero coordinates in the model vectors). It is generally a good idea to combine model sparsity with sparse input data representation.
Here is sample code that illustrates the use of the `sparsify()` method:

```python
clf = SGDRegressor(penalty='elasticnet', l1_ratio=0.25)
clf.fit(X_train, y_train).sparsify()
clf.predict(X_test)
```

In this example we prefer the `elasticnet` penalty as it is often a good compromise between model compactness and prediction power. One can also further tune the `l1_ratio` parameter (in combination with the regularization strength `alpha`) to control this tradeoff.

A typical benchmark on synthetic data yields a >30% decrease in latency when both the model and input are sparse (with 0.000024 and 0.027400 non-zero coefficients ratio respectively). Your mileage may vary depending on the sparsity and size of your data and model. Furthermore, sparsifying can be very useful to reduce the memory usage of predictive models deployed on production servers.

### Model Reshaping

Model reshaping consists in selecting only a portion of the available features to fit a model. In other words, if a model discards features during the learning phase we can then strip those from the input. This has several benefits. Firstly it reduces memory (and therefore time) overhead of the model itself. It also allows to discard explicit feature selection components in a pipeline once we know which features to keep from a previous run. Finally, it can help reduce processing time and I/O usage upstream in the data access and feature extraction layers by not collecting and building features that are discarded by the model. For instance if the raw data come from a database, it can make it possible to write simpler and faster queries or reduce I/O usage by making the queries return lighter records. At the moment, reshaping needs to be performed manually in scikit-learn. In the case of sparse input (particularly in CSR format), it is generally sufficient to not generate the relevant features, leaving their columns empty.

### Links

- scikit-learn developer performance documentation
- Scipy sparse matrix formats documentation
CHAPTER
FOUR

GENERAL EXAMPLES

General-purpose and introductory examples for the scikit.

4.1 Plotting Cross-Validated Predictions

This example shows how to use `cross_val_predict` to visualize prediction errors.

```python
from sklearn import datasets
from sklearn.model_selection import cross_val_predict
from sklearn import linear_model
import matplotlib.pyplot as plt
```
Isotonic Regression

An illustration of the isotonic regression on generated data. The isotonic regression finds a non-decreasing approximation of a function while minimizing the mean squared error on the training data. The benefit of such a model is that it does not assume any form for the target function such as linearity. For comparison a linear regression is also presented.

```python
print(__doc__)

# Author: Nelle Varoquaux <nelle.varoquaux@gmail.com>
# Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.collections import LineCollection
from sklearn.linear_model import LinearRegression
from sklearn.isotonic import IsotonicRegression
from sklearn.utils import check_random_state

n = 100
x = np.arange(n)
rs = check_random_state(0)
y = rs.randnint(-50, 50, size=(n,)) + 50. + np.log1 + np.arcange(n))

Fit IsotonicRegression and LinearRegression models

ir = IsotonicRegression()
y_ = ir.fit_transform(x, y)

lr = LinearRegression()
lr.fit(x[:, np.newaxis], y)  # x needs to be 2d for LinearRegression
```

Total running time of the script: (0 minutes 0.116 seconds)

Download Python source code: plot_cv_predict.py
Download IPython notebook: plot_cv_predict.ipynb
```python
segments = [[[i, y[i]], [i, y_[i]]] for i in range(n)]
lc = LineCollection(segments, zorder=0)
lc.set_array(np.ones(len(y)))
lc.set_linewidths(0.5 * np.ones(n))

fig = plt.figure()
plt.plot(x, y, 'r.', markersize=12)
plt.plot(x, y_, 'g.-', markersize=12)
plt.plot(x, lr.predict(x[:, np.newaxis]), 'b-')
plt.gca().add_collection(lc)
plt.legend(('Data', 'Isotonic Fit', 'Linear Fit'), loc='lower right')
plt.title('Isotonic regression')
plt.show()
```

Total running time of the script: (0 minutes 0.093 seconds)

Download Python source code: plot_isotonic_regression.py
Download IPython notebook: plot_isotonic_regression.ipynb
4.3 Concatenating multiple feature extraction methods

In many real-world examples, there are many ways to extract features from a dataset. Often it is beneficial to combine several methods to obtain good performance. This example shows how to use FeatureUnion to combine features obtained by PCA and univariate selection.

Combining features using this transformer has the benefit that it allows cross validation and grid searches over the whole process.

The combination used in this example is not particularly helpful on this dataset and is only used to illustrate the usage of FeatureUnion.

```python
# Author: Andreas Mueller <amueller@ais.uni-bonn.de>
# License: BSD 3 clause
from sklearn.pipeline import Pipeline, FeatureUnion
from sklearn.model_selection import GridSearchCV
from sklearn.svm import SVC
from sklearn.datasets import load_iris
from sklearn.decomposition import PCA
from sklearn.feature_selection import SelectKBest
iris = load_iris()
X, y = iris.data, iris.target

# This dataset is way too high-dimensional. Better do PCA:
pca = PCA(n_components=2)

# Maybe some original features were good, too?
selection = SelectKBest(k=1)

# Build estimator from PCA and Univariate selection:
combined_features = FeatureUnion([('pca', pca), ('univ_select', selection)])

# Use combined features to transform dataset:
X_features = combined_features.fit(X, y).transform(X)

svm = SVC(kernel="linear")

# Do grid search over k, n_components and C:
pipeline = Pipeline([('features', combined_features), ('svm', svm)])
param_grid = dict(features__pca__n_components=[1, 2, 3],

features__univ_select__k=[1, 2],

svm__C=[0.1, 1, 10])

grid_search = GridSearchCV(pipeline, param_grid=param_grid, verbose=10)
grid_search.fit(X, y)
print(grid_search.best_estimator_)
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: feature_stacker.py

Download IPython notebook: feature_stacker.ipynb
4.4 Pipelining: chaining a PCA and a logistic regression

The PCA does an unsupervised dimensionality reduction, while the logistic regression does the prediction. We use a GridSearchCV to set the dimensionality of the PCA.

```python
print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model, decomposition, datasets
from sklearn.pipeline import Pipeline
from sklearn.model_selection import GridSearchCV

logistic = linear_model.LogisticRegression()
pca = decomposition.PCA()
pipe = Pipeline(steps=[('pca', pca), ('logistic', logistic)])
digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target

Plot the PCA spectrum

pca.fit(X_digits)
plt.figure(1, figsize=(4, 3))
plt.clf()
plt.axes([.2, .2, .7, .7])
plt.plot(pca.explained_variance_, linewidth=2)
plt.axis('tight')
plt.xlabel('n_components')
plt.ylabel('explained_variance_')
```

4.4. Pipelining: chaining a PCA and a logistic regression 557
Prediction

```python
n_components = [20, 40, 64]
Cs = np.logspace(-4, 4, 3)

#Parameters of pipelines can be set using '__' separated parameter names:
estimator = GridSearchCV(pipe,
                dict(pca__n_components=n_components,
                     logistic__C=Cs))
estimator.fit(X_digits, y_digits)

plt.axvline(estimator.best_estimator_.named_steps['pca'].n_components,
            linestyle=':', label='n_components chosen')
plt.legend(prop=dict(size=12))
plt.show()
```
4.5 Selecting dimensionality reduction with Pipeline and GridSearchCV

This example constructs a pipeline that does dimensionality reduction followed by prediction with a support vector classifier. It demonstrates the use of GridSearchCV and Pipeline to optimize over different classes of estimators in a single CV run – unsupervised PCA and NMF dimensionality reductions are compared to univariate feature selection during the grid search.
from __future__ import print_function, division
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import load_digits
from sklearn.model_selection import GridSearchCV
from sklearn.pipeline import Pipeline
from sklearn.svm import LinearSVC
from sklearn.decomposition import PCA, NMF
from sklearn.feature_selection import SelectKBest, chi2

pipe = Pipeline([
    ('reduce_dim', PCA()),
    ('classify', LinearSVC())
])
N_FEATURES_OPTIONS = [2, 4, 8]
C_OPTIONS = [1, 10, 100, 1000]
param_grid = [
    {'reduce_dim': [PCA(iterated_power=7), NMF()],
     'reduce_dim__n_components': N_FEATURES_OPTIONS,
     'classify__C': C_OPTIONS}
]
{'classify__C': C_OPTIONS}
},

{'reduce_dim': [SelectKBest(chi2)],
'reduce_dim__k': N_FEATURES_OPTIONS,
'classify__C': C_OPTIONS}
},
]
reducer_labels = ['PCA', 'NMF', 'KBest(chi2)']

grid = GridSearchCV(pipe, cv=3, n_jobs=2, param_grid=param_grid)
digits = load_digits()
grid.fit(digits.data, digits.target)

mean_scores = np.array(grid.cv_results_['mean_test_score'])
# scores are in the order of param_grid iteration, which is alphabetical
mean_scores = mean_scores.reshape(len(C_OPTIONS), -1, len(N_FEATURES_OPTIONS))
# select score for best C
mean_scores = mean_scores.max(axis=0)
bar_offsets = (np.arange(len(N_FEATURES_OPTIONS)) *
(len(reducer_labels) + 1) + .5)

plt.figure()
COLORS = 'bgrcmyk'
for i, (label, reducer_scores) in enumerate(zip(reducer_labels, mean_scores)):
    plt.bar(bar_offsets + i, reducer_scores, label=label, color=COLORS[i])

plt.title("Comparing feature reduction techniques")
plt.xlabel('Reduced number of features')
plt.xticks(bar_offsets + len(reducer_labels) / 2, N_FEATURES_OPTIONS)
plt.ylabel('Digit classification accuracy')
plt.ylim((0, 1))
plt.legend(loc='upper left')
plt.show()

Total running time of the script: (0 minutes 37.027 seconds)

Download Python source code: plot_compare_reduction.py
Download IPython notebook: plot_compare_reduction.ipynb

4.6. Imputing missing values before building an estimator

This example shows that imputing the missing values can give better results than discarding the samples containing
any missing value. Imputing does not always improve the predictions, so please check via cross-validation. Sometimes
dropping rows or using marker values is more effective.

Missing values can be replaced by the mean, the median or the most frequent value using the strategy hyper-
parameter. The median is a more robust estimator for data with high magnitude variables which could dominate
results (otherwise known as a ‘long tail’).

Script output:

Score with the entire dataset = 0.56
Score without the samples containing missing values = 0.48
Score after imputation of the missing values = 0.55
In this case, imputing helps the classifier get close to the original score.

```python
import numpy as np

from sklearn.datasets import load_boston
from sklearn.ensemble import RandomForestRegressor
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import Imputer
from sklearn.model_selection import cross_val_score

rng = np.random.RandomState(0)

dataset = load_boston()
X_full, y_full = dataset.data, dataset.target
n_samples = X_full.shape[0]
n_features = X_full.shape[1]

# Estimate the score on the entire dataset, with no missing values
estimator = RandomForestRegressor(random_state=0, n_estimators=100)
score = cross_val_score(estimator, X_full, y_full).mean()
print("Score with the entire dataset = \%.2f" % score)

# Add missing values in 75% of the lines
missing_rate = 0.75
n_missing_samples = np.floor(n_samples * missing_rate)
missing_samples = np.hstack((np.zeros(n_samples - n_missing_samples,
dtype=np.bool),
np.ones(n_missing_samples,
dtype=np.bool)))
rng.shuffle(missing_samples)
missing_features = rng.randint(0, n_features, n_missing_samples)

# Estimate the score without the lines containing missing values
X_filtered = X_full[~missing_samples, :]
y_filtered = y_full[~missing_samples]
estimator = RandomForestRegressor(random_state=0, n_estimators=100)
score = cross_val_score(estimator, X_filtered, y_filtered).mean()
print("Score without the samples containing missing values = \%.2f" % score)

# Estimate the score after imputation of the missing values
X_missing = X_full.copy()
X_missing[np.where(missing_samples)[0], missing_features] = 0
y_missing = y_full.copy()
estimator = Pipeline(
    [('imputer', Imputer(missing_values=0,
                           strategy="mean",
                           axis=0)),
     ('forest', RandomForestRegressor(random_state=0,
                                       n_estimators=100))])
score = cross_val_score(estimator, X_missing, y_missing).mean()
print("Score after imputation of the missing values = \%.2f" % score)
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: missing_values.py

Download IPython notebook: missing_values.ipynb
4.7 Face completion with a multi-output estimators

This example shows the use of multi-output estimator to complete images. The goal is to predict the lower half of a face given its upper half.

The first column of images shows true faces. The next columns illustrate how extremely randomized trees, k nearest neighbors, linear regression and ridge regression complete the lower half of those faces.
import numpy as np
import matplotlib.pyplot as plt

from sklearn.datasets import fetch_olivetti_faces
from sklearn.utils.validation import check_random_state

from sklearn.ensemble import ExtraTreesRegressor
from sklearn.neighbors import KNeighborsRegressor
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import RidgeCV

# Load the faces datasets
data = fetch_olivetti_faces()
targets = data.target

data = data.images.reshape((len(data.images), -1))
train = data[targets < 30]
test = data[targets >= 30]  # Test on independent people

# Test on a subset of people
n_faces = 5
rng = check_random_state(4)
face_ids = rng.randint(test.shape[0], size=(n_faces, ))
test = test[face_ids, :]
n_pixels = data.shape[1]

# Upper half of the faces
X_train = train[:, :(n_pixels + 1) // 2]
# Lower half of the faces
y_train = train[:, n_pixels // 2:]
X_test = test[:, :(n_pixels + 1) // 2]
y_test = test[:, n_pixels // 2:]

# Fit estimators
ESTIMATORS = {
    "Extra trees": ExtraTreesRegressor(n_estimators=10, max_features=32,
                                          random_state=0),
    "K-nn": KNeighborsRegressor(),
    "Linear regression": LinearRegression(),
    "Ridge": RidgeCV(),
}

y_test_predict = dict()
for name, estimator in ESTIMATORS.items():
    estimator.fit(X_train, y_train)
    y_test_predict[name] = estimator.predict(X_test)

# Plot the completed faces
image_shape = (64, 64)

n_cols = 1 + len(ESTIMATORS)
plt.figure(figsize=(2. * n_cols, 2.26 * n_faces))
plt.suptitle("Face completion with multi-output estimators", size=16)

for i in range(n_faces):
    true_face = np.hstack((X_test[i], y_test[i]))

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4.8 Multilabel classification

This example simulates a multi-label document classification problem. The dataset is generated randomly based on the following process:

- pick the number of labels: $n \sim \text{Poisson}(n\_labels)$
- $n$ times, choose a class $c: c \sim \text{Multinomial}(\theta)$
- pick the document length: $k \sim \text{Poisson}(\text{length})$
- $k$ times, choose a word: $w \sim \text{Multinomial}(\theta_c)$

In the above process, rejection sampling is used to make sure that $n$ is more than 2, and that the document length is never zero. Likewise, we reject classes which have already been chosen. The documents that are assigned to both classes are plotted surrounded by two colored circles.

The classification is performed by projecting to the first two principal components found by PCA and CCA for visualisation purposes, followed by using the `sklearn.multiclass.OneVsRestClassifier` metaclassifier using two SVCs with linear kernels to learn a discriminative model for each class. Note that PCA is used to perform an unsupervised dimensionality reduction, while CCA is used to perform a supervised one.

Note: in the plot, “unlabeled samples” does not mean that we don’t know the labels (as in semi-supervised learning) but that the samples simply do not have a label.
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_multilabel_classification
from sklearn.multiclass import OneVsRestClassifier
from sklearn.svm import SVC
from sklearn.preprocessing import LabelBinarizer
from sklearn.decomposition import PCA
from sklearn.cross_decomposition import CCA

def plot_hyperplane(clf, min_x, max_x, linestyle, label):
    # get the separating hyperplane
    w = clf.coef_[0]
    a = -w[0] / w[1]
    xx = np.linspace(min_x - 5, max_x + 5)  # make sure the line is long enough
    yy = a * xx - (clf.intercept_[0]) / w[1]
    plt.plot(xx, yy, linestyle, label=label)

def plot_subfigure(X, Y, subplot, title, transform):
    if transform == "pca":
        X = PCA(n_components=2).fit_transform(X)
    elif transform == "cca":
        # code for CCA
```

---

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X = CCA(n_components=2).fit(X, Y).transform(X)

else:
    raise ValueError

min_x = np.min(X[:, 0])
max_x = np.max(X[:, 0])

min_y = np.min(X[:, 1])
max_y = np.max(X[:, 1])

classif = OneVsRestClassifier(SVC(kernel='linear'))
classif.fit(X, Y)

plt.subplot(2, 2, subplot)
plt.title(title)

zero_class = np.where(Y[:, 0])
one_class = np.where(Y[:, 1])
plt.scatter(X[:, 0], X[:, 1], s=40, c='gray')
plt.scatter(X[zero_class, 0], X[zero_class, 1], s=160, edgecolors='b',
            facecolors='none', linewidths=2, label='Class 1')
plt.scatter(X[one_class, 0], X[one_class, 1], s=80, edgecolors='orange',
            facecolors='none', linewidths=2, label='Class 2')

plot_hyperplane(classif.estimators_[0], min_x, max_x, 'k--',
                 'Boundary
for class 1')
plot_hyperplane(classif.estimators_[1], min_x, max_x, 'k-.',
                 'Boundary
for class 2')

plt.xticks(())
plt.yticks(())
plt.xlim(min_x - .5 * max_x, max_x + .5 * max_x)
plt.ylim(min_y - .5 * max_y, max_y + .5 * max_y)
if subplot == 2:
    plt.xlabel('First principal component')
    plt.ylabel('Second principal component')
    plt.legend(loc="upper left")

plt.figure(figsize=(8, 6))

X, Y = make_multilabel_classification(n_classes=2, n_labels=1,
                                      allow_unlabeled=True,
                                      random_state=1)

plot_subfigure(X, Y, 1, "With unlabeled samples + CCA", "cca")
plot_subfigure(X, Y, 2, "With unlabeled samples + PCA", "pca")

X, Y = make_multilabel_classification(n_classes=2, n_labels=1,
                                      allow_unlabeled=False,
                                      random_state=1)

plot_subfigure(X, Y, 3, "Without unlabeled samples + CCA", "cca")
plot_subfigure(X, Y, 4, "Without unlabeled samples + PCA", "pca")

plt.subplots_adjust(.04, .02, .97, .94, .09, .2)
plt.show()
4.9 The Johnson-Lindenstrauss bound for embedding with random projections

The Johnson-Lindenstrauss lemma states that any high dimensional dataset can be randomly projected into a lower dimensional Euclidean space while controlling the distortion in the pairwise distances.

4.9.1 Theoretical bounds

The distortion introduced by a random projection \( p \) is asserted by the fact that \( p \) is defining an \( \epsilon \)-embedding with good probability as defined by:

\[
(1 - \epsilon)\|u - v\|^2 < \|p(u) - p(v)\|^2 < (1 + \epsilon)\|u - v\|^2
\]

Where \( u \) and \( v \) are any rows taken from a dataset of shape \([n_{\text{samples}}, n_{\text{features}}]\) and \( p \) is a projection by a random Gaussian \( \mathcal{N}(0, 1) \) matrix with shape \([n_{\text{components}}, n_{\text{features}}]\) (or a sparse Achlioptas matrix).

The minimum number of components to guarantees the \( \epsilon \)-embedding is given by:

\[
n_{\text{components}} \geq 4\log(n_{\text{samples}})/(\epsilon^2/2 - \epsilon^3/3)
\]

The first plot shows that with an increasing number of samples \( n_{\text{samples}} \), the minimal number of dimensions \( n_{\text{components}} \) increased logarithmically in order to guarantee an \( \epsilon \)-embedding.

The second plot shows that an increase of the admissible distortion \( \epsilon \) allows to reduce drastically the minimal number of dimensions \( n_{\text{components}} \) for a given number of samples \( n_{\text{samples}} \).

4.9.2 Empirical validation

We validate the above bounds on the digits dataset or on the 20 newsgroups text document (TF-IDF word frequencies) dataset:

- for the digits dataset, some 8x8 gray level pixels data for 500 handwritten digits pictures are randomly projected to spaces for various larger number of dimensions \( n_{\text{components}} \).
- for the 20 newsgroups dataset some 500 documents with 100k features in total are projected using a sparse random matrix to smaller euclidean spaces with various values for the target number of dimensions \( n_{\text{components}} \).

The default dataset is the digits dataset. To run the example on the twenty newsgroups dataset, pass the –twenty-newsgroups command line argument to this script.

For each value of \( n_{\text{components}} \), we plot:

- 2D distribution of sample pairs with pairwise distances in original and projected spaces as x and y axis respectively.
- 1D histogram of the ratio of those distances (projected / original).

We can see that for low values of \( n_{\text{components}} \) the distribution is wide with many distorted pairs and a skewed distribution (due to the hard limit of zero ratio on the left as distances are always positives) while for larger values of \( n_{\text{components}} \) the distortion is controlled and the distances are well preserved by the random projection.
4.9.3 Remarks

According to the JL lemma, projecting 500 samples without too much distortion will require at least several thousands dimensions, irrespective of the number of features of the original dataset.

Hence using random projections on the digits dataset which only has 64 features in the input space does not make sense: it does not allow for dimensionality reduction in this case.

On the twenty newsgroups on the other hand the dimensionality can be decreased from 56436 down to 10000 while reasonably preserving pairwise distances.
Histogram of pairwise distance rates for n_components=300

Pairwise distances distribution for n_components=1000

Histogram of pairwise distance rates for n_components=1000

Pairwise distances distribution for n_components=10000
Embedding 500 samples with dim 64 using various random projections
Projected 500 samples from 64 to 300 in 0.018s
Random matrix with size: 0.029MB
Mean distances rate: 1.02 (0.09)
Projected 500 samples from 64 to 1000 in 0.037s
Random matrix with size: 0.097MB
Mean distances rate: 1.00 (0.04)
Projected 500 samples from 64 to 10000 in 0.441s
Random matrix with size: 0.964MB
Mean distances rate: 1.00 (0.02)

```
print(__doc__)

import sys
from time import time
import numpy as np
import matplotlib.pyplot as plt
from sklearn.random_projection import johnson_lindenstrauss_min_dim
from sklearn.random_projection import SparseRandomProjection
from sklearn.datasets import fetch_20newsgroups_vectorized
from sklearn.datasets import load_digits
from sklearn.metrics.pairwise import euclidean_distances

# Part 1: plot the theoretical dependency between n_components_min and n_samples
# range of admissible distortions
eps_range = np.linspace(0.1, 0.99, 5)
colors = plt.cm.Blues(np.linspace(0.3, 1.0, len(eps_range)))

# range of number of samples (observation) to embed
n_samples_range = np.logspace(1, 9, 9)

plt.figure()
for eps, color in zip(eps_range, colors):
    min_n_components = johnson_lindenstrauss_min_dim(n_samples_range, eps=eps)
```
```python
plt.loglog(n_samples_range, min_n_components, color=color)
plt.legend(["eps = %0.1f" % eps for eps in eps_range], loc="lower right")
plt.xlabel("Number of observations to eps-embed")
plt.ylabel("Minimum number of dimensions")
plt.title("Johnson-Lindenstrauss bounds: \nn_samples vs n_components")

# range of admissible distortions
eps_range = np.linspace(0.01, 0.99, 100)

# range of number of samples (observation) to embed
n_samples_range = np.logspace(2, 6, 5)
colors = plt.cm.Blues(np.linspace(0.3, 1.0, len(n_samples_range)))

plt.figure()
for n_samples, color in zip(n_samples_range, colors):
    min_n_components = johnson_lindenstrauss_min_dim(n_samples, eps=eps_range)
    plt.semilogy(eps_range, min_n_components, color=color)
plt.legend(["n_samples = %d" % n for n in n_samples_range], loc="upper right")
plt.xlabel("Distortion eps")
plt.ylabel("Minimum number of dimensions")
plt.title("Johnson-Lindenstrauss bounds: \nn_components vs eps")

# Part 2: perform sparse random projection of some digits images which are
# quite low dimensional and dense or documents of the 20 newsgroups dataset
# which is both high dimensional and sparse

if '--twenty-newsgroups' in sys.argv:
    # Need an internet connection hence not enabled by default
    data = fetch_20newsgroups_vectorized().data[:500]
else:
    data = load_digits().data[:500]
n_samples, n_features = data.shape
print("Embedding %d samples with dim %d using various random projections" % (n_samples, n_features))
n_components_range = np.array([300, 1000, 10000])
dists = euclidean_distances(data, squared=True).ravel()

    # select only non-identical samples pairs
nonzero = dists != 0
dists = dists[nonzero]

for n_components in n_components_range:
    t0 = time()
    rp = SparseRandomProjection(n_components=n_components)
    projected_data = rp.fit_transform(data)
    print("Projected %d samples from %d to %d in %0.3fs" %
          (n_samples, n_features, n_components, time() - t0))
    if hasattr(rp, 'components_'):
        n_bytes = rp.components_.data.nbytes
        n_bytes += rp.components_.indices.nbytes
        print("Random matrix with size: %0.3fMB" % (n_bytes / 1e6))

    projected_dists = euclidean_distances(
        projected_data, squared=True).ravel()[nonzero]
```
4.10 Comparison of kernel ridge regression and SVR

Both kernel ridge regression (KRR) and SVR learn a non-linear function by employing the kernel trick, i.e., they learn a linear function in the space induced by the respective kernel which corresponds to a non-linear function in the original space. They differ in the loss functions (ridge versus epsilon-insensitive loss). In contrast to SVR, fitting a KRR can be done in closed-form and is typically faster for medium-sized datasets. On the other hand, the learned model is non-sparse and thus slower than SVR at prediction-time.

This example illustrates both methods on an artificial dataset, which consists of a sinusoidal target function and strong noise added to every fifth datapoint. The first figure compares the learned model of KRR and SVR when both complexity/regularization and bandwidth of the RBF kernel are optimized using grid-search. The learned functions are very similar; however, fitting KRR is approx. seven times faster than fitting SVR (both with grid-search). However, prediction of 100000 target values is more than three times faster with SVR since it has learned a sparse model using only approx. 1/3 of the 100 training datapoints as support vectors.

The next figure compares the time for fitting and prediction of KRR and SVR for different sizes of the training set. Fitting KRR is faster than SVR for medium-sized training sets (less than 1000 samples); however, for larger training sets SVR scales better. With regard to prediction time, SVR is faster than KRR for all sizes of the training set because of the learned sparse solution. Note that the degree of sparsity and thus the prediction time depends on the parameters epsilon and C of the SVR.

# Authors: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
# License: BSD 3 clause
from __future__ import division
import time
import numpy as np

from sklearn.svm import SVR
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import learning_curve
from sklearn.kernel_ridge import KernelRidge
import matplotlib.pyplot as plt

rng = np.random.RandomState(0)

Generate sample data

X = 5 * rng.rand(10000, 1)
y = np.sin(X).ravel()

# Add noise to targets
y[::5] += 3 * (0.5 - rng.rand(X.shape[0] // 5))

X_plot = np.linspace(0, 5, 100000)[:, None]

Fit regression model

train_size = 100
svr = GridSearchCV(SVR(kernel='rbf', gamma=0.1), cv=5,
    param_grid={"C": [1e0, 1e1, 1e2, 1e3],
        "gamma": np.logspace(-2, 2, 5))

kr = GridSearchCV(KernelRidge(kernel='rbf', gamma=0.1), cv=5,
    param_grid={"alpha": [1e0, 0.1, 1e-2, 1e-3],
        "gamma": np.logspace(-2, 2, 5))

t0 = time.time()
svr.fit(X[:train_size], y[:train_size])
svr_fit = time.time() - t0
print("SVR complexity and bandwidth selected and model fitted in %.3f s" % svr_fit)

t0 = time.time()
kr.fit(X[:train_size], y[:train_size])
kr_fit = time.time() - t0
print("KRR complexity and bandwidth selected and model fitted in %.3f s" % kr_fit)

sv_ratio = svr.best_estimator_.support_.shape[0] / train_size
print("Support vector ratio: %.3f" % sv_ratio)

t0 = time.time()
y_svr = svr.predict(X_plot)
svr_predict = time.time() - t0
print("SVR prediction for %d inputs in %.3f s"  % (X_plot.shape[0], svr_predict))

t0 = time.time()
y_kr = kr.predict(X_plot)
KRR predict = time.time() - t0
print("KRR prediction for %d inputs in %.3f s"   
   % (X_plot.shape[0], kr_predict))

Out:

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Complexity and Bandwidth</th>
<th>Train Time</th>
<th>Prediction Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVR</td>
<td>and model fitted</td>
<td>0.912</td>
<td>0.147</td>
</tr>
<tr>
<td>KRR</td>
<td>and model fitted</td>
<td>0.547</td>
<td>0.393</td>
</tr>
</tbody>
</table>

Support vectors ratio: 0.320

Look at the results

```python
sv_ind = svr.best_estimator_.support_
plt.scatter(X[sv_ind], y[sv_ind], c='r', s=50, label='SVR support vectors',
            zorder=2)
plt.scatter(X[:100], y[:100], c='k', label='data', zorder=1)
plt.hold('on')
plt.plot(X_plot, y_svr, c='r',
         label='SVR (fit: %.3fs, predict: %.3fs)' % (svr_fit, svr_predict))
plt.plot(X_plot, y_kr, c='g',
         label='KRR (fit: %.3fs, predict: %.3fs)' % (kr_fit, kr_predict))
plt.xlabel('data')
plt.ylabel('target')
plt.title('SVR versus Kernel Ridge')
plt.legend()
```

# Visualize training and prediction time
plt.figure()

```python
# Generate sample data
X = 5 * rng.rand(10000, 1)
y = np.sin(X).ravel()
y[::5] += 3 * (0.5 - rng.rand(X.shape[0] // 5))
sizes = np.logspace(1, 4, 7, dtype=np.int)
for name, estimator in {"KRR": KernelRidge(kernel='rbf', alpha=0.1, 
gamma=10),
"SVR": SVR(kernel='rbf', C=1e1, gamma=10)}.items():
    train_time = []
    test_time = []
    for train_test_size in sizes:
        t0 = time.time()
        estimator.fit(X[:train_test_size], y[:train_test_size])
        train_time.append(time.time() - t0)
    t0 = time.time()
    estimator.predict(X_plot[:1000])
    test_time.append(time.time() - t0)

plt.plot(sizes, train_time, 'o-', color="r" if name == "SVR" else "g",
         label="%s (train)" % name)
plt.plot(sizes, test_time, 'o--', color="r" if name == "SVR" else "g",
         label="%s (test)" % name)
plt.xscale("log")
plt.yscale("log")
plt.xlabel("Train size")
```

4.10. Comparison of kernel ridge regression and SVR 575
plt.ylabel("Time (seconds)")
plt.title('Execution Time')
plt.legend(loc="best")

# Visualize learning curves
plt.figure()

svr = SVR(kernel='rbf', C=1e1, gamma=0.1)
kr = KernelRidge(kernel='rbf', alpha=0.1, gamma=0.1)
train_sizes, train_scores_svr, test_scores_svr = \
    learning_curve(svr, X[:100], y[:100], train_sizes=np.linspace(0.1, 1, 10),
                   scoring="neg_mean_squared_error", cv=10)
train_sizes_abs, train_scores_kr, test_scores_kr = \
    learning_curve(kr, X[:100], y[:100], train_sizes=np.linspace(0.1, 1, 10),
                   scoring="neg_mean_squared_error", cv=10)

plt.plot(train_sizes, -test_scores_svr.mean(1), 'o-', color="r",
         label="SVR")
plt.plot(train_sizes, -test_scores_kr.mean(1), 'o-', color="g",
         label="KRR")
plt.xlabel("Train size")
plt.ylabel("Mean Squared Error")
plt.title('Learning curves')
plt.legend(loc="best")
plt.show()
4.11 Feature Union with Heterogeneous Data Sources

Datasets can often contain components of that require different feature extraction and processing pipelines. This scenario might occur when:

1. Your dataset consists of heterogeneous data types (e.g. raster images and text captions)
2. Your dataset is stored in a Pandas DataFrame and different columns require different processing pipelines.

This example demonstrates how to use `sklearn.feature_extraction.FeatureUnion` on a dataset containing different types of features. We use the 20-newsgroups dataset and compute standard bag-of-words features for the subject line and body in separate pipelines as well as ad hoc features on the body. We combine them (with weights) using a FeatureUnion and finally train a classifier on the combined set of features.

The choice of features is not particularly helpful, but serves to illustrate the technique.

```python
# Author: Matt Terry <matt.terry@gmail.com>
#
# License: BSD 3 clause
from __future__ import print_function
import numpy as np

from sklearn.base import BaseEstimator, TransformerMixin
from sklearn.datasets import fetch_20newsgroups
from sklearn.datasets import strip_newsgroup_footer
from sklearn.datasets import strip_newsgroup_quoting
from sklearn.decomposition import TruncatedSVD
from sklearn.feature_extraction import DictVectorizer
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.metrics import classification_report
from sklearn.pipeline import FeatureUnion
from sklearn.pipeline import Pipeline
from sklearn.svm import SVC

class ItemSelector(BaseEstimator, TransformerMixin):
    def __init__(self, attr_name):
        self.attr_name = attr_name

    def fit(self, X, y=None):
        return self

    def transform(self, D):
        return D[self.attr_name]
```

Total running time of the script: (0 minutes 56.535 seconds)
Download Python source code: plot_kernel_ridge_regression.py
Download IPython notebook: plot_kernel_ridge_regression.ipynb
For data grouped by feature, select subset of data at a provided key.

The data is expected to be stored in a 2D data structure, where the first index is over features and the second is over samples. i.e.

```
>> len(data[key]) == n_samples
```

Please note that this is the opposite convention to scikit-learn feature matrices (where the first index corresponds to sample).

ItemSelector only requires that the collection implement getitem (data[key]). Examples include: a dict of lists, 2D numpy array, Pandas DataFrame, numpy record array, etc.

```
>> data = {'a': [1, 5, 2, 5, 2, 8],
        'b': [9, 4, 1, 4, 1, 3]}
>> ds = ItemSelector(key='a')
>> data['a'] == ds.transform(data)
```

ItemSelector is not designed to handle data grouped by sample. (e.g. a list of dicts). If your data is structured this way, consider a transformer along the lines of `sklearn.feature_extraction.DictVectorizer`.

```
def __init__(self, key):
    self.key = key

def fit(self, x, y=None):
    return self

def transform(self, data_dict):
    return data_dict[self.key]
```

```
class TextStats(BaseEstimator, TransformerMixin):
    """Extract features from each document for DictVectorizer""

def fit(self, x, y=None):
    return self

def transform(self, posts):
    return [{'length': len(text),
            'num_sentences': text.count('.'),
            } for text in posts]
```

```
class SubjectBodyExtractor(BaseEstimator, TransformerMixin):
    """Extract the subject & body from a usenet post in a single pass.
    Takes a sequence of strings and produces a dict of sequences. Keys are 'subject' and 'body'.
    ""

def fit(self, x, y=None):
    return self
```
def transform(self, posts):
    features = np.recarray(shape=(len(posts),),
                          dtype=[('subject', object), ('body', object)])
    for i, text in enumerate(posts):
        headers, _, bod = text.partition('\n
')
        bod = strip_newsgroup_footer(bod)
        bod = strip_newsgroup_quoting(bod)
        features['body'][i] = bod
        prefix = 'Subject:'
        sub = ''
        for line in headers.split('\\n'):
            if line.startswith(prefix):
                sub = line[len(prefix):]
                break
        features['subject'][i] = sub
    return features

pipeline = Pipeline([
    # Extract the subject & body
    ('subjectbody', SubjectBodyExtractor()),
    # Use FeatureUnion to combine the features from subject and body
    ('union', FeatureUnion(
        transformer_list=[
        # Pipeline for pulling features from the post's subject line
        ('subject', Pipeline([
            ('selector', ItemSelector(key='subject')),
            ('tfidf', TfidfVectorizer(min_df=50)),
        ])),
        # Pipeline for standard bag-of-words model for body
        ('body_bow', Pipeline([
            ('selector', ItemSelector(key='body')),
            ('tfidf', TfidfVectorizer()),
            ('best', TruncatedSVD(n_components=50)),
        ])),
        # Pipeline for pulling ad hoc features from post's body
        ('body_stats', Pipeline([
            ('selector', ItemSelector(key='body')),
            ('stats', TextStats()), # returns a list of dicts
            ('vect', DictVectorizer()), # list of dicts -> feature matrix
        ])),
        ],
        transformer_weights={
            'subject': 0.8,
            'body_bow': 0.5,
            'body_stats': 1.0,
        }),
])

4.11. Feature Union with Heterogeneous Data Sources
# Use a SVC classifier on the combined features
('svc', SVC(kernel='linear')),
]

# limit the list of categories to make running this example faster.
categories = ['alt.atheism', 'talk.religion.misc']

train = fetch_20newsgroups(random_state=1,
                           subset='train',
                           categories=categories,
                         )
test = fetch_20newsgroups(random_state=1,
                           subset='test',
                           categories=categories,
                           )

pipeline.fit(train.data, train.target)
y = pipeline.predict(test.data)
print(classification_report(y, test.target))

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: hetero_feature_union.py
Download IPython notebook: hetero_feature_union.ipynb

## 4.12 Explicit feature map approximation for RBF kernels

An example illustrating the approximation of the feature map of an RBF kernel.

It shows how to use RBFSampler and Nystroem to approximate the feature map of an RBF kernel for classification with an SVM on the digits dataset. Results using a linear SVM in the original space, a linear SVM using the approximate mappings and using a kernelized SVM are compared. Timings and accuracy for varying amounts of Monte Carlo samplings (in the case of RBFSampler, which uses random Fourier features) and different sized subsets of the training set (for Nystroem) for the approximate mapping are shown.

Please note that the dataset here is not large enough to show the benefits of kernel approximation, as the exact SVM is still reasonably fast.

Sampling more dimensions clearly leads to better classification results, but comes at a greater cost. This means there is a tradeoff between runtime and accuracy, given by the parameter n_components. Note that solving the Linear SVM and also the approximate kernel SVM could be greatly accelerated by using stochastic gradient descent via sklearn.linear_model.SGDClassifier. This is not easily possible for the case of the kernelized SVM.

The second plot visualized the decision surfaces of the RBF kernel SVM and the linear SVM with approximate kernel maps. The plot shows decision surfaces of the classifiers projected onto the first two principal components of the data. This visualization should be taken with a grain of salt since it is just an interesting slice through the decision surface in 64 dimensions. In particular note that a datapoint (represented as a dot) does not necessarily be classified into the region it is lying in, since it will not lie on the plane that the first two principal components span.

The usage of RBFSampler and Nystroem is described in detail in Kernel Approximation.
# Explicit feature map approximation for RBF kernels

## 4.12. Explicit feature map approximation for RBF kernels

```python
print(__doc__)

# Author: Gael Varoquaux <gael dot varoquaux at normalesup dot org>
# Andreas Mueller <amueller@ais.uni-bonn.de>
# License: BSD 3 clause

# Standard scientific Python imports
import matplotlib.pyplot as plt
import numpy as np
from time import time

# Import datasets, classifiers and performance metrics
from sklearn import datasets, svm, pipeline
from sklearn.kernel_approximation import (RBFSampler, Nystroem)
from sklearn.decomposition import PCA

# The digits dataset
digits = datasets.load_digits(n_class=9)

# To apply an classifier on this data, we need to flatten the image, to
turn the data in a (samples, feature) matrix:
n_samples = len(digits.data)
data = digits.data / 16.
data -= data.mean(axis=0)

# We learn the digits on the first half of the digits
data_train, targets_train = (data[:n_samples // 2],
                           digits.target[:n_samples // 2])
```

4.12. Explicit feature map approximation for RBF kernels
# Now predict the value of the digit on the second half:
data_test, targets_test = (data[n_samples // 2:],
    digits.target[n_samples // 2:])
# data_test = scaler.transform(data_test)

# Create a classifier: a support vector classifier
kernel_svm = svm.SVC(gamma=.2)
linear_svm = svm.LinearSVC()

# create pipeline from kernel approximation
# and linear svm
feature_map_fourier = RBFSampler(gamma=.2, random_state=1)
feature_map_nystroem = Nystroem(gamma=.2, random_state=1)
fourier_approx_svm = pipeline.Pipeline(["feature_map", feature_map_fourier],
    ("svm", svm.LinearSVC()))
nystroem_approx_svm = pipeline.Pipeline(["feature_map", feature_map_nystroem],
    ("svm", svm.LinearSVC()))

# fit and predict using linear and kernel svm:
kernel_svm_time = time()
kernel_svm.fit(data_train, targets_train)
kernel_svm_score = kernel_svm.score(data_test, targets_test)
kernel_svm_time = time() - kernel_svm_time

linear_svm_time = time()
linear_svm.fit(data_train, targets_train)
linear_svm_score = linear_svm.score(data_test, targets_test)
linear_svm_time = time() - linear_svm_time

sample_sizes = 30 * np.arange(1, 10)
fourier_scores = []
nystroem_scores = []
fourier_times = []
nystroem_times = []

for D in sample_sizes:
    fourier_approx_svm.set_params(feature_map__n_components=D)
nystroem_approx_svm.set_params(feature_map__n_components=D)
    start = time()
nystroem_approx_svm.fit(data_train, targets_train)
nystroem_times.append(time() - start)

    start = time()
    fourier_approx_svm.fit(data_train, targets_train)
    fourier_times.append(time() - start)

    fourier_score = fourier_approx_svm.score(data_test, targets_test)
nystroem_score = nystroem_approx_svm.score(data_test, targets_test)
nystroem_scores.append(nystroem_score)
fourier_scores.append(fourier_score)

# plot the results:
plt.figure(figsize=(8, 8))
accuracy = plt.subplot(211)
accuracy = plt.subplot(212)
# second y axis for timeings
timescale = plt.subplot(212)

accuracy.plot(sample_sizes, nystroem_scores, label="Nystroem approx. kernel")
timescale.plot(sample_sizes, nystroem_times, '--', label='Nystroem approx. kernel')

accuracy.plot(sample_sizes, fourier_scores, label="Fourier approx. kernel")
timescale.plot(sample_sizes, fourier_times, '--', label='Fourier approx. kernel')

# horizontal lines for exact rbf and linear kernels:
accuracy.plot([sample_sizes[0], sample_sizes[-1]], [linear_svm_score, linear_svm_score], label="linear svm")
timescale.plot([sample_sizes[0], sample_sizes[-1]], [linear_svm_time, linear_svm_time], '--', label='linear svm')

accuracy.plot([sample_sizes[0], sample_sizes[-1]], [kernel_svm_score, kernel_svm_score], label="rbf svm")
timescale.plot([sample_sizes[0], sample_sizes[-1]], [kernel_svm_time, kernel_svm_time], '--', label='rbf svm')

# vertical line for dataset dimensionality = 64
accuracy.plot([64, 64], [0.7, 1], label="n_features")

# legends and labels
accuracy.set_title("Classification accuracy")
timescale.set_title("Training times")
accuracy.set_xlim(sample_sizes[0], sample_sizes[-1])
accuracy.set_xticks(())
accuracy.set_ylim(np.min(fourier_scores), 1)
timescale.set_xlabel("Sampling steps = transformed feature dimension")
accuracy.set_ylabel("Classification accuracy")
timescale.set_ylabel("Training time in seconds")
accuracy.legend(loc='best')
timescale.legend(loc='best')

# visualize the decision surface, projected down to the first
# two principal components of the dataset
pca = PCA(n_components=8).fit(data_train)
X = pca.transform(data_train)

# Generate grid along first two principal components
multiples = np.arange(-2, 2, 0.1)

# steps along first component
first = multiples[:, np.newaxis] * pca.components_[0, :]

# steps along second component
second = multiples[:, np.newaxis] * pca.components_[1, :]

# combine
grid = first[np.newaxis, :, :] + second[:, np.newaxis, :]
flat_grid = grid.reshape(-1, data.shape[1])

# title for the plots
titles = ['SVC with rbf kernel',
          'SVC (linear kernel) with Fourier rbf feature map
           n_components=100',
          'SVC (linear kernel) with Nystroem rbf feature map
           n_components=100']

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plt.tight_layout()
plt.figure(figsize=(12, 5))

# predict and plot
for i, clf in enumerate((kernel_svm, nystroem_approx_svm, fourier_approx_svm)):
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    plt.subplot(1, 3, i + 1)
    Z = clf.predict(flat_grid)

    # Put the result into a color plot
    Z = Z.reshape(grid.shape[:-1])
    plt.contourf(multiples, multiples, Z, cmap=plt.cm.Paired)
    plt.axis('off')

    # Plot also the training points
    plt.scatter(X[:, 0], X[:, 1], c=targets_train, cmap=plt.cm.Paired)

    plt.title(titles[i])
plt.tight_layout()
plt.show()

Total running time of the script: (0 minutes 2.322 seconds)

Download Python source code: plot_kernel_approximation.py

Download IPython notebook: plot_kernel_approximation.ipynb
Examples based on real world datasets

Applications to real-world problems with some medium sized datasets or interactive user interface.

5.1 Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation

This is an example of applying Non-negative Matrix Factorization and Latent Dirichlet Allocation on a corpus of documents and extract additive models of the topic structure of the corpus. The output is a list of topics, each represented as a list of terms (weights are not shown).

The default parameters (n_samples / n_features / n_topics) should make the example runnable in a couple of tens of seconds. You can try to increase the dimensions of the problem, but be aware that the time complexity is polynomial in NMF. In LDA, the time complexity is proportional to (n_samples * iterations).

```python
# Author: Olivier Grisel <olivier.grisel@ensta.org>
# Lars Buitinck
# Chyi-Kwei Yau <chyikwei.yau@gmail.com>
# License: BSD 3 clause

from __future__ import print_function
from time import time

from sklearn.feature_extraction.text import TfidfVectorizer, CountVectorizer
from sklearn.decomposition import NMF, LatentDirichletAllocation
from sklearn.datasets import fetch_20newsgroups

n_samples = 2000
n_features = 1000
n_topics = 10
n_top_words = 20

def print_top_words(model, feature_names, n_top_words):
    for topic_idx, topic in enumerate(model.components_):
        print("Topic #{}:
        " % topic_idx)
        print(" ".join([feature_names[i]
            for i in topic.argsort()[:-n_top_words - 1:-1]]))
        print()

# Load the 20 newsgroups dataset and vectorize it. We use a few heuristics
# to filter out useless terms early on: the posts are stripped of headers,
# footers and quoted replies, and common English words, words occurring in
```
print("Loading dataset...")
t0 = time()
dataset = fetch_20newsgroups(shuffle=True, random_state=1,
    remove=('headers', 'footers', 'quotes'))
data_samples = dataset.data[:n_samples]
print("done in %.3fs." % (time() - t0))

# Only one document or in at least 95% of the documents are removed.

# Use tf-idf features for NMF.
print("Extracting tf-idf features for NMF...")
tfidf_vectorizer = TfidfVectorizer(max_df=0.95, min_df=2,
    max_features=n_features,
    stop_words='english')
t0 = time()
tfidf = tfidf_vectorizer.fit_transform(data_samples)
print("done in %.3fs." % (time() - t0))

# Use tf (raw term count) features for LDA.
print("Extracting tf features for LDA...")
tf_vectorizer = CountVectorizer(max_df=0.95, min_df=2,
    max_features=n_features,
    stop_words='english')
t0 = time()
tf = tf_vectorizer.fit_transform(data_samples)
print("done in %.3fs." % (time() - t0))

# Fit the NMF model
print("Fitting the NMF model with tf-idf features, ",
    "n_samples=%d and n_features=%d..." % (n_samples, n_features))
t0 = time()
nmf = NMF(n_components=n_topics, random_state=1,
    alpha=.1, l1_ratio=.5).fit(tfidf)
print("done in %.3fs." % (time() - t0))

print("nTopics in NMF model:")
tfidf_feature_names = tfidf_vectorizer.get_feature_names()
print_top_words(nmf, tfidf_feature_names, n_top_words)

print("Fitting LDA models with tf features, ",
    "n_samples=%d and n_features=%d..." % (n_samples, n_features))
lda = LatentDirichletAllocation(n_topics=n_topics, max_iter=5,
    learning_method='online',
    learning_offset=50.,
    random_state=0)
t0 = time()
lda.fit(tf)
print("done in %.3fs." % (time() - t0))

print("nTopics in LDA model:")
tf_feature_names = tf_vectorizer.get_feature_names()
print_top_words(lda, tf_feature_names, n_top_words)

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: topics_extraction_with_nmf_lda.py
5.2 Outlier detection on a real data set

This example illustrates the need for robust covariance estimation on a real data set. It is useful both for outlier detection and for a better understanding of the data structure.

We selected two sets of two variables from the Boston housing data set as an illustration of what kind of analysis can be done with several outlier detection tools. For the purpose of visualization, we are working with two-dimensional examples, but one should be aware that things are not so trivial in high-dimension, as it will be pointed out.

In both examples below, the main result is that the empirical covariance estimate, as a non-robust one, is highly influenced by the heterogeneous structure of the observations. Although the robust covariance estimate is able to focus on the main mode of the data distribution, it sticks to the assumption that the data should be Gaussian distributed, yielding some biased estimation of the data structure, but yet accurate to some extent. The One-Class SVM does not assume any parametric form of the data distribution and can therefore model the complex shape of the data much better.

5.2.1 First example

The first example illustrates how robust covariance estimation can help concentrating on a relevant cluster when another one exists. Here, many observations are confounded into one and break down the empirical covariance estimation. Of course, some screening tools would have pointed out the presence of two clusters (Support Vector Machines, Gaussian Mixture Models, univariate outlier detection, ...). But had it been a high-dimensional example, none of these could be applied that easily.

5.2.2 Second example

The second example shows the ability of the Minimum Covariance Determinant robust estimator of covariance to concentrate on the main mode of the data distribution: the location seems to be well estimated, although the covariance is hard to estimate due to the banana-shaped distribution. Anyway, we can get rid of some outlying observations. The One-Class SVM is able to capture the real data structure, but the difficulty is to adjust its kernel bandwidth parameter so as to obtain a good compromise between the shape of the data scatter matrix and the risk of over-fitting the data.
```python
import numpy as np
from sklearn.covariance import EllipticEnvelope
from sklearn.svm import OneClassSVM
import matplotlib.pyplot as plt
import matplotlib.font_manager
from sklearn.datasets import load_boston

# Get data
X1 = load_boston()[:, [8, 10]]  # two clusters
X2 = load_boston()[:, [5, 12]]  # "banana"-shaped

# Define "classifiers" to be used
classifiers = {
    "Empirical Covariance": EllipticEnvelope(support_fraction=1., contamination=0.261),
    "Robust Covariance (Minimum Covariance Determinant)": EllipticEnvelope(contamination=0.261),
    "OCSVM": OneClassSVM(nu=0.261, gamma=0.05)}

colors = ['m', 'g', 'b']
legend1 = {}
legend2 = {}

# Learn a frontier for outlier detection with several classifiers
for i, (clf_name, clf) in enumerate(classifiers.items()):
    plt.figure(1)
    clf.fit(X1)
    Z1 = clf.decision_function(np.c_[xx1.ravel(), yy1.ravel()])
    Z1 = Z1.reshape(xx1.shape)
    legend1[clf_name] = plt.contour(xx1, yy1, Z1, levels=[0], linewidths=2, colors=colors[i])
    plt.figure(2)
    clf.fit(X2)
    Z2 = clf.decision_function(np.c_[xx2.ravel(), yy2.ravel()])
    Z2 = Z2.reshape(xx2.shape)
    legend2[clf_name] = plt.contour(xx2, yy2, Z2, levels=[0], linewidths=2, colors=colors[i])
```

---

**print(__doc__)**

# Author: Virgile Fritsch <virgile.fritsch@inria.fr>
# License: BSD 3 clause

```python
import numpy as np
from sklearn.covariance import EllipticEnvelope
from sklearn.svm import OneClassSVM
import matplotlib.pyplot as plt
import matplotlib.font_manager
from sklearn.datasets import load_boston

# Get data
X1 = load_boston()[:, [8, 10]]  # two clusters
X2 = load_boston()[:, [5, 12]]  # "banana"-shaped

# Define "classifiers" to be used
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    "Robust Covariance (Minimum Covariance Determinant)": EllipticEnvelope(contamination=0.261),
    "OCSVM": OneClassSVM(nu=0.261, gamma=0.05)}

colors = ['m', 'g', 'b']
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# Learn a frontier for outlier detection with several classifiers
for i, (clf_name, clf) in enumerate(classifiers.items()):
    plt.figure(1)
    clf.fit(X1)
    Z1 = clf.decision_function(np.c_[xx1.ravel(), yy1.ravel()])
    Z1 = Z1.reshape(xx1.shape)
    legend1[clf_name] = plt.contour(xx1, yy1, Z1, levels=[0], linewidths=2, colors=colors[i])
    plt.figure(2)
    clf.fit(X2)
    Z2 = clf.decision_function(np.c_[xx2.ravel(), yy2.ravel()])
    Z2 = Z2.reshape(xx2.shape)
    legend2[clf_name] = plt.contour(xx2, yy2, Z2, levels=[0], linewidths=2, colors=colors[i])
```
# Plot the results (= shape of the data points cloud)
plt.figure(1)
plt.title("Outlier detection on a real data set (boston housing)")
plt.scatter(X1[:, 0], X1[:, 1], color='black')
bbox_args = dict(boxstyle="round", fc="0.8")
arrow_args = dict(arrowstyle="->")
plt.annotate("several confounded points", xy=(24, 19),
    xycoords="data", textcoords="data",
    xytext=(13, 10), bbox=bbox_args, arrowprops=arrow_args)
plt.xlim((xx1.min(), xx1.max()))
plt.ylim((yy1.min(), yy1.max()))
plt.legend((legend1_values_list[0].collections[0],
    legend1_values_list[1].collections[0],
    legend1_values_list[2].collections[0]),
    (legend1_keys_list[0], legend1_keys_list[1], legend1_keys_list[2]),
    loc="upper center",
    prop=matplotlib.font_manager.FontProperties(size=12))
plt.ylabel("accessibility to radial highways")
plt.xlabel("pupil-teacher ratio by town")

legend2_values_list = list(legend2.values())
legend2_keys_list = list(legend2.keys())
plt.figure(2)
plt.title("Outlier detection on a real data set (boston housing)")
plt.scatter(X2[:, 0], X2[:, 1], color='black')
plt.xlim((xx2.min(), xx2.max()))
plt.ylim((yy2.min(), yy2.max()))
plt.legend((legend2_values_list[0].collections[0],
    legend2_values_list[1].collections[0],
    legend2_values_list[2].collections[0]),
    (legend2_keys_list[0], legend2_keys_list[1], legend2_keys_list[2]),
    loc="upper center",
    prop=matplotlib.font_manager.FontProperties(size=12))
plt.ylabel("% lower status of the population")
plt.xlabel("average number of rooms per dwelling")
plt.show()

Total running time of the script: (0 minutes 4.336 seconds)

Download Python source code: plot_outlier_detection_housing.py
Download IPython notebook: plot_outlier_detection_housing.ipynb

5.3 Compressive sensing: tomography reconstruction with L1 prior (Lasso)

This example shows the reconstruction of an image from a set of parallel projections, acquired along different angles. Such a dataset is acquired in computed tomography (CT).

Without any prior information on the sample, the number of projections required to reconstruct the image is of the
order of the linear size \( l \) of the image (in pixels). For simplicity we consider here a sparse image, where only pixels on the boundary of objects have a non-zero value. Such data could correspond for example to a cellular material. Note however that most images are sparse in a different basis, such as the Haar wavelets. Only \( 1/7 \) projections are acquired, therefore it is necessary to use prior information available on the sample (its sparsity): this is an example of compressive sensing.

The tomography projection operation is a linear transformation. In addition to the data-fidelity term corresponding to a linear regression, we penalize the L1 norm of the image to account for its sparsity. The resulting optimization problem is called the Lasso. We use the class `sklearn.linear_model.Lasso`, that uses the coordinate descent algorithm. Importantly, this implementation is more computationally efficient on a sparse matrix, than the projection operator used here.

The reconstruction with L1 penalization gives a result with zero error (all pixels are successfully labeled with 0 or 1), even if noise was added to the projections. In comparison, an L2 penalization (`sklearn.linear_model.Ridge`) produces a large number of labeling errors for the pixels. Important artifacts are observed on the reconstructed image, contrary to the L1 penalization. Note in particular the circular artifact separating the pixels in the corners, that have contributed to fewer projections than the central disk.

```python
def _weights(x, dx=1, orig=0):
    x = np.ravel(x)
    floor_x = np.floor((x - orig) / dx)
    alpha = (x - orig - floor_x * dx) / dx
    return np.hstack((floor_x, floor_x + 1)), np.hstack((1 - alpha, alpha))

def _generate_center_coordinates(l_x):
    X, Y = np.mgrid[:l_x, :l_x].astype(np.float64)
```
center = l_x / 2.
X += 0.5 - center
Y += 0.5 - center
return X, Y

def build_projection_operator(l_x, n_dir):
    """Compute the tomography design matrix."

    Parameters
    ----------
    l_x : int
        linear size of image array
    n_dir : int
        number of angles at which projections are acquired.

    Returns
    -------
    p : sparse matrix of shape (n_dir l_x, l_x**2)
        """
    X, Y = _generate_center_coordinates(l_x)
    angles = np.linspace(0, np.pi, n_dir, endpoint=False)
    data_inds, weights, camera_inds = [], [], []
    data_unravel_indices = np.arange(l_x ** 2)
    for i, angle in enumerate(angles):
        Xrot = np.cos(angle) * X - np.sin(angle) * Y
        inds, w = _weights(Xrot, dx=1, orig=X.min())
        mask = np.logical_and(inds >= 0, inds < l_x)
        weights += list(w[mask])
        camera_inds += list(inds[mask] + i * l_x)
        data_inds += list(data_unravel_indices[mask])
    proj_operator = sparse.coo_matrix((weights, (camera_inds, data_inds)))
    return proj_operator

def generate_synthetic_data():
    """Synthetic binary data"
    """
    rs = np.random.RandomState(0)
    n_pts = 36
    x, y = np.ogrid[0:1, 0:1]
    mask_outer = (x - 1 / 2) ** 2 + (y - 1 / 2) ** 2 < (1 / 2) ** 2
    mask = np.zeros((l, l))
    points = 1 * rs.rand(2, n_pts)
    mask[(points[0]).astype(np.int), (points[1]).astype(np.int)] = 1
    mask = ndimage.gaussian_filter(mask, sigma=l / n_pts)
    res = np.logical_and(mask > mask.mean(), mask_outer)
    return np.logical_xor(res, ndimage.binary_erosion(res))

# Generate synthetic images, and projections
l = 128
proj_operator = build_projection_operator(l, l / 7.)
data = generate_synthetic_data()
proj = proj_operator * data.ravel()[:, np.newaxis]
proj += 0.15 * np.random.randn(*proj.shape)

# Reconstruction with L2 (Ridge) penalization
rgr_ridge = Ridge(alpha=0.2)
rgr_ridge.fit(proj_operator, proj.ravel())
rec_l2 = rgr_ridge.coef_.reshape(l, l)

# Reconstruction with L1 (Lasso) penalization
# the best value of alpha was determined using cross validation
# with LassoCV
rgr_lasso = Lasso(alpha=0.001)
rgr_lasso.fit(proj_operator, proj.ravel())
rec_l1 = rgr_lasso.coef_.reshape(l, l)

plt.figure(figsize=(8, 3.3))
plt.subplot(131)
plt.imshow(data, cmap=plt.cm.gray, interpolation='nearest')
plt.axis('off')
plt.title('original image')
plt.subplot(132)
plt.imshow(rec_l2, cmap=plt.cm.gray, interpolation='nearest')
plt.title('L2 penalization')
plt.axis('off')
plt.subplot(133)
plt.imshow(rec_l1, cmap=plt.cm.gray, interpolation='nearest')
plt.title('L1 penalization')
plt.axis('off')
plt.subplots_adjust(hspace=0.01, wspace=0.01, top=1, bottom=0, left=0, right=1)
plt.show()

Total running time of the script: (0 minutes 9.272 seconds)

Download Python source code: plot_tomography_l1_reconstruction.py
Download IPython notebook: plot_tomography_l1_reconstruction.ipynb

5.4 Faces recognition example using eigenfaces and SVMs

The dataset used in this example is a preprocessed excerpt of the “Labeled Faces in the Wild”, aka LFW:


Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Name</th>
<th>prob_0</th>
<th>prob_1</th>
<th>prob_2</th>
<th>rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ariel Sharon</td>
<td>0.67</td>
<td>0.92</td>
<td>0.77</td>
<td>13</td>
</tr>
<tr>
<td>Colin Powell</td>
<td>0.75</td>
<td>0.78</td>
<td>0.76</td>
<td>60</td>
</tr>
<tr>
<td>Donald Rumsfeld</td>
<td>0.78</td>
<td>0.67</td>
<td>0.72</td>
<td>27</td>
</tr>
<tr>
<td>George W Bush</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>146</td>
</tr>
<tr>
<td>Gerhard Schroeder</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
<td>25</td>
</tr>
<tr>
<td>Hugo Chavez</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
<td>15</td>
</tr>
<tr>
<td>Tony Blair</td>
<td>0.81</td>
<td>0.69</td>
<td>0.75</td>
<td>36</td>
</tr>
<tr>
<td>avg / total</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>322</td>
</tr>
</tbody>
</table>
from __future__ import print_function
from time import time
import logging
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.model_selection import GridSearchCV
from sklearn.datasets import fetch_lfw_people
from sklearn.metrics import classification_report
from sklearn.metrics import confusion_matrix
from sklearn.decomposition import PCA
from sklearn.svm import SVC

print(__doc__)
# Display progress logs on stdout
logging.basicConfig(level=logging.INFO, format='%(asctime)s %(message)s')

Download the data, if not already on disk and load it as numpy arrays

lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)

# introspect the images arrays to find the shapes (for plotting)
n_samples, h, w = lfw_people.images.shape

# for machine learning we use the 2 data directly (as relative pixel
# positions info is ignored by this model)
X = lfw_people.data
n_features = X.shape[1]

# the label to predict is the id of the person
y = lfw_people.target
target_names = lfw_people.target_names
n_classes = target_names.shape[0]

print("Total dataset size:")
print("n_samples: %d" % n_samples)
print("n_features: %d" % n_features)
print("n_classes: %d" % n_classes)

Split into a training set and a test set using a stratified k fold

# split into a training and testing set
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.25, random_state=42)

Compute a PCA (eigenfaces) on the face dataset (treated as unlabeled dataset): unsupervised feature extraction / dimensionality reduction

n_components = 150

print("Extracting the top %d eigenfaces from %d faces" % (n_components, X_train.shape[0]))
t0 = time()
pca = PCA(n_components=n_components, svd_solver='randomized',
           whiten=True).fit(X_train)
print("done in \$0.3fs" % (time() - t0))
eigenfaces = pca.components_.reshape((n_components, h, w))

print("Projecting the input data on the eigenfaces orthonormal basis")
t0 = time()
X_train_pca = pca.transform(X_train)
X_test_pca = pca.transform(X_test)
print("done in \$0.3fs" % (time() - t0))

Train a SVM classification model

print("Fitting the classifier to the training set")
t0 = time()
param_grid = {'C': [1e3, 5e3, 1e4, 5e4, 1e5],
              'gamma': [0.0001, 0.0005, 0.001, 0.005, 0.01, 0.1], }
clf = GridSearchCV(SVC(kernel='rbf', class_weight='balanced'), param_grid)
clf = clf.fit(X_train_pca, y_train)
print("done in \$0.3fs" % (time() - t0))
print("Best estimator found by grid search:")
print(clf.best_estimator_)

Quantitative evaluation of the model quality on the test set

print("Predicting people's names on the test set")
t0 = time()
y_pred = clf.predict(X_test_pca)
print("done in \$0.3fs" % (time() - t0))
print(classification_report(y_test, y_pred, target_names=target_names))
print(confusion_matrix(y_test, y_pred, labels=range(n_classes)))

Qualitative evaluation of the predictions using matplotlib

def plot_gallery(images, titles, h, w, n_row=3, n_col=4):
    
    """Helper function to plot a gallery of portraits""
    plt.figure(figsize=(1.8 * n_col, 2.4 * n_row))
    plt.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.35)
    for i in range(n_row * n_col):
        plt.subplot(n_row, n_col, i + 1)
        plt.imshow(images[i].reshape((h, w)), cmap=plt.cm.gray)
        plt.title(titles[i], size=12)
        plt.xticks(()
        plt.yticks(()

# plot the result of the prediction on a portion of the test set

def title(y_pred, y_test, target_names, i):
    pred_name = target_names[y_pred[i]].rsplit(' ', 1)[-1]
    true_name = target_names[y_test[i]].rsplit(' ', 1)[-1]
    return 'predicted: %s
true: %s' % (pred_name, true_name)

prediction_titles = [title(y_pred, y_test, target_names, i)
                     for i in range(y_pred.shape[0])]

plot_gallery(X_test, prediction_titles, h, w)
# plot the gallery of the most significative eigenfaces

eigenface_titles = ["eigenface %d" % i for i in range(eigenfaces.shape[0])]
pplot_gallery(eigenfaces, eigenface_titles, h, w)

plt.show()

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: face_recognition.py

Download IPython notebook: face_recognition.ipynb

5.5 Model Complexity Influence

Demonstrate how model complexity influences both prediction accuracy and computational performance. The dataset is the Boston Housing dataset (resp. 20 Newsgroups) for regression (resp. classification). For each class of models we make the model complexity vary through the choice of relevant model parameters and measure the influence on both computational performance (latency) and predictive power (MSE or Hamming Loss).

print(__doc__)

# Author: Eustache Diemert <eustache@diemert.fr>
# License: BSD 3 clause

import time
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.axes_grid1.parasite_axes import host_subplot
from mpl_toolkits.axisartist.axislines import Axes
from scipy.sparse.csr import csr_matrix
from sklearn import datasets
from sklearn.utils import shuffle
from sklearn.metrics import mean_squared_error
from sklearn.svm.classes import NuSVR
from sklearn.ensemble.gradient_boosting import GradientBoostingRegressor
from sklearn.linear_model.stochastic_gradient import SGDClassifier
from sklearn.metrics import hamming_loss

Routines

# initialize random generator
np.random.seed(0)

def generate_data(case, sparse=False):
    """Generate regression/classification data."""
    bunch = None
    if case == 'regression':
        bunch = datasets.load_boston()
    elif case == 'classification':
        bunch = datasets.fetch_20newsgroups_vectorized(subset='all')
    X, y = shuffle(bunch.data, bunch.target)
    offset = int(X.shape[0] * 0.8)
```python
X_train, y_train = X[:offset], y[:offset]
X_test, y_test = X[offset:], y[offset:]
if sparse:
    X_train = csr_matrix(X_train)
    X_test = csr_matrix(X_test)
else:
    X_train = np.array(X_train)
    X_test = np.array(X_test)
y_test = np.array(y_test)
y_train = np.array(y_train)
data = {'X_train': X_train, 'X_test': X_test, 'y_train': y_train,
               'y_test': y_test}
return data

def benchmark_influence(conf):
    
    Benchmark influence of :changing_param: on both MSE and latency.
    
    prediction_times = []
prediction_powers = []
complexities = []
for param_value in conf['changing_param_values']:
    conf['tuned_params'][conf['changing_param']] = param_value
    estimator = conf['estimator'](**conf['tuned_params'])
    print("Benchmarking %s" % estimator)
    estimator.fit(conf['data']['X_train'], conf['data']['y_train'])
    conf['postfit_hook'](estimator)
    complexity = conf['complexity_computer'](estimator)
    complexities.append(complexity)
    start_time = time.time()
    for _ in range(conf['n_samples']):
        y_pred = estimator.predict(conf['data']['X_test'])
        elapsed_time = (time.time() - start_time) / float(conf['n_samples'])
        prediction_times.append(elapsed_time)
pred_score = conf['prediction_performance_computer'](
            conf['data']['y_test'], y_pred)
        prediction_powers.append(pred_score)
        print("Complexity: %d | %s: %.4f | Pred. Time: %fs\n" %
               (complexity, conf['prediction_performance_label'],
                pred_score, elapsed_time))
    return prediction_powers, prediction_times, complexities

def plot_influence(conf, mse_values, prediction_times, complexities):
    
    Plot influence of model complexity on both accuracy and latency.
    
    plt.figure(figsize=(12, 6))
    host = host_subplot(111, axes_class=Axes)
    plt.subplots_adjust(right=0.75)
    par1 = host.twinx()
    host.set_xlabel('Model Complexity (%s) %s' %
                   (conf['complexity_label'],))
    y1_label = conf['prediction_performance_label']
    y2_label = "Time (s)"
    host.set_ylabel(y1_label)
    paralysis = y1_label
    par1.set_ylabel(y2_label)
    par1.set_xlabel(par1)
    p1, = host.plot(complexities, mse_values, 'b-', label="prediction error")
```

Chapter 5. Examples based on real world datasets
```python
def _count_nonzero_coefficients(estimator):
    a = estimator.coef_.toarray()
    return np.count_nonzero(a)
```

```python
regression_data = generate_data('regression')
classification_data = generate_data('classification', sparse=True)
configurations = [
    {'estimator': SGDClassifier,
     'tuned_params': {'penalty': 'elasticnet', 'alpha': 0.001, 'loss':
                      'modified_huber', 'fit_intercept': True},
     'changing_param': 'l1_ratio',
     'changing_param_values': [0.25, 0.5, 0.75, 0.9],
     'complexity_label': 'non_zero coefficients',
     'complexity_computer': _count_nonzero_coefficients,
     'prediction_performance_computer': hamming_loss,
     'prediction_performance_label': 'Hamming Loss (Misclassification Ratio)',
     'postfit_hook': lambda x: x.sparsify(),
     'data': classification_data,
     'n_samples': 30},
    {'estimator': NuSVR,
     'tuned_params': {'C': 1e3, 'gamma': 2 ** -15},
     'changing_param': 'nu',
     'changing_param_values': [0.1, 0.25, 0.5, 0.75, 0.9],
     'complexity_label': 'n_support_vectors',
     'complexity_computer': lambda x: len(x.support_vectors_),
     'data': regression_data,
     'postfit_hook': lambda x: x,
     'prediction_performance_computer': mean_squared_error,
     'prediction_performance_label': 'MSE',
     'n_samples': 30},
    {'estimator': GradientBoostingRegressor,
     'tuned_params': {'loss': 'ls'},
     'changing_param': 'n_estimators',
     'changing_param_values': [10, 50, 100, 200, 500],
     'complexity_label': 'n_trees',
     'complexity_computer': lambda x: x.n_estimators,
     'data': regression_data,
     'postfit_hook': lambda x: x,
     'prediction_performance_computer': mean_squared_error,
     'prediction_performance_label': 'MSE',
     'n_samples': 30},
]
```

```
for conf in configurations:
    prediction_performances, prediction_times, complexities = \
        benchmark_influence(conf)
    plot_influence(conf, prediction_performances, prediction_times,
                   complexities)
```

5.5. Model Complexity Influence
Out:

Benchmarking SGDClassifier(alpha=0.001, average=False, class_weight=None, epsilon=0.1, eta0=0.0, fit_intercept=True, l1_ratio=0.25, learning_rate='optimal', loss='modified_huber', n_iter=5, n_jobs=1, penalty='elasticnet', power_t=0.5, random_state=None, shuffle=True, verbose=0, warm_start=False)
Complexity: 4454 | Hamming Loss (Misclassification Ratio): 0.2501 | Pred. Time: 0.026299s

Benchmarking SGDClassifier(alpha=0.001, average=False, class_weight=None, epsilon=0.1, eta0=0.0, fit_intercept=True, l1_ratio=0.5, learning_rate='optimal', loss='modified_huber', n_iter=5, n_jobs=1, penalty='elasticnet', power_t=0.5, random_state=None, shuffle=True, verbose=0, warm_start=False)
Complexity: 1624 | Hamming Loss (Misclassification Ratio): 0.2923 | Pred. Time: 0.019267s

Benchmarking SGDClassifier(alpha=0.001, average=False, class_weight=None, epsilon=0.1, eta0=0.0, fit_intercept=True, l1_ratio=0.75, learning_rate='optimal', loss='modified_huber', n_iter=5, n_jobs=1, penalty='elasticnet', power_t=0.5, random_state=None, shuffle=True, verbose=0, warm_start=False)
Complexity: 873 | Hamming Loss (Misclassification Ratio): 0.3191 | Pred. Time: 0.015463s

Benchmarking SGDClassifier(alpha=0.001, average=False, class_weight=None, epsilon=0.1,
<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Complexity</th>
<th>MSE</th>
<th>Predict. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>NuSVR</td>
<td>C=1000.0, cache_size=200, coef0=0.0, degree=3, gamma=3.0517578125e-05, kernel='rbf', max_iter=-1, nu=0.1, shrinking=True, tol=0.001, verbose=False</td>
<td>655</td>
<td>31.8133s</td>
<td>2013982s</td>
</tr>
<tr>
<td>NuSVR</td>
<td>C=1000.0, cache_size=200, coef0=0.0, degree=3, gamma=3.0517578125e-05, kernel='rbf', max_iter=-1, nu=0.25, shrinking=True, tol=0.001, verbose=False</td>
<td>69</td>
<td>25.6140s</td>
<td>0.000363s</td>
</tr>
<tr>
<td>NuSVR</td>
<td>C=1000.0, cache_size=200, coef0=0.0, degree=3, gamma=3.0517578125e-05, kernel='rbf', max_iter=-1, nu=0.5, shrinking=True, tol=0.001, verbose=False</td>
<td>136</td>
<td>22.3315s</td>
<td>0.000647s</td>
</tr>
<tr>
<td>NuSVR</td>
<td>C=1000.0, cache_size=200, coef0=0.0, degree=3, gamma=3.0517578125e-05, kernel='rbf', max_iter=-1, nu=0.75, shrinking=True, tol=0.001, verbose=False</td>
<td>243</td>
<td>21.3679s</td>
<td>0.001103s</td>
</tr>
<tr>
<td>NuSVR</td>
<td>C=1000.0, cache_size=200, coef0=0.0, degree=3, gamma=3.0517578125e-05, kernel='rbf', max_iter=-1, nu=0.9, shrinking=True, tol=0.001, verbose=False</td>
<td>350</td>
<td>21.0915s</td>
<td>0.001559s</td>
</tr>
<tr>
<td>NuSVR</td>
<td>C=1000.0, cache_size=200, coef0=0.0, degree=3, gamma=3.0517578125e-05, kernel='rbf', max_iter=-1, nu=1.0, shrinking=True, tol=0.001, verbose=False</td>
<td>404</td>
<td>20.9915s</td>
<td>0.001788s</td>
</tr>
<tr>
<td>GradientBoostingRegressor</td>
<td>alpha=0.9, criterion='friedman_mse', init=None, learning_rate=0.1, loss='ls', max_depth=3, max_features=None, max_leaf_nodes=None, min_impurity_split=1e-07, min_samples_leaf=1, min_samples_split=2, min_weight_fraction_leaf=0.0, n_estimators=10, presort='auto', random_state=None, subsample=1.0, verbose=0, warm_start=False</td>
<td>10</td>
<td>28.9793s</td>
<td>5.5. Model Complexity Influence</td>
</tr>
</tbody>
</table>
5.6 Species distribution modeling

Modeling species’ geographic distributions is an important problem in conservation biology. In this example we model the geographic distribution of two south american mammals given past observations and 14 environmental variables. Since we have only positive examples (there are no unsuccessful observations), we cast this problem as a density estimation problem and use the OneClassSVM provided by the package sklearn.svm as our modeling tool. The dataset is provided by Phillips et. al. (2006). If available, the example uses basemap to plot the coast lines and national boundaries of South America.

The two species are:

- “Bradypus variegatus”, the Brown-throated Sloth.
- “Microryzomys minutus”, also known as the Forest Small Rice Rat, a rodent that lives in Peru, Colombia, Ecuador, Peru, and Venezuela.

5.6.1 References

Modeling distribution of species 'bradypus variegatus'
- fit OneClassSVM ... done.
- plot coastlines from coverage
  - predict species distribution

Area under the ROC curve : 0.868443

Modeling distribution of species 'microryzomys minutus'
- fit OneClassSVM ... done.
- plot coastlines from coverage
  - predict species distribution

Area under the ROC curve : 0.993919

time elapsed: 7.57s
scikit-learn user guide, Release 0.18.2

#
# License: BSD 3 clause
from __future__ import print_function
from time import time
import numpy as np
import matplotlib.pyplot as plt
from
from
from
from

sklearn.datasets.base import Bunch
sklearn.datasets import fetch_species_distributions
sklearn.datasets.species_distributions import construct_grids
sklearn import svm, metrics

# if basemap is available, we'll use it.
# otherwise, we'll improvise later...
try:
from mpl_toolkits.basemap import Basemap
basemap = True
except ImportError:
basemap = False
print(__doc__)

def create_species_bunch(species_name, train, test, coverages, xgrid, ygrid):
"""Create a bunch with information about a particular organism
This will use the test/train record arrays to extract the
data specific to the given species name.
"""
bunch = Bunch(name=' '.join(species_name.split("_")[:2]))
species_name = species_name.encode('ascii')
points = dict(test=test, train=train)
for label, pts in points.items():
# choose points associated with the desired species
pts = pts[pts['species'] == species_name]
bunch['pts_%s' % label] = pts
# determine coverage values for each of the training & testing points
ix = np.searchsorted(xgrid, pts['dd long'])
iy = np.searchsorted(ygrid, pts['dd lat'])
bunch['cov_%s' % label] = coverages[:, -iy, ix].T
return bunch

def plot_species_distribution(species=("bradypus_variegatus_0",
"microryzomys_minutus_0")):
"""
Plot the species distribution.
"""
if len(species) > 2:
print("Note: when more than two species are provided,"
" only the first two will be used")

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Chapter 5. Examples based on real world datasets


t0 = time()

# Load the compressed data
data = fetch_species_distributions()

# Set up the data grid
xgrid, ygrid = construct_grids(data)

# The grid in x,y coordinates
X, Y = np.meshgrid(xgrid, ygrid[::-1])

# create a bunch for each species
BV_bunch = create_species_bunch(species[0],
                                 data.train, data.test,
                                 data.coverages, xgrid, ygrid)

MM_bunch = create_species_bunch(species[1],
                                 data.train, data.test,
                                 data.coverages, xgrid, ygrid)

# background points (grid coordinates) for evaluation
np.random.seed(13)
background_points = np.c_[np.random.randint(low=0, high=data.Ny,
                          size=10000),
                          np.random.randint(low=0, high=data.Nx,
                          size=10000)].T

# We'll make use of the fact that coverages[6] has measurements at all
# land points. This will help us decide between land and water.
land_reference = data.coverages[6]

# Fit, predict, and plot for each species.
for i, species in enumerate([BV_bunch, MM_bunch]):
    print("_" * 80)
    print("Modeling distribution of species '%s'" % species.name)

    # Standardize features
    mean = species.cov_train.mean(axis=0)
    std = species.cov_train.std(axis=0)
    train_cover_std = (species.cov_train - mean) / std

    # Fit OneClassSVM
    print(" - fit OneClassSVM ... ", end='')
    clf = svm.OneClassSVM(nu=0.1, kernel="rbf", gamma=0.5)
    clf.fit(train_cover_std)
    print("done.")

    # Plot map of South America
    plt.subplot(1, 2, i + 1)
    if basemap:
        print(" - plot coastlines using basemap")
        m = Basemap(projection='cyl', llcrnrlat=Y.min(),
                          urcrnrlat=Y.max(), llcrnrlon=X.min(),
                          urcrnrlon=X.max(), resolution='c')
        m.drawcoastlines()
        m.drawcountries()
    else:
        print(" - plot coastlines from coverage")
        plt.contour(X, Y, land_reference,
 levels=[-9999], colors="k",
linestyles="solid")
plt.xticks([])
plt.yticks([])

print(" - predict species distribution")

# Predict species distribution using the training data
Z = np.ones((data.Ny, data.Nx), dtype=np.float64)

# We'll predict only for the land points.
idx = np.where(land_reference > -9999)
coverages_land = data.coverages[:, idx[0], idx[1]].T

pred = clf.decision_function((coverages_land - mean) / std)[:, 0]
Z *= pred.min()
Z[idx[0], idx[1]] = pred

levels = np.linspace(Z.min(), Z.max(), 25)
Z[land_reference == -9999] = -9999

# plot contours of the prediction
plt.contourf(X, Y, Z, levels=levels, cmap=plt.cm.Reds)
plt.colorbar(format='%.2f')

# scatter training/testing points
plt.scatter(species.pts_train['dd long'], species.pts_train['dd lat'],
s=2 ** 2, c='black',
marker='^', label='train')
plt.scatter(species.pts_test['dd long'], species.pts_test['dd lat'],
s=2 ** 2, c='black',
marker='x', label='test')
plt.legend()
plt.title(species.name)
plt.axis('equal')

# Compute AUC with regards to background points
pred_background = Z[background_points[0], background_points[1]]
pred_test = clf.decision_function((species.cov_test - mean)
/ std)[:, 0]
scores = np.r_[pred_test, pred_background]
y = np.r_[np.ones(pred_test.shape), np.zeros(pred_background.shape)]

fpr, tpr, thresholds = metrics.roc_curve(y, scores)
roc_auc = metrics.auc(fpr, tpr)
plt.text(-35, -70, "AUC: %.3f" % roc_auc, ha="right")

print("Area under the ROC curve : %.2f" % roc_auc)

plot_species_distribution()
plt.show()
5.7 Visualizing the stock market structure

This example employs several unsupervised learning techniques to extract the stock market structure from variations in historical quotes.

The quantity that we use is the daily variation in quote price: quotes that are linked tend to cofluctuate during a day.

5.7.1 Learning a graph structure

We use sparse inverse covariance estimation to find which quotes are correlated conditionally on the others. Specifically, sparse inverse covariance gives us a graph, that is a list of connection. For each symbol, the symbols that it is connected too are those useful to explain its fluctuations.

5.7.2 Clustering

We use clustering to group together quotes that behave similarly. Here, amongst the various clustering techniques available in the scikit-learn, we use Affinity Propagation as it does not enforce equal-size clusters, and it can choose automatically the number of clusters from the data.

Note that this gives us a different indication than the graph, as the graph reflects conditional relations between variables, while the clustering reflects marginal properties: variables clustered together can be considered as having a similar impact at the level of the full stock market.

5.7.3 Embedding in 2D space

For visualization purposes, we need to lay out the different symbols on a 2D canvas. For this we use Manifold learning techniques to retrieve 2D embedding.

5.7.4 Visualization

The output of the 3 models are combined in a 2D graph where nodes represents the stocks and edges the:

- cluster labels are used to define the color of the nodes
- the sparse covariance model is used to display the strength of the edges
- the 2D embedding is used to position the nodes in the plan

This example has a fair amount of visualization-related code, as visualization is crucial here to display the graph. One of the challenge is to position the labels minimizing overlap. For this we use an heuristic based on the direction of the nearest neighbor along each axis.

```python
print(__doc__)

# Author: Gael Varoquaux gael.varoquaux@normalesup.org
# License: BSD 3 clause

from datetime import datetime
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.collections import LineCollection
from six.moves.urllib.request import urlopen
```
Retrieve the data from Internet

```python
def quotes_historical_google(symbol, date1, date2):
    """Get the historical data from Google finance.

    Parameters
    ----------
    symbol : str
        Ticker symbol to query for, for example ""DELL"".
    date1 : datetime.datetime
        Start date.
    date2 : datetime.datetime
        End date.

    Returns
    -------
    X : array
        The columns are `date` -- datetime, `open`, `high`, `low`, `close` and `volume` of type float.
    """
    params = urlencode(
        {'q': symbol,
         'startdate': date1.strftime('%b %d, %Y'),
         'enddate': date2.strftime('%b %d, %Y'),
         'output': 'csv'}
    )
    url = 'http://www.google.com/finance/historical?' + params
    response = urlopen(url)
    dtype = {
        'names': ['date', 'open', 'high', 'low', 'close', 'volume'],
        'formats': ['object', 'f4', 'f4', 'f4', 'f4', 'f4']
    }
    converters = {0: lambda s: datetime.strptime(s.decode(), '%d-%b-%y')}
    return np.genfromtxt(response, delimiter=',', skip_header=1,
                         dtype=dtype, converters=converters,
                         missing_values='-', filling_values=-1)
```

# Choose a time period reasonably calm (not too long ago so that we get
# high-tech firms, and before the 2008 crash)
d1 = datetime(2003, 1, 1)
d2 = datetime(2008, 1, 1)
symbol_dict = {
    'TOT': 'Total',
    'XOM': 'Exxon',
    'CVX': 'Chevron',
    'COP': 'ConocoPhillips',
    'VLO': 'Valero Energy',
    'MSFT': 'Microsoft',
    'IBM': 'IBM',
    'TWX': 'Time Warner',
    'CMCSA': 'Comcast',
    'CVC': 'Cablevision',
    'YHOO': 'Yahoo',
}

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symbols, names = np.array(list(symbol_dict.items())).T

quotes = [
    quotes_historical_google(symbol, d1, d2) for symbol in symbols
]

close_prices = np.vstack([q['close'] for q in quotes])
open_prices = np.vstack([q['open'] for q in quotes])

# The daily variations of the quotes are what carry most information
variation = close_prices - open_prices
Learn a graphical structure from the correlations

```python
edge_model = covariance.GraphLassoCV()

# standardize the time series: using correlations rather than covariance
# is more efficient for structure recovery
X = variation.copy().T
X /= X.std(axis=0)
edge_model.fit(X)
```

Cluster using affinity propagation

```python
_, labels = cluster.affinity_propagation(edge_model.covariance_)
n_labels = labels.max()

for i in range(n_labels + 1):
    print('Cluster %i: %s % ((i + 1), ', ', '.join(names[labels == i])))
```

Out:

```
Cluster 1: American express
Cluster 2: Boeing
Cluster 3: Pepsi, Coca Cola, Kellogg
Cluster 4: Navistar
Cluster 5: Apple, Amazon, Yahoo
Cluster 6: GlaxoSmithKline, Novartis, Sanofi-Aventis
Cluster 7: ConocoPhillips, Chevron, Total, Valero Energy, Exxon
Cluster 8: Time Warner
Cluster 9: Cablevision
Cluster 10: Sony, Caterpillar, Canon, Toyota, Honda, Xerox, Unilever
Cluster 11: Kimberly-Clark, Colgate-Palmolive, Procter Gamble
Cluster 12: Ryder, Goldman Sachs, Wal-Mart, General Electrics, Pfizer, Marriott, 3M, → Comcast, Wells Fargo, DuPont de Nemours, CVS, Bank of America, AIG, Home Depot, → Ford, JPMorgan Chase, McDonald’s
Cluster 13: Microsoft, SAP, IBM, Texas Instruments, HP, Dell, Cisco
Cluster 14: Raytheon, General Dynamics, Northrop Grumman
```

Find a low-dimension embedding for visualization: find the best position of the nodes (the stocks) on a 2D plane

```python
# We use a dense eigen_solver to achieve reproducibility (arpack is
# initiated with random vectors that we don't control). In addition, we
# use a large number of neighbors to capture the large-scale structure.
node_position_model = manifold.LocallyLinearEmbedding(
    n_components=2, eigen_solver='dense', n_neighbors=6)

embedding = node_position_model.fit_transform(X.T).T
```

Visualization

```python
plt.figure(1, facecolor='w', figsize=(10, 8))
plt.clf()
ax = plt.axes([0., 0., 1., 1.])
plt.axis('off')

# Display a graph of the partial correlations
partial_correlations = edge_model.precision_.copy()
d = 1 / np.sqrt(np.diag(partial_correlations))
partial_correlations *= d
partial_correlations *= d[:, np.newaxis]
```
non_zero = (np.abs(np.triu(partial_correlations, k=1)) > 0.02)

# Plot the nodes using the coordinates of our embedding
plt.scatter(embedding[0], embedding[1], s=100 * d ** 2, c=labels,
            cmap=plt.cm.spectral)

# Plot the edges
start_idx, end_idx = np.where(non_zero)
segments = [[embedding[:, start], embedding[:, stop]]
             for start, stop in zip(start_idx, end_idx)]
values = np.abs(partial_correlations[non_zero])
lc = LineCollection(segments,
                    zorder=0, cmap=plt.cm.hot_r,
                    norm=plt.Normalize(0, .7 * values.max()))
lc.set_array(values)
lc.set_linewidths(15 * values)
al.add_collection(lc)

# Add a label to each node. The challenge here is that we want to
# position the labels to avoid overlap with other labels
for index, (name, label, (x, y)) in enumerate(zip(names, labels, embedding.T):
    dx = x - embedding[0]
dx[index] = 1
dy = y - embedding[1]
dy[index] = 1
this_dx = dx[np.argmin(np.abs(dy))]
this_dy = dy[np.argmin(np.abs(dx))]
if this_dx > 0:
    horizontalalignment = 'left'
x = x + .002
else:
    horizontalalignment = 'right'
x = x - .002
if this_dy > 0:
    verticalalignment = 'bottom'
y = y + .002
else:
    verticalalignment = 'top'
y = y - .002
plt.text(x, y, name, size=10,
          horizontalalignment=horizontalalignment,
          verticalalignment=verticalalignment,
          bbox=dict(facecolor='w',
                     edgecolor=plt.cm.spectral(label / float(n_labels)),
                     alpha=.6))
plt.xlim(embedding[0].min() - .15 * embedding[0].ptp(),
         embedding[0].max() + .10 * embedding[0].ptp(),)
plt.ylim(embedding[1].min() - .03 * embedding[1].ptp(),
         embedding[1].max() + .03 * embedding[1].ptp(),)
plt.show()
5.8 Wikipedia principal eigenvector

A classical way to assert the relative importance of vertices in a graph is to compute the principal eigenvector of the adjacency matrix so as to assign to each vertex the values of the components of the first eigenvector as a centrality score:

https://en.wikipedia.org/wiki/Eigenvector_centrality

On the graph of webpages and links those values are called the PageRank scores by Google.

The goal of this example is to analyze the graph of links inside wikipedia articles to rank articles by relative importance according to this eigenvector centrality.

The traditional way to compute the principal eigenvector is to use the power iteration method:

https://en.wikipedia.org/wiki/Power_iteration

Here the computation is achieved thanks to Martinsson’s Randomized SVD algorithm implemented in the scikit.
The graph data is fetched from the DBpedia dumps. DBpedia is an extraction of the latent structured data of the Wikipedia content.

```python
# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: BSD 3 clause

from __future__ import print_function
from bz2 import BZ2File
import os
from datetime import datetime
from pprint import pprint
from time import time

import numpy as np
from scipy import sparse
from sklearn.decomposition import randomized_svd
from sklearn.externals.joblib import Memory
from sklearn.externals.six.moves.urllib.request import urlopen
from sklearn.externals.six import iteritems

print(__doc__)

Where to download the data, if not already on disk

```bash
redirects_url = "http://downloads.dbpedia.org/3.5.1/en/redirects_en.nt.bz2"
redirects_filename = redirects_url.rsplit(”/”, 1)[1]

page_links_url = "http://downloads.dbpedia.org/3.5.1/en/page_links_en.nt.bz2"
page_links_filename = page_links_url.rsplit(”/”, 1)[1]

resources = [(redirects_url, redirects_filename),
              (page_links_url, page_links_filename),
]
```

```python
for url, filename in resources:
    if not os.path.exists(filename):
        print("Downloading data from ‘%s’, please wait..." % url)
        opener = urlopen(url)
        open(filename, ’wb’).write(opener.read())
        print()
```

Loading the redirect files

```python
memory = Memory(cachedir=".")

def index(redirects, index_map, k):
    """Find the index of an article name after redirect resolution""
    k = redirects.get(k, k)
    return index_map.setdefault(k, len(index_map))
```

```python
DBPEDIARESOURCEPREFIXLEN = len("http://dbpedia.org/resource/")
SHORTNAME_SLICE = slice(DBPEDIARESOURCEPREFIXLEN + 1, -1)
```

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```python
def short_name(nt_uri):
    """Remove the < and > URI markers and the common URI prefix""
    return nt_uri[SHORTNAME_SLICE]

def get_redirects(redirects_filename):
    """Parse the redirections and build a transitivity closed map out of it""
    redirects = {}
    print("Parsing the NT redirect file")
    for l, line in enumerate(BZ2File(redirects_filename)):
        split = line.split()
        if len(split) != 4:
            print("ignoring malformed line: " + line)
            continue
        redirects[short_name(split[0])] = short_name(split[2])
        if l % 1000000 == 0:
            print("[%s] line: %08d % (datetime.now().isoformat(), l))

    # compute the transitive closure
    print("Computing the transitive closure of the redirect relation")
    for l, source in enumerate(redirects.keys()):
        transitive_target = None
        target = redirects[source]
        seen = set([source])
        while True:
            transitive_target = target
            target = redirects.get(target)
            if target is None or target in seen:
                break
            seen.add(target)
            redirects[source] = transitive_target
        if l % 1000000 == 0:
            print("[\$s] line: %08d % (datetime.now().isoformat(), l))

    return redirects

# disabling joblib as the pickling of large dicts seems much too slow
#@memory.cache
def get_adjacency_matrix(redirects_filename, page_links_filename, limit=None):
    """Extract the adjacency graph as a scipy sparse matrix

    Redirects are resolved first.

    Returns X, the scipy sparse adjacency matrix, redirects as python
dict from article names to article names and index_map a python dict
from article names to python int (article indexes).
""

    print("Computing the redirect map")
    redirects = get_redirects(redirects_filename)

    print("Computing the integer index map")
    index_map = dict()
    links = list()
    for l, line in enumerate(BZ2File(page_links_filename)):
```

---

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split = line.split()
if len(split) != 4:
    print("ignoring malformed line: " + line)
    continue
i = index(redirects, index_map, short_name(split[0]))
j = index(redirects, index_map, short_name(split[2]))
links.append((i, j))
if l % 1000000 == 0:
    print("[%s] line: %08d" % (datetime.now().isoformat(), l))

if limit is not None and l >= limit - 1:
    break
print("Computing the adjacency matrix")
X = sparse.lil_matrix((len(index_map), len(index_map)), dtype=np.float32)
for i, j in links:
    X[i, j] = 1.0
del links
print("Converting to CSR representation")
X = X.tocsr()
print("CSR conversion done")
return X, redirects, index_map

# stop after 5M links to make it possible to work in RAM
X, redirects, index_map = get_adjacency_matrix(
    redirects_filename, page_links_filename, limit=5000000)
names = dict((i, name) for name, i in iteritems(index_map))

print("Computing the principal singular vectors using randomized_svd")
t0 = time()
U, s, V = randomized_svd(X, 5, n_iter=3)
print("done in $0.3fs" % (time() - t0))

# print the names of the wikipedia related strongest components of the
# principal singular vector which should be similar to the highest eigenvector
print("Top wikipedia pages according to principal singular vectors")
pprint([names[i] for i in np.abs(U.T[0]).argsort()[-10:]])
pprint([names[i] for i in np.abs(V[0]).argsort()[-10:]])

def centrality_scores(X, alpha=0.85, max_iter=100, tol=1e-10):
    ""
    Power iteration computation of the principal eigenvector
    ""
    This method is also known as Google PageRank and the implementation
    is based on the one from the NetworkX project (BSD licensed too)
    with copyrights by:
    Aric Hagberg <hagberg@lanl.gov>
    Dan Schult <dschult@colgate.edu>
    Pieter Swart <swart@lanl.gov>
    ""
    n = X.shape[0]
    X = X.copy()
    incoming_counts = np.asarray(X.sum(axis=1)).ravel()
    print("Normalizing the graph")
    for i in incoming_counts.nonzero()[0]:

5.8. Wikipedia principal eigenvector
X.data[X.indptr[i]:X.indptr[i + 1]] *= 1.0 / incoming_counts[i]
dangle = np.asarray(np.where(X.sum(axis=1) == 0, 1.0 / n, 0)).ravel()

scores = np.ones(n, dtype=np.float32) / n  # initial guess
for i in range(max_iter):
    print("power iteration \#\%d \% i")
    prev_scores = scores
    scores = (alpha * (scores * X + np.dot(dangle, prev_scores))
    + (1 - alpha) * prev_scores.sum() / n)
    # check convergence: normalized l_inf norm
    scores_max = np.abs(scores).max()
    if scores_max == 0.0:
        scores_max = 1.0
    err = np.abs(scores - prev_scores).max() / scores_max
    print("error: \%0.6f \% err")
    if err < n * tol:
        return scores

print("Computing principal eigenvector score using a power iteration method")
t0 = time()
scores = centrality_scores(X, max_iter=100, tol=1e-10)
print("done in \$0.3fs \% (time() - t0)")
pprint([names[i] for i in np.abs(scores).argsort()[-10:]])

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: wikipedia_principal_eigenvector.py
Download IPython notebook: wikipedia_principal_eigenvector.ipynb

5.9 Libsvm GUI

A simple graphical frontend for Libsvm mainly intended for didactic purposes. You can create data points by point and click and visualize the decision region induced by different kernels and parameter settings.

To create positive examples click the left mouse button; to create negative examples click the right button.

If all examples are from the same class, it uses a one-class SVM.

from __future__ import division, print_function

print(__doc__)

# Author: Peter Prettenhoer <peter.prettenhofer@gmail.com>
# License: BSD 3 clause

import matplotlib
matplotlib.use('TkAgg')

import matplotlib
matplotliblib.use('TkAgg')

from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg
from matplotlib.backends.backend_tkagg import Figure

from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg
from matplotlib.backends.backend_tkagg import NavigationToolbar2TkAgg
from matplotlib.figure import Figure

from matplotlib.contour import ContourSet
try:
    import tkinter as Tk
except ImportError:
    # Backward compat for Python 2
    import Tkinter as Tk
import sys
import numpy as np
from sklearn import svm
from sklearn.datasets import dump_svmlight_file
from sklearn.externals.six.moves import xrange

y_min, y_max = -50, 50
x_min, x_max = -50, 50

class Model(object):
    """The Model which hold the data. It implements the
    observable in the observer pattern and notifies the
    registered observers on change event.""

    def __init__(self):
        self.observers = []
        self.surface = None
        self.data = []
        self.cls = None
        self.surface_type = 0

    def changed(self, event):
        """Notify the observers. ""
        for observer in self.observers:
            observer.update(event, self)

    def add_observer(self, observer):
        """Register an observer. ""
        self.observers.append(observer)

    def set_surface(self, surface):
        self.surface = surface

    def dump_svmlight_file(self, file):
        data = np.array(self.data)
        X = data[:, 0:2]
        y = data[:, 2]
        dump_svmlight_file(X, y, file)

class Controller(object):
    def __init__(self, model):
        self.model = model
        self.kernel = Tk.IntVar()
        self.surface_type = Tk.IntVar()
        # Whether or not a model has been fitted
        self.fitted = False

    def fit(self):
print("fit the model")
train = np.array(self.model.data)
X = train[:, 0:2]
y = train[:, 2]

C = float(self.complexity.get())
gamma = float(self.gamma.get())
coef0 = float(self.coef0.get())
deregree = int(self.degree.get())
kern_map = {0: "linear", 1: "rbf", 2: "poly"}
if len(np.unique(y)) == 1:
    clf = svm.OneClassSVM(kernel=kern_map[self.kernel.get()],
                          gamma=gamma, coef0=coef0, degree=degree)
    clf.fit(X)
else:
    clf = svm.SVC(kernel=kern_map[self.kernel.get()], C=C,
                  gamma=gamma, coef0=coef0, degree=degree)
    clf.fit(X, y)
if hasattr(clf, 'score'):
    print("Accuracy:", clf.score(X, y) * 100)
X1, X2, Z = self.decision_surface(clf)
sel.model.clf = clf
sel.model.set_surface((X1, X2, Z))
sel.model.surface_type = sel.surface_type.get()
sel.fitted = True
sel.model.changed("surface")

def decision_surface(self, cls):
    delta = 1
    x = np.arange(x_min, x_max + delta, delta)
y = np.arange(y_min, y_max + delta, delta)
X1, X2 = np.meshgrid(x, y)
Z = cls.decision_function(np.c_[X1.ravel(), X2.ravel()])
Z = Z.reshape(X1.shape)
return X1, X2, Z

def clear_data(self):
    sel.model.data = []
    sel.fitted = False
    sel.model.changed("clear")

def add_example(self, x, y, label):
    sel.model.data.append((x, y, label))
    sel.model.changed("example_added")

    # update decision surface if already fitted.
    self.refit()

def refit(self):
    """Refit the model if already fitted. ""
    if self.fitted:
        self.fit()

class View(object):
    """Test docstring. ""
    def __init__(self, root, controller):
        f = Figure()
ax = f.add_subplot(111)
ax.set_xticks([])
ax.set_yticks([])
ax.set_xlim((x_min, x_max))
ax.set_ylim((y_min, y_max))
canvas = FigureCanvasTkAgg(f, master=root)
canvas.show()
canvas.get_tk_widget().pack(side=Tk.TOP, fill=Tk.BOTH, expand=1)
canvas._tkcanvas.pack(side=Tk.TOP, fill=Tk.BOTH, expand=1)
canvas.mpl_connect('button_press_event', self.onclick)
toolbar = NavigationToolbar2TkAgg(canvas, root)
toolbar.update()
self.controllbar = ControllBar(root, controller)
self.f = f
self.ax = ax
self.canvas = canvas
self.controller = controller
self.contours = []
self.c_labels = None
self.plot_kernels()

def plot_kernels(self):
    self.ax.text(-50, -60, "Linear: $u^T v$"
    self.ax.text(-20, -60, "RBF: $\exp (-\gamma \| u-v \|^2)$")
    self.ax.text(10, -60, "Poly: $(\gamma \, u^T v + r)^d$")

def onclick(self, event):
    if event.xdata and event.ydata:
        if event.button == 1:
            self.controller.add_example(event.xdata, event.ydata, 1)
        elif event.button == 3:
            self.controller.add_example(event.xdata, event.ydata, -1)

def update_example(self, model, idx):
    x, y, l = model.data[idx]
    if l == 1:
        color = 'w'
    elif l == -1:
        color = 'k'
    self.ax.plot([x], [y], "%so" % color, scalex=0.0, scaley=0.0)

def update(self, event, model):
    if event == "examples_loaded":
        for i in xrange(len(model.data)):
            self.update_example(model, i)
    if event == "example_added":
        self.update_example(model, -1)
    if event == "clear":
        self.ax.clear()
        self.ax.set_xticks([])
        self.ax.set_yticks([])
        self.contours = []
        self.c_labels = None
        self.plot_kernels()
    if event == "surface":
self.remove_surface()
self.plot_support_vectors(model.clf.support_vectors_)
self.plot_decision_surface(model.surface, model.surface_type)

self.canvas.draw()

def remove_surface(self):
    """Remove old decision surface."""
    if len(self.contours) > 0:
        for contour in self.contours:
            if isinstance(contour, ContourSet):
                for lineset in contour.collections:
                    lineset.remove()
            else:
                contour.remove()
    self.contours = []

def plot_support_vectors(self, support_vectors):
    """Plot the support vectors by placing circles over the
    corresponding data points and adds the circle collection
to the contours list."""
    cs = self.ax.scatter(support_vectors[:, 0], support_vectors[:, 1],
                         s=80, edgecolors="k", facecolors="none")
    self.contours.append(cs)

def plot_decision_surface(self, surface, type):
    X1, X2, Z = surface
    if type == 0:
        levels = [-1.0, 0.0, 1.0]
        linestyles = ['dashed', 'solid', 'dashed']
        colors = "k"
        self.contours.append(self.ax.contour(X1, X2, Z, levels,
                                              colors=colors,
                                              linestyles=linestyles))
    elif type == 1:
        self.contours.append(self.ax.contourf(X1, X2, Z, 10,
                                               cmap=matplotlib.cm.bone,
                                               origin='lower', alpha=0.85))
        self.contours.append(self.ax.contour(X1, X2, Z, [0.0], colors='k',
                                              linestyles=['solid']))
    else:
        raise ValueError("surface type unknown")

class ControllBar(object):
    def __init__(self, root, controller):
        fm = Tk.Frame(root)
        kernel_group = Tk.Frame(fm)
        Tk.Radiobutton(kernel_group, text="Linear", variable=controller.kernel,
                        value=0, command=controller.refit).pack(anchor=Tk.W)
        Tk.Radiobutton(kernel_group, text="RBF", variable=controller.kernel,
                        value=1, command=controller.refit).pack(anchor=Tk.W)
        Tk.Radiobutton(kernel_group, text="Poly", variable=controller.kernel,
                        value=2, command=controller.refit).pack(anchor=Tk.W)
        kernel_group.pack(side=Tk.LEFT)

        valbox = Tk.Frame(fm)
        controller.complexity = Tk.StringVar()
controller.complexity.set("1.0")
c = Tk.Frame(valbox)
Tk.Label(c, text="C:", anchor="e", width=7).pack(side=Tk.LEFT)
Tk.Entry(c, width=6, textvariable=controller.complexity).pack(
    side=Tk.LEFT)
c.pack()

controller.gamma = Tk.StringVar()
controller.gamma.set("0.01")
g = Tk.Frame(valbox)
Tk.Label(g, text="gamma:", anchor="e", width=7).pack(side=Tk.LEFT)
Tk.Entry(g, width=6, textvariable=controller.gamma).pack(side=Tk.LEFT)
g.pack()

controller.degree = Tk.StringVar()
controller.degree.set("3")
d = Tk.Frame(valbox)
Tk.Label(d, text="degree:", anchor="e", width=7).pack(side=Tk.LEFT)
Tk.Entry(d, width=6, textvariable=controller.degree).pack(side=Tk.LEFT)
d.pack()

controller.coef0 = Tk.StringVar()
controller.coef0.set("0")
r = Tk.Frame(valbox)
Tk.Label(r, text="coef0:", anchor="e", width=7).pack(side=Tk.LEFT)
Tk.Entry(r, width=6, textvariable=controller.coef0).pack(side=Tk.LEFT)
r.pack()

valbox.pack(side=Tk.LEFT)
cmap_group = Tk.Frame(fm)
Tk.Radiobutton(cmap_group, text="Hyperplanes", variable=controller.surface_type, value=0,
    command=controller.refit).pack(anchor=Tk.W)
Tk.Radiobutton(cmap_group, text="Surface", variable=controller.surface_type, value=1,
    command=controller.refit).pack(anchor=Tk.W)
cmap_group.pack(side=Tk.LEFT)

train_button = Tk.Button(fm, text='Fit', width=5,
    command=controller.fit)
train_button.pack()
fm.pack(side=Tk.LEFT)
Tk.Button(fm, text='Clear', width=5,
    command=controller.clear_data).pack(side=Tk.LEFT)


def get_parser():
    from optparse import OptionParser
    op = OptionParser()
    op.add_option("--output",
        action="store", type="str", dest="output",
        help="Path where to dump data.")
    return op

def main(argv):
    op = get_parser()
opts, args = op.parse_args(argv[1:])
root = Tk.Tk()
model = Model()
controller = Controller(model)
root.wm_title("Scikit-learn Libsvm GUI")
view = View(root, controller)
model.add_observer(view)
Tk.mainloop()

if opts.output:
    model.dump_svmlight_file(opts.output)

if __name__ == '__main__':
    main(sys.argv)

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: svm_gui.py
Download IPython notebook: svm_gui.ipynb

5.10 Prediction Latency

This is an example showing the prediction latency of various scikit-learn estimators.

The goal is to measure the latency one can expect when doing predictions either in bulk or atomic (i.e. one by one) mode.

The plots represent the distribution of the prediction latency as a boxplot.

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# License: BSD 3 clause

from __future__ import print_function
from collections import defaultdict
import time
gc
import numpy as np
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from scipy.stats import scoreatpercentile
from sklearn.datasets.samples_generator import make_regression
from sklearn.ensemble import RandomForestRegressor
from sklearn.linear_model import Ridge
from sklearn.linear_model import SGDRegressor
from sklearn.svm import SVR
from sklearn.utils import shuffle

def _not_in_sphinx():
    # Hack to detect whether we are running by the sphinx builder
    return '__file__' in globals()
def atomic_benchmark_estimator(estimator, X_test, verbose=False):
    """Measure runtime prediction of each instance."""
    n_instances = X_test.shape[0]
    runtimes = np.zeros(n_instances, dtype=np.float)
    for i in range(n_instances):
        instance = X_test[[i], :]
        start = time.time()
        estimator.predict(instance)
        runtimes[i] = time.time() - start
    if verbose:
        print("atomic_benchmark runtimes:", min(runtimes), scoreatpercentile(
            runtimes, 50), max(runtimes))
    return runtimes

def bulk_benchmark_estimator(estimator, X_test, n_bulk_repeats, verbose):
    """Measure runtime prediction of the whole input."""
    n_instances = X_test.shape[0]
    runtimes = np.zeros(n_bulk_repeats, dtype=np.float)
    for i in range(n_bulk_repeats):
        start = time.time()
        estimator.predict(X_test)
        runtimes[i] = time.time() - start
    runtimes = np.array(list(map(lambda x: x / float(n_instances), runtimes)))
    if verbose:
        print("bulk_benchmark runtimes:", min(runtimes), scoreatpercentile(
            runtimes, 50), max(runtimes))
    return runtimes

def benchmark_estimator(estimator, X_test, n_bulk_repeats=30, verbose=False):
    """Measure runtimes of prediction in both atomic and bulk mode.

    Parameters
    ----------
    estimator : already trained estimator supporting `predict()`
    X_test : test input
    n_bulk_repeats : how many times to repeat when evaluating bulk mode

    Returns
    -------
    atomic_runtimes, bulk_runtimes : a pair of `np.array` which contain the runtimes in seconds.
    """
    atomic_runtimes = atomic_benchmark_estimator(estimator, X_test, verbose)
    bulk_runtimes = bulk_benchmark_estimator(estimator, X_test, n_bulk_repeats, verbose)
    return atomic_runtimes, bulk_runtimes

def generate_dataset(n_train, n_test, n_features, noise=0.1, verbose=False):
    """Generate a regression dataset with the given parameters."""
    if verbose:
        print("generating dataset...")
    X, y, coef = make_regression(n_samples=n_train + n_test,
random_seed = 13
X_train, X_test, y_train, y_test = train_test_split(
    X, y, train_size=n_train, random_state=random_seed)
X_train, y_train = shuffle(X_train, y_train, random_state=random_seed)

X_scaler = StandardScaler()
X_train = X_scaler.fit_transform(X_train)
X_test = X_scaler.transform(X_test)

y_scaler = StandardScaler()
y_train = y_scaler.fit_transform(y_train[:, None])[:, 0]
y_test = y_scaler.transform(y_test[:, None])[:, 0]

gc.collect()
if verbose:
    print("ok")
return X_train, y_train, X_test, y_test

def boxplot_runtimes(runtimes, pred_type, configuration):
    ""
    Plot a new `Figure` with boxplots of prediction runtimes.
    Parameters
    ----------
    runtimes : list of `np.array` of latencies in micro-seconds
    cls_names : list of estimator class names that generated the runtimes
    pred_type : 'bulk' or 'atomic'
    ""
    fig, ax1 = plt.subplots(figsize=(10, 6))
    bp = plt.boxplot(runtimes, )

    cls_infos = ['%s
    estimator_conf['complexity_computer']('estimator_conf['instance'],
    estimator_conf['complexity_label']}) for
    estimator_conf in configuration['estimators']]
    plt.setp(ax1, xticklabels=cls_infos)
    plt.setp(bp['boxes'], color='black')
    plt.setp(bp['whiskers'], color='black')
    plt.setp(bp['fliers'], color='red', marker='+')

    ax1.yaxis.grid(True, linestyle='-', which='major', color='lightgrey',
                   alpha=0.5)
    ax1.set_axisbelow(True)
    ax1.set_title('Prediction Time per Instance - %s, %d feats.' % (pred_type.capitalize(),
        configuration['n_features']))
    ax1.set_ylabel('Prediction Time (us)')

    plt.show()
```python

def benchmark(configuration):
    """Run the whole benchmark."""
    X_train, y_train, X_test, y_test = generate_dataset(
        configuration['n_train'], configuration['n_test'],
        configuration['n_features'])

    stats = {}
    for estimator_conf in configuration['estimators']:
        print("Benchmarking ", estimator_conf['instance'])
        estimator_conf['instance'].fit(X_train, y_train)
        gc.collect()
        a, b = benchmark_estimator(estimator_conf['instance'], X_test)
        stats[estimator_conf['name']] = {'atomic': a, 'bulk': b}

    cls_names = [estimator_conf['name'] for estimator_conf in configuration["estimators"]]
    runtimes = [1e6 * stats[clf_name]['atomic'] for clf_name in cls_names]
    boxplot_runtimes(runtimes, 'atomic', configuration)
    runtimes = [1e6 * stats[clf_name]['bulk'] for clf_name in cls_names]
    boxplot_runtimes(runtimes, 'bulk (%d)' % configuration['n_test'], configuration)


def n_feature_influence(estimators, n_train, n_test, n_features, percentile):
    """Estimate influence of the number of features on prediction time.

    Parameters
    ----------
    estimators : dict of (name (str), estimator) to benchmark
    n_train : nber of training instances (int)
    n_test : nber of testing instances (int)
    n_features : list of feature-space dimensionality to test (int)
    percentile : percentile at which to measure the speed (int [0-100])

    Returns:
    -------
    percentiles : dict(name, dict(n_features, percentile_perf_in_us))
    """

    percentiles = defaultdict(lambda: defaultdict())
    for n in n_features:
        print("benchmarking with %d features" % n)
        X_train, y_train, X_test, y_test = generate_dataset(n_train, n_test, n)
        for cls_name, estimator in estimators.items():
            estimator.fit(X_train, y_train)
            gc.collect()
            runtimes = bulk_benchmark_estimator(estimator, X_test, 30, False)
            percentiles[cls_name][n] = 1e6 * scoreatpercentile(runtimes,
                                                                  percentile)

    return percentiles


def plot_n_features_influence(percentiles, percentile):
    fig, ax1 = plt.subplots(figsize=(10, 6))
```

5.10. Prediction Latency
colors = ['r', 'g', 'b']
for i, cls_name in enumerate(percentiles.keys()):
    x = np.array(sorted([n for n in percentiles[cls_name].keys()]))
    y = np.array([percentiles[cls_name][n] for n in x])
    plt.plot(x, y, color=colors[i], )
ax1.yaxis.grid(True, linestyle='-', which='major', color='lightgrey', alpha=0.5)
ax1.set_axisbelow(True)
ax1.set_title('Evolution of Prediction Time with #Features')
ax1.set_xlabel('#Features')
ax1.set_ylabel('Prediction Time at %d%%-ile (us)' % percentile)
plt.show()

def benchmark_throughputs(configuration, duration_secs=0.1):
    """benchmark throughput for different estimators."
    X_train, y_train, X_test, y_test = generate_dataset(
        configuration['n_train'], configuration['n_test'],
        configuration['n_features'])
    throughputs = dict()
    for estimator_config in configuration['estimators']:
        estimator_config['instance'].fit(X_train, y_train)
        start_time = time.time()
        n_predictions = 0
        while (time.time() - start_time) < duration_secs:
            estimator_config['instance'].predict(X_test[[0]])
            n_predictions += 1
        throughputs[estimator_config['name']] = n_predictions / duration_secs
    return throughputs

def plot_benchmark_throughput(throughputs, configuration):
    fig, ax = plt.subplots(figsize=(10, 6))
    colors = ['r', 'g', 'b']
    cls_infos = ['%s
    estimator_config['complexity_computer'](estimator_config['instance']),
    estimator_config['complexity_label']) for estimator_config in configuration['estimators']]
    cls_values = [throughputs[estimator_config['name']] for estimator_config in configuration['estimators']]
    plt.bar(range(len(throughputs)), cls_values, width=0.5, color=colors)
    ax.set_xticks(np.linspace(0.25, len(throughputs) - 0.75, len(throughputs)))
    ax.set_xticklabels(cls_infos, fontsize=10)
    ymax = max(cls_values) * 1.2
    ax.set_ylim((0, ymax))
    ax.set_ylabel('Throughput (predictions/sec)')
    ax.set_title('Prediction Throughput for different estimators (%d ' 'features)' % configuration['n_features'])
    plt.show()

main code
start_time = time.time()
# benchmark bulk/atomic prediction speed for various regressors
configuration = {
    'n_train': int(1e3),
"n_test": int(1e2),
"n_features": int(1e2),
'estimators": [
  {'name': 'Linear Model',
   'instance': SGDRegressor(penalty='elasticnet', alpha=0.01,
                          l1_ratio=0.25, fit_intercept=True),
   'complexity_label': 'non-zero coefficients',
   'complexity_computer': lambda clf: np.count_nonzero(clf.coef_),
  }
  {'name': 'RandomForest',
   'instance': RandomForestRegressor(),
   'complexity_label': 'estimators',
   'complexity_computer': lambda clf: clf.n_estimators,
  }
  {'name': 'SVR',
   'instance': SVR(kernel='rbf'),
   'complexity_label': 'support vectors',
   'complexity_computer': lambda clf: len(clf.support_vectors_),
  }
]
benchmark(configuration)

# benchmark n_features influence on prediction speed
percentile = 90
percentiles = n_feature_influence({'ridge': Ridge()},
                                   configuration['n_train'],
                                   configuration['n_test'],
                                   [100, 250, 500], percentile)
plot_n_features_influence(percentiles, percentile)

# benchmark throughput
throughputs = benchmark_throughputs(configuration)
plot_benchmark_throughput(throughputs, configuration)

stop_time = time.time()
print("example run in %.2fs" % (stop_time - start_time))
Out:

Benchmarking SGDRegressor(alpha=0.01, average=False, epsilon=0.1, eta0=0.01, fit_intercept=True, l1_ratio=0.25, learning_rate='invscaling', loss='squared_loss', n_iter=5, penalty='elasticnet', power_t=0.25, random_state=None, shuffle=True, verbose=0, warm_start=False)

Benchmarking RandomForestRegressor(bootstrap=True, criterion='mse', max_depth=None, max_features='auto', max_leaf_nodes=None, min_impurity_split=1e-07, min_samples_leaf=1, min_samples_split=2, min_weight_fraction_leaf=0.0, n_estimators=10, n_jobs=1, oob_score=False, random_state=None, verbose=0, warm_start=False)

Benchmarking SVR(C=1.0, cache_size=200, coef0=0.0, degree=3, epsilon=0.1, gamma='auto', kernel='rbf', max_iter=-1, shrinking=True, tol=0.001, verbose=False)

benchmarks with 100 features
benchmarking with 250 features
benchmarking with 500 features
example run in 3.64s

Total running time of the script: (0 minutes 3.642 seconds)

Download Python source code: plot_prediction_latency.py
Download IPython notebook: plot_prediction_latency.ipynb

5.11 Out-of-core classification of text documents

This is an example showing how scikit-learn can be used for classification using an out-of-core approach: learning from data that doesn’t fit into main memory. We make use of an online classifier, i.e., one that supports the partial_fit method, that will be fed with batches of examples. To guarantee that the features space remains the same over time we leverage a HashingVectorizer that will project each example into the same feature space. This is especially useful in the case of text classification where new features (words) may appear in each batch.
The dataset used in this example is Reuters-21578 as provided by the UCI ML repository. It will be automatically downloaded and uncompressed on first run.

The plot represents the learning curve of the classifier: the evolution of classification accuracy over the course of the mini-batches. Accuracy is measured on the first 1000 samples, held out as a validation set.

To limit the memory consumption, we queue examples up to a fixed amount before feeding them to the learner.

```python
from __future__ import print_function
from glob import glob
import itertools
import os.path
import re
import tarfile
import time
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import rcParams
from sklearn.externals.six.moves import html_parser
from sklearn.externals.six.moves import urllib
from sklearn.datasets import get_data_home
from sklearn.feature_extraction.text import HashingVectorizer
from sklearn.linear_model import SGDClassifier
from sklearn.linear_model import PassiveAggressiveClassifier
from sklearn.linear_model import Perceptron
from sklearn.naive_bayes import MultinomialNB

def _not_in_sphinx():
    # Hack to detect whether we are running by the sphinx builder
    return '__file__' in globals()

5.11.1 Reuters Dataset related routines

```python
class ReutersParser(html_parser.HTMLParser):
    """Utility class to parse a SGML file and yield documents one at a time."""

    def __init__(self, encoding='latin-1'):
        html_parser.HTMLParser.__init__(self)
        self._reset()
        self.encoding = encoding

        def handle_starttag(self, tag, attrs):
            method = 'start_' + tag
            getattr(self, method, lambda x: None)(attrs)

        def handle_endtag(self, tag):
            method = 'end_' + tag
            getattr(self, method, lambda: None)()

```python
5.11. Out-of-core classification of text documents
def _reset(self):
    self.in_title = 0
    self.in_body = 0
    self.in_topics = 0
    self.in_topic_d = 0
    self.title = ""
    self.body = ""
    self.topics = []
    self.topic_d = ""

def parse(self, fd):
    self.docs = []
    for chunk in fd:
        self.feed(chunk.decode(self.encoding))
        for doc in self.docs:
            yield doc
    self.docs = []
    self.close()

def handle_data(self, data):
    if self.in_body:
        self.body += data
    elif self.in_title:
        self.title += data
    elif self.in_topic_d:
        self.topic_d += data

def start_reuters(self, attributes):
    pass

def end_reuters(self):
    self.body = re.sub(r'\s+', r' ', self.body)
    self.docs.append({'title': self.title,
                      'body': self.body,
                      'topics': self.topics})
    self._reset()

def start_title(self, attributes):
    self.in_title = 1

def end_title(self):
    self.in_title = 0

def start_body(self, attributes):
    self.in_body = 1

def end_body(self):
    self.in_body = 0

def start_topics(self, attributes):
    self.in_topics = 1

def end_topics(self):
    self.in_topics = 0

def start_d(self, attributes):
    self.in_topic_d = 1
def end_d(self):
    self.in_topic_d = 0
    self.topics.append(self.topic_d)
    self.topic_d = ""

def stream_reuters_documents(data_path=None):
    """Iterate over documents of the Reuters dataset.

    The Reuters archive will automatically be downloaded and uncompressed if
    the `data_path` directory does not exist.

    Documents are represented as dictionaries with 'body' (str),
    'title' (str), 'topics' (list(str)) keys.
    """

        'reuters21578-mld/reuters21578.tar.gz')
    ARCHIVE_FILENAME = 'reuters21578.tar.gz'

    if data_path is None:
        data_path = os.path.join(get_data_home(), "reuters")
    if not os.path.exists(data_path):
        """Download the dataset."""
        print("downloading dataset (once and for all) into $s" %
            data_path)
        os.mkdir(data_path)

        def progress(blocknum, bs, size):
            total_sz_mb = '%.2f MB' % (size / 1e6)
            current_sz_mb = '%.2f MB' % ((blocknum * bs) / 1e6)
            if _not_in_sphinx():
                print('\rdownloaded $s / $s % (current_sz_mb, total_sz_mb),
                    end=''')

            archive_path = os.path.join(data_path, ARCHIVE_FILENAME)
            urllib.request.urlretrieve(DOWNLOAD_URL, filename=archive_path,
                report_hook=progress)
        if _not_in_sphinx():
            print('\r', end='')
        print("untarring Reuters dataset...")
        tarfile.open(archive_path, 'r:gz').extractall(data_path)
        print("done.")

    parser = ReutersParser()
    for filename in glob(os.path.join(data_path, "*.sgm")):
        for doc in parser.parse(open(filename, 'rb')):
            yield doc

5.11.2 Main

Create the vectorizer and limit the number of features to a reasonable maximum

vectorizer = HashingVectorizer(decode_error='ignore', n_features=2 ** 18,
    non_negative=True)
# Iterator over parsed Reuters SGML files.
data_stream = stream_reuters_documents()

# We learn a binary classification between the "acq" class and all the others.
# "acq" was chosen as it is more or less evenly distributed in the Reuters
# files. For other datasets, one should take care of creating a test set with
# a realistic portion of positive instances.
all_classes = np.array([0, 1])
positive_class = 'acq'

# Here are some classifiers that support the `partial_fit` method
partial_fit_classifiers = {
    'SGD': SGDClassifier(),
    'Perceptron': Perceptron(),
    'NB Multinomial': MultinomialNB(alpha=0.01),
    'Passive-Aggressive': PassiveAggressiveClassifier(),
}

def get_minibatch(doc_iter, size, pos_class=positive_class):
    """Extract a minibatch of examples, return a tuple X_text, y.
    Note: size is before excluding invalid docs with no topics assigned.
    """
    data = [(u'{title}


{body}'.format(**doc), pos_class in doc['topics'])
        for doc in itertools.islice(doc_iter, size)
        if doc['topics']]
    if not len(data):
        return np.asarray([], dtype=int), np.asarray([], dtype=int)
    X_text, y = zip(*data)
    return X_text, np.asarray(y, dtype=int)

def iter_minibatches(doc_iter, minibatch_size):
    """Generator of minibatches.""
    X_text, y = get_minibatch(doc_iter, minibatch_size)
    while len(X_text):
        yield X_text, y
        X_text, y = get_minibatch(doc_iter, minibatch_size)

# test data statistics
test_stats = {'n_test': 0, 'n_test_pos': 0}

# First we hold out a number of examples to estimate accuracy
n_test_documents = 1000
tick = time.time()
X_test_text, y_test = get_minibatch(data_stream, 1000)
parsing_time = time.time() - tick
tick = time.time()
X_test = vectorizer.transform(X_test_text)
vectorizing_time = time.time() - tick
test_stats['n_test'] += len(y_test)
test_stats['n_test_pos'] += sum(y_test)
print("Test set is \%d documents (\%d positive)" % (len(y_test), sum(y_test)))
def progress(cls_name, stats):
    """Report progress information, return a string."""
    duration = time.time() - stats['t0']
    s = "%20s classifier : \t %s
    s += "%(n_train)6d train docs (%(n_train_pos)6d positive) " % stats
    s += "%(n_test)6d test docs (%(n_test_pos)6d positive) " % test_stats
    s += "accuracy: %(accuracy).3f " % stats
    s += "in %0.2f s (%5d docs/s)" % (duration, stats['n_train'] / duration)
    return s

cls_stats = {}

for cls_name in partial_fit_classifiers:
    stats = {'n_train': 0, 'n_train_pos': 0,
             'accuracy': 0.0, 'accuracy_history': [(0, 0)],
             'runtime_history': [(0, 0)], 'total_fit_time': 0.0}
    cls_stats[cls_name] = stats

get_minibatch(data_stream, n_test_documents)
# Discard test set

# We will feed the classifier with mini-batches of 1000 documents; this means
# we have at most 1000 docs in memory at any time. The smaller the document
# batch, the bigger the relative overhead of the partial fit methods.
minibatch_size = 1000

# Create the data_stream that parses Reuters SGML files and iterates on
# documents as a stream.
minibatch_iterators = iter_minibatches(data_stream, minibatch_size)
total_vect_time = 0.0

# Main loop : iterate on mini-batches of examples
for i, (X_train_text, y_train) in enumerate(minibatch_iterators):
    tick = time.time()
    X_train = vectorizer.transform(X_train_text)
    total_vect_time += time.time() - tick

    for cls_name, cls in partial_fit_classifiers.items():
        tick = time.time()
        # update estimator with examples in the current mini-batch
        cls.partial_fit(X_train, y_train, classes=all_classes)

        # accumulate test accuracy stats
        cls_stats[cls_name]['total_fit_time'] += time.time() - tick
        cls_stats[cls_name]['n_train'] += X_train.shape[0]
        cls_stats[cls_name]['n_train_pos'] += sum(y_train)
        tick = time.time()
        cls_stats[cls_name]['accuracy'] = cls.score(X_test, y_test)
        cls_stats[cls_name]['prediction_time'] = time.time() - tick
        acc_history = (cls_stats[cls_name]['accuracy'],
                        cls_stats[cls_name]['n_train'])
        cls_stats[cls_name]['accuracy_history'].append(acc_history)
        run_history = (cls_stats[cls_name]['accuracy'],
                       total_vect_time + cls_stats[cls_name]['total_fit_time'])

5.11. Out-of-core classification of text documents
```python
cls_stats[cls_name]['runtime_history'].append(run_history)

if i % 3 == 0:
    print(progress(cls_name, cls_stats[cls_name]))
if i % 3 == 0:
    print('\\n')
```

<table>
<thead>
<tr>
<th>Test set</th>
<th>is 878 documents (108 positive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive-Aggressive classifier :</td>
<td>962 train docs ( 132 positive) 878 test</td>
</tr>
<tr>
<td>docs ( 108 positive) accuracy: 0.913 in 1.78s ( 539 docs/s)</td>
<td></td>
</tr>
<tr>
<td>Perceptron classifier :</td>
<td>962 train docs ( 132 positive) 878 test</td>
</tr>
<tr>
<td>docs ( 108 positive) accuracy: 0.921 in 1.79s ( 538 docs/s)</td>
<td></td>
</tr>
<tr>
<td>SGD classifier :</td>
<td>962 train docs ( 132 positive) 878 test</td>
</tr>
<tr>
<td>docs ( 108 positive) accuracy: 0.925 in 1.79s ( 536 docs/s)</td>
<td></td>
</tr>
<tr>
<td>NB Multinomial classifier :</td>
<td>962 train docs ( 132 positive) 878 test</td>
</tr>
<tr>
<td>docs ( 108 positive) accuracy: 0.877 in 1.83s ( 526 docs/s)</td>
<td></td>
</tr>
</tbody>
</table>

| Passive-Aggressive classifier : | 3911 train docs ( 517 positive) 878 test |
| docs ( 108 positive) accuracy: 0.946 in 7.29s ( 536 docs/s) | |
| Perceptron classifier : | 3911 train docs ( 517 positive) 878 test |
| docs ( 108 positive) accuracy: 0.926 in 7.30s ( 535 docs/s) | |
| SGD classifier : | 3911 train docs ( 517 positive) 878 test |
| docs ( 108 positive) accuracy: 0.945 in 7.30s ( 533 docs/s) | |
| NB Multinomial classifier : | 3911 train docs ( 517 positive) 878 test |
| docs ( 108 positive) accuracy: 0.885 in 7.33s ( 533 docs/s) | |

| Passive-Aggressive classifier : | 6821 train docs ( 891 positive) 878 test |
| docs ( 108 positive) accuracy: 0.951 in 10.85s ( 628 docs/s) | |
| Perceptron classifier : | 6821 train docs ( 891 positive) 878 test |
| docs ( 108 positive) accuracy: 0.949 in 10.85s ( 628 docs/s) | |
| SGD classifier : | 6821 train docs ( 891 positive) 878 test |
| docs ( 108 positive) accuracy: 0.938 in 10.85s ( 628 docs/s) | |
| NB Multinomial classifier : | 6821 train docs ( 891 positive) 878 test |
| docs ( 108 positive) accuracy: 0.899 in 10.89s ( 626 docs/s) | |

| Passive-Aggressive classifier : | 9759 train docs ( 1276 positive) 878 test |
| docs ( 108 positive) accuracy: 0.964 in 14.26s ( 684 docs/s) | |
| Perceptron classifier : | 9759 train docs ( 1276 positive) 878 test |
| docs ( 108 positive) accuracy: 0.950 in 14.27s ( 684 docs/s) | |
| SGD classifier : | 9759 train docs ( 1276 positive) 878 test |
| docs ( 108 positive) accuracy: 0.958 in 14.27s ( 683 docs/s) | |
| NB Multinomial classifier : | 9759 train docs ( 1276 positive) 878 test |
| docs ( 108 positive) accuracy: 0.909 in 14.30s ( 682 docs/s) | |

| Passive-Aggressive classifier : | 11680 train docs ( 1499 positive) 878 test |
| docs ( 108 positive) accuracy: 0.951 in 17.31s ( 674 docs/s) | |
| Perceptron classifier : | 11680 train docs ( 1499 positive) 878 test |
| docs ( 108 positive) accuracy: 0.951 in 17.31s ( 674 docs/s) | |
| SGD classifier : | 11680 train docs ( 1499 positive) 878 test |
| docs ( 108 positive) accuracy: 0.951 in 17.32s ( 674 docs/s) | |
| NB Multinomial classifier : | 11680 train docs ( 1499 positive) 878 test |
| docs ( 108 positive) accuracy: 0.916 in 17.35s ( 673 docs/s) | |
5.11.3 Plot results

```python
def plot_accuracy(x, y, x_legend):
    """Plot accuracy as a function of x.""
    x = np.array(x)
    y = np.array(y)
    plt.title('Classification accuracy as a function of %s' % x_legend)
    plt.xlabel('%s' % x_legend)
    plt.ylabel('Accuracy')
    plt.grid(True)
    plt.plot(x, y)

rcParams['legend.fontsize'] = 10
cls_names = list(sorted(cls_stats.keys()))

# Plot accuracy evolution
plt.figure()
for _, stats in sorted(cls_stats.items()):
    # Plot accuracy evolution with #examples
    accuracy, n_examples = zip(*stats['accuracy_history'])
    plot_accuracy(n_examples, accuracy, "training examples (#)")
    ax = plt.gca()
    ax.set_xlim((0.8, 1))
    plt.legend(cls_names, loc='best')

plt.figure()
for _, stats in sorted(cls_stats.items()):
    # Plot accuracy evolution with runtime
    accuracy, runtime = zip(*stats['runtime_history'])
    plot_accuracy(runtime, accuracy, 'runtime (s)'
    ax = plt.gca()
    ax.set_xlim((0.8, 1))
    plt.legend(cls_names, loc='best')
```

# Plot fitting times
plt.figure()
fig = plt.gcf()
cls_runtime = []
for cls_name, stats in sorted(cls_stats.items()):
    cls_runtime.append(stats['total_fit_time'])

cls_runtime.append(total_vect_time)
cls_names.append('Vectorization')
bar_colors = ['b', 'g', 'r', 'c', 'm', 'y']

ax = plt.subplot(111)
rectangles = plt.bar(range(len(cls_names)), cls_runtime, width=0.5,
                     color=bar_colors)

ax.set_xticks(np.linspace(0.25, len(cls_names) - 0.75, len(cls_names)))
ax.set_xticklabels(cls_names, fontsize=10)
ymax = max(cls_runtime) * 1.2
ax.set_ylim((0, ymax))
ax.set_ylabel('runtime (s)')
ax.set_title('Training Times')
def autolabel(rectangles):
    for rect in rectangles:
        height = rect.get_height()
        ax.text(rect.get_x() + rect.get_width() / 2.,
                1.05 * height, '%.4f' % height,
                ha='center', va='bottom')

autolabel(rectangles)
plt.show()

# Plot prediction times
plt.figure()
cls_names = list(sorted(cls_stats.keys()))
for cls_name, stats in sorted(cls_stats.items()):
    cls_runtime.append(stats['prediction_time'])
cls_runtime.append(parsing_time)
cls_names.append('Read/Parse\n+Feat.Extr.'
cls_runtime.append(vectorizing_time)
cls_names.append('Hashing\n+Vect.')

ax = plt.subplot(111)
rectangles = plt.bar(range(len(cls_names)), cls_runtime, width=0.5,
                     color=bar_colors)

ax.set_xticks(np.linspace(0.25, len(cls_names) - 0.75, len(cls_names)))
ax.set_xticklabels(cls_names, fontsize=8)
plt.setp(plt.xticks()[1], rotation=30)
ymax = max(cls_runtime) * 1.2
ax.set_ylim((0, ymax))
ax.set_ylabel('runtime (s)')
ax.set_title('Prediction Times (%d instances)' % n_test_documents)
autolabel(rectangles)
plt.show()
5.11. Out-of-core classification of text documents
Total running time of the script: (0 minutes 25.914 seconds)
Download Python source code: plot_out_of_core_classification.py
Download IPython notebook: plot_out_of_core_classification.ipynb
Examples concerning the `sklearn.cluster.bicluster` module.

### 6.1 A demo of the Spectral Co-Clustering algorithm

This example demonstrates how to generate a dataset and bicluster it using the Spectral Co-Clustering algorithm.

The dataset is generated using the `make_biclusters` function, which creates a matrix of small values and implants bicluster with large values. The rows and columns are then shuffled and passed to the Spectral Co-Clustering algorithm. Rearranging the shuffled matrix to make biclusters contiguous shows how accurately the algorithm found the biclusters.
print(__doc__)

# Author: Kemal Eren <kemal@kemaleren.com>
# License: BSD 3 clause

import numpy as np
from matplotlib import pyplot as plt

from sklearn.datasets import make_biclusters
from sklearn.datasets import samples_generator as sg
from sklearn.cluster.bicluster import SpectralCoclustering

Out:

consensus score: 1.000
from sklearn.metrics import consensus_score

data, rows, columns = make_biclusters(
    shape=(300, 300), n_clusters=5, noise=5,
    shuffle=False, random_state=0)

plt.matshow(data, cmap=plt.cm.Blues)
plt.title("Original dataset")

data, row_idx, col_idx = sg._shuffle(data, random_state=0)
plt.matshow(data, cmap=plt.cm.Blues)
plt.title("Shuffled dataset")

model = SpectralCoclustering(n_clusters=5, random_state=0)
model.fit(data)
score = consensus_score(model.biclusters_,
    (rows[:, row_idx], columns[:, col_idx]))

print("consensus score: {:.3f}".format(score))

fit_data = data[np.argsort(model.row_labels_)]
fit_data = fit_data[:, np.argsort(model.column_labels_)]

plt.matshow(fit_data, cmap=plt.cm.Blues)
plt.title("After biclustering; rearranged to show biclusters")

plt.show()

Total running time of the script: (0 minutes 0.226 seconds)

Download Python source code: plot_spectral_coclustering.py

Download IPython notebook: plot_spectral_coclustering.ipynb

6.2 A demo of the Spectral Biclustering algorithm

This example demonstrates how to generate a checkerboard dataset and bicluster it using the Spectral Biclustering algorithm.

The data is generated with the make_checkerboard function, then shuffled and passed to the Spectral Biclustering algorithm. The rows and columns of the shuffled matrix are rearranged to show the biclusters found by the algorithm.

The outer product of the row and column label vectors shows a representation of the checkerboard structure.
Chapter 6. Biclustering
Out:

```
consensus score: 1.0
```

```python
print(__doc__)

# Author: Kemal Eren <kemal@kemaleren.com>
# License: BSD 3 clause

import numpy as np
from matplotlib import pyplot as plt

from sklearn.datasets import make_checkerboard
from sklearn.datasets import samples_generator as sg
from sklearn.cluster.bicluster import SpectralBiclustering
from sklearn.metrics import consensus_score

n_clusters = (4, 3)
data, rows, columns = make_checkerboard(
    shape=(300, 300), n_clusters=n_clusters, noise=10,
    shuffle=False, random_state=0)

plt.matshow(data, cmap=plt.cm.Blues)
plt.title("Original dataset")

data, row_idx, col_idx = sg._shuffle(data, random_state=0)
plt.matshow(data, cmap=plt.cm.Blues)
plt.title("Shuffled dataset")

model = SpectralBiclustering(n_clusters=n_clusters, method='log',
    random_state=0)
model.fit(data)
score = consensus_score(model.biclusters_,
    (rows[:, row_idx], columns[:, col_idx]))
```

6.2. A demo of the Spectral Biclustering algorithm
print("consensus score: {:.1f}".format(score))

fit_data = data[np.argsort(model.row_labels_)]
fit_data = fit_data[:, np.argsort(model.column_labels_)]

plt.matshow(fit_data, cmap=plt.cm.Blues)
plt.title("After biclustering; rearranged to show biclusters")

plt.matshow(np.outer(np.sort(model.row_labels_) + 1,
                      np.sort(model.column_labels_) + 1),
            cmap=plt.cm.Blues)
plt.title("Checkerboard structure of rearranged data")

plt.show()

Total running time of the script: (0 minutes 0.740 seconds)
Download Python source code: plot_spectral_biclustering.py
Download IPython notebook: plot_spectral_biclustering.ipynb

6.3 Biclustering documents with the Spectral Co-clustering algorithm

This example demonstrates the Spectral Co-clustering algorithm on the twenty newsgroups dataset. The ‘comp.os.ms-windows.misc’ category is excluded because it contains many posts containing nothing but data.

The TF-IDF vectorized posts form a word frequency matrix, which is then biclustered using Dhillon’s Spectral Co-Clustering algorithm. The resulting document-word biclusters indicate subsets words used more often in those subsets documents.

For a few of the best biclusters, its most common document categories and its ten most important words get printed. The best biclusters are determined by their normalized cut. The best words are determined by comparing their sums inside and outside the bicluster.

For comparison, the documents are also clustered using MiniBatchKMeans. The document clusters derived from the biclusters achieve a better V-measure than clusters found by MiniBatchKMeans.

Output:

Vectorizing...
Coclustering...
Done in 9.53s. V-measure: 0.4455
MiniBatchKMeans...
Done in 12.00s. V-measure: 0.3309

Best biclusters:
----------------
bicluster 0 : 1951 documents, 4373 words
categories : 23% talk.politics.guns, 19% talk.politics.misc, 14% sci.med
words : gun, guns, geb, banks, firearms, drugs, gordon, clinton, cdt, amendment

bicluster 1 : 1165 documents, 3304 words
categories : 29% talk.politics.mideast, 26% soc.religion.christian, 25% alt.atheism
words : god, jesus, christians, atheists, kent, sin, morality, belief, resurrection, marriage
bicluster 2 : 2219 documents, 2830 words
categories : 18% comp.sys.mac.hardware, 16% comp.sys.ibm.pc.hardware, 16% comp.
→graphics
words : voltage, dsp, board, receiver, circuit, shipping, packages, stereo, stereo,
→compression, package

bicluster 3 : 1860 documents, 2745 words
categories : 26% rec.motorcycles, 23% rec.autos, 13% misc.forsale
words : bike, car, dod, engine, motorcycle, ride, honda, cars, bmw, bikes

bicluster 4 : 12 documents, 155 words
categories : 100% rec.sport.hockey
words : scorer, unassisted, reichel, semak, sweeney, kovalenko, ricci, audette,
→momesso, nedved

from __future__ import print_function
from collections import defaultdict
import operator
import re
from time import time
import numpy as np
from sklearn.cluster.bicluster import SpectralCoclustering
from sklearn.cluster import MiniBatchKMeans
from sklearn.externals.six import iteritems
from sklearn.datasets.twenty_newsgroups import fetch_20newsgroups
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.metrics.cluster import v_measure_score

def number_aware_tokenizer(doc):
    """ Tokenizer that maps all numeric tokens to a placeholder."

    For many applications, tokens that begin with a number are not directly
    useful, but the fact that such a token exists can be relevant. By applying
    this form of dimensionality reduction, some methods may perform better.
    ""
    token_pattern = re.compile(u'(\b\w+(\b\w+)*)')
tokens = token_pattern.findall(doc)
tokens = ["#NUMBER" if token[0] in "0123456789_" else token
          for token in tokens]
    return tokens

# exclude 'comp.os.ms-windows.misc'
categories = ["alt.atheism", "comp.graphics",
              "comp.sys.ibm.pc.hardware", "comp.sys.mac.hardware",
              "comp.windows.x", "misc.forsale", "rec.autos",
              "rec.motorcycles", "rec.sport.baseball",
              "rec.sport.hockey", "sci.crypt", "sci.electronics",
              "sci.med", "sci.space", "soc.religion.christian",
              "talk.politics.guns", "talk.politics.mideast",
              "talk.politics.misc", "talk.religion.misc"]
newsgroups = fetch_20newsgroups(categories=categories)
y_true = newsgroups.target

vectorizer = TfidfVectorizer(stop_words='english', min_df=5,
                           tokenizer=number_aware_tokenizer)
cocluster = SpectralCoclustering(n_clusters=len(categories),
                           svd_method='arpack', random_state=0)
kmeans = MiniBatchKMeans(n_clusters=len(categories), batch_size=20000,
                          random_state=0)

print("Vectorizing...")
X = vectorizer.fit_transform(newsgroups.data)

print("Coclustering...")
start_time = time()
cocluster.fit(X)
y_cocluster = cocluster.row_labels_
print("Done in {:.2f}s. V-measure: {:.4f}".format(
    time() - start_time,
    v_measure_score(y_cocluster, y_true)))

print("MiniBatchKMeans...")
start_time = time()
y_kmeans = kmeans.fit_predict(X)
print("Done in {:.2f}s. V-measure: {:.4f}".format(
    time() - start_time,
    v_measure_score(y_kmeans, y_true)))

feature_names = vectorizer.get_feature_names()
document_names = list(newsgroups.target_names[i] for i in newsgroups.target)

def bicluster_ncut(i):
    rows, cols = cocluster.get_indices(i)
    if not (np.any(rows) and np.any(cols)):
        import sys
        return sys.float_info.max
    row_complement = np.nonzero(np.logical_not(cocluster.rows_[i]))[0]
    col_complement = np.nonzero(np.logical_not(cocluster.columns_[i]))[0]
    # Note: the following is identical to X[rows[:, np.newaxis], cols].sum() but
    # much faster in scipy <= 0.16
    weight = X[rows[:, cols].sum()]
    cut = (X[row_complement[:, cols].sum() + X[rows[:, col_complement].sum()])
    return cut / weight

def most_common(d):
    """Items of a defaultdict(int) with the highest values.
    Like Counter.most_common in Python >=2.7.
    """
    return sorted(iteritems(d), key=operator.itemgetter(1), reverse=True)

bicluster_ncuts = list(bicluster_ncut(i) for i in range(len(newsgroups.target_names)))
best_idx = np.argsort(bicluster_ncuts)[:5]
print()  
print("Best biclusters:")  
print("----------------")  
for idx, cluster in enumerate(best_idx):  
    n_rows, n_cols = cocluster.get_shape(cluster)  
    cluster_docs, cluster_words = cocluster.get_indices(cluster)  
    if not len(cluster_docs) or not len(cluster_words):  
        continue  
    
    # categories  
    counter = defaultdict(int)  
    for i in cluster_docs:  
        counter[document_names[i]] += 1  
    cat_string = ", ".join("{:.0f}% {}".format(float(c) / n_rows * 100, name)  
    for name, c in most_common(counter)[:3])  

    # words  
    out_of_cluster_docs = cocluster.row_labels_ != cluster  
    out_of_cluster_docs = np.where(out_of_cluster_docs)[0]  
    word_col = X[:, cluster_words]  
    word_scores = np.array(word_col[cluster_docs, :].sum(axis=0) -  
                           word_col[out_of_cluster_docs, :].sum(axis=0))  
    word_scores = word_scores.ravel()  
    important_words = list(feature_names[cluster_words[i]]  
                           for i in word_scores.argsort()[:-11:-1])  

    print("bicluster {} : {} documents, {} words".format(  
        idx, n_rows, n_cols))  
    print("categories : ".format(cat_string))  
    print("words : {}\n".format(', '.join(important_words)))

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: bicluster_newsgroups.py
Download IPython notebook: bicluster_newsgroups.ipynb
Examples illustrating the calibration of predicted probabilities of classifiers.

### 7.1 Comparison of Calibration of Classifiers

Well calibrated classifiers are probabilistic classifiers for which the output of the predict_proba method can be directly interpreted as a confidence level. For instance a well calibrated (binary) classifier should classify the samples such that among the samples to which it gave a predict_proba value close to 0.8, approx. 80% actually belong to the positive class.

LogisticRegression returns well calibrated predictions as it directly optimizes log-loss. In contrast, the other methods return biased probabilities, with different biases per method:

- **GaussianNaiveBayes** tends to push probabilities to 0 or 1 (note the counts in the histograms). This is mainly because it makes the assumption that features are conditionally independent given the class, which is not the case in this dataset which contains 2 redundant features.

- **RandomForestClassifier** shows the opposite behavior: the histograms show peaks at approx. 0.2 and 0.9 probability, while probabilities close to 0 or 1 are very rare. An explanation for this is given by Niculescu-Mizil and Caruana [1]: “Methods such as bagging and random forests that average predictions from a base set of models can have difficulty making predictions near 0 and 1 because variance in the underlying base models will bias predictions that should be near zero or one away from these values. Because predictions are restricted to the interval [0,1], errors caused by variance tend to be one-sided near zero and one. For example, if a model should predict p = 0 for a case, the only way bagging can achieve this is if all bagged trees predict zero. If we add noise to the trees that bagging is averaging over, this noise will cause some trees to predict values larger than 0 for this case, thus moving the average prediction of the bagged ensemble away from 0. We observe this effect most strongly with random forests because the base-level trees trained with random forests have relatively high variance due to feature subsetting.” As a result, the calibration curve shows a characteristic sigmoid shape, indicating that the classifier could trust its “intuition” more and return probabilities closer to 0 or 1 typically.

- **Support Vector Classification (SVC)** shows an even more sigmoid curve as the RandomForestClassifier, which is typical for maximum-margin methods (compare Niculescu-Mizil and Caruana [1]), which focus on hard samples that are close to the decision boundary (the support vectors).

![References](print(__doc__))

# Author: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
# License: BSD Style.
scikit-learn user guide, Release 0.18.2

import numpy as np
np.random.seed(0)
import matplotlib.pyplot as plt
from
from
from
from
from
from

sklearn import datasets
sklearn.naive_bayes import GaussianNB
sklearn.linear_model import LogisticRegression
sklearn.ensemble import RandomForestClassifier
sklearn.svm import LinearSVC
sklearn.calibration import calibration_curve

X, y = datasets.make_classification(n_samples=100000, n_features=20,
n_informative=2, n_redundant=2)
train_samples = 100

# Samples used for training the models

X_train = X[:train_samples]
X_test = X[train_samples:]
y_train = y[:train_samples]
y_test = y[train_samples:]
# Create classifiers
lr = LogisticRegression()
gnb = GaussianNB()
svc = LinearSVC(C=1.0)
rfc = RandomForestClassifier(n_estimators=100)

Plot calibration plots
plt.figure(figsize=(10, 10))
ax1 = plt.subplot2grid((3, 1), (0, 0), rowspan=2)
ax2 = plt.subplot2grid((3, 1), (2, 0))
ax1.plot([0, 1], [0, 1], "k:", label="Perfectly calibrated")
for clf, name in [(lr, 'Logistic'),
(gnb, 'Naive Bayes'),
(svc, 'Support Vector Classification'),
(rfc, 'Random Forest')]:
clf.fit(X_train, y_train)
if hasattr(clf, "predict_proba"):
prob_pos = clf.predict_proba(X_test)[:, 1]
else: # use decision function
prob_pos = clf.decision_function(X_test)
prob_pos = \
(prob_pos - prob_pos.min()) / (prob_pos.max() - prob_pos.min())
fraction_of_positives, mean_predicted_value = \
calibration_curve(y_test, prob_pos, n_bins=10)
ax1.plot(mean_predicted_value, fraction_of_positives, "s-",
label="%s" % (name, ))
ax2.hist(prob_pos, range=(0, 1), bins=10, label=name,
histtype="step", lw=2)
ax1.set_ylabel("Fraction of positives")
ax1.set_ylim([-0.05, 1.05])
ax1.legend(loc="lower right")

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ax1.set_title('Calibration plots (reliability curve)')

ax2.set_xlabel("Mean predicted value")
ax2.set_ylabel("Count")
ax2.legend(loc="upper center", ncol=2)

plt.tight_layout()
plt.show()
7.2 Probability Calibration curves

When performing classification one often wants to predict not only the class label, but also the associated probability. This probability gives some kind of confidence on the prediction. This example demonstrates how to display how well calibrated the predicted probabilities are and how to calibrate an uncalibrated classifier.

The experiment is performed on an artificial dataset for binary classification with 100,000 samples (1,000 of them are used for model fitting) with 20 features. Of the 20 features, only 2 are informative and 10 are redundant. The first figure shows the estimated probabilities obtained with logistic regression, Gaussian naive Bayes, and Gaussian naive Bayes with both isotonic calibration and sigmoid calibration. The calibration performance is evaluated with Brier score, reported in the legend (the smaller the better). One can observe here that logistic regression is well calibrated while raw Gaussian naive Bayes performs very badly. This is because of the redundant features which violate the assumption of feature-independence and result in an overly confident classifier, which is indicated by the typical transposed-sigmoid curve.

Calibration of the probabilities of Gaussian naive Bayes with isotonic regression can fix this issue as can be seen from the nearly diagonal calibration curve. Sigmoid calibration also improves the brier score slightly, albeit not as strongly as the non-parametric isotonic regression. This can be attributed to the fact that we have plenty of calibration data such that the greater flexibility of the non-parametric model can be exploited.

The second figure shows the calibration curve of a linear support-vector classifier (LinearSVC). LinearSVC shows the opposite behavior as Gaussian naive Bayes: the calibration curve has a sigmoid curve, which is typical for an under-confident classifier. In the case of LinearSVC, this is caused by the margin property of the hinge loss, which lets the model focus on hard samples that are close to the decision boundary (the support vectors).

Both kinds of calibration can fix this issue and yield nearly identical results. This shows that sigmoid calibration can deal with situations where the calibration curve of the base classifier is sigmoid (e.g., for LinearSVC) but not where it is transposed-sigmoid (e.g., Gaussian naive Bayes).
Out:

Logistic:
   Brier: 0.099
   Precision: 0.872
   Recall: 0.851
   F1: 0.862

Naive Bayes:
   Brier: 0.118
   Precision: 0.857
   Recall: 0.876
   F1: 0.867

Naive Bayes + Isotonic:
   Brier: 0.098
   Precision: 0.883
   Recall: 0.836
   F1: 0.859

Naive Bayes + Sigmoid:
   Brier: 0.109
   Precision: 0.861
   Recall: 0.871
   F1: 0.866

Logistic:
   Brier: 0.099
   Precision: 0.872
   Recall: 0.851
   F1: 0.862

SVC:
   Brier: 0.163
   Precision: 0.872
   Recall: 0.852
   F1: 0.862

SVC + Isotonic:
Brier: 0.100
Precision: 0.853
Recall: 0.878
F1: 0.865

SVC + Sigmoid:
Brier: 0.099
Precision: 0.874
Recall: 0.849
F1: 0.861

print(__doc__)

# Author: Alexandre Gramfort <alexandre.gramfort@telecom-paristech.fr>
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# License: BSD Style.
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn.naive_bayes import GaussianNB
from sklearn.svm import LinearSVC
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import (brier_score_loss, precision_score, recall_score,
                             f1_score)
from sklearn.calibration import CalibratedClassifierCV, calibration_curve
from sklearn.model_selection import train_test_split

# Create dataset of classification task with many redundant and few
# informative features
X, y = datasets.make_classification(n_samples=100000, n_features=20,
                                   n_informative=2, n_redundant=10,
                                   random_state=42)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.99,
                                                    random_state=42)

def plot_calibration_curve(est, name, fig_index):
    """Plot calibration curve for est w/o and with calibration. """
    # Calibrated with isotonic calibration
    isotonic = CalibratedClassifierCV(est, cv=2, method='isotonic')

    # Calibrated with sigmoid calibration
    sigmoid = CalibratedClassifierCV(est, cv=2, method='sigmoid')

    # Logistic regression with no calibration as baseline
    lr = LogisticRegression(C=1., solver='lbfgs')

    fig = plt.figure(fig_index, figsize=(10, 10))
    ax1 = plt.subplot2grid((3, 1), (0, 0), rowspan=2)
    ax2 = plt.subplot2grid((3, 1), (2, 0))
for clf, name in [(lr, 'Logistic'),
    (est, name),
    (isotonic, name + ' + Isotonic'),
    (sigmoid, name + ' + Sigmoid')]:
    clf.fit(X_train, y_train)
    y_pred = clf.predict(X_test)
    if hasattr(clf, "predict_proba"):
        prob_pos = clf.predict_proba(X_test)[:, 1]
    else:
        prob_pos = clf.decision_function(X_test)
    prob_pos = (prob_pos - prob_pos.min()) / (prob_pos.max() - prob_pos.min())
    clf_score = brier_score_loss(y_test, prob_pos, pos_label=y.max())
    print("%s" % name)
    print("Brier: %1.3f" % clf_score)
    print("Precision: %1.3f" % precision_score(y_test, y_pred))
    print("Recall: %1.3f" % recall_score(y_test, y_pred))
    print("F1: %1.3f" % f1_score(y_test, y_pred))

    fraction_of_positives, mean_predicted_value = calibration_curve(y_test, prob_pos, n_bins=10)
    ax1.plot(mean_predicted_value, fraction_of_positives, "s-", label="%s (%1.3f)" % (name, clf_score))

    ax2.hist(prob_pos, range=(0, 1), bins=10, label=name, histtype="step", lw=2)

    ax1.set_ylabel("Fraction of positives")
    ax1.set_ylim([-0.05, 1.05])
    ax1.legend(loc="lower right")
    ax1.set_title('Calibration plots (reliability curve)')

    ax2.set_xlabel("Mean predicted value")
    ax2.set_ylabel("Count")
    ax2.legend(loc="upper center", ncol=2)

    plt.tight_layout()

    # Plot calibration curve for Gaussian Naive Bayes
    plot_calibration_curve(GaussianNB(), "Naive Bayes", 1)

    # Plot calibration curve for Linear SVC
    plot_calibration_curve(LinearSVC(), "SVC", 2)

    plt.show()
7.3 Probability calibration of classifiers

When performing classification you often want to predict not only the class label, but also the associated probability. This probability gives you some kind of confidence on the prediction. However, not all classifiers provide well-calibrated probabilities, some being over-confident while others being under-confident. Thus, a separate calibration of predicted probabilities is often desirable as a postprocessing. This example illustrates two different methods for this calibration and evaluates the quality of the returned probabilities using Brier’s score (see https://en.wikipedia.org/wiki/Brier_score).

Compared are the estimated probability using a Gaussian naive Bayes classifier without calibration, with a sigmoid calibration, and with a non-parametric isotonic calibration. One can observe that only the non-parametric model is able to provide a probability calibration that returns probabilities close to the expected 0.5 for most of the samples belonging to the middle cluster with heterogeneous labels. This results in a significantly improved Brier score.

```python
print(__doc__)

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# License: BSD Style.

import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm
from sklearn.datasets import make_blobs
from sklearn.naive_bayes import GaussianNB
from sklearn.metrics import brier_score_loss
from sklearn.calibration import CalibratedClassifierCV
from sklearn.model_selection import train_test_split

n_samples = 50000
n_bins = 3 # use 3 bins for calibration_curve as we have 3 clusters here

# Generate 3 blobs with 2 classes where the second blob contains
# half positive samples and half negative samples. Probability in this
# blob is therefore 0.5.
centers = [(-5, -5), (0, 0), (5, 5)]
X, y = make_blobs(n_samples=n_samples, n_features=2, cluster_std=1.0,
                  centers.centers, shuffle=False, random_state=42)
y[:n_samples // 2] = 0
y[n_samples // 2:] = 1
sample_weight = np.random.RandomState(42).rand(y.shape[0])

# split train, test for calibration
X_train, X_test, y_train, y_test, sw_train, sw_test = train_test_split(X, y, sample_weight, test_size=0.9, random_state=42)

# Gaussian Naive-Bayes with no calibration
clf = GaussianNB()
clf.fit(X_train, y_train)  # GaussianNB itself does not support sample-weights
prob_pos_clf = clf.predict_proba(X_test)[:, 1]

# Gaussian Naive-Bayes with isotonic calibration
```
clf_isotonic = CalibratedClassifierCV(clf, cv=2, method='isotonic')
clf_isotonic.fit(X_train, y_train, sw_train)
prob_pos_isotonic = clf_isotonic.predict_proba(X_test)[:, 1]

# Gaussian Naive-Bayes with sigmoid calibration
clf_sigmoid = CalibratedClassifierCV(clf, cv=2, method='sigmoid')
clf_sigmoid.fit(X_train, y_train, sw_train)
prob_pos_sigmoid = clf_sigmoid.predict_proba(X_test)[:, 1]

print("Brier scores: (the smaller the better)")

clf_score = brier_score_loss(y_test, prob_pos_clf, sw_test)
print("No calibration: \$1.3f" % clf_score)

clf_isotonic_score = brier_score_loss(y_test, prob_pos_isotonic, sw_test)
print("With isotonic calibration: \$1.3f" % clf_isotonic_score)

clf_sigmoid_score = brier_score_loss(y_test, prob_pos_sigmoid, sw_test)
print("With sigmoid calibration: \$1.3f" % clf_sigmoid_score)

Out:

Brier scores: (the smaller the better)
No calibration: 0.104
With isotonic calibration: 0.084
With sigmoid calibration: 0.109

Plot the data and the predicted probabilities

plt.figure()
y_unique = np.unique(y)
colors = cm.rainbow(np.linspace(0.0, 1.0, y_unique.size))
for this_y, color in zip(y_unique, colors):
    this_X = X_train[y_train == this_y]
    this_sw = sw_train[y_train == this_y]
    plt.scatter(this_X[:, 0], this_X[:, 1], s=this_sw * 50, c=color, alpha=0.5,
                label="Class $%s$" % this_y)
plt.legend(loc="best")
plt.title("Data")
plt.figure()
order = np.lexsort((prob_pos_clf, ))
plt.plot(prob_pos_clf[order], 'r', label='No calibration ($\$1.3f$" % clf_score)
plt.plot(prob_pos_isotonic[order], 'g', linewidth=3,
         label='Isotonic calibration ($\$1.3f$" % clf_isotonic_score)
plt.plot(prob_pos_sigmoid[order], 'b', linewidth=3,
         label='Sigmoid calibration ($\$1.3f$" % clf_sigmoid_score)
plt.plot(np.linspace(0, y_test.size, 51)[1::2],
         y_test[order].reshape(25, -1).mean(1),
         'k', linewidth=3, label=r'Empirical')
plt.ylim([-0.05, 1.05])
plt.xlabel("Instances sorted according to predicted probability "
           "(uncalibrated GNB)"
plt.ylabel("P(y=1)"
plt.legend(loc="upper left")
plt.title("Gaussian naive Bayes probabilities")
plt.show()
7.4 Probability Calibration for 3-class classification

This example illustrates how sigmoid calibration changes predicted probabilities for a 3-class classification problem. Illustrated is the standard 2-simplex, where the three corners correspond to the three classes. Arrows point from the probability vectors predicted by an uncalibrated classifier to the probability vectors predicted by the same classifier after sigmoid calibration on a hold-out validation set. Colors indicate the true class of an instance (red: class 1, green: class 2, blue: class 3).

The base classifier is a random forest classifier with 25 base estimators (trees). If this classifier is trained on all 800 training datapoints, it is overly confident in its predictions and thus incurs a large log-loss. Calibrating an identical classifier, which was trained on 600 datapoints, with method=’sigmoid’ on the remaining 200 datapoints reduces the confidence of the predictions, i.e., moves the probability vectors from the edges of the simplex towards the center. This calibration results in a lower log-loss. Note that an alternative would have been to increase the number of base estimators which would have resulted in a similar decrease in log-loss.
Out:

```
Log-loss of
* uncalibrated classifier trained on 800 datapoints: 1.280
* classifier trained on 600 datapoints **and** calibrated on 200 datapoint: 0.534
```

```
print(__doc__)
# Author: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
# License: BSD Style.

import matplotlib.pyplot as plt
import numpy as np
from sklearn.datasets import make_blobs
from sklearn.ensemble import RandomForestClassifier
from sklearn.calibration import CalibratedClassifierCV
from sklearn.metrics import log_loss

np.random.seed(0)
```

7.4. Probability Calibration for 3-class classification
# Generate data
X, y = make_blobs(n_samples=1000, n_features=2, random_state=42, cluster_std=5.0)
X_train, y_train = X[:600], y[:600]
X_valid, y_valid = X[600:800], y[600:800]
X_train_valid, y_train_valid = X[:800], y[:800]
X_test, y_test = X[800:], y[800:]

# Train uncalibrated random forest classifier on whole train and validation data and evaluate on test data
clf = RandomForestClassifier(n_estimators=25)
clf.fit(X_train_valid, y_train_valid)
clf_probs = clf.predict_proba(X_test)
score = log_loss(y_test, clf_probs)

# Train random forest classifier, calibrate on validation data and evaluate on test data
clf = RandomForestClassifier(n_estimators=25)
clf.fit(X_train, y_train)
clf_probs = clf.predict_proba(X_test)
sig_clf = CalibratedClassifierCV(clf, method="sigmoid", cv="prefit")
sig_clf.fit(X_valid, y_valid)
sig_clf_probs = sig_clf.predict_proba(X_test)
sig_score = log_loss(y_test, sig_clf_probs)

# Plot changes in predicted probabilities via arrows
plt.figure(0)
for i in range(clf_probs.shape[0]):
    plt.arrow(clf_probs[i, 0], clf_probs[i, 1],
             sig_clf_probs[i, 0] - clf_probs[i, 0],
             sig_clf_probs[i, 1] - clf_probs[i, 1],
             color=colors[y_test[i]], head_width=1e-2)

# Plot perfect predictions
plt.plot([1.0], [0.0], 'ro', ms=20, label="Class 1")
plt.plot([0.0], [1.0], 'go', ms=20, label="Class 2")
plt.plot([0.0], [0.0], 'bo', ms=20, label="Class 3")

# Plot boundaries of unit simplex
plt.plot([0.0, 1.0, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0], [0.0, 0.0, 1.0, 0.0, 0.0, 1.0, 0.0, 0.0], 'k', label="Simplex")

# Annotate points on the simplex
plt.annotate(r'($\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$)',
             xy=(1.0/3, 1.0/3), xytext=(1.0/3, .23), xycoords='data',
             arrowprops=dict(facecolor='black', shrink=0.05),
             horizontalalignment='center', verticalalignment='center')
plt.plot([1.0/3], [1.0/3], 'ko', ms=5)
plt.annotate(r'($\frac{1}{2}$, $0$, $\frac{1}{2}$)',
             xy=(.5, .0), xytext=(.5, .1), xycoords='data',
             arrowprops=dict(facecolor='black', shrink=0.05),
             horizontalalignment='center', verticalalignment='center')
plt.annotate(r'($0$, $\frac{1}{2}$, $\frac{1}{2}$)',
             xy=(.0, .5), xytext=(.1, .5), xycoords='data',
             arrowprops=dict(facecolor='black', shrink=0.05),
             horizontalalignment='center', verticalalignment='center')
plt.annotate(r'($\frac{1}{2}$, $\frac{1}{2}$, $0$)',
             xy=(.5, .5), xytext=(.6, .6), xycoords='data',
             arrowprops=dict(facecolor='black', shrink=0.05),
             horizontalalignment='center', verticalalignment='center')
```python
plt.annotate(r'($0$, $0$, $1$)

plt.annotate(r'($1$, $0$, $0$)

plt.annotate(r'($0$, $1$, $0$)

# Add grid
plt.grid("off")

for x in [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
    plt.plot([0, x], [x, 0], 'k', alpha=0.2)
    plt.plot([0, 0 + (1-x)/2], [x, x + (1-x)/2], 'k', alpha=0.2)
    plt.plot([x, x + (1-x)/2], [0, 0 + (1-x)/2], 'k', alpha=0.2)

plt.title("Change of predicted probabilities after sigmoid calibration")
plt.xlabel("Probability class 1")
plt.ylabel("Probability class 2")
plt.xlim(-0.05, 1.05)
plt.ylim(-0.05, 1.05)
plt.legend(loc="best")

print("Log-loss of")
print("* uncalibrated classifier trained on 800 datapoints: %.3f " % score)
print("* classifier trained on 600 datapoints and calibrated on "
"200 datapoint: %.3f" % sig_score)

# Illustrate calibrator
plt.figure(1)
# generate grid over 2-simplex
pld = np.linspace(0, 1, 20)
p0, p1 = np.meshgrid(pld, pld)
p2 = 1 - p0 - p1
p = np.c_[p0.ravel(), p1.ravel(), p2.ravel()]
p = p[p[:, 2] >= 0]

calibrated_classifier = sig_clf.calibrated_classifiers_[0]
prediction = np.vstack([calibrator.predict(this_p)
for calibrator, this_p in
zip(calibrated_classifier.calibrators_, p.T)]).T
prediction /= prediction.sum(axis=1)[:, None]

# Plot modifications of calibrator
for i in range(prediction.shape[0]):
    plt.arrow(p[i, 0], p[i, 1],
    prediction[i, 0] - p[i, 0], prediction[i, 1] - p[i, 1],
    head_width=1e-2, color=colors[np.argmax(p[i])])

# Plot boundaries of unit simplex
plt.plot([0.0, 1.0, 0.0, 0.0], [0.0, 0.0, 1.0, 0.0], 'k', label="Simplex")

plt.grid("off")
```

---

7.4. Probability Calibration for 3-class classification
for x in [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
    plt.plot([0, x], [x, 0], 'k', alpha=0.2)
    plt.plot([0, 0 + (1-x)/2], [x, x + (1-x)/2], 'k', alpha=0.2)
    plt.plot([x, x + (1-x)/2], [0, 0 + (1-x)/2], 'k', alpha=0.2)
plt.title("Illustration of sigmoid calibrator")
plt.xlabel("Probability class 1")
plt.ylabel("Probability class 2")
plt.xlim(-0.05, 1.05)
plt.ylim(-0.05, 1.05)
plt.show()
8.1 Recognizing hand-written digits

An example showing how the scikit-learn can be used to recognize images of hand-written digits.
This example is commented in the tutorial section of the user manual.
Classification report for classifier SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, decision_function_shape=None, degree=3, gamma=0.001, kernel='rbf', max_iter=-1, probability=False, random_state=None, shrinking=True, tol=0.001, verbose=False):

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<th>precision</th>
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<th>f1-score</th>
<th>support</th>
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</tr>
<tr>
<td>9</td>
<td>0.93</td>
<td>0.98</td>
<td>0.95</td>
<td>92</td>
</tr>
</tbody>
</table>

avg / total 0.97 0.97 0.97 899

Confusion matrix:
[[87 0 0 0 1 0 0 0 0 0]
 [ 0 88 1 0 0 0 0 0 1 1]
 [ 0 0 85 1 0 0 0 0 0 0]
 [ 0 0 0 79 0 3 0 4 5 0]
 [ 0 0 0 0 88 0 0 0 0 4]
 [ 0 0 0 0 0 88 1 0 0 2]
 [ 0 1 0 0 0 0 90 0 0 0]
 [ 0 0 0 0 0 1 0 88 0 0]
 [ 0 0 0 0 0 0 0 0 88 0]
 [ 0 0 0 1 0 1 0 0 0 90]]
images_and_labels = list(zip(digits.images, digits.target))
for index, (image, label) in enumerate(images_and_labels[:4]):
    plt.subplot(2, 4, index + 1)
    plt.axis('off')
    plt.imshow(image, cmap=plt.cm.gray_r, interpolation='nearest')
    plt.title('Training: %i' % label)

# To apply a classifier on this data, we need to flatten the image, to
# turn the data in a (samples, feature) matrix:
N = len(digits.images)
data = digits.images.reshape((N, -1))

# Create a classifier: a support vector classifier
classifier = svm.SVC(gamma=0.001)

# We learn the digits on the first half of the digits
classifier.fit(data[:N // 2], digits.target[:N // 2])

# Now predict the value of the digit on the second half:
expected = digits.target[N // 2:]
predicted = classifier.predict(data[N // 2:])

print("Classification report for classifier %s:
      precision, recall, f1-score,
      support
%s"
      % (classifier, metrics.classification_report(expected, predicted)))
print("Confusion matrix:
%s" % metrics.confusion_matrix(expected, predicted))

images_and_predictions = list(zip(digits.images[N // 2:], predicted))
for index, (image, prediction) in enumerate(images_and_predictions[:4]):
    plt.subplot(2, 4, index + 5)
    plt.axis('off')
    plt.imshow(image, cmap=plt.cm.gray_r, interpolation='nearest')
    plt.title('Prediction: %i' % prediction)

plt.show()

Total running time of the script: (0 minutes 0.752 seconds)

Download Python source code: plot_digits_classification.py
Download IPython notebook: plot_digits_classification.ipynb

8.2 Normal and Shrinkage Linear Discriminant Analysis for classification

Shows how shrinkage improves classification.
from __future__ import division

import numpy as np
import matplotlib.pyplot as plt

from sklearn.datasets import make_blobs
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis

n_train = 20  # samples for training
n_test = 200  # samples for testing
n_averages = 50  # how often to repeat classification
n_features_max = 75  # maximum number of features
step = 4  # step size for the calculation

def generate_data(n_samples, n_features):
    """Generate random blob-ish data with noisy features.

    This returns an array of input data with shape `(n_samples, n_features)` and an array of `n_samples` target labels.

    Only one feature contains discriminative information, the other features contain only noise.
    """

    X, y = make_blobs(n_samples=n_samples, n_features=1, centers=[[-2], [2]])
# add non-discriminative features
if n_features > 1:
    X = np.hstack([X, np.random.randn(n_samples, n_features - 1)])
return X, y

acc_clf1, acc_clf2 = [], []
n_features_range = range(1, n_features_max + 1, step)
for n_features in n_features_range:
    score_clf1, score_clf2 = 0, 0
    for _ in range(n_averages):  
        X, y = generate_data(n_train, n_features)
        clf1 = LinearDiscriminantAnalysis(solver='lsqr', shrinkage='auto').fit(X, y)
        clf2 = LinearDiscriminantAnalysis(solver='lsqr', shrinkage=None).fit(X, y)
        X, y = generate_data(n_test, n_features)
        score_clf1 += clf1.score(X, y)
        score_clf2 += clf2.score(X, y)

    acc_clf1.append(score_clf1 / n_averages)
    acc_clf2.append(score_clf2 / n_averages)

features_samples_ratio = np.array(n_features_range) / n_train
plt.plot(features_samples_ratio, acc_clf1, linewidth=2,
         label="Linear Discriminant Analysis with shrinkage", color='navy')
plt.plot(features_samples_ratio, acc_clf2, linewidth=2,
         label="Linear Discriminant Analysis", color='gold')
plt.xlabel('n_features / n_samples')
plt.ylabel('Classification accuracy')
plt.legend(loc=1, prop={'size': 12})
plt.suptitle('Linear Discriminant Analysis vs. shrinkage Linear Discriminant Analysis (1 discriminative feature)')
plt.show()

Total running time of the script: (0 minutes 5.999 seconds)
Download Python source code: plot_lda.py
Download IPython notebook: plot_lda.ipynb

8.3 Plot classification probability

Plot the classification probability for different classifiers. We use a 3 class dataset, and we classify it with a Support Vector classifier, L1 and L2 penalized logistic regression with either a One-Vs-Rest or multinomial setting, and Gaussian process classification.

The logistic regression is not a multiclass classifier out of the box. As a result it can identify only the first class.
classif_rate for GPC : 82.666667
classif_rate for L2 logistic (OvR) : 76.666667
classif_rate for L1 logistic : 79.333333
classif_rate for Linear SVC : 82.000000
classif_rate for L2 logistic (Multinomial) : 82.000000

print(__doc__)

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD 3 clause
import matplotlib.pyplot as plt
import numpy as np
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import RBF
from sklearn import datasets
iris = datasets.load_iris()
X = iris.data[:, 0:2]  # we only take the first two features for visualization
y = iris.target
n_features = X.shape[1]
C = 1.0
kernel = 1.0 * RBF([1.0, 1.0])  # for GPC

# Create different classifiers. The logistic regression cannot do
# multiclass out of the box.
classifiers = {'L1 logistic': LogisticRegression(C=C, penalty='l1'),
              'L2 logistic (OvR)': LogisticRegression(C=C, penalty='l2'),
              'Linear SVC': SVC(kernel='linear', C=C, probability=True,
                                random_state=0),
              'L2 logistic (Multinomial)': LogisticRegression(C=C, solver='lbfgs', multi_class='multinomial'),
              'GPC': GaussianProcessClassifier(kernel)
           }

n_classifiers = len(classifiers)
plt.figure(figsize=(3 * 2, n_classifiers * 2))
plt.subplots_adjust(bottom=.2, top=.95)
xx = np.linspace(3, 9, 100)
yy = np.linspace(1, 5, 100).T
xx, yy = np.meshgrid(xx, yy)
Xfull = np.c_[xx.ravel(), yy.ravel()]

for index, (name, classifier) in enumerate(classifiers.items()):

8.3. Plot classification probability
classifier.fit(X, y)

y_pred = classifier.predict(X)
classif_rate = np.mean(y_pred.ravel() == y.ravel()) * 100
print("classif_rate for %s : %f " % (name, classif_rate))

# View probabilities=
probas = classifier.predict_proba(Xfull)
n_classes = np.unique(y_pred).size
for k in range(n_classes):
    plt.subplot(n_classifiers, n_classes, index * n_classes + k + 1)
    plt.title("Class %d " % k)
    if k == 0:
        plt.ylabel(name)
imshow_handle = plt.imshow(probas[:, k].reshape((100, 100)),
                              extent=(3, 9, 1, 5), origin='lower')
    plt.xticks(()
plt.yticks(()
idx = (y_pred == k)
if idx.any():
    plt.scatter(X[idx, 0], X[idx, 1], marker='o', c='k')
ax = plt.axes([0.15, 0.04, 0.7, 0.05])
plt.title("Probability")
plt.colorbar(imshow_handle, cax=ax, orientation='horizontal')
plt.show()

Total running time of the script: (0 minutes 8.394 seconds)

Download Python source code: plot_classification_probability.py

Download IPython notebook: plot_classification_probability.ipynb

### 8.4 Classifier comparison

A comparison of a several classifiers in scikit-learn on synthetic datasets. The point of this example is to illustrate the nature of decision boundaries of different classifiers. This should be taken with a grain of salt, as the intuition conveyed by these examples does not necessarily carry over to real datasets.

Particularly in high-dimensional spaces, data can more easily be separated linearly and the simplicity of classifiers such as naive Bayes and linear SVMs might lead to better generalization than is achieved by other classifiers.

The plots show training points in solid colors and testing points semi-transparent. The lower right shows the classification accuracy on the test set.
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import make_moons, make_circles, make_classification
from sklearn.neural_network import MLPClassifier
from sklearn.neighbors import KNeighborsClassifier
from sklearn.svm import SVC
from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import RBF
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier, AdaBoostClassifier
from sklearn.naive_bayes import GaussianNB
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis

h = .02  # step size in the mesh


classifiers = [KNeighborsClassifier(3),
               SVC(kernel="linear", C=0.025),
               SVC(gamma=2, C=1),
               GaussianProcessClassifier(1.0 * RBF(1.0), warm_start=True),
               DecisionTreeClassifier(max_depth=5),
               RandomForestClassifier(max_depth=5, n_estimators=10, max_features=1),
               MLPClassifier(alpha=1),
               AdaBoostClassifier(),
               GaussianNB(),
               QuadraticDiscriminantAnalysis()]

X, y = make_classification(n_features=2, n_redundant=0, n_informative=2,
```python
random_state=1, n_clusters_per_class=1)
rng = np.random.RandomState(2)
X += 2 * rng.uniform(size=X.shape)
linearly_separable = (X, y)
datasets = [make_moons(noise=0.3, random_state=0),
    make_circles(noise=0.2, factor=0.5, random_state=1),
    linearly_separable]
figure = plt.figure(figsize=(27, 9))
i = 1
# iterate over datasets
for ds_cnt, ds in enumerate(datasets):
    # preprocess dataset, split into training and test part
    X, y = ds
    X = StandardScaler().fit_transform(X)
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=.4, random_state=42)
    x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
    y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
        np.arange(y_min, y_max, h))
    # just plot the dataset first
    cm = plt.cm.RdBu
    cm_bright = ListedColormap(['#FF0000', '#0000FF'])
    ax = plt.subplot(len(datasets), len(classifiers) + 1, i)
    if ds_cnt == 0:
        ax.set_title("Input data")
    # Plot the training points
    ax.scatter(X_train[:, 0], X_train[:, 1], c=y_train, cmap=cm_bright)
    # and testing points
    ax.scatter(X_test[:, 0], X_test[:, 1], c=y_test, cmap=cm_bright, alpha=0.6)
    ax.set_xlim(xx.min(), xx.max())
    ax.set_ylim(yy.min(), yy.max())
    ax.set_xticks(())
    ax.set_yticks(())
i += 1

# iterate over classifiers
for name, clf in zip(names, classifiers):  
    ax = plt.subplot(len(datasets), len(classifiers) + 1, i)
    clf.fit(X_train, y_train)
    score = clf.score(X_test, y_test)
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    if hasattr(clf, "decision_function"):
        Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
    else:
        Z = clf.predict_proba(np.c_[xx.ravel(), yy.ravel()])[::, 1]
    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    ax.contourf(xx, yy, Z, cmap=cm, alpha=.8)
```

Chapter 8. Classification
# Plot also the training points
ax.scatter(X_train[:, 0], X_train[:, 1], c=y_train, cmap=cm_bright)
# and testing points
ax.scatter(X_test[:, 0], X_test[:, 1], c=y_test, cmap=cm_bright, alpha=0.6)

ax.set_xlim(xx.min(), xx.max())
ax.set_ylim(yy.min(), yy.max())
ax.set_xticks(())
ax.set_yticks(()
if ds_cnt == 0:
    ax.set_title(name)
    ax.text(xx.max() - .3, yy.min() + .3, ('%.2f' % score).lstrip('0'),
            size=15, horizontalalignment='right')
i += 1

plt.tight_layout()
plt.show()

Total running time of the script: (0 minutes 11.439 seconds)
Download Python source code: plot_classifier_comparison.py
Download IPython notebook: plot_classifier_comparison.ipynb

## 8.5 Linear and Quadratic Discriminant Analysis with confidence ellipsoid

Plot the confidence ellipsoids of each class and decision boundary

```python
print(__doc__)

from scipy import linalg
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mpl
from matplotlib import colors
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis,
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
colormap
cmap = colors.LinearSegmentedColormap(
    'red_blue_classes',
    {'red': [(0, 1, 1), (1, 0.7, 0.7)],
    'green': [(0, 0.7, 0.7), (1, 0.7, 0.7)],
    'blue': [(0, 0.7, 0.7), (1, 1, 1)]})
plt.cm.register_cmap(cmap=cmap)
generate datasets
def dataset_fixed_cov():  '''Generate 2 Gaussians samples with the same covariance matrix'''
    n, dim = 300, 2
    np.random.seed(0)
```
```python
C = np.array([[0., -0.23], [0.83, .23]])
X = np.r_[np.dot(np.random.randn(n, dim), C),
         np.dot(np.random.randn(n, dim), C) + np.array([1, 1])]
y = np.hstack((np.zeros(n), np.ones(n)))
return X, y

def dataset_cov():
    '''Generate 2 Gaussians samples with different covariance matrices'''
    n, dim = 300, 2
    np.random.seed(0)
    C = np.array([[0., -1.], [2.5, .7]]) * 2.
    X = np.r_[np.dot(np.random.randn(n, dim), C),
              np.dot(np.random.randn(n, dim), C.T) + np.array([1, 4])]
y = np.hstack((np.zeros(n), np.ones(n)))
    return X, y

plot functions

def plot_data(lda, X, y, y_pred, fig_index):
    splot = plt.subplot(2, 2, fig_index)
    if fig_index == 1:
        plt.title('Linear Discriminant Analysis')
        plt.ylabel('Data with fixed covariance')
    elif fig_index == 2:
        plt.title('Quadratic Discriminant Analysis')
    elif fig_index == 3:
        plt.ylabel('Data with varying covariances')

    tp = (y == y_pred)  # True Positive
    tp0, tp1 = tp[y == 0], tp[y == 1]
    X0, X1 = X[y == 0], X[y == 1]
    X0_tp, X0_fp = X0[tp0], X0[~tp0]
    X1_tp, X1_fp = X1[tp1], X1[~tp1]

    alpha = 0.5

    # class 0: dots
    plt.plot(X0_tp[:, 0], X0_tp[:, 1], 'o', alpha=alpha,
             color='red')
    plt.plot(X0_fp[:, 0], X0_fp[:, 1], '*', alpha=alpha,
             color='#990000')  # dark red

    # class 1: dots
    plt.plot(X1_tp[:, 0], X1_tp[:, 1], 'o', alpha=alpha,
             color='blue')
    plt.plot(X1_fp[:, 0], X1_fp[:, 1], '*', alpha=alpha,
             color='#000099')  # dark blue

    # class 0 and 1 : areas
    nx, ny = 200, 100
    x_min, x_max = plt.xlim()
    y_min, y_max = plt.ylim()
    xx, yy = np.meshgrid(np.linspace(x_min, x_max, nx),
                         np.linspace(y_min, y_max, ny))
    Z = lda.predict_proba(np.c_[xx.ravel(), yy.ravel()])
    Z = Z[:, 1].reshape(xx.shape)
    plt.pcolormesh(xx, yy, Z, cmap='red_blue_classes',
```

norm=colors.Normalize(0., 1.))
plt.contour(xx, yy, Z, [0.5], linewidths=2., colors='k')

# means
plt.plot(lda.means_[0][0], lda.means_[0][1],
'o', color='black', markersize=10)
plt.plot(lda.means_[1][0], lda.means_[1][1],
'o', color='black', markersize=10)

return splot

def plot_ellipse(splot, mean, cov, color):
v, w = linalg.eigh(cov)
u = w[0] / linalg.norm(w[0])
angle = np.arctan(u[1] / u[0])
angle = 180 * angle / np.pi  # convert to degrees
# filled Gaussian at 2 standard deviation
ell = mpl.patches.Ellipse(mean, 2 * v[0] ** 0.5, 2 * v[1] ** 0.5,
180 + angle, facecolor=color, edgecolor='yellow',
linewidth=2, zorder=2)
ell.set_clip_box(splot.bbox)
ell.set_alpha(0.5)
splot.add_artist(ell)
splot.set_xticks(())
splot.set_yticks(())

def plot_lda_cov(lda, splot):
plot_ellipse(splot, lda.means_[0], lda.covariance_, 'red')
plot_ellipse(splot, lda.means_[1], lda.covariance_, 'blue')

def plot_qda_cov(qda, splot):
plot_ellipse(splot, qda.means_[0], qda.covariances_[0], 'red')
plot_ellipse(splot, qda.means_[1], qda.covariances_[1], 'blue')

for i, (X, y) in enumerate([dataset_fixed_cov(), dataset_cov()]):
    # Linear Discriminant Analysis
    lda = LinearDiscriminantAnalysis(solver="svd", store_covariance=True)
y_pred = lda.fit(X, y).predict(X)
splot = plot_data(lda, X, y, y_pred, fig_index=2 * i + 1)
plot_lda_cov(lda, splot)
plt.axis('tight')

    # Quadratic Discriminant Analysis
    qda = QuadraticDiscriminantAnalysis(store_covariances=True)
y_pred = qda.fit(X, y).predict(X)
splot = plot_data(qda, X, y, y_pred, fig_index=2 * i + 2)
plot_qda_cov(qda, splot)
plt.axis('tight')
plt.suptitle("Linear Discriminant Analysis vs Quadratic Discriminant Analysis")
plt.show()
Linear Discriminant Analysis vs Quadratic Discriminant Analysis

**Linear Discriminant Analysis**

Data with fixed covariance

**Quadratic Discriminant Analysis**

Data with varying covariances

**Total running time of the script:** (0 minutes 0.482 seconds)

Download Python source code: plot lda_qda.py

Download IPython notebook: plot lda_qda.ipynb
Examples concerning the `sklearn.cluster` module.

### 9.1 A demo of the mean-shift clustering algorithm

Reference:

```python
print(__doc__)

import numpy as np
from sklearn.cluster import MeanShift, estimate_bandwidth
from sklearn.datasets.samples_generator import make_blobs

Generate sample data

centers = [[1, 1], [-1, -1], [1, -1]]
X, _ = make_blobs(n_samples=10000, centers=centers, cluster_std=0.6)

Compute clustering with MeanShift

# The following bandwidth can be automatically detected using
bandwidth = estimate_bandwidth(X, quantile=0.2, n_samples=500)
ms = MeanShift(bandwidth=bandwidth, bin_seeding=True)
ms.fit(X)
labels = ms.labels_
cluster_centers = ms.cluster_centers_

labels_unique = np.unique(labels)
n_clusters_ = len(labels_unique)

print("number of estimated clusters : %d" % n_clusters_)

Out:

number of estimated clusters : 3

Plot result
import matplotlib.pyplot as plt
from itertools import cycle

plt.figure(1)
plt.clf()

colors = cycle('bgrcmykbgrcmykbgrcmykbgrcmyk')
for k, col in zip(range(n_clusters_), colors):
    my_members = labels == k
    cluster_center = cluster_centers[k]
    plt.plot(X[my_members, 0], X[my_members, 1], col + '.')
    plt.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,
             markeredgecolor='k', markersize=14)

plt.title('Estimated number of clusters: %d' % n_clusters_)
plt.show()
9.2 Feature agglomeration

These images show how similar features are merged together using feature agglomeration.

```
print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt

from sklearn import datasets, cluster
from sklearn.feature_extraction.image import grid_to_graph

digits = datasets.load_digits()
digits.data = np.reshape(digits.data, (len(digits.data), -1))
connectivity = grid_to_graph(*digits.data[0].shape)

agglo = cluster.FeatureAgglomeration(connectivity=connectivity,
                                      n_clusters=32)

agglo.fit(digits.data)

X_reduced = agglo.transform(digits.data)

X_restored = agglo.inverse_transform(X_reduced)

images_restored = np.reshape(X_restored, digits.data.shape)

fig, axes = plt.subplots(3, 4)

for i in range(4):
    axes[0, i].imshow(digits.data[i] * 255, cmap='gray')
    axes[0, i].set_title(digits.target[i])
    axes[1, i].imshow(images_restored[i], cmap='gray')
    axes[1, i].set_title('Agglomerated data')
    axes[2, i].imshow(digits.target[i] * 255, cmap='gray')
    axes[2, i].set_title('Labels')

plt.show()
```
9.3 Demonstration of k-means assumptions

This example is meant to illustrate situations where k-means will produce unintuitive and possibly unexpected clusters. In the first three plots, the input data does not conform to some implicit assumption that k-means makes and undesirable clusters are produced as a result. In the last plot, k-means returns intuitive clusters despite unevenly sized blobs.
import numpy as np
import matplotlib.pyplot as plt

from sklearn.cluster import KMeans
from sklearn.datasets import make_blobs

plt.figure(figsize=(12, 12))

n_samples = 1500
random_state = 170

9.3. Demonstration of k-means assumptions
X, y = make_blobs(n_samples=n_samples, random_state=random_state)

# Incorrect number of clusters
y_pred = KMeans(n_clusters=2, random_state=random_state).fit_predict(X)
plt.subplot(221)
plt.scatter(X[:, 0], X[:, 1], c=y_pred)
plt.title("Incorrect Number of Blobs")

# Anisotropically distributed data
transformation = [[ 0.60834549, -0.63667341], [-0.40887718, 0.85253229]]
X_aniso = np.dot(X, transformation)
y_pred = KMeans(n_clusters=3, random_state=random_state).fit_predict(X_aniso)
plt.subplot(222)
plt.scatter(X_aniso[:, 0], X_aniso[:, 1], c=y_pred)
plt.title("Anisotropically Distributed Blobs")

# Different variance
X_varied, y_varied = make_blobs(n_samples=n_samples, 
cluster_std=[1.0, 2.5, 0.5], 
random_state=random_state)
y_pred = KMeans(n_clusters=3, random_state=random_state).fit_predict(X_varied)
plt.subplot(223)
plt.scatter(X_varied[:, 0], X_varied[:, 1], c=y_pred)
plt.title("Unequal Variance")

# Unevenly sized blobs
X_filtered = np.vstack((X[y == 0][:500], X[y == 1][:100], X[y == 2][:10]))
y_pred = KMeans(n_clusters=3, random_state=random_state).fit_predict(X_filtered)
plt.subplot(224)
plt.scatter(X_filtered[:, 0], X_filtered[:, 1], c=y_pred)
plt.title("Unevenly Sized Blobs")
plt.show()

Total running time of the script: (0 minutes 0.364 seconds)

Download Python source code: plot_kmeans_assumptions.py
Download IPython notebook: plot_kmeans_assumptions.ipynb

9.4 A demo of structured Ward hierarchical clustering on a raccoon face image

Compute the segmentation of a 2D image with Ward hierarchical clustering. The clustering is spatially constrained in order for each segmented region to be in one piece.
import time as time
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
from sklearn.feature_extraction.image import grid_to_graph
from sklearn.cluster import AgglomerativeClustering
from sklearn.utils.testing import SkipTest
from sklearn.utils.fixes import sp_version

if sp_version < (0, 12):
    raise SkipTest("Skipping because SciPy version earlier than 0.12.0 and "
                   "thus does not include the scipy.misc.face() image.")

Generate data

try:
    face = sp.face(gray=True)
except AttributeError:
    # Newer versions of scipy have face in misc
    from scipy import misc
    face = misc.face(gray=True)

# Resize it to 10% of the original size to speed up the processing
face = sp.misc.imresize(face, 0.10) / 255.

X = np.reshape(face, (-1, 1))

Define the structure A of the data. Pixels connected to their neighbors.

connectivity = grid_to_graph(*face.shape)

Compute clustering

print("Compute structured hierarchical clustering...")
st = time.time()
n_clusters = 15  # number of regions
ward = AgglomerativeClustering(n_clusters=n_clusters, linkage='ward',
                                connectivity=connectivity)
ward.fit(X)
label = np.reshape(ward.labels_, face.shape)
print("Elapsed time: ", time.time() - st)
print("Number of pixels: ", label.size)
print("Number of clusters: ", np.unique(label).size)

Out:

Compute structured hierarchical clustering...
Elapsed time: 0.635977983475
Number of pixels: 7752
Number of clusters: 15

Plot the results on an image

plt.figure(figsize=(5, 5))
plt.imshow(face, cmap=plt.cm.gray)
for l in range(n_clusters):
    plt.contour(label == l, contours=1,
                colors=[plt.cm.spectral(l / float(n_clusters)), ])
plt.xticks(())
plt.yticks(())
plt.show()

Total running time of the script: (0 minutes 1.023 seconds)
Download Python source code: plot_face_ward_segmentation.py
Download IPython notebook: plot_face_ward_segmentation.ipynb

9.5 Online learning of a dictionary of parts of faces

This example uses a large dataset of faces to learn a set of 20 x 20 images patches that constitute faces.

From the programming standpoint, it is interesting because it shows how to use the online API of the scikit-learn to process a very large dataset by chunks. The way we proceed is that we load an image at a time and extract randomly 50 patches from this image. Once we have accumulated 500 of these patches (using 10 images), we run the `partial_fit` method of the online KMeans object, MiniBatchKMeans.

The verbose setting on the MiniBatchKMeans enables us to see that some clusters are reassigned during the successive
calls to partial-fit. This is because the number of patches that they represent has become too low, and it is better to choose a random new cluster.

```python
print(__doc__)

import time

import matplotlib.pyplot as plt
import numpy as np

from sklearn import datasets
from sklearn.cluster import MiniBatchKMeans
from sklearn.feature_extraction.image import extract_patches_2d

faces = datasets.fetch_olivetti_faces()

Learn the dictionary of images

```python
print('Learning the dictionary...')
rng = np.random.RandomState(0)
kmeans = MiniBatchKMeans(n_clusters=81, random_state=rng, verbose=True)
patch_size = (20, 20)

buffer = []
index = 1
t0 = time.time()

# The online learning part: cycle over the whole dataset 6 times
index = 0
for _ in range(6):
    for img in faces.images:
        data = extract_patches_2d(img, patch_size, max_patches=50,
                                   random_state=rng)
        data = np.reshape(data, (len(data), -1))
        buffer.append(data)
        index += 1
    if index % 10 == 0:
        data = np.concatenate(buffer, axis=0)
        data -= np.mean(data, axis=0)
        data /= np.std(data, axis=0)
        kmeans.partial_fit(data)
        buffer = []
    if index % 100 == 0:
        print('Partial fit of %i out of %i
% (index, 6 * len(faces.images)))

Out:

Learning the dictionary...
Partial fit of 100 out of 2400
Partial fit of 200 out of 2400
Partial fit of 300 out of 2400
Partial fit of 400 out of 2400
Partial fit of 500 out of 2400

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Partial fit of 600 out of 2400
Partial fit of 700 out of 2400
Partial fit of 800 out of 2400
Partial fit of 900 out of 2400
Partial fit of 1000 out of 2400
Partial fit of 1100 out of 2400
Partial fit of 1200 out of 2400
Partial fit of 1300 out of 2400
Partial fit of 1400 out of 2400
Partial fit of 1500 out of 2400
Partial fit of 1600 out of 2400
Partial fit of 1700 out of 2400
Partial fit of 1800 out of 2400
Partial fit of 1900 out of 2400
Partial fit of 2000 out of 2400
Partial fit of 2100 out of 2400
Partial fit of 2200 out of 2400
Partial fit of 2300 out of 2400
Partial fit of 2400 out of 2400
done in 3.15s.

Plot the results

```python
plt.figure(figsize=(4.2, 4))
for i, patch in enumerate(kmeans.cluster_centers_):
    plt.subplot(9, 9, i + 1)
    plt.imshow(patch.reshape(patch_size), cmap=plt.cm.gray,
               interpolation='nearest')
    plt.xticks(())
    plt.yticks(())

plt.suptitle('Patches of faces
Train time %.1fs on %d patches' %
             (dt, 8 * len(faces.images)), fontsize=16)
plt.subplots_adjust(0.08, 0.02, 0.92, 0.85, 0.08, 0.23)
plt.show()
```
9.6 Demo of affinity propagation clustering algorithm


```
print(__doc__)

from sklearn.cluster import AffinityPropagation
from sklearn import metrics
from sklearn.datasets.samples_generator import make_blobs

Generate sample data

centers = [[1, 1], [-1, -1], [1, -1]]
X, labels_true = make_blobs(n_samples=300, centers=center, cluster_std=0.5,
                         random_state=0)

Compute Affinity Propagation

af = AffinityPropagation(preference=-50).fit(X)
cluster_centers_indices = af.cluster_centers_indices_
labels = af.labels_

n_clusters_ = len(cluster_centers_indices)
```
Estimated number of clusters: 3
Homogeneity: 0.872
Completeness: 0.872
V-measure: 0.872
Adjusted Rand Index: 0.912
Adjusted Mutual Information: 0.871
Silhouette Coefficient: 0.753

Plot result

```python
import matplotlib.pyplot as plt
from itertools import cycle

plt.close('all')
plt.figure(1)
plt.clf()

colors = cycle('bgrcmykbgrcmykbgrcmykbgrcmyk')
for k, col in zip(range(n_clusters_), colors):
    class_members = labels == k
    cluster_center = X[cluster_centers_indices[k]]
    plt.plot(X[class_members, 0], X[class_members, 1], col + '.
    plt.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,
             markeredgecolor='k', markersize=14)
    for x in X[class_members]:
        plt.plot([cluster_center[0], x[0]], [cluster_center[1], x[1]], col)

plt.title('Estimated number of clusters: %d' % n_clusters_)
plt.show()
```
9.7 Hierarchical clustering: structured vs unstructured ward

Example builds a swiss roll dataset and runs hierarchical clustering on their position.

For more information, see *Hierarchical clustering*.

In a first step, the hierarchical clustering is performed without connectivity constraints on the structure and is solely based on distance, whereas in a second step the clustering is restricted to the k-Nearest Neighbors graph: it’s a hierarchical clustering with structure prior.

Some of the clusters learned without connectivity constraints do not respect the structure of the swiss roll and extend across different folds of the manifolds. On the opposite, when opposing connectivity constraints, the clusters form a nice parcellation of the swiss roll.
```python
print(__doc__)

import time as time
import numpy as np
import matplotlib.pyplot as plt
import mpl_toolkits.mplot3d.axes3d as p3
from sklearn.cluster import AgglomerativeClustering
from sklearn.datasets.samples_generator import make_swiss_roll

Generate data (swiss roll dataset)

n_samples = 1500
noise = 0.05
X, _ = make_swiss_roll(n_samples, noise)
# Make it thinner
X[:, 1] *= .5

Compute clustering

print("Compute unstructured hierarchical clustering...")
st = time.time()
ward = AgglomerativeClustering(n_clusters=6, linkage='ward').fit(X)
elapsed_time = time.time() - st
label = ward.labels_
print("Elapsed time: %.2fs" % elapsed_time)
print("Number of points: %i" % label.size)

Out:
Compute unstructured hierarchical clustering...
Elapsed time: 0.06s
Number of points: 1500

Plot result

fig = plt.figure()
ax = p3.Axes3D(fig)
ax.view_init(7, -80)
for l in np.unique(label):
    ax.plot3D(X[label == l, 0], X[label == l, 1], X[label == l, 2],
              'o', color=plt.cm.jet(np.float(l) / np.max(label + 1)))
plt.title('Without connectivity constraints (time %.2fs)' % elapsed_time)
```

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Define the structure $A$ of the data. Here a 10 nearest neighbors

```python
from sklearn.neighbors import kneighbors_graph
connectivity = kneighbors_graph(X, n_neighbors=10, include_self=False)
```

Compute clustering

```python
print("Compute structured hierarchical clustering...")
st = time.time()
ward = AgglomerativeClustering(n_clusters=6, connectivity=connectivity,
                             linkage='ward').fit(X)
elapsed_time = time.time() - st
label = ward.labels_
print("Elapsed time: %.2fs" % elapsed_time)
print("Number of points: %i" % label.size)
```

Out:

```
Compute structured hierarchical clustering...
Elapsed time: 0.16s
Number of points: 1500
```

Plot result

```python
fig = plt.figure()
ax = p3.Axes3D(fig)
```

9.7. Hierarchical clustering: structured vs unstructured ward
9.8 Agglomerative clustering with and without structure

This example shows the effect of imposing a connectivity graph to capture local structure in the data. The graph is simply the graph of 20 nearest neighbors.

Two consequences of imposing a connectivity can be seen. First clustering with a connectivity matrix is much faster.

Second, when using a connectivity matrix, average and complete linkage are unstable and tend to create a few clusters that grow very quickly. Indeed, average and complete linkage fight this percolation behavior by considering all the distances between two clusters when merging them. The connectivity graph breaks this mechanism. This effect is more
pronounced for very sparse graphs (try decreasing the number of neighbors in kneighbors_graph) and with complete linkage. In particular, having a very small number of neighbors in the graph, imposes a geometry that is close to that of single linkage, which is well known to have this percolation instability.

```python
# Authors: Gael Varoquaux, Nelle Varoquaux
# License: BSD 3 clause

import time
import matplotlib.pyplot as plt
import numpy as np

from sklearn.cluster import AgglomerativeClustering
from sklearn.neighbors import kneighbors_graph

# Generate sample data
n_samples = 1500
np.random.seed(0)
t = 1.5 * np.pi * (1 + 3 * np.random.rand(1, n_samples))
x = t * np.cos(t)
y = t * np.sin(t)
X = np.concatenate((x, y))
X += .7 * np.random.randn(2, n_samples)
X = X.T
```

9.8. Agglomerative clustering with and without structure
# Create a graph capturing local connectivity. Larger number of neighbors
# will give more homogeneous clusters to the cost of computation
# time. A very large number of neighbors gives more evenly distributed
# cluster sizes, but may not impose the local manifold structure of
# the data
knn_graph = kneighbors_graph(X, 30, include_self=False)

for connectivity in (None, knn_graph):
    for n_clusters in (30, 3):
        plt.figure(figsize=(10, 4))
        for index, linkage in enumerate(('average', 'complete', 'ward')):
            model = AgglomerativeClustering(linkage=linkage,
                                              connectivity=connectivity,
                                              n_clusters=n_clusters)
            t0 = time.time()
            model.fit(X)
            elapsed_time = time.time() - t0
            plt.scatter(X[:, 0], X[:, 1], c=model.labels_,
                        cmap=plt.cm.spectral)
            plt.title('linkage=%s (time %.2f s)' % (linkage, elapsed_time),
                      fontdict=dict(verticalalignment='top'))
            plt.axis('equal')
            plt.axis('off')
            plt.subplots_adjust(bottom=0, top=.89, wspace=0,
                                left=0, right=1)
            plt.suptitle('n_clusters=%i, connectivity=%r' %
                          (n_clusters, connectivity is not None), size=17)

plt.show()

Total running time of the script: (0 minutes 2.844 seconds)

Download Python source code: plot_agglomerative_clustering.py
Download IPython notebook: plot_agglomerative_clustering.ipynb

9.9 K-means Clustering

The plots display firstly what a K-means algorithm would yield using three clusters. It is then shown what the effect of a bad initialization is on the classification process: By setting n_init to only 1 (default is 10), the amount of times that the algorithm will be run with different centroid seeds is reduced. The next plot displays what using eight clusters would deliver and finally the ground truth.
print(__doc__) # Code source: Gaël Varoquaux # Modified for documentation by Jaques Grobler # License: BSD 3 clause
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

from sklearn.cluster import KMeans
from sklearn import datasets

np.random.seed(5)

centers = [[1, 1], [-1, -1], [1, -1]]
iris = datasets.load_iris()
X = iris.data
y = iris.target

estimators = {'k_means_iris_3': KMeans(n_clusters=3),
              'k_means_iris_8': KMeans(n_clusters=8),
              'k_means_iris_bad_init': KMeans(n_clusters=3, n_init=1,
                                             init='random')}

fignum = 1

9.9. K-means Clustering
```python
for name, est in estimators.items():
    fig = plt.figure(fignum, figsize=(4, 3))
    plt.clf()
    ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=48, azim=134)
    plt.cla()
    est.fit(X)
    labels = est.labels_
    ax.scatter(X[:, 3], X[:, 0], X[:, 2], c=labels.astype(np.float))
    ax.w_xaxis.set_ticklabels([])
    ax.w_yaxis.set_ticklabels([])
    ax.w_zaxis.set_ticklabels([])
    ax.set_xlabel('Petal width')
    ax.set_ylabel('Sepal length')
    ax.set_zlabel('Petal length')
    fignum = fignum + 1

# Plot the ground truth
fig = plt.figure(fignum, figsize=(4, 3))
plt.clf()
ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=48, azim=134)
plt.cla()
for name, label in [('Setosa', 0), ('Versicolour', 1), ('Virginica', 2)]:
    ax.text3D(X[y == label, 3].mean(), X[y == label, 0].mean() + 1.5,
             X[y == label, 2].mean(), name,
             horizontalalignment='center',
             bbox=dict(alpha=.5, edgecolor='w', facecolor='w'))

# Reorder the labels to have colors matching the cluster results
y = np.choose(y, [1, 2, 0]).astype(np.float)
ax.scatter(X[:, 3], X[:, 0], X[:, 2], c=y)

plt.show()
```

Total running time of the script: (0 minutes 0.415 seconds)

Download Python source code: plot_cluster_iris.py

Download IPython notebook: plot_cluster_iris.ipynb

### 9.10 Segmenting the picture of a raccoon face in regions

This example uses Spectral clustering on a graph created from voxel-to-voxel difference on an image to break this image into multiple partly-homogeneous regions.
This procedure (spectral clustering on an image) is an efficient approximate solution for finding normalized graph cuts.

There are two options to assign labels:

- with `kmeans` spectral clustering will cluster samples in the embedding space using a kmeans algorithm
- whereas `discrete` will iteratively search for the closest partition space to the embedding space.

```
print(__doc__)

# Author: Gael Varoquaux <gael.varoquaux@normalesup.org>, Brian Cheung
# License: BSD 3 clause

import time
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt

from sklearn.feature_extraction import image
from sklearn.cluster import spectral_clustering
from sklearn.utils.testing import SkipTest
from sklearn.utils.fixes import sp_version

if sp_version < (0, 12):
    raise SkipTest("Skipping because SciPy version earlier than 0.12.0 and "
    "thus does not include the scipy.misc.face() image.")

# load the raccoon face as a numpy array
try:
    face = sp.face(gray=True)
except AttributeError:
    # Newer versions of scipy have face in misc
    from scipy import misc
    face = misc.face(gray=True)

# Resize it to 10% of the original size to speed up the processing
face = sp.misc.imresize(face, 0.10) / 255.

# Convert the image into a graph with the value of the gradient on the
# edges.
graph = image.img_to_graph(face)

# Take a decreasing function of the gradient: an exponential
# The smaller beta is, the more independent the segmentation is of the
# actual image. For beta=1, the segmentation is close to a voronoi
beta = 5
eps = 1e-6
graph.data = np.exp(-beta * graph.data / graph.data.std()) + eps

# Apply spectral clustering (this step goes much faster if you have pyamg
# installed)
N_REGIONS = 25

Visualize the resulting regions

for assign_labels in ('kmeans', 'discrete'):
    t0 = time.time()
    labels = spectral_clustering(graph, n_clusters=N_REGIONS,
```
assign_labels=assign_labels, random_state=1)

t1 = time.time()
labels = labels.reshape(face.shape)

plt.figure(figsize=(5, 5))
plt.imshow(face, cmap=plt.cm.gray)
for l in range(N_REGIONS):
    plt.contour(labels == l, contours=1,
                colors=[plt.cm.spectral(l / float(N_REGIONS))])
plt.xticks(()
plt.yticks(()
title = 'Spectral clustering: %s, %.2fs' % (assign_labels, (t1 - t0))
print(title)
plt.title(title)
plt.show()
9.11 Demo of DBSCAN clustering algorithm

Finds core samples of high density and expands clusters from them.

```python
print(__doc__)

import numpy as np

from sklearn.cluster import DBSCAN
from sklearn import metrics
from sklearn.datasets.samples_generator import make_blobs
from sklearn.preprocessing import StandardScaler

Generate sample data

centers = [[1, 1], [-1, -1], [1, -1]]
X, labels_true = make_blobs(n_samples=750, centers=centers, cluster_std=0.4,
                            random_state=0)
X = StandardScaler().fit_transform(X)

Compute DBSCAN

db = DBSCAN(eps=0.3, min_samples=10).fit(X)
core_samples_mask = np.zeros_like(db.labels_, dtype=bool)
core_samples_mask[db.core_sample_indices_] = True
labels = db.labels_

# Number of clusters in labels, ignoring noise if present.
n_clusters_ = len(set(labels)) - (1 if -1 in labels else 0)

print('Estimated number of clusters: %d' % n_clusters_)
print("Homogeneity: \$0.3f\" % metrics.homogeneity_score(labels_true, labels))
print("Completeness: \$0.3f\" % metrics.completeness_score(labels_true, labels))
print("V-measure: \$0.3f\" % metrics.v_measure_score(labels_true, labels))
print("Adjusted Rand Index: \$0.3f\" % metrics.adjusted_rand_score(labels_true, labels))
print("Adjusted Mutual Information: \$0.3f\" % metrics.adjusted_mutual_info_score(labels_true, labels))
print("Silhouette Coefficient: \$0.3f\" % metrics.silhouette_score(X, labels))

Out:

Estimated number of clusters: 3
Homogeneity: 0.953
Completeness: 0.883
V-measure: 0.917
Adjusted Rand Index: 0.952
Adjusted Mutual Information: 0.883
Silhouette Coefficient: 0.626

Plot result

import matplotlib.pyplot as plt

# Black removed and is used for noise instead.
unique_labels = set(labels)
```
colors = [plt.cm.Spectral(each)
          for each in np.linspace(0, 1, len(unique_labels))]
for k, col in zip(unique_labels, colors):
    if k == -1:
        # Black used for noise.
        col = [0, 0, 0, 1]
    class_member_mask = (labels == k)
    xy = X[class_member_mask & core_samples_mask]
    plt.plot(xy[:, 0], xy[:, 1], 'o', markerfacecolor=tuple(col),
             markeredgecolor='k', markersize=14)
    xy = X[class_member_mask & ~core_samples_mask]
    plt.plot(xy[:, 0], xy[:, 1], 'o', markerfacecolor=tuple(col),
             markeredgecolor='k', markersize=6)
plt.title('Estimated number of clusters: %d' % n_clusters_)
plt.show()
9.12 Spectral clustering for image segmentation

In this example, an image with connected circles is generated and spectral clustering is used to separate the circles.

In these settings, the Spectral clustering approach solves the problem known as ‘normalized graph cuts’: the image is seen as a graph of connected voxels, and the spectral clustering algorithm amounts to choosing graph cuts defining regions while minimizing the ratio of the gradient along the cut, and the volume of the region.

As the algorithm tries to balance the volume (i.e., balance the region sizes), if we take circles with different sizes, the segmentation fails.

In addition, as there is no useful information in the intensity of the image, or its gradient, we choose to perform the spectral clustering on a graph that is only weakly informed by the gradient. This is close to performing a Voronoi partition of the graph.

In addition, we use the mask of the objects to restrict the graph to the outline of the objects. In this example, we are interested in separating the objects one from the other, and not from the background.

```python
print(__doc__)

# Authors: Emmanuelle Gouillart <emmanuelle.gouillart@normalesup.org>
#          Gael Varoquaux <gael.varoquaux@normalesup.org>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from sklearn.feature_extraction import image
from sklearn.cluster import spectral_clustering

l = 100
x, y = np.indices((l, l))

center1 = (28, 24)
center2 = (40, 50)
center3 = (67, 58)
center4 = (24, 70)

radius1, radius2, radius3, radius4 = 16, 14, 15, 14

circle1 = (x - center1[0])**2 + (y - center1[1])**2 < radius1**2
circle2 = (x - center2[0])**2 + (y - center2[1])**2 < radius2**2
circle3 = (x - center3[0])**2 + (y - center3[1])**2 < radius3**2
circle4 = (x - center4[0])**2 + (y - center4[1])**2 < radius4**2

4 circles

img = circle1 + circle2 + circle3 + circle4

# We use a mask that limits to the foreground: the problem that we are
# interested in here is not separating the objects from the background,
# but separating them one from the other.
mask = img.astype(bool)

img = img.astype(float)
img += 1 + 0.2 * np.random.randn(*img.shape)

# Convert the image into a graph with the value of the gradient on the
```
```python
# edges.
graph = image.img_to_graph(img, mask=mask)

# Take a decreasing function of the gradient: we take it weakly
# dependent from the gradient the segmentation is close to a voronoi
graph.data = np.exp(-graph.data / graph.data.std())

# Force the solver to be arpack, since amg is numerically
# unstable on this example
labels = spectral_clustering(graph, n_clusters=4, eigen_solver='arpack')
label_im = -np.ones(mask.shape)
label_im[mask] = labels

plt.matshow(img)
plt.matshow(label_im)
```

2 circles

```python
img = circle1 + circle2
mask = img.astype(bool)
```
img = img.astype(float)

img += 1 + 0.2 * np.random.randn(*img.shape)

graph = image.img_to_graph(img, mask=mask)
graph.data = np.exp(-graph.data / graph.data.std())

labels = spectral_clustering(graph, n_clusters=2, eigen_solver='arpack')
label_im = -np.ones(mask.shape)
label_im[mask] = labels

plt.matshow(img)
plt.matshow(label_im)
plt.show()
9.13 Vector Quantization Example

Face, a 1024 x 768 size image of a raccoon face, is used here to illustrate how \( k \)-means is used for vector quantization.

```python
print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause

import numpy as np
import scipy as sp
import matplotlib.pyplot as plt

from sklearn import cluster
from sklearn.utils.testing import SkipTest
from sklearn.utils.fixes import sp_version

if sp_version < (0, 12):
    raise SkipTest("Skipping because SciPy version earlier than 0.12.0 and"
                   "thus does not include the scipy.misc.face() image.")

try:
    face = sp.face(gray=True)
```
except AttributeError:
    # Newer versions of scipy have face in misc
    from scipy import misc
    face = misc.face(gray=True)

n_clusters = 5
np.random.seed(0)

X = face.reshape((-1, 1))  # We need an (n_sample, n_feature) array
k_means = cluster.KMeans(n_clusters=n_clusters, n_init=4)
k_means.fit(X)
values = k_means.cluster_centers_.squeeze()
labels = k_means.labels_

# create an array from labels and values
face_compressed = np.choose(labels, values)
face_compressed.shape = face.shape

vmin = face.min()
vmax = face.max()

# original face
plt.figure(1, figsize=(3, 2.2))
plt.imshow(face, cmap=plt.cm.gray, vmin=vmin, vmax=256)

# compressed face
plt.figure(2, figsize=(3, 2.2))
plt.imshow(face_compressed, cmap=plt.cm.gray, vmin=vmin, vmax=vmax)

# equal bins face
regular_values = np.linspace(0, 256, n_clusters + 1)
regular_labels = np.searchsorted(regular_values, face) - 1
regular_values = .5 * (regular_values[1:] + regular_values[:-1])  # mean
regular_face = np.choose(regular_labels.ravel(), regular_values, mode="clip")
regular_face.shape = face.shape
plt.figure(3, figsize=(3, 2.2))
plt.imshow(regular_face, cmap=plt.cm.gray, vmin=vmin, vmax=vmax)

# histogram
plt.figure(4, figsize=(3, 2.2))
plt.clf()
plt.axes([.01, .01, .98, .98])
plt.hist(X, bins=256, color='.5', edgecolor='.5')
plt.yticks(()
plt.xticks(regular_values)
values = np.sort(values)
for center_1, center_2 in zip(values[:-1], values[1:]):
    plt.axvline(.5 * (center_1 + center_2), color='b')
for center_1, center_2 in zip(regular_values[:-1], regular_values[1:]):
    plt.axvline(.5 * (center_1 + center_2), color='b', linestyle='--')
plt.show()
9.14 Various Agglomerative Clustering on a 2D embedding of digits

An illustration of various linkage option for agglomerative clustering on a 2D embedding of the digits dataset.

The goal of this example is to show intuitively how the metrics behave, and not to find good clusters for the digits. This is why the example works on a 2D embedding.

What this example shows us is the behavior “rich getting richer” of agglomerative clustering that tends to create uneven cluster sizes. This behavior is especially pronounced for the average linkage strategy, that ends up with a couple of singleton clusters.

Out:
Computing embedding
Done.
ward : 0.39s
average : 0.28s
complete : 0.26s
# Authors: Gael Varoquaux
# License: BSD 3 clause (C) INRIA 2014

```python
from time import time
import numpy as np
from scipy import ndimage
from matplotlib import pyplot as plt
from sklearn import manifold, datasets

digits = datasets.load_digits(n_class=10)
X = digits.data
y = digits.target
n_samples, n_features = X.shape

np.random.seed(0)

def nudge_images(X, y):
    # Having a larger dataset shows more clearly the behavior of the
    # methods, but we multiply the size of the dataset only by 2, as the
    # cost of the hierarchical clustering methods are strongly
    # super-linear in n_samples
    shift = lambda x: ndimage.shift(x.reshape((8, 8)), .3 * np.random.normal(size=2), mode='constant').ravel()
    X = np.concatenate([X, np.apply_along_axis(shift, 1, X)])
    Y = np.concatenate([y, y], axis=0)
    return X, Y

X, y = nudge_images(X, y)

# Visualize the clustering
```
# 2D embedding of the digits dataset
print("Computing embedding")
X_red = manifold.SpectralEmbedding(n_components=2).fit_transform(X)
print("Done.")

from sklearn.cluster import AgglomerativeClustering

for linkage in ('ward', 'average', 'complete'):
    clustering = AgglomerativeClustering(linkage=linkage, n_clusters=10)
    t0 = time()
    clustering.fit(X_red)
    print("%s: %.2fs" % (linkage, time() - t0))

    plot_clustering(X_red, X, clustering.labels_, "%s linkage" % linkage)

plt.show()

Total running time of the script: (0 minutes 21.258 seconds)

Download Python source code: plot_digits_linkage.py

Download IPython notebook: plot_digits_linkage.ipynb

## 9.15 Color Quantization using K-Means

Performs a pixel-wise Vector Quantization (VQ) of an image of the summer palace (China), reducing the number of colors required to show the image from 96,615 unique colors to 64, while preserving the overall appearance quality.

In this example, pixels are represented in a 3D-space and K-means is used to find 64 color clusters. In the image processing literature, the codebook obtained from K-means (the cluster centers) is called the color palette. Using a single byte, up to 256 colors can be addressed, whereas an RGB encoding requires 3 bytes per pixel. The GIF file format, for example, uses such a palette.

For comparison, a quantized image using a random codebook (colors picked up randomly) is also shown.

Original image (96,615 colors)
# Authors: Robert Layton <robertlayton@gmail.com>
# Olivier Grisel <olivier.grisel@ensta.org>
# Mathieu Blondel <mathieu@mblondel.org>
#
# License: BSD 3 clause

print(__doc__)
import numpy as np
import matplotlib.pyplot as plt
from sklearn.cluster import KMeans
from sklearn.metrics import pairwise_distances_argmin
from sklearn.datasets import load_sample_image
from sklearn.utils import shuffle
from time import time

9.15. Color Quantization using K-Means
n_colors = 64

# Load the Summer Palace photo
china = load_sample_image("china.jpg")

# Convert to floats instead of the default 8 bits integer coding. Dividing by # 255 is important so that plt.imshow behaves well on float data (need to # be in the range [0-1])
china = np.array(china, dtype=np.float64) / 255

# Load image and transform to a 2D numpy array.
w, h, d = original_shape = tuple(china.shape)
assert d == 3
image_array = np.reshape(china, (w * h, d))

print("Fitting model on a small sub-sample of the data")
t0 = time()
image_array_sample = shuffle(image_array, random_state=0)[:, :1000]
kmeans = KMeans(n_clusters=n_colors, random_state=0).fit(image_array_sample)
print("done in %0.3fs.\n" % (time() - t0))

# Get labels for all points
print("Predicting color indices on the full image (k-means")
t0 = time()
labels = kmeans.predict(image_array)
print("done in %0.3fs.\n" % (time() - t0))

codebook_random = shuffle(image_array, random_state=0)[:, :n_colors + 1]
print("Predicting color indices on the full image (random")
t0 = time()
labels_random = pairwise_distances_argmin(codebook_random, image_array, axis=0)
print("done in %0.3fs.\n" % (time() - t0))

def recreate_image(codebook, labels, w, h):
    """Recreate the (compressed) image from the code book & labels""
    d = codebook.shape[1]
    image = np.zeros((w, h, d))
    label_idx = 0
    for i in range(w):
        for j in range(h):
            image[i][j] = codebook[labels[label_idx]]
            label_idx += 1
    return image

# Display all results, alongside original image
plt.figure(1)
plt.clf()
ax = plt.axes([0, 0, 1, 1])
plt.axis('off')
plt.title('Original image (96,615 colors)')
plt.imshow(china)
plt.figure(2)
9.16 Agglomerative clustering with different metrics

Demonstrates the effect of different metrics on the hierarchical clustering.

The example is engineered to show the effect of the choice of different metrics. It is applied to waveforms, which can be seen as high-dimensional vector. Indeed, the difference between metrics is usually more pronounced in high dimension (in particular for euclidean and cityblock).

We generate data from three groups of waveforms. Two of the waveforms (waveform 1 and waveform 2) are proportional one to the other. The cosine distance is invariant to a scaling of the data, as a result, it cannot distinguish these two waveforms. Thus even with no noise, clustering using this distance will not separate out waveform 1 and 2.

We add observation noise to these waveforms. We generate very sparse noise: only 6% of the time points contain noise. As a result, the l1 norm of this noise (ie “cityblock” distance) is much smaller than it’s l2 norm (“euclidean” distance). This can be seen on the inter-class distance matrices: the values on the diagonal, that characterize the spread of the class, are much bigger for the Euclidean distance than for the cityblock distance.

When we apply clustering to the data, we find that the clustering reflects what was in the distance matrices. Indeed, for the Euclidean distance, the classes are ill-separated because of the noise, and thus the clustering does not separate the waveforms. For the cityblock distance, the separation is good and the waveform classes are recovered. Finally, the cosine distance does not separate at all waveform 1 and 2, thus the clustering puts them in the same cluster.
import matplotlib.pyplot as plt
import numpy as np
from sklearn.cluster import AgglomerativeClustering
from sklearn.metrics import pairwise_distances
np.random.seed(0)

# Generate waveform data
n_features = 2000
t = np.pi * np.linspace(0, 1, n_features)
def sqr(x):
    return np.sign(np.cos(x))

X = list()
y = list()
for i, (phi, a) in enumerate([(.5, .15), (.5, .6), (.3, .2))):
    for _ in range(30):
        phase_noise = .01 * np.random.randn()
        amplitude_noise = .04 * np.random.randn()
        additional_noise = 1 - 2 * np.random.rand(n_features)
        # Make the noise sparse
        additional_noise[np.abs(additional_noise) < .997] = 0

        # Create the waveform
        X.append(np.sin(t) * sqr(np.cos(t + phi))
                  + np.sin(t * a) * np.cos(t + phi))
        y.append(i)

plt.figure(figsize=(10, 5))
plt.plot(X[0], label='AgglomerativeClustering (affinity=euclidean)')
plt.plot(X[1], label='AgglomerativeClustering (affinity=cityblock)')
plt.legend()
```python
X.append(12 * (a + amplitude_noise) *
        (sqr(6 * (t + phi + phase_noise)))
        + additional_noise))
y.append(i)

X = np.array(X)
y = np.array(y)

n_clusters = 3
labels = ('Waveform 1', 'Waveform 2', 'Waveform 3')

# Plot the ground-truth labelling
plt.figure()
plt.axes([0, 0, 1, 1])
for l, c, n in zip(range(n_clusters), 'rgb',
                   labels):
    lines = plt.plot(X[y == l].T, c=c, alpha=.5)
    lines[0].set_label(n)
plt.legend(loc='best')
plt.axis('tight')
plt.axis('off')
plt.suptitle("Ground truth", size=20)

# Plot the distances
for index, metric in enumerate(['cosine', 'euclidean', 'cityblock']):
    avg_dist = np.zeros((n_clusters, n_clusters))
    plt.figure(figsize=(5, 4.5))
    for i in range(n_clusters):
        for j in range(n_clusters):
            avg_dist[i, j] = pairwise_distances(X[y == i], X[y == j],
                                                metric=metric).mean()
    avg_dist /= avg_dist.max()
    for i in range(n_clusters):
        for j in range(n_clusters):
            plt.text(i, j, '%5.3f' % avg_dist[i, j],
                      verticalalignment='center',
                      horizontalalignment='center')
    plt.imshow(avg_dist, interpolation='nearest', cmap=plt.cm.gnuplot2,
               vmin=0)
    plt.xticks(range(n_clusters), labels, rotation=45)
    plt.yticks(range(n_clusters), labels)
    plt.colorbar()
    plt.suptitle("Interclass %s distances" % metric, size=18)
    plt.tight_layout()

# Plot clustering results
for index, metric in enumerate(['cosine', 'euclidean', 'cityblock']):
    model = AgglomerativeClustering(n_clusters=n_clusters,
                                     linkage="average", affinity=metric)
    model.fit(X)
    plt.figure()
```

---

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Chapter 9. Clustering
for l, c in zip(np.arange(model.n_clusters), 'rgbk'):
    plt.plot(X[model.labels_ == l].T, c=c, alpha=.5)
plt.axis('tight')
plt.axis('off')
plt.suptitle("AgglomerativeClustering(affinity=%s)" % metric, size=20)

plt.show()

9.17 Comparison of the K-Means and MiniBatchKMeans clustering algorithms

We want to compare the performance of the MiniBatchKMeans and KMeans: the MiniBatchKMeans is faster, but
gives slightly different results (see Mini Batch K-Means).

We will cluster a set of data, first with KMeans and then with MiniBatchKMeans, and plot the results. We will also
plot the points that are labelled differently between the two algorithms.

Generate sample data

np.random.seed(0)
batch_size = 45
centers = [[1, 1], [-1, -1], [1, -1]]
n_clusters = len(centers)
X, labels_true = make_blobs(n_samples=3000, centers=centers, cluster_std=0.7)

Compute clustering with Means

k_means = KMeans(init='k-means++', n_clusters=3, n_init=10)
t0 = time.time()
k_means.fit(X)
t_batch = time.time() - t0

Compute clustering with MiniBatchKMeans

mbk = MiniBatchKMeans(init='k-means++', n_clusters=3, batch_size=batch_size,
                      n_init=10, max_no_improvement=10, verbose=0)
t0 = time.time()
mbk.fit(X)
t_mini_batch = time.time() - t0

Plot result

```python
fig = plt.figure(figsize=(8, 3))
fig.subplots_adjust(left=0.02, right=0.98, bottom=0.05, top=0.9)
colors = ['#4EACC5', '#FF9C34', '#4E9A06']

# We want to have the same colors for the same cluster from the
# MiniBatchKMeans and the KMeans algorithm. Let's pair the cluster centers per
# closest one.
k_means_cluster_centers = np.sort(k_means.cluster_centers_, axis=0)
mbk_means_cluster_centers = np.sort(mbk.cluster_centers_, axis=0)
k_means_labels = pairwise_distances_argmin(X, k_means.cluster_centers)
mbk_means_labels = pairwise_distances_argmin(X, mbk.cluster_centers)
order = pairwise_distances_argmin(k_means.cluster_centers,
    mbk_means_cluster_centers)

# KMeans
ax = fig.add_subplot(1, 3, 1)
for k, col in zip(range(n_clusters), colors):
    my_members = k_means_labels == k
    cluster_center = k_means_cluster_centers[k]
    ax.plot(X[my_members, 0], X[my_members, 1], 'w',
        markerfacecolor=col, marker='.'
    )
    ax.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,
        markeredgecolor='k', markersize=6
    )
ax.set_xticks(())
ax.set_yticks(())
plt.text(-3.5, 1.8, 'train time: %.2f
  inertia: %f' % (t_batch, k_means.inertia_))

# MiniBatchKMeans
ax = fig.add_subplot(1, 3, 2)
for k, col in zip(range(n_clusters), colors):
    my_members = mbk_means_labels == order[k]
    cluster_center = mbk_means_cluster_centers[order[k]]
    ax.plot(X[my_members, 0], X[my_members, 1], 'w',
        markerfacecolor=col, marker='.'
    )
    ax.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,
        markeredgecolor='k', markersize=6
    )
ax.set_xticks(())
ax.set_yticks(())
plt.text(-3.5, 1.8, 'train time: %.2f
  inertia: %f' % (t_mini_batch, mbk.inertia_))

# Initialise the different array to all False
different = (mbk_means_labels == 4)
ax = fig.add_subplot(1, 3, 3)
for k in range(n_clusters):
    different += ((k_means_labels == k) != (mbk_means_labels == order[k]))
identic = np.logical_not(different)
```
ax.plot(X[identic, 0], X[identic, 1], 'w', markerfacecolor='#bbbbbb', marker='.'
ax.plot(X[different, 0], X[different, 1], 'w', markerfacecolor='m', marker='.'
ax.set_title('Difference')
ax.set_xticks(()
ax.set_yticks()
plt.show()

KMeans

train time: 0.08s
inertia: 2470.576891

MiniBatchKMeans

train time: 9.07s
inertia: 2476.644432

Difference

Total running time of the script: (0 minutes 0.344 seconds)

Download Python source code: plot_mini_batch_kmeans.py
Download IPython notebook: plot_mini_batch_kmeans.ipynb

9.18 Feature agglomeration vs. univariate selection

This example compares 2 dimensionality reduction strategies:

- univariate feature selection with Anova
- feature agglomeration with Ward hierarchical clustering

Both methods are compared in a regression problem using a BayesianRidge as supervised estimator.
from sklearn.linear_model import BayesianRidge
from sklearn.pipeline import Pipeline
from sklearn.externals.joblib import Memory
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import KFold

Generate data

n_samples = 200
size = 40  # image size
roi_size = 15
snr = 5.
np.random.seed(0)
mask = np.ones([size, size], dtype=np.bool)
coef = np.zeros((size, size))
coef[0:roi_size, 0:roi_size] = -1.
coef[-roi_size:, -roi_size:] = 1.
X = np.random.randn(n_samples, size ** 2)
for x in X:  # smooth data
    x[:] = ndimage.gaussian_filter(x.reshape(size, size), sigma=1.0).ravel()
X -= X.mean(axis=0)
X /= X.std(axis=0)
y = np.dot(X, coef.ravel())
noise = np.random.randn(y.shape[0])
noise_coef = (linalg.norm(y, 2) / np.exp(snr / 20.)) / linalg.norm(noise, 2)
y += noise_coef * noise  # add noise

Compute the coef's of a Bayesian Ridge with GridSearch

cv = KFold(2)  # cross-validation generator for model selection
ridge = BayesianRidge()
cachdir = tempfile.mkdtemp()
mem = Memory(cachdir=cachdir, verbose=1)

# Ward agglomeration followed by BayesianRidge
connectivity = grid_to_graph(n_x=size, n_y=size)
ward = FeatureAgglomeration(n_clusters=10, connectivity=connectivity,
                            memory=mem)
clf = Pipeline([('ward', ward), ('ridge', ridge)])
# Select the optimal number of parcels with grid search
clf = GridSearchCV(clf, {'ward__n_clusters': [10, 20, 30]}, n_jobs=1, cv=cv)
clf.fit(X, y)  # set the best parameters
coef_ = clf.best_estimator_.steps[-1][1].coef_
coef_ = clf.best_estimator_.steps[0][1].inverse_transform(coef_)
coef_agglomeration_ = coef_.reshape(size, size)

# Anova univariate feature selection followed by BayesianRidge
f_regression = mem.cache(feature_selection.f_regression)  # caching function
anova = feature_selection.SelectPercentile(f_regression)
clf = Pipeline([('anova', anova), ('ridge', ridge)])
# Select the optimal percentage of features with grid search
clf = GridSearchCV(clf, {'anova__percentile': [5, 10, 20]}, cv=cv)
clf.fit(X, y)  # set the best parameters
coef_ = clf.best_estimator_.steps[-1][1].coef_
coef_ = clf.best_estimator_.steps[0][1].inverse_transform(coef_.reshape(1, -1))
coef_selection_ = coef_.reshape(size, size)

Out:

```
[Memory] Calling sklearn.cluster.hierarchical.ward_tree...
ward_tree(array([[ -0.451933, ..., -0.675318],
                   [ 0.275706, ..., -1.085711]],
                  dtype='float64', shape=(2, 8)),
                  n_clusters=None)
ward_tree - 0.2s, 0.0min
```

```
[Memory] Calling sklearn.cluster.hierarchical.ward_tree...
ward_tree(array([[ 0.905206, ..., 0.161245],
                   [-0.849835, ..., -1.091621]],
                  dtype='float64', shape=(2, 8)),
                  n_clusters=None)
ward_tree - 0.2s, 0.0min
```

```
[Memory] Calling sklearn.cluster.hierarchical.ward_tree...
ward_tree(array([[ 0.905206, ..., -0.849835],
                   [ 0.161245, ..., -1.091621]],
                  dtype='float64', shape=(2, 8)),
                  n_clusters=None)
ward_tree - 0.2s, 0.0min
```

```
[Memory] Calling sklearn.feature_selection.univariate_selection.f_regression...
f_regression(array([[ -0.451933, ..., 0.275706],
                      [-0.675318, ..., -1.085711]],
                     dtype='float64', shape=(2, 8)))
f_regression - 0.0s, 0.0min
```

```
[Memory] Calling sklearn.feature_selection.univariate_selection.f_regression...
f_regression(array([[ 0.905206, ..., -0.849835],
                      [-0.849835, ..., -1.091621]],
                     dtype='float64', shape=(2, 8)))
f_regression - 0.0s, 0.0min
```

```
[Memory] Calling sklearn.feature_selection.univariate_selection.f_regression...
f_regression(array([[ 0.905206, ..., -0.849835],
                      [-0.675318, ..., -1.085711]],
                     dtype='float64', shape=(2, 8)))
f_regression - 0.0s, 0.0min
```

Inverse the transformation to plot the results on an image

```
plt.close('all')
plt.figure(figsize=(7.3, 2.7))
plt.subplot(1, 3, 1)
plt.imshow(coef, interpolation="nearest", cmap=plt.cm.RdBu_r)
plt.title("True weights")
plt.subplot(1, 3, 2)
```

9.18. Feature agglomeration vs. univariate selection
9.19 Compare BIRCH and MiniBatchKMeans

This example compares the timing of Birch (with and without the global clustering step) and MiniBatchKMeans on a synthetic dataset having 100,000 samples and 2 features generated using make_blobs.

If \( \text{n\_clusters} \) is set to None, the data is reduced from 100,000 samples to a set of 158 clusters. This can be viewed as a preprocessing step before the final (global) clustering step that further reduces these 158 clusters to 100 clusters.
Birch without global clustering as the final step took 4.80 seconds
n_clusters : 158
Birch with global clustering as the final step took 4.86 seconds
n_clusters : 100
Time taken to run MiniBatchKMeans 4.99 seconds

# Authors: Manoj Kumar <manojkumarsivaraj334@gmail.com>
# Alexandre Gramfort <alexandre.gramfort@telecom-paristech.fr>
# License: BSD 3 clause

print(__doc__)

from itertools import cycle
from time import time
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from sklearn.preprocessing import StandardScaler
from sklearn.cluster import Birch, MiniBatchKMeans
from sklearn.datasets.samples_generator import make_blobs

# Generate centers for the blobs so that it forms a 10 X 10 grid.
xx = np.linspace(-22, 22, 10)
yy = np.linspace(-22, 22, 10)
xx, yy = np.meshgrid(xx, yy)
n_centres = np.hstack((np.ravel(xx)[:, np.newaxis],
n.ravel(yy)[:, np.newaxis]))

# Generate blobs to do a comparison between MiniBatchKMeans and Birch.
X, y = make_blobs(n_samples=100000, centers=n_centres, random_state=0)

# Use all colors that matplotlib provides by default.
colors_ = cycle(colors.cnames.keys())

fig = plt.figure(figsize=(12, 4))
fig.subplots_adjust(left=0.04, right=0.98, bottom=0.1, top=0.9)

# Compute clustering with Birch with and without the final clustering step
# and plot.
birch_models = [Birch(threshold=1.7, n_clusters=None),
               Birch(threshold=1.7, n_clusters=100)]
final_step = ['without global clustering', 'with global clustering']

for ind, (birch_model, info) in enumerate(zip(birch_models, final_step)):
    t = time()
    birch_model.fit(X)
    time_ = time() - t
    print("Birch $s as the final step took $0.2f seconds" % (info, (time() - t)))
# Plot result
labels = birch_model.labels_
centroids = birch_model.subcluster_centers_
n_clusters = np.unique(labels).size
print("n_clusters : %d" % n_clusters)

ax = fig.add_subplot(1, 3, ind + 1)
for this_centroid, k, col in zip(centroids, range(n_clusters), colors_):
    mask = labels == k
    ax.plot(X[mask, 0], X[mask, 1], 'w', markerfacecolor=col, marker='.'
    if birch_model.n_clusters is None:
        ax.plot(this_centroid[0], this_centroid[1], '+', markerfacecolor=col,
                markeredgecolor='k', markersize=5)
ax.set_xlim([-25, 25])
ax.set_ylim([-25, 25])
ax.set_title('Birch %s' % info)

# Compute clustering with MiniBatchKMeans.
mbk = MiniBatchKMeans(init='k-means++', n_clusters=100, batch_size=100,
                      n_init=10, max_no_improvement=10, verbose=0,
                      random_state=0)
t0 = time()
mbk.fit(X)
t_mini_batch = time() - t0
print("Time taken to run MiniBatchKMeans $0.2f$ seconds" % t_mini_batch)
mbk_means_labels_unique = np.unique(mbk.labels_)

ax = fig.add_subplot(1, 3, 3)
for this_centroid, k, col in zip(mbk.cluster_centers_, range(n_clusters), colors_):
    mask = mbk.labels_ == k
    ax.plot(X[mask, 0], X[mask, 1], 'w', markerfacecolor=col, marker='.'
    ax.plot(this_centroid[0], this_centroid[1], '+', markeredgecolor='k',
            markersize=5)
ax.set_xlim([-25, 25])
ax.set_ylim([-25, 25])
ax.set_title("MiniBatchKMeans")
ax.set_autoscaley_on(False)
plt.show()

Total running time of the script: (0 minutes 16.043 seconds)

Download Python source code: plot_birch_vs_minibatchkmeans.py
Download IPython notebook: plot_birch_vs_minibatchkmeans.ipynb

## 9.20 Empirical evaluation of the impact of k-means initialization

Evaluate the ability of k-means initializations strategies to make the algorithm convergence robust as measured by the relative standard deviation of the inertia of the clustering (i.e. the sum of distances to the nearest cluster center).

The first plot shows the best inertia reached for each combination of the model (KMeans or MiniBatchKMeans) and the init method (init="random" or init="kmeans++") for increasing values of the n_init parameter that controls the number of initializations.
The second plot demonstrate one single run of the MiniBatchKMeans estimator using a init="random" and n_init=1. This run leads to a bad convergence (local optimum) with estimated centers stuck between ground truth clusters.

The dataset used for evaluation is a 2D grid of isotropic Gaussian clusters widely spaced.

```
print(__doc__)

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm

from sklearn.utils import shuffle
from sklearn.utils import check_random_state
```

```
9.20. Empirical evaluation of the impact of k-means initialization
```
from sklearn.cluster import MiniBatchKMeans
from sklearn.cluster import KMeans

random_state = np.random.RandomState(0)

# Number of run (with randomly generated dataset) for each strategy so as
# to be able to compute an estimate of the standard deviation
n_runs = 5

# k-means models can do several random inits so as to be able to trade
# CPU time for convergence robustness
n_init_range = np.array([1, 5, 10, 15, 20])

# Datasets generation parameters
n_samples_per_center = 100
grid_size = 3
scale = 0.1
n_clusters = grid_size ** 2

def make_data(random_state, n_samples_per_center, grid_size, scale):
    random_state = check_random_state(random_state)
    centers = np.array([[i, j]
                        for i in range(grid_size)
                        for j in range(grid_size)])
    n_clusters_true, n_features = centers.shape

    noise = random_state.normal(
        scale=scale, size=(n_samples_per_center, centers.shape[1]))

    X = np.concatenate([c + noise for c in centers])
    y = np.concatenate([[i] * n_samples_per_center
                        for i in range(n_clusters_true)])

    return shuffle(X, y, random_state=random_state)

# Part 1: Quantitative evaluation of various init methods

fig = plt.figure()
plots = []
legends = []

cases = [
    (KMeans, 'k-means++', {}),
    (KMeans, 'random', {}),
    (MiniBatchKMeans, 'k-means++', {'max_no_improvement': 3}),
    (MiniBatchKMeans, 'random', {'max_no_improvement': 3, 'init_size': 500}),
]

for factory, init, params in cases:
    print("Evaluation of \$s with \$s init\" % (factory.__name__, init))
    inertia = np.empty((len(n_init_range), n_runs))

    for run_id in range(n_runs):
        X, y = make_data(run_id, n_samples_per_center, grid_size, scale)

        for i, n_init in enumerate(n_init_range):
            km = factory(n_clusters=n_clusters, init=init, random_state=run_id,
                         n_init=n_init, **params).fit(X)
            inertia[i, run_id] = km.inertia_
p = plt.errorbar(n_init_range, inertia.mean(axis=1), inertia.std(axis=1))
plots.append(p[0])
legends.append("%s with %s init" % (factory.__name__, init))

plt.xlabel('n_init')
plt.ylabel('inertia')
plt.legend(plots, legends)
plt.title("Mean inertia for various k-means init across %d runs" % n_runs)

# Part 2: Qualitative visual inspection of the convergence

X, y = make_data(random_state, n_samples_per_center, grid_size, scale)
kmean = MiniBatchKMeans(n_clusters=n_clusters, init='random', n_init=1,
                       random_state=random_state).fit(X)

fig = plt.figure()
for k in range(n_clusters):
    my_members = kmean.labels_ == k
    color = cm.spectral(float(k) / n_clusters, 1)
    plt.plot(X[my_members, 0], X[my_members, 1], 'o', marker='.', c=color)
    cluster_center = kmean.cluster_centers_[k]
    plt.plot(cluster_center[0], cluster_center[1], 'o',
             markerfacecolor=color, markeredgecolor='k', markersize=6)
plt.title("Example cluster allocation with a single random init\n" "with MiniBatchKMeans")

plt.show()

Total running time of the script: (0 minutes 3.249 seconds)

Download Python source code: plot_kmeans_stability_low_dim_dense.py
Download IPython notebook: plot_kmeans_stability_low_dim_dense.ipynb

9.21 Adjustment for chance in clustering performance evaluation

The following plots demonstrate the impact of the number of clusters and number of samples on various clustering performance evaluation metrics.

Non-adjusted measures such as the V-Measure show a dependency between the number of clusters and the number of samples: the mean V-Measure of random labeling increases significantly as the number of clusters is closer to the total number of samples used to compute the measure.

Adjusted for chance measure such as ARI display some random variations centered around a mean score of 0.0 for any number of samples and clusters.

Only adjusted measures can hence safely be used as a consensus index to evaluate the average stability of clustering algorithms for a given value of k on various overlapping sub-samples of the dataset.
Out:

Computing adjusted_rand_score for 10 values of n_clusters and n_samples=100 done in 0.041s
Computing v_measure_score for 10 values of n_clusters and n_samples=100 done in 0.061s
Computing adjusted_mutual_info_score for 10 values of n_clusters and n_samples=100 done in 0.442s
Computing mutual_info_score for 10 values of n_clusters and n_samples=100 done in 0.051s
Computing adjusted_rand_score for 10 values of n_clusters and n_samples=1000 done in 0.062s
Computing v_measure_score for 10 values of n_clusters and n_samples=1000 done in 0.079s
Computing adjusted_mutual_info_score for 10 values of n_clusters and n_samples=1000 done in 0.286s
Computing mutual_info_score for 10 values of n_clusters and n_samples=1000 done in 0.065s

print(__doc__)

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: BSD 3 clause
import numpy as np
import matplotlib.pyplot as plt
from time import time
from sklearn import metrics

def uniform_labelings_scores(score_func, n_samples, n_clusters_range,
                             fixed_n_classes=None, n_runs=5, seed=42):
    ""
    Compute score for 2 random uniform cluster labelings.
    Both random labelings have the same number of clusters for each value
    possible value in `n_clusters_range`.
    When fixed_n_classes is not None the first labeling is considered a ground
    truth class assignment with fixed number of classes.
    ""
    random_labels = np.random.RandomState(seed).randint
    scores = np.zeros((len(n_clusters_range), n_runs))
    if fixed_n_classes is not None:
        labels_a = random_labels(low=0, high=fixed_n_classes, size=n_samples)
        for i, k in enumerate(n_clusters_range):
            for j in range(n_runs):
                if fixed_n_classes is None:
                    labels_a = random_labels(low=0, high=k, size=n_samples)
                    labels_b = random_labels(low=0, high=k, size=n_samples)
                    scores[i, j] = score_func(labels_a, labels_b)
    return scores

score_funcs = [
    metrics.adjusted_rand_score,
    metrics.v_measure_score,
    metrics.adjusted_mutual_info_score,
    metrics.mutual_info_score,
]

# 2 independent random clusterings with equal cluster number
n_samples = 100
n_clusters_range = np.linspace(2, n_samples, 10).astype(np.int)

plt.figure(1)
plots = []
names = []
for score_func in score_funcs:
    print("Computing %s for %d values of n_clusters and n_samples=%d" %
          (score_func.__name__, len(n_clusters_range), n_samples))
    t0 = time()
    scores = uniform_labelings_scores(score_func, n_samples, n_clusters_range)
    print("done in %0.3fs" % (time() - t0))
    plots.append(plt.errorbar(n_clusters_range, np.median(scores, axis=1), scores.std(axis=1))[0])
    names.append(score_func.__name__)
In this example we compare the various initialization strategies for K-means in terms of runtime and quality of the results.

As the ground truth is known here, we also apply different cluster quality metrics to judge the goodness of fit of the cluster labels to the ground truth.

Cluster quality metrics evaluated (see Clustering performance evaluation for definitions and discussions of the metrics):

### 9.22 A demo of K-Means clustering on the handwritten digits data
Shorthand | full name
--- | ---
homo | homogeneity score
compl | completeness score
v-meas | V measure
ARI | adjusted Rand index
AMI | adjusted mutual information
silhouette | silhouette coefficient

```
print(__doc__)

from time import time
import numpy as np
import matplotlib.pyplot as plt

from sklearn import metrics
from sklearn.cluster import KMeans
from sklearn.datasets import load_digits
from sklearn.decomposition import PCA
from sklearn.preprocessing import scale

np.random.seed(42)
digits = load_digits()
data = scale(digits.data)
n_samples, n_features = data.shape
n_digits = len(np.unique(digits.target))
labels = digits.target

sample_size = 300

print("n_digits: {} \
      n_samples: {} \ 
      n_features: {}"
% (n_digits, n_samples, n_features))

print(79 * '_')
print('% 9s %8s %8s %8s %8s %8s %8s %8s %8s %8s
      % (name, time, inertia, homo, compl, v-meas, ARI, AMI, silhouette)

def bench_k_means(estimator, name, data):
t0 = time()
estimator.fit(data)

print('% 9s %8.2f %8i %8.3f %8.3f %8.3f %8.3f %8.3f %8.3f
      % (name, time - t0, estimator.inertia_,
         metrics.homogeneity_score(labels, estimator.labels_),
         metrics.completeness_score(labels, estimator.labels_),
         metrics.v_measure_score(labels, estimator.labels_),
         metrics.adjusted_rand_score(labels, estimator.labels_),
         metrics.adjusted_mutual_info_score(labels, estimator.labels_),
         metrics.silhouette_score(data, estimator.labels_,
                                   metric='euclidean',
                                   sample_size=sample_size)))

bench_k_means(KMeans(init='k-means++', n_clusters=n_digits, n_init=10),
              name="k-means++", data=data)
bench_k_means(KMeans(init='random', n_clusters=n_digits, n_init=10),
              name="random", data=data)
```

9.22. A demo of K-Means clustering on the handwritten digits data
```python
# in this case the seeding of the centers is deterministic, hence we run the
# kmeans algorithm only once with n_init=1
pcas = PCA(n_components=n_digits).fit(data)
bench_k_means(KMeans(init=pcas.components_, n_clusters=n_digits, n_init=1),
            name="PCA-based",
            data=data)

print(79 * '_')
```

Out:

```

<table>
<thead>
<tr>
<th>n_digits: 10, n_samples 1797, n_features 64</th>
</tr>
</thead>
<tbody>
<tr>
<td>init</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>k-means++</td>
</tr>
<tr>
<td>random</td>
</tr>
<tr>
<td>PCA-based</td>
</tr>
</tbody>
</table>
```

Visualize the results on PCA-reduced data

```python
reduced_data = PCA(n_components=2).fit_transform(data)
kmeans = KMeans(n_clusters=n_digits, n_init=10)
kmeans.fit(reduced_data)

# Step size of the mesh. Decrease to increase the quality of the VQ.
h = .02

# Plot the decision boundary. For that, we will assign a color to each
x_min, x_max = reduced_data[:, 0].min() - 1, reduced_data[:, 0].max() + 1
y_min, y_max = reduced_data[:, 1].min() - 1, reduced_data[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

# Obtain labels for each point in mesh. Use last trained model.
Z = kmeans.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
plt.figure(1)
plt.clf()
plt.imshow(Z, interpolation='nearest', extent=(xx.min(), xx.max(), yy.min(), yy.max()),
cmap=plt.cm.Paired,
aspect='auto', origin='lower')

plt.plot(reduced_data[:, 0], reduced_data[:, 1], 'k.', markersize=2)

# Plot the centroids as a white X
centroids = kmeans.cluster_centers_
plt.scatter(centroids[:, 0], centroids[:, 1], marker='x', s=169, linewidths=3,
            color='w', zorder=10)

plt.title("'K-means clustering on the digits dataset (PCA-reduced data)'
          'Centroids are marked with white cross'")
plt.xlabel('x_min', x_max)
plt.yticks(range(0, 10))
plt.yticks()```

728 Chapter 9. Clustering
9.23 Comparing different clustering algorithms on toy datasets

This example aims at showing characteristics of different clustering algorithms on datasets that are “interesting” but still in 2D. The last dataset is an example of a ‘null’ situation for clustering: the data is homogeneous, and there is no good clustering.

While these examples give some intuition about the algorithms, this intuition might not apply to very high dimensional data.

The results could be improved by tweaking the parameters for each clustering strategy, for instance setting the number of clusters for the methods that needs this parameter specified. Note that affinity propagation has a tendency to create many clusters. Thus in this example its two parameters (damping and per-point preference) were set to mitigate this behavior.
import time

import numpy as np
import matplotlib.pyplot as plt

from sklearn import cluster, datasets
from sklearn.neighbors import kneighbors_graph
from sklearn.preprocessing import StandardScaler

np.random.seed(0)

# Generate datasets. We choose the size big enough to see the scalability
# of the algorithms, but not too big to avoid too long running times
n_samples = 1500
noisy_circles = datasets.make_circles(n_samples=n_samples, factor=.5,
    noise=.05)
noisy_moons = datasets.make_moons(n_samples=n_samples, noise=.05)
blobs = datasets.make_blobs(n_samples=n_samples, random_state=8)
o_structure = np.random.rand(n_samples, 2), None

colors = np.array([x for x in 'bgrcmykbgrcmykbgrcmykbgrcmykBgrcmyk'])
colors = np.hstack([colors] * 20)

clustering_names = ['MiniBatchKMeans', 'AffinityPropagation', 'MeanShift',
                  'SpectralClustering', 'Ward', 'AgglomerativeClustering',
                  'DBSCAN', 'Birch']

plt.figure(figsize=(len(clustering_names) * 2 + 3, 9.5))
plt.subplots_adjust(left=.02, right=.98, bottom=.001, top=.96, wspace=.05,
                    hspace=.01)

plot_num = 1
datasets = [noisy_circles, noisy_moons, blobs, no_structure]
for i_dataset, dataset in enumerate(datasets):
    X, y = dataset
    # normalize dataset for easier parameter selection
    X = StandardScaler().fit_transform(X)

    # estimate bandwidth for mean shift
    bandwidth = cluster.estimate_bandwidth(X, quantile=0.3)

    # connectivity matrix for structured Ward
    connectivity = kneighbors_graph(X, n_neighbors=10, include_self=False)
    # make connectivity symmetric
    connectivity = 0.5 * (connectivity + connectivity.T)

    # create clustering estimators
    ms = cluster.MeanShift(bandwidth=bandwidth, bin_seeding=True)
two_means = cluster.MiniBatchKMeans(n_clusters=2)
ward = cluster.AgglomerativeClustering(n_clusters=2, linkage='ward',
    connectivity=connectivity)
spectral = cluster.SpectralClustering(n_clusters=2,
    eigen_solver='arpack',
    affinity="nearest_neighbors")
dbscan = cluster.DBSCAN(eps=.2)
affinity_propagation = cluster.AffinityPropagation(damping=.9,
    preference=-200)

    average_linkage = cluster.AgglomerativeClustering(
        linkage="average", affinity="cityblock", n_clusters=2,
        connectivity=connectivity)

    birch = cluster.Birch(n_clusters=2)
clustering_algorithms = [
    two_means, affinity_propagation, ms, spectral, ward, average_linkage,
    dbscan, birch]

for name, algorithm in zip(clustering_names, clustering_algorithms):
    # predict cluster memberships
    t0 = time.time()
    algorithm.fit(X)
    t1 = time.time()
    if hasattr(algorithm, 'labels_ '):
        y_pred = algorithm.labels_.astype(np.int)
    else:
        y_pred = algorithm.predict(X)

    # plot
    plt.subplot(4, len(clustering_algorithms), plot_num)
    if i_dataset == 0:
        plt.title(name, size=18)
    plt.scatter(X[:, 0], X[:, 1], color=colors[y_pred].tolist(), s=10)

    if hasattr(algorithm, 'cluster_centers_ '):
        centers = algorithm.cluster_centers_
        center_colors = colors[len(centers)]
        plt.scatter(centers[:, 0], centers[:, 1], s=100, c=center_colors)
    plt.xlim(-2, 2)
    plt.ylim(-2, 2)
    plt.xticks(())
9.24 Selecting the number of clusters with silhouette analysis on KMeans clustering

Silhouette analysis can be used to study the separation distance between the resulting clusters. The silhouette plot displays a measure of how close each point in one cluster is to points in the neighboring clusters and thus provides a way to assess parameters like number of clusters visually. This measure has a range of [-1, 1].

Silhouette coefficients (as these values are referred to as) near +1 indicate that the sample is far away from the neighboring clusters. A value of 0 indicates that the sample is on or very close to the decision boundary between two neighboring clusters and negative values indicate that those samples might have been assigned to the wrong cluster.

In this example the silhouette analysis is used to choose an optimal value for n_clusters. The silhouette plot shows that the n_clusters value of 3, 5 and 6 are a bad pick for the given data due to the presence of clusters with below average silhouette scores and also due to wide fluctuations in the size of the silhouette plots. Silhouette analysis is more ambivalent in deciding between 2 and 4.

Also from the thickness of the silhouette plot the cluster size can be visualized. The silhouette plot for cluster 0 when n_clusters is equal to 2, is bigger in size owing to the grouping of the 3 sub clusters into one big cluster. However when the n_clusters is equal to 4, all the plots are more or less of similar thickness and hence are of similar sizes as can be also verified from the labelled scatter plot on the right.
For n_clusters = 2 The average silhouette_score is: 0.704978749608
For n_clusters = 3 The average silhouette_score is: 0.588200401213
For n_clusters = 4 The average silhouette_score is: 0.650518663273
For n_clusters = 5 The average silhouette_score is: 0.563764690262
For n_clusters = 6 The average silhouette_score is: 0.450466629437
range_n_clusters = [2, 3, 4, 5, 6]

for n_clusters in range_n_clusters:
    # Create a subplot with 1 row and 2 columns
    fig, (ax1, ax2) = plt.subplots(1, 2)
    fig.set_size_inches(18, 7)

    # The 1st subplot is the silhouette plot
    # The silhouette coefficient can range from -1, 1 but in this example all
    # lie within [-0.1, 1]
    ax1.set_xlim([-0.1, 1])
    ax1.set_ylim([0, len(X) + (n_clusters + 1) * 10])

    # Initialize the clusterer with n_clusters value and a random generator
    # seed of 10 for reproducibility.
    clusterer = KMeans(n_clusters=n_clusters, random_state=10)
    cluster_labels = clusterer.fit_predict(X)

    # The silhouette_score gives the average value for all the samples.
    # This gives a perspective into the density and separation of the formed
    # clusters
    silhouette_avg = silhouette_score(X, cluster_labels)
    print("For n_clusters =", n_clusters,
        "The average silhouette score is :", silhouette_avg)

    # Compute the silhouette scores for each sample
    sample_silhouette_values = silhouette_samples(X, cluster_labels)

    y_lower = 10
    for i in range(n_clusters):
        # Aggregate the silhouette scores for samples belonging to
        # cluster i, and sort them
        ith_cluster_silhouette_values = \
            sample_silhouette_values[cluster_labels == i]

        ith_cluster_silhouette_values.sort()

        size_cluster_i = ith_cluster_silhouette_values.shape[0]
        y_upper = y_lower + size_cluster_i

        color = cm.spectral(float(i) / n_clusters)
        ax1.fill_betweenx(np.arange(y_lower, y_upper),
            0, ith_cluster_silhouette_values,
            facecolor=color, edgecolor=color, alpha=0.7)

        # Label the silhouette plots with their cluster numbers at the middle
        ax1.text(-0.05, y_lower + 0.5 * size_cluster_i, str(i))

        # Compute the new y_lower for next plot
        y_lower = y_upper + 10  # 10 for the 0 samples

ax1.set_title("The silhouette plot for the various clusters.")
ax1.set_xlabel("The silhouette coefficient values")
a1.set_ylabel("Cluster label")
# The vertical line for average silhouette score of all the values
ax1.axvline(x=silhouette_avg, color="red", linestyle="--")

ax1.set_yticks([])  # Clear the yaxis labels / ticks
ax1.set_xticks([-0.1, 0, 0.2, 0.4, 0.6, 0.8, 1])

# 2nd Plot showing the actual clusters formed
colors = cm.spectral(cluster_labels.astype(float) / n_clusters)
ax2.scatter(X[:, 0], X[:, 1], marker='.', s=30, lw=0, alpha=0.7,
colors)

# Labeling the clusters
centers = clusterer.cluster_centers_
# Draw white circles at cluster centers
ax2.scatter(centers[:, 0], centers[:, 1],
            marker='o', c="white", alpha=1, s=200)

for i, c in enumerate(centers):
    ax2.scatter(c[0], c[1], marker='$%d$' % i, alpha=1, s=50)

ax2.set_title("The visualization of the clustered data.")
ax2.set_xlabel("Feature space for the 1st feature")
ax2.set_ylabel("Feature space for the 2nd feature")

plt.suptitle("Silhouette analysis for KMeans clustering on sample data "
"with n_clusters = %d" % n_clusters),
            fontsize=14, fontweight='bold')

plt.show()

Total running time of the script: (0 minutes 1.232 seconds)

Download Python source code: plot_kmeans_silhouette_analysis.py
Download IPython notebook: plot_kmeans_silhouette_analysis.ipynb
Examples concerning the sklearn.covariance module.

10.1 Ledoit-Wolf vs OAS estimation

The usual covariance maximum likelihood estimate can be regularized using shrinkage. Ledoit and Wolf proposed a close formula to compute the asymptotically optimal shrinkage parameter (minimizing a MSE criterion), yielding the Ledoit-Wolf covariance estimate.

Chen et al. proposed an improvement of the Ledoit-Wolf shrinkage parameter, the OAS coefficient, whose convergence is significantly better under the assumption that the data are Gaussian.

This example, inspired from Chen’s publication [1], shows a comparison of the estimated MSE of the LW and OAS methods, using Gaussian distributed data.


```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from scipy.linalg import toeplitz, cholesky
from sklearn.covariance import LedoitWolf, OAS
np.random.seed(0)

n_features = 100
# simulation covariance matrix (AR(1) process)
r = 0.1
real_cov = toeplitz(r ** np.arange(n_features))
coloring_matrix = cholesky(real_cov)

n_samples_range = np.arange(6, 31, 1)
repeat = 100
lw_mse = np.zeros((n_samples_range.size, repeat))
oa_mse = np.zeros((n_samples_range.size, repeat))
lw_shrinkage = np.zeros((n_samples_range.size, repeat))
oa_shrinkage = np.zeros((n_samples_range.size, repeat))
for i, n_samples in enumerate(n_samples_range):
    for j in range(repeat):
        X = np.dot(
            np.random.normal(size=(n_samples, n_features)), coloring_matrix.T)
```
lw = LedoitWolf(store_precision=False, assume_centered=True)
lw.fit(X)
lw_mse[i, j] = lw.error_norm(real_cov, scaling=False)
lw_shrinkage[i, j] = lw.shrinkage_

oa = OAS(store_precision=False, assume_centered=True)
oa.fit(X)
oa_mse[i, j] = oa.error_norm(real_cov, scaling=False)
oa_shrinkage[i, j] = oa.shrinkage_

# plot MSE
plt.subplot(2, 1, 1)
plt.errorbar(n_samples_range, lw_mse.mean(1), yerr=lw_mse.std(1),
             label='Ledoit-Wolf', color='navy', lw=2)
plt.errorbar(n_samples_range, oa_mse.mean(1), yerr=oa_mse.std(1),
             label='OAS', color='darkorange', lw=2)
plt.ylabel("Squared error")
plt.legend(loc="upper right")
plt.title("Comparison of covariance estimators")
plt.xlim(5, 31)

# plot shrinkage coefficient
plt.subplot(2, 1, 2)
plt.errorbar(n_samples_range, lw_shrinkage.mean(1), yerr=lw_shrinkage.std(1),
             label='Ledoit-Wolf', color='navy', lw=2)
plt.errorbar(n_samples_range, oa_shrinkage.mean(1), yerr=oa_shrinkage.std(1),
             label='OAS', color='darkorange', lw=2)
plt.xlabel("n_samples")
plt.ylabel("Shrinkage")
plt.legend(loc="lower right")
plt.ylim(plt.ylim()[0], 1. + (plt.ylim()[1] - plt.ylim()[0]) / 10.)
plt.xlim(5, 31)

plt.show()
Using the GraphLasso estimator to learn a covariance and sparse precision from a small number of samples.

To estimate a probabilistic model (e.g. a Gaussian model), estimating the precision matrix, that is the inverse covariance matrix, is as important as estimating the covariance matrix. Indeed a Gaussian model is parametrized by the precision matrix.

To be in favorable recovery conditions, we sample the data from a model with a sparse inverse covariance matrix. In addition, we ensure that the data is not too much correlated (limiting the largest coefficient of the precision matrix) and that there a no small coefficients in the precision matrix that cannot be recovered. In addition, with a small number of observations, it is easier to recover a correlation matrix rather than a covariance, thus we scale the time series.

Here, the number of samples is slightly larger than the number of dimensions, thus the empirical covariance is still invertible. However, as the observations are strongly correlated, the empirical covariance matrix is ill-conditioned and as a result its inverse –the empirical precision matrix– is very far from the ground truth.

If we use l2 shrinkage, as with the Ledoit-Wolf estimator, as the number of samples is small, we need to shrink a lot. As a result, the Ledoit-Wolf precision is fairly close to the ground truth precision, that is not far from being diagonal,
but the off-diagonal structure is lost.  
The l1-penalized estimator can recover part of this off-diagonal structure. It learns a sparse precision. It is not able to recover the exact sparsity pattern: it detects too many non-zero coefficients. However, the highest non-zero coefficients of the l1 estimated correspond to the non-zero coefficients in the ground truth. Finally, the coefficients of the l1 precision estimate are biased toward zero: because of the penalty, they are all smaller than the corresponding ground truth value, as can be seen on the figure.

Note that, the color range of the precision matrices is tweaked to improve readability of the figure. The full range of values of the empirical precision is not displayed.

The alpha parameter of the GraphLasso setting the sparsity of the model is set by internal cross-validation in the GraphLassoCV. As can be seen on figure 2, the grid to compute the cross-validation score is iteratively refined in the neighborhood of the maximum.

```python
print(__doc__)
# author: Gael Varoquaux <gael.varoquaux@inria.fr>
# License: BSD 3 clause
# Copyright: INRIA
import numpy as np
from scipy import linalg
from sklearn.datasets import make_sparse_spd_matrix
from sklearn.covariance import GraphLassoCV, ledoit_wolf
import matplotlib.pyplot as plt

Generate the data

```python
n_samples = 60
n_features = 20

prng = np.random.RandomState(1)
prec = make_sparse_spd_matrix(n_features, alpha=.98,
    smallest_coef=.4,
    largest_coef=.7,
    random_state=prng)

cov = linalg.inv(prec)
d = np.sqrt(np.diag(cov))
cov /= d
cov /= d[:, np.newaxis]
prec *= d
prec *= d[:, np.newaxis]
X = prng.multivariate_normal(np.zeros(n_features), cov, size=n_samples)
X -= X.mean(axis=0)
X /= X.std(axis=0)

Estimate the covariance

```python
emp Cov = np.dot(X.T, X) / n_samples

model = GraphLassoCV()
model.fit(X)
cov_ = model.covariance_
prec_ = model.precision_

lw_cov_, _ = ledoit_wolf(X)
lw_prec_ = linalg.inv(lw_cov_)

Plot the results

```
### 10.2. Sparse inverse covariance estimation

```python
plt.figure(figsize=(10, 6))
plt.subplots_adjust(left=0.02, right=0.98)

# plot the covariances
covs = [('Empirical', emp_cov), ('Ledoit-Wolf', lw_cov_),
        ('GraphLasso', cov_), ('True', cov)]
vmax = cov_.max()
for i, (name, this_cov) in enumerate(covs):
    plt.subplot(2, 4, i + 1)
    plt.imshow(this_cov, interpolation='nearest', vmin=-vmax, vmax=vmax,
               cmap=plt.cm.RdBu_r)
    plt.xticks(())
    plt.yticks(()
    plt.title('%s covariance' % name)

# plot the precisions
precs = [('Empirical', linalg.inv(emp_cov)), ('Ledoit-Wolf', lw_prec_),
         ('GraphLasso', prec_), ('True', prec)]
vmax = .9 * prec_.max()
for i, (name, this_prec) in enumerate(precs):
    ax = plt.subplot(2, 4, i + 5)
    plt.imshow(np.ma.masked_equal(this_prec, 0),
               interpolation='nearest', vmin=-vmax, vmax=vmax,
               cmap=plt.cm.RdBu_r)
    plt.xticks(())
    plt.yticks(()
    plt.title('%s precision' % name)
    ax.set_axis_bgcolor('.7')

# plot the model selection metric
plt.figure(figsize=(4, 3))
plt.axes([.2, .15, .75, .7])
plt.plot(model.cv_alphas_, np.mean(model.grid_scores, axis=1), 'o-')
plt.axvline(model.alpha_, color='.5')
plt.title('Model selection')
plt.ylabel('Cross-validation score')
plt.xlabel('alpha')
plt.show()
```
10.3 Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood

When working with covariance estimation, the usual approach is to use a maximum likelihood estimator, such as the `sklearn.covariance.EmpiricalCovariance`. It is unbiased, i.e. it converges to the true (population) covariance when given many observations. However, it can also be beneficial to regularize it, in order to reduce its variance; this, in turn, introduces some bias. This example illustrates the simple regularization used in `Shrunk Covariance` estimators. In particular, it focuses on how to set the amount of regularization, i.e. how to choose the bias-variance trade-off.

Here we compare 3 approaches:

- Setting the parameter by cross-validating the likelihood on three folds according to a grid of potential shrinkage parameters.
- A close formula proposed by Ledoit and Wolf to compute the asymptotically optimal regularization parameter (minimizing a MSE criterion), yielding the `sklearn.covariance.LedoitWolf` covariance estimate.
- An improvement of the Ledoit-Wolf shrinkage, the `sklearn.covariance.OAS`, proposed by Chen et al. Its convergence is significantly better under the assumption that the data are Gaussian, in particular for small samples.

To quantify estimation error, we plot the likelihood of unseen data for different values of the shrinkage parameter. We also show the choices by cross-validation, or with the LedoitWolf and OAS estimates.

Note that the maximum likelihood estimate corresponds to no shrinkage, and thus performs poorly. The Ledoit-Wolf estimate performs really well, as it is close to the optimal and is computational not costly. In this example, the OAS estimate is a bit further away. Interestingly, both approaches outperform cross-validation, which is significantly most computationally costly.

```
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from scipy import linalg

from sklearn.covariance import LedoitWolf, OAS, ShrunkCovariance,
    log_likelihood, empirical_covariance

from sklearn.model_selection import GridSearchCV

Generate sample data
n_features, n_samples = 40, 20
np.random.seed(42)
base_X_train = np.random.normal(size=(n_samples, n_features))
base_X_test = np.random.normal(size=(n_samples, n_features))

# Color samples
coloring_matrix = np.random.normal(size=(n_features, n_features))
X_train = np.dot(base_X_train, coloring_matrix)
X_test = np.dot(base_X_test, coloring_matrix)

Compute the likelihood on test data

Compute different approaches to setting the parameter

Plot results

10.3. Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood
plt.vlines(lw.shrinkage_, ymin, -loglik_lw, color='magenta', linewidth=3, label='Ledoit-Wolf estimate')

# OAS likelihood
plt.vlines(oa.shrinkage_, ymin, -loglik_oa, color='purple', linewidth=3, label='OAS estimate')

# best CV estimator likelihood
plt.vlines(cv.best_estimator_.shrinkage, ymin, -cv.best_estimator_.score(X_test), color='cyan', linewidth=3, label='Cross-validation best estimate')

plt.ylim(ymin, ymax)
plt.xlim(xmin, xmax)
plt.legend()

plt.show()
10.4 Outlier detection with several methods.

When the amount of contamination is known, this example illustrates three different ways of performing *Novelty and Outlier Detection*:

- based on a robust estimator of covariance, which is assuming that the data are Gaussian distributed and performs better than the One-Class SVM in that case.
- using the One-Class SVM and its ability to capture the shape of the data set, hence performing better when the data is strongly non-Gaussian, i.e. with two well-separated clusters;
- using the Isolation Forest algorithm, which is based on random forests and hence more adapted to large-dimensional settings, even if it performs quite well in the examples below.

The ground truth about inliers and outliers is given by the points colors while the orange-filled area indicates which points are reported as inliers by each method.

Here, we assume that we know the fraction of outliers in the datasets. Thus rather than using the ‘predict’ method of the objects, we set the threshold on the decision function to separate out the corresponding fraction.

```python
print(__doc__)

import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import matplotlib.font_manager

from sklearn import svm
def sklearn.covariance import EllipticEnvelope
def sklearn.ensemble import IsolationForest

rng = np.random.RandomState(42)

# Example settings
n_samples = 200
outliers_fraction = 0.25
clusters_separation = [0, 1, 2]
```
# define two outlier detection tools to be compared
classifiers = {
    "One-Class SVM": svm.OneClassSVM(nu=0.95 * outliers_fraction + 0.05,
        kernel="rbf", gamma=0.1),
    "Robust covariance": EllipticEnvelope(contamination=outliers_fraction),
    "Isolation Forest": IsolationForest(max_samples=n_samples,
        contamination=outliers_fraction,
        random_state=rng)
}

# Compare given classifiers under given settings
xx, yy = np.meshgrid(np.linspace(-7, 7, 500), np.linspace(-7, 7, 500))
n_inliers = int((1. - outliers_fraction) * n_samples)
n_outliers = int(outliers_fraction * n_samples)
ground_truth = np.ones(n_samples, dtype=int)
ground_truth[-n_outliers:] = -1

# Fit the problem with varying cluster separation
for i, offset in enumerate(clusters_separation):
    np.random.seed(42)
    # Data generation
    X1 = 0.3 * np.random.randn(n_inliers // 2, 2) - offset
    X2 = 0.3 * np.random.randn(n_inliers // 2, 2) + offset
    X = np.r_[X1, X2]
    # Add outliers
    X = np.r_[X, np.random.uniform(low=-6, high=6, size=(n_outliers, 2))]

    # Fit the model
    plt.figure(figsize=(10.8, 3.6))
    for i, (clf_name, clf) in enumerate(classifiers.items()):
        # fit the data and tag outliers
        clf.fit(X)
        scores_pred = clf.decision_function(X)
        threshold = stats.scoreatpercentile(scores_pred,
            100 * outliers_fraction)
        y_pred = clf.predict(X)
        n_errors = (y_pred != ground_truth).sum()

        # plot the levels lines and the points
        Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
        Z = Z.reshape(xx.shape)
        subplot = plt.subplot(1, 3, i + 1)
        subplot.contourf(xx, yy, Z, levels=np.linspace(Z.min(), threshold, 7),
            cmap=plt.cm.Blues_r)
        a = subplot.contour(xx, yy, Z, levels=[threshold],
            linewidths=2, colors='red')
        b = subplot.scatter(X[:-n_outliers, 0], X[:-n_outliers, 1], c='white')
        c = subplot.scatter(X[-n_outliers:, 0], X[-n_outliers:, 1], c='black')
        subplot.axis('tight')
        subplot.legend([a.collections[0], b, c],
            ['learned decision function', 'true inliers', 'true outliers'],
            prop=matplotlib.font_manager.FontProperties(size=11),
            loc='lower right')
        subplot.set_title("%d. %s (errors: %d)" % (i + 1, clf_name, n_errors))
        subplot.set_xlim((-7, 7))
        subplot.set_ylim((-7, 7))
        plt.subplots_adjust(0.04, 0.1, 0.96, 0.92, 0.1, 0.26)
10.5 Robust covariance estimation and Mahalanobis distances relevance

An example to show covariance estimation with the Mahalanobis distances on Gaussian distributed data.

For Gaussian distributed data, the distance of an observation $x_i$ to the mode of the distribution can be computed using its Mahalanobis distance:

$$d(\mu, \Sigma)(x_i)^2 = (x_i - \mu)'\Sigma^{-1}(x_i - \mu)$$

where $\mu$ and $\Sigma$ are the location and the covariance of the underlying Gaussian distribution.

In practice, $\mu$ and $\Sigma$ are replaced by some estimates. The usual covariance maximum likelihood estimate is very sensitive to the presence of outliers in the data set and therefore, the corresponding Mahalanobis distances are. One would better have to use a robust estimator of covariance to guarantee that the estimation is resistant to “erroneous” observations in the data set and that the associated Mahalanobis distances accurately reflect the true organization of the observations.

The Minimum Covariance Determinant estimator is a robust, high-breakdown point (i.e. it can be used to estimate the covariance matrix of highly contaminated datasets, up to $\frac{n_{\text{samples}} - n_{\text{outliers}} - 1}{2}$ outliers) estimator of covariance. The idea is to find $\frac{n_{\text{samples}} + n_{\text{outliers}} + 1}{2}$ observations whose empirical covariance has the smallest determinant, yielding a “pure” subset of observations from which to compute standards estimates of location and covariance.

The Minimum Covariance Determinant estimator (MCD) has been introduced by P.J. Rousseeuw in [1].

This example illustrates how the Mahalanobis distances are affected by outlying data: observations drawn from a contaminating distribution are not distinguishable from the observations coming from the real, Gaussian distribution that one may want to work with. Using MCD-based Mahalanobis distances, the two populations become distinguishable. Associated applications are outliers detection, observations ranking, clustering, ... For visualization purpose, the cubic root of the Mahalanobis distances are represented in the boxplot, as Wilson and Hilferty suggest [2]


```python
# add some outliers
outliers_cov = np.eye(n_features)
outliers_cov[np.arange(1, n_features), np.arange(1, n_features)] = 7.
X[-n_outliers:] = np.dot(np.random.randn(n_outliers, n_features), outliers_cov)

# fit a Minimum Covariance Determinant (MCD) robust estimator to data
robust_cov = MinCovDet().fit(X)

# compare estimators learnt from the full data set with true parameters
emp_cov = EmpiricalCovariance().fit(X)

Display results

fig = plt.figure()
plt.subplots_adjust(hspace=-.1, wspace=.4, top=.95, bottom=.05)

# Show data set
subfig1 = plt.subplot(3, 1, 1)
inlier_plot = subfig1.scatter(X[:, 0], X[:, 1],
    color='black', label='inliers')
outlier_plot = subfig1.scatter(X[:, 0][-n_outliers:], X[:, 1][-n_outliers:],
    color='red', label='outliers')
subfig1.set_xlim(subfig1.get_xlim()[0], 11.)
subfig1.set_title("Mahalanobis distances of a contaminated data set:")

# Show contours of the distance functions
xx, yy = np.meshgrid(np.linspace(plt.xlim()[0], plt.xlim()[1], 100),
    np.linspace(plt.ylim()[0], plt.ylim()[1], 100))
zz = np.c_[xx.ravel(), yy.ravel()]
mahal_emp_cov = emp_cov.mahalanobis(zz)
mahal_emp_cov = mahal_emp_cov.reshape(xx.shape)
emp_cov_contour = subfig1.contour(xx, yy, np.sqrt(mahal_emp_cov),
    cmap=plt.cm.PuBu_r,
    linestyles='dashed')
mahal_robust_cov = robust_cov.mahalanobis(zz)
mahal_robust_cov = mahal_robust_cov.reshape(xx.shape)
robust_contour = subfig1.contour(xx, yy, np.sqrt(mahal_robust_cov),
    cmap=plt.cm.YlOrBr_r, linestyles='dotted')

subfig1.legend([emp_cov_contour.collections[1], robust_contour.collections[1],
    inlier_plot, outlier_plot],
    ['MLE dist', 'robust dist', 'inliers', 'outliers'],
    loc="upper right", borderaxespad=0)
plt.xticks(()
plt.yticks(()

# Plot the scores for each point
emp_mahal = emp_cov.mahalanobis(X - np.mean(X, 0)) ** (0.33)
subfig2 = plt.subplot(2, 2, 3)
subfig2.boxplot([emp_mahal[:-n_outliers], emp_mahal[-n_outliers:]],
    widths=.25)
subfig2.plot(1.26 * np.ones(n_samples - n_outliers),
    emp_mahal[:-n_outliers], '+k', markeredgewidth=1)
subfig2.plot(2.26 * np.ones(n_outliers),
    emp_mahal[-n_outliers:], '+k', markeredgewidth=1)
subfig2.axes.set_xticklabels(('inliers', 'outliers'), size=15)
subfig2.set_xlabel("$\sqrt{3} \times \text{(Mahalanobis dist.)}$", size=16)```

---

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subfig2.set_title("1. from non-robust estimates
(Maximum Likelihood)\n")
plt.yticks(()

robust_mahal = robust_cov.mahalanobis(X - robust_cov.location_) ** (0.33)
subfig3 = plt.subplot(2, 2, 4)
subfig3.boxplot([robust_mahal[:-n_outliers], robust_mahal[-n_outliers:]],
                widths=.25)
subfig3.plot(1.26 * np.ones(n_samples - n_outliers),
             robust_mahal[:-n_outliers], '+k', markeredgewidth=1)
subfig3.plot(2.26 * np.ones(n_outliers),
             robust_mahal[-n_outliers:], '+k', markeredgewidth=1)
subfig3.axes.set_xticklabels(('inliers', 'outliers'), size=15)
subfig3.set_ylabel(r"$\sqrt[3]{\text{Mahal. dist.}}$", size=16)
subfig3.set_title("2. from robust estimates
(Minimum Covariance Determinant)\n")
plt.yticks(()

plt.show()

Total running time of the script: (0 minutes 0.342 seconds)

Download Python source code: plot_mahalanobis_distances.py
Download IPython notebook: plot_mahalanobis_distances.ipynb
10.6 Robust vs Empirical covariance estimate

The usual covariance maximum likelihood estimate is very sensitive to the presence of outliers in the data set. In such a case, it would be better to use a robust estimator of covariance to guarantee that the estimation is resistant to “erroneous” observations in the data set.

10.6.1 Minimum Covariance Determinant Estimator

The Minimum Covariance Determinant estimator is a robust, high-breakdown point (i.e. it can be used to estimate the covariance matrix of highly contaminated datasets, up to \(\frac{n_{\text{samples}} - n_{\text{features}} - 1}{2}\) outliers) estimator of covariance. The idea is to find \(n_{\text{samples}} + \frac{n_{\text{features}} + 1}{2}\) observations whose empirical covariance has the smallest determinant, yielding a “pure” subset of observations from which to compute standards estimates of location and covariance. After a correction step aiming at compensating the fact that the estimates were learned from only a portion of the initial data, we end up with robust estimates of the data set location and covariance.

The Minimum Covariance Determinant estimator (MCD) has been introduced by P.J. Rousseeuw in \(^1\).

10.6.2 Evaluation

In this example, we compare the estimation errors that are made when using various types of location and covariance estimates on contaminated Gaussian distributed data sets:

- The mean and the empirical covariance of the full dataset, which break down as soon as there are outliers in the data set
- The robust MCD, that has a low error provided \(n_{\text{samples}} > 5n_{\text{features}}\)
- The mean and the empirical covariance of the observations that are known to be good ones. This can be considered as a “perfect” MCD estimation, so one can trust our implementation by comparing to this case.

10.6.3 References

print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
import matplotlib.font_manager

from sklearn.covariance import EmpiricalCovariance, MinCovDet

# example settings
n_samples = 80
n_features = 5
repeat = 10

range_n_outliers = np.concatenate((np.linspace(0, n_samples / 8, 5), np.linspace(n_samples / 8, n_samples / 2, 5)[1:-1])).astype(np.int)

# definition of arrays to store results
err_loc_mcd = np.zeros((range_n_outliers.size, repeat))
err_cov_mcd = np.zeros((range_n_outliers.size, repeat))
err_loc_emp_full = np.zeros((range_n_outliers.size, repeat))
err_cov_emp_full = np.zeros((range_n_outliers.size, repeat))
err_loc_emp_pure = np.zeros((range_n_outliers.size, repeat))

10.6. Robust vs Empirical covariance estimate
err_cov_emp_pure = np.zeros((range_n_outliers.size, repeat))

# computation
for i, n_outliers in enumerate(range_n_outliers):
    for j in range(repeat):
        rng = np.random.RandomState(i * j)

        # generate data
        X = rng.randn(n_samples, n_features)
        # add some outliers
        outliers_index = rng.permutation(n_samples)[:n_outliers]
        outliers_offset = 10. * 
                         (np.random.randint(2, size=(n_outliers, n_features)) - 0.5)
        X[outliers_index] += outliers_offset

        inliers_mask = np.ones(n_samples).astype(bool)
        inliers_mask[outliers_index] = False

        # fit a Minimum Covariance Determinant (MCD) robust estimator to data
        mcd = MinCovDet().fit(X)

        # compare raw robust estimates with the true location and covariance
        err_loc_mcd[i, j] = np.sum(mcd.location_ ** 2)
        err_cov_mcd[i, j] = mcd.error_norm(np.eye(n_features))

        # compare estimators learned from the full data set with true
        # parameters
        err_loc_emp_full[i, j] = np.sum(X.mean(0) ** 2)
        err_cov_emp_full[i, j] = EmpiricalCovariance().fit(X).error_norm(np.eye(n_features))

        # compare with an empirical covariance learned from a pure data set
        # (i.e. "perfect" mcd)
        pure_X = X[inliers_mask]
        pure_location = pure_X.mean(0)
        pure_emp_cov = EmpiricalCovariance().fit(pure_X)
        err_loc_emp_pure[i, j] = np.sum(pure_location ** 2)
        err_cov_emp_pure[i, j] = pure_emp_cov.error_norm(np.eye(n_features))

# Display results
font_prop = matplotlib.font_manager.FontProperties(size=11)
plt.subplot(2, 1, 1)
lw = 2
plt.errorbar(range_n_outliers, err_loc_mcd.mean(1),
             yerr=err_loc_mcd.std(1) / np.sqrt(repeat),
             label="Robust location", lw=lw, color='m')
plt.errorbar(range_n_outliers, err_loc_emp_full.mean(1),
             yerr=err_loc_emp_full.std(1) / np.sqrt(repeat),
             label="Full data set mean", lw=lw, color='green')
plt.errorbar(range_n_outliers, err_loc_emp_pure.mean(1),
             yerr=err_loc_emp_pure.std(1) / np.sqrt(repeat),
             label="Pure data set mean", lw=lw, color='black')
plt.title("Influence of outliers on the location estimation")
plt.ylabel(r"Error ($||\mu - \hat{\mu}||_2^2$)"")
plt.legend(loc="upper left", prop=font_prop)

plt.subplot(2, 1, 2)
x_size = range_n_outliers.size
plt.errorbar(range_n_outliers, err_cov_mcd.mean(1),
             yerr=err_cov_mcd.std(1) / np.sqrt(repeat),
             label="Robust covariance", lw=lw, color='m')
plt.errorbar(range_n_outliers, err_cov_emp_full.mean(1),
             yerr=err_cov_emp_full.std(1) / np.sqrt(repeat),
             label="Full data set mean", lw=lw, color='green')
plt.errorbar(range_n_outliers, err_cov_emp_pure.mean(1),
             yerr=err_cov_emp_pure.std(1) / np.sqrt(repeat),
             label="Pure data set mean", lw=lw, color='black')
plt.title("Influence of outliers on the covariance estimation")
plt.ylabel(r"Error ($||\Sigma - \hat{\Sigma}||_2$)"")
plt.legend(loc="upper left", prop=font_prop)
yerr=err_cov_mcd.std(1),
label="Robust covariance (mcd)", color='m')
plt.errorbar(range_n_outliers[:\(\frac{x\_size}{5} + 1\)]),
    err_cov_emp_full.mean(1)[:\(\frac{x\_size}{5} + 1\)],
    yerr=err_cov_emp_full.std(1)[:\(\frac{x\_size}{5} + 1\)],
    label="Full data set empirical covariance", color='green')
plt.plot(range_n_outliers[\(\frac{x\_size}{5}\):\(\frac{x\_size}{2} - 1\)],
    err_cov_emp_full.mean(1)[\(\frac{x\_size}{5}\):\(\frac{x\_size}{2} - 1\)],
    color='green', ls='--')
plt.errorbar(range_n_outliers, err_cov_emp_pure.mean(1),
    yerr=err_cov_emp_pure.std(1),
    label="Pure data set empirical covariance", color='black')
plt.title("Influence of outliers on the covariance estimation")
plt.xlabel("Amount of contamination (%)")
plt.ylabel("RMSE")
plt.legend(loc="upper center", prop=font_prop)
plt.show()
Examples concerning the `sklearn.cross_decomposition` module.

### 11.1 Compare cross decomposition methods

Simple usage of various cross decomposition algorithms:
- `PLSCanonical` - PLSRegression, with multivariate response, a.k.a. PLS2 - PLSRegression, with univariate response, a.k.a. PLS1 - CCA

Given 2 multivariate covarying two-dimensional datasets, X, and Y, PLS extracts the 'directions of covariance', i.e. the components of each datasets that explain the most shared variance between both datasets. This is apparent on the scatterplot matrix display: components 1 in dataset X and dataset Y are maximally correlated (points lie around the first diagonal). This is also true for components 2 in both dataset, however, the correlation across datasets for different components is weak: the point cloud is very spherical.

```python
print(__doc__)
import numpy as np
import matplotlib.pyplot as plt
from sklearn.cross_decomposition import PLSCanonical, PLSRegression, CCA

Dataset based latent variables model

```n = 500
# 2 latents vars:
l1 = np.random.normal(size=n)
l2 = np.random.normal(size=n)
latents = np.array([l1, l1, l2, l2]).T
X = latents + np.random.normal(size=4 * n).reshape((n, 4))
Y = latents + np.random.normal(size=4 * n).reshape((n, 4))

X_train = X[:n // 2]
Y_train = Y[:n // 2]
X_test = X[n // 2:]
Y_test = Y[n // 2:]

print("Corr(X)")
print(np.round(np.corrcoef(X.T), 2))
print("Corr(Y")
print(np.round(np.corrcoef(Y.T), 2))
```

Out:
Canonical (symmetric) PLS

```python
# Transform data
# ~~~~~~~~~~~~~~
plsca = PLSCanonical(n_components=2)
plsca.fit(X_train, Y_train)
X_train_r, Y_train_r = plsca.transform(X_train, Y_train)
X_test_r, Y_test_r = plsca.transform(X_test, Y_test)

# Scatter plot of scores
# ~~~~~~~~~~~~~~~~~~~~~~
# 1) On diagonal plot X vs Y scores on each component
plt.figure(figsize=(12, 8))
plt.subplot(221)
plt.plot(X_train_r[:, 0], Y_train_r[:, 0], "ob", label="train")
plt.plot(X_test_r[:, 0], Y_test_r[:, 0], "or", label="test")
plt.xlabel("x scores")
plt.ylabel("y scores")
plt.title('Comp. 1: X vs Y (test corr = %.2f)'
np.corrcoef(X_test_r[:, 0], Y_test_r[:, 0])[0, 1])
plt.xticks(())
plt.yticks(())
plt.legend(loc="best")
plt.subplot(222)
plt.plot(X_train_r[:, 1], X_train_r[:, 1], "*b", label="train")
plt.plot(X_test_r[:, 1], X_test_r[:, 1], "*r", label="test")
plt.xlabel("X comp. 1")
plt.ylabel("X comp. 2")
plt.title('X comp. 1 vs X comp. 2 (test corr = %.2f)'
np.corrcoef(X_test_r[:, 1], X_test_r[:, 1])[0, 1])
plt.xticks()
plt.yticks()
plt.legend(loc="best")
```

Corr(X)
```
[[ 1.  0.52 -0.03  0. ]
 [ 0.52 1.  0.02 -0.01]
[-0.03 0.02 1.  0.45]
 [ 0. -0.01 0.45 1. ]]
```

Corr(Y)
```
[[ 1.  0.52  0.01 -0.01]
 [ 0.52 1.  0.  0.06]
[ 0.01 0.  1.  0.52]
[-0.01 0.06 0.52 1. ]]
```
```python
plt.subplot(221)
plt.plot(Y_train_r[:, 0], Y_train_r[:, 1], "*b", label="train")
plt.plot(Y_test_r[:, 0], Y_test_r[:, 1], "*r", label="test")
plt.xlabel("Y comp. 1")
plt.ylabel("Y comp. 2")
plt.title('Y comp. 1 vs Y comp. 2, (test corr = %.2f)'
                 % np.corrcoef(Y_test_r[:, 0], Y_test_r[:, 1])[0, 1])
plt.legend(loc="best")
plt.xticks(())
plt.yticks(())
plt.show()
```

PLS regression, with multivariate response, a.k.a. PLS2

```python
n = 1000
q = 3
p = 10
X = np.random.normal(size=n * p).reshape((n, p))
B = np.array([[1, 2] + [0] * (p - 2)] * q).T
# each Yj = 1*X1 + 2*X2 + noize
Y = np.dot(X, B) + np.random.normal(size=n * q).reshape((n, q)) + 5
pls2 = PLSRegression(n_components=3)
pls2.fit(X, Y)
print("True B (such that: Y = XB + Err)")
print(B)
# compare pls2.coef_ with B
print("Estimated B")
print(np.round(pls2.coef_, 1))
```
pls2.predict(X)

Out:

```
True  B (such  that:  Y = XB + Err)
[[1 1 1]
[2 2 2]
[0 0 0]
[0 0 0]
[0 0 0]
[0 0 0]
[0 0 0]
[0 0 0]
[0 0 0]]
```

Estimated B
[[ 1.  1.  1. ]
[ 2.  2.  2. ]
[-0.  0. -0. ]
[-0. -0. -0. ]
[-0. -0. -0. ]
[ 0. -0.1 -0. ]
[-0. -0.1 0. ]
[-0.  0. -0. ]
[ 0.  0.  0. ]
[ 0.  0.  0. ]]

PLS regression, with univariate response, a.k.a. PLS1

```
n = 1000
p = 10
X = np.random.normal(size=n * p).reshape((n, p))
y = X[:, 0] + 2 * X[:, 1] + np.random.normal(size=n * 1) + 5
pls1 = PLSRegression(n_components=3)
pls1.fit(X, y)
print("Estimated betas")
print(np.round(pls1.coef_, 1))
```

Out:

```
Estimated betas
[[ 1. ]
[ 2. ]
[-0. ]
[ 0. ]
[-0. ]
[ 0. ]
[-0.1]
[-0.1]
[-0. ]
[ 0. ]]
```

CCA (PLS mode B with symmetric deflation)

```
cca = CCA(n_components=2)
cca.fit(X_train, Y_train)
```
X_train_r, Y_train_r = plsca.transform(X_train, Y_train)
X_test_r, Y_test_r = plsca.transform(X_test, Y_test)

Total running time of the script: (0 minutes 0.322 seconds)
Download Python source code: plot_compare_cross_decomposition.py
Download IPython notebook: plot_compare_cross_decomposition.ipynb
Examples concerning the \texttt{sklearn.datasets} module.

### 12.1 The Digit Dataset

This dataset is made up of 1797 8x8 images. Each image, like the one shown below, is of a hand-written digit. In order to utilize an 8x8 figure like this, we’d have to first transform it into a feature vector with length 64.

See here for more information about this dataset.

```python
print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause

from sklearn import datasets

import matplotlib.pyplot as plt

# Load the digits dataset
digits = datasets.load_digits()
```

![Digit Dataset Example](image-url)
# Display the first digit
plt.figure(1, figsize=(3, 3))
plt.imshow(digits.images[-1], cmap=plt.cm.gray_r, interpolation='nearest')
plt.show()

Total running time of the script: (0 minutes 0.158 seconds)

Download Python source code: plot_digits_last_image.py
Download IPython notebook: plot_digits_last_image.ipynb

12.2 The Iris Dataset

This data sets consists of 3 different types of irises' (Setosa, Versicolour, and Virginica) petal and sepal length, stored in a 150x4 numpy.ndarray

The rows being the samples and the columns being: Sepal Length, Sepal Width, Petal Length and Petal Width. The below plot uses the first two features. See here for more information on this dataset.

print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from sklearn import datasets
from sklearn.decomposition import PCA

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
Y = iris.target

x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5

plt.figure(2, figsize=(8, 6))
plt.clf()
# Plot the training points
plt.scatter(X[:, 0], X[:, 1], c=Y, cmap=plt.cm.Paired)
plt.xlabel('Sepal length')
plt.ylabel('Sepal width')
plt.xlim(x_min, x_max)
plt.ylim(y_min, y_max)
plt.xticks(())
plt.yticks(())

# To get a better understanding of interaction of the dimensions
# plot the first three PCA dimensions
fig = plt.figure(1, figsize=(8, 6))
ax = Axes3D(fig, elev=-150, azim=110)
X_reduced = PCA(n_components=3).fit_transform(iris.data)
ax.scatter(X_reduced[:, 0], X_reduced[:, 1], X_reduced[:, 2], c=Y,
          cmap=plt.cm.Paired)
ax.set_title("First three PCA directions")
ax.set_xlabel("1st eigenvector")
ax.w_xaxis.set_ticklabels([])
ax.set_ylabel("2nd eigenvector")
ax.w_yaxis.set_ticklabels([])
ax.set_zlabel("3rd eigenvector")
ax.w_zaxis.set_ticklabels([])
plt.show()
For `make_classification`, three binary and two multi-class classification datasets are generated, with different numbers of informative features and clusters per class.

```python
print(__doc__)

import matplotlib.pyplot as plt
from sklearn.datasets import make_classification
from sklearn.datasets import make_blobs
from sklearn.datasets import make_gaussian_quantiles

plt.figure(figsize=(8, 8))
plt.subplots_adjust(bottom=.05, top=.9, left=.05, right=.95)
plt.subplot(321)
```

```python
plt.figure(figsize=(8, 8))
plt.subplots_adjust(bottom=.05, top=.9, left=.05, right=.95)
plt.subplot(321)
```
12.4 Plot randomly generated multilabel dataset

This illustrates the `datasets.make_multilabel_classification` dataset generator. Each sample consists of counts of two features (up to 50 in total), which are differently distributed in each of two classes.

Points are labeled as follows, where Y means the class is present:
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Red</td>
</tr>
<tr>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Blue</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Yellow</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Purple</td>
</tr>
<tr>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Orange</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Green</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Brown</td>
</tr>
</tbody>
</table>

A star marks the expected sample for each class; its size reflects the probability of selecting that class label.

The left and right examples highlight the `n_labels` parameter: more of the samples in the right plot have 2 or 3 labels.

Note that this two-dimensional example is very degenerate: generally the number of features would be much greater than the “document length”, while here we have much larger documents than vocabulary. Similarly, with `n_classes > n_features`, it is much less likely that a feature distinguishes a particular class.

```python
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt

from sklearn.datasets import make_multilabel_classification as make_ml_clf
```

Out:

```
The data was generated from (random_state=54):
Class   P(C)   P(w0|C)   P(w1|C)
red     0.43   0.39     0.61
blue    0.38   0.01     0.99
yellow  0.19   0.59     0.41
```
COLORS = np.array(['!', '#FF3333', '#0198E1', '#BF5FFF', '#FCD116', '#FF7216', '#4DBD33', '#87421F'])
# Use same random seed for multiple calls to make_multilabel_classification to
# ensure same distributions
RANDOM_SEED = np.random.randint(2 ** 10)

def plot_2d(ax, n_labels=1, n_classes=3, length=50):
    X, Y, p_c, p_w_c = make_ml_clf(n_samples=150, n_features=2,
        n_classes=n_classes, n_labels=n_labels,
        length=length, allow_unlabeled=False,
        return_distributions=True,
        random_state=RANDOM_SEED)

    ax.scatter(X[:, 0], X[:, 1], color=COLORS.take((Y * [1, 2, 4]).sum(axis=1)),
               marker='.',
    ax.scatter(p_w_c[0] * length, p_w_c[1] * length,
               marker='*', linewidth=.5, edgecolor='black',
               s=20 + 1500 * p_c ** 2,
               color=COLORS.take([1, 2, 4]))
    ax.set_xlabel('Feature 0 count')
    return p_c, p_w_c

_ = (ax1, ax2) = plt.subplots(1, 2, sharex='row', sharey='row', figsize=(8, 4))
plt.subplots_adjust(bottom=.15)

p_c, p_w_c = plot_2d(ax1, n_labels=1)
ax1.set_title('n_labels=1, length=50')
ax1.set_ylabel('Feature 1 count')
plot_2d(ax2, n_labels=3)
ax2.set_title('n_labels=3, length=50')
ax2.set_xlim(left=0, auto=True)
ax2.set_ylim(bottom=0, auto=True)
plt.show()

print('The data was generated from (random_state=%d):' % RANDOM_SEED)
print('Class', 'P(C)', 'P(w0|C)', 'P(w1|C)', sep='	')
for k, p, p_w in zip(['red', 'blue', 'yellow'], p_c, p_w_c.T):
    print('%s\t%0.2f\t%0.2f\t%0.2f' % (k, p, p_w[0], p_w[1]))

Total running time of the script: (0 minutes 0.157 seconds)
Download Python source code: plot_random_multilabel_dataset.py
Download IPython notebook: plot_random_multilabel_dataset.ipynb

12.4. Plot randomly generated multilabel dataset 767
Examples concerning the `sklearn.decomposition` module.

### 13.1 PCA example with Iris Data-set

Principal Component Analysis applied to the Iris dataset.
See [here](#) for more information on this dataset.

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from sklearn import decomposition
from sklearn import datasets

print(__doc__)

# Code source: Gaël Varoquaux
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

from sklearn import decomposition
from sklearn import datasets
```
```python
np.random.seed(5)

centers = [[1, 1], [-1, -1], [1, -1]]
iris = datasets.load_iris()
X = iris.data
y = iris.target

fig = plt.figure(1, figsize=(4, 3))
plt.clf()
ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=48, azim=134)

plt.cla()
pca = decomposition.PCA(n_components=3)
pca.fit(X)
X = pca.transform(X)

for name, label in [('Setosa', 0), ('Versicolour', 1), ('Virginica', 2)]:
    ax.text3D(X[y == label, 0].mean(),
              X[y == label, 1].mean() + 1.5,
              X[y == label, 2].mean(), name,
              horizontalalignment='center',
              bbox=dict(alpha=.5, edgecolor='w', facecolor='w'))

# Reorder the labels to have colors matching the cluster results
y = np.choose(y, [1, 2, 0]).astype(np.float)
ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=y, cmap=plt.cm.spectral)

ax.w_xaxis.set_ticklabels([])
ax.w_yaxis.set_ticklabels([])
ax.w_zaxis.set_ticklabels([])

plt.show()
```

**Total running time of the script:** (0 minutes 0.119 seconds)

**Download Python source code:** plot_pca_iris.py

**Download IPython notebook:** plot_pca_iris.ipynb

### 13.2 Incremental PCA

Incremental principal component analysis (IPCA) is typically used as a replacement for principal component analysis (PCA) when the dataset to be decomposed is too large to fit in memory. IPCA builds a low-rank approximation for the input data using an amount of memory which is independent of the number of input data samples. It is still dependent on the input data features, but changing the batch size allows for control of memory usage.

This example serves as a visual check that IPCA is able to find a similar projection of the data to PCA (to a sign flip), while only processing a few samples at a time. This can be considered a “toy example”, as IPCA is intended for large datasets which do not fit in main memory, requiring incremental approaches.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import load_iris
from sklearn.decomposition import PCA, IncrementalPCA

iris = load_iris()
X = iris.data
y = iris.target

n_components = 2
ipca = IncrementalPCA(n_components=n_components, batch_size=10)
X_ipca = ipca.fit_transform(X)
pca = PCA(n_components=n_components)
X_pca = pca.fit_transform(X)

colors = ['navy', 'turquoise', 'darkorange']

for X_transformed, title in [(X_ipca, "Incremental PCA"), (X_pca, "PCA")]:
    plt.figure(figsize=(8, 8))
    for color, i, target_name in zip(colors, [0, 1, 2], iris.target_names):
        plt.scatter(X_transformed[y == i, 0], X_transformed[y == i, 1],
                    color=color, lw=2, label=target_name)

    if "Incremental" in title:
        err = np.abs(np.abs(X_pca) - np.abs(X_ipca)).mean()
        plt.title(title + " of iris dataset
        Mean absolute unsigned error "
                   "%.6f" % err)
    else:
        plt.title(title + " of iris dataset")
    plt.legend(loc="best", shadow=False, scatterpoints=1)
    plt.axis([-4, 4, -1.5, 1.5])

plt.show()

Total running time of the script: (0 minutes 0.184 seconds)
Download Python source code: plot_incremental_pca.py
Download IPython notebook: plot_incremental_pca.ipynb

13.3 Comparison of LDA and PCA 2D projection of Iris dataset

The Iris dataset represents 3 kind of Iris flowers (Setosa, Versicolour and Virginica) with 4 attributes: sepal length, sepal width, petal length and petal width.

Principal Component Analysis (PCA) applied to this data identifies the combination of attributes (principal components, or directions in the feature space) that account for the most variance in the data. Here we plot the different samples on the 2 first principal components.

Linear Discriminant Analysis (LDA) tries to identify attributes that account for the most variance between classes. In particular, LDA, in contrast to PCA, is a supervised method, using known class labels.
import matplotlib.pyplot as plt

from sklearn import datasets
from sklearn.decomposition import PCA
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis

iris = datasets.load_iris()
X = iris.data
y = iris.target

target_names = iris.target_names

pca = PCA(n_components=2)
X_r = pca.fit(X).transform(X)

lda = LinearDiscriminantAnalysis(n_components=2)
X_r2 = lda.fit(X, y).transform(X)

# Percentage of variance explained for each components
print('explained variance ratio (first two components): %s' % str(pca.explained_variance_ratio_))

plt.figure()

for color, i, target_name in zip(colors, [0, 1, 2], target_names):
    plt.scatter(X_r[y == i, 0], X_r[y == i, 1], color=color, alpha=.8, lw=lw,
                label=target_name)
plt.legend(loc='best', shadow=False, scatterpoints=1)
plt.title('PCA of IRIS dataset')
13.4 Blind source separation using FastICA

An example of estimating sources from noisy data.

Independent component analysis (ICA) is used to estimate sources given noisy measurements. Imagine 3 instruments playing simultaneously and 3 microphones recording the mixed signals. ICA is used to recover the sources i.e. what is played by each instrument. Importantly, PCA fails at recovering our instruments since the related signals reflect non-Gaussian processes.

```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from scipy import signal
from sklearn.decomposition import FastICA, PCA

Generate sample data

np.random.seed(0)
n_samples = 2000
time = np.linspace(0, 8, n_samples)
s1 = np.sin(2 * time)  # Signal 1 : sinusoidal signal
s2 = np.sign(np.sin(3 * time))  # Signal 2 : square signal
s3 = signal.sawtooth(2 * np.pi * time)  # Signal 3: saw tooth signal
S = np.c_[s1, s2, s3]
S += 0.2 * np.random.normal(size=S.shape)  # Add noise
S /= S.std(axis=0)  # Standardize data

# Mix data
A = np.array([[1, 1, 1], [0.5, 2, 1.0], [1.5, 1.0, 2.0]])  # Mixing matrix
X = np.dot(S, A.T)  # Generate observations

# Compute ICA
ica = FastICA(n_components=3)
S_ = ica.fit_transform(X)  # Reconstruct signals
A_ = ica.mixing_  # Get estimated mixing matrix

# We can 'prove' that the ICA model applies by reverting the unmixing.
assert np.allclose(X, np.dot(S_, A_.T) + ica.mean_)
```
For comparison, compute PCA:
```
pca = PCA(n_components=3)
H = pca.fit_transform(X)  # Reconstruct signals based on orthogonal components
```

Plot results:
```
plt.figure()
models = [X, S, S_, H]
names = ['Observations (mixed signal)',
         'True Sources',
         'ICA recovered signals',
         'PCA recovered signals']
colors = ['red', 'steelblue', 'orange']

for ii, (model, name) in enumerate(zip(models, names), 1):
    plt.subplot(4, 1, ii)
    plt.title(name)
    for sig, color in zip(model.T, colors):
        plt.plot(sig, color=color)

plt.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.46)
plt.show()
```
13.5 FastICA on 2D point clouds

This example illustrates visually in the feature space a comparison by results using two different component analysis techniques. 

*Independent component analysis (ICA)* vs *Principal component analysis (PCA)*.

Representing ICA in the feature space gives the view of ‘geometric ICA’: ICA is an algorithm that finds directions in the feature space corresponding to projections with high non-Gaussianity. These directions need not be orthogonal in the original feature space, but they are orthogonal in the whitened feature space, in which all directions correspond to the same variance.

PCA, on the other hand, finds orthogonal directions in the raw feature space that correspond to directions accounting for maximum variance.

Here we simulate independent sources using a highly non-Gaussian process, 2 student T with a low number of degrees of freedom (top left figure). We mix them to create observations (top right figure). In this raw observation space, directions identified by PCA are represented by orange vectors. We represent the signal in the PCA space, after whitening by the variance corresponding to the PCA vectors (lower left). Running ICA corresponds to finding a rotation in this space to identify the directions of largest non-Gaussianity (lower right).

```python
print(__doc__)

# Authors: Alexandre Gramfort, Gael Varoquaux
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA, FastICA

Generate sample data

rng = np.random.RandomState(42)
S = rng.standard_t(1.5, size=(20000, 2))
S[:, 0] *= 2.

# Mix data
A = np.array([[1, 1], [0, 2]])  # Mixing matrix
X = np.dot(S, A.T)  # Generate observations

pca = PCA()
S_pca_ = pca.fit(X).transform(X)

ica = FastICA(random_state=rng)
S_ica_ = ica.fit(X).transform(X)  # Estimate the sources
S_ica_ /= S_ica_.std(axis=0)

Plot results
```
def plot_samples(S, axis_list=None):
    plt.scatter(S[:, 0], S[:, 1], s=2, marker='o', zorder=10,
                color='steelblue', alpha=0.5)
    if axis_list is not None:
        colors = ['orange', 'red']
        for color, axis in zip(colors, axis_list):
            axis /= axis.std()
            x_axis, y_axis = axis
            # Trick to get legend to work
            plt.plot(0.1 * x_axis, 0.1 * y_axis, linewidth=2, color=color)
            plt.quiver(0, 0, x_axis, y_axis, zorder=11, width=0.01, scale=6,
                       color=color)
    plt.hlines(0, -3, 3)
    plt.vlines(0, -3, 3)
    plt.xlim(-3, 3)
    plt.ylim(-3, 3)
    plt.xlabel('x')
    plt.ylabel('y')

plt.figure()
plt.subplot(2, 2, 1)
plot_samples(S / S.std())
plt.title('True Independent Sources')

axis_list = [pca.components_.T, ica.mixing_]
plt.subplot(2, 2, 2)
plot_samples(X / np.std(X), axis_list=axis_list)
legend = plt.legend(['PCA', 'ICA'], loc='upper right')
legend.set_zorder(100)
plt.title('Observations')

plt.subplot(2, 2, 3)
plot_samples(S_pca_ / np.std(S_pca_, axis=0))
plt.title('PCA recovered signals')

plt.subplot(2, 2, 4)
plot_samples(S_ica_ / np.std(S_ica_))
plt.title('ICA recovered signals')

plt.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.36)
plt.show()
13.6 Kernel PCA

This example shows that Kernel PCA is able to find a projection of the data that makes data linearly separable.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA, KernelPCA
from sklearn.datasets import make_circles

np.random.seed(0)
X, y = make_circles(n_samples=400, factor=.3, noise=.05)
kpca = KernelPCA(kernel="rbf", fit_inverse_transform=True, gamma=10)
X_kpca = kpca.fit_transform(X)
X_back = kpca.inverse_transform(X_kpca)
pca = PCA()
X_pca = pca.fit_transform(X)

# Plot results
plt.figure()
plt.subplot(2, 2, 1, aspect='equal')
plt.title("Original space")
reds = y == 0
blues = y == 1

plt.plot(X[reds, 0], X[reds, 1], "ro")
plt.plot(X[blues, 0], X[blues, 1], "bo")
plt.xlabel("$x_1$")
plt.ylabel("$x_2$")

X1, X2 = np.meshgrid(np.linspace(-1.5, 1.5, 50), np.linspace(-1.5, 1.5, 50))
X_grid = np.array([np.ravel(X1), np.ravel(X2)]).T
# projection on the first principal component (in the phi space)
Z_grid = kpca.transform(X_grid)[:, 0].reshape(X1.shape)
plt.contour(X1, X2, Z_grid, colors='grey', linewidths=1, origin='lower')

plt.subplot(2, 2, 2, aspect='equal')
plt.plot(X_pca[reds, 0], X_pca[reds, 1], "ro")
plt.plot(X_pca[blues, 0], X_pca[blues, 1], "bo")
plt.title("Projection by PCA")
plt.xlabel("1st principal component")
plt.ylabel("2nd component")

plt.subplot(2, 2, 3, aspect='equal')
plt.plot(X_kpca[reds, 0], X_kpca[reds, 1], "ro")
plt.plot(X_kpca[blues, 0], X_kpca[blues, 1], "bo")
plt.title("Projection by KPCA")
plt.xlabel("1st principal component in space induced by $\phi$")
plt.ylabel("2nd component")

plt.subplot(2, 2, 4, aspect='equal')
plt.plot(X_back[reds, 0], X_back[reds, 1], "ro")
plt.plot(X_back[blues, 0], X_back[blues, 1], "bo")
plt.title("Original space after inverse transform")
plt.xlabel("$x_1$")
plt.ylabel("$x_2$")

plt.subplots_adjust(0.02, 0.10, 0.98, 0.94, 0.04, 0.35)
plt.show()

Total running time of the script: (0 minutes 0.499 seconds)
Download Python source code: plot_kernel_pca.py
Download IPython notebook: plot_kernel_pca.ipynb

13.7 Principal components analysis (PCA)

These figures aid in illustrating how a point cloud can be very flat in one direction—which is where PCA comes in to choose a direction that is not flat.

print(__doc__)

# Authors: Gael Varoquaux
# Jaques Grobler
# Kevin Hughes
Create the data

e = np.exp(1)
np.random.seed(4)

def pdf(x):
    return 0.5 * (stats.norm(scale=0.25 / e).pdf(x) + stats.norm(scale=4 / e).pdf(x))

y = np.random.normal(scale=0.5, size=(30000))
x = np.random.normal(scale=0.5, size=(30000))
z = np.random.normal(scale=0.1, size=len(x))

density = pdf(x) * pdf(y)
pdf_z = pdf(5 * z)
density *= pdf_z

a = x + y
b = 2 * y
c = a - b + z

norm = np.sqrt(a.var() + b.var())
a /= norm
b /= norm

Plot the figures

def plot_figs(fig_num, elev, azim):
    fig = plt.figure(fig_num, figsize=(4, 3))
    plt.clf()
    ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=elev, azim=azim)

    ax.scatter(a[::10], b[::10], c[::10], c=density[::10], marker='+', alpha=.4)

    Y = np.c_[a, b, c]
    # Using SciPy’s SVD, this would be:
    # _, pca_score, V = scipy.linalg.svd(Y, full_matrices=False)

    pca = PCA(n_components=3)
pca.fit(Y)
pca_score = pca.explained_variance_ratio_
    V = pca.components_

    x_pca_axis, y_pca_axis, z_pca_axis = V.T * pca_score / pca_score.min()

    x_pca_axis, y_pca_axis, z_pca_axis = 3 * V.T
    x_pca_plane = np.r_[x_pca_axis[::2], - x_pca_axis[1::1]]
Probabilistic PCA and Factor Analysis are probabilistic models. The consequence is that the likelihood of new data can be used for model selection and covariance estimation. Here we compare PCA and FA with cross-validation on low rank data corrupted with homoscedastic noise (noise variance is the same for each feature) or heteroscedastic noise (noise variance is the different for each feature). In a second step we compare the model likelihood to the likelihoods obtained from shrinkage covariance estimators.

One can observe that with homoscedastic noise both FA and PCA succeed in recover the size of the low rank subspace. The likelihood with PCA is higher than FA in this case. However PCA fails and overestimates the rank
when heteroscedastic noise is present. Under appropriate circumstances the low rank models are more likely than shrinkage models.

The automatic estimation from Automatic Choice of Dimensionality for PCA. NIPS 2000: 598-604 by Thomas P. Minka is also compared.

```python
# Authors: Alexandre Gramfort
# Denis A. Engemann
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from scipy import linalg
from sklearn.decomposition import PCA, FactorAnalysis
from sklearn.covariance import ShrunkCovariance, LedoitWolf
from sklearn.model_selection import cross_val_score
from sklearn.model_selection import GridSearchCV

print(__doc__)

Create the data

n_samples, n_features, rank = 1000, 50, 10
sigma = 1.
rng = np.random.RandomState(42)
U, _, _ = linalg.svd(rng.randn(n_features, n_features))
X = np.dot(rng.randn(n_samples, rank), U[:, :rank].T)

# Adding homoscedastic noise
X_homo = X + sigma * rng.randn(n_samples, n_features)

# Adding heteroscedastic noise
sigmas = sigma * rng.rand(n_features) + sigma / 2.
X_hetero = X + rng.randn(n_samples, n_features) * sigmas

Fit the models

n_components = np.arange(0, n_features, 5)  # options for n_components

def compute_scores(X):
    pca = PCA(svd_solver='full')
    fa = FactorAnalysis()

    pca_scores, fa_scores = [], []
    for n in n_components:
        pca.n_components = n
        fa.n_components = n
        pca_scores.append(np.mean(cross_val_score(pca, X)))
        fa_scores.append(np.mean(cross_val_score(fa, X)))

    return pca_scores, fa_scores

def shrunk_cov_score(X):
    shrinkages = np.logspace(-2, 0, 30)
    cv = GridSearchCV(ShrunkCovariance(), {'shrinkage': shrinkages})
    return np.mean(cross_val_score(cv.fit(X).best_estimator_, X))
```

13.8. Model selection with Probabilistic PCA and Factor Analysis (FA)
def lw_score(X):
    return np.mean(cross_val_score(LedoitWolf(), X))

for X, title in [(X_homo, 'Homoscedastic Noise'),
                 (X_hetero, 'Heteroscedastic Noise')]:
    pca_scores, fa_scores = compute_scores(X)
    n_components_pca = n_components[np.argmax(pca_scores)]
    n_components_fa = n_components[np.argmax(fa_scores)]

    pca = PCA(svd_solver='full', n_components='mle')
    pca.fit(X)
    n_components_pca_mle = pca.n_components_

    print("best n_components by PCA CV = %d" % n_components_pca)
    print("best n_components by FactorAnalysis CV = %d" % n_components_fa)
    print("best n_components by PCA MLE = %d" % n_components_pca_mle)

    plt.figure()
    plt.plot(n_components, pca_scores, 'b', label='PCA scores')
    plt.plot(n_components, fa_scores, 'r', label='FA scores')
    plt.axvline(rank, color='g', label='TRUTH: %d' % rank, linestyle='--')
    plt.axvline(n_components_pca, color='b',
                label='PCA CV: %d' % n_components_pca, linestyle='--')
    plt.axvline(n_components_fa, color='r',
                label='FactorAnalysis CV: %d' % n_components_fa,
                linestyle='--')
    plt.axvline(n_components_pca_mle, color='k',
                label='PCA MLE: %d' % n_components_pca_mle, linestyle='--')

    # compare with other covariance estimators
    plt.axhline(shrunk_cov_score(X), color='violet',
                label='Shrunk Covariance MLE', linestyle='-.')
    plt.axhline(lw_score(X), color='orange',
                label='LedoitWolf MLE' % n_components_pca_mle, linestyle='-.')

    plt.xlabel('nb of components')
    plt.ylabel('CV scores')
    plt.legend(loc='lower right')
    plt.title(title)

    plt.show()
Out:

- best n_components by PCA CV = 10
- best n_components by FactorAnalysis CV = 10
- best n_components by PCA MLE = 10
- best n_components by PCA CV = 40
- best n_components by FactorAnalysis CV = 10
- best n_components by PCA MLE = 38

Total running time of the script: (0 minutes 12.618 seconds)

Download Python source code: plot_pca_vs_fa_model_selection.py
Download IPython notebook: plot_pca_vs_fa_model_selection.ipynb

13.9 Sparse coding with a precomputed dictionary

Transform a signal as a sparse combination of Ricker wavelets. This example visually compares different sparse coding methods using the `sklearn.decomposition.SparseCoder` estimator. The Ricker (also known as Mexican hat or the second derivative of a Gaussian) is not a particularly good kernel to represent piecewise constant signals like this one. It can therefore be seen how much adding different widths of atoms matters and it therefore motivates learning the dictionary to best fit your type of signals.

The richer dictionary on the right is not larger in size, heavier subsampling is performed in order to stay on the same order of magnitude.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import SparseCoder

def ricker_function(resolution, center, width):
    """Discrete sub-sampled Ricker (Mexican hat) wavelet"""
    x = np.linspace(0, resolution - 1, resolution)
    x = ((2 / ((np.sqrt(3 * width) * np.pi ** 1 / 4))
         * (1 - ((x - center) ** 2 / width ** 2))
         * np.exp((-((x - center) ** 2) / (2 * width ** 2))))
    return x

def ricker_matrix(width, resolution, n_components):
    """Dictionary of Ricker (Mexican hat) wavelets"""
    centers = np.linspace(0, resolution - 1, n_components)
    D = np.empty((n_components, resolution))
    for i, center in enumerate(centers):
        D[i] = ricker_function(resolution, center, width)
    D /= np.sqrt(np.sum(D ** 2, axis=1))[:, np.newaxis]
    return D

resolution = 1024
subsampling = 3  # subsampling factor
width = 100
n_components = resolution // subsampling

# Compute a wavelet dictionary
D_fixed = ricker_matrix(width=width, resolution=resolution,
                        n_components=n_components)
D_multi = np.r_[tuple(ricker_matrix(width=w, resolution=resolution,
                                    n_components=n_components // 5)
                      for w in (10, 50, 100, 500, 1000))]
# Generate a signal
y = np.linspace(0, resolution - 1, resolution)
first_quarter = y < resolution / 4
y[first_quarter] = 3.
y[np.logical_not(first_quarter)] = -1.

# List the different sparse coding methods in the following format:
# (title, transform_algorithm, transform_alpha, transform_n_nozero_coefs)
estimators = [('OMP', 'omp', None, 15, 'navy'),
              ('Lasso', 'lasso_cd', 2, None, 'turquoise'),
              ]
lw = 2

plt.figure(figsize=(13, 6))
for subplot, (D, title) in enumerate(zip((D_fixed, D_multi),
                                        ('fixed width', 'multiple widths'))):
    plt.subplot(1, 2, subplot + 1)
    plt.title('Sparse coding against %s dictionary' % title)
    plt.plot(y, lw=lw, linestyle='--', label='Original signal')
    # Do a wavelet approximation
    for title, algo, alpha, n_nonzero, color in estimators:
        coder = SparseCoder(dictionary=D, transform_n_nonzero_coefs=n_nonzero,
                              transform_algorithm=algo)
        x = coder.transform(y.reshape(1, -1))
        density = len(np.flatnonzero(x))
        x = np.ravel(np.dot(x, D))
        squared_error = np.sum((y - x) ** 2)
        plt.plot(x, color=color, lw=lw,
                  label='\(\text{%s}: %d \text{ non-zero coefs, %.2f error}\)'
                  % (title, density, squared_error))

    # Soft thresholding debiasing
    coder = SparseCoder(dictionary=D, transform_algorithm='threshold',
                         transform_alpha=20)
    x = coder.transform(y.reshape(1, -1))
    _, idx = np.where(x != 0)
    x[0, idx], _, _, _ = np.linalg.lstsq(D[idx, :].T, y)
    x = np.ravel(np.dot(x, D))
    squared_error = np.sum((y - x) ** 2)
    plt.plot(x, color='darkorange', lw=lw,
              label='Thresholding w/ debiasing: %d non-zero coefs, %.2f error'
              % (len(idx), squared_error))

plt.axis('tight')
plt.legend(shadow=False, loc='best')
plt.subplots_adjust(.04, .07, .97, .90, .09, .2)
plt.show()

Total running time of the script: (0 minutes 0.366 seconds)

Download Python source code: plot_sparse_coding.py
Download IPython notebook: plot_sparse_coding.ipynb

### 13.10 Faces dataset decompositions

This example applies to *The Olivetti faces dataset* different unsupervised matrix decomposition (dimension reduction) methods from the module *sklearn.decomposition* (see the documentation chapter *Decomposing signals in...*).
components (matrix factorization problems)).

print(__doc__)

# Authors: Vlad Niculae, Alexandre Gramfort
# License: BSD 3 clause

import logging
from time import time
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_olivetti_faces
from sklearn.cluster import MiniBatchKMeans
from sklearn import decomposition

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO,
                    format='%(asctime)s %(levelname)s %(message)s')

n_row, n_col = 2, 3
n_components = n_row * n_col
image_shape = (64, 64)
rng = RandomState(0)

Load faces data

dataset = fetch_olivetti_faces(shuffle=True, random_state=rng)
faces = dataset.data

n_samples, n_features = faces.shape

# global centering
faces_centered = faces - faces.mean(axis=0)

# local centering
faces_centered -= faces_centered.mean(axis=1).reshape(n_samples, -1)

print("Dataset consists of \%d faces" % n_samples)

Out:

Dataset consists of 400 faces

def plot_gallery(title, images, n_col=n_col, n_row=n_row):
    plt.figure(figsize=(2.*n_col, 2.26 * n_row))
    plt.suptitle(title, size=16)
    for i, comp in enumerate(images):
        plt.subplot(n_row, n_col, i + 1)
        vmax = max(comp.max(), -comp.min())
        plt.imshow(comp.reshape(image_shape), cmap=plt.cm.gray,
                   interpolation='nearest',
                   vmin=-vmax, vmax=vmax)
        plt.xticks(()
        plt.yticks(()
    plt.subplots_adjust(0.01, 0.05, 0.99, 0.93, 0.04, 0.)
List of the different estimators, whether to center and transpose the problem, and whether the transformer uses the clustering API.

```python
estimators = [
    ('Eigenfaces - PCA using randomized SVD',
     decomposition.PCA(n_components=n_components, svd_solver='randomized',
                     whiten=True),
     True),

    ('Non-negative components - NMF',
     decomposition.NMF(n_components=n_components, init='nndsvda', tol=5e-3),
     False),

    ('Independent components - FastICA',
     decomposition.FastICA(n_components=n_components, whiten=True),
     True),

    ('Sparse comp. - MiniBatchSparsePCA',
     decomposition.MiniBatchSparsePCA(n_components=n_components, alpha=0.8,
                                        n_iter=100, batch_size=3,
                                        random_state=rng),
     True),

    ('MiniBatchDictionaryLearning',
     decomposition.MiniBatchDictionaryLearning(n_components=15, alpha=0.1,
                                               n_iter=50, batch_size=3,
                                               random_state=rng),
     True),

    ('Cluster centers - MiniBatchKMeans',
     MiniBatchKMeans(n_clusters=n_components, tol=1e-3, batch_size=20,
                     max_iter=50, random_state=rng),
     True),

    ('Factor Analysis components - FA',
     decomposition.FactorAnalysis(n_components=n_components, max_iter=2),
     True),
]
```

Plot a sample of the input data

```python
plot_gallery("First centered Olivetti faces", faces_centered[:n_components])
```
First centered Olivetti faces

Do the estimation and plot it

```python
for name, estimator, center in estimators:
    print("Extracting the top $\%d$ %s...") % (n_components, name)
    t0 = time()
    data = faces
    if center:
        data = faces_centered
    estimator.fit(data)
    train_time = (time() - t0)
    print("done in $%.3fs" % train_time)
    if hasattr(estimator, 'cluster_centers_'):
        components_ = estimator.cluster_centers_
    else:
        components_ = estimator.components_
        if (hasattr(estimator, 'noise_variance_') and
            estimator.noise_variance_.shape != ()):  
            plot_gallery("Pixelwise variance",
                         estimator.noise_variance_.reshape(1, -1), n_col=1, n_row=1)
        plot_gallery('$s - Train time $%.1f$' % (name, train_time),
                     components_[:n_components])

plt.show()
```
13.10. Faces dataset decompositions

genfaces - PCA using randomized SVD - Train time 0.1s

• Non-negative components - NMF - Train time 0.7s

• Independent components - FastICA - Train time 0.3s

• Sparse comp. - MiniBatchSparsePCA - Train time 1.1s
Extracting the top 6 Eigenfaces - PCA using randomized SVD... done in 0.094s
Extracting the top 6 Non-negative components - NMF... done in 0.660s
Extracting the top 6 Independent components - FastICA...
13.11 Image denoising using dictionary learning

An example comparing the effect of reconstructing noisy fragments of a raccoon face image using firstly online Dictionary Learning and various transform methods.

The dictionary is fitted on the distorted left half of the image, and subsequently used to reconstruct the right half. Note that even better performance could be achieved by fitting to an undistorted (i.e. noiseless) image, but here we start from the assumption that it is not available.

A common practice for evaluating the results of image denoising is by looking at the difference between the reconstruction and the original image. If the reconstruction is perfect this will look like Gaussian noise.

It can be seen from the plots that the results of Orthogonal Matching Pursuit (OMP) with two non-zero coefficients is a bit less biased than when keeping only one (the edges look less prominent). It is in addition closer from the ground truth in Frobenius norm.

The result of Least Angle Regression is much more strongly biased: the difference is reminiscent of the local intensity value of the original image.

Thresholding is clearly not useful for denoising, but it is here to show that it can produce a suggestive output with very high speed, and thus be useful for other tasks such as object classification, where performance is not necessarily related to visualisation.

```python
print(__doc__)

from time import time
import matplotlib.pyplot as plt
import numpy as np
import scipy as sp

from sklearn.decomposition import MiniBatchDictionaryLearning
from sklearn.feature_extraction.image import extract_patches_2d
from sklearn.feature_extraction.image import reconstruct_from_patches_2d
from sklearn.utils.testing import SkipTest
from sklearn.utils.fixes import sp_version

if sp_version < (0, 12):
    raise SkipTest("Skipping because SciPy version earlier than 0.12.0 and "
    "thus does not include the scipy.misc.face() image.")
```

13.11. Image denoising using dictionary learning 793
try:
    from scipy import misc
    face = misc.face(gray=True)
except AttributeError:
    # Old versions of scipy have face in the top level package
    face = sp.face(gray=True)

# Convert from uint8 representation with values between 0 and 255 to a floating point representation with values between 0 and 1.
face = face / 255

# downsample for higher speed
face = face[::2, ::2] + face[1::2, ::2] + face[::2, 1::2] + face[1::2, 1::2]
face /= 4.0
height, width = face.shape

# Distort the right half of the image
print('Distorting image...')
distorted = face.copy()
distorted[:, width // 2:] += 0.075 * np.random.randn(height, width // 2)

# Extract all reference patches from the left half of the image
print('Extracting reference patches...')
t0 = time()
patch_size = (7, 7)
data = extract_patches_2d(distorted[:, :width // 2], patch_size)
data = data.reshape(data.shape[0], -1)
data -= np.mean(data, axis=0)
data /= np.std(data, axis=0)
print('done in %.2f s.' % (time() - t0))

Out:

Distorting image...
Extracting reference patches...
done in 0.07s.

Learn the dictionary from reference patches

print('Learning the dictionary...')
t0 = time()
dico = MiniBatchDictionaryLearning(n_components=100, alpha=1, n_iter=500)
V = dico.fit(data).components_
dt = time() - t0
print('done in %.2fs.' % dt)

plt.figure(figsize=(4.2, 4))
for i, comp in enumerate(V[:100]):
    plt.subplot(10, 10, i + 1)
    plt.imshow(comp.reshape(patch_size), cmap=plt.cm.gray_r, interpolation='nearest')
    plt.xticks(())
    plt.yticks(())
plt.suptitle('Dictionary learned from face patches
Train time %.1f s on %d patches' % (dt, len(data)), fontsize=16)
plt.subplots_adjust(0.08, 0.02, 0.92, 0.85, 0.08, 0.23)
Dictionary learned from face patches
Train time 7.7s on 94500 patches

Out:

Learning the dictionary...
done in 7.73s.

Display the distorted image

def show_with_diff(image, reference, title):
    """Helper function to display denoising"""
    plt.figure(figsize=(5, 3.3))
    plt.subplot(1, 2, 1)
    plt.title('Image')
    plt.imshow(image, vmin=0, vmax=1, cmap=plt.cm.gray,
               interpolation='nearest')
    plt.xticks(())
    plt.yticks(())
    plt.subplot(1, 2, 2)
    difference = image - reference
    plt.title('Difference (norm: %.2f)' % np.sqrt(np.sum(difference ** 2)))
    plt.imshow(difference, vmin=-0.5, vmax=0.5, cmap=plt.cm.PuOr,
               interpolation='nearest')
    plt.xticks(())
    plt.yticks(())
    plt.suptitle(title, size=16)
    plt.subplots_adjust(0.02, 0.02, 0.98, 0.79, 0.02, 0.2)

show_with_diff(distorted, face, 'Distorted image')
Extract noisy patches and reconstruct them using the dictionary

```python
print('Extracting noisy patches...')
t0 = time()
data = extract_patches_2d(distorted[:, width // 2:], patch_size)
data = data.reshape(data.shape[0], -1)
intercept = np.mean(data, axis=0)
data -= intercept
print('done in %.2fs.' % (time() - t0))
transform_algorithms = [
    ('Orthogonal Matching Pursuit
     1 atom', 'omp',
     {'transform_n_nonzero_coefs': 1}),
    ('Orthogonal Matching Pursuit
     2 atoms', 'omp',
     {'transform_n_nonzero_coefs': 2}),
    ('Least-angle regression
     5 atoms', 'lars',
     {'transform_n_nonzero_coefs': 5}),
    ('Thresholding
     alpha=0.1', 'threshold',
     {'transform_alpha': .1})
]
reconstructions = {}
for title, transform_algorithm, kwargs in transform_algorithms:
    print(title + '...')
    reconstructions[title] = face.copy()
t0 = time()
dico.set_params(transform_algorithm=transform_algorithm, **kwargs)
    code = dico.transform(data)
    patches = np.dot(code, V)
    patches += intercept
    patches = patches.reshape(len(data), *patch_size)
    if transform_algorithm == 'threshold':
        patches -= patches.min()
        patches /= patches.max()
    reconstructions[title][..., width // 2:] = reconstruct_from_patches_2d(
        patches, (height, width // 2))
dt = time() - t0
```

Extract noisy patches and reconstruct them using the dictionary.
print('done in %.2fs.' % dt)
show_with_diff(reconstructions[title], face,
    title + ' (time: %.1fs)' % dt)

plt.show()

Orthogonal Matching Pursuit
1 atom (time: 9.8s)

<table>
<thead>
<tr>
<th>Image</th>
<th>Difference (norm: 15.80)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="" /></td>
<td><img src="image2" alt="" /></td>
</tr>
</tbody>
</table>

Orthogonal Matching Pursuit
2 atoms (time: 20.4s)

<table>
<thead>
<tr>
<th>Image</th>
<th>Difference (norm: 14.54)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="" /></td>
<td><img src="image4" alt="" /></td>
</tr>
</tbody>
</table>

Least-angle regression
5 atoms (time: 96.2s)

<table>
<thead>
<tr>
<th>Image</th>
<th>Difference (norm: 23.69)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="" /></td>
<td><img src="image6" alt="" /></td>
</tr>
</tbody>
</table>

Thresholding
alpha=0.1 (time: 1.2s)

<table>
<thead>
<tr>
<th>Image</th>
<th>Difference (norm: 22.39)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7" alt="" /></td>
<td><img src="image8" alt="" /></td>
</tr>
</tbody>
</table>

Out:

Extracting noisy patches...
done in 0.02s.
Orthogonal Matching Pursuit
1 atom...
done in 9.76s.
Orthogonal Matching Pursuit
2 atoms...
Least-angle regression
5 atoms...
done in 96.18s.
Thresholding
  alpha=0.1...
done in 1.17s.

**Total running time of the script:** (2 minutes 22.129 seconds)

**Download Python source code:** plot_image_denoising.py

**Download IPython notebook:** plot_image_denoising.ipynb
Examples concerning the `sklearn.ensemble` module.

14.1 Decision Tree Regression with AdaBoost

A decision tree is boosted using the AdaBoost.R2 [1] algorithm on a 1D sinusoidal dataset with a small amount of Gaussian noise. 299 boosts (300 decision trees) is compared with a single decision tree regressor. As the number of boosts is increased the regressor can fit more detail.
```python
print(__doc__)

# Author: Noel Dawe <noel.dawe@gmail.com>
#
# License: BSD 3 clause

# importing necessary libraries
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeRegressor
from sklearn.ensemble import AdaBoostRegressor

# Create the dataset
rng = np.random.RandomState(1)
X = np.linspace(0, 6, 100)[:, np.newaxis]
y = np.sin(X).ravel() + np.sin(6 * X).ravel() + rng.normal(0, 0.1, X.shape[0])

# Fit regression model
regr_1 = DecisionTreeRegressor(max_depth=4)
regr_2 = AdaBoostRegressor(DecisionTreeRegressor(max_depth=4),
                           n_estimators=300, random_state=rng)

regr_1.fit(X, y)
regr_2.fit(X, y)

# Predict
y_1 = regr_1.predict(X)
y_2 = regr_2.predict(X)

# Plot the results
plt.figure()
plt.scatter(X, y, c="k", label="training samples")
plt.plot(X, y_1, c="g", label="n_estimators=1", linewidth=2)
plt.plot(X, y_2, c="r", label="n_estimators=300", linewidth=2)
plt.xlabel("data")
plt.ylabel("target")
plt.title("Boosted Decision Tree Regression")
plt.legend()
plt.show()
```

Total running time of the script: (0 minutes 0.749 seconds)

Download Python source code: plot_adaboost_regression.py
Download IPython notebook: plot_adaboost_regression.ipynb

### 14.2 Pixel importances with a parallel forest of trees

This example shows the use of forests of trees to evaluate the importance of the pixels in an image classification task (faces). The hotter the pixel, the more important.

The code below also illustrates how the construction and the computation of the predictions can be parallelized within multiple jobs.
Out:

```
Fitting ExtraTreesClassifier on faces data with 1 cores...
done in 3.359s
```

```
print(__doc__)
from time import time
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_olivetti_faces
from sklearn.ensemble import ExtraTreesClassifier
```
# Number of cores to use to perform parallel fitting of the forest model
n_jobs = 1

# Load the faces dataset
data = fetch_olivetti_faces()
X = data.images.reshape((len(data.images), -1))
y = data.target

mask = y < 5  # Limit to 5 classes
X = X[mask]
y = y[mask]

# Build a forest and compute the pixel importances
print("Fitting ExtraTreesClassifier on faces data with %d cores..." % n_jobs)
t0 = time()
forest = ExtraTreesClassifier(n_estimators=1000,
                               max_features=128,
                               n_jobs=n_jobs,
                               random_state=0)
forest.fit(X, y)
print("done in %0.3fs" % (time() - t0))
importances = forest.feature_importances_
importances = importances.reshape(data.images[0].shape)

# Plot pixel importances
plt.matshow(importances, cmap=plt.cm.hot)
plt.title("Pixel importances with forests of trees")
plt.show()

Total running time of the script: (0 minutes 4.096 seconds)

Download Python source code: plot_forest_importances_faces.py
Download IPython notebook: plot_forest_importances_faces.ipynb

### 14.3 IsolationForest example

An example using IsolationForest for anomaly detection.

The IsolationForest ‘isolates’ observations by randomly selecting a feature and then randomly selecting a split value between the maximum and minimum values of the selected feature.

Since recursive partitioning can be represented by a tree structure, the number of splittings required to isolate a sample is equivalent to the path length from the root node to the terminating node.

This path length, averaged over a forest of such random trees, is a measure of normality and our decision function.

Random partitioning produces noticeable shorter paths for anomalies. Hence, when a forest of random trees collectively produce shorter path lengths for particular samples, they are highly likely to be anomalies.
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.ensemble import IsolationForest
rng = np.random.RandomState(42)

# Generate train data
X = 0.3 * rng.randn(100, 2)
X_train = np.r_[X + 2, X - 2]
# Generate some regular novel observations
X = 0.3 * rng.randn(20, 2)
X_test = np.r_[X + 2, X - 2]
# Generate some abnormal novel observations
X_outliers = rng.uniform(low=-4, high=4, size=(20, 2))

# fit the model
clf = IsolationForest(max_samples=100, random_state=rng)
clf.fit(X_train)
y_pred_train = clf.predict(X_train)
y_pred_test = clf.predict(X_test)
y_pred_outliers = clf.predict(X_outliers)

# plot the line, the samples, and the nearest vectors to the plane
xx, yy = np.meshgrid(np.linspace(-5, 5, 50), np.linspace(-5, 5, 50))
```
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)

plt.title("IsolationForest")
plt.contourf(xx, yy, Z, cmap=plt.cm.Blues_r)

b1 = plt.scatter(X_train[:, 0], X_train[:, 1], c='white')
b2 = plt.scatter(X_test[:, 0], X_test[:, 1], c='green')
c = plt.scatter(X_outliers[:, 0], X_outliers[:, 1], c='red')
plt.axis('tight')
plt.xlim((-5, 5))
plt.ylim((-5, 5))
plt.legend([b1, b2, c],
           ['training observations',
            'new regular observations',
            'new abnormal observations'],
           loc='upper left')
plt.show()

Total running time of the script: (0 minutes 0.568 seconds)
Download Python source code: plot_isolation_forest.py
Download IPython notebook: plot_isolation_forest.ipynb

14.4 Feature importances with forests of trees

This examples shows the use of forests of trees to evaluate the importance of features on an artificial classification task. The red bars are the feature importances of the forest, along with their inter-trees variability.

As expected, the plot suggests that 3 features are informative, while the remaining are not.
Feature importances with forests of trees

```
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification
from sklearn.ensemble import ExtraTreesClassifier

print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification
from sklearn.ensemble import ExtraTreesClassifier
```

Out:

```
Feature ranking:
1. feature 1 (0.295902)
2. feature 2 (0.208351)
3. feature 0 (0.177632)
4. feature 3 (0.047121)
5. feature 6 (0.046303)
6. feature 8 (0.046013)
7. feature 7 (0.045575)
8. feature 4 (0.044614)
9. feature 9 (0.044577)
10. feature 5 (0.043912)
```
# Build a classification task using 3 informative features
X, y = make_classification(n_samples=1000,
    n_features=10,
    n_informative=3,
    n_redundant=0,
    n_repeated=0,
    n_classes=2,
    random_state=0,
    shuffle=False)

# Build a forest and compute the feature importances
forest = ExtraTreesClassifier(n_estimators=250,
    random_state=0)
forest.fit(X, y)
importances = forest.feature_importances_
std = np.std([tree.feature_importances_ for tree in forest.estimators_],
    axis=0)
indices = np.argsort(importances)[::-1]

# Print the feature ranking
print("Feature ranking:")
for f in range(X.shape[1]):
    print("%d. feature %d (%f)" % (f + 1, indices[f], importances[indices[f]]))

# Plot the feature importances of the forest
plt.figure()
plt.title("Feature importances")
plt.bar(range(X.shape[1]), importances[indices], color="r", yerr=std[indices], align="center")
plt.xticks(range(X.shape[1]), indices)
plt.xlim([-1, X.shape[1]])
plt.show()
from itertools import product

import numpy as np
import matplotlib.pyplot as plt

from sklearn import datasets
from sklearn.tree import DecisionTreeClassifier
from sklearn.neighbors import KNeighborsClassifier
from sklearn.svm import SVC
from sklearn.ensemble import VotingClassifier

# Loading some example data
iris = datasets.load_iris()
X = iris.data[:, [0, 2]]
y = iris.target

# Training classifiers
clf1 = DecisionTreeClassifier(max_depth=4)
clf2 = KNeighborsClassifier(n_neighbors=7)
clf3 = SVC(kernel='rbf', probability=True)
eclf = VotingClassifier(estimators=[('dt', clf1), ('knn', clf2),

14.5. Plot the decision boundaries of a VotingClassifier
clf1.fit(X, y)
clf2.fit(X, y)
clf3.fit(X, y)
eclf.fit(X, y)

# Plotting decision regions
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, 0.1),
                     np.arange(y_min, y_max, 0.1))
f, axarr = plt.subplots(2, 2, sharex='col', sharey='row', figsize=(10, 8))
for idx, clf, tt in zip(product([0, 1], [0, 1]),
                        [clf1, clf2, clf3, eclf],
                        ['Decision Tree (depth=4)', 'KNN (k=7)',
                         'Kernel SVM', 'Soft Voting']):
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    axarr[idx[0], idx[1]].contourf(xx, yy, Z, alpha=0.4)
    axarr[idx[0], idx[1]].scatter(X[:, 0], X[:, 1], c=y, alpha=0.8)
    axarr[idx[0], idx[1]].set_title(tt)
plt.show()

Total running time of the script: (0 minutes 0.326 seconds)

Download Python source code: plot_voting_decision_regions.py
Download IPython notebook: plot_voting_decision_regions.ipynb

14.6 Comparing random forests and the multi-output meta estimator

An example to compare multi-output regression with random forest and the multioutput.MultiOutputRegressor metaestimator.

This example illustrates the use of the multioutput.MultiOutputRegressor meta-estimator to perform multi-output regression. A random forest regressor is used, which supports multi-output regression natively, so the results can be compared.

The random forest regressor will only ever predict values within the range of observations or closer to zero for each of the targets. As a result the predictions are biased towards the centre of the circle.

Using a single underlying feature the model learns both the x and y coordinate as output.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.ensemble import RandomForestRegressor
from sklearn.model_selection import train_test_split
from sklearn.multioutput import MultiOutputRegressor

# Create a random dataset
rng = np.random.RandomState(1)
X = np.sort(200 * rng.rand(600, 1) - 100, axis=0)
y = np.array([np.pi * np.sin(X).ravel(), np.pi * np.cos(X).ravel()]).T
y += (0.5 - rng.rand(*y.shape))

X_train, X_test, y_train, y_test = train_test_split(X, y,
    train_size=400,
    random_state=4)

max_depth = 30
regr_multirf = MultiOutputRegressor(RandomForestRegressor(max_depth=max_depth,
    random_state=0))
regr_multirf.fit(X_train, y_train)

trees = RandomForestRegressor(max_depth=max_depth, random_state=2)
trees.fit(X_train, y_train)

# Predict on new data
y_multirf = regr_multirf.predict(X_test)
y_rf = regr_rf.predict(X_test)

# Plot the results
plt.figure()
s = 50
alpha = 0.4
plt.scatter(y_test[:, 0], y_test[:, 1],
c="navy", s=s, marker="s", alpha=alpha, label="Data")
plt.scatter(y_multirf[:, 0], y_multirf[:, 1],
c="cornflowerblue", s=s, alpha=alpha,
label="Multi RF score=%.2f" % regr_multirf.score(X_test, y_test))
plt.scatter(y_rf[:, 0], y_rf[:, 1],
c="c", s=s, marker="^", alpha=alpha,
label="RF score=%.2f" % regr_rf.score(X_test, y_test))
plt.xlim([-6, 6])
plt.ylim([-6, 6])
plt.xlabel("target 1")
plt.ylabel("target 2")
plt.title("Comparing random forests and the multi-output meta estimator")
plt.legend()
plt.show()

---

**Total running time of the script:** (0 minutes 0.213 seconds)

**Download Python source code:** plot_random_forest_regression_multioutput.py

**Download IPython notebook:** plot_random_forest_regression_multioutput.ipynb

### 14.7 Gradient Boosting regression

Demonstrate Gradient Boosting on the Boston housing dataset.

This example fits a Gradient Boosting model with least squares loss and 500 regression trees of depth 4.

```python
print(__doc__)

# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt

from sklearn import ensemble
from sklearn import datasets
from sklearn import shuffle
from sklearn import metrics

Load data
```
boston = datasets.load_boston()
X, y = shuffle(boston.data, boston.target, random_state=13)
X = X.astype(np.float32)
offset = int(X.shape[0] * 0.9)
X_train, y_train = X[:offset], y[:offset]
X_test, y_test = X[offset:], y[offset:]

Fit regression model

params = {'n_estimators': 500, 'max_depth': 4, 'min_samples_split': 2,
          'learning_rate': 0.01, 'loss': 'ls'}
clf = ensemble.GradientBoostingRegressor(**params)
clf.fit(X_train, y_train)
mse = mean_squared_error(y_test, clf.predict(X_test))
print("MSE: %.4f" % mse)

Out:
MSE: 6.6213

Plot training deviance

# compute test set deviance
test_score = np.zeros((params['n_estimators'],), dtype=np.float64)

for i, y_pred in enumerate(clf.staged_predict(X_test)):
    test_score[i] = clf.loss_(y_test, y_pred)

plt.figure(figsize=(12, 6))
plt.plot(np.arange(params['n_estimators']) + 1, clf.train_score_, 'b-',
         label='Training Set Deviance')
plt.plot(np.arange(params['n_estimators']) + 1, test_score, 'r-',
         label='Test Set Deviance')
plt.legend(loc='upper right')
plt.xlabel('Boosting Iterations')
plt.ylabel('Deviance')

14.7. Gradient Boosting regression 811
Plot feature importance

```python
feature_importance = clf.feature_importances_
# make importances relative to max importance
feature_importance = 100.0 * (feature_importance / feature_importance.max())
sorted_idx = np.argsort(feature_importance)
pos = np.arange(sorted_idx.shape[0]) + .5
plt.subplot(1, 2, 2)
plt.barh(pos, feature_importance[sorted_idx], align='center')
plt.yticks(pos, boston.feature_names[sorted_idx])
plt.xlabel('Relative Importance')
plt.title('Variable Importance')
plt.show()
```
14.8 Prediction Intervals for Gradient Boosting Regression

This example shows how quantile regression can be used to create prediction intervals.
import numpy as np
import matplotlib.pyplot as plt

from sklearn.ensemble import GradientBoostingRegressor

np.random.seed(1)

def f(x):
    """The function to predict."""
    return x * np.sin(x)

#----------------------------------------------------------------------
# First the noiseless case
X = np.atleast_2d(np.random.uniform(0, 10.0, size=100)).T
X = X.astype(np.float32)
# Observations
y = f(X).ravel()

dy = 1.5 + 1.0 * np.random.random(y.shape)
noise = np.random.normal(0, dy)
y += noise
y = y.astype(np.float32)

# Mesh the input space for evaluations of the real function, the prediction and
```python
# its MSE
xx = np.atleast_2d(np.linspace(0, 10, 1000)).T
xx = xx.astype(np.float32)
alpha = 0.95

clf = GradientBoostingRegressor(loss='quantile', alpha=alpha,
                                 n_estimators=250, max_depth=3,
                                 learning_rate=.1, min_samples_leaf=9,
                                 min_samples_split=9)

clf.fit(X, y)

# Make the prediction on the meshed x-axis
y_upper = clf.predict(xx)

clf.set_params(alpha=1.0 - alpha)
clf.fit(X, y)

# Make the prediction on the meshed x-axis
y_lower = clf.predict(xx)

clf.set_params(loss='ls')
clf.fit(X, y)

# Make the prediction on the meshed x-axis
y_pred = clf.predict(xx)

# Plot the function, the prediction and the 90% confidence interval based on
# the MSE
fig = plt.figure()
plt.plot(xx, f(xx), 'g:', label=u'$f(x) = x\sin(x)$')
plt.plot(X, y, 'b.', markersize=10, label=u'Observations')
plt.plot(xx, y_pred, 'r-', label=u'Prediction')
plt.plot(xx, y_upper, 'k-')
plt.plot(xx, y_lower, 'k-')
plt.fill(np.concatenate([xx, xx[::-1]]),
         np.concatenate([y_upper, y_lower[::-1]]),
         alpha=.5, fc='b', ec='None', label='90% prediction interval')
plt.xlabel('$x$')
plt.ylabel('$f(x)$')
plt.ylim(-10, 20)
plt.legend(loc='upper left')
plt.show()

Total running time of the script: (0 minutes 0.312 seconds)
Download Python source code: plot_gradient_boosting_quantile.py
Download IPython notebook: plot_gradient_boosting_quantile.ipynb

14.9 Plot class probabilities calculated by the VotingClassifier

Plot the class probabilities of the first sample in a toy dataset predicted by three different classifiers and averaged by
the VotingClassifier.

First, three exemplary classifiers are initialized (LogisticRegression, GaussianNB, and RandomForestClassifier) and
```
used to initialize a soft-voting `VotingClassifier` with weights `[1, 1, 5]`, which means that the predicted probabilities of the `RandomForestClassifier` count 5 times as much as the weights of the other classifiers when the averaged probability is calculated.

To visualize the probability weighting, we fit each classifier on the training set and plot the predicted class probabilities for the first sample in this example dataset.

```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LogisticRegression
from sklearn.naive_bayes import GaussianNB
from sklearn.ensemble import RandomForestClassifier, VotingClassifier

clf1 = LogisticRegression(random_state=123)
clf2 = RandomForestClassifier(random_state=123)
clf3 = GaussianNB()
X = np.array([[-1.0, -1.0], [-1.2, -1.4], [-3.4, -2.2], [1.1, 1.2]])
y = np.array([1, 1, 2, 2])
eclf = VotingClassifier(estimators=[('lr', clf1), ('rf', clf2), ('gnb', clf3)],
                        voting='soft',
                        weights=[1, 1, 5])
```
# predict class probabilities for all classifiers
probas = [c.fit(X, y).predict_proba(X) for c in (clf1, clf2, clf3, eclf)]

# get class probabilities for the first sample in the dataset
class1_1 = [pr[0, 0] for pr in probas]
class2_1 = [pr[0, 1] for pr in probas]

# plotting
N = 4  # number of groups
ind = np.arange(N)  # group positions
width = 0.35  # bar width

fig, ax = plt.subplots()

# bars for classifier 1-3
p1 = ax.bar(ind, np.hstack((class1_1[:-1], [0])), width, color='green')
p2 = ax.bar(ind + width, np.hstack((class2_1[:-1], [0])), width, color='lightgreen')

# bars for VotingClassifier
p3 = ax.bar(ind, [0, 0, 0, class1_1[-1]], width, color='blue')
p4 = ax.bar(ind + width, [0, 0, 0, class2_1[-1]], width, color='steelblue')

# plot annotations
plt.axvline(2.8, color='k', linestyle='dashed')
ax.set_xticks(ind + width)
ax.set_xticklabels(['LogisticRegression\nweight 1', 'GaussianNB\nweight 1', 'RandomForestClassifier\nweight 5', 'VotingClassifier\n(average probabilities)'], rotation=40, ha='right')

plt.ylim([0, 1])
plt.title('Class probabilities for sample 1 by different classifiers')
plt.legend([p1[0], p2[0]], ['class 1', 'class 2'], loc='upper left')
plt.show()

Total running time of the script: (0 minutes 0.235 seconds)
Download Python source code: plot_voting_probas.py
Download IPython notebook: plot_voting_probas.ipynb

14.10 Gradient Boosting regularization

Illustration of the effect of different regularization strategies for Gradient Boosting. The example is taken from Hastie et al 2009.

The loss function used is binomial deviance. Regularization via shrinkage (learning_rate < 1.0) improves performance considerably. In combination with shrinkage, stochastic gradient boosting (subsample < 1.0) can produce more accurate models by reducing the variance via bagging. Subsampling without shrinkage usually does poorly. Another strategy to reduce the variance is by subsampling the features analogous to the random splits in Random Forests (via the max_features parameter).
import numpy as np
import matplotlib.pyplot as plt
from sklearn import ensemble
from sklearn import datasets

X, y = datasets.make_hastie_10_2(n_samples=12000, random_state=1)
X = X.astype(np.float32)

# map labels from {-1, 1} to {0, 1}
labels, y = np.unique(y, return_inverse=True)

X_train, X_test = X[:2000], X[2000:]
y_train, y_test = y[:2000], y[2000:]

original_params = {'n_estimators': 1000, 'max_leaf_nodes': 4, 'max_depth': None,
                    'random_state': 2,
                    'min_samples_split': 5}
plt.figure()

for label, color, setting in [('No shrinkage', 'orange',
{'learning_rate': 1.0, 'subsample': 1.0}),
{'learning_rate=0.1', 'turquoise',
{'learning_rate': 0.1, 'subsample': 1.0}),
{'subsample=0.5', 'blue',
{'learning_rate': 1.0, 'subsample': 0.5}),
{'learning_rate=0.1, subsample=0.5', 'gray',
{'learning_rate': 0.1, 'subsample': 0.5}),
{'learning_rate=0.1, max_features=2', 'magenta',
{'learning_rate': 0.1, 'max_features': 2})]:

params = dict(original_params)
params.update(setting)

clf = ensemble.GradientBoostingClassifier(**params)
clf.fit(X_train, y_train)

# compute test set deviance
test_deviance = np.zeros((params['n_estimators']), dtype=np.float64)

for i, y_pred in enumerate(clf.staged_decision_function(X_test)):
# clf.loss_ assumes that y_test[i] in {0, 1}
test_deviance[i] = clf.loss_(y_test, y_pred)

plt.plot((np.arange(test_deviance.shape[0]) + 1)[::5], test_deviance[::5], '-

plt.legend(loc='upper left')
plt.xlabel('Boosting Iterations')
plt.ylabel('Test Set Deviance')

plt.show()

Total running time of the script: (0 minutes 12.004 seconds)

Download Python source code: plot_gradient_boosting_regularization.py
Download IPython notebook: plot_gradient_boosting_regularization.ipynb

14.11 OOB Errors for Random Forests

The RandomForestClassifier is trained using bootstrap aggregation, where each new tree is fit from a boot-
strap sample of the training observations \( z_i = (x_i, y_i) \). The out-of-bag (OOB) error is the average error for each \( z_i \) calculated using predictions from the trees that do not contain \( z_i \) in their respective bootstrap sample. This allows the RandomForestClassifier to be fit and validated whilst being trained [1].

The example below demonstrates how the OOB error can be measured at the addition of each new tree during train-
ing. The resulting plot allows a practitioner to approximate a suitable value of n_estimators at which the error stabilizes.
import matplotlib.pyplot as plt

from collections import OrderedDict
from sklearn.datasets import make_classification
from sklearn.ensemble import RandomForestClassifier, ExtraTreesClassifier

# Author: Kian Ho <hui.kian.ho@gmail.com>
# Gilles Louppe <g.louppe@gmail.com>
# Andreas Mueller <amueller@ais.uni-bonn.de>
#
# License: BSD 3 Clause

print(__doc__)

RANDOM_STATE = 123

# Generate a binary classification dataset.
X, y = make_classification(n_samples=500, n_features=25,
                           n_clusters_per_class=1, n_informative=15,
                           random_state=RANDOM_STATE)

# NOTE: Setting the 'warm_start' construction parameter to 'True' disables
# support for parallelized ensembles but is necessary for tracking the OOB
# error trajectory during training.
ensemble_clfs = [
    ("RandomForestClassifier, max_features='sqrt'",
     RandomForestClassifier(max_features='sqrt',
                            n_estimators=n_estimators,
                            random_state=RANDOM_STATE,
                            warm_start=True),
    ("RandomForestClassifier, max_features='log2'",
     RandomForestClassifier(max_features='log2',
                            n_estimators=n_estimators,
                            random_state=RANDOM_STATE,
                            warm_start=True),
    ("RandomForestClassifier, max_features=None",
     RandomForestClassifier(max_features=None,
                            n_estimators=n_estimators,
                            random_state=RANDOM_STATE,
                            warm_start=True)]}
RandomForestClassifier(warm_start=True, oob_score=True,
max_features="sqrt",
random_state=RANDOM_STATE)),
("RandomForestClassifier, max_features='log2'",
RandomForestClassifier(warm_start=True, max_features='log2',
oob_score=True,
random_state=RANDOM_STATE)),
("RandomForestClassifier, max_features=None",
RandomForestClassifier(warm_start=True, max_features=None,
oob_score=True,
random_state=RANDOM_STATE))
]

# Map a classifier name to a list of (<n_estimators>, <error rate>) pairs.
error_rate = OrderedDict((label, []) for label, _ in ensemble_clfs)

# Range of `n_estimators` values to explore.
min_estimators = 15
max_estimators = 175

for label, clf in ensemble_clfs:
    for i in range(min_estimators, max_estimators + 1):
        clf.set_params(n_estimators=i)
        clf.fit(X, y)

        # Record the OOB error for each `n_estimators=i` setting.
        oob_error = 1 - clf.oob_score_
        error_rate[label].append((i, oob_error))

# Generate the "OOB error rate" vs. "n_estimators" plot.
for label, clf_err in error_rate.items():
    xs, ys = zip(*clf_err)
    plt.plot(xs, ys, label=label)
plt.xlim(min_estimators, max_estimators)
plt.xlabel("n_estimators")
plt.ylabel("OOB error rate")
plt.legend(loc="upper right")
plt.show()

Total running time of the script: (0 minutes 10.994 seconds)

Download Python source code: plot_ensemble_oob.py
Download IPython notebook: plot_ensemble_oob.ipynb

### 14.12 Two-class AdaBoost

This example fits an AdaBoosted decision stump on a non-linearly separable classification dataset composed of two “Gaussian quantiles” clusters (see sklearn.datasets.make_gaussian_quantiles) and plots the decision boundary and decision scores. The distributions of decision scores are shown separately for samples of class A and B. The predicted class label for each sample is determined by the sign of the decision score. Samples with decision scores greater than zero are classified as B, and are otherwise classified as A. The magnitude of a decision score determines the degree of likeness with the predicted class label. Additionally, a new dataset could be constructed containing a desired purity of class B, for example, by only selecting samples with a decision score above some value.
# Construct dataset
X1, y1 = make_gaussian_quantiles(cov=2.,
                               n_samples=200, n_features=2,
                               n_classes=2, random_state=1)
X2, y2 = make_gaussian_quantiles(mean=(3, 3), cov=1.5,
                               n_samples=300, n_features=2,
                               n_classes=2, random_state=1)
X = np.concatenate((X1, X2))
y = np.concatenate((y1, -y2 + 1))

# Create and fit an AdaBoosted decision tree
bdt = AdaBoostClassifier(DecisionTreeClassifier(max_depth=1),
                          algorithm="SAMME",
                          n_estimators=200)
bdt.fit(X, y)

plot_colors = "br"
plot_step = 0.02
class_names = "AB"

plt.figure(figsize=(10, 5))
# Plot the decision boundaries
plt.subplot(121)
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
                     np.arange(y_min, y_max, plot_step))

Z = bdt.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
cs = plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)
plt.axis("tight")

# Plot the training points
for i, n, c in zip(range(2), class_names, plot_colors):
    idx = np.where(y == i)
    plt.scatter(X[idx, 0], X[idx, 1],
                c=c, cmap=plt.cm.Paired,
                label="Class %s" % n)
plt.xlim(x_min, x_max)
plt.ylim(y_min, y_max)
plt.legend(loc='upper right')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Decision Boundary')

# Plot the two-class decision scores
twoclass_output = bdt.decision_function(X)
plot_range = (twoclass_output.min(), twoclass_output.max())
plt.subplot(122)
for i, n, c in zip(range(2), class_names, plot_colors):
    plt.hist(twoclass_output[y == i],
             bins=10,
             range=plot_range,
             facecolor=c,
             label='Class %s' % n,
             alpha=.5)
x1, x2, y1, y2 = plt.axis()
plt.axis((x1, x2, y1, y2 * 1.2))
plt.legend(loc='upper right')
plt.ylabel('Samples')
plt.xlabel('Score')
plt.title('Decision Scores')
plt.tight_layout()
plt.subplots_adjust(wspace=0.35)
plt.show()
14.13 Hashing feature transformation using Totally Random Trees

RandomTreesEmbedding provides a way to map data to a very high-dimensional, sparse representation, which might be beneficial for classification. The mapping is completely unsupervised and very efficient.

This example visualizes the partitions given by several trees and shows how the transformation can also be used for non-linear dimensionality reduction or non-linear classification.

Points that are neighboring often share the same leaf of a tree and therefore share large parts of their hashed representation. This allows to separate two concentric circles simply based on the principal components of the transformed data with truncated SVD.

In high-dimensional spaces, linear classifiers often achieve excellent accuracy. For sparse binary data, BernoulliNB is particularly well-suited. The bottom row compares the decision boundary obtained by BernoulliNB in the transformed space with an ExtraTreesClassifier forests learned on the original data.

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_circles
```

![Original Data (2d)](image1)
![Truncated SVD reduction (2d) of transformed data (74d)](image2)

![Naive Bayes on Transformed data](image3)
![ExtraTrees predictions](image4)
```python
from sklearn.ensemble import RandomTreesEmbedding, ExtraTreesClassifier
from sklearn.decomposition import TruncatedSVD
from sklearn.naive_bayes import BernoulliNB

# make a synthetic dataset
X, y = make_circles(factor=0.5, random_state=0, noise=0.05)

# use RandomTreesEmbedding to transform data
hasher = RandomTreesEmbedding(n_estimators=10, random_state=0, max_depth=3)
X_transformed = hasher.fit_transform(X)

# Visualize result after dimensionality reduction using truncated SVD
svd = TruncatedSVD(n_components=2)
X_reduced = svd.fit_transform(X_transformed)

# Learn a Naive Bayes classifier on the transformed data
nb = BernoulliNB()
nb.fit(X_transformed, y)

# Learn an ExtraTreesClassifier for comparison
trees = ExtraTreesClassifier(max_depth=3, n_estimators=10, random_state=0)
trees.fit(X, y)

# scatter plot of original and reduced data
fig = plt.figure(figsize=(9, 8))
ax = plt.subplot(221)
ax.scatter(X[:, 0], X[:, 1], c=y, s=50)
ax.set_title("Original Data (2d)"
ax.set_xticks(()
ax.set_yticks(()

ax = plt.subplot(222)
ax.scatter(X_reduced[:, 0], X_reduced[:, 1], c=y, s=50)
ax.set_title("Truncated SVD reduction (2d) of transformed data (%d)" %
X_transformed.shape[1])
ax.set_xticks(()
ax.set_yticks(()

# Plot the decision in original space. For that, we will assign a color to each
# point in the mesh [x_min, x_max]x[y_min, y_max].
h = .01
x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

# transform grid using RandomTreesEmbedding
transformed_grid = hasher.transform(np.c_[xx.ravel(), yy.ravel()])
y_grid_pred = nb.predict_proba(transformed_grid)[:, 1]

ax = plt.subplot(223)
ax.set_title("Naive Bayes on Transformed data")
ax.pcolormesh(xx, yy, y_grid_pred.reshape(xx.shape))
ax.scatter(X[:, 0], X[:, 1], c=y, s=50)
ax.set_ylim(-1.4, 1.4)
ax.set_xlim(-1.4, 1.4)
```

Partial dependence plots show the dependence between the target function and a set of ‘target’ features, marginalizing over the values of all other features (the complement features). Due to the limits of human perception the size of the target feature set must be small (usually, one or two) thus the target features are usually chosen among the most important features (see `feature_importances_`).

This example shows how to obtain partial dependence plots from a `GradientBoostingRegressor` trained on the California housing dataset. The example is taken from 1.

The plot shows four one-way and one two-way partial dependence plots. The target variables for the one-way PDP are: median income (`MedInc`), avg. occupants per household (`AvgOccup`), median house age (`HouseAge`), and avg. rooms per household (`AveRooms`).

We can clearly see that the median house price shows a linear relationship with the median income (top left) and that the house price drops when the avg. occupants per household increases (top middle). The top right plot shows that the house age in a district does not have a strong influence on the (median) house price; so does the average rooms per household. The tick marks on the x-axis represent the deciles of the feature values in the training data.

Partial dependence plots with two target features enable us to visualize interactions among them. The two-way partial dependence plot shows the dependence of median house price on joint values of house age and avg. occupants per household. We can clearly see an interaction between the two features: For an avg. occupancy greater than two, the house price is nearly independent of the house age, whereas for values less than two there is a strong dependence on age.

---


2. For classification you can think of it as the regression score before the link function.
Out:

Training GBRT...
done.
Convenience plot with `partial_dependence_plots`
Custom 3d plot via `partial_dependence`

```python
from __future__ import print_function
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt

from mpl_toolkits.mplot3d import Axes3D

from sklearn.model_selection import train_test_split
from sklearn.ensemble import GradientBoostingRegressor
from sklearn.ensemble.partial_dependence import plot_partial_dependence
from sklearn.ensemble.partial_dependence import partial_dependence
from sklearn.datasets.california_housing import fetch_california_housing

def main():

```
cal_housing = fetch_california_housing()

# split 80/20 train-test
X_train, X_test, y_train, y_test = train_test_split(cal_housing.data, cal_housing.target, test_size=0.2, random_state=1)

names = cal_housing.feature_names

print("Training GBRT...")
clf = GradientBoostingRegressor(n_estimators=100, max_depth=4, learning_rate=0.1, loss='huber', random_state=1)
clf.fit(X_train, y_train)
print(" done.")

print('Convenience plot with `partial_dependence_plots`')
features = [0, 5, 1, 2, (5, 1)]
fig, axs = plot_partial_dependence(clf, X_train, features, feature_names=names, n_jobs=3, grid_resolution=50)
fig.suptitle('Partial dependence of house value on nonlocation features
for the California housing dataset')
plt.subplots_adjust(top=0.9)  # tight_layout causes overlap with suptitle

print('Custom 3d plot via `partial_dependence`')
fig = plt.figure()
target_feature = (1, 5)
pdp, axes = partial_dependence(clf, target_feature, X=X_train, grid_resolution=50)
XX, YY = np.meshgrid(axes[0], axes[1])
Z = pdp[0].reshape(list(map(np.size, axes))).T
ax = Axes3D(fig)
surf = ax.plot_surface(XX, YY, Z, rstride=1, cstride=1, cmap=plt.cm.BuPu)
ax.set_xlabel(names[target_feature[0]])
ax.set_ylabel(names[target_feature[1]])
ax.set_zlabel('Partial dependence')
# pretty init view
ax.view_init(elev=22, azim=122)
plt.colorbar(surf)
plt.suptitle('Partial dependence of house value on median age and ' + 'average occupancy')
plt.subplots_adjust(top=0.9)
plt.show()

# Needed on Windows because plot_partial_dependence uses multiprocessing
if __name__ == '__main__':
    main()
14.15 Discrete versus Real AdaBoost

This example is based on Figure 10.2 from Hastie et al 2009 [1] and illustrates the difference in performance between the discrete SAMME [2] boosting algorithm and real SAMME.R boosting algorithm. Both algorithms are evaluated on a binary classification task where the target Y is a non-linear function of 10 input features.

Discrete SAMME AdaBoost adapts based on errors in predicted class labels whereas real SAMME.R uses the predicted class probabilities.

```python
print(__doc__)

# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>,
# Noel Dawe <noel.dawe@gmail.com>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn.tree import DecisionTreeClassifier
from sklearn.metrics import zero_one_loss
from sklearn.ensemble import AdaBoostClassifier
```

![Graph showing error rate vs. n_estimators for discrete and real AdaBoost](image)

14.15. Discrete versus Real AdaBoost
n_estimators = 400
# A learning rate of 1. may not be optimal for both SAMME and SAMME.R
learning_rate = 1.

X, y = datasets.make_hastie_10_2(n_samples=12000, random_state=1)

X_test, y_test = X[2000:], y[2000:]
X_train, y_train = X[:2000], y[:2000]

dt_stump = DecisionTreeClassifier(max_depth=1, min_samples_leaf=1)
dt_stump.fit(X_train, y_train)
dt_stump_err = 1.0 - dt_stump.score(X_test, y_test)

dt = DecisionTreeClassifier(max_depth=9, min_samples_leaf=1)
dt.fit(X_train, y_train)
dt_err = 1.0 - dt.score(X_test, y_test)

ada_discrete = AdaBoostClassifier(
    base_estimator=dt_stump,
    learning_rate=learning_rate,
    n_estimators=n_estimators,
    algorithm="SAMME")
ada_discrete.fit(X_train, y_train)

ada_real = AdaBoostClassifier(
    base_estimator=dt_stump,
    learning_rate=learning_rate,
    n_estimators=n_estimators,
    algorithm="SAMME.R")
ada_real.fit(X_train, y_train)

fig = plt.figure()
ax = fig.add_subplot(111)

ax.plot([1, n_estimators], [dt_stump_err] * 2, 'k-,
    label='Decision Stump Error')
ax.plot([1, n_estimators], [dt_err] * 2, 'k--',
    label='Decision Tree Error')

ada_discrete_err = np.zeros((n_estimators,))
for i, y_pred in enumerate(ada_discrete.staged_predict(X_test)):
    ada_discrete_err[i] = zero_one_loss(y_pred, y_test)

ada_discrete_err_train = np.zeros((n_estimators,))
for i, y_pred in enumerate(ada_discrete.staged_predict(X_train)):
    ada_discrete_err_train[i] = zero_one_loss(y_pred, y_train)

ada_real_err = np.zeros((n_estimators,))
for i, y_pred in enumerate(ada_real.staged_predict(X_test)):
    ada_real_err[i] = zero_one_loss(y_pred, y_test)

ada_real_err_train = np.zeros((n_estimators,))
for i, y_pred in enumerate(ada_real.staged_predict(X_train)):
    ada_real_err_train[i] = zero_one_loss(y_pred, y_train)

ax.plot(np.arange(n_estimators) + 1, ada_discrete_err,
    label='Discrete AdaBoost Test Error',
    color='red')
```
ax.plot(np.arange(n_estimators) + 1, ada_discrete_err_train,
         label='Discrete AdaBoost Train Error',
         color='blue')
ax.plot(np.arange(n_estimators) + 1, ada_real_err,
         label='Real AdaBoost Test Error',
         color='orange')
ax.plot(np.arange(n_estimators) + 1, ada_real_err_train,
         label='Real AdaBoost Train Error',
         color='green')
ax.set_ylim((0.0, 0.5))
ax.set_xlabel('n_estimators')
ax.set_ylabel('error rate')
leg = ax.legend(loc='upper right', fancybox=True)
leg.get_frame().set_alpha(0.7)
plt.show()
```

Total running time of the script: (0 minutes 6.416 seconds)

Download Python source code: plot_adaboost_hastie_10_2.py
Download IPython notebook: plot_adaboost_hastie_10_2.ipynb

### 14.16 Multi-class AdaBoosted Decision Trees

This example reproduces Figure 1 of Zhu et al [1] and shows how boosting can improve prediction accuracy on a multi-class problem. The classification dataset is constructed by taking a ten-dimensional standard normal distribution and defining three classes separated by nested concentric ten-dimensional spheres such that roughly equal numbers of samples are in each class (quantiles of the $\chi^2$ distribution).

The performance of the SAMME and SAMME.R [1] algorithms are compared. SAMME.R uses the probability estimates to update the additive model, while SAMME uses the classifications only. As the example illustrates, the SAMME.R algorithm typically converges faster than SAMME, achieving a lower test error with fewer boosting iterations. The error of each algorithm on the test set after each boosting iteration is shown on the left, the classification error on the test set of each tree is shown in the middle, and the boost weight of each tree is shown on the right. All trees have a weight of one in the SAMME.R algorithm and therefore are not shown.

```
print(__doc__)
```
from sklearn.externals.six.moves import zip
import matplotlib.pyplot as plt
from sklearn.datasets import make_gaussian_quantiles
from sklearn.ensemble import AdaBoostClassifier
from sklearn.metrics import accuracy_score
from sklearn.tree import DecisionTreeClassifier

X, y = make_gaussian_quantiles(n_samples=13000, n_features=10,
                               n_classes=3, random_state=1)

n_split = 3000
X_train, X_test = X[:n_split], X[n_split:]
y_train, y_test = y[:n_split], y[n_split:]

bdt_real = AdaBoostClassifier(
    DecisionTreeClassifier(max_depth=2),
    n_estimators=600,
    learning_rate=1)

bdt_discrete = AdaBoostClassifier(
    DecisionTreeClassifier(max_depth=2),
    n_estimators=600,
    learning_rate=1.5,
    algorithm="SAMME")

bdt_real.fit(X_train, y_train)
bdt_discrete.fit(X_train, y_train)

real_test_errors = []
discrete_test_errors = []

for real_test_predict, discrete_train_predict in zip(
    bdt_real.staged_predict(X_test), bdt_discrete.staged_predict(X_test)):
    real_test_errors.append(1. - accuracy_score(real_test_predict, y_test))
    discrete_test_errors.append(1. - accuracy_score(discrete_train_predict, y_test))

n_trees_discrete = len(bdt_discrete)
n_trees_real = len(bdt_real)

# Boosting might terminate early, but the following arrays are always
# n_estimators long. We crop them to the actual number of trees here:
discrete_estimator_errors = bdt_discrete.estimator_errors_[:n_trees_discrete]
real_estimator_errors = bdt_real.estimator_errors_[:n_trees_real]
discrete_estimator_weights = bdt_discrete.estimator_weights_[:n_trees_discrete]

plt.figure(figsize=(15, 5))
plt.subplot(131)
Transform your features into a higher dimensional, sparse space. Then train a linear model on these features.

First fit an ensemble of trees (totally random trees, a random forest, or gradient boosted trees) on the training set. Then each leaf of each tree in the ensemble is assigned a fixed arbitrary feature index in a new feature space. These leaf indices are then encoded in a one-hot fashion.

Each sample goes through the decisions of each tree of the ensemble and ends up in one leaf per tree. The sample is encoded by setting feature values for these leaves to 1 and the other feature values to 0.

The resulting transformer has then learned a supervised, sparse, high-dimensional categorical embedding of the data.
import numpy as np
np.random.seed(10)

import matplotlib.pyplot as plt

from sklearn.datasets import make_classification
from sklearn.linear_model import LogisticRegression
from sklearn.ensemble import (RandomTreesEmbedding, RandomForestClassifier, GradientBoostingClassifier)
from sklearn.preprocessing import OneHotEncoder
from sklearn.model_selection import train_test_split
from sklearn.metrics import roc_curve
from sklearn.pipeline import make_pipeline

n_estimator = 10
X, y = make_classification(n_samples=80000)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.5)
# It is important to train the ensemble of trees on a different subset
# of the training data than the linear regression model to avoid
# overfitting, in particular if the total number of leaves is
# similar to the number of training samples
X_train, X_train_lr, y_train, y_train_lr = train_test_split(X_train, y_train, test_size=0.5)
Unsupervised transformation based on totally random trees

```python
rt = RandomTreesEmbedding(max_depth=3, n_estimators=n_estimator,
                          random_state=0)

rt_lm = LogisticRegression()
pipeline = make_pipeline(rt, rt_lm)
pipeline.fit(X_train, y_train)
y_pred_rt = pipeline.predict_proba(X_test)[:, 1]
fpr_rt_lm, tpr_rt_lm, _ = roc_curve(y_test, y_pred_rt)
```

Supervised transformation based on random forests

```python
rf = RandomForestClassifier(max_depth=3, n_estimators=n_estimator)
rf_enc = OneHotEncoder()
rf_lm = LogisticRegression()
rf.fit(X_train, y_train)
rf_enc.fit(rf.apply(X_train))
rf_lm.fit(rf_enc.transform(rf.apply(X_train_lr)), y_train_lr)
y_pred_rf_lm = rf_lm.predict_proba(rf_enc.transform(rf.apply(X_test)))[:, 1]
fpr_rf_lm, tpr_rf_lm, _ = roc_curve(y_test, y_pred_rf_lm)
```

The gradient boosted model by itself

```python
grd = GradientBoostingClassifier(n_estimators=n_estimator)
grd_enc = OneHotEncoder()
grd_lm = LogisticRegression()
grd.fit(X_train, y_train)
grd_enc.fit(grd.apply(X_train)[:, :, 0])
grd_lm.fit(grd_enc.transform(grd.apply(X_train_lr)[:, :, 0]), y_train_lr)
y_pred_grd_lm = grd_lm.predict_proba(
grd_enc.transform(grd.apply(X_test)[:, :, 0]))[:, 1]
fpr_grd_lm, tpr_grd_lm, _ = roc_curve(y_test, y_pred_grd_lm)
```

The random forest model by itself

```python
y_pred_rf = rf.predict_proba(X_test)[:, 1]
fpr_rf, tpr_rf, _ = roc_curve(y_test, y_pred_rf)
```

14.17. Feature transformations with ensembles of trees
14.18 Gradient Boosting Out-of-Bag estimates

Out-of-bag (OOB) estimates can be a useful heuristic to estimate the “optimal” number of boosting iterations. OOB estimates are almost identical to cross-validation estimates but they can be computed on-the-fly without the need for repeated model fitting. OOB estimates are only available for Stochastic Gradient Boosting (i.e. `subsample < 1.0`), the estimates are derived from the improvement in loss based on the examples not included in the bootstrap sample (the so-called out-of-bag examples). The OOB estimator is a pessimistic estimator of the true test loss, but remains a fairly good approximation for a small number of trees.

The figure shows the cumulative sum of the negative OOB improvements as a function of the boosting iteration. As you can see, it tracks the test loss for the first hundred iterations but then diverges in a pessimistic way. The figure also shows the performance of 3-fold cross validation which usually gives a better estimate of the test loss but is computationally more demanding.
Out:

Accuracy: 0.6840

print(__doc__)

# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>
#
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt

from sklearn import ensemble
from sklearn.model_selection import KFold
from sklearn.model_selection import train_test_split

# Generate data (adapted from G. Ridgeway's gbm example)
n_samples = 1000
random_state = np.random.RandomState(13)
x1 = random_state.uniform(size=n_samples)
x2 = random_state.uniform(size=n_samples)
x3 = random_state.randint(0, 4, size=n_samples)

p = 1 / (1.0 + np.exp(-(np.sin(3 * x1) - 4 * x2 + x3)))
y = random_state.binomial(1, p, size=n_samples)

X = np.c_[x1, x2, x3]
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.5, random_state=9)

# Fit classifier with out-of-bag estimates
params = {'n_estimators': 1200, 'max_depth': 3, 'subsample': 0.5,
          'learning_rate': 0.01, 'min_samples_leaf': 1, 'random_state': 3}
clf = ensemble.GradientBoostingClassifier(**params)
clf.fit(X_train, y_train)
acc = clf.score(X_test, y_test)
print("Accuracy: {:.4f}".format(acc))

n_estimators = params['n_estimators']
x = np.arange(n_estimators) + 1

# Estimate best n_estimator using cross-validation
for i, y_pred in enumerate(clf.staged_decision_function(X_test)):
    score[i] = clf.loss_(y_test, y_pred)

# Tune best n_estimators using cross validation
def heldout_score(clf, X_test, y_test):
    return score

def cv_estimate(n_splits=3):
    cv = KFold(n_splits=n_splits)
    cv_clf = ensemble.GradientBoostingClassifier(**params)
    val_scores = np.zeros((n_estimators,), dtype=np.float64)
    for train, test in cv.split(X_train, y_train):
        cv_clf.fit(X_train[train], y_train[train])
        val_scores += heldout_score(cv_clf, X_train[test], y_train[test])
    val_scores /= n_splits
    return val_scores

# Estimate best n_estimators using cross-validation
cv_score = cv_estimate(3)

# Compute best n_estimators for test data
test_score = heldout_score(clf, X_test, y_test)

cumsum = -np.cumsum(clf.oob_improvement_)

# min loss according to OOB
oob_best_iter = x[np.argmin(cumsum)]

# min loss according to test (normalize such that first loss is 0)
test_score -= test_score[0]
test_best_iter = x[np.argmin(test_score)]

# min loss according to cv (normalize such that first loss is 0)
cv_score -= cv_score[0]
cv_best_iter = x[np.argmin(cv_score)]

# color brew for the three curves
oob_color = list(map(lambda x: x / 256.0, (190, 174, 212)))
test_color = list(map(lambda x: x / 256.0, (127, 201, 127)))
cv_color = list(map(lambda x: x / 256.0, (253, 192, 134)))

# plot curves and vertical lines for best iterations
plt.plot(x, cumsum, label='OOB loss', color=oob_color)
plt.plot(x, test_score, label='Test loss', color=test_color)
plt.plot(x, cv_score, label='CV loss', color=cv_color)
plt.axvline(x=oob_best_iter, color=oob_color)
plt.axvline(x=test_best_iter, color=test_color)
plt.axvline(x=cv_best_iter, color=cv_color)

# add three vertical lines to xticks
xticks = plt.xticks()
xticks_pos = np.array(xticks[0].tolist() + [oob_best_iter, cv_best_iter, test_best_iter])
xticks_label = np.array(list(map(lambda t: int(t), xticks[0])) + ['OOB', 'CV', 'Test'])
ind = np.argsort(xticks_pos)
xticks_pos = xticks_pos[ind]
xticks_label = xticks_label[ind]
plt.xticks(xticks_pos, xticks_label)

plt.legend(loc='upper right')
plt.ylabel('normalized loss')
plt.xlabel('number of iterations')
plt.show()
here corresponds to the width of the beam of predictions (in light red) of the individual estimators. The larger the variance, the more sensitive are the predictions for \( x \) to small changes in the training set. The bias term corresponds to the difference between the average prediction of the estimator (in cyan) and the best possible model (in dark blue). On this problem, we can thus observe that the bias is quite low (both the cyan and the blue curves are close to each other) while the variance is large (the red beam is rather wide).

The lower left figure plots the pointwise decomposition of the expected mean squared error of a single decision tree. It confirms that the bias term (in blue) is low while the variance is large (in green). It also illustrates the noise part of the error which, as expected, appears to be constant and around 0.01.

The right figures correspond to the same plots but using instead a bagging ensemble of decision trees. In both figures, we can observe that the bias term is larger than in the previous case. In the upper right figure, the difference between the average prediction (in cyan) and the best possible model is larger (e.g., notice the offset around \( x=2 \)). In the lower right figure, the bias curve is also slightly higher than in the lower left figure. In terms of variance however, the beam of predictions is narrower, which suggests that the variance is lower. Indeed, as the lower right figure confirms, the variance term (in green) is lower than for single decision trees. Overall, the bias-variance decomposition is therefore no longer the same. The tradeoff is better for bagging: averaging several decision trees fit on bootstrap copies of the dataset slightly increases the bias term but allows for a larger reduction of the variance, which results in a lower overall mean squared error (compare the red curves int the lower figures). The script output also confirms this intuition. The total error of the bagging ensemble is lower than the total error of a single decision tree, and this difference indeed mainly stems from a reduced variance.

For further details on bias-variance decomposition, see section 7.3 of \(^1\).

14.19.1 References

```python
print(__doc__)

# Author: Gilles Louppe <g.louppe@gmail.com>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt

from sklearn.ensemble import BaggingRegressor
from sklearn.tree import DecisionTreeRegressor

# Settings
n_repeat = 50  # Number of iterations for computing expectations
```

Out:

```
Tree: 0.0255 (error) = 0.0003 (bias^2) + 0.0152 (var) + 0.0098 (noise)
Bagging(Tree): 0.0196 (error) = 0.0004 (bias^2) + 0.0092 (var) + 0.0098 (noise)
```
n_train = 50  # Size of the training set
n_test = 1000  # Size of the test set
noise = 0.1  # Standard deviation of the noise
np.random.seed(0)

# Change this for exploring the bias-variance decomposition of other estimators. This should work well for estimators with high variance (e.g., decision trees or KNN), but poorly for estimators with low variance (e.g., linear models).
estimators = [('Tree', DecisionTreeRegressor()), ('Bagging(Tree)', BaggingRegressor(DecisionTreeRegressor()))]
n_estimators = len(estimators)

# Generate data

def f(x):
    x = x.ravel()
    return np.exp(-x ** 2) + 1.5 * np.exp(-(x - 2) ** 2)

def generate(n_samples, noise, n_repeat=1):
    X = np.random.rand(n_samples) * 10 - 5
    X = np.sort(X)
    if n_repeat == 1:
        y = f(X) + np.random.normal(0.0, noise, n_samples)
    else:
        y = np.zeros((n_samples, n_repeat))
        for i in range(n_repeat):
            y[:, i] = f(X) + np.random.normal(0.0, noise, n_samples)
    X = X.reshape((n_samples, 1))
    return X, y

X_train = []
y_train = []
for i in range(n_repeat):
    X, y = generate(n_samples=n_train, noise=noise)
    X_train.append(X)
y_train.append(y)

X_test, y_test = generate(n_samples=n_test, noise=noise, n_repeat=n_repeat)

# Loop over estimators to compare
for n, (name, estimator) in enumerate(estimators):
    # Compute predictions
    y_predict = np.zeros((n_test, n_repeat))
    for i in range(n_repeat):
        estimator.fit(X_train[i], y_train[i])
        y_predict[:, i] = estimator.predict(X_test)

    # Bias^2 + Variance + Noise decomposition of the mean squared error
    y_error = np.zeros(n_test)
for i in range(n_repeat):
    for j in range(n_repeat):
        y_error += (y_test[:, j] - y_predict[:, i]) ** 2

y_error /= (n_repeat * n_repeat)

y_noise = np.var(y_test, axis=1)
y_bias = (f(X_test) - np.mean(y_predict, axis=1)) ** 2
y_var = np.var(y_predict, axis=1)

print("{0}: \{1:.4f\} (error) = \{2:.4f\} (bias^2) "
      " + \{3:.4f\} (var) + \{4:.4f\} (noise)".format(name,
                  np.mean(y_error),
                  np.mean(y_bias),
                  np.mean(y_var),
                  np.mean(y_noise)))

# Plot figures
plt.subplot(2, n_estimators, n + 1)
plt.plot(X_test, f(X_test), "b", label="$f(x)$")
plt.plot(X_train[0], y_train[0], ".b", label="LS ~ $y = f(x)+noise$")

for i in range(n_repeat):
    if i == 0:
        plt.plot(X_test, y_predict[:, i], "r", label="$\hat{y}(x)$")
    else:
        plt.plot(X_test, y_predict[:, i], "r", alpha=0.05)

plt.plot(X_test, np.mean(y_predict, axis=1), "c",
         label="$\mathbb{E}_{LS} \hat{y}(x)$")
plt.xlim([-5, 5])
plt.title(name)

if n == 0:
    plt.legend(loc="upper left", prop={"size": 11})

plt.subplot(2, n_estimators, n_estimators + n + 1)
plt.plot(X_test, y_error, "r", label="error(x)")
plt.plot(X_test, y_bias, "b", label="bias^2(x)")
plt.plot(X_test, y_var, "g", label="variance(x)")
plt.plot(X_test, y_noise, "c", label="noise(x)")

plt.xlim([-5, 5])
plt.ylim([0, 0.1])

if n == 0:
    plt.legend(loc="upper left", prop={"size": 11})

plt.show()}
Plot the decision surfaces of ensembles of trees on the iris dataset

Plot the decision surfaces of forests of randomized trees trained on pairs of features of the iris dataset.

This plot compares the decision surfaces learned by a decision tree classifier (first column), by a random forest classifier (second column), by an extra-trees classifier (third column) and by an AdaBoost classifier (fourth column).

In the first row, the classifiers are built using the sepal width and the sepal length features only, on the second row using the petal length and sepal length only, and on the third row using the petal width and the petal length only.

In descending order of quality, when trained (outside of this example) on all 4 features using 30 estimators and scored using 10 fold cross validation, we see:

<table>
<thead>
<tr>
<th>Model</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ExtraTreesClassifier()</td>
<td>0.95</td>
</tr>
<tr>
<td>RandomForestClassifier()</td>
<td>0.94</td>
</tr>
<tr>
<td>AdaBoost(DecisionTree(max_depth=3))</td>
<td>0.94</td>
</tr>
<tr>
<td>DecisionTree(max_depth=None)</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Increasing max_depth for AdaBoost lowers the standard deviation of the scores (but the average score does not improve).

See the console’s output for further details about each model.

In this example you might try to:

1. vary the max_depth for the DecisionTreeClassifier and AdaBoostClassifier, perhaps try max_depth=3 for the DecisionTreeClassifier or max_depth=None for AdaBoostClassifier
2. vary n_estimators

It is worth noting that RandomForests and ExtraTrees can be fitted in parallel on many cores as each tree is built independently of the others. AdaBoost’s samples are built sequentially and so do not use multiple cores.
Classifiers on feature subsets of the Iris dataset

DecisionTree with features [0, 1] has a score of 0.926666666667
RandomForest with 30 estimators with features [0, 1] has a score of 0.926666666667
ExtraTrees with 30 estimators with features [0, 1] has a score of 0.926666666667
AdaBoost with 30 estimators with features [0, 1] has a score of 0.84
DecisionTree with features [0, 2] has a score of 0.993333333333
RandomForest with 30 estimators with features [0, 2] has a score of 0.993333333333
ExtraTrees with 30 estimators with features [0, 2] has a score of 0.993333333333
AdaBoost with 30 estimators with features [0, 2] has a score of 0.993333333333
DecisionTree with features [2, 3] has a score of 0.993333333333
RandomForest with 30 estimators with features [2, 3] has a score of 0.993333333333
ExtraTrees with 30 estimators with features [2, 3] has a score of 0.993333333333
AdaBoost with 30 estimators with features [2, 3] has a score of 0.993333333333

print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn import clone
from sklearn.datasets import load_iris
from sklearn.ensemble import RandomForestClassifier, ExtraTreesClassifier, AdaBoostClassifier
from sklearn.externals.six.moves import xrange
from sklearn.tree import DecisionTreeClassifier

# Parameters
n_classes = 3
n_estimators = 30
plot_colors = "ryb"
cmap = plt.cm.RdYlBu
plot_step = 0.02  # fine step width for decision surface contours
plot_step_coarser = 0.5  # step widths for coarse classifier guesses
RANDOM_SEED = 13  # fix the seed on each iteration

# Load data
iris = load_iris()

plot_idx = 1

models = [DecisionTreeClassifier(max_depth=None),
          RandomForestClassifier(n_estimators=n_estimators),
          ExtraTreesClassifier(n_estimators=n_estimators),
          AdaBoostClassifier(DecisionTreeClassifier(max_depth=3),
                              n_estimators=n_estimators)]

for pair in ([0, 1], [0, 2], [2, 3]):
    for model in models:
        # We only take the two corresponding features
        X = iris.data[:, pair]
y = iris.target

        # Shuffle
        idx = np.arange(X.shape[0])
np.random.seed(RANDOM_SEED)
np.random.shuffle(idx)
        X = X[idx]
y = y[idx]

        # Standardize
        mean = X.mean(axis=0)
        std = X.std(axis=0)
        X = (X - mean) / std

        # Train
        clf = clone(model)
        clf = model.fit(X, y)

        scores = clf.score(X, y)
        # Create a title for each column and the console by using str() and
        # slicing away useless parts of the string
        model_title = str(type(model)).split(".")[1][:-2][:-len("Classifier")]
        model_details = model_title
        if hasattr(model, "estimators_"):
            model_details += " with {} estimators".format(len(model.estimators_))
        print(model_details + " with features", pair, "has a score of", scores)

        plt.subplot(3, 4, plot_idx)
if plot_idx <= len(models):
    # Add a title at the top of each column
    plt.title(model_title)

# Now plot the decision boundary using a fine mesh as input to a
# filled contour plot
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
                     np.arange(y_min, y_max, plot_step))

# Plot either a single DecisionTreeClassifier or alpha blend the
# decision surfaces of the ensemble of classifiers
if isinstance(model, DecisionTreeClassifier):
    Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    cs = plt.contourf(xx, yy, Z, cmap=cmap)
else:
    # Choose alpha blend level with respect to the number of estimators
    # that are in use (noting that AdaBoost can use fewer estimators
    # than its maximum if it achieves a good enough fit early on)
    estimator_alpha = 1.0 / len(model.estimators_)
    for tree in model.estimators_:
        Z = tree.predict(np.c_[xx.ravel(), yy.ravel()])
        Z = Z.reshape(xx.shape)
        cs = plt.contourf(xx, yy, Z, alpha=estimator_alpha, cmap=cmap)

    # Build a coarser grid to plot a set of ensemble classifications
    # to show how these are different to what we see in the decision
    # surfaces. These points are regularly space and do not have a black outline
    xx_coarser, yy_coarser = np.meshgrid(np.arange(x_min, x_max, plot_step_coarser),
                                         np.arange(y_min, y_max, plot_step_coarser))
    Z_points_coarser = model.predict(np.c_[xx_coarser.ravel(), yy_coarser.ravel()]).reshape(xx_coarser.shape)
    cs_points = plt.scatter(xx_coarser, yy_coarser, s=15, c=Z_points_coarser, cmap=cmap, edgecolors="none")

    # Plot the training points, these are clustered together and have a
    # black outline
    for i, c in zip(xrange(n_classes), plot_colors):
        idx = np.where(y == i)
        plt.scatter(X[idx, 0], X[idx, 1], c=c, label=iris.target_names[i],
                    cmap=cmap)

plot_idx += 1  # move on to the next plot in sequence
plt.suptitle("Classifiers on feature subsets of the Iris dataset")
plt.axis("tight")
plt.show()

Total running time of the script: (0 minutes 10.421 seconds)
Download Python source code: plot_forest_iris.py
Download IPython notebook: plot_forest_iris.ipynb
Exercises for the tutorials

15.1 Digits Classification Exercise

A tutorial exercise regarding the use of classification techniques on the Digits dataset. This exercise is used in the Classification part of the Supervised learning: predicting an output variable from high-dimensional observations section of the A tutorial on statistical-learning for scientific data processing.

```python
print(__doc__)

from sklearn import datasets, neighbors, linear_model

digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target

n_samples = len(X_digits)

X_train = X_digits[:.9 * n_samples]
y_train = y_digits[:.9 * n_samples]
X_test = X_digits[.9 * n_samples:]
y_test = y_digits[.9 * n_samples:]

knn = neighbors.KNeighborsClassifier()
logistic = linear_model.LogisticRegression()

print('KNN score: %f' % knn.fit(X_train, y_train).score(X_test, y_test))
print('LogisticRegression score: %f' % logistic.fit(X_train, y_train).score(X_test, y_test))
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: digits_classification_exercise.py
Download IPython notebook: digits_classification_exercise.ipynb

15.2 Cross-validation on Digits Dataset Exercise

A tutorial exercise using Cross-validation with an SVM on the Digits dataset.
This exercise is used in the Cross-validation generators part of the Model selection: choosing estimators and their parameters section of the A tutorial on statistical-learning for scientific data processing.
15.3 SVM Exercise

A tutorial exercise for using different SVM kernels.

This exercise is used in the Using kernels part of the Supervised learning: predicting an output variable from high-dimensional observations section of the A tutorial on statistical-learning for scientific data processing.
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets, svm

iris = datasets.load_iris()
X = iris.data
y = iris.target

X = X[y != 0, :2]
y = y[y != 0]
n_sample = len(X)

np.random.seed(0)
order = np.random.permutation(n_sample)
X = X[order]
y = y[order].astype(np.float)

X_train = X[:int(.9 * n_sample)]
y_train = y[:int(.9 * n_sample)]
X_test = X[int(.9 * n_sample):]
y_test = y[int(.9 * n_sample):]

# fit the model
for fig_num, kernel in enumerate(('linear', 'rbf', 'poly')):
    clf = svm.SVC(kernel=kernel, gamma=10)
    clf.fit(X_train, y_train)
    plt.figure(fig_num)
    plt.clf()
    plt.scatter(X[:, 0], X[:, 1], c=y, zorder=10, cmap=plt.cm.Paired)
    # Circle out the test data
    plt.scatter(X_test[:, 0], X_test[:, 1], s=80, facecolors='none', zorder=10)
    plt.axis('tight')
    x_min = X[:, 0].min()
    x_max = X[:, 0].max()
    y_min = X[:, 1].min()
    y_max = X[:, 1].max()
    XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
    Z = clf.decision_function(np.c_[XX.ravel(), YY.ravel()])
    # Put the result into a color plot
    Z = Z.reshape(XX.shape)
    plt.pcolormesh(XX, YY, Z > 0, cmap=plt.cm.Paired)
    plt.contour(XX, YY, Z, colors=['k', 'k', 'k'],
                linestyles=['--', '-', '--'], levels=[-.5, 0, .5])
    plt.title(kernel)
plt.show()

Total running time of the script: (0 minutes 6.988 seconds)
15.4 Cross-validation on diabetes Dataset Exercise

A tutorial exercise which uses cross-validation with linear models.

This exercise is used in the Cross-validated estimators part of the Model selection: choosing estimators and their parameters section of the A tutorial on statistical-learning for scientific data processing.

```python
from __future__ import print_function
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt

from sklearn import datasets
from sklearn.linear_model import LassoCV
from sklearn.linear_model import Lasso
from sklearn.model_selection import KFold
from sklearn.model_selection import cross_val_score

diabetes = datasets.load_diabetes()
X = diabetes.data[:150]
y = diabetes.target[:150]

lasso = Lasso(random_state=0)
alphas = np.logspace(-4, -0.5, 30)

scores = list()
scores_std = list()
n_folds = 3

for alpha in alphas:
    lasso.alpha = alpha
    this_scores = cross_val_score(lasso, X, y, cv=n_folds, n_jobs=1)
    scores.append(np.mean(this_scores))
    scores_std.append(np.std(this_scores))

scores, scores_std = np.array(scores), np.array(scores_std)

plt.figure().set_size_inches(8, 6)
plt.semilogx(alphas, scores)
# plot error lines showing +/- std. errors of the scores
std_error = scores_std / np.sqrt(n_folds)
plt.semilogx(alphas, scores + std_error, 'b--')
plt.semilogx(alphas, scores - std_error, 'b--')

# alpha=0.2 controls the translucency of the fill color
plt.fill_between(alphas, scores + std_error, scores - std_error, alpha=0.2)

plt.ylabel('CV score +/- std error')
plt.xlabel('alpha')
```

plt.axhline(np.max(scores), linestyle='--', color='.5')
plt.xlim([alphas[0], alphas[-1]])

Bonus: how much can you trust the selection of alpha?

```python
# To answer this question we use the LassoCV object that sets its alpha
# parameter automatically from the data by internal cross-validation (i.e. it
# performs cross-validation on the training data it receives).
# We use external cross-validation to see how much the automatically obtained
# alphas differ across different cross-validation folds.
lasso_cv = LassoCV(alphas=alphas, random_state=0)
k_fold = KFold(3)

print("Answer to the bonus question:",
      "how much can you trust the selection of alpha?")
print()
print("Alpha parameters maximising the generalization score on different")
print("subsets of the data:")
for k, (train, test) in enumerate(k_fold.split(X, y)):
    lasso_cv.fit(X[train], y[train])
    print("[fold {0}] alpha: {1:.5f}, score: {2:.5f}".
          format(k, lasso_cv.alpha_, lasso_cv.score(X[test], y[test])))
print()
print("Answer: Not very much since we obtained different alphas for different")
print("subsets of the data and moreover, the scores for these alphas differ")
```
Answer to the bonus question: how much can you trust the selection of alpha?

Alpha parameters maximising the generalization score on different subsets of the data:
[fold 0] alpha: 0.10405, score: 0.53573  
[fold 1] alpha: 0.05968, score: 0.16278  
[fold 2] alpha: 0.10405, score: 0.44437

Answer: Not very much since we obtained different alphas for different subsets of the data and moreover, the scores for these alphas differ quite substantially.

Total running time of the script: (0 minutes 0.457 seconds)

Download Python source code: plot_cv_diabetes.py

Download IPython notebook: plot_cv_diabetes.ipynb
Examples concerning the `sklearn.feature_selection` module.

### 16.1 Pipeline Anova SVM

Simple usage of Pipeline that runs successively a univariate feature selection with anova and then a C-SVM of the selected features.

```python
print(__doc__)

from sklearn import svm
from sklearn.datasets import samples_generator
from sklearn.feature_selection import SelectKBest, f_regression
from sklearn.pipeline import make_pipeline

# import some data to play with
X, y = samples_generator.make_classification(
    n_features=20, n_informative=3, n_redundant=0, n_classes=4,
    n_clusters_per_class=2)

# ANOVA SVM-C
# 1) anova filter, take 3 best ranked features
anova_filter = SelectKBest(f_regression, k=3)

# 2) svm
clf = svm.SVC(kernel='linear')

anova_svm = make_pipeline(anova_filter, clf)
anova_svm.fit(X, y)
anova_svm.predict(X)
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: feature_selection_pipeline.py
Download IPython notebook: feature_selection_pipeline.ipynb

### 16.2 Recursive feature elimination

A recursive feature elimination example showing the relevance of pixels in a digit classification task.
Note: See also *Recursive feature elimination with cross-validation*

```python
print(__doc__)

from sklearn.svm import SVC
from sklearn.datasets import load_digits
from sklearn.feature_selection import RFE
import matplotlib.pyplot as plt

# Load the digits dataset
digits = load_digits()
X = digits.images.reshape((len(digits.images), -1))
y = digits.target

# Create the RFE object and rank each pixel
svc = SVC(kernel="linear", C=1)
```
rfe = RFE(estimator=svc, n_features_to_select=1, step=1)
rfe.fit(X, y)
ranking = rfe.ranking_.reshape(digits.images[0].shape)

# Plot pixel ranking
plt.matshow(ranking, cmap=plt.cm.Blues)
plt.colorbar()
plt.title("Ranking of pixels with RFE")
plt.show()

Total running time of the script: (0 minutes 4.869 seconds)
Download Python source code: plot_rfe_digits.py
Download IPython notebook: plot_rfe_digits.ipynb

16.3 Comparison of F-test and mutual information

This example illustrates the differences between univariate F-test statistics and mutual information.

We consider 3 features x_1, x_2, x_3 distributed uniformly over [0, 1], the target depends on them as follows:
y = x_1 + \sin(6 \times \pi \times x_2) + 0.1 \times N(0, 1), that is the third features is completely irrelevant.

The code below plots the dependency of y against individual x_i and normalized values of univariate F-tests statistics and mutual information.

As F-test captures only linear dependency, it rates x_1 as the most discriminative feature. On the other hand, mutual information can capture any kind of dependency between variables and it rates x_2 as the most discriminative feature, which probably agrees better with our intuitive perception for this example. Both methods correctly marks x_3 as irrelevant.

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.feature_selection import f_regression, mutual_info_regression

np.random.seed(0)
X = np.random.rand(1000, 3)
y = X[:, 0] + np.sin(6 * np.pi * X[:, 1]) + 0.1 * np.random.randn(1000)

f_test, _ = f_regression(X, y)
```
f_test /= np.max(f_test)
mi = mutual_info_regression(X, y)
mi /= np.max(mi)

plt.figure(figsize=(15, 5))
for i in range(3):
    plt.subplot(1, 3, i + 1)
    plt.scatter(X[::, i], y)
    plt.xlabel(f"x_{{}}\)\".format(i + 1), fontsize=14)
    if i == 0:
        plt.ylabel("y", fontsize=14)
    plt.title(f"F-test={:.2f}, MI={:.2f}".format(f_test[i], mi[i]),
              fontsize=16)
plt.show()

Total running time of the script: (0 minutes 0.226 seconds)
Download Python source code: plot_f_test_vs_mi.py
Download IPython notebook: plot_f_test_vs_mi.ipynb

16.4 Recursive feature elimination with cross-validation

A recursive feature elimination example with automatic tuning of the number of features selected with cross-validation.
Out:

Optimal number of features : 3

```python
print(__doc__)

import matplotlib.pyplot as plt
from sklearn.svm import SVC
from sklearn.model_selection import StratifiedKFold
from sklearn.feature_selection import RFECV
from sklearn.datasets import make_classification

# Build a classification task using 3 informative features
X, y = make_classification(n_samples=1000, n_features=25, n_informative=3,
    n_redundant=2, n_repeated=0, n_classes=8,
    n_clusters_per_class=1, random_state=0)

# Create the RFE object and compute a cross-validated score.
svc = SVC(kernel="linear")
# The "accuracy" scoring is proportional to the number of correct
# classifications
```
rfecv = RFECV(estimator=svc, step=1, cv=StratifiedKFold(2),
               scoring='accuracy')
rfecv.fit(X, y)

print("Optimal number of features : %d" % rfecv.n_features_)

# Plot number of features VS. cross-validation scores
plt.figure()
plt.xlabel("Number of features selected")
plt.ylabel("Cross validation score (nb of correct classifications)")
plt.plot(range(1, len(rfecv.grid_scores_) + 1), rfecv.grid_scores_)
plt.show()

Total running time of the script: (0 minutes 2.340 seconds)

Download Python source code: plot_rfe_with_cross_validation.py
Download IPython notebook: plot_rfe_with_cross_validation.ipynb

16.5 Feature selection using SelectFromModel and LassoCV

Use SelectFromModel meta-transformer along with Lasso to select the best couple of features from the Boston dataset.

Features selected from Boston using SelectFromModel with threshold 0.750.
# Author: Manoj Kumar <mks542@nyu.edu>
# License: BSD 3 clause

print(__doc__)

import matplotlib.pyplot as plt
import numpy as np

from sklearn.datasets import load_boston
from sklearn.feature_selection import SelectFromModel
from sklearn.linear_model import LassoCV

# Load the boston dataset.
boston = load_boston()
X, y = boston['data'], boston['target']

# We use the base estimator LassoCV since the L1 norm promotes sparsity of features.
clf = LassoCV()

# Set a minimum threshold of 0.25
sfm = SelectFromModel(clf, threshold=0.25)
sfm.fit(X, y)
n_features = sfm.transform(X).shape[1]

# Reset the threshold till the number of features equals two.
# Note that the attribute can be set directly instead of repeatedly
# fitting the metatransformer.
while n_features > 2:
sfm.threshold += 0.1
X_transform = sfm.transform(X)
n_features = X_transform.shape[1]

# Plot the selected two features from X.
plt.title("Features selected from Boston using SelectFromModel with ", "threshold %0.3f." % sfm.threshold)
feature1 = X_transform[:, 0]
feature2 = X_transform[:, 1]
plt.plot(feature1, feature2, 'r.')
plt.xlabel("Feature number 1")
plt.ylabel("Feature number 2")
plt.ylim([np.min(feature2), np.max(feature2)])
plt.show()

Total running time of the script: (0 minutes 0.105 seconds)

Download Python source code: plot_select_from_model_boston.py
Download IPython notebook: plot_select_from_model_boston.ipynb

16.6 Univariate Feature Selection

An example showing univariate feature selection.

Noisy (non informative) features are added to the iris data and univariate feature selection is applied. For each feature, we plot the p-values for the univariate feature selection and the corresponding weights of an SVM. We can see that univariate feature selection selects the informative features and that these have larger SVM weights.
In the total set of features, only the 4 first ones are significant. We can see that they have the highest score with univariate feature selection. The SVM assigns a large weight to one of these features, but also Selects many of the non-informative features. Applying univariate feature selection before the SVM increases the SVM weight attributed to the significant features, and will thus improve classification.

```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets, svm
from sklearn.feature_selection import SelectPercentile, f_classif

import some data to play with

# The iris dataset
iris = datasets.load_iris()

# Some noisy data not correlated
E = np.random.uniform(0, 0.1, size=(len(iris.data), 20))

# Add the noisy data to the informative features
X = np.hstack((iris.data, E))
y = iris.target

plt.figure(1)
plt.clf()
plt.clf()
X_indices = np.arange(X.shape[-1])
```
Univariate feature selection with F-test for feature scoring. We use the default selection function: the 10% most significant features.

```python
selector = SelectPercentile(f_classif, percentile=10)
selector.fit(X, y)
scores = -np.log10(selector.pvalues_)
scores /= scores.max()
plt.bar(X_indices - .45, scores, width=.2,
       label=r'Univariate score ($-\text{Log}(p\text{-value})$)', color='darkorange')
```

Univariate Feature Selection
Compare to the weights of an SVM

```python
clf = svm.SVC(kernel='linear')
clf.fit(X, y)

svm_weights = (clf.coef_ ** 2).sum(axis=0)
svm_weights /= svm_weights.max()

plt.bar(X_indices - .25, svm_weights, width=.2, label='SVM weight',
        color='navy')

clf_selected = svm.SVC(kernel='linear')
clf_selected.fit(selector.transform(X), y)

svm_weights_selected = (clf_selected.coef_ ** 2).sum(axis=0)
svm_weights_selected /= svm_weights_selected.max()

plt.bar(X_indices[selector.get_support()] - .05, svm_weights_selected,
        width=.2, label='SVM weights after selection', color='c')

plt.title("Comparing feature selection")
plt.xlabel('Feature number')
plt.yticks(())
plt.axis('tight')
plt.legend(loc='upper right')
```
16.7 Test with permutations the significance of a classification score

In order to test if a classification score is significative a technique in repeating the classification procedure after randomizing, permuting, the labels. The p-value is then given by the percentage of runs for which the score obtained is greater than the classification score obtained in the first place.

```python
# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD 3 clause
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn.svm import SVC
from sklearn.model_selection import StratifiedKFold
```
from sklearn.model_selection import permutation_test_score
from sklearn import datasets

Loading a dataset

iris = datasets.load_iris()
X = iris.data
y = iris.target
n_classes = np.unique(y).size

# Some noisy data not correlated
random = np.random.RandomState(seed=0)
E = random.normal(size=(len(X), 2200))

# Add noisy data to the informative features for make the task harder
X = np.c_[X, E]

svm = SVC(kernel='linear')
cv = StratifiedKFold(2)

classifier = svm

cv = StratifiedKFold(2)
score, permutation_scores, pvalue = permutation_test_score(
    svm, X, y, scoring="accuracy", cv=cv, n_permutations=100, n_jobs=1)

print("Classification score %s (pvalue : %s)" % (score, pvalue))

Out:

Classification score 0.513333333333 (pvalue : 0.00990099009901)

View histogram of permutation scores

plt.hist(permutation_scores, 20, label='Permutation scores')

ylim = plt.ylim()  # BUG: vlines(..., linestyle='--') fails on older versions of matplotlib

plt.vlines(score, ylim[0], ylim[1], linestyle='--', color='g', linewidth=3, label='Classification Score
' (pvalue %s)' % pvalue)

plt.vlines(1.0 / n_classes, ylim[0], ylim[1], linestyle='--', color='k', linewidth=3, label='Luck')

plt.plot(2 * [score], ylim, 'g', label='Classification Score
' (pvalue %s)' % pvalue)

plt.plot(2 * [1. / n_classes], ylim, 'k', label='Luck')

plt.ylabel('Score')
plt.show()
16.7. Test with permutations the significance of a classification score
Examples concerning the `sklearn.gaussian_process` module.

### 17.1 Illustration of Gaussian process classification (GPC) on the XOR dataset

This example illustrates GPC on XOR data. Compared are a stationary, isotropic kernel (RBF) and a non-stationary kernel (DotProduct). On this particular dataset, the DotProduct kernel obtains considerably better results because the class-boundaries are linear and coincide with the coordinate axes. In general, stationary kernels often obtain better results.

```python
316**2 * RBF(length_scale=1.25)  
Log-Marginal-Likelihood:-23.674

316**2 * DotProduct(sigma_0=0.0104) ** 2  
Log-Marginal-Likelihood:-9.284
```

```python
print(__doc__)

# Authors: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
#
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
```
from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import RBF, DotProduct

xx, yy = np.meshgrid(np.linspace(-3, 3, 50),
                     np.linspace(-3, 3, 50))
rng = np.random.RandomState(0)
X = rng.randn(200, 2)
Y = np.logical_xor(X[:, 0] > 0, X[:, 1] > 0)

# fit the model
plt.figure(figsize=(10, 5))
kernels = [1.0 * RBF(length_scale=1.0), 1.0 * DotProduct(sigma_0=1.0)**2]
for i, kernel in enumerate(kernels):
    clf = GaussianProcessClassifier(kernel=kernel, warm_start=True).fit(X, Y)

    # plot the decision function for each datapoint on the grid
    Z = clf.predict_proba(np.vstack((xx.ravel(), yy.ravel())).T)[:, 1]
    Z = Z.reshape(xx.shape)
    plt.subplot(1, 2, i + 1)
    image = plt.imshow(Z, interpolation='nearest',
                       extent=(xx.min(), xx.max(), yy.min(), yy.max()),
                       aspect='auto', origin='lower', cmap=plt.cm.PuOr_r)
    contours = plt.contour(xx, yy, Z, levels=[0], linewidths=2,
                            linetypes='--')
    plt.scatter(X[:, 0], X[:, 1], s=30, c=Y, cmap=plt.cm.Paired)
    plt.xticks(())
    plt.yticks(()
    plt.axis([-3, 3, -3, 3])
    plt.colorbar(image)
    plt.title("%s
    Log-Marginal-Likelihood: %.3f"
              % (clf.kernel_, clf.log_marginal_likelihood(clf.kernel_.theta)),
              fontsize=12)
plt.tight_layout()
plt.show()
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import RBF

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
y = np.array(iris.target, dtype=int)

h = .02  # step size in the mesh

kernel = 1.0 * RBF([1.0])
gpc_rbf_isotropic = GaussianProcessClassifier(kernel=kernel).fit(X, y)
kernel = 1.0 * RBF([1.0, 1.0])
gpc_rbf_anisotropic = GaussianProcessClassifier(kernel=kernel).fit(X, y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

titles = ['Isotropic RBF', 'Anisotropic RBF']

plt.figure(figsize=(10, 5))
for i, clf in enumerate((gpc_rbf_isotropic, gpc_rbf_anisotropic)):
    Z = clf.predict_proba(xx.ravel()[:, None], yy.ravel())
    Z = Z[:, 1].reshape(xx.shape)
    plt.subplot(1, 2, i + 1)
    plt.pcolormesh(xx, yy, Z)
    plt.title(titles[i])
    plt.xticks(())
    plt.yticks(())
```

17.2. Gaussian process classification (GPC) on iris dataset
Both kernel ridge regression (KRR) and Gaussian process regression (GPR) learn a target function by employing internally the “kernel trick”. KRR learns a linear function in the space induced by the respective kernel which corresponds to a non-linear function in the original space. The linear function in the kernel space is chosen based on the mean-squared error loss with ridge regularization. GPR uses the kernel to define the covariance of a prior distribution over the target functions and uses the observed training data to define a likelihood function. Based on Bayes theorem, a (Gaussian) posterior distribution over target functions is defined, whose mean is used for prediction.

A major difference is that GPR can choose the kernel’s hyperparameters based on gradient-ascent on the marginal likelihood function while KRR needs to perform a grid search on a cross-validated loss function (mean-squared error loss). A further difference is that GPR learns a generative, probabilistic model of the target function and can thus provide meaningful confidence intervals and posterior samples along with the predictions while KRR only provides predictions.

This example illustrates both methods on an artificial dataset, which consists of a sinusoidal target function and strong noise. The figure compares the learned model of KRR and GPR based on an ExpSineSquared kernel, which is suited for learning periodic functions. The kernel’s hyperparameters control the smoothness (l) and periodicity of the kernel (p). Moreover, the noise level of the data is learned explicitly by GPR by an additional WhiteKernel component in the kernel and by the regularization parameter alpha of KRR.

The figure shows that both methods learn reasonable models of the target function. GPR correctly identifies the periodicity of the function to be roughly 2*pi (6.28), while KRR chooses the doubled periodicity 4*pi. Besides that, GPR provides reasonable confidence bounds on the prediction which are not available for KRR. A major difference between the two methods is the time required for fitting and predicting: while fitting KRR is fast in principle, the grid-search for hyperparameter optimization scales exponentially with the number of hyperparameters (“curse of dimensionality”). The gradient-based optimization of the parameters in GPR does not suffer from this exponential scaling and is thus considerably faster on this example with 3-dimensional hyperparameter space. The time for predicting is similar; however, generating the variance of the predictive distribution of GPR takes considerably longer than just predicting the mean.
print(__doc__)

# Authors: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
# License: BSD 3 clause

import time
import numpy as np
import matplotlib.pyplot as plt

from sklearn.kernel_ridge import KernelRidge
from sklearn.model_selection import GridSearchCV
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import WhiteKernel, ExpSineSquared

rng = np.random.RandomState(0)

# Generate sample data
X = 15 * rng.rand(100, 1)
y = np.sin(X).ravel()
y += 3 * (0.5 - rng.rand(X.shape[0]))

17.3. Comparison of kernel ridge and Gaussian process regression
# Fit KernelRidge with parameter selection based on 5-fold cross validation

```python
param_grid = {
    "alpha": [1e0, 1e-1, 1e-2, 1e-3],
    "kernel": [ExpSineSquared(l, p)
                for l in np.logspace(-2, 2, 10)
                for p in np.logspace(0, 2, 10)]
}

kr = GridSearchCV(KernelRidge(), cv=5, param_grid=param_grid)
stime = time.time()
kr.fit(X, y)

print("Time for KRR fitting: %3.3f" % (time.time() - stime))

gp_kernel = ExpSineSquared(1.0, 5.0, periodicity_bounds=(1e-2, 1e1)) +
            WhiteKernel(1e-1)

gpr = GaussianProcessRegressor(kernel=gp_kernel)
stime = time.time()
gpr.fit(X, y)

print("Time for GPR fitting: %3.3f" % (time.time() - stime))

# Predict using kernel ridge

X_plot = np.linspace(0, 20, 10000)[:, None]
stime = time.time()
y_kr = kr.predict(X_plot)

print("Time for KRR prediction: %3.3f" % (time.time() - stime))

# Predict using kernel ridge

time = time.time()
y_gpr = gpr.predict(X_plot, return_std=False)

print("Time for GPR prediction: %3.3f" % (time.time() - stime))

time = time.time()
y_gpr, y_std = gpr.predict(X_plot, return_std=True)

print("Time for GPR prediction with standard-deviation: %3.3f"
      % (time.time() - stime))

# Plot results

plt.figure(figsize=(10, 5))
lw = 2
plt.scatter(X, y, c='k', label='data')
plt.plot(X_plot, np.sin(X_plot), color='navy', lw=lw, label='True')
plt.plot(X_plot, y_kr, color='turquoise', lw=lw,
         label='KRR (%s) % kr.best_params_' % kr.best_params_)
plt.plot(X_plot, y_gpr, color='darkorange', lw=lw,
         label='GPR (%s) % gpr.kernel_'
         % gpr.kernel_)
plt.fill_between(X_plot[:, 0], y_gpr - y_std, y_gpr + y_std, color='darkorange',
                 alpha=0.2)
plt.xlabel('data')
plt.ylabel('target')
plt.xlim(0, 20)
plt.ylim(-4, 4)
plt.title('GPR versus Kernel Ridge')
plt.legend(loc="best", scatterpoints=1, prop={"size": 8})
plt.show()
```

Total running time of the script: (0 minutes 11.581 seconds)

Download Python source code: plot_compare_gpr_krr.py
Download IPython notebook: plot_compare_gpr_krr.ipynb
17.4 Gaussian process regression (GPR) on Mauna Loa CO2 data.

This example is based on Section 5.4.3 of “Gaussian Processes for Machine Learning” [RW2006]. It illustrates an example of complex kernel engineering and hyperparameter optimization using gradient ascent on the log-marginal-likelihood. The data consists of the monthly average atmospheric CO2 concentrations (in parts per million by volume (ppmv)) collected at the Mauna Loa Observatory in Hawaii, between 1958 and 1997. The objective is to model the CO2 concentration as a function of the time t.

The kernel is composed of several terms that are responsible for explaining different properties of the signal:

• a long term, smooth rising trend is to be explained by an RBF kernel. The RBF kernel with a large length-scale enforces this component to be smooth; it is not enforced that the trend is rising which leaves this choice to the GP. The specific length-scale and the amplitude are free hyperparameters.

• a seasonal component, which is to be explained by the periodic ExpSineSquared kernel with a fixed periodicity of 1 year. The length-scale of this periodic component, controlling its smoothness, is a free parameter. In order to allow decaying away from exact periodicity, the product with an RBF kernel is taken. The length-scale of this RBF component controls the decay time and is a further free parameter.

• smaller, medium term irregularities are to be explained by a RationalQuadratic kernel component, whose length-scale and alpha parameter, which determines the diffuseness of the length-scales, are to be determined. According to [RW2006], these irregularities can better be explained by a RationalQuadratic than an RBF kernel component, probably because it can accommodate several length-scales.

• a “noise” term, consisting of an RBF kernel contribution, which shall explain the correlated noise components such as local weather phenomena, and a WhiteKernel contribution for the white noise. The relative amplitudes and the RBF’s length scale are further free parameters.

Maximizing the log-marginal-likelihood after subtracting the target’s mean yields the following kernel with an LML of -83.214:

```
34.4**2 * RBF(length_scale=41.8)
+ 3.27**2 * RBF(length_scale=180) * ExpSineSquared(length_scale=1.44,
    periodicity=1)
+ 0.446**2 * RationalQuadratic(alpha=17.7, length_scale=0.957)
+ 0.197**2 * RBF(length_scale=0.138) + WhiteKernel(noise_level=0.0336)
```

Thus, most of the target signal (34.4ppm) is explained by a long-term rising trend (length-scale 41.8 years). The periodic component has an amplitude of 3.27ppm, a decay time of 180 years and a length-scale of 1.44. The long decay time indicates that we have a locally very close to periodic seasonal component. The correlated noise has an amplitude of 0.197ppm with a length scale of 0.138 years and a white-noise contribution of 0.197ppm. Thus, the overall noise level is very small, indicating that the data can be very well explained by the model. The figure shows also that the model makes very confident predictions until around 2015.
scikit-learn user guide, Release 0.18.2

Out:

GPML kernel: 66**2 * RBF(length_scale=67) + 2.4**2 * RBF(length_scale=90) *
→ ExpSineSquared(length_scale=1.3, periodicity=1) + 0.66**2 *
→ RationalQuadratic(alpha=0.78, length_scale=1.2) + 0.18**2 * RBF(length_scale=0.134)
→ WhiteKernel(noise_level=0.0361)
Log-marginal-likelihood: -87.034

Learned kernel: 34.5**2 * RBF(length_scale=41.8) + 3.27**2 * RBF(length_scale=180) *
→ ExpSineSquared(length_scale=1.44, periodicity=1) + 0.446**2 *
→ RationalQuadratic(alpha=17.6, length_scale=0.957) + 0.197**2 * RBF(length_scale=0.138) + WhiteKernel(noise_level=0.0336)
Log-marginal-likelihood: -83.214

print(__doc__)

# Authors: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
#
# License: BSD 3 clause

import numpy as np

Chapter 17. Gaussian Process for Machine Learning
from matplotlib import pyplot as plt
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, WhiteKernel, RationalQuadratic, ExpSineSquared
from sklearn.datasets import fetch_mldata

data = fetch_mldata('mauna-loa-atmospheric-co2').data
X = data[:, [1]]
y = data[:, 0]

# Kernel with parameters given in GPML book
k1 = 66.0**2 * RBF(length_scale=67.0)  # long term smooth rising trend
k2 = 2.4**2 * RBF(length_scale=90.0) * ExpSineSquared(length_scale=1.3, periodicity=1.0)  # seasonal component
k3 = 0.66**2 * RationalQuadratic(length_scale=1.2, alpha=0.78)
k4 = 0.18**2 * RBF(length_scale=0.134) + WhiteKernel(noise_level=0.19**2)  # noise terms
kernel_gpml = k1 + k2 + k3 + k4

gp = GaussianProcessRegressor(kernel=kernel_gpml, alpha=0,
                              optimizer=None, normalize_y=True)
gp.fit(X, y)

print("GPML kernel: %s" % gp.kernel_)
print("Log-marginal-likelihood: %.3f" % gp.log_marginal_likelihood(gp.kernel_.theta))

# Kernel with optimized parameters
k1 = 50.0**2 * RBF(length_scale=50.0)  # long term smooth rising trend
k2 = 2.0**2 * RBF(length_scale=100.0) * ExpSineSquared(length_scale=1.0, periodicity=1.0,
                                        periodicity_bounds="fixed")  # seasonal component
k3 = 0.5**2 * RationalQuadratic(length_scale=1.0, alpha=1.0)
k4 = 0.1**2 * RBF(length_scale=0.1) + WhiteKernel(noise_level=0.1**2,
                                        noise_level_bounds=(1e-3, np.inf))  # noise terms
kernel = k1 + k2 + k3 + k4

gp = GaussianProcessRegressor(kernel=kernel, alpha=0,
                              normalize_y=True)
gp.fit(X, y)

print("Learned kernel: %s" % gp.kernel_)
print("Log-marginal-likelihood: %.3f" % gp.log_marginal_likelihood(gp.kernel_.theta))

X_ = np.linspace(X.min(), X.max() + 30, 1000)[:, np.newaxis]
y_pred, y_std = gp.predict(X_, return_std=True)

# Illustration
plt.scatter(X, y, c='k')
plt.plot(X_, y_pred)
plt.fill_between(X_[:, 0], y_pred - y_std, y_pred + y_std,
17.5 Illustration of prior and posterior Gaussian process for different kernels

This example illustrates the prior and posterior of a GPR with different kernels. Mean, standard deviation, and 10 samples are shown for both prior and posterior.
17.5. Illustration of prior and posterior Gaussian process for different kernels
import numpy as np
from matplotlib import pyplot as plt
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, Matern, RationalQuadratic, ExpSineSquared, DotProduct, ConstantKernel

kernels = [1.0 * RBF(length_scale=1.0, length_scale_bounds=(1e-1, 10.0)),
           1.0 * RationalQuadratic(length_scale=1.0, alpha=0.1),
           1.0 * ExpSineSquared(length_scale=1.0, periodicity=3.0,
                                length_scale_bounds=(0.1, 10.0),
                                periodicity_bounds=(1.0, 10.0)),
           ConstantKernel(0.1, (0.01, 10.0))
           * (DotProduct(sigma_0=1.0, sigma_0_bounds=(0.0, 10.0)) ** 2),
           1.0 * Matern(length_scale=1.0, length_scale_bounds=(1e-1, 10.0),
                         nu=1.5)]

for fig_index, kernel in enumerate(kernels):
    # Specify Gaussian Process
    gp = GaussianProcessRegressor(kernel=kernel)

    # Plot prior
    plt.figure(fig_index, figsize=(8, 8))
    plt.subplot(2, 1, 1)
    X_ = np.linspace(0, 5, 100)
    y_mean, y_std = gp.predict(X_[:, np.newaxis], return_std=True)
    plt.plot(X_, y_mean, 'k', lw=3, zorder=9)
    plt.fill_between(X_, y_mean - y_std, y_mean + y_std,
                     alpha=0.5, color='k')
    y_samples = gp.sample_y(X_[:, np.newaxis], 10)
plt.plot(X_, y_samples, lw=1)
plt.xlim(0, 5)
plt.ylim(-3, 3)
plt.title("Prior (kernel: %s)" % kernel, fontsize=12)

# Generate data and fit GP
rng = np.random.RandomState(4)
X = rng.uniform(0, 5, 10)[:, np.newaxis]
y = np.sin((X[:, 0] - 2.5) ** 2)
gp.fit(X, y)

# Plot posterior
plt.subplot(2, 1, 2)
X_ = np.linspace(0, 5, 100)
y_mean, y_std = gp.predict(X_[:, np.newaxis], return_std=True)
plt.plot(X_, y_mean, 'k', lw=3, zorder=9)
plt.fill_between(X_, y_mean - y_std, y_mean + y_std,
                 alpha=0.5, color='k')
y_samples = gp.sample_y(X_[:, np.newaxis], 10)
plt.plot(X_, y_samples, lw=1)
plt.scatter(X[:, 0], y, c='r', s=50, zorder=10)
plt.xlim(0, 5)
plt.ylim(-3, 3)
plt.title("Posterior (kernel: %s)\nLog-Likelihood: %.3f" % (gp.kernel_,
                           gp.log_marginal_likelihood(gp.kernel_.theta)),
                 fontsize=12)
plt.tight_layout()
plt.show()

Total running time of the script: (0 minutes 2.178 seconds)
Download Python source code: plot_gpr_prior_posterior.py
Download IPython notebook: plot_gpr_prior_posterior.ipynb

17.6 Iso-probability lines for Gaussian Processes classification (GPC)

A two-dimensional classification example showing iso-probability lines for the predicted probabilities.
Learned kernel: $0.0256^2 \times \text{DotProduct}(\sigma_0=5.72)^2$

```python
print(__doc__)
```

```
# Author: Vincent Dubourg <vincent.dubourg@gmail.com>
# Adapted to GaussianProcessClassifier:
#     Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
# License: BSD 3 clause

import numpy as np
from matplotlib import pyplot as plt
from matplotlib import cm

from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import DotProduct, ConstantKernel as C

# A few constants
lim = 8
```
```python
def g(x):
    """The function to predict (classification will then consist in predicting
    whether g(x) <= 0 or not)"
    return 5. - x[:, 1] - .5 * x[:, 0] ** 2.

# Design of experiments
X = np.array([[-4.61611719, -6.00099547],
              [4.10469096, 5.32782448],
              [0.00000000, -0.50000000],
              [-6.17289014, -4.69847430],
              [1.31093060, -6.93271427],
              [-5.03823144, 3.10584743],
              [-2.87600388, 6.74310541],
              [5.21301203, 4.26386883]])

# Observations
y = np.array(g(X) > 0, dtype=int)

# Instantiate and fit Gaussian Process Model
kernel = C(0.1, (1e-5, np.inf)) * DotProduct(sigma_0=0.1) ** 2
gp = GaussianProcessClassifier(kernel=kernel)
gp.fit(X, y)
print("Learned kernel: %s " % gp.kernel_)

# Evaluate real function and the predicted probability
res = 50
x1, x2 = np.meshgrid(np.linspace(-lim, lim, res),
                     np.linspace(-lim, lim, res))
xx = np.vstack([x1.reshape(x1.size), x2.reshape(x2.size)]).T
y_true = g(xx)
y_prob = gp.predict_proba(xx)[:, 1]
y_true = y_true.reshape((res, res))
y_prob = y_prob.reshape((res, res))

# Plot the probabilistic classification iso-values
fig = plt.figure(1)
ax = fig.gca()
ax.axes.set_aspect('equal')
plt.xticks([])
plt.yticks([])
plt.xlabel('$x_1$')
plt.ylabel('$x_2$')
cax = plt.imshow(y_prob, cmap=cm.gray_r, alpha=0.8,
                 extent=(-lim, lim, -lim, lim))
norm = plt.matplotlib.colors.Normalize(vmin=0., vmax=0.9)
cb = plt.colorbar(cax, ticks=[0., 0.2, 0.4, 0.6, 0.8, 1.], norm=norm)
cb.set_label('$\mathbb{P}\left[G(\mathbf{x}) \leq 0\right]$')
plt.clim(0, 1)
plt.plot(X[y <= 0, 0], X[y <= 0, 1], 'r.', markersize=12)
plt.plot(X[y > 0, 0], X[y > 0, 1], 'b.', markersize=12)
```

17.6. Iso-probability lines for Gaussian Processes classification (GPC)
17.7 Probabilistic predictions with Gaussian process classification (GPC)

This example illustrates the predicted probability of GPC for an RBF kernel with different choices of the hyperparameters. The first figure shows the predicted probability of GPC with arbitrarily chosen hyperparameters and with the hyperparameters corresponding to the maximum log-marginal-likelihood (LML).

While the hyperparameters chosen by optimizing LML have a considerably larger LML, they perform slightly worse according to the log-loss on test data. The figure shows that this is because they exhibit a steep change of the class probabilities at the class boundaries (which is good) but have predicted probabilities close to 0.5 far away from the class boundaries (which is bad) This undesirable effect is caused by the Laplace approximation used internally by GPC.

The second figure shows the log-marginal-likelihood for different choices of the kernel’s hyperparameters, highlighting the two choices of the hyperparameters used in the first figure by black dots.
Out:

Log Marginal Likelihood (initial): -17.598
Log Marginal Likelihood (optimized): -3.875
Accuracy: 1.000 (initial) 1.000 (optimized)
Log-loss: 0.214 (initial) 0.319 (optimized)

print(__doc__)

# Authors: Jan Hendrik Metzen <jhm@informatik.uni-bremen.de>
#
# License: BSD 3 clause

import numpy as np
from matplotlib import pyplot as plt
from sklearn.metrics.classification import accuracy_score, log_loss
from sklearn.gaussian_process import GaussianProcessClassifier
from sklearn.gaussian_process.kernels import RBF

# Generate data
train_size = 50
rng = np.random.RandomState(0)
X = rng.uniform(0, 5, 100)[:, np.newaxis]
y = np.array(X[:, 0] > 2.5, dtype=int)

# Specify Gaussian Processes with fixed and optimized hyperparameters
gp_fix = GaussianProcessClassifier(kernel=1.0 * RBF(length_scale=1.0), optimizer=None)
gp_fix.fit(X[:train_size], y[:train_size])

gp_opt = GaussianProcessClassifier(kernel=1.0 * RBF(length_scale=1.0))
gp_opt.fit(X[:train_size], y[:train_size])

print("Log Marginal Likelihood (initial): ".3f" % gp_fix.log_marginal_likelihood(gp_fix.kernel_.theta))
print("Log Marginal Likelihood (optimized): %.3f"
      % gp_opt.log_marginal_likelihood(gp_opt.kernel_.theta))

print("Accuracy: %.3f (initial) %.3f (optimized)"
      % (accuracy_score(y[:train_size], gp_fix.predict(X[:train_size])),
         accuracy_score(y[:train_size], gp_opt.predict(X[:train_size])))
print("Log-loss: %.3f (initial) %.3f (optimized)"
      % (log_loss(y[:train_size], gp_fix.predict_proba(X[:train_size])[:, 1]),
         log_loss(y[:train_size], gp_opt.predict_proba(X[:train_size])[:, 1])))

# Plot posteriors
plt.figure(0)
plt.scatter(X[:train_size, 0], y[:train_size], c='k', label="Train data")
plt.scatter(X[train_size:, 0], y[train_size:], c='g', label="Test data")
X_ = np.linspace(0, 5, 100)
plt.plot(X_, gp_fix.predict_proba(X_[:, np.newaxis])[:, 1], 'r',
         label="Initial kernel: %s" % gp_fix.kernel_)
plt.plot(X_, gp_opt.predict_proba(X_[:, np.newaxis])[:, 1], 'b',
         label="Optimized kernel: %s" % gp_opt.kernel_)
plt.xlabel("Feature")
plt.ylabel("Class 1 probability")
plt.xlim(0, 5)
plt.ylim(-0.25, 1.5)
plt.legend(loc="best")

# Plot LML landscape
plt.figure(1)
theta0 = np.logspace(0, 8, 30)
theta1 = np.logspace(-1, 1, 29)
Theta0, Theta1 = np.meshgrid(theta0, theta1)
LML = [[gp_opt.log_marginal_likelihood(np.log([Theta0[i, j], Theta1[i, j]]))
        for i in range(Theta0.shape[0])]
       for j in range(Theta0.shape[1])]
LML = np.array(LML).T
plt.plot(np.exp(gp_fix.kernel_.theta)[0], np.exp(gp_fix.kernel_.theta)[1],
         'ko', zorder=10)
plt.plot(np.exp(gp_opt.kernel_.theta)[0], np.exp(gp_opt.kernel_.theta)[1],
         'ko', zorder=10)
plt.pcolor(Theta0, Theta1, LML)
plt.xscale("log")
plt.yscale("log")
plt.xlabel("Magnitude")
plt.ylabel("Length-scale")
plt.title("Log-marginal-likelihood")
plt.show()
17.8 Gaussian process regression (GPR) with noise-level estimation

This example illustrates that GPR with a sum-kernel including a WhiteKernel can estimate the noise level of data. An illustration of the log-marginal-likelihood (LML) landscape shows that there exist two local maxima of LML. The first corresponds to a model with a high noise level and a large length scale, which explains all variations in the data by noise. The second one has a smaller noise level and shorter length scale, which explains most of the variation by the noise-free functional relationship. The second model has a higher likelihood; however, depending on the initial value for the hyperparameters, the gradient-based optimization might also converge to the high-noise solution. It is thus important to repeat the optimization several times for different initializations.
# First run
plt.figure(0)
kernel = 1.0 * RBF(length_scale=100.0, length_scale_bounds=(1e-2, 1e3)) \
    + WhiteKernel(noise_level=1, noise_level_bounds=(1e-10, 1e+1))
gp = GaussianProcessRegressor(kernel=kernel, alpha=0.0).fit(X, y)
X_ = np.linspace(0, 5, 100)
y_mean, y_cov = gp.predict(X_[:, np.newaxis], return_cov=True)
plt.plot(X_, y_mean, 'k', lw=3, zorder=9)
plt.fill_between(X_, y_mean - np.sqrt(np.diag(y_cov)), 
                 y_mean + np.sqrt(np.diag(y_cov)), alpha=0.5, color='k')
plt.plot(X_, 0.5*np.sin(3*X_), 'r', lw=3, zorder=9)
plt.scatter(X[:, 0], y, c='r', s=50, zorder=10)
plt.title("Initial: \$%s\nOptimum: \$%s\nLog-Marginal-Likelihood: \$s" % (kernel, gp.kernel_, gp.log_marginal_likelihood(gp.kernel_.theta)))
plt.tight_layout()

# Second run
plt.figure(1)
kernel = 1.0 * RBF(length_scale=1.0, length_scale_bounds=(1e-2, 1e3)) \
    + WhiteKernel(noise_level=1e-5, noise_level_bounds=(1e-10, 1e+1))
gp = GaussianProcessRegressor(kernel=kernel, alpha=0.0).fit(X, y)
X_ = np.linspace(0, 5, 100)
y_mean, y_cov = gp.predict(X_[:, np.newaxis], return_cov=True)
plt.plot(X_, y_mean, 'k', lw=3, zorder=9)
plt.fill_between(X_, y_mean - np.sqrt(np.diag(y_cov)), 
                 y_mean + np.sqrt(np.diag(y_cov)), alpha=0.5, color='k')
plt.plot(X_, 0.5*np.sin(3*X_), 'r', lw=3, zorder=9)
plt.scatter(X[:, 0], y, c='r', s=50, zorder=10)
plt.title("Initial: \$%s\nOptimum: \$%s\nLog-Marginal-Likelihood: \$s" % (kernel, gp.kernel_, gp.log_marginal_likelihood(gp.kernel_.theta)))
plt.tight_layout()

# Plot LML landscape
plt.figure(2)
theta0 = np.logspace(-2, 3, 49)
theta0, Theta1 = np.meshgrid(theta0, thetal)
LML = [[gp.log_marginal_likelihood(np.log([0.36, Theta0[i, j], Theta1[i, j]]))
    for i in range(Theta0.shape[0])]
     for j in range(Theta0.shape[1])]
LML = np.array(LML).T

vmin, vmax = (-LML).min(), (-LML).max()
vmax = 50
plt.contour(Theta0, Theta1, -LML,
    levels=np.logspace(np.log10(vmin), np.log10(vmax), 50),
    norm=LogNorm(vmin=vmin, vmax=vmax))
plt.colorbar()
plt.xscale("log")
plt.yscale("log")
plt.xlabel("Length-scale")
plt.ylabel("Noise-level")
plt.title("Log-marginal-likelihood")
plt.tight_layout()
plt.show()

Total running time of the script: (0 minutes 10.482 seconds)
Download Python source code: plot_gpr_noisy.py
Download IPython notebook: plot_gpr_noisy.ipynb

17.9 Gaussian Processes regression: basic introductory example

A simple one-dimensional regression example computed in two different ways:

1. A noise-free case
2. A noisy case with known noise-level per datapoint

In both cases, the kernel’s parameters are estimated using the maximum likelihood principle.

The figures illustrate the interpolating property of the Gaussian Process model as well as its probabilistic nature in the form of a pointwise 95% confidence interval.

Note that the parameter alpha is applied as a Tikhonov regularization of the assumed covariance between the training points.
import numpy as np
from matplotlib import pyplot as plt

from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, ConstantKernel as C

np.random.seed(1)

def f(x):
    """The function to predict."""
    return x * np.sin(x)

# First the noiseless case
X = np.atleast_2d([1., 3., 5., 6., 7., 8.]).T
y = f(X).ravel()

# Observations
x = np.linspace(0, 10, 1000)

# Mesh the input space for evaluations of the real function, the prediction and
# its MSE
x = np.atleast_2d(np.linspace(0, 10, 1000)).T

# Instantiate a Gaussian Process model
kernel = C(1.0, (1e-3, 1e3)) * RBF(10, (1e-2, 1e2))
gp = GaussianProcessRegressor(kernel=kernel, n_restarts_optimizer=9)

# Fit to data using Maximum Likelihood Estimation of the parameters
gp.fit(X, y)

# Make the prediction on the meshed x-axis (ask for MSE as well)
y_pred, sigma = gp.predict(x, return_std=True)

# Plot the function, the prediction and the 95% confidence interval based on
# the MSE
```python
fig = plt.figure()
plt.plot(x, f(x), 'r:', label=u'$f(x) = x\sin(x)$'
plt.plot(X, y, 'r.', markersize=10, label=u'Observations')
plt.plot(x, y_pred, 'b-', label=u'Prediction')
plt.fill(np.concatenate([x, x[::-1]]),
         np.concatenate([y_pred - 1.9600 * sigma,
                         (y_pred + 1.9600 * sigma)[::-1]]),
        alpha=.5, fc='b', ec='None', label='95% confidence interval')
plt.xlabel('$x$')
plt.ylabel('$f(x)$')
plt.ylim(-10, 20)
plt.legend(loc='upper left')
#
now the noisy case
X = np.linspace(0.1, 9.9, 20)
X = np.atleast_2d(X).T
y = f(X).ravel()
dy = .5 + 1.0 * np.random.random(y.shape)
noise = np.random.normal(0, dy)
y += noise

# Instantiate a Gaussian Process model
gp = GaussianProcessRegressor(kernel=kernel, alpha=(dy / y) ** 2,
                               n_restarts_optimizer=10)

# Fit to data using Maximum Likelihood Estimation of the parameters
gp.fit(X, y)

# Make the prediction on the meshed x-axis (ask for MSE as well)
y_pred, sigma = gp.predict(x, return_std=True)

# Plot the function, the prediction and the 95% confidence interval based on
# the MSE
fig = plt.figure()
plt.plot(x, f(x), 'r:', label=u'$f(x) = x\sin(x)$'
plt.errorbar(X.ravel(), y, dy, fmt='r.', markersize=10, label=u'Observations')
plt.plot(x, y_pred, 'b-', label=u'Prediction')
plt.fill(np.concatenate([x, x[::-1]]),
         np.concatenate([y_pred - 1.9600 * sigma,
                         (y_pred + 1.9600 * sigma)[::-1]]),
        alpha=.5, fc='b', ec='None', label='95% confidence interval')
plt.xlabel('$x$')
plt.ylabel('$f(x)$')
plt.ylim(-10, 20)
plt.legend(loc='upper left')
plt.show()
```

Total running time of the script: (0 minutes 1.011 seconds)

Download Python source code: plot_gpr_noisy_targets.py

Download IPython notebook: plot_gpr_noisy_targets.ipynb
Examples concerning the `sklearn.linear_model` module.

### 18.1 Lasso path using LARS

Computes Lasso Path along the regularization parameter using the LARS algorithm on the diabetes dataset. Each color represents a different feature of the coefficient vector, and this is displayed as a function of the regularization parameter.
Computing regularization path using the LARS ...

```python
print(__doc__)

# Author: Fabian Pedregosa <fabian.pedregosa@inria.fr>
# Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt

from sklearn import linear_model
from sklearn import datasets

diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

print("Computing regularization path using the LARS ...")
alphas, _, coefs = linear_model.lars_path(X, y, method='lasso', verbose=True)

xx = np.sum(np.abs(coefs.T), axis=1)
xx /= xx[-1]

plt.plot(xx, coefs.T)
ymin, ymax = plt.ylim()
plt.vlines(xx, ymin, ymax, linestyle='dashed')
plt.xlabel('|coef| / max|coef|')
plt.ylabel('Coefficients')
plt.title('LASSO Path')
plt.axis('tight')
plt.show()
```

Total running time of the script: (0 minutes 0.110 seconds)

Download Python source code: plot_lasso_lars.py

Download IPython notebook: plot_lasso_lars.ipynb

18.2 Plot Ridge coefficients as a function of the regularization

Shows the effect of collinearity in the coefficients of an estimator.

**Ridge** Regression is the estimator used in this example. Each color represents a different feature of the coefficient vector, and this is displayed as a function of the regularization parameter.

This example also shows the usefulness of applying Ridge regression to highly ill-conditioned matrices. For such matrices, a slight change in the target variable can cause huge variances in the calculated weights. In such cases, it is useful to set a certain regularization (alpha) to reduce this variation (noise).
When alpha is very large, the regularization effect dominates the squared loss function and the coefficients tend to zero. At the end of the path, as alpha tends toward zero and the solution tends towards the ordinary least squares, coefficients exhibit big oscillations. In practise it is necessary to tune alpha in such a way that a balance is maintained between both.

```
# Author: Fabian Pedregosa -- <fabian.pedregosa@inria.fr>
# License: BSD 3 clause

print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model

# X is the 10x10 Hilbert matrix
X = 1. / (np.arange(1, 11) + np.arange(0, 10)[:, np.newaxis])
y = np.ones(10)

Compute paths

n_alphas = 200
alphas = np.logspace(-10, -2, n_alphas)
clf = linear_model.Ridge(fit_intercept=False)

coefs = []
for a in alphas:
    clf.set_params(alpha=a)
    clf.fit(X, y)
    coefs.append(clf.coef_)

Display results

ax = plt.gca()
ax.plot(alphas, coefs)
ax.set_xscale('log')
ax.set_xlim(ax.get_xlim()[::-1])  # reverse axis
plt.xlabel('alpha')
plt.ylabel('weights')
plt.title('Ridge coefficients as a function of the regularization')
plt.axis('tight')
plt.show()
```
18.3 Path with L1- Logistic Regression

Computes path on IRIS dataset.

```python
print(__doc__)

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD 3 clause

from datetime import datetime
import numpy as np
import matplotlib.pyplot as plt

from sklearn import linear_model
from sklearn import datasets
from sklearn.svm import l1_min_c

iris = datasets.load_iris()
```
Demo path functions

```python
X = iris.data
y = iris.target

X = X[y != 2]
y = y[y != 2]
X -= np.mean(X, 0)

cs = l1_min_c(X, y, loss='log') * np.logspace(0, 3)

print("Computing regularization path ...")
start = datetime.now()
clf = linear_model.LogisticRegression(C=1.0, penalty='l1', tol=1e-6)
coefs_ = []
for c in cs:
    clf.set_params(C=c)
    clf.fit(X, y)
    coefs_.append(clf.coef_.ravel().copy())
print("This took ", datetime.now() - start)

coefs_ = np.array(coefs_)
plt.plot(np.log10(cs), coefs_)
ymin, ymax = plt.ylim()
plt.xlabel('log(C)')
plt.ylabel('Coefficients')
plt.title('Logistic Regression Path')
plt.axis('tight')
plt.show()
```
18.4 SGD: Maximum margin separating hyperplane

Plot the maximum margin separating hyperplane within a two-class separable dataset using a linear Support Vector Machines classifier trained using SGD.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import SGDClassifier
from sklearn.datasets.samples_generator import make_blobs

# we create 50 separable points
X, Y = make_blobs(n_samples=50, centers=2, random_state=0, cluster_std=0.60)

# fit the model
clf = SGDClassifier(loss="hinge", alpha=0.01, n_iter=200, fit_intercept=True)
clf.fit(X, Y)

# plot the line, the points, and the nearest vectors to the plane
xx = np.linspace(-1, 5, 10)
yy = np.linspace(-1, 5, 10)
X1, X2 = np.meshgrid(xx, yy)
Z = np.empty(X1.shape)
for (i, j), val in np.ndenumerate(X1):
    x1 = val
    x2 = X2[i, j]
    p = clf.decision_function([[x1, x2]])
    Z[i, j] = p

levels = [-1.0, 0.0, 1.0]
linestyles = ['dashed', 'solid', 'dashed']
colors = 'k'
plt.contour(X1, X2, Z, levels, colors=colors, linestyles=linestyles)
plt.scatter(X[:, 0], X[:, 1], c=Y, cmap=plt.cm.Paired)
plt.axis('tight')
plt.show()

Total running time of the script: (0 minutes 0.069 seconds)
Download Python source code: plot_sgd_separating_hyperplane.py
Download IPython notebook: plot_sgd_separating_hyperplane.ipynb

18.5 SGD: convex loss functions

A plot that compares the various convex loss functions supported by sklearn.linear_model.SGDClassifier.

![Plot of convex loss functions](image)

```
print(__doc__)
import numpy as np
import matplotlib.pyplot as plt
```
def modified_huber_loss(y_true, y_pred):
    z = y_pred * y_true
    loss = -4 * z
    loss[z >= -1] = (1 - z[z >= -1]) ** 2
    loss[z >= 1.] = 0
    return loss

xmin, xmax = -4, 4
xx = np.linspace(xmin, xmax, 100)
lw = 2
plt.plot([xmin, 0, 0, xmax], [1, 1, 0, 0], color='gold', lw=lw,
         label="Zero-one loss")
plt.plot(xx, np.where(xx < 1, 1 - xx, 0), color='teal', lw=lw,
         label="Hinge loss")
plt.plot(xx, -np.minimum(xx, 0), color='yellowgreen', lw=lw,
         label="Perceptron loss")
plt.plot(xx, np.log2(1 + np.exp(-xx)), color='cornflowerblue', lw=lw,
         label="Log loss")
plt.plot(xx, np.where(xx < 1, 1 - xx, 0) ** 2, color='orange', lw=lw,
         label="Squared hinge loss")
plt.plot(xx, modified_huber_loss(xx, 1), color='darkorchid', lw=lw,
         linestyle='--', label="Modified Huber loss")
plt.ylim((0, 8))
plt.legend(loc="upper right")
plt.xlabel(r"Decision function $f(x)$")
plt.ylabel("$L(y, f(x))$")
plt.show()

Total running time of the script: (0 minutes 0.068 seconds)

Download Python source code: plot_sgd_loss_functions.py
Download IPython notebook: plot_sgd_loss_functions.ipynb

18.6 Plot Ridge coefficients as a function of the L2 regularization

Ridge Regression is the estimator used in this example. Each color in the left plot represents one different dimension of the coefficient vector, and this is displayed as a function of the regularization parameter. The right plot shows how exact the solution is. This example illustrates how a well defined solution is found by Ridge regression and how regularization affects the coefficients and their values. The plot on the right shows how the difference of the coefficients from the estimator changes as a function of regularization.

In this example the dependent variable Y is set as a function of the input features: $y = X*w + c$. The coefficient vector w is randomly sampled from a normal distribution, whereas the bias term c is set to a constant.

As alpha tends toward zero the coefficients found by Ridge regression stabilize towards the randomly sampled vector w. For big alpha (strong regularisation) the coefficients are smaller (eventually converging at 0) leading to a simpler and biased solution. These dependencies can be observed on the left plot.

The right plot shows the mean squared error between the coefficients found by the model and the chosen vector w. Less regularised models retrieve the exact coefficients (error is equal to 0), stronger regularised models increase the error.

Please note that in this example the data is non-noisy, hence it is possible to extract the exact coefficients.
import matplotlib.pyplot as plt
import numpy as np
from sklearn.datasets import make_regression
from sklearn.linear_model import Ridge
from sklearn.metrics import mean_squared_error

clf = Ridge()
X, y, w = make_regression(n_samples=10, n_features=10, coef=True,
                          random_state=1, bias=3.5)

coefs = []
errors = []

alphas = np.logspace(-6, 6, 200)

# Train the model with different regularisation strengths
for a in alphas:
    clf.set_params(alpha=a)
    clf.fit(X, y)
    coefs.append(clf.coef_)
    errors.append(mean_squared_error(clf.coef_, w))

# Display results
plt.figure(figsize=(20, 6))

plt.subplot(121)
ax = plt.gca()
ax.plot(alphas, coefs)
ax.set_xscale('log')
plt.xlabel('alpha')
plt.ylabel('weights')
plt.title('Ridge coefficients as a function of the regularization')
plt.axis('tight')

plt.subplot(122)
ax = plt.gca()
ax.plot(alphas, errors)
ax.set_xscale('log')
plt.xlabel('alpha')
18.7 Ordinary Least Squares and Ridge Regression Variance

Due to the few points in each dimension and the straight line that linear regression uses to follow these points as well as it can, noise on the observations will cause great variance as shown in the first plot. Every line’s slope can vary quite a bit for each prediction due to the noise induced in the observations.

Ridge regression is basically minimizing a penalised version of the least-squared function. The penalising shrinks the value of the regression coefficients. Despite the few data points in each dimension, the slope of the prediction is much more stable and the variance in the line itself is greatly reduced, in comparison to that of the standard linear regression.

```python
print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model

X_train = np.c_[.5, 1].T
```
y_train = [.5, 1]
X_test = np.c_[0, 2].T

np.random.seed(0)

classifiers = dict(ols=linear_model.LinearRegression(),
                   ridge=linear_model.Ridge(alpha=.1))

fignum = 1
for name, clf in classifiers.items():
    fig = plt.figure(fignum, figsize=(4, 3))
    plt.clf()
    plt.title(name)
    ax = plt.axes([.12, .12, .8, .8])
    for n in range(6):
        this_X = .1 * np.random.normal(size=(2, 1)) + X_train
        clf.fit(this_X, y_train)
        ax.plot(X_test, clf.predict(X_test), color='.5')
        ax.scatter(this_X, y_train, s=3, c='.5', marker='o', zorder=10)

    clf.fit(X_train, y_train)
    ax.plot(X_test, clf.predict(X_test), linewidth=2, color='blue')
    ax.scatter(X_train, y_train, s=30, c='r', marker='+', zorder=10)

    ax.set_xticks(())
    ax.set_yticks(())
    ax.set_ylim((0, 1.6))
    ax.set_xlabel('X')
    ax.set_ylabel('y')
    ax.set_xlim(0, 2)
    fignum += 1

plt.show()

Total running time of the script: (0 minutes 0.266 seconds)
Download Python source code: plot_ols_ridge_variance.py
Download IPython notebook: plot_ols_ridge_variance.ipynb

18.8 Logistic function

Shown in the plot is how the logistic regression would, in this synthetic dataset, classify values as either 0 or 1, i.e. class one or two, using the logistic curve.
import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model

# this is our test set, it's just a straight line with some
# Gaussian noise
xmin, xmax = -5, 5
n_samples = 100
np.random.seed(0)
X = np.random.normal(size=n_samples)
y = (X > 0).astype(np.float)
X[X > 0] *= 4
X += .3 * np.random.normal(size=n_samples)
X = X[:, np.newaxis]

# run the classifier
clf = linear_model.LogisticRegression(C=1e5)
clf.fit(X, y)

# and plot the result
plt.figure(1, figsize=(4, 3))
plt.clf()
plt.scatter(X.ravel(), y, color='black', zorder=20)
X_test = np.linspace(-5, 10, 300)
def model(x):
    return 1 / (1 + np.exp(-x))
loss = model(X_test * clf.coef_ + clf.intercept_).ravel()
plt.plot(X_test, loss, color='red', linewidth=3)
ols = linear_model.LinearRegression()
ols.fit(X, y)
plt.plot(X_test, ols.coef_ * X_test + ols.intercept_, linewidth=1)
plt.axhline(.5, color='.5')
plt.ylabel('y')
plt.xlabel('X')
plt.xticks(range(-5, 10))
plt.yticks([0, 0.5, 1])
plt.ylim(-.25, 1.25)
plt.xlim(-4, 10)
plt.legend(('Logistic Regression Model', 'Linear Regression Model'),
            loc="lower right", fontsize='small')
plt.show()

Total running time of the script: (0 minutes 0.107 seconds)

Download Python source code: plot_logistic.py
Download IPython notebook: plot_logistic.ipynb

18.9 Polynomial interpolation

This example demonstrates how to approximate a function with a polynomial of degree n_degree by using ridge regression. Concretely, from n_samples 1d points, it suffices to build the Vandermonde matrix, which is n_samples x n_degree+1 and has the following form:

\[
\begin{bmatrix}
1, x_1, x_1 ** 2, x_1 ** 3, \\
1, x_2, x_2 ** 2, x_2 ** 3, \\
\end{bmatrix}, \ldots
\]

Intuitively, this matrix can be interpreted as a matrix of pseudo features (the points raised to some power). The matrix is akin to (but different from) the matrix induced by a polynomial kernel.

This example shows that you can do non-linear regression with a linear model, using a pipeline to add non-linear features. Kernel methods extend this idea and can induce very high (even infinite) dimensional feature spaces.
# 18.9. Polynomial interpolation

```python
def f(x):
    """function to approximate by polynomial interpolation""
    return x * np.sin(x)

# generate points used to plot
x_plot = np.linspace(0, 10, 100)

# generate points and keep a subset of them
x = np.linspace(0, 10, 100)
rng = np.random.RandomState(0)
rng.shuffle(x)
```
```python
x = np.sort(x[:20])
y = f(x)

# create matrix versions of these arrays
X = x[:, np.newaxis]
X_plot = x_plot[:, np.newaxis]

colors = ['teal', 'yellowgreen', 'gold']
lw = 2
plt.plot(x_plot, f(x_plot), color='cornflowerblue', linewidth=lw, label="ground truth")
plt.scatter(x, y, color='navy', s=30, marker='o', label="training points")

for count, degree in enumerate([3, 4, 5]):
    model = make_pipeline(PolynomialFeatures(degree), Ridge())
    model.fit(X, y)
    y_plot = model.predict(X_plot)
    plt.plot(x_plot, y_plot, color=colors[count], linewidth=lw,
              label="degree %d" % degree)

plt.legend(loc='lower left')
plt.show()
```

**Total running time of the script:** (0 minutes 0.072 seconds)

**Download Python source code:** plot_polynomial_interpolation.py

**Download IPython notebook:** plot_polynomial_interpolation.ipynb

## 18.10 Linear Regression Example

This example uses the only the first feature of the diabetes dataset, in order to illustrate a two-dimensional plot of this regression technique. The straight line can be seen in the plot, showing how linear regression attempts to draw a straight line that will best minimize the residual sum of squares between the observed responses in the dataset, and the responses predicted by the linear approximation.

The coefficients, the residual sum of squares and the variance score are also calculated.
Coefficients:
[ 938.23786125]
Mean squared error: 2548.07
Variance score: 0.47

import matplotlib.pyplot as plt
import numpy as np
from sklearn import datasets, linear_model

# Load the diabetes dataset
diabetes = datasets.load_diabetes()
# Use only one feature
diabetes_X = diabetes.data[:, np.newaxis, 2]

# Split the data into training/testing sets
diabetes_X_train = diabetes_X[:-20]
diabetes_X_test = diabetes_X[-20:]

# Split the targets into training/testing sets
diabetes_y_train = diabetes.target[:-20]
diabetes_y_test = diabetes.target[-20:]

# Create linear regression object
regr = linear_model.LinearRegression()

# Train the model using the training sets
regr.fit(diabetes_X_train, diabetes_y_train)

# The coefficients
print('Coefficients: 
', regr.coef_)

# The mean squared error
print("Mean squared error: %2f" % np.mean((regr.predict(diabetes_X_test) - diabetes_y_test) ** 2))
# Explained variance score: 1 is perfect prediction
print('Variance score: %.2f' % regr.score(diabetes_X_test, diabetes_y_test))

# Plot outputs
plt.scatter(diabetes_X_test, diabetes_y_test, color='black')
plt.plot(diabetes_X_test, regr.predict(diabetes_X_test), color='blue', linewidth=3)
plt.xticks(())
plt.yticks(())
plt.show()

Total running time of the script: (0 minutes 0.097 seconds)

Download Python source code: plot_ols.py
Download IPython notebook: plot_ols.ipynb

18.11 Logistic Regression 3-class Classifier

Show below is a logistic-regression classifiers decision boundaries on the iris dataset. The datapoints are colored according to their labels.
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model, datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
Y = iris.target

h = .02  # step size in the mesh

logreg = linear_model.LogisticRegression(C=1e5)

# we create an instance of Neighbours Classifier and fit the data.
logreg.fit(X, Y)

# Plot the decision boundary. For that, we will assign a color to each
# point in the mesh [x_min, x_max]x[y_min, y_max].
x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

Z = logreg.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
plt.pcolormesh(xx, yy, Z, cmap=plt.cm.Paired)

# Plot also the training points
plt.scatter(X[:, 0], X[:, 1], c=Y, edgecolors='k', cmap=plt.cm.Paired)
plt.xlabel('Sepal length')
print(__doc__)
```

18.11. Logistic Regression 3-class Classifier
18.12 SGD: Weighted samples

Plot decision function of a weighted dataset, where the size of points is proportional to its weight.
# we create 20 points
np.random.seed(0)
X = np.r_[np.random.randn(10, 2) + [1, 1], np.random.randn(10, 2)]
y = [1] * 10 + [-1] * 10
sample_weight = 100 * np.abs(np.random.randn(20))
# and assign a bigger weight to the last 10 samples
sample_weight[:10] *= 10

# plot the weighted data points
xx, yy = np.meshgrid(np.linspace(-4, 5, 500), np.linspace(-4, 5, 500))
plt.figure()
plt.scatter(X[:, 0], X[:, 1], c=y, s=sample_weight, alpha=0.9,
cmap=plt.cm.bone)

## fit the unweighted model
clf = linear_model.SGDClassifier(alpha=0.01, n_iter=100)
clf.fit(X, y)
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
no_weights = plt.contour(xx, yy, Z, levels=[0], linestyles=['solid'])

## fit the weighted model
clf = linear_model.SGDClassifier(alpha=0.01, n_iter=100)
clf.fit(X, y, sample_weight=sample_weight)
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
samples_weights = plt.contour(xx, yy, Z, levels=[0], linestyles=['dashed'])

plt.legend([no_weights.collections[0], samples_weights.collections[0]],
           ['no weights', 'with weights'], loc="lower left")
plt.xticks(())
plt.yticks(())
plt.show()

Total running time of the script: (0 minutes 0.110 seconds)
Download Python source code: plot_sgd_weighted_samples.py
Download IPython notebook: plot_sgd_weighted_samples.ipynb

18.13 Lasso on dense and sparse data

We show that linear_model.Lasso provides the same results for dense and sparse data and that in the case of sparse data the speed is improved.

print(__doc__)

from time import time
from scipy import sparse
from scipy import linalg

from sklearn.datasets.samples_generator import make_regression
from sklearn.linear_model import Lasso

The two Lasso implementations on Dense data
print("--- Dense matrices")
X, y = make_regression(n_samples=200, n_features=5000, random_state=0)
X_sp = sparse.coo_matrix(X)

alpha = 1
sparse_lasso = Lasso(alpha=alpha, fit_intercept=False, max_iter=1000)
dense_lasso = Lasso(alpha=alpha, fit_intercept=False, max_iter=1000)

t0 = time()
sparse_lasso.fit(X_sp, y)
print("Sparse Lasso done in $fs \% (time() - t0)"

t0 = time()
dense_lasso.fit(X, y)
print("Dense Lasso done in $fs \% (time() - t0)"

print("Distance between coefficients : $s"
    % linalg.norm(sparse_lasso.coef_ - dense_lasso.coef_))

The two Lasso implementations on Sparse data

print("--- Sparse matrices")
Xs = X.copy()
Xs[Xs < 2.5] = 0.0
Xs = sparse.coo_matrix(Xs)
Xs = Xs.tocsc()

print("Matrix density : $s \%% % (Xs.nnz / float(X.size) * 100))

alpha = 0.1
sparse_lasso = Lasso(alpha=alpha, fit_intercept=False, max_iter=10000)
dense_lasso = Lasso(alpha=alpha, fit_intercept=False, max_iter=10000)

t0 = time()
sparse_lasso.fit(Xs, y)
print("Sparse Lasso done in $fs \% (time() - t0)"

t0 = time()
dense_lasso.fit(Xs.toarray(), y)
print("Dense Lasso done in $fs \% (time() - t0)"

print("Distance between coefficients : $s"
    % linalg.norm(sparse_lasso.coef_ - dense_lasso.coef_))

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: lasso_dense_vs_sparse_data.py
Download IPython notebook: lasso_dense_vs_sparse_data.ipynb

18.14 Lasso and Elastic Net for Sparse Signals

Estimates Lasso and Elastic-Net regression models on a manually generated sparse signal corrupted with an additive noise. Estimated coefficients are compared with the ground-truth.
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import r2_score

# generate some sparse data to play with

np.random.seed(42)
n_samples, n_features = 50, 200
X = np.random.randn(n_samples, n_features)
coef = 3 * np.random.randn(n_features)
inds = np.arange(n_features)
np.random.shuffle(inds)
coef[inds[10:]] = 0  # sparsify coef
y = np.dot(X, coef)

# add noise
y += 0.01 * np.random.normal((n_samples,))

# Split data in train set and test set
n_samples = X.shape[0]
X_train, y_train = X[:n_samples // 2], y[:n_samples // 2]
X_test, y_test = X[n_samples // 2:], y[n_samples // 2:]

# Lasso

from sklearn.linear_model import Lasso
alpha = 0.1
lasso = Lasso(alpha=alpha)
y_pred_lasso = lasso.fit(X_train, y_train).predict(X_test)
r2_score_lasso = r2_score(y_test, y_pred_lasso)
print(lasso)
print("r^2 on test data : %f" % r2_score_lasso)

# ElasticNet

from sklearn.linear_model import ElasticNet
enet = ElasticNet(alpha=alpha, l1_ratio=0.7)
y_pred_enet = enet.fit(X_train, y_train).predict(X_test)
r2_score_enet = r2_score(y_test, y_pred_enet)
print(enet)
print("r^2 on test data : %f" % r2_score_enet)
plt.plot(enet.coef_, color='lightgreen', linewidth=2,
```

ElasticNet(alpha=0.1, copy_X=True, fit_intercept=True, l1_ratio=0.7, max_iter=1000, normalize=False, positive=False, precompute=False, random_state=None, selection='cyclic', tol=0.0001, warm_start=False)

r^2 on test data : 0.240176

Total running time of the script: (0 minutes 0.202 seconds)
Download Python source code: plot_lasso_and_elasticnet.py
Download IPython notebook: plot_lasso_and_elasticnet.ipynb
18.15 Sparsity Example: Fitting only features 1 and 2

Features 1 and 2 of the diabetes-dataset are fitted and plotted below. It illustrates that although feature 2 has a strong coefficient on the full model, it does not give us much regarding $y$ when compared to just feature 1.

```python
print(__doc__)

# Code source: Gaël Varoquaux
# Modified for documentation by Jaques Grobler
# License: BSD 3 clause

import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
from sklearn import datasets, linear_model

diabetes = datasets.load_diabetes()
indices = (0, 1)

X_train = diabetes.data[:-20, indices]
X_test = diabetes.data[-20:, indices]
y_train = diabetes.target[:-20]
y_test = diabetes.target[-20:]

ols = linear_model.LinearRegression()
ols.fit(X_train, y_train)

Plot the figure

```python
def plot_figs(fig_num, elev, azim, X_train, clf):
    fig = plt.figure(fig_num, figsize=(4, 3))
    plt.clf()
    ax = Axes3D(fig, elev=elev, azim=azim)

    ax.scatter(X_train[:, 0], X_train[:, 1], y_train, c='k', marker='+')
    ax.plot_surface(np.array([[-.1, -.1], [.15, .15]]),
                    np.array([[-.1, .15], [-.1, .15]]),
                    clf.predict(np.array([[-.1, -.1, .15, .15],
                                            [-.1, .15, -.1, .15]]).T
                                ).reshape((2, 2)),
                    alpha=.5)
    ax.set_xlabel('X_1')
    ax.set_ylabel('X_2')
    ax.set_zlabel('Y')
    ax.w_xaxis.set_ticklabels([])
    ax.w_yaxis.set_ticklabels([])
    ax.w_zaxis.set_ticklabels([])

#Generate the three different figures from different views

elev = 43.5
azim = -110
plot_figs(1, elev, azim, X_train, ols)

elev = -.5
azim = 0
plot_figs(2, elev, azim, X_train, ols)
```
18.16 Joint feature selection with multi-task Lasso

The multi-task lasso allows to fit multiple regression problems jointly enforcing the selected features to be the same across tasks. This example simulates sequential measurements, each task is a time instant, and the relevant features vary in amplitude over time while being the same. The multi-task lasso imposes that features that are selected at one time point are select for all time point. This makes feature selection by the Lasso more stable.
import numpy as np

from sklearn.linear_model import MultiTaskLasso, Lasso

rng = np.random.RandomState(42)

# Generate some 2D coefficients with sine waves with random frequency and phase
n_samples, n_features, n_tasks = 100, 30, 40
n_relevant_features = 5
coef = np.zeros((n_tasks, n_features))
times = np.linspace(0, 2 * np.pi, n_tasks)
for k in range(n_relevant_features):
    coef[:, k] = np.sin((1. + rng.randn(1)) * times + 3 * rng.randn(1))
X = rng.randn(n_samples, n_features)
Y = np.dot(X, coef.T) + rng.randn(n_samples, n_tasks)
coef_lasso_ = np.array([Lasso(alpha=0.5).fit(X, y).coef_
                        for y in Y.T])
coef_multi_task_lasso_ = MultiTaskLasso(alpha=1.).fit(X, Y).coef_

Plot support and time series

fig = plt.figure(figsize=(8, 5))
plt.subplot(1, 2, 1)
plt.spy(coef_lasso_)
plt.xlabel('Feature')
plt.ylabel('Time (or Task)')
plt.text(10, 5, 'Lasso')
plt.subplot(1, 2, 2)
plt.spy(coef_multi_task_lasso_)
plt.xlabel('Feature')
plt.ylabel('Time (or Task)')
plt.text(10, 5, 'MultiTaskLasso')
fig.suptitle('Coefficient non-zero location')

feature_to_plot = 0
plt.figure()
lw = 2
plt.plot(coef[:, feature_to_plot], color='seagreen', linewidth=lw, label='Ground truth')
plt.plot(coef_lasso_[:, feature_to_plot], color='cornflowerblue', linewidth=lw, label='Lasso')
plt.plot(coef_multi_task_lasso_[:, feature_to_plot], color='gold', linewidth=lw, label='MultiTaskLasso')
plt.legend(loc='upper center')
plt.axis('tight')
plt.ylim([-1.1, 1.1])
plt.show()
18.17 Comparing various online solvers

An example showing how different online solvers perform on the hand-written digits dataset.
18.17. Comparing various online solvers

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn.model_selection import train_test_split
from sklearn.linear_model import SGDClassifier, Perceptron
from sklearn.linear_model import PassiveAggressiveClassifier
from sklearn.linear_model import LogisticRegression
```

Out:

```
training SGD
training ASGD
training Perceptron
training Passive-Aggressive I
training Passive-Aggressive II
training SAG
```
```python
heldout = [0.95, 0.90, 0.75, 0.50, 0.01]
rounds = 20
digits = datasets.load_digits()
X, y = digits.data, digits.target
classifiers = [
    ("SGD", SGDClassifier()),
    ("ASGD", SGDClassifier(average=True)),
    ("Perceptron", Perceptron()),
    ("Passive-Aggressive I", PassiveAggressiveClassifier(loss='hinge', C=1.0)),
    ("Passive-Aggressive II", PassiveAggressiveClassifier(loss='squared_hinge', C=1.0)),
    ("SAG", LogisticRegression(solver='sag', tol=1e-1, C=1.e4 / X.shape[0]))
]
xx = 1. - np.array(heldout)
for name, clf in classifiers:
    print("training %s" % name)
    rng = np.random.RandomState(42)
    yy = []
    for i in heldout:
        yy_ = []
        for r in range(rounds):
            X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=i, random_state=rng)
            clf.fit(X_train, y_train)
            y_pred = clf.predict(X_test)
            yy_.append(1 - np.mean(y_pred == y_test))
        yy.append(np.mean(yy_))
    plt.plot(xx, yy, label=name)
plt.legend(loc="upper right")
plt.xlabel("Proportion train")
plt.ylabel("Test Error Rate")
plt.show()
```

Total running time of the script: (0 minutes 13.852 seconds)

Download Python source code: plot_sgd_comparison.py
Download IPython notebook: plot_sgd_comparison.ipynb

### 18.18 Robust linear model estimation using RANSAC

In this example we see how to robustly fit a linear model to faulty data using the RANSAC algorithm.
import numpy as np
from matplotlib import pyplot as plt
from sklearn import linear_model, datasets

n_samples = 1000
n_outliers = 50

X, y, coef = datasets.make_regression(n_samples=n_samples, n_features=1,
                                      n_informative=1, noise=10,
                                      coef=True, random_state=0)

np.random.seed(0)

Out:

Estimated coefficients (true, normal, RANSAC):
82.1903908408 [ 54.17236387] [ 82.08533159]
X[:n_outliers] = 3 + 0.5 * np.random.normal(size=(n_outliers, 1))
y[:n_outliers] = -3 + 10 * np.random.normal(size=n_outliers)

# Fit line using all data
model = linear_model.LinearRegression()
model.fit(X, y)

# Robustly fit linear model with RANSAC algorithm
model_ransac = linear_model.RANSACRegressor(linear_model.LinearRegression())
model_ransac.fit(X, y)
inlier_mask = model_ransac.inlier_mask_
outlier_mask = np.logical_not(inlier_mask)

# Predict data of estimated models
line_X = np.arange(-5, 5)
line_y = model.predict(line_X[:, np.newaxis])
line_y_ransac = model_ransac.predict(line_X[:, np.newaxis])

# Compare estimated coefficients
print("Estimated coefficients (true, normal, RANSAC):")
print(coef, model.coef_, model_ransac.estimator_.coef_)

lw = 2
plt.scatter(X[inlier_mask], y[inlier_mask], color='yellowgreen', marker='.', label='Inliers')
plt.scatter(X[outlier_mask], y[outlier_mask], color='gold', marker='.', label='Outliers')
plt.plot(line_X, line_y, color='navy', linestyle='-', linewidth=lw, label='Linear regressor')
plt.plot(line_X, line_y_ransac, color='cornflowerblue', linestyle='-', linewidth=lw, label='RANSAC regressor')
plt.legend(loc='lower right')
plt.show()

Total running time of the script: (0 minutes 0.130 seconds)

Download Python source code: plot_ransac.py
Download IPython notebook: plot_ransac.ipynb

18.19 HuberRegressor vs Ridge on dataset with strong outliers

Fit Ridge and HuberRegressor on a dataset with outliers.

The example shows that the predictions in ridge are strongly influenced by the outliers present in the dataset. The Huber regressor is less influenced by the outliers since the model uses the linear loss for these. As the parameter epsilon is increased for the Huber regressor, the decision function approaches that of the ridge.
# Authors: Manoj Kumar mks542@nyu.edu
# License: BSD 3 clause

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_regression
from sklearn.linear_model import HuberRegressor, Ridge

# Generate toy data.
rng = np.random.RandomState(0)
X, y = make_regression(n_samples=20, n_features=1, random_state=0, noise=4.0,
                       bias=100.0)

# Add four strong outliers to the dataset.
X_outliers = rng.normal(0, 0.5, size=(4, 1))
y_outliers = rng.normal(0, 2.0, size=4)
X_outliers[:2, :] += X.max() + X.mean() / 4.
y_outliers[:2] += y.min() - y.mean() / 4.
y_outliers[2:] += y.max() + y.mean() / 4.
X = np.vstack((X, X_outliers))
y = np.concatenate((y, y_outliers))
plt.plot(X, y, 'b.')
```
# Fit the huber regressor over a series of epsilon values.
colors = ['r-', 'b-', 'y-', 'm-']

x = np.linspace(X.min(), X.max(), 7)
epsilon_values = [1.35, 1.5, 1.75, 1.9]
for k, epsilon in enumerate(epsilon_values):
    huber = HuberRegressor(fit_intercept=True, alpha=0.0, max_iter=100,
                           epsilon=epsilon)
    huber.fit(X, y)
    coef_ = huber.coef_ * x + huber.intercept_
    plt.plot(x, coef_, colors[k], label="huber loss, \$\epsilon =$ %s" % epsilon)

# Fit a ridge regressor to compare it to huber regressor.
ridge = Ridge(fit_intercept=True, alpha=0.0, random_state=0, normalize=True)
ridge.fit(X, y)
coef_ridge = ridge.coef_
coef_ = ridge.coef_ * x + ridge.intercept_
plt.plot(x, coef_, 'g-', label="ridge regression")

plt.title("Comparison of HuberRegressor vs Ridge")
plt.xlabel("X")
plt.ylabel("y")
plt.legend(loc=0)
plt.show()

Total running time of the script: (0 minutes 0.092 seconds)
Download Python source code: plot_huber_vs_ridge.py
Download IPython notebook: plot_huber_vs_ridge.ipynb

## 18.20 SGD: Penalties

Plot the contours of the three penalties.

All of the above are supported by sklearn.linear_model.stochastic_gradient.
from __future__ import division
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt

def l1(xs):
    return np.array([np.sqrt((1 - np.sqrt(x ** 2.0)) ** 2.0) for x in xs])

def l2(xs):
    return np.array([np.sqrt(1.0 - x ** 2.0) for x in xs])

def el(xs, z):
    return np.array([(2 - 2 * x - 2 * z + 4 * x * z -
                      (4 * z ** 2 - 8 * x * z ** 2 + 8 * x ** 2 * z ** 2 - 16 * x ** 2 * z ** 3 + 8 * x * z ** 3 + 4 * x ** 2 * z ** 4) ** (1. / 2) - 2 * x * z ** 2) / (2 - 4 * z) for x in xs])

def cross(ext):

18.20. SGD: Penalties
pl.plot([-ext, ext], [0, 0], "k-")
pl.plot([0, 0], [-ext, ext], "k-")

xs = np.linspace(0, 1, 100)
alpha = 0.501  # 0.5 division through zero

cross(1.2)

l1_color = "navy"
l2_color = "c"
elastic_net_color = "darkorange"
lw = 2

pl.plot(xs, l1(xs), color=l1_color, label="L1", lw=lw)
pl.plot(xs, -1.0 * l1(xs), color=l1_color, lw=lw)
pl.plot(-1 * xs, l1(xs), color=l1_color, lw=lw)
pl.plot(-1 * xs, -1.0 * l1(xs), color=l1_color, lw=lw)

pl.plot(xs, l2(xs), color=l2_color, label="L2", lw=lw)
pl.plot(xs, -1.0 * l2(xs), color=l2_color, lw=lw)
pl.plot(-1 * xs, l2(xs), color=l2_color, lw=lw)
pl.plot(-1 * xs, -1.0 * l2(xs), color=l2_color, lw=lw)

pl.plot(xs, el(xs, alpha), color=elastic_net_color, label="Elastic Net", lw=lw)
pl.plot(xs, -1.0 * el(xs, alpha), color=elastic_net_color, lw=lw)
pl.plot(-1 * xs, el(xs, alpha), color=elastic_net_color, lw=lw)
pl.plot(-1 * xs, -1.0 * el(xs, alpha), color=elastic_net_color, lw=lw)

pl.xlabel(r"$w_0$")
pl.ylabel(r"$w_1$")
pl.legend()

pl.axis("equal")
pl.show()

Total running time of the script: (0 minutes 0.080 seconds)
Download Python source code: plot_sgd_penalties.py
Download IPython notebook: plot_sgd_penalties.ipynb

## 18.21 Bayesian Ridge Regression

Computes a Bayesian Ridge Regression on a synthetic dataset.

See Bayesian Ridge Regression for more information on the regressor.

Compared to the OLS (ordinary least squares) estimator, the coefficient weights are slightly shifted toward zeros, which stabilizes them.

As the prior on the weights is a Gaussian prior, the histogram of the estimated weights is Gaussian.

The estimation of the model is done by iteratively maximizing the marginal log-likelihood of the observations.
Generating simulated data with Gaussian weights

```python
np.random.seed(0)
n_samples, n_features = 100, 100
X = np.random.randn(n_samples, n_features)  # Create Gaussian data
# Create weights with a precision lambda_ of 4.
lambda_ = 4.
w = np.zeros(n_features)
# Only keep 10 weights of interest
relevant_features = np.random.randint(0, n_features, 10)
for i in relevant_features:
    w[i] = stats.norm.rvs(loc=0, scale=1. / np.sqrt(lambda_))  # Create noise with a precision alpha of 50.
alpha_ = 50.
noise = stats.norm.rvs(loc=0, scale=1. / np.sqrt(alpha_), size=n_samples)  # Create the target
y = np.dot(X, w) + noise
```

Fit the Bayesian Ridge Regression and an OLS for comparison

```python
clf = BayesianRidge(compute_score=True)
clf.fit(X, y)
ols = LinearRegression()
ols.fit(X, y)
```

Plot true weights, estimated weights and histogram of the weights

```python
lw = 2
plt.figure(figsize=(6, 5))
plt.title("Weights of the model")
plt.plot(clf.coef_, color='lightgreen', linewidth=lw,
         label="Bayesian Ridge estimate")
plt.plot(w, color='gold', linewidth=lw, label="Ground truth")
plt.plot(ols.coef_, color='navy', linestyle='--', label="OLS estimate")
plt.xlabel("Features")
plt.ylabel("Values of the weights")
plt.legend(loc="best", prop=dict(size=12))

plt.figure(figsize=(6, 5))
plt.title("Histogram of the weights")
plt.hist(clf.coef_, bins=n_features, color='gold', log=True)
plt.scatter(clf.coef_[relevant_features], 5 * np.ones(len(relevant_features)),
            color='navy', label="Relevant features")
plt.xlabel("Features")
plt.ylabel("Values of the weights")
plt.legend(loc="upper left")

plt.figure(figsize=(6, 5))
plt.title("Marginal log-likelihood")
plt.plot(clf.scores_, color='navy', linewidth=lw)
plt.ylabel("Score")
plt.xlabel("Iterations")
```
plt.show()
18.22 Automatic Relevance Determination Regression (ARD)

Fit regression model with Bayesian Ridge Regression.

See Bayesian Ridge Regression for more information on the regressor.

Compared to the OLS (ordinary least squares) estimator, the coefficient weights are slightly shifted toward zeros, which stabilises them.

The histogram of the estimated weights is very peaked, as a sparsity-inducing prior is implied on the weights.

The estimation of the model is done by iteratively maximizing the marginal log-likelihood of the observations.

```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from scipy import stats
from sklearn.linear_model import ARDRegression, LinearRegression

Generating simulated data with Gaussian weights

```Parameters of the example```
np.random.seed(0)
n_samples, n_features = 100, 100
```Create Gaussian data```
X = np.random.randn(n_samples, n_features)
```Create weights with a precision lambda_ of 4.```
lambda_ = 4.
w = np.zeros(n_features)
```Only keep 10 weights of interest```
relevant_features = np.random.randint(0, n_features, 10)
for i in relevant_features:
    w[i] = stats.norm.rvs(loc=0, scale=1. / np.sqrt(lambda_))
```Create noise with a precision alpha of 50.```
alpha_ = 50.
noise = stats.norm.rvs(loc=0, scale=1. / np.sqrt(alpha_), size=n_samples)
```Create the target```
y = np.dot(X, w) + noise

Fit the ARD Regression

```python
clf = ARDRegression(compute_score=True)
clf.fit(X, y)
ols = LinearRegression()
ols.fit(X, y)
```

Plot the true weights, the estimated weights and the histogram of the weights

```python
plt.figure(figsize=(6, 5))
plt.title("Weights of the model")
plt.plot(clf.coef_, color='darkblue', linestyle='-', linewidth=2,
         label="ARD estimate")
plt.plot(ols.coef_, color='yellowgreen', linestyle=':', linewidth=2,
         label="OLS estimate")
plt.plot(w, color='orange', linestyle='--', linewidth=2,
         label="Ground truth")
plt.xlabel("Features")
```
```python
plt.ylabel("Values of the weights")
plt.legend(loc=1)

plt.figure(figsize=(6, 5))
plt.title("Histogram of the weights")
plt.hist(clf.coef_, bins=n_features, color='navy', log=True)
plt.scatter(clf.coef_[relevant_features], 5 * np.ones(len(relevant_features)),
            color='gold', marker='o', label="Relevant features")
plt.ylabel("Features")
plt.xlabel("Values of the weights")
plt.legend(loc=1)

plt.figure(figsize=(6, 5))
plt.title("Marginal log-likelihood")
plt.plot(clf.scores_, color='navy', linewidth=2)
plt.ylabel("Score")
plt.xlabel("Iterations")
plt.show()
```
18.23 Orthogonal Matching Pursuit

Using orthogonal matching pursuit for recovering a sparse signal from a noisy measurement encoded with a dictionary

```python
import __doc__
import matplotlib.pyplot as plt
import numpy as np
from sklearn.linear_model import OrthogonalMatchingPursuit
from sklearn.linear_model import OrthogonalMatchingPursuitCV
from sklearn.datasets import make_sparse_coded_signal

n_components, n_features = 512, 100
n_nonzero_coefs = 17

# generate the data

y, X, w = make_sparse_coded_signal(n_samples=1,
                                  n_components=n_components,
                                  n_features=n_features,
                                  n_nonzero_coefs=n_nonzero_coefs,
                                  random_state=0)

idx, = w.nonzero()  # distort the clean signal

y_noisy = y + 0.05 * np.random.randn(len(y))

# plot the sparse signal
```

Total running time of the script: (0 minutes 0.406 seconds)

Download Python source code: plot_ard.py
Download IPython notebook: plot_ard.ipynb
```python
plt.figure(figsize=(7, 7))
plt.subplot(4, 1, 1)
plt.xlim(0, 512)
plt.title("Sparse signal")
plt.stem(idx, w[idx])

# plot the noise-free reconstruction
omp = OrthogonalMatchingPursuit(n_nonzero_coefs=n_nonzero_coefs)
omp.fit(X, y)
coef = omp.coef_
idx_r, = coef.nonzero()
plt.subplot(4, 1, 2)
plt.xlim(0, 512)
```

![Sparse signal plot](image)
plt.title("Recovered signal from noise-free measurements")
plt.stem(idx_r, coef[idx_r])

# plot the noisy reconstruction

cmp.fit(X, y_noisy)
coef = omp.coef_
idx_r, = coef.nonzero()
plt.subplot(4, 1, 3)
plt.xlim(0, 512)
plt.title("Recovered signal from noisy measurements")
plt.stem(idx_r, coef[idx_r])

# plot the noisy reconstruction with number of non-zeros set by CV
```python
omp_cv = OrthogonalMatchingPursuitCV()
omp_cv.fit(X, y_noisy)
coef = omp_cv.coef_
idx_r, = coef.nonzero()
plt.subplot(4, 1, 4)
plt.xlim(0, 512)
plt.title("Recovered signal from noisy measurements with CV")
plt.stem(idx_r, coef[idx_r])
plt.subplots_adjust(0.06, 0.04, 0.94, 0.90, 0.20, 0.38)
plt.suptitle('Sparse signal recovery with Orthogonal Matching Pursuit',
             fontsize=16)
plt.show()
```
Sparse signal recovery with Orthogonal Matching Pursuit

18.24 Plot multi-class SGD on the iris dataset

Plot decision surface of multi-class SGD on iris dataset. The hyperplanes corresponding to the three one-versus-all (OVA) classifiers are represented by the dashed lines.
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn.linear_model import SGDClassifier

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
                      # avoid this ugly slicing by using a two-dim dataset
y = iris.target
colors = "bry"

# shuffle
idx = np.arange(X.shape[0])
np.random.seed(13)
np.random.shuffle(idx)
X = X[idx]
y = y[idx]

# standardize
mean = X.mean(axis=0)
std = X.std(axis=0)
X = (X - mean) / std
scikit-learn user guide, Release 0.18.2

h = .02  # step size in the mesh

clf = SGDClassifier(alpha=0.001, n_iter=100).fit(X, y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

# Plot the decision boundary. For that, we will assign a color to each
# point in the mesh [x_min, x_max]x[y_min, y_max].
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
# Put the result into a color plot
Z = Z.reshape(xx.shape)
cs = plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)
plt.axis('tight')

# Plot also the training points
for i, color in zip(clf.classes_, colors):
    idx = np.where(y == i)
    plt.scatter(X[idx, 0], X[idx, 1], c=color, label=iris.target_names[i],
                cmap=plt.cm.Paired)
plt.title("Decision surface of multi-class SGD")
plt.axis('tight')

# Plot the three one-against-all classifiers
xmin, xmax = plt.xlim()
ymin, ymax = plt.ylim()
coef = clf.coef_
intercept = clf.intercept_

def plot_hyperplane(c, color):
    def line(x0):
        return -(x0 * coef[c, 0]) - intercept[c] / coef[c, 1]
    plt.plot([xmin, xmax], [line(xmin), line(xmax)],
              ls="--", color=color)

for i, color in zip(clf.classes_, colors):
    plot_hyperplane(i, color)
plt.legend()
plt.show()

Total running time of the script: (0 minutes 0.143 seconds)

Download Python source code: plot_sgd_iris.py
Download IPython notebook: plot_sgd_iris.ipynb

18.25 Theil-Sen Regression

Computes a Theil-Sen Regression on a synthetic dataset.

See Theil-Sen estimator: generalized-median-based estimator for more information on the regressor.
Compared to the OLS (ordinary least squares) estimator, the Theil-Sen estimator is robust against outliers. It has a breakdown point of about 29.3% in case of a simple linear regression which means that it can tolerate arbitrary corrupted data (outliers) of up to 29.3% in the two-dimensional case.

The estimation of the model is done by calculating the slopes and intercepts of a subpopulation of all possible combinations of \( p \) subsample points. If an intercept is fitted, \( p \) must be greater than or equal to \( n_{\text{features}} + 1 \). The final slope and intercept is then defined as the spatial median of these slopes and intercepts.

In certain cases Theil-Sen performs better than RANSAC which is also a robust method. This is illustrated in the second example below where outliers with respect to the x-axis perturb RANSAC. Tuning the residual_threshold parameter of RANSAC remedies this but in general a priori knowledge about the data and the nature of the outliers is needed. Due to the computational complexity of Theil-Sen it is recommended to use it only for small problems in terms of number of samples and features. For larger problems the max_subpopulation parameter restricts the magnitude of all possible combinations of \( p \) subsample points to a randomly chosen subset and therefore also limits the runtime. Therefore, Theil-Sen is applicable to larger problems with the drawback of losing some of its mathematical properties since it then works on a random subset.

```python
# Author: Florian Wilhelm -- <florian.wilhelm@gmail.com>
# License: BSD 3 clause

import time
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression, TheilSenRegressor
from sklearn.linear_model import RANSACRegressor

print(__doc__)

estimators = [('OLS', LinearRegression()),
              ('Theil-Sen', TheilSenRegressor(random_state=42)),
              ('RANSAC', RANSACRegressor(random_state=42)),
             ]

colors = {'OLS': 'turquoise', 'Theil-Sen': 'gold', 'RANSAC': 'lightgreen'}
lw = 2

Outliers only in the y direction

np.random.seed(0)
n_samples = 200
# Linear model y = 3*x + N(2, 0.1**2)
x = np.random.randn(n_samples)
w = 3.
c = 2.
noise = 0.1 * np.random.randn(n_samples)
y = w * x + c + noise
# 10% outliers
y[-20:] += -20 * x[-20:]
X = x[:, np.newaxis]

plt.scatter(x, y, color='indigo', marker='x', s=40)
line_x = np.array([-3, 3])

for name, estimator in estimators:
    t0 = time.time()
    estimator.fit(X, y)
    elapsed_time = time.time() - t0
    y_pred = estimator.predict(line_x.reshape(2, 1))
    plt.plot(line_x, y_pred, color=colors[name], linewidth=lw,
             label='{} (fit time: {:.2fs})'.format(name, elapsed_time))
```

# Chapter 18. Generalized Linear Models
Outliers in the X direction

```
np.random.seed(0)
# Linear model y = 3*x + N(2, 0.1**2)
x = np.random.randn(n_samples)
noise = 0.1 * np.random.randn(n_samples)
y = 3 * x + 2 + noise
# 10% outliers
x[-20:] = 9.9
y[-20:] += 22
X = x[:, np.newaxis]

plt.figure()
plt.scatter(x, y, color='indigo', marker='x', s=40)

line_x = np.array([-3, 10])
for name, estimator in estimators:
t0 = time.time()
estimator.fit(X, y)
elapsed_time = time.time() - t0
y_pred = estimator.predict(line_x.reshape(2, 1))```

```
plt.legend(loc='upper left')
plt.title("Corrupt y")```
Comparison of the sparsity (percentage of zero coefficients) of solutions when L1 and L2 penalty are used for different values of C. We can see that large values of C give more freedom to the model. Conversely, smaller values of C constrain the model more. In the L1 penalty case, this leads to sparser solutions.

We classify 8x8 images of digits into two classes: 0-4 against 5-9. The visualization shows coefficients of the models for varying C.
L1 penalty

C = 100.00

Sparsity with L1 penalty: 6.25%
Score with L1 penalty: 0.9104
Sparsity with L2 penalty: 4.69%
Score with L2 penalty: 0.9098

C = 1.00

Sparsity with L1 penalty: 10.94%
Score with L1 penalty: 0.9104
Sparsity with L2 penalty: 4.69%
Score with L2 penalty: 0.9093

C = 0.01

Sparsity with L1 penalty: 85.94%
Score with L1 penalty: 0.8614
Sparsity with L2 penalty: 4.69%
Score with L2 penalty: 0.8915

print(__doc__)

# Authors: Alexandre Gramfort <alexandre.gramfort@inria.fr>
import numpy as np
import matplotlib.pyplot as plt

from sklearn.linear_model import LogisticRegression
from sklearn import datasets
from sklearn.preprocessing import StandardScaler

digits = datasets.load_digits()
X, y = digits.data, digits.target
X = StandardScaler().fit_transform(X)

# classify small against large digits
y = (y > 4).astype(np.int)

# Set regularization parameter
for i, C in enumerate((100, 1, 0.01)):
    # turn down tolerance for short training time
    clf_l1_LR = LogisticRegression(C=C, penalty='l1', tol=0.01)
    clf_l2_LR = LogisticRegression(C=C, penalty='l2', tol=0.01)
    clf_l1_LR.fit(X, y)
    clf_l2_LR.fit(X, y)
    coef_l1_LR = clf_l1_LR.coef_.ravel()
    coef_l2_LR = clf_l2_LR.coef_.ravel()

    # coef_l1_LR contains zeros due to the
    # L1 sparsity inducing norm

    sparsity_l1_LR = np.mean(coef_l1_LR == 0) * 100
    sparsity_l2_LR = np.mean(coef_l2_LR == 0) * 100

    print("C=%.2f" % C)
    print("Sparsity with L1 penalty: %.2f%%" % sparsity_l1_LR)
    print("score with L1 penalty: %.4f" % clf_l1_LR.score(X, y))
    print("Sparsity with L2 penalty: %.2f%%" % sparsity_l2_LR)
    print("score with L2 penalty: %.4f" % clf_l2_LR.score(X, y))

    l1_plot = plt.subplot(3, 2, 2 * i + 1)
    l2_plot = plt.subplot(3, 2, 2 * (i + 1))
    if i == 0:
        l1_plot.set_title("L1 penalty")
        l2_plot.set_title("L2 penalty")

    l1_plot.imshow(np.abs(coef_l1_LR.reshape(8, 8)), interpolation='nearest',
                   cmap='binary', vmax=1, vmin=0)
    l2_plot.imshow(np.abs(coef_l2_LR.reshape(8, 8)), interpolation='nearest',
                   cmap='binary', vmax=1, vmin=0)

plt.text(-8, 3, "C = %.2f" % C)

l1_plot.set_xticks({})
l1_plot.set_yticks({})
l2_plot.set_xticks({})
l2_plot.set_yticks({})
18.27. Plot multinomial and One-vs-Rest Logistic Regression

Plot decision surface of multinomial and One-vs-Rest Logistic Regression. The hyperplanes corresponding to the three One-vs-Rest (OVR) classifiers are represented by the dashed lines.

Out:

```
training score : 0.995 (multinomial)
training score : 0.976 (ovr)
```
from sklearn.datasets import make_blobs
from sklearn.linear_model import LogisticRegression

centers = [[-5, 0], [0, 1.5], [5, -1]]
X, y = make_blobs(n_samples=1000, centers=centers, random_state=40)
transformation = [[0.4, 0.2], [-0.4, 1.2]]
X = np.dot(X, transformation)

for multi_class in ('multinomial', 'ovr'):
    clf = LogisticRegression(solver='sag', max_iter=100, random_state=42,
                             multi_class=multi_class).fit(X, y)

    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)

    plt.figure()
    plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)
    plt.title("Decision surface of LogisticRegression (%s)" % multi_class)
    plt.axis('tight')

    # Plot also the training points
    colors = "bry"
    for i, color in zip(clf.classes_, colors):
        idx = np.where(y == i)
        plt.scatter(X[idx, 0], X[idx, 1], c=color, cmap=plt.cm.Paired)

    # Plot the three one-against-all classifiers
    xmin, xmax = plt.xlim()
    ymin, ymax = plt.ylim()
    coef = clf.coef_
    intercept = clf.intercept_

    def plot_hyperplane(c, color):
        def line(x0):
            return -(x0 * coef[c, 0] - intercept[c]) / coef[c, 1]
        plt.plot([xmin, xmax], [line(xmin), line(xmax)], ls="--", color=color)

    for i, color in zip(clf.classes_, colors):
18.28 Robust linear estimator fitting

Here a sine function is fit with a polynomial of order 3, for values close to zero.

Robust fitting is demoed in different situations:

- No measurement errors, only modelling errors (fitting a sine with a polynomial)
- Measurement errors in X
- Measurement errors in y

The median absolute deviation to non corrupt new data is used to judge the quality of the prediction.

What we can see that:

- RANSAC is good for strong outliers in the y direction
- TheilSen is good for small outliers, both in direction X and y, but has a break point above which it performs worse than OLS.
- The scores of HuberRegressor may not be compared directly to both TheilSen and RANSAC because it does not attempt to completely filter the outliers but lessen their effect.
```python
from matplotlib import pyplot as plt
import numpy as np
from sklearn.linear_model import (LinearRegression, TheilSenRegressor, RANSACRegressor, HuberRegressor)
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import make_pipeline

np.random.seed(42)
X = np.random.normal(size=400)
y = np.sin(X)
# Make sure that it X is 2D
X = X[:, np.newaxis]
X_test = np.random.normal(size=200)
y_test = np.sin(X_test)
X_test = X_test[:, np.newaxis]
```

Chapter 18. Generalized Linear Models
```python
y_errors = y.copy()
y_errors[::3] = 3

X_errors = X.copy()
X_errors[::3] = 3

y_errors_large = y.copy()
y_errors_large[::3] = 10

X_errors_large = X.copy()
X_errors_large[::3] = 10

estimators = [('OLS', LinearRegression()),
              ('Theil-Sen', TheilSenRegressor(random_state=42)),
              ('RANSAC', RANSACRegressor(random_state=42)),
              ('HuberRegressor', HuberRegressor())]

colors = {'OLS': 'turquoise', 'Theil-Sen': 'gold', 'RANSAC': 'lightgreen',
          'HuberRegressor': 'black'}

linestyle = {'OLS': '-', 'Theil-Sen': '-.', 'RANSAC': '--', 'HuberRegressor': '--'}

lw = 3

x_plot = np.linspace(X.min(), X.max())
for title, this_X, this_y in [('Modeling Errors Only', X, y),
                             ('Corrupt X, Small Deviants', X_errors, y),
                             ('Corrupt y, Small Deviants', X, y_errors),
                             ('Corrupt X, Large Deviants', X_errors_large, y),
                             ('Corrupt y, Large Deviants', X, y_errors_large)]:
    plt.figure(figsize=(5, 4))
    plt.plot(this_X[:, 0], this_y, 'b+')
    for name, estimator in estimators:
        model = make_pipeline(PolynomialFeatures(3), estimator)
        model.fit(this_X, this_y)
        mse = mean_squared_error(model.predict(X_test), y_test)
        y_plot = model.predict(x_plot[:, np.newaxis])
        plt.plot(x_plot, y_plot, color=colors[name], linestyle=linestyle[name],
                 linewidth=lw, label='{}: error = {:.3f}'.format(name, mse))

    legend_title = 'Error of Mean Absolute Deviation to Non-corrupt Data'
    legend = plt.legend(loc='upper right', frameon=False, title=legend_title,
                        prop=dict(size='x-small'))
    plt.xlim(-4, 10.2)
    plt.ylim(-2, 10.2)
    plt.title(title)
    plt.show()
```

Total running time of the script: (0 minutes 5.835 seconds)

Download Python source code: plot_robust_fit.py
Download IPython notebook: plot_robust_fit.ipynb

### 18.29 Lasso and Elastic Net

Lasso and elastic net (L1 and L2 penalisation) implemented using a coordinate descent.

---

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The coefficients can be forced to be positive.

Out:

```
Computing regularization path using the lasso...
Computing regularization path using the positive lasso...
Computing regularization path using the elastic net...
Computing regularization path using the positive elastic net...
```
from itertools import cycle
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import lasso_path, enet_path
from sklearn import datasets
diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target
X /= X.std(axis=0)  # Standardize data (easier to set the l1_ratio parameter)

# Compute paths
eps = 5e-3  # the smaller it is the longer is the path

print("Computing regularization path using the lasso...")
alphas_lasso, coefs_lasso, _ = lasso_path(X, y, eps, fit_intercept=False)

print("Computing regularization path using the positive lasso...")
alphas_positive_lasso, coefs_positive_lasso, _ = lasso_path(X, y, eps, positive=True, fit_intercept=False)

print("Computing regularization path using the elastic net...")
alphas_enet, coefs_enet, _ = enet_path(X, y, eps=eps, l1_ratio=0.8, fit_intercept=False)

print("Computing regularization path using the positive elastic net...")
alphas_positive_enet, coefs_positive_enet, _ = enet_path(X, y, eps=eps, l1_ratio=0.8, positive=True, fit_intercept=False)

# Display results
plt.figure(1)
ax = plt.gca()
colors = cycle(['b', 'r', 'g', 'c', 'k'])
neg_log_alphas_lasso = -np.log10(alphas_lasso)
neg_log_alphas_enet = -np.log10(alphas_enet)
for coef_l, coef_e, c in zip(coefs_lasso, coefs_enet, colors):
    l1 = plt.plot(neg_log_alphas_lasso, coef_l, c=c)
    l2 = plt.plot(neg_log_alphas_enet, coef_e, linestyle='--', c=c)
plt.xlabel('-Log(alpha)')
plt.ylabel('coefficients')
plt.title('Lasso and Elastic-Net Paths')
plt.legend((l1[-1], l2[-1]), ('Lasso', 'Elastic-Net'), loc='lower left')
plt.axis('tight')

plt.figure(2)
ax = plt.gca()
neg_log_alphas_positive_lasso = -np.log10(alphas_positive_lasso)

18.29. Lasso and Elastic Net
for coef_l, coef_pl, c in zip(coefs_lasso, coefs_positive_lasso, colors):
    l1 = plt.plot(neg_log_alphas_lasso, coef_l, c=c)
    l2 = plt.plot(neg_log_alphas_positive_lasso, coef_pl, linestyle='--', c=c)
plt.xlabel('-Log(alpha)')
plt.ylabel('coefficients')
plt.title('Lasso and positive Lasso')
plt.legend((l1[-1], l2[-1]), ('Lasso', 'positive Lasso'), loc='lower left')
plt.axis('tight')
plt.figure(3)
ax = plt.gca()
neg_log_alphas_positive_enet = -np.log10(alphas_positive_enet)
for (coef_e, coef_pe, c) in zip(coefs_enet, coefs_positive_enet, colors):
    l1 = plt.plot(neg_log_alphas_enet, coef_e, c=c)
    l2 = plt.plot(neg_log_alphas_positive_enet, coef_pe, linestyle='--', c=c)
plt.xlabel('-Log(alpha)')
plt.ylabel('coefficients')
plt.title('Elastic-Net and positive Elastic-Net')
plt.legend((l1[-1], l2[-1]), ('Elastic-Net', 'positive Elastic-Net'),
           loc='lower left')
plt.axis('tight')
plt.show()

Total running time of the script: (0 minutes 0.377 seconds)

Download Python source code: plot_lasso_coordinate_descent_path.py
Download IPython notebook: plot_lasso_coordinate_descent_path.ipynb

18.30 Lasso model selection: Cross-Validation / AIC / BIC

Use the Akaike information criterion (AIC), the Bayes Information criterion (BIC) and cross-validation to select an optimal value of the regularization parameter alpha of the Lasso estimator.

Results obtained with LassoLarsIC are based on AIC/BIC criteria.

Information-criterion based model selection is very fast, but it relies on a proper estimation of degrees of freedom, are derived for large samples (asymptotic results) and assume the model is correct, i.e. that the data are actually generated by this model. They also tend to break when the problem is badly conditioned (more features than samples).

For cross-validation, we use 20-fold with 2 algorithms to compute the Lasso path: coordinate descent, as implemented by the LassoCV class, and Lars (least angle regression) as implemented by the LassoLarsCV class. Both algorithms give roughly the same results. They differ with regards to their execution speed and sources of numerical errors.

Lars computes a path solution only for each kink in the path. As a result, it is very efficient when there are only of few kinks, which is the case if there are few features or samples. Also, it is able to compute the full path without setting any meta parameter. On the opposite, coordinate descent compute the path points on a pre-specified grid (here we use the default). Thus it is more efficient if the number of grid points is smaller than the number of kinks in the path. Such a strategy can be interesting if the number of features is really large and there are enough samples to select a large amount. In terms of numerical errors, for heavily correlated variables, Lars will accumulate more errors, while the coordinate descent algorithm will only sample the path on a grid.

Note how the optimal value of alpha varies for each fold. This illustrates why nested-cross validation is necessary when trying to evaluate the performance of a method for which a parameter is chosen by cross-validation: this choice
of parameter may not be optimal for unseen data.

```python
print(__doc__)

# Author: Olivier Grisel, Gael Varoquaux, Alexandre Gramfort
# License: BSD 3 clause

import time

import numpy as np
import matplotlib.pyplot as plt

from sklearn.linear_model import LassoCV, LassoLarsCV, LassoLarsIC
from sklearn import datasets

diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

rng = np.random.RandomState(42)
X = np.c_[X, rng.randn(X.shape[0], 14)]  # add some bad features

# normalize data as done by Lars to allow for comparison
X /= np.sqrt(np.sum(X ** 2, axis=0))

LassoLarsIC: least angle regression with BIC/AIC criterion

model_bic = LassoLarsIC(criterion='bic')
t1 = time.time()
model_bic.fit(X, y)
t_bic = time.time() - t1
alpha_bic_ = model_bic.alpha_

model_aic = LassoLarsIC(criterion='aic')
model_aic.fit(X, y)
alpha_aic_ = model_aic.alpha_

def plot_ic_criterion(model, name, color):
    alpha_ = model.alpha_
    alphas_ = model.alphas_
    criterion_ = model.criterion_
    plt.plot(-np.log10(alphas_), criterion_, '--', color=color, linewidth=3, label='%s criterion' % name)
    plt.axvline(-np.log10(alpha_), color=color, linewidth=3, label='alpha: %s estimate' % name)
    plt.xlabel('-log(alpha)')
    plt.ylabel('criterion')

plt.figure()
plot_ic_criterion(model_aic, 'AIC', 'b')
plot_ic_criterion(model_bic, 'BIC', 'r')
plt.legend()
plt.title('Information-criterion for model selection (training time %.3fs)' % t_bic)
```

18.30. Lasso model selection: Cross-Validation / AIC / BIC
LassoCV: coordinate descent

```python
# Compute paths
print("Computing regularization path using the coordinate descent lasso...")
t1 = time.time()
model = LassoCV(cv=20).fit(X, y)
t_lasso_cv = time.time() - t1

# Display results
m_log_alphas = -np.log10(model.alphas_)
plt.figure()
ymin, ymax = 2300, 3800
plt.plot(m_log_alphas, model.mse_path_, ':')
plt.plot(m_log_alphas, model.mse_path_.mean(axis=-1), 'k',
         label='Average across the folds', linewidth=2)
plt.axvline(-np.log10(model.alpha_), linestyle='--', color='k',
           label='alpha: CV estimate')
plt.legend()
plt.xlabel('-log(alpha)')
plt.ylabel('Mean square error')
plt.title('Mean square error on each fold: coordinate descent' ' (train time: %.2fs)' % t_lasso_cv)
plt.axis('tight')
```

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plt.ylim(ymin, ymax)

Computing regularization path using the coordinate descent lasso...

LassoLarsCV: least angle regression

# Compute paths
print("Computing regularization path using the Lars lasso...")
t1 = time.time()
model = LassoLarsCV(cv=20).fit(X, y)
t_lasso_lars_cv = time.time() - t1

# Display results
m_log_alphas = -np.log10(model.cv_alphas_)

plt.figure()
plt.plot(m_log_alphas, model.cv_mse_path_, ':')
plt.plot(m_log_alphas, model.cv_mse_path_.mean(axis=-1), 'k',
         label='Average across the folds', linewidth=2)
plt.axvline(-np.log10(model.alpha_), linestyle='--', color='k',
            label='alpha CV')
plt.legend()
Given a small number of observations, we want to recover which features of X are relevant to explain y. For this sparse linear models can outperform standard statistical tests if the true model is sparse, i.e. if a small fraction of the features are relevant.
As detailed in the compressive sensing notes, the ability of L1-based approach to identify the relevant variables depends on the sparsity of the ground truth, the number of samples, the number of features, the conditioning of the design matrix on the signal subspace, the amount of noise, and the absolute value of the smallest non-zero coefficient [Wainwright2006](http://statistics.berkeley.edu/sites/default/files/tech-reports/709.pdf).

Here we keep all parameters constant and vary the conditioning of the design matrix. For a well-conditioned design matrix (small mutual incoherence) we are exactly in compressive sensing conditions (i.i.d Gaussian sensing matrix), and L1-recovery with the Lasso performs very well. For an ill-conditioned matrix (high mutual incoherence), regressors are very correlated, and the Lasso randomly selects one. However, randomized-Lasso can recover the ground truth well.

In each situation, we first vary the alpha parameter setting the sparsity of the estimated model and look at the stability scores of the randomized Lasso. This analysis, knowing the ground truth, shows an optimal regime in which relevant features stand out from the irrelevant ones. If alpha is chosen too small, non-relevant variables enter the model. On the opposite, if alpha is selected too large, the Lasso is equivalent to stepwise regression, and thus brings no advantage over a univariate F-test.

In a second time, we set alpha and compare the performance of different feature selection methods, using the area under curve (AUC) of the precision-recall.
print(__doc__)

# Author: Alexandre Gramfort and Gael Varoquaux
# License: BSD 3 clause

import warnings
import matplotlib.pyplot as plt
import numpy as np
from scipy.linalg import pinvh
from sklearn.linear_model import RandomizedLasso, lasso_stability_path, LassoLarsCV
from sklearn.feature_selection import f_regression
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import auc, precision_recall_curve
from sklearn.ensemble import ExtraTreesRegressor
from sklearn.utils.extmath import pinvh
from sklearn.exceptions import ConvergenceWarning

def mutual_incoherence(X_relevant, X_irelevant):
  '''
  Mutual incoherence, as defined by formula (26a) of [Wainwright2006].
  '''
  projector = np.dot(np.dot(X_irelevant.T, X_relevant),
                    pinvh(np.dot(X_relevant.T, X_relevant)))
  return np.max(np.abs(projector).sum(axis=1))
for conditioning in (1, 1e-4):
    ###########################################################################
    # Simulate regression data with a correlated design
    n_features = 501
    n_relevant_features = 3
    noise_level = .2
    coef_min = .2
    # The Donoho-Tanner phase transition is around n_samples=25: below we
    # will completely fail to recover in the well-conditioned case
    n_samples = 25
    block_size = n_relevant_features

    rng = np.random.RandomState(42)

    # The coefficients of our model
    coef = np.zeros(n_features)
    coef[:n_relevant_features] = coef_min + rng.rand(n_relevant_features)

    # The correlation of our design: variables correlated by blocs of 3
    corr = np.zeros((n_features, n_features))
    for i in range(0, n_features, block_size):
        corr[i:i + block_size, i:i + block_size] = 1 - conditioning
    corr.flat[:n_features + 1] = 1
    corr = linalg.cholesky(corr)

    # Our design
    X = rng.normal(size=(n_samples, n_features))
    X = np.dot(X, corr)
    # Keep [Wainwright2006] (26c) constant
    X[:n_relevant_features] /= np.abs(linalg.svdvals(X[:n_relevant_features])).max()
    X = StandardScaler().fit_transform(X.copy())

    # The output variable
    y = np.dot(X, coef)
    y /= np.std(y)
    # We scale the added noise as a function of the average correlation
    # between the design and the output variable
    y += noise_level * rng.normal(size=n_samples)
    mi = mutual_incoherence(X[:, :n_relevant_features],
                            X[:, n_relevant_features:])

    ###########################################################################
    # Plot stability selection path, using a high eps for early stopping
    # of the path, to save computation time
    alpha_grid, scores_path = lasso_stability_path(X, y, random_state=42,
                                                   eps=0.05)

    plt.figure()
    # We plot the path as a function of alpha/alpha_max to the power 1/3: the
    # power 1/3 scales the path less brutally than the log, and enables to
    # see the progression along the path
    hg = plt.plot(alpha_grid[1:] ** .333, scores_path[coef != 0].T[1:], 'r')
    hb = plt.plot(alpha_grid[1:] ** .333, scores_path[coef == 0].T[1:], 'k')
    ymin, ymax = plt.ylim()
    plt.xlabel(r'$(\alpha / \alpha_{max})^{1/3}$')
    plt.ylabel('Stability score: proportion of times selected')
plt.title('Stability Scores Path - Mutual incoherence: %.1f' % mi)
plt.axis('tight')
plt.legend((hg[0], hb[0]), ('relevant features', 'irrelevant features'),
    loc='best')

###########################################################################
# Plot the estimated stability scores for a given alpha
#
# Use 6-fold cross-validation rather than the default 3-fold: it leads to
# a better choice of alpha:
# Stop the user warnings outputs- they are not necessary for the example
# as it is specifically set up to be challenging.
with warnings.catch_warnings():
    warnings.simplefilter('ignore', UserWarning)
    warnings.simplefilter('ignore', ConvergenceWarning)
lars_cv = LassoLarsCV(cv=6).fit(X, y)

# Run the RandomizedLasso: we use a paths going down to .1*alpha_max
# to avoid exploring the regime in which very noisy variables enter
# the model
alphas = np.linspace(lars_cv.alphas_[0], .1 * lars_cv.alphas_[0], 6)
clf = RandomizedLasso(alpha=alphas, random_state=42).fit(X, y)
trees = ExtraTreesRegressor(100).fit(X, y)

# Compare with F-score
F, _ = f_regression(X, y)

plt.figure()
for name, score in [('F-test', F),
                    ('Stability selection', clf.scores_),
                    ('Lasso coefs', np.abs(lars_cv.coef_)),
                    ('Trees', trees.feature_importances_),
                    ]:
    precision, recall, thresholds = precision_recall_curve(coef != 0,
                                                             score)
    plt.semilogy(np.maximum(score / np.max(score), 1e-4),
                 label='%s AUC: %.3f' % (name, auc(recall, precision)))

plt.plot(np.where(coef != 0)[0], [2e-4] * n_relevant_features, 'mo',
          label='Ground truth')
plt.xlabel('Features')
plt.ylabel('Score')
# Plot only the 100 first coefficients
plt.xlim(0, 100)
plt.legend(loc='best')
plt.title('Feature selection scores - Mutual incoherence: %.1f' % mi)
plt.show()
Examples concerning the `sklearn.manifold` module.

### 19.1 Swiss Roll reduction with LLE

An illustration of Swiss Roll reduction with locally linear embedding

```python
Computing LLE embedding
Done. Reconstruction error: 9.45483e-08
```
# Locally linear embedding of the swiss roll

```python
from sklearn import manifold, datasets
X, color = datasets.samples_generator.make_swiss_roll(n_samples=1500)

print("Computing LLE embedding")
X_r, err = manifold.locally_linear_embedding(X, n_neighbors=12,
                                          n_components=2)
print("Done. Reconstruction error: %g" % err)

# Plot result
fig = plt.figure()
try:
    ax = fig.add_subplot(211, projection='3d')
    ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=color, cmap=plt.cm.Spectral)
except:
    ax = fig.add_subplot(211)
    ax.scatter(X[:, 0], X[:, 2], c=color, cmap=plt.cm.Spectral)

ax.set_title("Original data")
ax = fig.add_subplot(212)
ax.scatter(X_r[:, 0], X_r[:, 1], c=color, cmap=plt.cm.Spectral)
plt.axis('tight')
plt.xticks([]), plt.yticks([])
plt.title('Projected data')
plt.show()
```

Total running time of the script: (0 minutes 0.472 seconds)

Download Python source code: plot_swissroll.py

Download IPython notebook: plot_swissroll.ipynb

## 19.2 Multi-dimensional scaling

An illustration of the metric and non-metric MDS on generated noisy data. The reconstructed points using the metric MDS and non metric MDS are slightly shifted to avoid overlapping.
```python
# Author: Nelle Varoquaux <nelle.varoquaux@gmail.com>
# License: BSD

print(__doc__)

import numpy as np
from matplotlib import pyplot as plt
from matplotlib.collections import LineCollection
from sklearn import manifold
from sklearn.metrics import euclidean_distances
from sklearn.decomposition import PCA

n_samples = 20
seed = np.random.RandomState(seed=3)
X_true = seed.randint(0, 20, 2 * n_samples).astype(np.float)
X_true = X_true.reshape((n_samples, 2))  # Center the data
X_true -= X_true.mean()
similarities = euclidean_distances(X_true)

# Add noise to the similarities
noise = np.random.rand(n_samples, n_samples)
noise = noise + noise.T
noise[np.arange(noise.shape[0]), np.arange(noise.shape[0])] = 0

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```
similarities += noise

mds = manifold.MDS(n_components=2, max_iter=3000, eps=1e-9, random_state=seed,
                    dissimilarity="precomputed", n_jobs=1)
pos = mds.fit(similarities).embedding_

nmds = manifold.MDS(n_components=2, metric=False, max_iter=3000, eps=1e-12,
                     dissimilarity="precomputed", random_state=seed, n_jobs=1,
                     n_init=1)
npos = nmds.fit_transform(similarities, init=pos)

# Rescale the data
pos *= np.sqrt((X_true ** 2).sum()) / np.sqrt((pos ** 2).sum())
npos *= np.sqrt((X_true ** 2).sum()) / np.sqrt((npos ** 2).sum())

# Rotate the data
clf = PCA(n_components=2)
X_true = clf.fit_transform(X_true)
pos = clf.fit_transform(pos)
npos = clf.fit_transform(npos)

fig = plt.figure(1)
ax = plt.axes([0., 0., 1., 1.])
s = 100
plt.scatter(X_true[:, 0], X_true[:, 1], color='navy', s=s, lw=0,
            label='True Position')
plt.scatter(pos[:, 0], pos[:, 1], color='turquoise', s=s, lw=0, label='MDS')
plt.scatter(npos[:, 0], npos[:, 1], color='darkorange', s=s, lw=0, label='NMDS')
plt.legend(scatterpoints=1, loc='best', shadow=False)
similarities = similarities.max() / similarities * 100
similarities[np.isinf(similarities)] = 0

# Plot the edges
start_idx, end_idx = np.where(pos)
segments = [(X_true[i, :], X_true[j, :])
            for i in range(len(pos))
            for j in range(len(pos))]
values = np.abs(similarities)
lc = LineCollection(segments,
                    zorder=0, cmap=plt.cm.Blues,
                    norm=plt.Normalize(0, values.max()))
lc.set_array(similarities.flatten())
lc.set_linewidths(0.5 * np.ones(len(segments)))
ax.add_collection(lc)

plt.show()

Total running time of the script: (0 minutes 0.191 seconds)
Download Python source code: plot_mds.py
Download IPython notebook: plot_mds.ipynb
19.3 Comparison of Manifold Learning methods

An illustration of dimensionality reduction on the S-curve dataset with various manifold learning methods. For a discussion and comparison of these algorithms, see the manifold module page.

For a similar example, where the methods are applied to a sphere dataset, see Manifold Learning methods on a severed sphere

Note that the purpose of the MDS is to find a low-dimensional representation of the data (here 2D) in which the distances respect well the distances in the original high-dimensional space, unlike other manifold-learning algorithms, it does not seeks an isotropic representation of the data in the low-dimensional space.

Out:

```
standard: 0.2 sec
ltsa: 0.35 sec
hessian: 0.48 sec
modified: 0.39 sec
Isomap: 0.47 sec
MDS: 2.4 sec
SpectralEmbedding: 0.21 sec
t-SNE: 3.8 sec
```

# Author: Jake Vanderplas -- <vanderplas@astro.washington.edu>

```
print(__doc__)

from time import time
```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.ticker import NullFormatter

from sklearn import manifold, datasets

# Next line to silence pyflakes. This import is needed.
Axes3D

n_points = 1000
X, color = datasets.samples_generator.make_s_curve(n_points, random_state=0)
n_neighbors = 10
n_components = 2

fig = plt.figure(figsize=(15, 8))
plt.suptitle("Manifold Learning with %i points, %i neighbors" 
% (1000, n_neighbors), fontsize=14)

try:
    # compatibility matplotlib < 1.0
    ax = fig.add_subplot(251, projection='3d')
    ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=color, cmap=plt.cm.Spectral)
    ax.view_init(4, -72)
except:
    ax = fig.add_subplot(251, projection='3d')
    plt.scatter(X[:, 0], X[:, 2], c=color, cmap=plt.cm.Spectral)

methods = ['standard', 'ltsa', 'hessian', 'modified']
labels = ['LLE', 'LTSA', 'Hessian LLE', 'Modified LLE']

for i, method in enumerate(methods):
    t0 = time()
    Y = manifold.LocallyLinearEmbedding(n_neighbors, n_components,
                                         eigen_solver='auto',
                                         method=method).fit_transform(X)
    t1 = time()
    print("%s: %.2g sec" % (methods[i], t1 - t0))

    ax = fig.add_subplot(252 + i)
    plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
    plt.title("%s (%.2g sec)" % (labels[i], t1 - t0))
    ax.xaxis.set_major_formatter(NullFormatter())
    ax.yaxis.set_major_formatter(NullFormatter())
    plt.axis('tight')

print("Isomap: %.2g sec" % (t1 - t0))

Y = manifold.Isomap(n_neighbors, n_components).fit_transform(X)
t1 = time()
print("Isomap: %.2g sec" % (t1 - t0))

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An application of the different Manifold learning techniques on a spherical data-set. Here one can see the use of dimensionality reduction in order to gain some intuition regarding the manifold learning methods. Regarding the dataset, the poles are cut from the sphere, as well as a thin slice down its side. This enables the manifold learning techniques to ‘spread it open’ whilst projecting it onto two dimensions.

For a similar example, where the methods are applied to the S-curve dataset, see Comparison of Manifold Learning methods.

Note that the purpose of the MDS is to find a low-dimensional representation of the data (here 2D) in which the distances respect well the distances in the original high-dimensional space, unlike other manifold-learning algorithms.
it does not seek a representation of the data in the low-dimensional space. Here the manifold problem
matches fairly that of representing a flat map of the Earth, as with map projection.

Out:

```plaintext
standard: 0.15 sec
ltsa: 0.23 sec
hessian: 0.35 sec
modified: 0.28 sec
ISO: 0.26 sec
MDS: 1.1 sec
Spectral Embedding: 0.17 sec
t-SNE: 2.4 sec
```

```python
# Author: Jaques Grobler <jaques.grobler@inria.fr>
# License: BSD 3 clause

print(__doc__)

from time import time
import numpy as np
import matplotlib.pyplot as plt
import mpl_toolkits.mplot3d.axes3d as Axes3D
from matplotlib.ticker import NullFormatter
from sklearn import manifold
from sklearn.utils import check_random_state

# Next line to silence pyflakes.
```
Axes3D

# Variables for manifold learning.
n_neighbors = 10
n_samples = 1000

# Create our sphere.
random_state = check_random_state(0)
p = random_state.rand(n_samples) * (2 * np.pi - 0.55)
t = random_state.rand(n_samples) * np.pi

# Sever the poles from the sphere.
indices = (t < (np.pi - (np.pi / 8))) & (t > ((np.pi / 8)))
colors = p[indices]
x, y, z = np.sin(t[indices]) * np.cos(p[indices]), np.sin(t[indices]) * np.cos(p[indices]), np.cos(t[indices])

# Plot our dataset.
fig = plt.figure(figsize=(15, 8))
plt.suptitle("Manifold Learning with %i points, %i neighbors" % (1000, n_neighbors), fontsize=14)
ax = fig.add_subplot(251, projection='3d')
ax.scatter(x, y, z, c=colors, cmap=plt.cm.rainbow)
try:
    # compatibility matplotlib < 1.0
    ax.view_init(40, -10)
except:
    pass

sphere_data = np.array([x, y, z]).T

# Perform Locally Linear Embedding Manifold learning
methods = ['standard', 'ltsa', 'hessian', 'modified']
labels = ['LLE', 'LTSA', 'Hessian LLE', 'Modified LLE']

for i, method in enumerate(methods):
    t0 = time()
    trans_data = manifold\
    .LocallyLinearEmbedding(n_neighbors, 2, method=method).fit_transform(sphere_data).T
    t1 = time()
    print("%s: %.2g sec" % (methods[i], t1 - t0))
    ax = fig.add_subplot(252 + i)
    plt.scatter(trans_data[0], trans_data[1], c=colors, cmap=plt.cm.rainbow)
    plt.title("%s (%.2g sec)" % (labels[i], t1 - t0))
    ax.xaxis.set_major_formatter(NullFormatter())
    ax.yaxis.set_major_formatter(NullFormatter())
    plt.axis('tight')

# Perform Isomap Manifold learning.
t0 = time()
trans_data = manifold.Isomap(n_neighbors, n_components=2)\
    .fit_transform(sphere_data).T
for k, d in enumerate([1, 2, 3]):
    t0 = time()
    trans_data = manifold.Isomap(n_neighbors, n_components=k)\
        .fit_transform(sphere_data).T
    t1 = time()
    print("$s: %.2g sec" % ('ISO', t1 - t0))

19.4. Manifold Learning methods on a severed sphere
ax = fig.add_subplot(257)
plt.scatter(trans_data[0], trans_data[1], c=colors, cmap=plt.cm.rainbow)
plt.title("%s (%.2g sec)" % ('Isomap', t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

# Perform Multi-dimensional scaling.
t0 = time()
mds = manifold.MDS(2, max_iter=100, n_init=1)
trans_data = mds.fit_transform(sphere_data).T
t1 = time()
print("MDS: %.2g sec" % (t1 - t0))

ax = fig.add_subplot(258)
plt.scatter(trans_data[0], trans_data[1], c=colors, cmap=plt.cm.rainbow)
plt.title("MDS (%.2g sec)" % (t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

# Perform Spectral Embedding.
t0 = time()
se = manifold.SpectralEmbedding(n_components=2,
                               n_neighbors=n_neighbors)
trans_data = se.fit_transform(sphere_data).T
t1 = time()
print("Spectral Embedding: %.2g sec" % (t1 - t0))

ax = fig.add_subplot(259)
plt.scatter(trans_data[0], trans_data[1], c=colors, cmap=plt.cm.rainbow)
plt.title("Spectral Embedding (%.2g sec)" % (t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

# Perform t-distributed stochastic neighbor embedding.
t0 = time()
	sne = manifold.TSNE(n_components=2, init='pca', random_state=0)
trans_data = tsne.fit_transform(sphere_data).T
t1 = time()
print("t-SNE: %.2g sec" % (t1 - t0))

ax = fig.add_subplot(2, 5, 10)
plt.scatter(trans_data[0], trans_data[1], c=colors, cmap=plt.cm.rainbow)
plt.title("t-SNE (%.2g sec)" % (t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

plt.show()
19.5 Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...

An illustration of various embeddings on the digits dataset.

The RandomTreesEmbedding, from the sklearn.ensemble module, is not technically a manifold embedding method, as it learn a high-dimensional representation on which we apply a dimensionality reduction method. However, it is often useful to cast a dataset into a representation in which the classes are linearly-separable.

t-SNE will be initialized with the embedding that is generated by PCA in this example, which is not the default setting. It ensures global stability of the embedding, i.e., the embedding does not depend on random initialization.
19.5. Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...
Computing random projection
Computing PCA projection
Computing Linear Discriminant Analysis projection
Computing Isomap embedding
Done.
Computing LLE embedding
Done. Reconstruction error: 1.63544e-06
Computing modified LLE embedding
Done. Reconstruction error: 0.360832
Computing Hessian LLE embedding
Done. Reconstruction error: 0.212807
Computing LTSA embedding
Done. Reconstruction error: 0.212806
Computing MDS embedding
Done. Stress: 150446492.243191
Computing Totally Random Trees embedding
Computing Spectral embedding
Computing t-SNE embedding
print(__doc__)
from time import time
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import offsetbox
from sklearn import (manifold, datasets, decomposition, ensemble,
                     discriminant_analysis, random_projection)

digits = datasets.load_digits(n_class=6)
X = digits.data
y = digits.target
n_samples, n_features = X.shape
n_neighbors = 30

# Scale and visualize the embedding vectors

def plot_embedding(X, title=None):
    x_min, x_max = np.min(X, 0), np.max(X, 0)
    X = (X - x_min) / (x_max - x_min)
    plt.figure()
    ax = plt.subplot(111)
    for i in range(X.shape[0]):
        plt.text(X[i, 0], X[i, 1], str(digits.target[i]),
                 color=plt.cm.Set1(y[i] / 10.),
                 fontdict={'weight': 'bold', 'size': 9})

    if hasattr(offsetbox, 'AnnotationBbox'):
        # only print thumbnails with matplotlib > 1.0
        shown_images = np.array([[1., 1.]])  # just something big
        for i in range(digits.data.shape[0]):
            dist = np.sum((X[i] - shown_images) ** 2, 1)
            if np.min(dist) < 4e-3:
                # don't show points that are too close
                continue
            shown_images = np.r_[shown_images, [X[i]]]
            imagebox = offsetbox.AnnotationBbox(
                offsetbox.OffsetImage(digits.images[i], cmap=plt.cm.gray_r),
                X[i])
            ax.add_artist(imagebox)
    plt.xticks([]), plt.yticks([])
    if title is not None:
        plt.title(title)

# Plot images of the digits
n_img_per_row = 20
img = np.zeros((10 * n_img_per_row, 10 * n_img_per_row))
for i in range(n_img_per_row):
    ix = 10 * i + 1
    for j in range(n_img_per_row):
iy = 10 * j + 1
img[ix:ix + 8, iy:iy + 8] = X[i * n_img_per_row + j].reshape((8, 8))

plt.imshow(img, cmap=plt.cm.binary)
plt.xticks([])
plt.yticks([])
plt.title('A selection from the 64-dimensional digits dataset')

#----------------------------------------------------------------------
# Random 2D projection using a random unitary matrix
print("Computing random projection")
rp = random_projection.SparseRandomProjection(n_components=2, random_state=42)
X_projected = rp.fit_transform(X)
plot_embedding(X_projected, "Random Projection of the digits")

#----------------------------------------------------------------------
# Projection on to the first 2 principal components
print("Computing PCA projection")
t0 = time()
X_pca = decomposition.TruncatedSVD(n_components=2).fit_transform(X)
plot_embedding(X_pca,
"Principal Components projection of the digits (time %.2fs)" %
(time() - t0))

#----------------------------------------------------------------------
# Projection on to the first 2 linear discriminant components
print("Computing Linear Discriminant Analysis projection")
X2 = X.copy()
X2.flat[::X.shape[1] + 1] += 0.01  # Make X invertible

print("Principal Components projection of the digits (time %.2fs)" %
(time() - t0))

#----------------------------------------------------------------------
# Isomap projection of the digits dataset
print("Computing Isomap embedding")
t0 = time()
X_iso = manifold.Isomap(n_neighbors, n_components=2).fit_transform(X)

print("Isomap projection of the digits (time %.2fs)" %
(time() - t0))

#----------------------------------------------------------------------
# Locally linear embedding of the digits dataset
print("Computing LLE embedding")
clf = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
method="standard")
t0 = time()
X_lle = clf.fit_transform(X)

print("Done. Reconstruction error: \$g\" % clf.reconstruction_error_)
plot_embedding(X_lle,
            "Locally Linear Embedding of the digits (time \$%.2fs)\" %
            (time() - t0))

#----------------------------------------------------------------------
# Modified Locally linear embedding of the digits dataset

print("Computing modified LLE embedding")
clf = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                       method='modified')
t0 = time()
X_mlle = clf.fit_transform(X)

print("Done. Reconstruction error: \$g\" % clf.reconstruction_error_)
plot_embedding(X_mlle,
            "Modified Locally Linear Embedding of the digits (time \$%.2fs)\" %
            (time() - t0))

#----------------------------------------------------------------------
# Hessian LLE embedding of the digits dataset

print("Computing Hessian LLE embedding")
clf = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                       method='hessian')
t0 = time()
X_hlle = clf.fit_transform(X)

print("Done. Reconstruction error: \$g\" % clf.reconstruction_error_)
plot_embedding(X_hlle,
            "Hessian Locally Linear Embedding of the digits (time \$%.2fs)\" %
            (time() - t0))

#----------------------------------------------------------------------
# LTSA embedding of the digits dataset

print("Computing LTSA embedding")
clf = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                       method='ltsa')
t0 = time()
X_ltsa = clf.fit_transform(X)

print("Done. Reconstruction error: \$g\" % clf.reconstruction_error_)
plot_embedding(X_ltsa,
            "Local Tangent Space Alignment of the digits (time \$%.2fs)\" %
            (time() - t0))

#----------------------------------------------------------------------
# MDS embedding of the digits dataset

print("Computing MDS embedding")
clf = manifold.MDS(n_components=2, n_init=1, max_iter=100)
t0 = time()
X_mds = clf.fit_transform(X)

print("Done. Stress: \$f\" % clf.stress_)
plot_embedding(X_mds,
            "MDS embedding of the digits (time \$%.2fs)\" %
            (time() - t0))

#----------------------------------------------------------------------
# Random Trees embedding of the digits dataset

19.5. Manifold learning on handwritten digits: Locally Linear Embedding, Isomap... 979
print("Computing Totally Random Trees embedding")
hasher = ensemble.RandomTreesEmbedding(n_estimators=200, random_state=0,
                                          max_depth=5)

t0 = time()
X_transformed = hasher.fit_transform(X)
pca = decomposition.TruncatedSVD(n_components=2)
X_reduced = pca.fit_transform(X_transformed)

plot_embedding(X_reduced,
               "Random forest embedding of the digits (time %.2fs)" %
               (time() - t0))

#----------------------------------------------------------------------
# Spectral embedding of the digits dataset
print("Computing Spectral embedding")
embedder = manifold.SpectralEmbedding(n_components=2, random_state=0,
                                       eigen_solver="arpack")

t0 = time()
X_se = embedder.fit_transform(X)

plot_embedding(X_se,
               "Spectral embedding of the digits (time %.2fs)" %
               (time() - t0))

#----------------------------------------------------------------------
# t-SNE embedding of the digits dataset
print("Computing t-SNE embedding")
tsne = manifold.TSNE(n_components=2, init='pca', random_state=0)
t0 = time()
X_tsne = tsne.fit_transform(X)

plot_embedding(X_tsne,
               "t-SNE embedding of the digits (time %.2fs)" %
               (time() - t0))

plt.show()
Examples concerning the `sklearn.mixture` module.

### 20.1 Density Estimation for a Gaussian mixture

Plot the density estimation of a mixture of two Gaussians. Data is generated from two Gaussians with different centers and covariance matrices.

```python
going to the code here...```
from sklearn import mixture

n_samples = 300

# generate random sample, two components
np.random.seed(0)

# generate spherical data centered on (20, 20)
shifted_gaussian = np.random.randn(n_samples, 2) + np.array([20, 20])

# generate zero centered stretched Gaussian data
C = np.array([[0., -0.7], [3.5, .7]])
stretched_gaussian = np.dot(np.random.randn(n_samples, 2), C)

# concatenate the two datasets into the final training set
X_train = np.vstack([shifted_gaussian, stretched_gaussian])

# fit a Gaussian Mixture Model with two components
clf = mixture.GaussianMixture(n_components=2, covariance_type='full')
clf.fit(X_train)

# display predicted scores by the model as a contour plot
x = np.linspace(-20., 30.)
y = np.linspace(-20., 40.)
X, Y = np.meshgrid(x, y)
XX = np.array([X.ravel(), Y.ravel()]).T
Z = -clf.score_samples(XX)
Z = Z.reshape(X.shape)
CS = plt.contour(X, Y, Z, norm=LogNorm(vmin=1.0, vmax=1000.0),
                 levels=np.logspace(0, 3, 10))
CB = plt.colorbar(CS, shrink=0.8, extend='both')
plt.scatter(X_train[:, 0], X_train[:, 1], .8)
plt.title('Negative log-likelihood predicted by a GMM')
plt.axis('tight')
plt.show()
model is that it can fit full covariance matrices effectively even when there are less examples per cluster than there are dimensions in the data, due to regularization properties of the inference algorithm.

```
import itertools
import numpy as np
from scipy import linalg
import matplotlib.pyplot as plt
import matplotlib as mpl
from sklearn import mixture

color_iter = itertools.cycle(['navy', 'c', 'cornflowerblue', 'gold', 'darkorange'])

def plot_results(X, Y_, means, covariances, index, title):
    splot = plt.subplot(2, 1, 1 + index)
    for i, (mean, covar, color) in enumerate(zip(
        means, covariances, color_iter):
        v, w = linalg.eigh(covar)
        v = 2. * np.sqrt(2.) * np.sqrt(v)
        u = w[0] / linalg.norm(w[0])
        # as the DP will not use every component it has access to
        # unless it needs it, we shouldn't plot the redundant
        # components.
```
```python
if not np.any(Y_ == i):
    continue
plt.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)

# Plot an ellipse to show the Gaussian component
angle = np.arctan(u[1] / u[0])
angle = 180. * angle / np.pi # convert to degrees
ell = mpl.patches.Ellipse(mean, v[0], v[1], 180. + angle, color=color)
ell.set_clip_box(splot.bbox)
ell.set_alpha(0.5)
splot.add_artist(ell)

plt.xlim(-9., 5.)
plt.ylim(-3., 6.)
plt.xticks(()
plt.yticks(()
plt.title(title)

# Number of samples per component
n_samples = 500

# Generate random sample, two components
np.random.seed(0)
C = np.array([[0., -0.1], [1.7, .4]])
X = np.r_[np.dot(np.random.randn(n_samples, 2), C),
          .7 * np.random.randn(n_samples, 2) + np.array([-6, 3])]

# Fit a Gaussian mixture with EM using five components
gmm = mixture.GaussianMixture(n_components=5, covariance_type='full').fit(X)
plot_results(X, gmm.predict(X), gmm.means_, gmm.covariances_, 0,
             'Gaussian Mixture')

# Fit a Dirichlet process Gaussian mixture using five components
dpgmm = mixture.BayesianGaussianMixture(n_components=5,
                                         covariance_type='full').fit(X)
plot_results(X, dpgmm.predict(X), dpgmm.means_, dpgmm.covariances_, 1,
             'Bayesian Gaussian Mixture with a Dirichlet process prior')

plt.show()
```

Total running time of the script: (0 minutes 0.443 seconds)

Download Python source code: plot_gmm.py
Download IPython notebook: plot_gmm.ipynb

### 20.3 Gaussian Mixture Model Selection

This example shows that model selection can be performed with Gaussian Mixture Models using information-theoretic
criteria (BIC). Model selection concerns both the covariance type and the number of components in the model. In that
case, AIC also provides the right result (not shown to save time), but BIC is better suited if the problem is to identify
the right model. Unlike Bayesian procedures, such inferences are prior-free.

In that case, the model with 2 components and full covariance (which corresponds to the true generative model) is
selected.
import numpy as np
import itertools
from scipy import linalg
import matplotlib.pyplot as plt
import matplotlib as mpl
from sklearn import mixture
print(__doc__)

# Number of samples per component
n_samples = 500

# Generate random sample, two components
np.random.seed(0)
C = np.array([[0., -0.1], [1.7, .4]])
X = np.r_[np.dot(np.random.randn(n_samples, 2), C),
          .7 * np.random.randn(n_samples, 2) + np.array([-6, 3])]

lowest_bic = np.infty
bic = []
n_components_range = range(1, 7)
cv_types = ['spherical', 'tied', 'diag', 'full']
for cv_type in cv_types:
    for n_components in n_components_range:
        gmm = mixture.GaussianMixture(n_components=n_components,
                                      covariance_type=cv_type)
        gmm.fit(X)
        bic.append(-gmm.bic(X))

bic = np.array(bic)
 bic = [bic[i] - bic.min() for i in range(bic.shape[0])]
 bic = bic / bic.max()

fig, ax = plt.subplots()
ax.set(tight_layout=True)
ax.plot(n_components_range, bic, 'o-')
ax.set(xlim=(1, 7), ylim=(5000, 9000), xlabel='Number of components',
       ylabel='BIC score per model')
ax.grid()
# Fit a Gaussian mixture with EM

gmm = mixture.GaussianMixture(n_components=n_components,
covariance_type=cv_type)

gmm.fit(X)
bic.append(gmm.bic(X))

if bic[-1] < lowest_bic:
    lowest_bic = bic[-1]
    best_gmm = gmm

bic = np.array(bic)
color_iter = itertools.cycle(['navy', 'turquoise', 'cornflowerblue',
'darkorange'])
clf = best_gmm
bars = []

# Plot the BIC scores
spl = plt.subplot(2, 1, 1)
for i, (cv_type, color) in enumerate(zip(cv_types, color_iter)):
xpos = np.array(n_components_range) + .2 * (i - 2)
bars.append(plt.bar(xpos, bic[i * len(n_components_range):
    (i + 1) * len(n_components_range)],
    width=.2, color=color))
plt.xticks(n_components_range)
plt.ylim([bic.min() * 1.01 - .01 * bic.max(), bic.max()])
plt.title('BIC score per model')
xpos = np.mod(bic.argmin(), len(n_components_range)) + .65 +
    .2 * np.floor(bic.argmin() / len(n_components_range))
plt.text(xpos, bic.min() * 0.97 + .03 * bic.max(), '*', fontsize=14)
spl.set_xlabel('Number of components')
spl.legend([b[0] for b in bars], cv_types)

# Plot the winner
splot = plt.subplot(2, 1, 2)
Y_ = clf.predict(X)
for i, (mean, cov, color) in enumerate(zip(clf.means_, clf.covariances_,
color_iter)):
    v, w = linalg.eigh(cov)
    if not np.any(Y_ == i):
        continue
    plt.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)

    # Plot an ellipse to show the Gaussian component
    angle = np.arctan2(w[0][1], w[0][0])  # convert to degrees
    v = 2. * np.sqrt(2.) * np.sqrt(v)
    ell = mpl.patches.Ellipse(mean, v[0], v[1], 180. + angle, color=color)
    ell.set_clip_box(splot.bbox)
    ell.set_alpha(.5)
splot.add_artist(ell)

plt.xticks(()
plt.yticks(()
plt.title('Selected GMM: full model, 2 components')
plt.subplots_adjust(hspace=.35, bottom=.02)
plt.show()
Download Python source code: plot_gmm_selection.py
Download IPython notebook: plot_gmm_selection.ipynb

20.4 GMM covariances

Demonstration of several covariances types for Gaussian mixture models.

See Gaussian mixture models for more information on the estimator.

Although GMM are often used for clustering, we can compare the obtained clusters with the actual classes from the dataset. We initialize the means of the Gaussians with the means of the classes from the training set to make this comparison valid.

We plot predicted labels on both training and held out test data using a variety of GMM covariance types on the iris dataset. We compare GMMs with spherical, diagonal, full, and tied covariance matrices in increasing order of performance. Although one would expect full covariance to perform best in general, it is prone to overfitting on small datasets and does not generalize well to held out test data.

On the plots, train data is shown as dots, while test data is shown as crosses. The iris dataset is four-dimensional. Only the first two dimensions are shown here, and thus some points are separated in other dimensions.
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np

from sklearn import datasets
from sklearn.mixture import GaussianMixture
from sklearn.model_selection import StratifiedKFold

print(__doc__)

colors = ['navy', 'turquoise', 'darkorange']
def make_ellipses(gmm, ax):
    for n, color in enumerate(colors):
        if gmm.covariance_type == 'full':
            covariances = gmm.covariances_[n][:2, :2]
        elif gmm.covariance_type == 'tied':
            covariances = gmm.covariances_[:2, :2]
        elif gmm.covariance_type == 'diag':
            covariances = np.diag(gmm.covariances_[n][:2])
        elif gmm.covariance_type == 'spherical':
            covariances = np.eye(gmm.means_.shape[1]) * gmm.covariances_[n]
        v, w = np.linalg.eigh(covariances)
        u = w[0] / np.linalg.norm(w[0])
        angle = np.arctan2(u[1], u[0])
        angle = 180 * angle / np.pi # convert to degrees
        v = 2. * np.sqrt(2.) * np.sqrt(v)
        ell = mpl.patches.Ellipse(gmm.means_[n, :2], v[0], v[1],
                                   180 + angle, color=color)
        ell.set_clip_box(ax.bbox)
        ell.set_alpha(0.5)
        ax.add_artist(ell)

iris = datasets.load_iris()

# Break up the dataset into non-overlapping training (75%) and testing # (25%) sets.
skf = StratifiedKFold(n_splits=4)
# Only take the first fold.
train_index, test_index = next(iter(skb.split(iris.data, iris.target)))

X_train = iris.data[train_index]
y_train = iris.target[train_index]
X_test = iris.data[test_index]
y_test = iris.target[test_index]
n_classes = len(np.unique(y_train))

# Try GMMS using different types of covariances.
estimators = dict((cov_type, GaussianMixture(n_components=n_classes,
                                           covariance_type=cov_type, max_iter=20, random_state=0))
                   for cov_type in ['spherical', 'diag', 'tied', 'full'])
n_estimators = len(estimators)

plt.figure(figsize=(3 * n_estimators // 2, 6))
plt.subplots_adjust(bottom=.01, top=0.95, hspace=.15, wspace=.05,
                    left=.01, right=.99)

for index, (name, estimator) in enumerate(estimators.items()):
    # Since we have class labels for the training data, we can
    # initialize the GMM parameters in a supervised manner.
estimator.means_init = np.array([X_train[y_train == i].mean(axis=0)
                                     for i in range(n_classes)])

    # Train the other parameters using the EM algorithm.
estimator.fit(X_train)
This example demonstrates the behavior of Gaussian mixture models fit on data that was not sampled from a mixture of Gaussian random variables. The dataset is formed by 100 points loosely spaced following a noisy sine curve. There is therefore no ground truth value for the number of Gaussian components.

The first model is a classical Gaussian Mixture Model with 10 components fit with the Expectation-Maximization algorithm.

The second model is a Bayesian Gaussian Mixture Model with a Dirichlet process prior fit with variational inference. The low value of the concentration prior makes the model favor a lower number of active components. This models “decides” to focus its modeling power on the big picture of the structure of the dataset: groups of points with alternating directions modeled by non-diagonal covariance matrices. Those alternating directions roughly capture the alternating nature of the original sine signal.

The third model is also a Bayesian Gaussian mixture model with a Dirichlet process prior but this time the value of the concentration prior is higher giving the model more liberty to model the fine-grained structure of the data. The result is a mixture with a larger number of active components that is similar to the first model where we arbitrarily decided to fix the number of components to 10.
Which model is the best is a matter of subjective judgement: do we want to favor models that only capture the big picture to summarize and explain most of the structure of the data while ignoring the details or do we prefer models that closely follow the high density regions of the signal?

The last two panels show how we can sample from the last two models. The resulting samples distributions do not look exactly like the original data distribution. The difference primarily stems from the approximation error we made by using a model that assumes that the data was generated by a finite number of Gaussian components instead of a continuous noisy sine curve.

```python
import itertools
import numpy as np
from scipy import linalg
import matplotlib.pyplot as plt
import matplotlib as mpl
```

20.5. Gaussian Mixture Model Sine Curve
from sklearn import mixture

print(__doc__)

color_iter = itertools.cycle(['navy', 'c', 'cornflowerblue', 'gold', 'darkorange'])

def plot_results(X, Y, means, covariances, index, title):
splot = plt.subplot(5, 1, 1 + index)
    for i, (mean, covar, color) in enumerate(zip(means, covariances, color_iter)):
        v, w = linalg.eigh(covar)
        v = 2. * np.sqrt(2.) * np.sqrt(v)
        u = w[0] / linalg.norm(w[0])
        # as the DP will not use every component it has access to
        # unless it needs it, we shouldn't plot the redundant
        # components.
        if not np.any(Y == i):
            continue
        plt.scatter(X[Y == i, 0], X[Y == i, 1], .8, color=color)
        # Plot an ellipse to show the Gaussian component
        angle = np.arctan(u[1] / u[0])
        angle = 180. * angle / np.pi  # convert to degrees
        ell = mpl.patches.Ellipse(mean, v[0], v[1], 180. + angle, color=color)
        ell.set_clip_box(splot.bbox)
        ell.set_alpha(0.5)
        splot.add_artist(ell)
    plt.xlim(-6., 4. * np.pi - 6.)
    plt.ylim(-5., 5.)
    plt.title(title)
    plt.xticks(())
    plt.yticks(())

def plot_samples(X, Y, n_components, index, title):
splot = plt.subplot(5, 1, 4 + index)
    for i, color in zip(range(n_components), color_iter):
        # as the DP will not use every component it has access to
        # unless it needs it, we shouldn't plot the redundant
        # components.
        if not np.any(Y == i):
            continue
        plt.scatter(X[Y == i, 0], X[Y == i, 1], .8, color=color)
    plt.xlim(-6., 4. * np.pi - 6.)
    plt.ylim(-5., 5.)
    plt.title(title)
    plt.xticks(())
    plt.yticks(())

# Parameters
n_samples = 100
# Generate random sample following a sine curve
np.random.seed(0)
X = np.zeros((n_samples, 2))
step = 4. * np.pi / n_samples
for i in range(X.shape[0]):
    x = i * step - 6.
    X[i, 0] = x + np.random.normal(0, 0.1)
    X[i, 1] = 3. * (np.sin(x) + np.random.normal(0, .2))

plt.figure(figsize=(10, 10))
plt.subplots_adjust(bottom=.04, top=0.95, hspace=.2, wspace=.05,
left=.03, right=.97)

# Fit a Gaussian mixture with EM using ten components
    gmm = mixture.GaussianMixture(n_components=10, covariance_type='full',
        max_iter=100).fit(X)
    plot_results(X, gmm.predict(X), gmm.means_, gmm.covariances_, 0,
        'Expectation-maximization')

dpgmm = mixture.BayesianGaussianMixture(  
    n_components=10, covariance_type='full', weight_concentration_prior=1e-2,
    weight_concentration_prior_type='dirichlet_process',
    mean_precision_prior=1e-2, covariance_prior=1e0 * np.eye(2),
    init_params="random", max_iter=100, random_state=2).fit(X)
    plot_results(X, dpgmm.predict(X), dpgmm.means_, dpgmm.covariances_, 1,
        "Bayesian Gaussian mixture models with a Dirichlet process prior "
        r"for $\gamma_0=0.01$.")

    X_s, y_s = dpgmm.sample(n_samples=2000)
    plot_samples(X_s, y_s, dpgmm.n_components, 0,
        "Gaussian mixture with a Dirichlet process prior "
        r"for $\gamma_0=0.01$ sampled with $2000$ samples.")

dpgmm = mixture.BayesianGaussianMixture(  
    n_components=10, covariance_type='full', weight_concentration_prior=1e+2,
    weight_concentration_prior_type='dirichlet_process',
    mean_precision_prior=1e-2, covariance_prior=1e0 * np.eye(2),
    init_params="kmeans", max_iter=100, random_state=2).fit(X)
    plot_results(X, dpgmm.predict(X), dpgmm.means_, dpgmm.covariances_, 2,
        "Bayesian Gaussian mixture models with a Dirichlet process prior "
        r"for $\gamma_0=100$")

    X_s, y_s = dpgmm.sample(n_samples=2000)
    plot_samples(X_s, y_s, dpgmm.n_components, 1,
        "Gaussian mixture with a Dirichlet process prior "
        r"for $\gamma_0=100$ sampled with $2000$ samples.")

plt.show()
20.6 Concentration Prior Type Analysis of Variation Bayesian Gaussian Mixture

This example plots the ellipsoids obtained from a toy dataset (mixture of three Gaussians) fitted by the `BayesianGaussianMixture` class models with a Dirichlet distribution prior (`weight_concentration_prior_type='dirichlet_distribution'`) and a Dirichlet process prior (`weight_concentration_prior_type='dirichlet_process'`). On each figure, we plot the results for three different values of the weight concentration prior.

The `BayesianGaussianMixture` class can adapt its number of mixture components automatically. The parameter `weight_concentration_prior` has a direct link with the resulting number of components with non-zero weights. Specifying a low value for the concentration prior will make the model put most of the weight on few components set the remaining components weights very close to zero. High values of the concentration prior will allow a larger number of components to be active in the mixture.

The Dirichlet process prior allows to define an infinite number of components and automatically selects the correct number of components: it activates a component only if it is necessary.

On the contrary the classical finite mixture model with a Dirichlet distribution prior will favor more uniformly weighted components and therefore tends to divide natural clusters into unnecessary sub-components.

```python
# Author: Thierry Guillemot <thierry.guillemot.work@gmail.com>
# License: BSD 3 clause

import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec

from sklearn.mixture import BayesianGaussianMixture

print(__doc__)
```

```python
from sklearn.mixture import BayesianGaussianMixture
```

```python
print(__doc__)
```
```python
def plot_ellipses(ax, weights, means, covars):
    for n in range(means.shape[0]):
        eig_vals, eig_vecs = np.linalg.eigh(covars[n])
        unit_eig_vec = eig_vecs[0] / np.linalg.norm(eig_vecs[0])
        angle = np.arctan2(unit_eig_vec[1], unit_eig_vec[0])
        # Ellipse needs degrees
        angle = 180 * angle / np.pi
        # eigenvector normalization
        eig_vals = 2 * np.sqrt(2) * np.sqrt(eig_vals)
        ell = mpl.patches.Ellipse(means[n], eig_vals[0], eig_vals[1],
                                   180 + angle)
        ell.set_clip_box(ax.bbox)
        ell.set_alpha(weights[n])
        ell.set_facecolor('#56B4E9')
        ax.add_artist(ell)

def plot_results(ax1, ax2, estimator, X, y, title, plot_title=False):
    ax1.set_title(title)
    ax1.scatter(X[:, 0], X[:, 1], s=5, marker='o', color=colors[y], alpha=0.8)
    ax1.set_xlim(-2., 2.)
    ax1.set_ylim(-3., 3.)
    ax1.set_xticks(())
    ax1.set_yticks(())
    plot_ellipses(ax1, estimator.weights_, estimator.means_,
                  estimator.covariances_)
    for k, w in enumerate(estimator.weights_):
        ax2.bar(k - .45, w, width=0.9, color='#56B4E9', zorder=3)
        ax2.text(k, w + 0.007, "%.1f%%" % (w * 100.),
                  horizontalalignment='center')
    ax2.set_xlim(-.6, 2 * n_components - .4)
    ax2.set_ylim(0., 1.1)
    ax2.tick_params(axis='y', which='both', left='off',
                    right='off', labelleft='off')
    ax2.tick_params(axis='x', which='both', top='off')
    if plot_title:
        ax1.set_ylabel('Estimated Mixtures')
        ax2.set_ylabel('Weight of each component')

# Parameters of the dataset
random_state, n_components, n_features = 2, 3, 2
colors = np.array([ '#0072B2', '#F0E442', '#D55E00' ])
covars = np.array([[[.7, .0], [.0, .1]],
                   [[.5, .0], [.0, .1]],
                   [[.5, .0], [.0, .1]]])
samples = np.array([200, 500, 200])
means = np.array([ [.0, -.70],
                  [.0, .0],
                  [.0, .70]])

# mean_precision_prior = 0.8 to minimize the influence of the prior
estimators = [
    ('Finite mixture with a Dirichlet distribution\nprior and "
``````
Infinite mixture with a Dirichlet process prior and $\gamma_0=$
BayesianGaussianMixture(
    weight_concentration_prior_type="dirichlet_process",
    n_components=2 * n_components, reg_covar=0, init_params='random',
    max_iter=1500, mean_precision_prior=.8,
    random_state=random_state), [1, 1000, 100000])

# Generate data
rng = np.random.RandomState(random_state)
X = np.vstack([
    rng.multivariate_normal(means[j], covars[j], samples[j])
    for j in range(n_components)])
y = np.concatenate([j * np.ones(samples[j], dtype=int)
    for j in range(n_components)])

# Plot results in two different figures
for (title, estimator, concentrations_prior) in estimators:
    plt.figure(figsize=(4.7 * 3, 8))
    plt.subplots_adjust(bottom=.04, top=0.90, hspace=.05, wspace=.05,
                        left=.03, right=.99)
    gs = gridspec.GridSpec(3, len(concentrations_prior))
    for k, concentration in enumerate(concentrations_prior):
        estimator.weight_concentration_prior = concentration
        estimator.fit(X)
        plot_results(plt.subplot(gs[0:2, k]), plt.subplot(gs[2, k]), estimator,
                     X, y, r"%.1e" % (title, concentration),
                     plot_title=k == 0)
plt.show()
Examples related to the `sklearn.model_selection` module.

### 21.1 Plotting Validation Curves

In this plot you can see the training scores and validation scores of an SVM for different values of the kernel parameter gamma. For very low values of gamma, you can see that both the training score and the validation score are low. This is called underfitting. Medium values of gamma will result in high values for both scores, i.e. the classifier is performing fairly well. If gamma is too high, the classifier will overfit, which means that the training score is good but the validation score is poor.
21.2 Underfitting vs. Overfitting

This example demonstrates the problems of underfitting and overfitting and how we can use linear regression with polynomial features to approximate nonlinear functions. The plot shows the function that we want to approximate, which is a part of the cosine function. In addition, the samples from the real function and the approximations of different models are displayed. The models have polynomial features of different degrees. We can see that a linear function (polynomial with degree 1) is not sufficient to fit the training samples. This is called underfitting. A polynomial of degree 4 approximates the true function almost perfectly. However, for higher degrees the model will overfit the training data, i.e. it learns the noise of the training data. We evaluate quantitatively overfitting / underfitting by using cross-validation. We calculate the mean squared error (MSE) on the validation set, the higher, the less likely the model generalizes correctly from the training data.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_val_score

np.random.seed(0)
n_samples = 30
degrees = [1, 4, 15]
true_fun = lambda X: np.cos(1.5 * np.pi * X)
X = np.sort(np.random.rand(n_samples))
y = true_fun(X) + np.random.randn(n_samples) * 0.1

plt.figure(figsize=(14, 5))
for i in range(len(degrees)):
    ax = plt.subplot(1, len(degrees), i + 1)
    plt.setp(ax, xticks=(), yticks=())

    polynomial_features = PolynomialFeatures(degree=degrees[i],
                                               include_bias=False)
    linear_regression = LinearRegression()
    pipeline = Pipeline(["polynomial_features", polynomial_features],
                        ["linear_regression", linear_regression])
    pipeline.fit(X[:, np.newaxis], y)
    # Evaluate the models using cross-validation
    scores = cross_val_score(pipeline, X[:, np.newaxis], y,
                              scoring="neg_mean_squared_error", cv=10)

    X_test = np.linspace(0, 1, 100)
    plt.plot(X_test, pipeline.predict(X_test[:, np.newaxis]), label="Model")
    plt.plot(X_test, true_fun(X_test), label="True function")
    plt.scatter(X, y, label="Samples")
    plt.xlabel("x")
    plt.ylabel("y")
    plt.xlim(0, 1)
    plt.ylim(-2, 2)
21.3 Train error vs Test error

Illustration of how the performance of an estimator on unseen data (test data) is not the same as the performance on training data. As the regularization increases the performance on train decreases while the performance on test is optimal within a range of values of the regularization parameter. The example with an Elastic-Net regression model and the performance is measured using the explained variance a.k.a. $R^2$.

```python
print(__doc__)

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD 3 clause
import numpy as np
from sklearn import linear_model

Generate sample data

```r
n_samples_train, n_samples_test, n_features = 75, 150, 500
np.random.seed(0)
coef = np.random.randn(n_features)
coef[50:] = 0.0  # only the top 10 features are impacting the model
X = np.random.randn(n_samples_train + n_samples_test, n_features)
y = np.dot(X, coef)

# Split train and test data
X_train, X_test = X[:n_samples_train], X[n_samples_train:]
y_train, y_test = y[:n_samples_train], y[n_samples_train:]

Compute train and test errors

```r
alphas = np.logspace(-5, 1, 60)
enet = linear_model.ElasticNet(l1_ratio=0.7)
train_errors = list()
test_errors = list()
for alpha in alphas:
    net.set_params(alpha=alpha)
    net.fit(X_train, y_train)
    train_errors.append(net.score(X_train, y_train))
    test_errors.append(net.score(X_test, y_test))

i_alpha_optim = np.argmax(test_errors)
alpha_optim = alphas[i_alpha_optim]
print("Optimal regularization parameter : %s" % alpha_optim)

# Estimate the coef_ on full data with optimal regularization parameter
```
enet.set_params(alpha=alpha_optim)
coef_ = enet.fit(X, y).coef_

Out:

Optimal regularization parameter : 0.000335292414925

Plot results functions

```python
import matplotlib.pyplot as plt
plt.subplot(2, 1, 1)
plt.semilogx(alphas, train_errors, label='Train')
plt.semilogx(alphas, test_errors, label='Test')
plt.vlines(alpha_optim, plt.ylim()[0], np.max(test_errors), color='k',
        linewidth=3, label='Optimum on test')
plt.legend(loc='lower left')
plt.ylim([0, 1.2])
plt.xlabel('Regularization parameter')
plt.ylabel('Performance')

# Show estimated coef_ vs true coef
plt.subplot(2, 1, 2)
plt.plot(coef, label='True coef')
plt.plot(coef_, label='Estimated coef')
plt.legend()
plt.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.26)
plt.show()
```
21.4 Receiver Operating Characteristic (ROC) with cross validation

Example of Receiver Operating Characteristic (ROC) metric to evaluate classifier output quality using cross-validation. ROC curves typically feature true positive rate on the Y axis, and false positive rate on the X axis. This means that the top left corner of the plot is the “ideal” point - a false positive rate of zero, and a true positive rate of one. This is not very realistic, but it does mean that a larger area under the curve (AUC) is usually better.

The “steepness” of ROC curves is also important, since it is ideal to maximize the true positive rate while minimizing the false positive rate.

This example shows the ROC response of different datasets, created from K-fold cross-validation. Taking all of these curves, it is possible to calculate the mean area under curve, and see the variance of the curve when the training set is split into different subsets. This roughly shows how the classifier output is affected by changes in the training data, and how different the splits generated by K-fold cross-validation are from one another.
print(__doc__)

import numpy as np
from scipy import interp
import matplotlib.pyplot as plt
from itertools import cycle
from sklearn import svm, datasets
from sklearn.metrics import roc_curve, auc
from sklearn.model_selection import StratifiedKFold

Data IO and generation

# import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target
X, y = X[y != 2], y[y != 2]
n_samples, n_features = X.shape

# Add noisy features
random_state = np.random.RandomState(0)
X = np.c_[X, random_state.randn(n_samples, 200 * n_features)]

Classification and ROC analysis

# Run classifier with cross-validation and plot ROC curves
cv = StratifiedKFold(n_splits=6)
classifier = svm.SVC(kernel='linear', probability=True,
                     random_state=random_state)
mean_tpr = 0.0
mean_fpr = np.linspace(0, 1, 100)

colors = cycle(['cyan', 'indigo', 'seagreen', 'yellow', 'blue', 'darkorange'])
lw = 2

i = 0
for (train, test), color in zip(cv.split(X, y), colors):
    probas_ = classifier.fit(X[train], y[train]).predict_proba(X[test])
    # Compute ROC curve and area the curve
    fpr, tpr, thresholds = roc_curve(y[test], probas_[:, 1])
    mean_tpr += interp(mean_fpr, fpr, tpr)
    mean_tpr[0] = 0.0
    roc_auc = auc(fpr, tpr)
    plt.plot(fpr, tpr, lw=lw, color=color,
             label='ROC fold %d (area = %0.2f)' % (i, roc_auc))
    i += 1
plt.plot([0, 1], [0, 1], linestyle='--', lw=lw, color='k',
         label='Luck')
mean_tpr /= cv.get_n_splits(X, y)
mean_tpr[-1] = 1.0

---

See also sklearn.metrics.auc_score, sklearn.model_selection.cross_val_score, Receiver Operating Characteristic (ROC),
mean_auc = auc(mean_fpr, mean_tpr)
plt.plot(mean_fpr, mean_tpr, color='g', linestyle='--',
         label='Mean ROC (area = %0.2f)' % mean_auc, lw=lw)
plt.xlim([-0.05, 1.05])
plt.ylim([-0.05, 1.05])
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Receiver operating characteristic example')
plt.legend(loc="lower right")
plt.show()
The performance of the selected hyper-parameters and trained model is then measured on a dedicated evaluation set that was not used during the model selection step.

More details on tools available for model selection can be found in the sections on Cross-validation: evaluating estimator performance and Tuning the hyper-parameters of an estimator.

```python
from __future__ import print_function

from sklearn import datasets
from sklearn.model_selection import train_test_split
from sklearn.model_selection import GridSearchCV
from sklearn.metrics import classification_report
from sklearn.svm import SVC

print(__doc__)

# Loading the Digits dataset
digits = datasets.load_digits()

# To apply an classifier on this data, we need to flatten the image, to
# turn the data in a (samples, feature) matrix:
n_samples = len(digits.images)
X = digits.images.reshape((n_samples, -1))
y = digits.target

# Split the dataset in two equal parts
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.5, random_state=0)

# Set the parameters by cross-validation
tuned_parameters = [{'kernel': ['rbf'], 'gamma': [1e-3, 1e-4],
                    'C': [1, 10, 100, 1000]},
                   {'kernel': ['linear'], 'C': [1, 10, 100, 1000]}]

scores = ['precision', 'recall']

for score in scores:
    print("# Tuning hyper-parameters for %s" % score)
    print()
    clf = GridSearchCV(SVC(C=1), tuned_parameters, cv=5,
                       scoring='%s_macro' % score)
    clf.fit(X_train, y_train)

    print("Best parameters set found on development set:")
    print()
    print(clf.best_params_)
    print("Grid scores on development set:")
    print()
    means = clf.cv_results_['mean_test_score']
    stds = clf.cv_results_['std_test_score']
    for mean, std, params in zip(means, stds, clf.cv_results_['params']):
        print("%0.3f (+/-%0.03f) for %r" % (mean, std * 2, params))
    print()

    print("Detailed classification report:")
    print()
```

21.5. Parameter estimation using grid search with cross-validation 1005
print("The model is trained on the full development set.")
print("The scores are computed on the full evaluation set.")
print()
y_true, y_pred = y_test, clf.predict(X_test)
print(classification_report(y_true, y_pred))
print()

# Note the problem is too easy: the hyperparameter plateau is too flat and the
# output model is the same for precision and recall with ties in quality.

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: grid_search_digits.py
Download IPython notebook: grid_search_digits.ipynb

## 21.6 Confusion matrix

Example of confusion matrix usage to evaluate the quality of the output of a classifier on the iris data set. The diagonal elements represent the number of points for which the predicted label is equal to the true label, while off-diagonal elements are those that are mislabeled by the classifier. The higher the diagonal values of the confusion matrix the better, indicating many correct predictions.

The figures show the confusion matrix with and without normalization by class support size (number of elements in each class). This kind of normalization can be interesting in case of class imbalance to have a more visual interpretation of which class is being misclassified.

Here the results are not as good as they could be as our choice for the regularization parameter C was not the best. In real life applications this parameter is usually chosen using Tuning the hyper-parameters of an estimator.
Out:

Confusion matrix, without normalization

\[
\begin{bmatrix}
13 & 0 & 0 \\
0 & 10 & 6 \\
0 & 0 & 9 \\
\end{bmatrix}
\]

Normalized confusion matrix

\[
\begin{bmatrix}
1.0 & 0.0 & 0.0 \\
0.0 & 0.62 & 0.38 \\
0.0 & 0.0 & 1.0 \\
\end{bmatrix}
\]

```python
print(__doc__)

import itertools
import numpy as np
import matplotlib.pyplot as plt

from sklearn import svm, datasets
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix

# import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target
class_names = iris.target_names

# Split the data into a training set and a test set
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)

# Run classifier, using a model that is too regularized (C too low) to see
# the impact on the results
classifier = svm.SVC(kernel='linear', C=0.01)
y_pred = classifier.fit(X_train, y_train).predict(X_test)

def plot_confusion_matrix(cm, classes, normalize=False,
                          cmap=plt.cm.Blues):
    # Plot non-normalized confusion matrix
    plt.figure()
    plt.imshow(cm, interpolation='nearest', cmap=cmap)
    plt.title(f'Normalized confusion matrix

Confusion matrix, without normalization

\[
\begin{bmatrix}
13 & 0 & 0 \\
0 & 10 & 6 \\
0 & 0 & 9 \\
\end{bmatrix}
\]

Normalized confusion matrix

\[
\begin{bmatrix}
1.0 & 0.0 & 0.0 \\
0.0 & 0.62 & 0.38 \\
0.0 & 0.0 & 1.0 \\
\end{bmatrix}
\]

```
This function prints and plots the confusion matrix. Normalization can be applied by setting 'normalize=True'.

```python
plt.imshow(cm, interpolation='nearest', cmap=cmap)
plt.title(title)
plt.colorbar()
tick_marks = np.arange(len(classes))
plt.xticks(tick_marks, classes, rotation=45)
plt.yticks(tick_marks, classes)

if normalize:
    cm = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]
    print("Normalized confusion matrix")
else:
    print('Confusion matrix, without normalization')

print(cm)

thresh = cm.max() / 2.
for i, j in itertools.product(range(cm.shape[0]), range(cm.shape[1])):
    plt.text(j, i, cm[i, j],
             horizontalalignment="center",
             color="white" if cm[i, j] > thresh else "black")

plt.tight_layout()
plt.ylabel('True label')
plt.xlabel('Predicted label')
```

Total running time of the script: (0 minutes 0.302 seconds)

Download Python source code: plot_confusion_matrix.py
Download IPython notebook: plot_confusion_matrix.ipynb
21.7 Comparing randomized search and grid search for hyperparameter estimation

Compare randomized search and grid search for optimizing hyperparameters of a random forest. All parameters that influence the learning are searched simultaneously (except for the number of estimators, which poses a time / quality tradeoff).

The randomized search and the grid search explore exactly the same space of parameters. The result in parameter settings is quite similar, while the run time for randomized search is drastically lower.

The performance is slightly worse for the randomized search, though this is most likely a noise effect and would not carry over to a held-out test set.

Note that in practice, one would not search over this many different parameters simultaneously using grid search, but pick only the ones deemed most important.

```python
print(__doc__)

import numpy as np
from time import time
from scipy.stats import randint as sp_randint
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import RandomizedSearchCV
from sklearn.datasets import load_digits
from sklearn.ensemble import RandomForestClassifier

# get some data
digits = load_digits()
X, y = digits.data, digits.target

# build a classifier
clf = RandomForestClassifier(n_estimators=20)

# Utility function to report best scores
def report(results, n_top=3):
    for i in range(1, n_top + 1):
        candidates = np.flatnonzero(results['rank_test_score'] == i)
        for candidate in candidates:
            print("Model with rank: {0}".format(i))
            print("Mean validation score: {0:.3f} (std: {1:.3f})".format(
                results['mean_test_score'][candidate],
                results['std_test_score'][candidate]))
            print("Parameters: {0}".format(results['params'][candidate]))
            print("")

# specify parameters and distributions to sample from
param_dist = {"max_depth": [3, None],
              "max_features": sp_randint(1, 11),
              "min_samples_split": sp_randint(1, 11),
              "min_samples_leaf": sp_randint(1, 11),
              "bootstrap": [True, False],
              "criterion": ["gini", "entropy"]}

# run randomized search
```
```python
n_iter_search = 20
random_search = RandomizedSearchCV(clf, param_distributions=param_dist,
                                     n_iter=n_iter_search)

start = time()
random_search.fit(X, y)
print("RandomizedSearchCV took %.2f seconds for %d candidates" % ((time() - start), n_iter_search))
report(random_search.cv_results_)

# use a full grid over all parameters
param_grid = {
    "max_depth": [3, None],
    "max_features": [1, 3, 10],
    "min_samples_split": [1, 3, 10],
    "min_samples_leaf": [1, 3, 10],
    "bootstrap": [True, False],
    "criterion": ["gini", "entropy"]
}

# run grid search
grid_search = GridSearchCV(clf, param_grid=param_grid)
start = time()
grid_search.fit(X, y)
print("GridSearchCV took %.2f seconds for %d candidate parameter settings." % (time() - start, len(grid_search.cv_results_['params'])))
report(grid_search.cv_results_)
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: randomized_search.py

Download IPython notebook: randomized_search.ipynb

### 21.8 Nested versus non-nested cross-validation

This example compares non-nested and nested cross-validation strategies on a classifier of the iris data set. Nested cross-validation (CV) is often used to train a model in which hyperparameters also need to be optimized. Nested CV estimates the generalization error of the underlying model and its (hyper)parameter search. Choosing the parameters that maximize non-nested CV biases the model to the dataset, yielding an overly-optimistic score.

Model selection without nested CV uses the same data to tune model parameters and evaluate model performance. Information may thus “leak” into the model and overfit the data. The magnitude of this effect is primarily dependent on the size of the dataset and the stability of the model. See Cawley and Talbot \(^1\) for an analysis of these issues.

To avoid this problem, nested CV effectively uses a series of train/validation/test set splits. In the inner loop, the score is approximately maximized by fitting a model to each training set, and then directly maximized in selecting (hyper)parameters over the validation set. In the outer loop, generalization error is estimated by averaging test set scores over several dataset splits.

The example below uses a support vector classifier with a non-linear kernel to build a model with optimized hyperparameters by grid search. We compare the performance of non-nested and nested CV strategies by taking the difference between their scores.

See Also:

- Cross-validation: evaluating estimator performance
- Tuning the hyper-parameters of an estimator

References:

```python
from sklearn.datasets import load_iris
from matplotlib import pyplot as plt
from sklearn.svm import SVC
from sklearn.model_selection import GridSearchCV, cross_val_score, KFold
import numpy as np
```

Out:

Average difference of 0.007742 with std. dev. of 0.007688.
# Number of random trials
NUM_TRIALS = 30

# Load the dataset
iris = load_iris()
X_iris = iris.data
y_iris = iris.target

# Set up possible values of parameters to optimize over
p_grid = {"C": [1, 10, 100],
          "gamma": [.01, .1]}

# We will use a Support Vector Classifier with "rbf" kernel
svr = SVC(kernel="rbf")

# Arrays to store scores
non_nested_scores = np.zeros(NUM_TRIALS)
nested_scores = np.zeros(NUM_TRIALS)

# Loop for each trial
for i in range(NUM_TRIALS):
    # Choose cross-validation techniques for the inner and outer loops,
    # independently of the dataset.
    # E.g "LabelKFold", "LeaveOneOut", "LeaveOneLabelOut", etc.
    inner_cv = KFold(n_splits=4, shuffle=True, random_state=i)
    outer_cv = KFold(n_splits=4, shuffle=True, random_state=i)

    # Non_nested parameter search and scoring
    clf = GridSearchCV(estimator=svr, param_grid=p_grid, cv=inner_cv)
    clf.fit(X_iris, y_iris)
    non_nested_scores[i] = clf.best_score_

    # Nested CV with parameter optimization
    nested_score = cross_val_score(clf, X=X_iris, y=y_iris, cv=outer_cv)
    nested_scores[i] = nested_score.mean()

score_difference = non_nested_scores - nested_scores
print("Average difference of {0:.6f} with std. dev. of {1:.6f}.").format(score_difference.mean(), score_difference.std()))

# Plot scores on each trial for nested and non-nested CV
plt.figure()
plt.subplot(211)
non_nested_scores_line, = plt.plot(non_nested_scores, color='r')
nested_line, = plt.plot(nested_scores, color='b')
plt.ylabel("score", fontsize="14")
plt.legend([non_nested_scores_line, nested_line],
           ["Non-Nested CV", "Nested CV"],
           bbox_to_anchor=(0, .4, .5, 0))
plt.title("Non-Nested and Nested Cross Validation on Iris Dataset",
          x=.5, y=1.1, fontsize="15")

# Plot bar chart of the difference.
plt.subplot(212)
21.9 Sample pipeline for text feature extraction and evaluation

The dataset used in this example is the 20 newsgroups dataset which will be automatically downloaded and then cached and reused for the document classification example.

You can adjust the number of categories by giving their names to the dataset loader or setting them to None to get the 20 of them.

Here is a sample output of a run on a quad-core machine:

| Loading 20 newsgroups dataset for categories: |
| ['alt.atheism', 'talk.religion.misc'] |
| 1427 documents |
| 2 categories |

Performing grid search...

pipeline: ['vect', 'tfidf', 'clf']
parameters:

{'clf__alpha': (1.0000000000000000e-05, 9.9999999999999995e-07),
 'clf__n_iter': (10, 50, 80),
 'clf__penalty': ('l2', 'elasticnet'),
 'tfidf__use_idf': (True, False),
 'vect__max_n': (1, 2),
 'vect__max_df': (0.5, 0.75, 1.0),
 'vect__max_features': (None, 5000, 10000, 50000)}

done in 1737.030s

Best score: 0.940

Best parameters set:

clf__alpha: 9.9999999999999995e-07
clf__n_iter: 50
clf__penalty: 'elasticnet'
tfidf__use_idf: True
vect__max_n: 2
vect__max_df: 0.75
vect__max_features: 50000

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# Peter Prettenhofer <peter.prettenhofer@gmail.com>
# Mathieu Blondel <mathieu@mblondel.org>
# License: BSD 3 clause
from __future__ import print_function
from pprint import pprint
from time import time
import logging
from sklearn.datasets import fetch_20newsgroups
from sklearn.feature_extraction.text import CountVectorizer
from sklearn.feature_extraction.text import TfidfTransformer
from sklearn.linear_model import SGDClassifier
from sklearn.model_selection import GridSearchCV
from sklearn.pipeline import Pipeline

print(__doc__)

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO,
                     format='%(asctime)s %(levelname)s %(message)s')

Load some categories from the training set

categories = [
    'alt.atheism',
    'talk.religion.misc',
]
# Uncomment the following to do the analysis on all the categories
# categories = None

print("Loading 20 newsgroups dataset for categories:")
print(categories)

data = fetch_20newsgroups(subset='train', categories=categories)
print("%d documents" % len(data.filenames))
print("%d categories" % len(data.target_names))
print()

define a pipeline combining a text feature extractor with a simple classifier

pipeline = Pipeline([
    ('vect', CountVectorizer()),
    ('tfidf', TfidfTransformer()),
    ('clf', SGDClassifier()),
])

# uncommenting more parameters will give better exploring power but will
# increase processing time in a combinatorial way
parameters = {
    'vect__max_df': (0.5, 0.75, 1.0),
    # 'vect__max_features': (None, 5000, 10000, 50000),
    'vect__ngram_range': ((1, 1), (1, 2)),  # unigrams or bigrams
    # 'tfidf__use_idf': (True, False),
    # 'tfidf__norm': ('l1', 'l2'),
    'clf__alpha': (0.00001, 0.000001),
    'clf__penalty': ('l2', 'elasticnet'),
    # 'clf__n_iter': (10, 50, 80),
}

if __name__ == '__main__':

# multiprocessing requires the fork to happen in a __main__ protected block

# find the best parameters for both the feature extraction and the classifier
grid_search = GridSearchCV(pipeline, parameters, n_jobs=-1, verbose=1)

print("Performing grid search...")
print("pipeline:", [name for name, _ in pipeline.steps])
print("parameters:")
pprint(parameters)
t0 = time()
grid_search.fit(data.data, data.target)
print("done in \%0.3f s" % (time() - t0))
print()

print("Best score: \%0.3f" % grid_search.best_score_)
print("Best parameters set:")
best_parameters = grid_search.best_estimator_.get_params()
for param_name in sorted(parameters.keys()):
    print("\t\$s: \$r" % (param_name, best_parameters[param_name]))

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: grid_search_text_feature_extraction.py
Download IPython notebook: grid_search_text_feature_extraction.ipynb

## 21.10 Precision-Recall

Example of Precision-Recall metric to evaluate classifier output quality.

In information retrieval, precision is a measure of result relevancy, while recall is a measure of how many truly relevant results are returned. A high area under the curve represents both high recall and high precision, where high precision relates to a low false positive rate, and high recall relates to a low false negative rate. High scores for both show that the classifier is returning accurate results (high precision), as well as returning a majority of all positive results (high recall).

A system with high recall but low precision returns many results, but most of its predicted labels are incorrect when compared to the training labels. A system with high precision but low recall is just the opposite, returning very few results, but most of its predicted labels are correct when compared to the training labels. An ideal system with high precision and high recall will return many results, with all results labeled correctly.

Precision ($P$) is defined as the number of true positives ($T_p$) over the number of true positives plus the number of false positives ($F_p$).

$$P = \frac{T_p}{T_p + F_p}$$

Recall ($R$) is defined as the number of true positives ($T_p$) over the number of true positives plus the number of false negatives ($F_n$).

$$R = \frac{T_p}{T_p + F_n}$$

These quantities are also related to the ($F_1$) score, which is defined as the harmonic mean of precision and recall.

$$F_1 = \frac{2 \times P \times R}{P + R}$$
It is important to note that the precision may not decrease with recall. The definition of precision ($\frac{T_p}{T_p + F_p}$) shows that lowering the threshold of a classifier may increase the denominator, by increasing the number of results returned. If the threshold was previously set too high, the new results may all be true positives, which will increase precision. If the previous threshold was about right or too low, further lowering the threshold will introduce false positives, decreasing precision.

Recall is defined as $\frac{T_p}{T_p + F_n}$, where $T_p + F_n$ does not depend on the classifier threshold. This means that lowering the classifier threshold may increase recall, by increasing the number of true positive results. It is also possible that lowering the threshold may leave recall unchanged, while the precision fluctuates.

The relationship between recall and precision can be observed in the stairstep area of the plot - at the edges of these steps a small change in the threshold considerably reduces precision, with only a minor gain in recall. See the corner at recall = .59, precision = .8 for an example of this phenomenon.

Precision-recall curves are typically used in binary classification to study the output of a classifier. In order to extend Precision-recall curve and average precision to multi-class or multi-label classification, it is necessary to binarize the output. One curve can be drawn per label, but one can also draw a precision-recall curve by considering each element of the label indicator matrix as a binary prediction (micro-averaging).

Note:
See also `sklearn.metrics.average_precision_score`, `sklearn.metrics.recall_score`, `sklearn.metrics.precision_score`, `sklearn.metrics.f1_score`
import matplotlib.pyplot as plt
import numpy as np
from itertools import cycle
from sklearn import svm, datasets
from sklearn.metrics import precision_recall_curve
from sklearn.metrics import average_precision_score
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import label_binarize
from sklearn.multiclass import OneVsRestClassifier

# import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target

# setup plot details
colors = cycle(['navy', 'turquoise', 'darkorange', 'cornflowerblue', 'teal'])
lw = 2

# Binarize the output
y = label_binarize(y, classes=[0, 1, 2])
n_classes = y.shape[1]

# Add noisy features
random_state = np.random.RandomState(0)
n_samples, n_features = X.shape
X = np.c_[X, random_state.randn(n_samples, 200 * n_features)]

# Split into training and test
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=.5,
                                                     random_state=random_state)

# Run classifier
classifier = OneVsRestClassifier(svm.SVC(kernel='linear', probability=True,
                                          random_state=random_state))
y_score = classifier.fit(X_train, y_train).decision_function(X_test)

# Compute Precision-Recall and plot curve
precision = dict()
recall = dict()
average_precision = dict()
for i in range(n_classes):
    precision[i], recall[i], _ = precision_recall_curve(y_test[:, i],
                                                        y_score[:, i])
    average_precision[i] = average_precision_score(y_test[:, i], y_score[:, i])

# Compute micro-average ROC curve and ROC area
precision['micro'], recall['micro'], _ = precision_recall_curve(y_test.ravel(),
                                                                  y_score.ravel())
average_precision['micro'] = average_precision_score(y_test, y_score,
                                                      average='micro')

# Plot Precision-Recall curve
plt.clf()
### 21.11 Receiver Operating Characteristic (ROC)

Example of Receiver Operating Characteristic (ROC) metric to evaluate classifier output quality.

ROC curves typically feature true positive rate on the Y axis, and false positive rate on the X axis. This means that the top left corner of the plot is the “ideal” point - a false positive rate of zero, and a true positive rate of one. This is not very realistic, but it does mean that a larger area under the curve (AUC) is usually better.

The “steepness” of ROC curves is also important, since it is ideal to maximize the true positive rate while minimizing the false positive rate.

#### 21.11.1 Multiclass settings

ROC curves are typically used in binary classification to study the output of a classifier. In order to extend ROC curve and ROC area to multi-class or multi-label classification, it is necessary to binarize the output. One ROC curve can be drawn per label, but one can also draw a ROC curve by considering each element of the label indicator matrix as a binary prediction (micro-averaging).

Another evaluation measure for multi-class classification is macro-averaging, which gives equal weight to the classification of each label.
scikit-learn user guide, Release 0.18.2

Note:
See also sklearn.metrics.roc_auc_score, Receiver Operating Characteristic (ROC) with cross validation.
print(__doc__)
import numpy as np
import matplotlib.pyplot as plt
from itertools import cycle
from
from
from
from
from
from

sklearn import svm, datasets
sklearn.metrics import roc_curve, auc
sklearn.model_selection import train_test_split
sklearn.preprocessing import label_binarize
sklearn.multiclass import OneVsRestClassifier
scipy import interp

# Import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target
# Binarize the output
y = label_binarize(y, classes=[0, 1, 2])
n_classes = y.shape[1]
# Add noisy features to make the problem harder
random_state = np.random.RandomState(0)
n_samples, n_features = X.shape
X = np.c_[X, random_state.randn(n_samples, 200 * n_features)]
# shuffle and split training and test sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=.5,
random_state=0)
# Learn to predict each class against the other
classifier = OneVsRestClassifier(svm.SVC(kernel='linear', probability=True,
random_state=random_state))
y_score = classifier.fit(X_train, y_train).decision_function(X_test)
# Compute ROC curve and ROC area for each class
fpr = dict()
tpr = dict()
roc_auc = dict()
for i in range(n_classes):
fpr[i], tpr[i], _ = roc_curve(y_test[:, i], y_score[:, i])
roc_auc[i] = auc(fpr[i], tpr[i])
# Compute micro-average ROC curve and ROC area
fpr["micro"], tpr["micro"], _ = roc_curve(y_test.ravel(), y_score.ravel())
roc_auc["micro"] = auc(fpr["micro"], tpr["micro"])

Plot of a ROC curve for a specific class
plt.figure()
lw = 2

21.11. Receiver Operating Characteristic (ROC)

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Plot ROC curves for the multiclass problem

```python
from sklearn.datasets import load_iris
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn.metrics import roc_curve, roc_auc_score

X, y = load_iris(return_X_y=True)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.33, random_state=42)

logreg = LogisticRegression(max_iter=10000)
logreg.fit(X_train, y_train)

y_pred_proba = logreg.predict_proba(X_test)

n_classes = y_pred_proba.shape[1]

fpr = dict()
tpr = dict()
roc_auc = dict()
for i in range(n_classes):
    fpr[i], tpr[i], _ = roc_curve(y_test == i, y_pred_proba[:, i])
    roc_auc[i] = roc_auc_score(y_test == i, y_pred_proba[:, i])

# Compute macro-average ROC curve and ROC area
# First aggregate all false positive rates
all_fpr = np.unique(np.concatenate([fpr[i] for i in range(n_classes)]))

# Then interpolate all ROC curves at this points
mean_tpr = np.zeros_like(all_fpr)
for i in range(n_classes):
    mean_tpr += interp(all_fpr, fpr[i], tpr[i])

mean_tpr /= n_classes

# Finally average it and compute AUC
mean_tpr /= n_classes
```

```
plt.plot([0, 1], [0, 1], color='navy', lw=lw, linestyle='--')
plt.xlim([0.0, 1.0])
plt.ylim([0.0, 1.05])
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Receiver operating characteristic example')
plt.legend(loc="lower right")
plt.show()
```
fpr["macro"] = all_fpr
tpr["macro"] = mean_tpr
roc_auc["macro"] = auc(fpr["macro"], tpr["macro"])

# Plot all ROC curves
plt.plot(fpr["micro"], tpr["micro"],
         label='micro-average ROC curve (area = {0:0.2f})'
            ''.format(roc_auc["micro"]),
         color='deeppink', linestyle=':', linewidth=4)
plt.plot(fpr["macro"], tpr["macro"],
         label='macro-average ROC curve (area = {0:0.2f})'
            ''.format(roc_auc["macro"]),
         color='navy', linestyle=':', linewidth=4)

colors = cycle(['aqua', 'darkorange', 'cornflowerblue'])
for i, color in zip(range(n_classes), colors):
    plt.plot(fpr[i], tpr[i], color=color, lw=lw,
             label='ROC curve of class {0} (area = {1:0.2f})'
                 ''.format(i, roc_auc[i]))
plt.plot([0, 1], [0, 1], 'k--', lw=lw)
plt.xlim([0.0, 1.0])
plt.ylim([0.0, 1.05])
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('Some extension of Receiver operating characteristic to multi-class')
plt.legend(loc="lower right")
plt.show()
21.12 Plotting Learning Curves

On the left side the learning curve of a naive Bayes classifier is shown for the digits dataset. Note that the training score and the cross-validation score are both not very good at the end. However, the shape of the curve can be found in more complex datasets very often: the training score is very high at the beginning and decreases and the cross-validation score is very low at the beginning and increases. On the right side we see the learning curve of an SVM with RBF kernel. We can see clearly that the training score is still around the maximum and the validation score could be increased with more training samples.
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn.naive_bayes import GaussianNB
from sklearn.svm import SVC
from sklearn.datasets import load_digits
from sklearn.model_selection import learning_curve
from sklearn.model_selection import ShuffleSplit

def plot_learning_curve(estimator, title, X, y, ylim=None, cv=None,
                        n_jobs=1, train_sizes=np.linspace(.1, 1.0, 5)):
    """
    Generate a simple plot of the test and training learning curve.

    Parameters
    ----------
    estimator : object type that implements the "fit" and "predict" methods
                An object of that type which is cloned for each validation.
    title : string
              Title for the chart.
    X : array-like, shape (n_samples, n_features)
        Training vector, where n_samples is the number of samples and
        n_features is the number of features.
    y : array-like, shape (n_samples) or (n_samples, n_outputs)
        Target relative to X
    ylim : tuple, shape (ymin, ymax), default: None
        Defines the y-axis range
    cv : int/iterable/object, default: None
        Determines the cross-validation splitting strategy.
        Possible inputs for int: cv=fold, must be at least 2.
        Possible inputs for object: an iterable:
        cv=result出于iterable of objects to use as cross-validation generators.
        If None, KFold with 5 splits is used.
    n_jobs : int, default: 1
        Number of jobs to run in parallel.
    train_sizes : array-like, shape (n_ticks,), dtype float or int
        Relative or absolute numbers of training examples that will be used to
        generate the learning curve. If None, 3 ** n_jobs points
        are used: np.linspace(0.1, 1.0, 5) by default.

    Returns
    -------
    train_sizes, train_scores, test_scores : float
        train_sizes of training sets
        log-average 5-fold validation score per training set
        (mean of the train scores on the log scale)
y : array-like, shape (n_samples) or (n_samples, n_features), optional
   Target relative to X for classification or regression;
   None for unsupervised learning.

ylim : tuple, shape (ymin, ymax), optional
   Defines minimum and maximum yvalues plotted.

cv : int, cross-validation generator or an iterable, optional
   Determines the cross-validation splitting strategy.
   Possible inputs for cv are:
   - None, to use the default 3-fold cross-validation,
   - integer, to specify the number of folds.
   - An object to be used as a cross-validation generator.
   - An iterable yielding train/test splits.

   For integer/None inputs, if 'y' is binary or multiclass,
   :class:`StratifiedKFold` used. If the estimator is not a classifier
   or if 'y' is neither binary nor multiclass, :class:`KFold` is used.

   Refer :ref:`User Guide <cross_validation>` for the various
cross-validators that can be used here.

n_jobs : integer, optional
   Number of jobs to run in parallel (default 1).

""
plt.figure()
plt.title(title)
if ylim is not None:
    plt ylim(*ylim)
plt.xlabel("Training examples")
plt.ylabel("Score")
train_sizes, train_scores, test_scores = learning_curve(
estimator, X, y, cv=cv, n_jobs=n_jobs, train_sizes=train_sizes)
train_scores_mean = np.mean(train_scores, axis=1)
train_scores_std = np.std(train_scores, axis=1)
test_scores_mean = np.mean(test_scores, axis=1)
test_scores_std = np.std(test_scores, axis=1)
plt.grid()

plt.fill_between(train_sizes, train_scores_mean - train_scores_std,
                 train_scores_mean + train_scores_std, alpha=0.1,
                 color="r")
plt.fill_between(train_sizes, test_scores_mean - test_scores_std,
                 test_scores_mean + test_scores_std, alpha=0.1, color="g")
plt.plot(train_sizes, train_scores_mean, 'o-', color="r",
         label="Training score")
plt.plot(train_sizes, test_scores_mean, 'o-', color="g",
         label="Cross-validation score")

plt.legend(loc="best")
return plt

digits = load_digits()
X, y = digits.data, digits.target

title = "Learning Curves (Naive Bayes)"
# Cross validation with 100 iterations to get smoother mean test and train score curves, each time with 20% data randomly selected as a validation set.
cv = ShuffleSplit(n_splits=100, test_size=0.2, random_state=0)
estimator = GaussianNB()
plot_learning_curve(estimator, title, X, y, ylim=(0.7, 1.01), cv=cv, n_jobs=4)

title = "Learning Curves (SVM, RBF kernel, $\gamma=0.001$)"
# SVC is more expensive so we do a lower number of CV iterations:
cv = ShuffleSplit(n_splits=10, test_size=0.2, random_state=0)
estimator = SVC(gamma=0.001)
plot_learning_curve(estimator, title, X, y, (0.7, 1.01), cv=cv, n_jobs=4)

plt.show()

Total running time of the script: (0 minutes 11.760 seconds)

Download Python source code: plot_learning_curve.py

Download IPython notebook: plot_learning_curve.ipynb
Examples concerning the `sklearn.neighbors` module.

### 22.1 Nearest Neighbors regression

Demonstrate the resolution of a regression problem using a k-Nearest Neighbor and the interpolation of the target using both barycenter and constant weights.

```python
print(__doc__)

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# Fabian Pedregosa <fabian.pedregosa@inria.fr>
# License: BSD 3 clause (C) INRIA

Generate sample data

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import neighbors

np.random.seed(0)
X = np.sort(5 * np.random.rand(40, 1), axis=0)
T = np.linspace(0, 5, 500)[:, np.newaxis]
y = np.sin(X).ravel()
y[::5] += 1 * (0.5 - np.random.rand(8))

Fit regression model

```python
n_neighbors = 5

for i, weights in enumerate(['uniform', 'distance']):
    knn = neighbors.KNeighborsRegressor(n_neighbors, weights=weights)
    y_ = knn.fit(X, y).predict(T)

    plt.subplot(2, 1, i + 1)
    plt.scatter(X, y, c='k', label='data')
    plt.plot(T, y_, c='g', label='prediction')
    plt.axis('tight')
    plt.legend()
    plt.xticks([i for i in range(0, 11)], range(0, 11))
    plt.title("KNeighborsRegressor \(k = \$i\), weights = \'$s'\)" % (n_neighbors,
```

```
Total running time of the script: (0 minutes 0.127 seconds)
Download Python source code: plot_regression.py
Download IPython notebook: plot_regression.ipynb

22.2 Nearest Neighbors Classification

Sample usage of Nearest Neighbors classification. It will plot the decision boundaries for each class.
```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn import neighbors, datasets

n_neighbors = 15

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
# avoid this ugly slicing by using a two-dim dataset
y = iris.target

h = .02  # step size in the mesh

# Create color maps
cmap_light = ListedColormap(['#FFAAAA', '#AAFFAA', '#AAAAFF'])
cmap_bold = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])

for weights in ['uniform', 'distance']:
    # we create an instance of Neighbours Classifier and fit the data.
    clf = neighbors.KNeighborsClassifier(n_neighbors, weights=weights)
    clf.fit(X, y)

    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
```
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                      np.arange(y_min, y_max, h))
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
plt.figure()
plt.pcolormesh(xx, yy, Z, cmap=cmap_light)

# Plot also the training points
plt.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap_bold)
plt.xlim(xx.min(), xx.max())
plt.ylim(yy.min(), yy.max())
plt.title("3-Class classification (k = \$%i\$, weights = '\$%s$')" % (n_neighbors, weights))
plt.show()

Total running time of the script: (0 minutes 0.403 seconds)

Download Python source code: plot_classification.py
Download IPython notebook: plot_classification.ipynb

### 22.3 Nearest Centroid Classification

Sample usage of Nearest Centroid classification. It will plot the decision boundaries for each class.
```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn import datasets
from sklearn.neighbors import NearestCentroid

n_neighbors = 15

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
                      # avoid this ugly slicing by using a two-dim dataset
y = iris.target

h = .02  # step size in the mesh

# Create color maps
cmap_light = ListedColormap(['#FFAAAA', '#AAFFAA', '#AAAAFF'])
cmap_bold = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])

for shrinkage in [None, .2]:
    # we create an instance of Neighbours Classifier and fit the data.
    clf = NearestCentroid(shrink_threshold=shrinkage)
    clf.fit(X, y)
    y_pred = clf.predict(X)
    print(shrinkage, np.mean(y == y_pred))

    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

    # Plot all points with appropriate color
    plt.xlim(x_min, x_max)
    plt.ylim(y_min, y_max)
    plt.pcolormesh(xx, yy, y_pred, cmap=cmap_light)
    # Plot training points
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap_bold, edgecolor='k', s=20)

    plt.title('3-Class classification (shrink_threshold=%0.2f)' % shrinkage)
plt.show()
```

Out:

```
None 0.813333333333
0.2 0.82
```
### 22.4 Kernel Density Estimation

This example shows how kernel density estimation (KDE), a powerful non-parametric density estimation technique, can be used to learn a generative model for a dataset. With this generative model in place, new samples can be drawn. These new samples reflect the underlying model of the data.
import numpy as np
import matplotlib.pyplot as plt

from sklearn.datasets import load_digits
from sklearn.neighbors import KernelDensity
from sklearn.decomposition import PCA
from sklearn.model_selection import GridSearchCV

# load the data
digits = load_digits()
data = digits.data

# project the 64-dimensional data to a lower dimension
pca = PCA(n_components=15, whiten=False)
data = pca.fit_transform(digits.data)

# use grid search cross-validation to optimize the bandwidth

Out:

best bandwidth: 3.79269019073
```python
params = {'bandwidth': np.logspace(-1, 1, 20)}
grid = GridSearchCV(KernelDensity(), params)
grid.fit(data)

print("best bandwidth: {0}".format(grid.best_estimator_.bandwidth))

# use the best estimator to compute the kernel density estimate
kde = grid.best_estimator_

# sample 44 new points from the data
new_data = kde.sample(44, random_state=0)
new_data = pca.inverse_transform(new_data)

# turn data into a 4x11 grid
new_data = new_data.reshape((4, 11, -1))
real_data = digits.data[:44].reshape((4, 11, -1))

# plot real digits and resampled digits
fig, ax = plt.subplots(9, 11, subplot_kw=dict(xticks=[], yticks=[]))
for j in range(11):
    ax[4, j].set_visible(False)
for i in range(4):
    im = ax[i, j].imshow(real_data[i, j].reshape((8, 8)),
                        cmap=pl.cm.binary, interpolation='nearest')
    im.set_clim(0, 16)
    im = ax[i + 5, j].imshow(new_data[i, j].reshape((8, 8)),
                            cmap=pl.cm.binary, interpolation='nearest')
    im.set_clim(0, 16)

ax[0, 5].set_title('Selection from the input data')
ax[5, 5].set_title('"New" digits drawn from the kernel density model')
plt.show()
```

Total running time of the script: (0 minutes 15.624 seconds)

Download Python source code: plot_digits_kde_sampling.py
Download IPython notebook: plot_digits_kde_sampling.ipynb

22.5 Kernel Density Estimate of Species Distributions

This shows an example of a neighbors-based query (in particular a kernel density estimate) on geospatial data, using a Ball Tree built upon the Haversine distance metric – i.e. distances over points in latitude/longitude. The dataset is provided by Phillips et. al. (2006). If available, the example uses basemap to plot the coast lines and national boundaries of South America.

This example does not perform any learning over the data (see Species distribution modeling for an example of classification based on the attributes in this dataset). It simply shows the kernel density estimate of observed data points in geospatial coordinates.

The two species are:

- “Bradypus variegatus”, the Brown-throated Sloth.
- “Microryzomys minutus”, also known as the Forest Small Rice Rat, a rodent that lives in Peru, Colombia, Ecuador, Peru, and Venezuela.
22.5.1 References


Out:

- computing KDE in spherical coordinates
- plot coastlines from coverage
- computing KDE in spherical coordinates
- plot coastlines from coverage

# Author: Jake Vanderplas <jakevdp@cs.washington.edu>
#
# License: BSD 3 clause

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_species_distributions
from sklearn.datasets.species_distributions import construct_grids
from sklearn.neighbors import KernelDensity
```

22.5. Kernel Density Estimate of Species Distributions
# if basemap is available, we'll use it.
# otherwise, we'll improvise later...
try:
    from mpl_toolkits.basemap import Basemap
    basemap = True
except ImportError:
    basemap = False

# Get matrices/arrays of species IDs and locations
data = fetch_species_distributions()
species_names = ['Bradypus Variegatus', 'Microryzomys Minutus']

Xtrain = np.vstack([data['train']['dd lat'],
    data['train']['dd long']]).T
ytrain = np.array([d.decode('ascii').startswith('micro')
    for d in data['train']['species']], dtype='int')
Xtrain *= np.pi / 180.

# Set up the data grid for the contour plot
xgrid, ygrid = construct_grids(data)
X, Y = np.meshgrid(xgrid[::5], ygrid[::5][::-1])
land_reference = data.coverages[6][::5, ::5]
land_mask = (land_reference > -9999).ravel()

xy = np.vstack([Y.ravel(), X.ravel()]).T
xy *= np.pi / 180.

# Plot map of South America with distributions of each species
fig = plt.figure()
fig.subplots_adjust(left=0.05, right=0.95, wspace=0.05)
for i in range(2):
    plt.subplot(1, 2, i + 1)
    # construct a kernel density estimate of the distribution
    print(" - computing KDE in spherical coordinates")
    kde = KernelDensity(bandwidth=0.04, metric='haversine',
        kernel='gaussian', algorithm='ball_tree')
    kde.fit(Xtrain[ytrain == i])
    # evaluate only on the land: -9999 indicates ocean
    Z = -9999 + np.zeros(land_mask.shape[0])
    Z[land_mask] = np.exp(kde.score_samples(xy))
    Z = Z.reshape(X.shape)
    # plot contours of the density
    levels = np.linspace(0, Z.max(), 25)
    plt.contourf(X, Y, Z, levels=levels, cmap=plt.cm.Reds)
    if basemap:
        print(" - plot coastlines using basemap")
        m = Basemap(projection='cyl', llcrnrlat=Y.min(),
            urcrnrlat=Y.max(), llcrnrlon=X.min(),
            urcrnrlon=X.max(), resolution='c')
        m.drawcoastlines()
        m.drawcountries()
```python
ele:
    print(" - plot coastlines from coverage")
    plt.contour(X, Y, land_reference,
        levels=[-9999], colors="k",
        linestyles="solid")
    plt.xticks([])
    plt.yticks([])
    plt.title(species_names[i])

plt.show()
```

**Total running time of the script:** (0 minutes 10.640 seconds)

**Download Python source code:** plot_species_kde.py

**Download IPython notebook:** plot_species_kde.ipynb

## 22.6 Simple 1D Kernel Density Estimation

This example uses the `sklearn.neighbors.KernelDensity` class to demonstrate the principles of Kernel Density Estimation in one dimension.

The first plot shows one of the problems with using histograms to visualize the density of points in 1D. Intuitively, a histogram can be thought of as a scheme in which a unit “block” is stacked above each point on a regular grid. As the top two panels show, however, the choice of gridding for these blocks can lead to wildly divergent ideas about the underlying shape of the density distribution. If we instead center each block on the point it represents, we get the estimate shown in the bottom left panel. This is a kernel density estimation with a “top hat” kernel. This idea can be generalized to other kernel shapes: the bottom-right panel of the first figure shows a Gaussian kernel density estimate over the same distribution.

Scikit-learn implements efficient kernel density estimation using either a Ball Tree or KD Tree structure, through the `sklearn.neighbors.KernelDensity` estimator. The available kernels are shown in the second figure of this example.

The third figure compares kernel density estimates for a distribution of 100 samples in 1 dimension. Though this example uses 1D distributions, kernel density estimation is easily and efficiently extensible to higher dimensions as well.
# Author: Jake Vanderplas <jakevdp@cs.washington.edu>

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
from sklearn.neighbors import KernelDensity

# Plot the progression of histograms to kernels
np.random.seed(1)
N = 20
X = np.concatenate((np.random.normal(0, 1, int(0.3 * N)),
                    np.random.normal(5, 1, int(0.7 * N))))[:, np.newaxis]
X_plot = np.linspace(-5, 10, 1000)[:, np.newaxis]
bins = np.linspace(-5, 10, 10)

fig, ax = plt.subplots(2, 2, sharex=True, sharey=True)
fig.subplots_adjust(hspace=0.05, wspace=0.05)

# histogram 1
ax[0, 0].hist(X[:, 0], bins=bins, fc='AAAAFF', normed=True)
ax[0, 0].text(-3.5, 0.31, "Histogram")

# histogram 2
ax[0, 1].hist(X[:, 0], bins=bins + 0.75, fc='AAAAFF', normed=True)
ax[0, 1].text(-3.5, 0.31, "Histogram, bins shifted")
```

---

Chapter 22. Nearest Neighbors
# tophat KDE
kde = KernelDensity(kernel='tophat', bandwidth=0.75).fit(X)
log_dens = kde.score_samples(X_plot)
ax[1, 0].fill(X_plot[:, 0], np.exp(log_dens), fc='#AAAAFF')
ax[1, 0].text(-3.5, 0.31, "Tophat Kernel Density")

# Gaussian KDE
kde = KernelDensity(kernel='gaussian', bandwidth=0.75).fit(X)
log_dens = kde.score_samples(X_plot)
ax[1, 1].fill(X_plot[:, 0], np.exp(log_dens), fc='#AAAAFF')
ax[1, 1].text(-3.5, 0.31, "Gaussian Kernel Density")

for axi in ax.ravel():
    axi.plot(X[:, 0], np.zeros(X.shape[0]) - 0.01, '+k')
    axi.set_xlim(-4, 9)
    axi.set_ylim(-0.02, 0.34)

for axi in ax[:, 0]:
    axi.set_ylabel('Normalized Density')

for axi in ax[1, :]:
    axi.set_xlabel('x')

#----------------------------------------------------------------------
# Plot all available kernels
X_plot = np.linspace(-6, 6, 1000)[:, None]
X_src = np.zeros((1, 1))

fig, ax = plt.subplots(2, 3, sharex=True, sharey=True)
fig.subplots_adjust(left=0.05, right=0.95, hspace=0.05, wspace=0.05)
def format_func(x, loc):
    if x == 0:
        return '0'
    elif x == 1:
        return 'h'
    elif x == -1:
        return '-h'
    else:
        return '%ih' % x

for i, kernel in enumerate(['gaussian', 'tophat', 'epanechnikov',
                            'exponential', 'linear', 'cosine']):
    axi = ax.ravel()[i]
    log_dens = KernelDensity(kernel=kernel).fit(X_src).score_samples(X_plot)
    axi.fill(X_plot[:, 0], np.exp(log_dens), '-k', fc='#AAAAFF')
    axi.text(-2.6, 0.95, kernel)
    axi.xaxis.set_major_formatter(plt.FuncFormatter(format_func))
    axi.xaxis.set_major_locator(plt.MultipleLocator(1))
    axi.yaxis.set_major_locator(plt.NullLocator())
    axi.yaxis.set_major_locator(plt.NullLocator())
    axi.set_ylim(0, 1.05)
    axi.set_xlim(-2.9, 2.9)
    ax[0, 1].set_title('Available Kernels')
# Plot a 1D density example

N = 100
np.random.seed(1)
X = np.concatenate((np.random.normal(0, 1, int(0.3 * N)),
                   np.random.normal(5, 1, int(0.7 * N))))[:, np.newaxis]

X_plot = np.linspace(-5, 10, 1000)[:, np.newaxis]
true_dens = (0.3 * norm(0, 1).pdf(X_plot[:, 0])
             + 0.7 * norm(5, 1).pdf(X_plot[:, 0]))

fig, ax = plt.subplots()
ax.fill(X_plot[:, 0], true_dens, fc='black', alpha=0.2,
       label='input distribution')

for kernel in ['gaussian', 'tophat', 'epanechnikov']:
    kde = KernelDensity(kernel=kernel, bandwidth=0.5).fit(X)
    log_dens = kde.score_samples(X_plot)
    ax.plot(X_plot[:, 0], np.exp(log_dens), '-',
             label="kernel = '{0}'".format(kernel))

ax.text(6, 0.38, "N={0} points".format(N))
ax.legend(loc='upper left')
ax.plot(X[:, 0], -0.005 - 0.01 * np.random.random(X.shape[0]), '+k')

ax.set_xlim(-4, 9)
ax.set_ylim(-0.02, 0.4)
plt.show()

Total running time of the script: (0 minutes 0.989 seconds)

Download Python source code: plot_kde_1d.py
Download IPython notebook: plot_kde_1d.ipynb

22.7 Hyper-parameters of Approximate Nearest Neighbors

This example demonstrates the behaviour of the accuracy of the nearest neighbor queries of Locality Sensitive Hashing Forest as the number of candidates and the number of estimators (trees) vary.

In the first plot, accuracy is measured with the number of candidates. Here, the term “number of candidates” refers to maximum bound for the number of distinct points retrieved from each tree to calculate the distances. Nearest neighbors are selected from this pool of candidates. Number of estimators is maintained at three fixed levels (1, 5, 10).

In the second plot, the number of candidates is fixed at 50. Number of trees is varied and the accuracy is plotted against those values. To measure the accuracy, the true nearest neighbors are required, therefore sklearn.neighbors.NearestNeighbors is used to compute the exact neighbors.

from __future__ import division
print(__doc__)

# Author: Maheshakya Wijewardena <maheshakya.108@cse.mrt.ac.lk>
#
# License: BSD 3 clause
import numpy as np
from sklearn.datasets.samples_generator import make_blobs
from sklearn.neighbors import LSHForest
from sklearn.neighbors import NearestNeighbors
import matplotlib.pyplot as plt

# Initialize size of the database, iterations and required neighbors.
n_samples = 10000
n_features = 100
n_queries = 30
rng = np.random.RandomState(42)

# Generate sample data
X, _ = make_blobs(n_samples=n_samples + n_queries,
                   n_features=n_features, centers=10,
                   random_state=0)
X_index = X[:n_samples]
X_query = X[n_samples:]

# Get exact neighbors
nbrs = NearestNeighbors(n_neighbors=1, algorithm='brute',
                         metric='cosine').fit(X_index)
neighbors_exact = nbrs.kneighbors(X_query, return_distance=False)

# Set `n_candidate` values
n_candidates_values = np.linspace(10, 500, 5).astype(np.int)
n_estimators_for_candidate_value = [1, 5, 10]
n_iter = 10
stds_accuracies = np.zeros((len(n_estimators_for_candidate_value),
                            n_candidates_values.shape[0]), dtype=float)
accuracies_c = np.zeros((len(n_estimators_for_candidate_value),
                         n_candidates_values.shape[0]), dtype=float)

# LSH Forest is a stochastic index: perform several iteration to estimate
# expected accuracy and standard deviation displayed as error bars in
# the plots
for j, value in enumerate(n_estimators_for_candidate_value):
    for i, n_candidates in enumerate(n_candidates_values):
        accuracy_c = []
        for seed in range(n_iter):
            lshf = LSHForest(n_estimators=value,
                             n_candidates=n_candidates, n_neighbors=1,
                             random_state=seed)

            # Build the LSH Forest index
            lshf.fit(X_index)
            # Get neighbors
            neighbors_approx = lshf.kneighbors(X_query,
                                                return_distance=False)
            accuracy_c.append(np.sum(np.equal(neighbors_approx,
                                               neighbors_exact)) / n_queries)

        stds_accuracies[j, i] = np.std(accuracy_c)
        accuracies_c[j, i] = np.mean(accuracy_c)

# Set `n_estimators` values
n_estimators_values = [1, 5, 10, 20, 30, 40, 50]
accuracies_trees = np.zeros(len(n_estimators_values), dtype=float)

# Calculate average accuracy for each value of 'n_estimators'
for i, n_estimators in enumerate(n_estimators_values):
    lshf = LSHForest(n_estimators=n_estimators, n_neighbors=1)
    # Build the LSH Forest index
    lshf.fit(X_index)
    # Get neighbors
    neighbors_approx = lshf.kneighbors(X_query, return_distance=False)
    accuracies_trees[i] = np.sum(np.equal(neighbors_approx, neighbors_exact))/n_queries

Plot the accuracy variation with n_candidates

plt.figure()
colors = ['c', 'm', 'y']
for i, n_estimators in enumerate(n_estimators_for_candidate_value):
    label = 'n_estimators = %d' % n_estimators
    plt.plot(n_candidates_values, accuracies_c[i, :], 'o-', c=colors[i], label=label)
    plt.errorbar(n_candidates_values, accuracies_c[i, :], stds_accuracies[i, :], c=colors[i])
plt.legend(loc='upper left', prop=dict(size='small'))
plt.ylim([0, 1.2])
plt.xlim(min(n_candidates_values), max(n_candidates_values))
plt.ylabel("Accuracy")
plt.xlabel("n_candidates")
plt.grid(which='both')
plt.title("Accuracy variation with n_candidates")

# Plot the accuracy variation with 'n_estimators'
plt.figure()
plt.scatter(n_estimators_values, accuracies_trees, c='k')
plt.plot(n_estimators_values, accuracies_trees, c='g')
plt.ylim([0, 1.2])
plt.xlim(min(n_estimators_values), max(n_estimators_values))
plt.ylabel("Accuracy")
plt.xlabel("n_estimators")
plt.grid(which='both')
plt.title("Accuracy variation with n_estimators")
plt.show()
22.8 Scalability of Approximate Nearest Neighbors

This example studies the scalability profile of approximate 10-neighbors queries using the LSHForest with `n_estimators=20` and `n_candidates=200` when varying the number of samples in the dataset.

The first plot demonstrates the relationship between query time and index size of LSHForest. Query time is compared with the brute force method in exact nearest neighbor search for the same index sizes. The brute force queries have a very predictable linear scalability with the index (full scan). LSHForest index have sub-linear scalability profile but can be slower for small datasets.

The second plot shows the speedup when using approximate queries vs brute force exact queries. The speedup tends to increase with the dataset size but should reach a plateau typically when doing queries on datasets with millions of samples and a few hundreds of dimensions. Higher dimensional datasets tends to benefit more from LSHForest indexing.

The break even point (speedup = 1) depends on the dimensionality and structure of the indexed data and the parameters of the LSHForest index.

The precision of approximate queries should decrease slowly with the dataset size. The speed of the decrease depends mostly on the LSHForest parameters and the dimensionality of the data.

```python
from __future__ import division
print(__doc__)

# Authors: Maheshakya Wijewardena <maheshakya.10@cse.mrt.ac.lk>
# Olivier Grisel <olivier.grisel@ensta.org>
# # License: BSD 3 clause

import time
import numpy as np
from sklearn.datasets.samples_generator import make_blobs
from sklearn.neighbors import LSHForest
from sklearn.neighbors import NearestNeighbors
import matplotlib.pyplot as plt
```

```
```
# Parameters of the study
n_samples_min = int(1e3)
n_samples_max = int(1e5)
n_features = 100
n_centers = 100
n_queries = 100
n_steps = 6
n_iter = 5

# Initialize the range of `n_samples`
n_samples_values = np.logspace(np.log10(n_samples_min), np.log10(n_samples_max), n_steps).astype(np.int)

# Generate some structured data
rng = np.random.RandomState(42)
all_data, _ = make_blobs(n_samples=n_samples_max + n_queries, n_features=n_features, centers=n_centers, shuffle=True, random_state=0)
queries = all_data[:n_queries]
index_data = all_data[n_queries:]

# Metrics to collect for the plots
average_times_exact = []
average_times_approx = []
std_times_approx = []
accuracies = []
std_accuracies = []
average_speedups = []
std_speedups = []

# Calculate the average query time
for n_samples in n_samples_values:
    X = index_data[:n_samples]
    # Initialize LSHForest for queries of a single neighbor
    lshf = LSHForest(n_estimators=20, n_candidates=200, n_neighbors=10).fit(X)
    nbrs = NearestNeighbors(algorithm='brute', metric='cosine', n_neighbors=10).fit(X)
    time_approx = []
    time_exact = []
    accuracy = []
    for i in range(n_iter):
        # pick one query at random to study query time variability in LSHForest
        query = queries[[rng.randint(0, n_queries)]]
        t0 = time.time()
        exact_neighbors = nbrs.kneighbors(query, return_distance=False)
        time_exact.append(time.time() - t0)
        t0 = time.time()
        approx_neighbors = lshf.kneighbors(query, return_distance=False)
        time_approx.append(time.time() - t0)
        accuracy.append(np.in1d(approx_neighbors, exact_neighbors).mean())

    average_time_exact = np.mean(time_exact)
average_time_auth = np.mean(time_auth)
speedup = np.array(time_exact) / np.array(time_auth)
average_speedup = np.mean(speedup)
mean_accuracy = np.mean(accuracy)
std_accuracy = np.std(accuracy)
print("Index size: %d, exact: %0.3f s, LSH: %0.3f s, speedup: %0.1f, "
"accuracy: %0.2f +/-%0.2f" %
(n_samples, average_time_exact, average_time_auth, average_speedup,
mean_accuracy, std_accuracy))
accuracies.append(mean_accuracy)
std_accuracies.append(std_accuracy)
average_times_exact.append(average_time_exact)
average_times_auth.append(average_time_auth)
std_times_auth.append(np.std(time_auth))
average_speedups.append(average_speedup)
std_speedups.append(np.std(speedup))

plt.figure()
plt.errorbar(n_samples_values, average_times_auth, yerr=std_times_auth,
fmt='o-', c='r', label='LSHForest')
plt.plot(n_samples_values, average_times_exact, c='b',
label="NearestNeighbors(algorithm='brute', metric='cosine')")
plt.legend(loc='upper left', prop=dict(size='small'))
plt.ylim(0, None)
plt.ylabel("Average query time in seconds")
plt.xlabel("n_samples")
plt.grid(which='both')
plt.title("Impact of index size on response time for first "
"nearest neighbors queries")

plt.figure()
plt.errorbar(n_samples_values, average_speedups, yerr=std_speedups,
fmt='o-', c='r')
plt.ylim(0, None)
plt.ylabel("Average speedup")
plt.xlabel("n_samples")
plt.grid(which='both')
plt.title("Speedup of the approximate NN queries vs brute force")

plt.figure()
plt.errorbar(n_samples_values, accuracies, std_accuracies, fmt='o-', c='c')
plt.ylim(0, 1.1)
plt.ylabel("precision@10")
plt.xlabel("n_samples")
plt.grid(which='both')
plt.title("precision of 10-nearest-neighbors queries with index size")
plt.show()
Out:

<table>
<thead>
<tr>
<th>Index size</th>
<th>exact time</th>
<th>LSHF time</th>
<th>speedup</th>
<th>accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.001s</td>
<td>0.017s</td>
<td>0.1</td>
<td>1.00 +/- 0.00</td>
</tr>
<tr>
<td>2511</td>
<td>0.003s</td>
<td>0.018s</td>
<td>0.2</td>
<td>1.00 +/- 0.00</td>
</tr>
<tr>
<td>6309</td>
<td>0.007s</td>
<td>0.020s</td>
<td>0.3</td>
<td>1.00 +/- 0.00</td>
</tr>
<tr>
<td>15848</td>
<td>0.018s</td>
<td>0.022s</td>
<td>0.9</td>
<td>1.00 +/- 0.00</td>
</tr>
<tr>
<td>39810</td>
<td>0.045s</td>
<td>0.020s</td>
<td>2.2</td>
<td>1.00 +/- 0.00</td>
</tr>
<tr>
<td>100000</td>
<td>0.132s</td>
<td>0.026s</td>
<td>5.1</td>
<td>0.96 +/- 0.05</td>
</tr>
</tbody>
</table>

Total running time of the script: (0 minutes 5.955 seconds)

Download Python source code: plot_approximate_nearest_neighbors_scalability.py

Download IPython notebook: plot_approximate_nearest_neighbors_scalability.ipynb
Examples concerning the `sklearn.neural_network` module.

### 23.1 Visualization of MLP weights on MNIST

Sometimes looking at the learned coefficients of a neural network can provide insight into the learning behavior. For example if weights look unstructured, maybe some were not used at all, or if very large coefficients exist, maybe regularization was too low or the learning rate too high.

This example shows how to plot some of the first layer weights in a MLPClassifier trained on the MNIST dataset.

The input data consists of 28x28 pixel handwritten digits, leading to 784 features in the dataset. Therefore the first layer weight matrix have the shape (784, `hidden_layer_sizes[0]`). We can therefore visualize a single column of the weight matrix as a 28x28 pixel image.

To make the example run faster, we use very few hidden units, and train only for a very short time. Training longer would result in weights with a much smoother spatial appearance.
Out:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32212731</td>
</tr>
<tr>
<td>2</td>
<td>0.15738787</td>
</tr>
<tr>
<td>3</td>
<td>0.11647274</td>
</tr>
<tr>
<td>4</td>
<td>0.09631113</td>
</tr>
<tr>
<td>5</td>
<td>0.08074513</td>
</tr>
<tr>
<td>6</td>
<td>0.07163224</td>
</tr>
<tr>
<td>7</td>
<td>0.06351392</td>
</tr>
<tr>
<td>8</td>
<td>0.05694146</td>
</tr>
<tr>
<td>9</td>
<td>0.05213487</td>
</tr>
<tr>
<td>10</td>
<td>0.04708320</td>
</tr>
</tbody>
</table>

Training set score: 0.985733
Test set score: 0.971000

```
print(__doc__)

import matplotlib.pyplot as plt
from sklearn.datasets import fetch_mldata
from sklearn.neural_network import MLPClassifier
```
mnist = fetch_mldata("MNIST original")
# rescale the data, use the traditional train/test split
X, y = mnist.data / 255., mnist.target
X_train, X_test = X[:60000], X[60000:]
y_train, y_test = y[:60000], y[60000:]

# mlp = MLPClassifier(hidden_layer_sizes=(100, 100), max_iter=400, alpha=1e-4,
# solver='sgd', verbose=10, tol=1e-4, random_state=1)
mlp = MLPClassifier(hidden_layer_sizes=(50,), max_iter=10, alpha=1e-4,
    solver='sgd', verbose=10, tol=1e-4, random_state=1,
    learning_rate_init=.1)

mlp.fit(X_train, y_train)
print("Training set score: \%f" % mlp.score(X_train, y_train))
print("Test set score: \%f" % mlp.score(X_test, y_test))

fig, axes = plt.subplots(4, 4)
# use global min / max to ensure all weights are shown on the same scale
vmin, vmax = mlp.coefs_[0].min(), mlp.coefs_[0].max()
for coef, ax in zip(mlp.coefs_[0].T, axes.ravel()):
    ax.matshow(coef.reshape(28, 28), cmap=plt.cm.gray, vmin=.5 * vmin,
               vmax=.5 * vmax)
    ax.set_xticks(())
    ax.set_yticks(())
plt.show()

Total running time of the script: (0 minutes 17.925 seconds)

Download Python source code: plot_mnist_filters.py
Download IPython notebook: plot_mnist_filters.ipynb

23.2 Restricted Boltzmann Machine features for digit classification

For greyscale image data where pixel values can be interpreted as degrees of blackness on a white background, like handwritten digit recognition, the Bernoulli Restricted Boltzmann machine model \(\text{(BernoulliRB)}\) can perform effective non-linear feature extraction.

In order to learn good latent representations from a small dataset, we artificially generate more labeled data by perturbing the training data with linear shifts of 1 pixel in each direction.

This example shows how to build a classification pipeline with a BernoulliRB feature extractor and a \texttt{LogisticRegression} classifier. The hyperparameters of the entire model (learning rate, hidden layer size, regularization) were optimized by grid search, but the search is not reproduced here because of runtime constraints.

Logistic regression on raw pixel values is presented for comparison. The example shows that the features extracted by the BernoulliRB help improve the classification accuracy.
```python
import matplotlib.pyplot as plt

from scipy.ndimage import convolve
from sklearn import linear_model, datasets, metrics
from sklearn.model_selection import train_test_split
from sklearn.neural_network import BernoulliRBM
from sklearn.pipeline import Pipeline

Setting up

def nudge_dataset(X, Y):
    
    """
    This produces a dataset 5 times bigger than the original one,
    by moving the 8x8 images in X around by 1px to left, right, down, up
    """
    direction_vectors = [
        [[0, 1, 0],
         [0, 0, 0],
         [0, 0, 0]],
        [[0, 0, 0],
         [1, 0, 0],
         [0, 0, 0]],
        [[0, 0, 0],
         [0, 0, 1],
         [0, 0, 0]],
        [[0, 0, 0],
         [0, 0, 0],
         [0, 1, 0]]
    
    shift = lambda x, w: convolve(x.reshape((8, 8)), mode='constant',
                                   weights=w).ravel()
    X = np.concatenate([X] +
                       [np.apply_along_axis(shift, 1, X, vector)
                        for vector in direction_vectors])
    Y = np.concatenate([Y for _ in range(5)], axis=0)
    return X, Y

# Load Data
digits = datasets.load_digits()
X = np.asarray(digits.data, 'float32')
X, Y = nudge_dataset(X, digits.target)
X = (X - np.min(X, 0)) / (np.max(X, 0) + 0.0001)  # 0-1 scaling
X_train, X_test, Y_train, Y_test = train_test_split(X, Y,
                                                       test_size=0.2,
                                                       random_state=0)

# Models we will use
logistic = linear_model.LogisticRegression()
rbm = BernoulliRBM(random_state=0, verbose=True)

classifier = Pipeline(steps=[('rbm', rbm), ('logistic', logistic)])

Training
```
# Hyper-parameters. These were set by cross-validation, # using a GridSearchCV. Here we are not performing cross-validation to # save time.
rbm.learning_rate = 0.06
rbm.n_iter = 20
# More components tend to give better prediction performance, but larger # fitting time
rbm.n_components = 100
logistic.C = 6000.0

# Training RBM-Logistic Pipeline
classifier.fit(X_train, Y_train)

# Training Logistic regression
logistic_classifier = linear_model.LogisticRegression(C=100.0)
logistic_classifier.fit(X_train, Y_train)

Out:

[ BernoulliRBM ] Iteration 1, pseudo-likelihood = -25.39, time = 0.40s
[ BernoulliRBM ] Iteration 2, pseudo-likelihood = -23.77, time = 0.52s
[ BernoulliRBM ] Iteration 3, pseudo-likelihood = -22.94, time = 0.51s
[ BernoulliRBM ] Iteration 4, pseudo-likelihood = -21.91, time = 0.51s
[ BernoulliRBM ] Iteration 5, pseudo-likelihood = -21.69, time = 0.51s
[ BernoulliRBM ] Iteration 6, pseudo-likelihood = -21.06, time = 0.51s
[ BernoulliRBM ] Iteration 7, pseudo-likelihood = -20.89, time = 0.50s
[ BernoulliRBM ] Iteration 8, pseudo-likelihood = -20.64, time = 0.50s
[ BernoulliRBM ] Iteration 9, pseudo-likelihood = -20.36, time = 0.50s
[ BernoulliRBM ] Iteration 10, pseudo-likelihood = -20.09, time = 0.50s
[ BernoulliRBM ] Iteration 11, pseudo-likelihood = -20.08, time = 0.50s
[ BernoulliRBM ] Iteration 12, pseudo-likelihood = -19.82, time = 0.50s
[ BernoulliRBM ] Iteration 13, pseudo-likelihood = -19.64, time = 0.50s
[ BernoulliRBM ] Iteration 14, pseudo-likelihood = -19.61, time = 0.50s
[ BernoulliRBM ] Iteration 15, pseudo-likelihood = -19.57, time = 0.51s
[ BernoulliRBM ] Iteration 16, pseudo-likelihood = -19.41, time = 0.50s
[ BernoulliRBM ] Iteration 17, pseudo-likelihood = -19.30, time = 0.50s
[ BernoulliRBM ] Iteration 18, pseudo-likelihood = -19.25, time = 0.50s
[ BernoulliRBM ] Iteration 19, pseudo-likelihood = -19.27, time = 0.50s
[ BernoulliRBM ] Iteration 20, pseudo-likelihood = -19.01, time = 0.50s

Evaluation

print() 
print("Logistic regression using RBM features:
metrics.classification_report(
    Y_test,
    classifier.predict(X_test)))")

print("Logistic regression using raw pixel features:
metrics.classification_report(
    Y_test,
    logistic_classifier.predict(X_test)))")

Out:

Logistic regression using RBM features:
   precision    recall  f1-score   support

23.2. Restricted Boltzmann Machine features for digit classification
Logistic regression using raw pixel features:

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.85</td>
<td>0.94</td>
<td>0.89</td>
<td>174</td>
</tr>
<tr>
<td>1</td>
<td>0.57</td>
<td>0.55</td>
<td>0.56</td>
<td>184</td>
</tr>
<tr>
<td>2</td>
<td>0.72</td>
<td>0.85</td>
<td>0.78</td>
<td>166</td>
</tr>
<tr>
<td>3</td>
<td>0.76</td>
<td>0.74</td>
<td>0.75</td>
<td>194</td>
</tr>
<tr>
<td>4</td>
<td>0.85</td>
<td>0.82</td>
<td>0.84</td>
<td>186</td>
</tr>
<tr>
<td>5</td>
<td>0.74</td>
<td>0.75</td>
<td>0.75</td>
<td>181</td>
</tr>
<tr>
<td>6</td>
<td>0.93</td>
<td>0.88</td>
<td>0.91</td>
<td>207</td>
</tr>
<tr>
<td>7</td>
<td>0.86</td>
<td>0.90</td>
<td>0.88</td>
<td>154</td>
</tr>
<tr>
<td>8</td>
<td>0.68</td>
<td>0.55</td>
<td>0.61</td>
<td>182</td>
</tr>
<tr>
<td>9</td>
<td>0.71</td>
<td>0.74</td>
<td>0.72</td>
<td>169</td>
</tr>
</tbody>
</table>

avg / total 0.77 0.77 0.77 1797

Plotting

```python
plt.figure(figsize=(4.2, 4))
for i, comp in enumerate(rbm.components_):
    plt.subplot(10, 10, i + 1)
    plt.imshow(comp.reshape((8, 8)), cmap=plt.cm.gray_r,
               interpolation='nearest')
    plt.xticks(())
    plt.yticks(())
plt.suptitle('100 components extracted by RBM', fontsize=16)
plt.subplots_adjust(0.08, 0.02, 0.92, 0.85, 0.08, 0.23)
plt.show()
```
23.3 Compare Stochastic learning strategies for MLPClassifier

This example visualizes some training loss curves for different stochastic learning strategies, including SGD and Adam. Because of time-constraints, we use several small datasets, for which L-BFGS might be more suitable. The general trend shown in these examples seems to carry over to larger datasets, however.

Note that those results can be highly dependent on the value of `learning_rate_init`. 

Total running time of the script: (0 minutes 36.804 seconds)

Download Python source code: plot_rbm_logistic_classification.py
Download IPython notebook: plot_rbm_logistic_classification.ipynb
learning on dataset iris
training: constant learning-rate
Training set score: 0.980000
Training set loss: 0.096922
training: constant with momentum
Training set score: 0.980000
Training set loss: 0.050260
training: constant with Nesterov's momentum
Training set score: 0.980000
Training set loss: 0.050277
training: inv-scaling learning-rate
Training set score: 0.360000
Training set loss: 0.979983
training: inv-scaling with momentum
Training set score: 0.860000
Training set loss: 0.504017
training: inv-scaling with Nesterov's momentum
Training set score: 0.860000
Training set loss: 0.504760
training: adam
Training set score: 0.980000
Training set loss: 0.046248

learning on dataset digits
training: constant learning-rate
Training set score: 0.956038
Training set loss: 0.243802
training: constant with momentum
Training set score: 0.992766
Training set loss: 0.041297
training: constant with Nesterov's momentum
Training set score: 0.993879
Training set loss: 0.042898
training: inv-scaling learning-rate
Training set score: 0.638843
Training set loss: 1.855465
training: inv-scaling with momentum
Training set score: 0.912632
Training set loss: 0.290584
training: inv-scaling with Nesterov's momentum
Training set score: 0.909293
Training set loss: 0.318387
training: adam
Training set score: 0.991653
Training set loss: 0.045934

learning on dataset circles
training: constant learning-rate
Training set score: 0.830000
Training set loss: 0.681498
training: constant with momentum
Training set score: 0.940000
Training set loss: 0.163712
training: constant with Nesterov's momentum
Training set score: 0.940000
Training set loss: 0.163012
training: inv-scaling learning-rate
Training set score: 0.500000
Training set loss: 0.692855
training: inv-scaling with momentum
Training set score: 0.510000
Training set loss: 0.688376
training: inv-scaling with Nesterov's momentum
Training set score: 0.500000
Training set loss: 0.688593
training: adam
Training set score: 0.930000
Training set loss: 0.159988

learning on dataset moons
training: constant learning-rate
Training set score: 0.850000
Training set loss: 0.342245
training: constant with momentum
Training set score: 0.850000
Training set loss: 0.345580
training: constant with Nesterov's momentum
Training set score: 0.850000
Training set loss: 0.336284
training: inv-scaling learning-rate
Training set score: 0.500000
Training set loss: 0.689729
training: inv-scaling with momentum
Training set score: 0.830000
Training set loss: 0.512595

23.3. Compare Stochastic learning strategies for MLPClassifier
training: inv-scaling with Nesterov's momentum
Training set score: 0.830000
Training set loss: 0.513034
training: adam
Training set score: 0.850000
Training set loss: 0.334243

```
print(__doc__)
import matplotlib.pyplot as plt
from sklearn.neural_network import MLPClassifier
from sklearn.preprocessing import MinMaxScaler
from sklearn.datasets import datasets

# different learning rate schedules and momentum parameters
params = [
    {'solver': 'sgd', 'learning_rate': 'constant', 'momentum': 0, 'learning_rate_init': 0.2},
    {'solver': 'sgd', 'learning_rate': 'constant', 'momentum': .9, 'nesterovs_momentum': False, 'learning_rate_init': 0.2},
    {'solver': 'sgd', 'learning_rate': 'constant', 'momentum': .9, 'nesterovs_momentum': True, 'learning_rate_init': 0.2},
    {'solver': 'sgd', 'learning_rate': 'invscaling', 'momentum': 0, 'learning_rate_init': 0.2},
    {'solver': 'sgd', 'learning_rate': 'invscaling', 'momentum': .9, 'nesterovs_momentum': False, 'learning_rate_init': 0.2},
    {'solver': 'sgd', 'learning_rate': 'invscaling', 'momentum': .9, 'nesterovs_momentum': True, 'learning_rate_init': 0.2},
    {'solver': 'adam', 'learning_rate_init': 0.01}]

labels = ["constant learning-rate", "constant with momentum", "constant with Nesterov's momentum", "inv-scaling learning-rate", "inv-scaling with momentum", "inv-scaling with Nesterov's momentum", "adam"]

plot_args = [{'c': 'red', 'linestyle': '-'},
             {'c': 'green', 'linestyle': '-'},
             {'c': 'blue', 'linestyle': '-'},
             {'c': 'red', 'linestyle': '--'},
             {'c': 'green', 'linestyle': '--'},
             {'c': 'blue', 'linestyle': '--'},
             {'c': 'black', 'linestyle': '--'}]

def plot_on_dataset(X, y, ax, name):
    # for each dataset, plot learning for each learning strategy
    print("learning on dataset %s" % name)
    ax.set_title(name)
    X = MinMaxScaler().fit_transform(X)
    mlpss = []
    if name == "digits":  # digits is larger but converges fairly quickly
        max_iter = 15
    else:
        max_iter = 400
for label, param in zip(labels, params):
    print("training: %s \% label")
    mlp = MLPClassifier(verbos=0, random_state=0,
        max_iter=max_iter, **param)
    mlp.fit(X, y)
    mps.append(mlp)
    print("Training set score: %f \% mlp.score(X, y)"))
    print("Training set loss: %f \% mlp.loss_"])
    for mlp, label, args in zip(mps, labels, plot_args):
        ax.plot(mlp.loss_curve_, label=label, **args)

fig, axes = plt.subplots(2, 2, figsize=(15, 10))
# load / generate some toy datasets
iris = datasets.load_iris()
digits = datasets.load_digits()
data_sets = [(iris.data, iris.target),
        (digits.data, digits.target),
        datasets.make_circles(noise=0.2, factor=0.5, random_state=1),
        datasets.make_moons(noise=0.3, random_state=0)]

for ax, data, name in zip(axes.ravel(), data_sets, ['iris', 'digits',
        'circles', 'moons']):
    plot_on_dataset(*data, ax=ax, name=name)
fig.legend(ax.get_lines(), labels=labels, ncol=3, loc="upper center")
plt.show()

Total running time of the script: (0 minutes 5.879 seconds)

Download Python source code: plot_mlp_training_curves.py
Download IPython notebook: plot_mlp_training_curves.ipynb

23.4 Varying regularization in Multi-layer Perceptron

A comparison of different values for regularization parameter ‘alpha’ on synthetic datasets. The plot shows that different alphas yield different decision functions.

Alpha is a parameter for regularization term, aka penalty term, that combats overfitting by constraining the size of the weights. Increasing alpha may fix high variance (a sign of overfitting) by encouraging smaller weights, resulting in a decision boundary plot that appears with lesser curvatures. Similarly, decreasing alpha may fix high bias (a sign of underfitting) by encouraging larger weights, potentially resulting in a more complicated decision boundary.
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import make_moons, make_circles, make_classification
from sklearn.neural_network import MLPClassifier

h = .02 # step size in the mesh

alphas = np.logspace(-5, 3, 5)
names = []
for i in alphas:
    names.append('alpha ' + str(i))

classifiers = []
for i in alphas:
    classifiers.append(MLPClassifier(alpha=i, random_state=1))

X, y = make_classification(n_features=2, n_redundant=0, n_informative=2, n_clusters_per_class=1, random_state=0)

rng = np.random.RandomState(2)
X += 2 * rng.uniform(size=X.shape)
linearly_separable = (X, y)
datasets = [make_moons(noise=0.3, random_state=0),
            make_circles(noise=0.2, factor=0.5, random_state=1),
            linearly_separable]

print(__doc__)
```python
# Just plot the dataset first
fig = plt.figure(figsize=(17, 9))
i = 1
for X, y in datasets:
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=.4)
    x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
    y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    ax = plt.subplot(len(datasets), len(classifiers) + 1, i)
    ax.scatter(X_train[:, 0], X_train[:, 1], c=y_train, cmap=cm_bright)
    ax.scatter(X_test[:, 0], X_test[:, 1], c=y_test, cmap=cm_bright, alpha=0.6)
    i += 1

# Plot the decision boundary. For that, we will assign a color to each point in the mesh [x_min, x_max]x[y_min, y_max].
for name, clf in zip(names, classifiers):
    ax = plt.subplot(len(datasets), len(classifiers) + 1, i)
    clf.fit(X_train, y_train)
    score = clf.score(X_test, y_test)
    if hasattr(clf, "decision_function"):  
        Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
    else:
        Z = clf.predict_proba(np.c_[xx.ravel(), yy.ravel()])[:, 1]
    ax.contourf(xx, yy, Z, cmap=cm, alpha=.8)
    ax.scatter(X_train[:, 0], X_train[:, 1], c=y_train, cmap=cm_bright)
    ax.scatter(X_test[:, 0], X_test[:, 1], c=y_test, cmap=cm_bright,
               alpha=0.6)
    ax.set_xlim(xx.min(), xx.max())
    ax.set_ylim(yy.min(), yy.max())
    ax.set_xticks(())
    ax.set_yticks(())
    ax.set_title(name)
    ax.text(xx.max() - .3, yy.min() + .3, ('%.2f' % score).lstrip('0'),
            size=15, horizontalalignment='right')
    i += 1
```

23.4. Varying regularization in Multi-layer Perceptron
```
figure.subplots_adjust(left=.02, right=.98)
plt.show()
```

**Total running time of the script:** (0 minutes 5.244 seconds)

**Download Python source code:** plot_mlp_alpha.py

**Download IPython notebook:** plot_mlp_alpha.ipynb
Examples concerning the `sklearn.preprocessing` module.

### 24.1 Using FunctionTransformer to select columns

Shows how to use a function transformer in a pipeline. If you know your dataset’s first principle component is irrelevant for a classification task, you can use the FunctionTransformer to select all but the first column of the PCA transformed data.

```python
import matplotlib.pyplot as plt
import numpy as np
from sklearn.model_selection import train_test_split

```
```python
from sklearn.decomposition import PCA
from sklearn.pipeline import make_pipeline
from sklearn.preprocessing import FunctionTransformer

def _generate_vector(shift=0.5, noise=15):
    return np.arange(1000) + (np.random.rand(1000) - shift) * noise

def generate_dataset():
    """This dataset is two lines with a slope ~ 1, where one has a y offset of ~100"""
    return np.vstack((
        np.vstack((
            _generate_vector(),
            _generate_vector() + 100,
        )).T,
        np.vstack((
            _generate_vector(),
            _generate_vector(),
        )).T,
    )).T,
    np.hstack((np.zeros(1000), np.ones(1000)))

def all_but_first_column(X):
    return X[:, 1:]

def drop_first_component(X, y):
    """Create a pipeline with PCA and the column selector and use it to transform the dataset."
    pipeline = make_pipeline(PCA(), FunctionTransformer(all_but_first_column),
    )
    X_train, X_test, y_train, y_test = train_test_split(X, y)
    pipeline.fit(X_train, y_train)
    return pipeline.transform(X_test), y_test

if __name__ == '__main__':
    X, y = generate_dataset()
    lw = 0
    plt.figure()
    plt.scatter(X[:, 0], X[:, 1], c=y, lw=lw)
    plt.figure()
    X_transformed, y_transformed = drop_first_component(*generate_dataset())
    plt.scatter(
        X_transformed[:, 0],
        np.zeros(len(X_transformed)),
        c=y_transformed,
        lw=lw,
        s=60
    )
    plt.show()
```
24.2 Robust Scaling on Toy Data

Making sure that each feature has approximately the same scale can be a crucial preprocessing step. However, when data contains outliers, StandardScaler can often be mislead. In such cases, it is better to use a scaler that is robust against outliers.

Here, we demonstrate this on a toy dataset, where one single datapoint is a large outlier.

Out:

```
Testset accuracy using standard scaler: 0.545
Testset accuracy using robust scaler: 0.705
```

```python
from __future__ import print_function
print(__doc__)

# Code source: Thomas Unterthiner
# License: BSD 3 clause

import matplotlib.pyplot as plt
import numpy as np
from sklearn.preprocessing import StandardScaler, RobustScaler

# Create training and test data
np.random.seed(42)
n_datapoints = 100
Cov = [[0.9, 0.0], [0.0, 20.0]]
mu1 = [100.0, -3.0]
mu2 = [101.0, -3.0]
X1 = np.random.multivariate_normal(mean=mu1, cov=Cov, size=n_datapoints)
```
```python
X2 = np.random.multivariate_normal(mean=mu2, cov=Cov, size=n_datapoints)
Y_train = np.hstack([-1]*n_datapoints, [1]*n_datapoints)
X_train = np.vstack((X1, X2))

X1 = np.random.multivariate_normal(mean=mu1, cov=Cov, size=n_datapoints)
X2 = np.random.multivariate_normal(mean=mu2, cov=Cov, size=n_datapoints)
Y_test = np.hstack([-1]*n_datapoints, [1]*n_datapoints)
X_test = np.vstack((X1, X2))

X_train[0, 0] = -1000  # a fairly large outlier

# Scale data
standard_scaler = StandardScaler()
Xtr_s = standard_scaler.fit_transform(X_train)
Xte_s = standard_scaler.transform(X_test)

robust_scaler = RobustScaler()
Xtr_r = robust_scaler.fit_transform(X_train)
Xte_r = robust_scaler.transform(X_test)

# Plot data
fig, ax = plt.subplots(1, 3, figsize=(12, 4))
ax[0].scatter(X_train[:, 0], X_train[:, 1],
              color=np.where(Y_train > 0, 'r', 'b'))
ax[1].scatter(Xtr_s[:, 0], Xtr_s[:, 1], color=np.where(Y_train > 0, 'r', 'b'))
ax[2].scatter(Xtr_r[:, 0], Xtr_r[:, 1], color=np.where(Y_train > 0, 'r', 'b'))
ax[0].set_title("Unscaled data")
ax[1].set_title("After standard scaling (zoomed in)")
ax[2].set_title("After robust scaling (zoomed in)")

# for the scaled data, we zoom in to the data center (outlier can't be seen!)
for a in ax[1:]:
a.set_xlim(-3, 3)
a.set_ylim(-3, 3)
plt.tight_layout()
plt.show()

# Classify using k-NN
from sklearn.neighbors import KNeighborsClassifier
knn = KNeighborsClassifier()
knn.fit(Xtr_s, Y_train)
acc_s = knn.score(Xte_s, Y_test)
print("Testset accuracy using standard scaler: \$%.3f\$ \% acc_s)
knn.fit(Xtr_r, Y_train)
acc_r = knn.score(Xte_r, Y_test)
print("Testset accuracy using robust scaler: \$%.3f\$ \% acc_r)
```

Total running time of the script: (0 minutes 0.284 seconds)

Download Python source code: plot_robust_scaling.py
Download IPython notebook: plot_robust_scaling.ipynb
Examples concerning the `sklearn.semi_supervised` module.

### 25.1 Label Propagation learning a complex structure

Example of LabelPropagation learning a complex internal structure to demonstrate “manifold learning”. The outer circle should be labeled “red” and the inner circle “blue”. Because both label groups lie inside their own distinct shape, we can see that the labels propagate correctly around the circle.

```python
print(__doc__)

# Authors: Clay Woolam <clay@woolam.org>
# Andreas Mueller <amueller@ais.uni-bonn.de>
# License: BSD

import numpy as np
import matplotlib.pyplot as plt
from sklearn.semi_supervised import LabelPropagation
from sklearn.datasets import make_circles

# generate ring with inner box
n_samples = 200
X, y = make_circles(n_samples=n_samples, shuffle=False)
outer, inner = 0, 1
labels = -np.ones(n_samples)
labels[0] = outer
labels[-1] = inner

label_spread = LabelPropagation(kernel='knn', alpha=1.0)
label_spread.fit(X, labels)

output_labels = label_spread.transduction_
plt.figure(figsize=(8.5, 4))
plt.scatter(X[labels == outer, 0], X[labels == outer, 1], color='navy',
            marker='s', lw=0, label="outer labeled", s=10)
plt.scatter(X[labels == inner, 0], X[labels == inner, 1], color='c',
            marker='s', lw=0, label="inner labeled", s=10)
plt.scatter(X[labels == -1, 0], X[labels == -1, 1], color='darkorange',
            label="unlabeled")
```

Learn with LabelSpreading

```python
label_spread = LabelPropagation(kernel='knn', alpha=1.0)
label_spread.fit(X, labels)
```

Plot output labels

```python
output_labels = label_spread.transduction_
plt.figure(figsize=(8.5, 4))
plt.scatter(X[labels == outer, 0], X[labels == outer, 1], color='navy',
            marker='s', lw=0, label="outer labeled", s=10)
plt.scatter(X[labels == inner, 0], X[labels == inner, 1], color='c',
            marker='s', lw=0, label="inner labeled", s=10)
plt.scatter(X[labels == -1, 0], X[labels == -1, 1], color='darkorange',
            label="unlabeled")
```
25.2 Label Propagation digits: Demonstrating performance

This example demonstrates the power of semisupervised learning by training a Label Spreading model to classify handwritten digits with sets of very few labels.

The handwritten digit dataset has 1797 total points. The model will be trained using all points, but only 30 will be labeled. Results in the form of a confusion matrix and a series of metrics over each class will be very good.

At the end, the top 10 most uncertain predictions will be shown.

```python
print(__doc__)
```
import numpy as np
import matplotlib.pyplot as plt

from scipy import stats
from sklearn import datasets
from sklearn.semi_supervised import label_propagation

from sklearn.metrics import confusion_matrix, classification_report

digits = datasets.load_digits()
rng = np.random.RandomState(0)
indices = np.arange(len(digits.data))
rng.shuffle(indices)

X = digits.data[indices[:330]]
y = digits.target[indices[:330]]
images = digits.images[indices[:330]]

n_total_samples = len(y)
n_labeled_points = 30

indices = np.arange(n_total_samples)
unlabeled_set = indices[n_labeled_points:]

# shuffle everything around
y_train = np.copy(y)
y_train[unlabeled_set] = -1

Learn with LabelSpreading

lp_model = label_propagation.LabelSpreading(gamma=0.25, max_iter=5)
lp_model.fit(X, y_train)
predicted_labels = lp_model.transduction_[unlabeled_set]
true_labels = y[unlabeled_set]

cm = confusion_matrix(true_labels, predicted_labels, labels=lp_model.classes_)

print("Label Spreading model: %d labeled & %d unlabeled points (%d total)" %
      (n_labeled_points, n_total_samples - n_labeled_points, n_total_samples))

print(classification_report(true_labels, predicted_labels))

print("Confusion matrix")
print(cm)

# calculate uncertainty values for each transduced distribution
pred_entropies = stats.distributions.entropy(lp_model.label_distributions_.T)

# pick the top 10 most uncertain labels
uncertainty_index = np.argsort(pred_entropies)[-10:]

Out:
Label Spreading model: 30 labeled & 300 unlabeled points (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.58</td>
<td>0.54</td>
<td>0.56</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>0.96</td>
<td>0.93</td>
<td>0.95</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>0.91</td>
<td>0.80</td>
<td>0.85</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>0.79</td>
<td>0.87</td>
<td>33</td>
</tr>
<tr>
<td>6</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>0.89</td>
<td>1.00</td>
<td>0.94</td>
<td>34</td>
</tr>
<tr>
<td>8</td>
<td>0.48</td>
<td>0.83</td>
<td>0.61</td>
<td>29</td>
</tr>
<tr>
<td>9</td>
<td>0.54</td>
<td>0.77</td>
<td>0.64</td>
<td>35</td>
</tr>
</tbody>
</table>

avg / total 0.73 0.77 0.74 300

Confusion matrix

```
[[23 0 0 0 0 0 0 0 0]
 [ 0 15 1 0 0 1 0 11 0]
 [ 0 0 27 0 0 0 2 0 0]
 [ 0 5 0 20 0 0 0 0 0]
 [ 0 0 0 26 0 0 1 6 0]
 [ 0 1 0 0 0 35 0 0 0]
 [ 0 0 0 0 0 0 34 0 0]
 [ 0 5 0 0 0 0 0 24 0]
 [ 0 0 0 2 1 0 2 3 27]]
```

plot

```python
f = plt.figure(figsize=(7, 5))
for index, image_index in enumerate(uncertainty_index):
    image = images[image_index]
    sub = f.add_subplot(2, 5, index + 1)
    sub.imshow(image, cmap=plt.cm.gray_r)
    plt.xticks([])
    plt.yticks([])
    sub.set_title('predict: %i
         true: %i' % (lp_model.transduction_[image_index], y[image_index]))

f.suptitle('Learning with small amount of labeled data')
plt.show()
```
25.3 Decision boundary of label propagation versus SVM on the Iris dataset

Comparison for decision boundary generated on iris dataset between Label Propagation and SVM.

This demonstrates Label Propagation learning a good boundary even with a small amount of labeled data.
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn import svm
from sklearn.semi_supervised import label_propagation

rng = np.random.RandomState(0)
iris = datasets.load_iris()
X = iris.data[:, :2]
y = iris.target

# step size in the mesh
h = .02

y_30 = np.copy(y)
y_30[rng.rand(len(y)) < 0.3] = -1
y_50 = np.copy(y)
y_50[rng.rand(len(y)) < 0.5] = -1

# we create an instance of SVM and fit out data. We do not scale our
# data since we want to plot the support vectors
ls30 = (label_propagation.LabelSpreading().fit(X, y_30), y_30)
ls50 = (label_propagation.LabelSpreading().fit(X, y_50), y_50)
ls100 = (label_propagation.LabelSpreading().fit(X, y), y)
rbf_svc = (svm.SVC(kernel='rbf').fit(X, y), y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

titles = ['Label Spreading 30\% data',
          'Label Spreading 50\% data',
          'Label Spreading 100\% data',
          'SVC with rbf kernel']

color_map = {-1: (1, 1, 1), 0: (0, 0, .9), 1: (1, 0, 0), 2: (.8, .6, 0)}

for i, (clf, y_train) in enumerate((ls30, ls50, ls100, rbf_svc)):
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    plt.subplot(2, 2, i + 1)
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)
    plt.axis('off')
    # Plot also the training points
    colors = [color_map[y] for y in y_train]
    plt.scatter(X[:, 0], X[:, 1], c=colors, cmap=plt.cm.Paired)
    plt.title(titles[i])
    plt.text(.90, .0, "Unlabeled points are colored white")
plt.show()
A plot will appear showing the top 5 most uncertain digits for each iteration of training. These may or may not contain mistakes, but we will train the next model with their true labels.

Active learning with Label Propagation.
Rows show 5 most uncertain labels to learn with the next model.

Out:

<table>
<thead>
<tr>
<th>Iteration 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Label Spreading model: labeled &amp; unlabeled (total)</td>
</tr>
<tr>
<td>precision</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
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<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>avg / total</td>
</tr>
</tbody>
</table>

Confusion matrix
[[26 1 0 0 1 0 1]
 [ 1 30 0 0 0 0 0]
 [ 0 0 17 6 0 2 10]]
Iteration 1
Label Spreading model: 15 labeled & 315 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
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<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.61</td>
<td>0.59</td>
<td>0.60</td>
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<td>0.71</td>
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<td>4</td>
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<tr>
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<td>0.84</td>
<td>0.66</td>
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<tr>
<td>avg / total</td>
<td>0.82</td>
<td>0.80</td>
<td>0.79</td>
<td>315</td>
</tr>
</tbody>
</table>

Confusion matrix

```
[[23 0 0 0 0 0 0 0 0 0]
 [0 17 1 0 2 0 0 1 7 1]
 [0 0 130 0 0 0 0 0 0 0]
 [0 0 0 15 0 0 0 10 2 0]
 [0 3 0 0 23 0 0 0 0 0]
 [0 0 0 0 1 16 6 0 2 10]
 [0 2 0 0 0 0 38 0 0 0]
 [0 0 2 0 1 0 0 33 0 0]
 [0 5 0 0 0 0 0 0 26 0]
 [0 0 0 0 2 2 0 0 3 30]]
```

Iteration 2
Label Spreading model: 20 labeled & 310 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.68</td>
<td>0.59</td>
<td>0.63</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>0.97</td>
<td>0.94</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
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<td>0.98</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>0.81</td>
<td>1.00</td>
<td>0.89</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.89</td>
<td>0.46</td>
<td>0.60</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>0.86</td>
<td>0.95</td>
<td>0.90</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>0.97</td>
<td>0.92</td>
<td>0.94</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>0.68</td>
<td>0.84</td>
<td>0.75</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.81</td>
<td>0.78</td>
<td>37</td>
</tr>
<tr>
<td>avg / total</td>
<td>0.85</td>
<td>0.84</td>
<td>0.83</td>
<td>310</td>
</tr>
</tbody>
</table>

Confusion matrix

```
[[23 0 0 0 0 0 0 0 0 0]
 [0 17 1 0 2 0 0 1 7 1]
 [0 0 130 0 0 0 0 0 0 0]
 [0 0 0 23 0 0 0 0 0 0]
 [0 0 0 0 23 0 0 0 0 0]
 [0 0 0 0 1 16 6 0 2 9]
 [0 2 0 0 0 0 38 0 0 0]
 [0 0 2 0 1 0 0 33 0 0]
 [0 5 0 0 0 0 0 0 26 0]
 [0 0 0 0 2 2 0 0 3 30]]
```
Iteration 3

Label Spreading model: 25 labeled & 305 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
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<td>0.85</td>
<td>0.77</td>
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</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.90</td>
<td>0.95</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>0.74</td>
<td>0.83</td>
<td>34</td>
</tr>
<tr>
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<td>0.95</td>
<td>0.97</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>1.00</td>
<td>0.95</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>0.83</td>
<td>0.81</td>
<td>0.82</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.83</td>
<td>0.79</td>
<td>36</td>
</tr>
</tbody>
</table>

avg / total 0.91 0.90 0.90 305

Confusion matrix

```
[[23 0 0 0 0 0 0 0 0 0]
 [0 23 0 0 0 0 0 0 4 0]
 [0 1 28 0 0 0 0 2 0 0]
 [0 0 0 23 0 0 0 0 0 0]
 [0 0 0 0 25 0 0 0 0 0]
 [0 0 0 0 0 25 0 0 0 9]
 [0 2 0 0 0 0 38 0 0 0]
 [0 0 0 0 0 0 35 0 0 0]
 [0 5 0 0 0 0 0 25 1 0]
 [0 2 0 0 0 1 0 32 0 0]]
```

Iteration 4

Label Spreading model: 30 labeled & 300 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
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<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.77</td>
<td>0.88</td>
<td>0.82</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.90</td>
<td>0.95</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.94</td>
<td>0.97</td>
<td>0.95</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.97</td>
<td>0.99</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>1.00</td>
<td>0.95</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>0.89</td>
<td>0.81</td>
<td>0.85</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.94</td>
<td>0.89</td>
<td>0.91</td>
<td>35</td>
</tr>
</tbody>
</table>

avg / total 0.94 0.94 0.94 300

Confusion matrix

```
[[23 0 0 0 0 0 0 0 0 0]
 [0 23 0 0 0 0 0 0 3 0]
 [0 1 28 0 0 0 0 2 0 0]
 [0 0 0 23 0 0 0 0 0 0]
 [0 0 0 0 25 0 0 0 0 0]
 [0 0 0 0 0 31 0 0 0 1]
 [0 1 0 0 0 0 38 0 0 0]
 [0 0 0 0 0 0 35 0 0 0]
 [0 5 0 0 0 0 0 25 1 0]
 [0 0 0 0 0 2 0 2 0 31]]
```
```python
print(__doc__)

# Authors: Clay Woolam <clay@woolam.org>
# License: BSD

import numpy as np
import matplotlib.pyplot as plt
from scipy import stats
from sklearn import datasets
from sklearn.semi_supervised import label_propagation
from sklearn.metrics import classification_report, confusion_matrix

digits = datasets.load_digits()
rng = np.random.RandomState(0)
indices = np.arange(len(digits.data))
rng.shuffle(indices)

X = digits.data[indices[:330]]
y = digits.target[indices[:330]]
images = digits.images[indices[:330]]

n_total_samples = len(y)
n_labeled_points = 10

unlabeled_indices = np.arange(n_total_samples)[n_labeled_points:]
f = plt.figure()

for i in range(5):
    y_train = np.copy(y)
y_train[unlabeled_indices] = -1

lp_model = label_propagation.LabelSpreading(gamma=0.25, max_iter=5)
lp_model.fit(X, y_train)
predicted_labels = lp_model.transduction_[unlabeled_indices]
true_labels = y[unlabeled_indices]

cm = confusion_matrix(true_labels, predicted_labels,
                      labels=lp_model.classes_)

print('Iteration %i %s' % (i, 70 * '_'))
print("Label Spreading model: %d labeled & %d unlabeled (%d total)"
      % (n_labeled_points, n_total_samples - n_labeled_points, n_total_samples))
print(classification_report(true_labels, predicted_labels))
print("Confusion matrix")
print(cm)

# compute the entropies of transduced label distributions
pred_entropies = stats.distributions.entropy(
    lp_model.label_distributions_.T)

# select five digit examples that the classifier is most uncertain about
```

25.4. Label Propagation digits active learning
uncertainty_index = np.argsort(pred_entropies)[-5:]

# keep track of indices that we get labels for
delete_indices = np.array([])

for index, image_index in enumerate(uncertainty_index):
    image = images[image_index]

    sub = f.add_subplot(5, 5, index + 1 + (5*i))
    sub.imshow(image, cmap=plt.cm.gray_r)
    sub.set_title('predict: %i, true: %i' % (lp_model.transduction_[image_index], y[image_index]), size=10)
    sub.axis('off')

    delete_index, = np.where(unlabeled_indices == image_index)
    delete_indices = np.concatenate((delete_indices, delete_index))

    unlabeled_indices = np.delete(unlabeled_indices, delete_indices)
    n_labeled_points += 5

f.suptitle("Active learning with Label Propagation.Rows show 5 most "
          "uncertain labels to learn with the next model.")
plt.subplots_adjust(0.12, 0.03, 0.9, 0.8, 0.2, 0.45)
plt.show()
Examples concerning the sklearn.svm module.

### 26.1 Support Vector Regression (SVR) using linear and non-linear kernels

Toy example of 1D regression using linear, polynomial and RBF kernels.

```python
print(__doc__)

import numpy as np
from sklearn.svm import SVR
import matplotlib.pyplot as plt

# Generate sample data
X = np.sort(5 * np.random.rand(40, 1), axis=0)
y = np.sin(X).ravel()

# Add noise to targets
y[::5] += 3 * (0.5 - np.random.rand(8))

# Fit regression model
svr_rbf = SVR(kernel='rbf', C=1e3, gamma=0.1)
svr_lin = SVR(kernel='linear', C=1e3)
svr_poly = SVR(kernel='poly', C=1e3, degree=2)
y_rbf = svr_rbf.fit(X, y).predict(X)
y_lin = svr_lin.fit(X, y).predict(X)
y_poly = svr_poly.fit(X, y).predict(X)

# Look at the results
lw = 2
plt.scatter(X, y, color='darkorange', label='data')
plt.hold('on')
plt.plot(X, y_rbf, color='navy', lw=lw, label='RBF model')
plt.plot(X, y_lin, color='c', lw=lw, label='Linear model')
plt.plot(X, y_poly, color='cornflowerblue', lw=lw, label='Polynomial model')
plt.xlabel('data')
plt.ylabel('target')
plt.title('Support Vector Regression')
```

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Perform binary classification using non-linear SVC with RBF kernel. The target to predict is a XOR of the inputs. The color map illustrates the decision function learned by the SVC.

26.2 Non-linear SVM

Total running time of the script: (0 minutes 0.778 seconds)
Download Python source code: plot_svm_regression.py
Download IPython notebook: plot_svm_regression.ipynb
```
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm

xx, yy = np.meshgrid(np.linspace(-3, 3, 500),
                     np.linspace(-3, 3, 500))
np.random.seed(0)
X = np.random.randn(300, 2)
Y = np.logical_xor(X[:, 0] > 0, X[:, 1] > 0)

# fit the model
clf = svm.NuSVC()
clf.fit(X, Y)

# plot the decision function for each datapoint on the grid
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
plt.imshow(Z, interpolation='nearest',
           extent=(xx.min(), xx.max(), yy.min(), yy.max()), aspect='auto',
           origin='lower', cmap=plt.cm.PuOr_r)
contours = plt.contour(xx, yy, Z, levels=[0], linewidths=2,
                       linetypes='--')
plt.scatter(X[:, 0], X[:, 1], s=30, c=Y, cmap=plt.cm.Paired)
```
26.3 SVM: Maximum margin separating hyperplane

Plot the maximum margin separating hyperplane within a two-class separable dataset using a Support Vector Machine classifier with linear kernel.
X = np.random.randn(20, 2) - [2, 2], np.random.randn(20, 2) + [2, 2]
Y = [0] * 20 + [1] * 20

# fit the model
clf = svm.SVC(kernel='linear')
clf.fit(X, Y)

# get the separating hyperplane
w = clf.coef_[0]
a = -w[0] / w[1]
xx = np.linspace(-5, 5)
yy = a * xx - (clf.intercept_[0]) / w[1]

# plot the parallels to the separating hyperplane that pass through the
# support vectors
b = clf.support_vectors_[0]
yy_down = a * xx + (b[1] - a * b[0])
b = clf.support_vectors_[-1]
yy_up = a * xx + (b[1] - a * b[0])

# plot the line, the points, and the nearest vectors to the plane
plt.plot(xx, yy, 'k-')
plt.plot(xx, yy_down, 'k--')
plt.plot(xx, yy_up, 'k--')
plt.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1],
        s=80, facecolors='none')
plt.scatter(X[:, 0], X[:, 1], c=Y, cmap=plt.cm.Paired)
plt.axis('tight')
plt.show()

Total running time of the script: (0 minutes 0.061 seconds)

Download Python source code: plot_separating_hyperplane.py
Download IPython notebook: plot_separating_hyperplane.ipynb

26.4 SVM: Separating hyperplane for unbalanced classes

Find the optimal separating hyperplane using an SVC for classes that are unbalanced.

We first find the separating plane with a plain SVC and then plot (dashed) the separating hyperplane with automatically correction for unbalanced classes.

Note: This example will also work by replacing SVC(kernel="linear") with SGDClassifier(loss="hinge"). Setting the loss parameter of the SGDClassifier equal to hinge will yield behaviour such as that of a SVC with a linear kernel.

For example try instead of the SVC:

clf = SGDClassifier(n_iter=100, alpha=0.01)
import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm  # from sklearn.linear_model import SGDClassifier

# we create 40 separable points
rng = np.random.RandomState(0)
n_samples_1 = 1000
n_samples_2 = 100
X = np.r_[1.5 * rng.randn(n_samples_1, 2),
          0.5 * rng.randn(n_samples_2, 2) + [2, 2]]
y = [0] * (n_samples_1) + [1] * (n_samples_2)

# fit the model and get the separating hyperplane
clf = svm.SVC(kernel='linear', C=1.0)
clf.fit(X, y)

w = clf.coef_[0]
a = -w[0] / w[1]
xx = np.linspace(-5, 5)
yy = a * xx - clf.intercept_[0] / w[1]

# get the separating hyperplane using weighted classes

print(__doc__)
wclf = svm.SVC(kernel='linear', class_weight={1: 10})
wclf.fit(X, y)

ww = wclf.coef_[0]
wa = -ww[0] / ww[1]
wy = wa * xx - wclf.intercept_[0] / ww[1]

# plot separating hyperplanes and samples
h0 = plt.plot(xx, yy, 'k-', label='no weights')
h1 = plt.plot(xx, wyy, 'k--', label='with weights')
plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Paired)
plt.legend()
plt.axis('tight')
plt.show()

Total running time of the script: (0 minutes 0.080 seconds)
Download Python source code: plot_separating_hyperplane_unbalanced.py
Download IPython notebook: plot_separating_hyperplane_unbalanced.ipynb

26.5 SVM-Anova: SVM with univariate feature selection

This example shows how to perform univariate feature selection before running a SVC (support vector classifier) to improve the classification scores.

print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm, datasets, feature_selection
from sklearn.model_selection import cross_val_score
from sklearn.pipeline import Pipeline

# Import some data to play with
digits = datasets.load_digits()
y = digits.target

# Throw away data, to be in the curse of dimension settings
y = y[:200]
X = digits.data[:200]
n_samples = len(y)
X = X.reshape((n_samples, -1))

# add 200 non-informative features
X = np.hstack((X, 2 * np.random.random((n_samples, 200))))

Create a feature-selection transform and an instance of SVM that we combine together to have an full-blown estimator

transform = feature_selection.SelectPercentile(feature_selection.f_classif)
clf = Pipeline([('anova', transform), ('svc', svm.SVC(C=1.0))])

Plot the cross-validation score as a function of percentile of features
score_means = list()
score_stds = list()
percentiles = (1, 3, 6, 10, 15, 20, 30, 40, 60, 80, 100)

for percentile in percentiles:
    clf.set_params(anova__percentile=percentile)
    # Compute cross-validation score using 1 CPU
    this_scores = cross_val_score(clf, X, y, n_jobs=1)
    score_means.append(this_scores.mean())
    score_stds.append(this_scores.std())

plt.errorbar(percentiles, score_means, np.array(score_stds))

plt.title('Performance of the SVM-Anova varying the percentile of features selected')
plt.xlabel('Percentile')
plt.ylabel('Prediction rate')
plt.axis('tight')
plt.show()
26.6 SVM with custom kernel

Simple usage of Support Vector Machines to classify a sample. It will plot the decision surface and the support vectors.

```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm, datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
# avoid this ugly slicing by using a two-dim dataset
Y = iris.target

def my_kernel(X, Y):
    """
    We create a custom kernel:

    \[
    k(X, Y) = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} Y.T
    \]
    """
```
M = np.array([[2, 0], [0, 1.0]])
return np.dot(np.dot(X, M), Y.T)

h = .02  # step size in the mesh

# we create an instance of SVM and fit out data.
clf = svm.SVC(kernel=my_kernel)
clf.fit(X, Y)

# Plot the decision boundary. For that, we will assign a color to each
# point in the mesh [x_min, x_max]x[y_min, y_max].
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
plt.pcolormesh(xx, yy, Z, cmap=plt.cm.Paired)

# Plot also the training points
plt.scatter(X[:, 0], X[:, 1], c=Y, cmap=plt.cm.Paired)
plt.title('3-Class classification using Support Vector Machine with custom'
' kernel')
plt.axis('tight')
plt.show()
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm

def plot_decision_function(classifier, sample_weight, axis, title):
    # plot the decision function
    xx, yy = np.meshgrid(np.linspace(-4, 5, 500), np.linspace(-4, 5, 500))
    Z = classifier.decision_function(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)

    # plot the line, the points, and the nearest vectors to the plane
    axis.contourf(xx, yy, Z, alpha=0.75, cmap=plt.cm.bone)
    axis.scatter(X[:, 0], X[:, 1], c=y, s=100 * sample_weight, alpha=0.9, cmap=plt.cm.bone)
    axis.axis('off')
    axis.set_title(title)

# we create 20 points
np.random.seed(0)
X = np.r_[np.random.randn(10, 2) + [1, 1], np.random.randn(10, 2)]
y = [1] * 10 + [-1] * 10
sample_weight_last_ten = abs(np.random.randn(len(X)))
sample_weight_constant = np.ones(len(X))
# and bigger weights to some outliers
sample_weight_last_ten[15:] *= 5
sample_weight_last_ten[9] *= 15

# for reference, first fit without class weights

# fit the model
clf_weights = svm.SVC()
clf_weights.fit(X, y, sample_weight=sample_weight_last_ten)
```

26.7. SVM: Weighted samples
```python
clf_no_weights = svm.SVC()
clf_no_weights.fit(X, y)

fig, axes = plt.subplots(1, 2, figsize=(14, 6))
plot_decision_function(clf_no_weights, sample_weight_constant, axes[0],
                      "Constant weights")
plot_decision_function(clf_weights, sample_weight_last_ten, axes[1],
                      "Modified weights")

plt.show()
```

Total running time of the script: (0 minutes 0.507 seconds)

Download Python source code: plot_weighted_samples.py
Download IPython notebook: plot_weighted_samples.ipynb

### 26.8 SVM-Kernels

Three different types of SVM-Kernels are displayed below. The polynomial and RBF are especially useful when the data-points are not linearly separable.
# License: BSD 3 clause

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm

# Our dataset and targets
X = np.c_[(.4, -.7),
          (-1.5, -1),
          (-1.4, -.9),
          (-1.3, -1.2),
          (-1.1, -.2),
          (-1.2, -.4),
          (-.5, 1.2),
          (-1.5, 2.1),
          (1, 1),
          # --
          (1.3, .8),
          (1.2, .5),
          (.2, -2),
          (.5, -2.4),
          (.2, -2.3),
          (0, -2.7),
          (1.3, 2.1)].T
Y = [0] * 8 + [1] * 8

# figure number
fignum = 1

# fit the model
for kernel in ('linear', 'poly', 'rbf'):
    clf = svm.SVC(kernel=kernel, gamma=2)
    clf.fit(X, Y)

    # plot the line, the points, and the nearest vectors to the plane
    plt.figure(fignum, figsize=(4, 3))
    plt.clf()
    plt.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1], s=80,
                facecolors='none', zorder=10)
    plt.scatter(X[:, 0], X[:, 1], c=Y, zorder=10, cmap=plt.cm.Paired)
    plt.axis('tight')
    x_min = -3
    x_max = 3
    y_min = -3
    y_max = 3
    XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
    Z = clf.decision_function(np.c_[XX.ravel(), YY.ravel()])
    Z = Z.reshape(XX.shape)
    plt.figure(fignum, figsize=(4, 3))
    plt.pcolormesh(XX, YY, Z > 0, cmap=plt.cm.Paired)
    plt.contour(XX, YY, Z, colors=[k', 'k', 'k'], linestyles=['-', '-', '-'],
                levels=[-.5, 0, .5])
```

26.8. SVM-Kernels 1089
plt.xlim(x_min, x_max)
plt.ylim(y_min, y_max)

plt.xticks(()
plt.yticks(()
fignum = fignum + 1
plt.show()

**26.9 SVM Margins Example**

The plots below illustrate the effect the parameter $C$ has on the separation line. A large value of $C$ basically tells our model that we do not have that much faith in our data’s distribution, and will only consider points close to line of separation.

A small value of $C$ includes more/all the observations, allowing the margins to be calculated using all the data in the area.
```python
X = np.r_[np.random.randn(20, 2) - [2, 2], np.random.randn(20, 2) + [2, 2]]
Y = [0] * 20 + [1] * 20

# figure number
fignum = 1

# fit the model
for name, penalty in {('unreg', 1), ('reg', 0.05)}:
    clf = svm.SVC(kernel='linear', C=penalty)
    clf.fit(X, Y)

    # get the separating hyperplane
    w = clf.coef_[0]
    a = -w[0] / w[1]
    xx = np.linspace(-5, 5)
    yy = a * xx - (clf.intercept_[0]) / w[1]

    # plot the parallels to the separating hyperplane that pass through the
    # support vectors
    margin = 1 / np.sqrt(np.sum(clf.coef_ ** 2))
    yy_down = yy + a * margin
    yy_up = yy - a * margin

    # plot the line, the points, and the nearest vectors to the plane
    plt.figure(fignum, figsize=(4, 3))
    plt.plot(xx, yy, 'k-')
    plt.plot(xx, yy_down, 'k--')
    plt.plot(xx, yy_up, 'k--')
    plt.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1], s=80, facecolors='none', zorder=10)
    plt.scatter(X[:, 0], X[:, 1], c=Y, zorder=10, cmap=plt.cm.Paired)
    plt.axis('tight')
    x_min = -4.8
    x_max = 4.2
    y_min = -6
    y_max = 6
    XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
    Z = clf.predict(np.c_[XX.ravel(), YY.ravel()])

    # Put the result into a color plot
    Z = Z.reshape(XX.shape)
    plt.figure(fignum, figsize=(4, 3))
    plt.pcolormesh(XX, YY, Z, cmap=plt.cm.Paired)
    plt.xlim(x_min, x_max)
    plt.ylim(y_min, y_max)
    plt.xticks()
    plt.yticks()
    fignum = fignum + 1

plt.show()
```
26.10 Plot different SVM classifiers in the iris dataset

Comparison of different linear SVM classifiers on a 2D projection of the iris dataset. We only consider the first 2 features of this dataset:

- Sepal length
- Sepal width

This example shows how to plot the decision surface for four SVM classifiers with different kernels.

The linear models `LinearSVC()` and `SVC(kernel='linear')` yield slightly different decision boundaries. This can be a consequence of the following differences:

- `LinearSVC` minimizes the squared hinge loss while `SVC` minimizes the regular hinge loss.
- `LinearSVC` uses the One-vs-All (also known as One-vs-Rest) multiclass reduction while `SVC` uses the One-vs-One multiclass reduction.

Both linear models have linear decision boundaries (intersecting hyperplanes) while the non-linear kernel models (polynomial or Gaussian RBF) have more flexible non-linear decision boundaries with shapes that depend on the kind of kernel and its parameters.

**Note:** while plotting the decision function of classifiers for toy 2D datasets can help get an intuitive understanding of their respective expressive power, be aware that those intuitions don’t always generalize to more realistic high-dimensional problems.
import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm, datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
# avoid this ugly slicing by using a two-dim dataset
y = iris.target

h = .02  # step size in the mesh

# we create an instance of SVM and fit out data. We do not scale our
# data since we want to plot the support vectors
C = 1.0  # SVM regularization parameter
svc = svm.SVC(kernel='linear', C=C).fit(X, y)
rbf_svc = svm.SVC(kernel='rbf', gamma=0.7, C=C).fit(X, y)
poly_svc = svm.SVC(kernel='poly', degree=3, C=C).fit(X, y)
lin_svc = svm.LinearSVC(C=C).fit(X, y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h),
                     sparse=False,      
                     indexing='xy')

# plot the decision boundaries
plt.subplot(2, 2, 1)
plt.contourf(xx, yy, svc.predict(np.c_[xx.ravel(), yy.ravel()]), cmap=plt.cm.Paired)
plt.axis('off')
plt.title('SVC with linear kernel')

plt.subplot(2, 2, 2)
plt.contourf(xx, yy, rbf_svc.predict(np.c_[xx.ravel(), yy.ravel()]), cmap=plt.cm.Paired)
plt.axis('off')
plt.title('LinearSVC (linear kernel)')

plt.subplot(2, 2, 3)
plt.contourf(xx, yy, poly_svc.predict(np.c_[xx.ravel(), yy.ravel()]), cmap=plt.cm.Paired)
plt.axis('off')
plt.title('SVC with RBF kernel')

plt.subplot(2, 2, 4)
plt.contourf(xx, yy, lin_svc.predict(np.c_[xx.ravel(), yy.ravel()]), cmap=plt.cm.Paired)
plt.axis('off')
plt.title('SVC with polynomial (degree 3) kernel')

plt.show()
np.arange(y_min, y_max, h))

titles = ['SVC with linear kernel',
        'LinearSVC (linear kernel)',
        'SVC with RBF kernel',
        'SVC with polynomial (degree 3) kernel']

for i, clf in enumerate((svc, lin_svc, rbf_svc, poly_svc)):
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    plt.subplot(2, 2, i + 1)
    plt.subplots_adjust(wspace=0.4, hspace=0.4)
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    plt.contourf(xx, yy, Z, cmap=plt.cm.coolwarm, alpha=0.8)
    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    plt.contourf(xx, yy, Z, cmap=plt.cm.coolwarm, alpha=0.8)
    # Plot also the training points
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.coolwarm)
    plt.xlabel('Sepal length')
    plt.ylabel('Sepal width')
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.xticks(())
    plt.yticks(())
    plt.title(titles[i])

plt.show()

Total running time of the script: (0 minutes 0.679 seconds)

Download Python source code: plot_iris.py
Download IPython notebook: plot_iris.ipynb

### 26.11 One-class SVM with non-linear kernel (RBF)

An example using a one-class SVM for novelty detection.

*One-class SVM* is an unsupervised algorithm that learns a decision function for novelty detection: classifying new data as similar or different to the training set.
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.font_manager
from sklearn import svm

tax, ty = np.meshgrid(np.linspace(-5, 5, 500), np.linspace(-5, 5, 500))
# Generate train data
X = 0.3 * np.random.randn(100, 2)
X_train = np.r_[X + 2, X - 2]
# Generate some regular novel observations
X = 0.3 * np.random.randn(20, 2)
X_test = np.r_[X + 2, X - 2]
# Generate some abnormal novel observations
X_outliers = np.random.uniform(low=-4, high=4, size=(20, 2))

# fit the model
clf = svm.OneClassSVM(nu=0.1, kernel="rbf", gamma=0.1)
clf.fit(X_train)
y_pred_train = clf.predict(X_train)
y_pred_test = clf.predict(X_test)
y_pred_outliers = clf.predict(X_outliers)
n_error_train = y_pred_train[y_pred_train == -1].size
n_error_test = y_pred_test[y_pred_test == -1].size
n_error_outliers = y_pred_outliers[y_pred_outliers == 1].size

print(__doc__)
# plot the line, the points, and the nearest vectors to the plane
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
plt.title("Novelty Detection")
plt.contourf(xx, yy, Z, levels=np.linspace(Z.min(), 0, 7), cmap=plt.cm.PuBu)
a = plt.contour(xx, yy, Z, levels=[0], linewidths=2, colors='darkred')
plt.contourf(xx, yy, Z, levels=[0, Z.max()], colors='palevioletred')
s = 40
b1 = plt.scatter(X_train[:, 0], X_train[:, 1], c='white', s=s)
b2 = plt.scatter(X_test[:, 0], X_test[:, 1], c='blueviolet', s=s)
c = plt.scatter(X_outliers[:, 0], X_outliers[:, 1], c='gold', s=s)
plt.axis('tight')
plt.xlim((-5, 5))
plt.ylim((-5, 5))
plt.legend([a.collections[0], b1, b2, c],
    ["learned frontier", "training observations",
     "new regular observations", "new abnormal observations"],
    loc="upper left", prop=matplotlib.font_manager.FontProperties(size=11))
plt.xlabel("error train: %d /200 ; errors novel regular: %d /40 ; "
"errors novel abnormal: %d /40"
% (n_error_train, n_error_test, n_error_outliers))
plt.show()

Total running time of the script: (0 minutes 0.291 seconds)

Download Python source code: plot_oneclass.py
Download IPython notebook: plot_oneclass.ipynb

26.12 Scaling the regularization parameter for SVCs

The following example illustrates the effect of scaling the regularization parameter when using Support Vector Machines for classification. For SVC classification, we are interested in a risk minimization for the equation:

\[ C \sum_{i=1,n} \mathcal{L}(f(x_i), y_i) + \Omega(w) \]

where

- \( C \) is used to set the amount of regularization
- \( \mathcal{L} \) is a loss function of our samples and our model parameters.
- \( \Omega \) is a penalty function of our model parameters

If we consider the loss function to be the individual error per sample, then the data-fit term, or the sum of the error for each sample, will increase as we add more samples. The penalization term, however, will not increase.

When using, for example, cross validation, to set the amount of regularization with \( C \), there will be a different amount of samples between the main problem and the smaller problems within the folds of the cross validation.

Since our loss function is dependent on the amount of samples, the latter will influence the selected value of \( C \). The question that arises is How do we optimally adjust \( C \) to account for the different amount of training samples?
The figures below are used to illustrate the effect of scaling our $C$ to compensate for the change in the number of samples, in the case of using an $l1$ penalty, as well as the $l2$ penalty.

### 26.12.1 $l1$-penalty case

In the $l1$ case, theory says that prediction consistency (i.e. that under given hypothesis, the estimator learned predicts as well as a model knowing the true distribution) is not possible because of the bias of the $l1$. It does say, however, that model consistency, in terms of finding the right set of non-zero parameters as well as their signs, can be achieved by scaling $C1$.

### 26.12.2 $l2$-penalty case

The theory says that in order to achieve prediction consistency, the penalty parameter should be kept constant as the number of samples grow.

### 26.12.3 Simulations

The two figures below plot the values of $C$ on the $x$-axis and the corresponding cross-validation scores on the $y$-axis, for several different fractions of a generated data-set.

In the $l1$ penalty case, the cross-validation-error correlates best with the test-error, when scaling our $C$ with the number of samples, $n$, which can be seen in the first figure.

For the $l2$ penalty case, the best result comes from the case where $C$ is not scaled.

---

**Note:**

Two separate datasets are used for the two different plots. The reason behind this is the $l1$ case works better on sparse data, while $l2$ is better suited to the non-sparse case.
print(__doc__)

# Author: Andreas Mueller <amueller@ais.uni-bonn.de>
#         Jaques Grobler <jaques.grobler@inria.fr>
# License: BSD 3 clause

import numpy as np
import matplotlib.pyplot as plt

from sklearn.svm import LinearSVC
from sklearn.model_selection import ShuffleSplit
from sklearn.model_selection import GridSearchCV
from sklearn.utils import check_random_state
from sklearn import datasets

rnd = check_random_state(1)

# set up dataset
n_samples = 100
n_features = 300

# 11 data (only 5 informative features)
X_1, y_1 = datasets.make_classification(n_samples=n_samples,
                                       n_features=n_features,
                                       n_informative=5,
                                       random_state=1)

# 12 data: non sparse, but less features
y_2 = np.sign(.5 - rnd.randn(n_samples))
X_2 = rnd.randn(n_samples, n_features // 5) + y_2[:, np.newaxis]
X_2 += 5 * rnd.randn(n_samples, n_features // 5)

clf_sets = [(LinearSVC(penalty='l1', loss='squared_hinge', dual=False,
                        tol=1e-3),
            np.logspace(-2.3, -1.3, 10), X_1, y_1),
26.13 RBF SVM parameters

This example illustrates the effect of the parameters gamma and C of the Radial Basis Function (RBF) kernel SVM. Intuitively, the gamma parameter defines how far the influence of a single training example reaches, with low values meaning ‘far’ and high values meaning ‘close’. The gamma parameters can be seen as the inverse of the radius of influence of samples selected by the model as support vectors.

The C parameter trades off misclassification of training examples against simplicity of the decision surface. A low C makes the decision surface smooth, while a high C aims at classifying all training examples correctly by giving the model freedom to select more samples as support vectors.

The first plot is a visualization of the decision function for a variety of parameter values on a simplified classification.
problem involving only 2 input features and 2 possible target classes (binary classification). Note that this kind of plot is not possible to do for problems with more features or target classes.

The second plot is a heatmap of the classifier’s cross-validation accuracy as a function of $C$ and $\gamma$. For this example we explore a relatively large grid for illustration purposes. In practice, a logarithmic grid from $10^{-3}$ to $10^{3}$ is usually sufficient. If the best parameters lie on the boundaries of the grid, it can be extended in that direction in a subsequent search.

Note that the heat map plot has a special colorbar with a midpoint value close to the score values of the best performing models so as to make it easy to tell them apart in the blink of an eye.

The behavior of the model is very sensitive to the $\gamma$ parameter. If $\gamma$ is too large, the radius of the area of influence of the support vectors only includes the support vector itself and no amount of regularization with $C$ will be able to prevent overfitting.

When $\gamma$ is very small, the model is too constrained and cannot capture the complexity or “shape” of the data. The region of influence of any selected support vector would include the whole training set. The resulting model will behave similarly to a linear model with a set of hyperplanes that separate the centers of high density of any pair of two classes.

For intermediate values, we can see on the second plot that good models can be found on a diagonal of $C$ and $\gamma$. Smooth models (lower $\gamma$ values) can be made more complex by selecting a larger number of support vectors (larger $C$ values) hence the diagonal of good performing models.

Finally one can also observe that for some intermediate values of $\gamma$ we get equally performing models when $C$ becomes very large: it is not necessary to regularize by limiting the number of support vectors. The radius of the RBF kernel alone acts as a good structural regularizer. In practice though it might still be interesting to limit the number of support vectors with a lower value of $C$ so as to favor models that use less memory and that are faster to predict.

We should also note that small differences in scores results from the random splits of the cross-validation procedure. Those spurious variations can be smoothed out by increasing the number of CV iterations $n_{\text{ splits }}$ at the expense of compute time. Increasing the value number of $C_{\text{ range }}$ and $\gamma_{\text{ range }}$ steps will increase the resolution of the hyper-parameter heat map.

```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import Normalize

from sklearn.svm import SVC
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import load_iris
from sklearn.model_selection import StratifiedShuffleSplit
from sklearn.model_selection import GridSearchCV

# Utility function to move the midpoint of a colormap to be around
# the values of interest.

class MidpointNormalize(Normalize):

    def __init__(self, vmin=None, vmax=None, midpoint=None, clip=False):
        self.midpoint = midpoint
        Normalize.__init__(self, vmin, vmax, clip)

    def __call__(self, value, clip=None):
        x, y = [self.vmin, self.midpoint, self.vmax], [0, 0.5, 1]
        return np.ma.masked_array(np.interp(value, x, y))
```

1100 Chapter 26. Support Vector Machines
Load and prepare data set
dataset for grid search

```python
iris = load_iris()
X = iris.data
y = iris.target

# Dataset for decision function visualization: we only keep the first two
# features in X and sub-sample the dataset to keep only 2 classes and
# make it a binary classification problem.

X_2d = X[:, :2]
X_2d = X_2d[y > 0]
y_2d = y[y > 0]
y_2d -= 1

# It is usually a good idea to scale the data for SVM training.
# We are cheating a bit in this example in scaling all of the data,
# instead of fitting the transformation on the training set and
# just applying it on the test set.

scaler = StandardScaler()
X = scaler.fit_transform(X)
X_2d = scaler.fit_transform(X_2d)
```

Train classifiers

For an initial search, a logarithmic grid with basis 10 is often helpful. Using a basis of 2, a finer tuning can be achieved but at a much higher cost.

```python
C_range = np.logspace(-2, 10, 13)
gamma_range = np.logspace(-9, 3, 13)
param_grid = dict(gamma=gamma_range, C=C_range)
cv = StratifiedShuffleSplit(n_splits=5, test_size=0.2, random_state=42)
grid = GridSearchCV(SVC(), param_grid=param_grid, cv=cv)
grid.fit(X, y)

print("The best parameters are %s with a score of %0.2f")
    % (grid.best_params_, grid.best_score_))

# Now we need to fit a classifier for all parameters in the 2d version
# (we use a smaller set of parameters here because it takes a while to train)

C_2d_range = [1e-2, 1, 1e2]
gamma_2d_range = [1e-1, 1, 1e1]
classifiers = []
for C in C_2d_range:
    for gamma in gamma_2d_range:
        clf = SVC(C=C, gamma=gamma)
        clf.fit(X_2d, y_2d)
        classifiers.append((C, gamma, clf))
```

Out:

```
The best parameters are {'C': 1.0, 'gamma': 0.10000000000000001} with a score of 0.97
```

Visualization
draw visualization of parameter effects

26.13. RBF SVM parameters
for (k, (C, gamma, clf)) in enumerate(classifiers):
    # evaluate decision function in a grid
    Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)

    # visualize decision function for these parameters
    plt.subplot(len(C_2d_range), len(gamma_2d_range), k + 1)
    plt.title("gamma=10^%d, C=10^%d" % (np.log10(gamma), np.log10(C)),
             size='medium')
    # visualize parameter's effect on decision function
    plt.pcolormesh(xx, yy, -Z, cmap=plt.cm.RdBu)
    plt.scatter(X_2d[:, 0], X_2d[:, 1], c=y_2d, cmap=plt.cm.RdBu_r)
    plt.xticks(())
    plt.yticks(())
    plt.axis('tight')

scores = grid.cv_results_['mean_test_score'].reshape(len(C_range),
                                                     len(gamma_range))

# Draw heatmap of the validation accuracy as a function of gamma and C

# The score are encoded as colors with the hot colormap which varies from dark
# red to bright yellow. As the most interesting scores are all located in the
# 0.92 to 0.97 range we use a custom normalizer to set the mid-point to 0.92 so
# as to make it easier to visualize the small variations of score values in the
# interesting range while not brutally collapsing all the low score values to
# the same color.

plt.figure(figsize=(8, 6))
plt.subplots_adjust(left=.2, right=0.95, bottom=0.15, top=0.95)
plt.imshow(scores, interpolation='nearest', cmap=plt.cm.hot,
           norm=MidpointNormalize(vmin=0.2, midpoint=0.92))
plt.xlabel('gamma')
plt.ylabel('C')
plt.colorbar()
plt.xticks(np.arange(len(gamma_range)), gamma_range, rotation=45)
plt.yticks(np.arange(len(C_range)), C_range)
plt.title('Validation accuracy')
plt.show()
Total running time of the script: (0 minutes 7.772 seconds)

Download Python source code: plot_rbf_parameters.py

Download IPython notebook: plot_rbf_parameters.ipynb
Examples concerning the `sklearn.feature_extraction.text` module.

### 27.1 FeatureHasher and DictVectorizer Comparison

Compares FeatureHasher and DictVectorizer by using both to vectorize text documents.

The example demonstrates syntax and speed only; it doesn’t actually do anything useful with the extracted vectors. See the example scripts `{document_classification_20newsgroups,clustering}.py` for actual learning on text documents.

A discrepancy between the number of terms reported for DictVectorizer and for FeatureHasher is to be expected due to hash collisions.

```python
# Author: Lars Buitinck
# License: BSD 3 clause

from __future__ import print_function
from collections import defaultdict
import re
import sys
from time import time

import numpy as np

from sklearn.datasets import fetch_20newsgroups
from sklearn.feature_extraction DictVectorizer, FeatureHasher

def n_nonzero_columns(X):
    """Returns the number of non-zero columns in a CSR matrix X."""
    return len(np.unique(X.nonzero()[1]))

def tokens(doc):
    """Extract tokens from doc.
    This uses a simple regex to break strings into tokens. For a more
    principled approach, see CountVectorizer or TfidfVectorizer.
    """
    return (tok.lower() for tok in re.findall(r"\w+", doc))

def token_figs(doc):
    """Extract a dict mapping tokens from doc to their frequencies."""
```
freq = defaultdict(int)
for tok in tokens(doc):
    freq[tok] += 1
return freq

categories = [
    'alt.atheism',
    'comp.graphics',
    'comp.sys.ibm.pc.hardware',
    'misc.forsale',
    'rec.autos',
    'sci.space',
    'talk.religion.misc',
]

# Uncomment the following line to use a larger set (11k+ documents)
#categories = None

print(__doc__)
print("Usage: $s [n_features_for_hashing]" % sys.argv[0])
print(" The default number of features is 2**18.")
print()
try:
    n_features = int(sys.argv[1])
except IndexError:
    n_features = 2 ** 18
except ValueError:
    print("not a valid number of features: $r" % sys.argv[1])
sys.exit(1)

print("Loading 20 newsgroups training data")
raw_data = fetch_20newsgroups(subset='train', categories=categories).data
data_size_mb = sum(len(s.encode('utf-8')) for s in raw_data) / 1e6
print("%d documents - %0.3f MB" % (len(raw_data), data_size_mb))
print()

print("DictVectorizer")
t0 = time()
vectorizer = DictVectorizer()
vectorizer.fit_transform(token_freqs(d) for d in raw_data)
duration = time() - t0
print("done in %fs at %0.3fMB/s" % (duration, data_size_mb / duration))
print("Found %d unique terms" % len(vectorizer.get_feature_names()))
print()

print("FeatureHasher on frequency dicts")
t0 = time()
hasher = FeatureHasher(n_features=n_features)
X = hasher.transform(token_freqs(d) for d in raw_data)
duration = time() - t0
print("done in %fs at %0.3fMB/s" % (duration, data_size_mb / duration))
print("Found %d unique terms" % n_nonzero_columns(X))
print()

print("FeatureHasher on raw tokens")
t0 = time()
27.2 Classification of text documents: using a MLComp dataset

This is an example showing how the scikit-learn can be used to classify documents by topics using a bag-of-words approach. This example uses a scipy.sparse matrix to store the features instead of standard numpy arrays.

The dataset used in this example is the 20 newsgroups dataset and should be downloaded from the http://mlcomp.org (free registration required):

http://mlcomp.org/datasets/379

Once downloaded unzip the archive somewhere on your filesystem. For instance in:

% mkdir -p ~/data/mlcomp
% cd ~/data/mlcomp
% unzip /path/to/dataset-379-20news-18828_XXXXX.zip

You should get a folder ~/data/mlcomp/379 with a file named metadata and subfolders raw, train and test holding the text documents organized by newsgroups.

Then set the MLCOMP_DATASETS_HOME environment variable pointing to the root folder holding the uncompressed archive:

% export MLCOMP_DATASETS_HOME="~/data/mlcomp"

Then you are ready to run this example using your favorite python shell:

% ipython examples/mlcomp_sparse_document_classification.py

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: BSD 3 clause

from __future__ import print_function

from time import time
import sys
import os
import numpy as np
import scipy.sparse as sp
import matplotlib.pyplot as plt

from sklearn.datasets import load_mlcomp
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.linear_model import SGDClassifier
from sklearn.metrics import confusion_matrix
from sklearn.metrics import classification_report

hasher = FeatureHasher(n_features=n_features, input_type="string")
X = hasher.transform(tokens(d) for d in raw_data)
duration = time() - t0
print("done in %fs at %0.3fMB/s" % (duration, data_size_mb / duration))
print("Found %d unique terms" % n_nonzero_columns(X))

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: hashing_vs_dict_vectorizer.py
Download IPython notebook: hashing_vs_dict_vectorizer.ipynb
from sklearn.naive_bayes import MultinomialNB

print(__doc__)

if 'MLCOMP_DATASETS_HOME' not in os.environ:
    print('MLCOMP_DATASETS_HOME not set; please follow the above instructions')
    sys.exit(0)

# Load the training set
print('Loading 20 newsgroups training set... ')
news_train = load_mlcomp('20news-18828', 'train')
print(news_train.DESCR)
print('%d documents' % len(news_train.filenames))
print('%d categories' % len(news_train.target_names))

print('Extracting features from the dataset using a sparse vectorizer')
t0 = time()
vectorizer = TfidfVectorizer(encoding='latin1')
X_train = vectorizer.fit_transform((open(f).read()
    for f in news_train.filenames))
print('done in %fs' % (time() - t0))
print('n_samples: %d, n_features: %d' % X_train.shape)
assert sp.issparse(X_train)
y_train = news_train.target

print('Loading 20 newsgroups test set... ')
news_test = load_mlcomp('20news-18828', 'test')
t0 = time()
print('done in %fs' % (time() - t0))

print('Predicting the labels of the test set...')
print('%d documents' % len(news_test.filenames))
print('%d categories' % len(news_test.target_names))

print('Extracting features from the dataset using the same vectorizer')
t0 = time()
X_test = vectorizer.transform((open(f).read() for f in news_test.filenames))
y_test = news_test.target
print('done in %fs' % (time() - t0))
print('n_samples: %d, n_features: %d' % X_test.shape)

Benchmark classifiers

def benchmark(clf_class, params, name):
    print('parameters:', params)
    t0 = time()
    clf = clf_class(**params).fit(X_train, y_train)
    print('done in %fs' % (time() - t0))

    if hasattr(clf, 'coef_'):
        print('Percentage of non zeros coef: %.2f%%' %
            (np.mean(clf.coef_ != 0) * 100))
    print('Predicting the outcomes of the testing set')
    t0 = time()
    pred = clf.predict(X_test)
    print('done in %fs' % (time() - t0))
print("Classification report on test set for classifier:")
print(clf)
print()
print(classification_report(y_test, pred,
    target_names=news_test.target_names))

cm = confusion_matrix(y_test, pred)
print("Confusion matrix:")
print(cm)

# Show confusion matrix
plt.matshow(cm)
plt.title('Confusion matrix of the %s classifier' % name)
plt.colorbar()

print("Testbenching a linear classifier...")
parameters = {
    'loss': 'hinge',
    'penalty': 'l2',
    'n_iter': 50,
    'alpha': 0.00001,
    'fit_intercept': True,
}
 benchmark(SGDClassifier, parameters, 'SGD')

print("Testbenching a MultinomialNB classifier...")
parameters = {'alpha': 0.01}
 benchmark(MultinomialNB, parameters, 'MultinomialNB')

plt.show()

Total running time of the script: (0 minutes 0.000 seconds)
Download Python source code: mlcomp_sparse_document_classification.py
Download IPython notebook: mlcomp_sparse_document_classification.ipynb

27.3 Clustering text documents using k-means

This is an example showing how the scikit-learn can be used to cluster documents by topics using a bag-of-words approach. This example uses a scipy.sparse matrix to store the features instead of standard numpy arrays.

Two feature extraction methods can be used in this example:

- TfidfVectorizer uses a in-memory vocabulary (a python dict) to map the most frequent words to features indices and hence compute a word occurrence frequency (sparse) matrix. The word frequencies are then reweighted using the Inverse Document Frequency (IDF) vector collected feature-wise over the corpus.

- HashingVectorizer hashes word occurrences to a fixed dimensional space, possibly with collisions. The word count vectors are then normalized to each have l2-norm equal to one (projected to the euclidean unit-ball) which seems to be important for k-means to work in high dimensional space.

HashingVectorizer does not provide IDF weighting as this is a stateless model (the fit method does nothing). When IDF weighting is needed it can be added by pipelining its output to a TfidfTransformer instance.
Two algorithms are demoed: ordinary k-means and its more scalable cousin minibatch k-means.

Additionally, latent semantic analysis can also be used to reduce dimensionality and discover latent patterns in the data.

It can be noted that k-means (and minibatch k-means) are very sensitive to feature scaling and that in this case the IDF weighting helps improve the quality of the clustering by quite a lot as measured against the “ground truth” provided by the class label assignments of the 20 newsgroups dataset.

This improvement is not visible in the Silhouette Coefficient which is small for both as this measure seem to suffer from the phenomenon called “Concentration of Measure” or “Curse of Dimensionality” for high dimensional datasets such as text data. Other measures such as V-measure and Adjusted Rand Index are information theoretic based evaluation scores: as they are only based on cluster assignments rather than distances, hence not affected by the curse of dimensionality.

Note: as k-means is optimizing a non-convex objective function, it will likely end up in a local optimum. Several runs with independent random init might be necessary to get a good convergence.
Load some categories from the training set

```python
categories = [
    'alt.atheism',
    'talk.religion.misc',
    'comp.graphics',
    'sci.space',
]
# Uncomment the following to do the analysis on all the categories
# categories = None

print("Loading 20 newsgroups dataset for categories:")
print(categories)

dataset = fetch_20newsgroups(subset='all', categories=categories,
                             shuffle=True, random_state=42)

print("%d documents" % len(dataset.data))
print("%d categories" % len(dataset.target_names))
print()

labels = dataset.target
true_k = np.unique(labels).shape[0]

print("Extracting features from the training dataset using a sparse vectorizer")
t0 = time()
if opts.use_hashing:
    if opts.use_idf:
        # Perform an IDF normalization on the output of HashingVectorizer
        hasher = HashingVectorizer(n_features=opts.n_features,
                                    stop_words='english', non_negative=True,
                                    norm=None, binary=False)
        vectorizer = make_pipeline(hasher, TfidfTransformer())
    else:
        vectorizer = HashingVectorizer(n_features=opts.n_features,
                                        stop_words='english',
                                        non_negative=False, norm='l2',
                                        binary=False)
    else:
        vectorizer = TfidfVectorizer(max_df=0.5, max_features=opts.n_features,
                                    min_df=2, stop_words='english',
            )
```
X = vectorizer.fit_transform(dataset.data)

print("done in %fs \(\text{time() - t0)\)}")
print("n_samples: %d, n_features: %d" \(\text{X.shape)\)}
print()

if opts.n_components:
    print("Performing dimensionality reduction using LSA")
    t0 = time()
    # Vectorizer results are normalized, which makes KMeans behave as
    # spherical k-means for better results. Since LSA/SVD results are
    # not normalized, we have to redo the normalization.
    svd = TruncatedSVD(opts.n_components)
    normalizer = Normalizer(copy=False)
    lsa = make_pipeline(svd, normalizer)
    X = lsa.fit_transform(X)

    print("done in %fs \(\text{time() - t0)\)}")

    explained_variance = svd.explained_variance_ratio_.sum()
    print("Explained variance of the SVD step: \(\text{explained_variance * 100)\)\)
    print()

Do the actual clustering

if opts.minibatch:
    km = MiniBatchKMeans(n_clusters=true_k, init='k-means++', n_init=1,
                      init_size=1000, batch_size=1000, verbose=opts.verbose)
else:
    km = KMeans(n_clusters=true_k, init='k-means++', max_iter=100, n_init=1,
                verbose=opts.verbose)

print("Clustering sparse data with %s" \(\text{km)\)
    t0 = time()
    km.fit(X)
    print("done in 0.3fs \(\text{time() - t0)\)}")
    print()""

print("Homogeneity: %0.3f" % metrics.homogeneity_score(labels, km.labels_))
print("Completeness: %0.3f" % metrics.completeness_score(labels, km.labels_))
print("V-measure: %0.3f" % metrics.v_measure_score(labels, km.labels_))
print("Adjusted Rand-Index: %0.3f" % metrics.adjusted_rand_score(labels, km.labels_))
print("Silhouette Coefficient: %0.3f" % metrics.silhouette_score(X, km.labels_, sample_size=1000))

print()

if not opts.use_hashing:
    print("Top terms per cluster:")
    if opts.n_components:
        original_space_centroids = svd.inverse_transform(km.cluster_centers_)

```python
order_centroids = original_space_centroids.argsort()[:, ::-1]

else:
    order_centroids = km.cluster_centers_.argsort()[:, ::-1]

terms = vectorizer.get_feature_names()
for i in range(true_k):
    print('{}Cluster {}:
'.format(' ' * 15, i))
    for ind in order_centroids[i, :10]:
        print('{}  {}
'.format(' ' * 15, terms[ind]))
    print()
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: `document_clustering.py`

Download IPython notebook: `document_clustering.ipynb`

## 27.4 Classification of text documents using sparse features

This is an example showing how scikit-learn can be used to classify documents by topics using a bag-of-words approach. This example uses a scipy.sparse matrix to store the features and demonstrates various classifiers that can efficiently handle sparse matrices.

The dataset used in this example is the 20 newsgroups dataset. It will be automatically downloaded, then cached.

The bar plot indicates the accuracy, training time (normalized) and test time (normalized) of each classifier.

```python
# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>
# Olivier Grisel <olivier.grisel@ensta.org>
# Mathieu Blondel <mathieu@mblondel.org>
# Lars Buitinck
# License: BSD 3 clause

from __future__ import print_function
import logging
import numpy as np
from optparse import OptionParser
import sys
from time import time
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_20newsgroups
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.feature_extraction.text import HashingVectorizer
from sklearn.feature_selection import SelectKBest, chi2
from sklearn.linear_model import RidgeClassifier
from sklearn.pipeline import Pipeline
from sklearn.svm import LinearSVC
from sklearn.linear_model import SGDClassifier
from sklearn.linear_model import Perceptron
from sklearn.linear_model import PassiveAggressiveClassifier
from sklearn.naive_bayes import BernoulliNB, MultinomialNB
from sklearn.neighbors import KNeighborsClassifier
from sklearn.neighbors import NearestCentroid
from sklearn.ensemble import RandomForestClassifier
from sklearn.utils.extmath import density
```
from sklearn import metrics

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO, format='%(asctime)s %(levelname)s %(message)s')

# parse commandline arguments
op = OptionParser()
op.add_option("--report", action="store_true", dest="print_report", help="Print a detailed classification report.")
op.add_option("--chi2_select", action="store", type="int", dest="select_chi2", help="Select some number of features using a chi-squared test")
op.add_option("--confusion_matrix", action="store_true", dest="print_cm", help="Print the confusion matrix.")
op.add_option("--top10", action="store_true", dest="print_top10", help="Print ten most discriminative terms per class for every classifier.")
op.add_option("--all_categories", action="store_true", dest="all_categories", help="Whether to use all categories or not.")
op.add_option("--use_hashing", action="store_true", help="Use a hashing vectorizer.")
op.add_option("--n_features", action="store", type=int, default=2 ** 16, help="n_features when using the hashing vectorizer.")
op.add_option("--filtered", action="store_true", help="Remove newsgroup information that is easily overfit: "headers, signatures, and quoting.")

(opts, args) = op.parse_args()
if len(args) > 0:
    op.error("this script takes no arguments.")
sys.exit(1)

print(__doc__)
op.print_help()
print()

Load some categories from the training set

if opts.all_categories:
categories = None
else:
categories = ['alt.atheism', 'talk.religion.misc', 'comp.graphics', 'sci.space',]
if opts.filtered:
    remove = ('headers', 'footers', 'quotes')
else:
    remove = ()

print("Loading 20 newsgroups dataset for categories:")
print(categories if categories else "all")

data_train = fetch_20newsgroups(subset='train', categories=categories,
                                   shuffle=True, random_state=42,
                                   remove=remove)

data_test = fetch_20newsgroups(subset='test', categories=categories,
                                 shuffle=True, random_state=42,
                                 remove=remove)

print('data loaded')

# order of labels in 'target_names' can be different from 'categories'
target_names = data_train.target_names

def size_mb(docs):
    return sum(len(s.encode('utf-8')) for s in docs) / 1e6

data_train_size_mb = size_mb(data_train.data)
data_test_size_mb = size_mb(data_test.data)

print("%d documents - %0.3fMB (training set)" % (len(data_train.data), data_train_size_mb))
print("%d documents - %0.3fMB (test set)" % (len(data_test.data), data_test_size_mb))
print("%d categories" % len(categories))

# split a training set and a test set
y_train, y_test = data_train.target, data_test.target

print("Extracting features from the training data using a sparse vectorizer")
t0 = time()
if opts.use_hashing:
    vectorizer = HashingVectorizer(stop_words='english', non_negative=True,
                                    n_features=opts.n_features)
    X_train = vectorizer.transform(data_train.data)
else:
    vectorizer = TfidfVectorizer(sublinear_tf=True, max_df=0.5,
                                  stop_words='english')
    X_train = vectorizer.fit_transform(data_train.data)

duration = time() - t0
print("done in %fs at %0.3fMB/s" % (duration, data_train_size_mb / duration))
print("n_samples: %d, n_features: %d" % X_train.shape)
print()

print("Extracting features from the test data using the same vectorizer")
t0 = time()
X_test = vectorizer.transform(data_test.data)
duration = time() - t0
print("done in %fs at %0.3fMB/s" % (duration, data_test_size_mb / duration))
print("n_samples: %d, n_features: %d" % X_test.shape)
print()

# mapping from integer feature name to original token string
if opts.use_hashing:
    feature_names = None
else:
    feature_names = vectorizer.get_feature_names()

if opts.select_chi2:
    print("Extracting \$d best features by a chi-squared test" %
          opts.select_chi2)
    t0 = time()
    ch2 = SelectKBest(chi2, k=opts.select_chi2)
    X_train = ch2.fit_transform(X_train, y_train)
    X_test = ch2.transform(X_test)
    if feature_names:
        # keep selected feature names
        feature_names = [feature_names[i] for i
                         in ch2.get_support(indices=True)]
    print("done in \$fs" % (time() - t0))
    print()

if feature_names:
    feature_names = np.asarray(feature_names)

def trim(s):
    """Trim string to fit on terminal (assuming 80-column display)""
    return s if len(s) <= 80 else s[:77] + "...

Benchmark classifiers

def benchmark(clf):
    print('_' * 80)
    print("Training: ")
    print(clf)
    t0 = time()
    clf.fit(X_train, y_train)
    train_time = time() - t0
    print("train time: \$0.3fs" % train_time)

    t0 = time()
    pred = clf.predict(X_test)
    test_time = time() - t0
    print("test time: \$0.3fs" % test_time)

    score = metrics.accuracy_score(y_test, pred)
    print("accuracy: \$0.3f" % score)

    if hasattr(clf, 'coef_'):
        print("dimensionality: \$d" % clf.coef_.shape[1])
        print("density: \$f" % density(clf.coef_))

        if opts.print_top10 and feature_names is not None:
            print("top 10 keywords per class:")
            for i, label in enumerate(target_names):
                top10 = np.argsort(clf.coef_[i])[-10:]
                print(trim("%s: %s" % (label, " ".join(feature_names[top10]))))
print()  
if opts.print_report:
    print("classification report:")
    print(metrics.classification_report(y_test, pred,  
            target_names=target_names))

if opts.print_cm:
    print("confusion matrix:")
    print(metrics.confusion_matrix(y_test, pred))

print()
clf_descr = str(clf).split('(')[0]
return clf_descr, score, train_time, test_time

results = []
for clf, name in [(RidgeClassifier(tol=1e-2, solver="lsqr"), "Ridge Classifier"),  
                  (Perceptron(n_iter=50), "Perceptron"),  
                  (PassiveAggressiveClassifier(n_iter=50), "Passive-Aggressive"),  
                  (KNeighborsClassifier(n_neighbors=10), "kNN"),  
                  (RandomForestClassifier(n_estimators=100), "Random forest")]:
    print('=' * 80)
    print(name)
    results.append(benchmark(clf))

for penalty in ["l2", "l1"]:  
    print('=' * 80)
    print("%s penalty" % penalty.upper())
    # Train Liblinear model
    results.append(benchmark(LinearSVC(loss='l2', penalty=penalty,  
                              dual=False, tol=1e-3)))

    # Train SGD model
    results.append(benchmark(SGDClassifier(alpha=.0001, n_iter=50,  
                              penalty=penalty)))

# Train SGD with Elastic Net penalty
print('=' * 80)
print("Elastic-Net penalty")
results.append(benchmark(SGDClassifier(alpha=.0001, n_iter=50,  
                              penalty="elasticnet")))

# Train NearestCentroid without threshold
print('=' * 80)
print("NearestCentroid (aka Rocchio classifier")  
results.append(benchmark(NearestCentroid()))

# Train sparse Naive Bayes classifiers
print('=' * 80)
print("Naive Bayes")
results.append(benchmark(MultinomialNB(alpha=.01)))
results.append(benchmark(BernoulliNB(alpha=.01)))

print('=' * 80)
print("LinearSVC with L1-based feature selection")
# The smaller C, the stronger the regularization.

27.4. Classification of text documents using sparse features
The more regularization, the more sparsity.

```python
results.append(benchmark(Pipeline(
    [{'feature_selection': LinearSVC(penalty="l1", dual=False, tol=1e-3)},
     {'classification': LinearSVC()}
    ]))

# make some plots

indices = np.arange(len(results))

results = [[x[i] for x in results] for i in range(4)]

clf_names, score, training_time, test_time = results
training_time = np.array(training_time) / np.max(training_time)
test_time = np.array(test_time) / np.max(test_time)

plt.figure(figsize=(12, 8))
plt.title("Score")
plt.barh(indices, score, .2, label="score", color='navy')
plt.barh(indices + .3, training_time, .2, label="training time",
         color='c')
plt.barh(indices + .6, test_time, .2, label="test time", color='darkorange')
plt.yticks(())
plt.legend(loc='best')
plt.subplots_adjust(left=.25)
plt.subplots_adjust(top=.95)
plt.subplots_adjust(bottom=.05)

for i, c in zip(indices, clf_names):
    plt.text(-.3, i, c)

plt.show()
```

Total running time of the script: (0 minutes 0.000 seconds)

Download Python source code: document_classification_20newsgroups.py

Download IPython notebook: document_classification_20newsgroups.ipynb
Examples concerning the `sklearn.tree` module.

28.1 Decision Tree Regression

A 1D regression with decision tree.

The decision trees is used to fit a sine curve with addition noisy observation. As a result, it learns local linear regressions approximating the sine curve.

We can see that if the maximum depth of the tree (controlled by the `max_depth` parameter) is set too high, the decision trees learn too fine details of the training data and learn from the noise, i.e. they overfit.
# Import the necessary modules and libraries
import numpy as np
from sklearn.tree import DecisionTreeRegressor
import matplotlib.pyplot as plt

# Create a random dataset
rng = np.random.RandomState(1)
X = np.sort(5 * rng.rand(80, 1), axis=0)
y = np.sin(X).ravel()
y[::5] += 3 * (0.5 - rng.rand(16))

# Fit regression model
regr_1 = DecisionTreeRegressor(max_depth=2)
regr_2 = DecisionTreeRegressor(max_depth=5)
oregr_1.fit(X, y)
oregr_2.fit(X, y)

# Predict
X_test = np.arange(0.0, 5.0, 0.01)[:, np.newaxis]
y_1 = regr_1.predict(X_test)
y_2 = regr_2.predict(X_test)

# Plot the results
plt.figure()
28.2 Multi-output Decision Tree Regression

An example to illustrate multi-output regression with decision tree.

The decision trees is used to predict simultaneously the noisy x and y observations of a circle given a single underlying feature. As a result, it learns local linear regressions approximating the circle.

We can see that if the maximum depth of the tree (controlled by the max_depth parameter) is set too high, the decision trees learn too fine details of the training data and learn from the noise, i.e. they overfit.
# Create a random dataset
rng = np.random.RandomState(1)
X = np.sort(200 * rng.rand(100, 1) - 100, axis=0)
y = np.array([np.pi * np.sin(X).ravel(), np.pi * np.cos(X).ravel()]).T
y[:5, :] += (0.5 - rng.rand(20, 2))

# Fit regression model
regr_1 = DecisionTreeRegressor(max_depth=2)
regr_2 = DecisionTreeRegressor(max_depth=5)
regr_3 = DecisionTreeRegressor(max_depth=8)
regr_1.fit(X, y)
regr_2.fit(X, y)
regr_3.fit(X, y)

# Predict
X_test = np.arange(-100.0, 100.0, 0.01)[:, np.newaxis]
y_1 = regr_1.predict(X_test)
y_2 = regr_2.predict(X_test)
y_3 = regr_3.predict(X_test)

# Plot the results
plt.figure()
s = 50
plt.scatter(y[:, 0], y[:, 1], c="navy", s=s, label="data")
plt.scatter(y_1[:, 0], y_1[:, 1], c="cornflowerblue", s=s, label="max_depth=2")
plt.scatter(y_2[:, 0], y_2[:, 1], c="c", s=s, label="max_depth=5")
plt.scatter(y_3[:, 0], y_3[:, 1], c="orange", s=s, label="max_depth=8")
plt.xlim([-6, 6])
plt.ylim([-6, 6])
plt.xlabel("target 1")
plt.ylabel("target 2")
plt.title("Multi-output Decision Tree Regression")
plt.legend()
plt.show()
# Parameters
n_classes = 3
plot_colors = "bry"
plot_step = 0.02

# Load data
iris = load_iris()

for pairidx, pair in enumerate([[0, 1], [0, 2], [0, 3],
                                [1, 2], [1, 3], [2, 3]]):
    # We only take the two corresponding features
    X = iris.data[:, pair]
y = iris.target

    # Train
    clf = DecisionTreeClassifier().fit(X, y)

    # Plot the decision boundary
28.4 Understanding the decision tree structure

The decision tree structure can be analysed to gain further insight on the relation between the features and the target to predict. In this example, we show how to retrieve:

- the binary tree structure;
- the depth of each node and whether or not it’s a leaf;
- the nodes that were reached by a sample using the decision_path method;
- the leaf that was reached by a sample using the apply method;
- the rules that were used to predict a sample;
- the decision path shared by a group of samples.

Out:

```python
plt.subplot(2, 3, pairidx + 1)

x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
                     np.arange(y_min, y_max, plot_step))

Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
cs = plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)
plt.xlabel(iris.feature_names[pair[0]])
plt.ylabel(iris.feature_names[pair[1]])
plt.axis("tight")

# Plot the training points
for i, color in zip(range(n_classes), plot_colors):  
    idx = np.where(y == i)
    plt.scatter(X[idx, 0], X[idx, 1], c=color, label=iris.target_names[i],
                cmap=plt.cm.Paired)

plt.axis("tight")
plt.suptitle("Decision surface of a decision tree using paired features")
plt.legend()
plt.show()
```

Total running time of the script: (0 minutes 0.514 seconds)

Download Python source code: plot_iris.py

Download IPython notebook: plot_iris.ipynb

Out:

The binary tree structure has 5 nodes and has the following tree structure:
node=0 test node: go to node 1 if X[:, 3] <= 0.800000011921s else to node 2.
    node=1 leaf node.
    node=2 test node: go to node 3 if X[:, 2] <= 4.94999998027s else to node 4.
        node=3 leaf node.
        node=4 leaf node.
Rules used to predict sample 0:

decision id node 4 : (X[0, -2] (= 1.5) > -2.0)

The following samples [0, 1] share the node [0 2] in the tree. It is 40.0 % of all nodes.

```python
import numpy as np

from sklearn.model_selection import train_test_split
from sklearn.datasets import load_iris
from sklearn.tree import DecisionTreeClassifier

iris = load_iris()
X = iris.data
y = iris.target
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=0)
estimator = DecisionTreeClassifier(max_leaf_nodes=3, random_state=0)
estimator.fit(X_train, y_train)

# The decision estimator has an attribute called tree_ which stores the entire
# tree structure and allows access to low level attributes. The binary tree
# tree_ is represented as a number of parallel arrays. The i-th element of each
# array holds information about the node 'i'. Node 0 is the tree's root. NOTE:
# Some of the arrays only apply to either leaves or split nodes, resp. In this
# case the values of nodes of the other type are arbitrary!
#
# Among those arrays, we have:
# - left_child, id of the left child of the node
# - right_child, id of the right child of the node
# - feature, feature used for splitting the node
# - threshold, threshold value at the node
#
# Using those arrays, we can parse the tree structure:

n_nodes = estimator.tree_.node_count
children_left = estimator.tree_.children_left
children_right = estimator.tree_.children_right
feature = estimator.tree_.feature
threshold = estimator.tree_.threshold

# The tree structure can be traversed to compute various properties such
# as the depth of each node and whether or not it is a leaf.
node_depth = np.zeros(shape=n_nodes, dtype=np.int64)
is_leaves = np.zeros(shape=n_nodes, dtype=bool)
stack = [(0, -1)]  # seed is the root node id and its parent depth
while len(stack) > 0:
    node_id, parent_depth = stack.pop()
    node_depth[node_id] = parent_depth + 1
    # If we have a test node
```

28.4. Understanding the decision tree structure
if (children_left[node_id] != children_right[node_id]):
    stack.append((children_left[node_id], parent_depth + 1))
    stack.append((children_right[node_id], parent_depth + 1))
else:
    is_leaves[node_id] = True

print("The binary tree structure has \$s\$ nodes and has ", "the following tree structure:", n_nodes)
for i in range(n_nodes):
    if is_leaves[i]:
        print("\$s\$ leaf node. \% (node_depth[i] * "\t", i))
    else:
        print("\$s\$ test node: go to node \$s\$ if X[:, \$s]\ <= \$s\$ else to "
        "node \$s\$.
        % (node_depth[i] * "\t", i, children_left[i], feature[i], threshold[i], children_right[i], )

print()

# First let's retrieve the decision path of each sample. The decision_path
# method allows to retrieve the node indicator functions. A non zero element of
# indicator matrix at the position (i, j) indicates that the sample i goes
# through the node j.

node_indicator = estimator.decision_path(X_test)

# Similarly, we can also have the leaves ids reached by each sample.

leave_id = estimator.apply(X_test)

# Now, it's possible to get the tests that were used to predict a sample or
# a group of samples. First, let's make it for the sample.

sample_id = 0
node_index = node_indicator.indices[node_indicator.indptr[sample_id]: node_indicator.indptr[sample_id + 1]]

print('Rules used to predict sample \$s\$: ' % sample_id)
for node_id in node_index:
    if leave_id[sample_id] != node_id:
        continue
    if (X_test[sample_id, feature[node_id]] <= threshold[node_id]):
        threshold_sign = "<="
    else:
        threshold_sign = ">"

print("decision id node \$s\$ : (X[\$s, \$s]\ (= \$s\$ \$s\$ \$s\$)" % (node_id, sample_id, feature[node_id], X_test[i, feature[node_id]], threshold_sign)
# For a group of samples, we have the following common node.
sample_ids = [0, 1]
common_nodes = (node_indicator.toarray()[sample_ids].sum(axis=0) ==
    len(sample_ids))

common_node_id = np.arange(n_nodes)[common_nodes]

print("The following samples \$s\ share the node \$s in the tree" %
    (sample_ids, common_node_id))
print("It is \$s % of all nodes." % (100 * len(common_node_id) / n_nodes,))

Total running time of the script: (0 minutes 0.005 seconds)

Download Python source code: plot_unveil_tree_structure.py
Download IPython notebook: plot_unveil_tree_structure.ipynb
This is the class and function reference of scikit-learn. Please refer to the full user guide for further details, as the class and function raw specifications may not be enough to give full guidelines on their uses.

29.1 sklearn.base: Base classes and utility functions

Base classes for all estimators.

29.1.1 Base classes

<table>
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<th>Class</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td><code>sklearn.base.BaseEstimator</code></td>
<td>Base class for all estimators in scikit-learn</td>
</tr>
<tr>
<td><code>sklearn.base.ClassifierMixin</code></td>
<td>Mixin class for all classifiers in scikit-learn.</td>
</tr>
<tr>
<td><code>sklearn.base.ClusterMixin</code></td>
<td>Mixin class for all cluster estimators in scikit-learn.</td>
</tr>
<tr>
<td><code>sklearn.base.RegressorMixin</code></td>
<td>Mixin class for all regression estimators in scikit-learn.</td>
</tr>
<tr>
<td><code>sklearn.base.TransformerMixin</code></td>
<td>Mixin class for all transformers in scikit-learn.</td>
</tr>
</tbody>
</table>

**sklearn.base.BaseEstimator**

class `sklearn.base.BaseEstimator`

Base class for all estimators in scikit-learn

**Notes**

All estimators should specify all the parameters that can be set at the class level in their `__init__` as explicit keyword arguments (no *args or **kwargs).

**Methods**

- `get_params([deep])` Get parameters for this estimator.
- `set_params(**params)` Set the parameters of this estimator.

```
    __init__(
        x.__init__(...) initializes x; see help(type(x)) for signature
    get_params (deep=True)
```
Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

```python
set_params(**params)
```

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :

**Examples using sklearn.base.BaseEstimator**

- Feature Union with Heterogeneous Data Sources

```python
sklearn.base.ClassifierMixin
```

**class sklearn.base.ClassifierMixin**

Mixin class for all classifiers in scikit-learn.

**Methods**

```python
score(X, y[, sample_weight])
```

Returns the mean accuracy on the given test data and labels.

```python
__init__()
```

x.__init__(...) initializes x; see help(type(x)) for signature

```python
score(X, y, sample_weight=None)
```

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X** : array-like, shape = (n_samples, n_features)
  Test samples.

- **y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.

- **sample_weight** : array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **score** : float
  Mean accuracy of self.predict(X) wrt. y.
sklearn.base.ClusterMixin

class sklearn.base.ClusterMixin
    Mixin class for all cluster estimators in scikit-learn.

    Methods

    fit_predict(X[, y])
        Performs clustering on X and returns cluster labels.

    __init__()
        x.__init__(...) initializes x; see help(type(x)) for signature

    fit_predict (X, y=None)
        Performs clustering on X and returns cluster labels.
    Parameters
    X : ndarray, shape (n_samples, n_features)
        Input data.
    Returns
    y : ndarray, shape (n_samples,)
        cluster labels

sklearn.base.RegressorMixin

class sklearn.base.RegressorMixin
    Mixin class for all regression estimators in scikit-learn.

    Methods

    score(X, y[, sample_weight])
        Returns the coefficient of determination R^2 of the prediction.

    __init__()
        x.__init__(...) initializes x; see help(type(x)) for signature

    score (X, y, sample_weight=None)
        Returns the coefficient of determination R^2 of the prediction.
    The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares
    ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares
    ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative
    (because the model can be arbitrarily worse). A constant model that always
    predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.
    Parameters
    X : array-like, shape = (n_samples, n_features)
        Test samples.
    y : array-like, shape = (n_samples) or (n_samples, n_outputs)
        True values for X.
    sample_weight : array-like, shape = [n_samples], optional
        Sample weights.
**Returnsscore**: float

\( R^2 \) of self.predict(X) wrt. y.

---

**sklearn.base.TransformerMixin**

**class sklearn.base.TransformerMixin**

Mixin class for all transformers in scikit-learn.

**Methods**

- **fit_transform(X[, y])**

  Fit to data, then transform it.

  - **__init__()**
    
    x.__init__(...) initializes x; see help(type(x)) for signature

  - **fit_transform(X, y=None, **fit_params)**

    Fit to data, then transform it.

    Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

    - **Parameters**
      
      - **X**: numpy array of shape [n_samples, n_features]  
        Training set.
      
      - **y**: numpy array of shape [n_samples]
        Target values.

    - **Returns**
      
      - **X_new**: numpy array of shape [n_samples, n_features_new]
        Transformed array.

**Examples using sklearn.base.TransformerMixin**

- **Feature Union with Heterogeneous Data Sources**

**29.1.2 Functions**

- **base.clone(estimator[, safe])**

  Constructs a new estimator with the same parameters.

- **sklearn.base.clone**

  **sklearn.base.clone(estimator, safe=True)**

  Constructs a new estimator with the same parameters.

  Clone does a deep copy of the model in an estimator without actually copying attached data. It yields a new estimator with the same parameters that has not been fit on any data.

  - **Parameters**
    
    - **estimator**: estimator object, or list, tuple or set of objects
      
      The estimator or group of estimators to be cloned
    
    - **safe**: boolean, optional
If safe is false, clone will fall back to a deepcopy on objects that are not estimators.

## 29.2 sklearn.cluster: Clustering

The `sklearn.cluster` module gathers popular unsupervised clustering algorithms.

**User guide:** See the Clustering section for further details.

### 29.2.1 Classes

- `cluster.AffinityPropagation([damping, ...])`: Perform Affinity Propagation Clustering of data.
- `cluster.AgglomerativeClustering([...])`: Agglomerative Clustering.
- `cluster.DBSCAN([eps, min_samples, metric, ...])`: Perform DBSCAN clustering from vector array or distance matrix.
- `cluster.FeatureAgglomeration([n_clusters, ...])`: Agglomerate features.
- `cluster.MeanShift([bandwidth, seeds, ...])`: Mean shift clustering using a flat kernel.
- `cluster.SpectralClustering([n_clusters, ...])`: Apply clustering to a projection to the normalized laplacian.

### sklearn.cluster.AffinityPropagation

**class sklearn.cluster.AffinityPropagation**

```
(damping=0.5, max_iter=200, convergence_iter=15, copy=True, preference=None, affinity='euclidean', verbose=False)
```

Perform Affinity Propagation Clustering of data.

Read more in the *User Guide*.

**Parameters**

- **damping**: float, optional, default: 0.5
  - Damping factor between 0.5 and 1.
- **convergence_iter**: int, optional, default: 15
  - Number of iterations with no change in the number of estimated clusters that stops the convergence.
- **max_iter**: int, optional, default: 200
  - Maximum number of iterations.
- **copy**: boolean, optional, default: True
  - Make a copy of input data.
- **preference**: array-like, shape (n_samples,) or float, optional
  - Preferences for each point - points with larger values of preferences are more likely to be chosen as exemplars. The number of exemplars, i.e., of clusters, is influenced by the input preferences value. If the preferences are not passed as arguments, they will be set to the median of the input similarities.
- **affinity**: string, optional, default='euclidean'
Which affinity to use. At the moment `precomputed` and `euclidean` are supported. `euclidean` uses the negative squared euclidean distance between points.

verbose : boolean, optional, default: False
Whether to be verbose.

Attributes

`cluster_centers_indices_` : array, shape (n_clusters,)
Indices of cluster centers

`cluster_centers_` : array, shape (n_clusters, n_features)
Cluster centers (if affinity != `precomputed`).

`labels_` : array, shape (n_samples,)
Labels of each point

`affinity_matrix_` : array, shape (n_samples, n_samples)
Stores the affinity matrix used in `fit`.

`n_iter_` : int
Number of iterations taken to converge.

Notes

See examples/cluster/plot_affinity_propagation.py for an example.
The algorithmic complexity of affinity propagation is quadratic in the number of points.

References


Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X[, y])</code></td>
<td>Create affinity matrix from negative euclidean distances, then apply affinity propagation clustering.</td>
</tr>
<tr>
<td><code>fit_predict(X[, y])</code></td>
<td>Performs clustering on X and returns cluster labels.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict the closest cluster each sample in X belongs to.</td>
</tr>
<tr>
<td><code>set_params(X=None)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (damping=0.5, max_iter=200, convergence_iter=15, copy=True, preference=None, affinity='euclidean', verbose=False)

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X, y=None)</code></td>
<td>Create affinity matrix from negative euclidean distances, then apply affinity propagation clustering.</td>
</tr>
<tr>
<td>ParametersX: array-like, shape (n_samples, n_features) or (n_samples, n_samples) :</td>
<td></td>
</tr>
<tr>
<td>Data matrix or, if affinity is <code>precomputed</code>, matrix of similarities / affinities.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit_predict(X, y=None)</code></td>
<td>Performs clustering on X and returns cluster labels.</td>
</tr>
</tbody>
</table>
Parameters

X : ndarray, shape (n_samples, n_features)
    Input data.

Returns

y : ndarray, shape (n_samples,)
    Cluster labels

get_params (deep=True)
    Get parameters for this estimator.

Parameters

depth : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

params : mapping of string to any
    Parameter names mapped to their values.

predict (X)
    Predict the closest cluster each sample in X belongs to.

Parameters

X : {array-like, sparse matrix}, shape (n_samples, n_features)
    New data to predict.

Returns

labels : array, shape (n_samples,)
    Index of the cluster each sample belongs to.

set_params (**params)
    Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns

self :

Examples using sklearn.cluster.AffinityPropagation

• Demo of affinity propagation clustering algorithm
• Comparing different clustering algorithms on toy datasets

sklearn.cluster.AgglomerativeClustering

class sklearn.cluster.AgglomerativeClustering (n_clusters=2, affinity='euclidean', memory=Memory(cachedir=None), connectivity=None, compute_full_tree='auto', linkage='ward', pooling_func=<function mean>)

Agglomerative Clustering

Recursively merges the pair of clusters that minimally increases a given linkage distance.

Read more in the User Guide.

Parameters

n_clusters : int, default=2
    The number of clusters to find.
connectivity : array-like or callable, optional

Connectivity matrix. Defines for each sample the neighboring samples following a given structure of the data. This can be a connectivity matrix itself or a callable that transforms the data into a connectivity matrix, such as derived from kneighbors_graph. Default is None, i.e., the hierarchical clustering algorithm is unstructured.

affinity : string or callable, default: “euclidean”

Metric used to compute the linkage. Can be “euclidean”, “l1”, “l2”, “manhattan”, “cosine”, or ‘precomputed’. If linkage is “ward”, only “euclidean” is accepted.

memory : Instance of joblib.Memory or string (optional)

Used to cache the output of the computation of the tree. By default, no caching is done. If a string is given, it is the path to the caching directory.

compute_full_tree : bool or ‘auto’ (optional)

Stop early the construction of the tree at n_clusters. This is useful to decrease computation time if the number of clusters is not small compared to the number of samples. This option is useful only when specifying a connectivity matrix. Note also that when varying the number of clusters and using caching, it may be advantageous to compute the full tree.

linkage : {“ward”, “complete”, “average”}, optional, default: “ward”

Which linkage criterion to use. The linkage criterion determines which distance to use between sets of observation. The algorithm will merge the pairs of cluster that minimize this criterion.

• ward minimizes the variance of the clusters being merged.
• average uses the average of the distances of each observation of the two sets.
• complete or maximum linkage uses the maximum distances between all observations of the two sets.

pooling_func : callable, default=np.mean

This combines the values of agglomerated features into a single value, and should accept an array of shape [M, N] and the keyword argument axis=1, and reduce it to an array of size [M].

Attributes

labels_ : array [n_samples]

cluster labels for each point

n_leaves_ : int

Number of leaves in the hierarchical tree.

n_components_ : int

The estimated number of connected components in the graph.

children_ : array-like, shape (n_nodes-1, 2)

The children of each non-leaf node. Values less than n_samples correspond to leaves of the tree which are the original samples. A node i greater than or equal to n_samples is a non-leaf node and has children children[fi - n_samples]. Alternatively at the i-th iteration, children[1][0] and children[1][1] are merged to form node n_samples + i
## Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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<tr>
<td><code>fit(X[, y])</code></td>
<td>Fit the hierarchical clustering on the data</td>
</tr>
<tr>
<td><code>fit_predict(X[, y])</code></td>
<td>Performs clustering on X and returns cluster labels.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__ (n_clusters=2, affinity='euclidean', memory=Memory(cachedir=None), connectivity=None, compute_full_tree='auto', linkage='ward', pooling_func=<function mean>)
```

```python
fit (X, y=None)
Fit the hierarchical clustering on the data

Parameters:
- X : array-like, shape = [n_samples, n_features]
  The samples a.k.a. observations.

Returns:
self :
```

```python
fit_predict (X, y=None)
Performs clustering on X and returns cluster labels.

Parameters:
- X : ndarray, shape (n_samples, n_features)
  Input data.

Returns:
- y : ndarray, shape (n_samples,)
  Cluster labels
```

```python
get_params (deep=True)
Get parameters for this estimator.

Parameters:
- deep : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params : mapping of string to any
  Parameter names mapped to their values.
```

```python
set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `component__parameter` so that it's possible to update each component of a nested object.

Returns:
self :
```

### Examples using `sklearn.cluster.AgglomerativeClustering`

- Agglomerative clustering with and without structure
- Agglomerative clustering with different metrics
- Comparing different clustering algorithms on toy datasets
- Various Agglomerative Clustering on a 2D embedding of digits
- A demo of structured Ward hierarchical clustering on a raccoon face image
Hierarchical clustering: structured vs unstructured ward

sklearn.cluster.Birch

class sklearn.cluster.Birch (threshold=0.5, branching_factor=50, n_clusters=3, compute_labels=True, copy=True)

Implements the Birch clustering algorithm.

Every new sample is inserted into the root of the Clustering Feature Tree. It is then clubbed together with the subcluster that has the centroid closest to the new sample. This is done recursively till it ends up at the subcluster of the leaf of the tree has the closest centroid.

Read more in the User Guide.

Parametersthreshold : float, default 0.5

The radius of the subcluster obtained by merging a new sample and the closest subcluster should be lesser than the threshold. Otherwise a new subcluster is started.

branching_factor : int, default 50

Maximum number of CF subclusters in each node. If a new samples enters such that the number of subclusters exceed the branching_factor then the node has to be split. The corresponding parent also has to be split and if the number of subclusters in the parent is greater than the branching factor, then it has to be split recursively.

n_clusters : int, instance of sklearn.cluster model, default 3

Number of clusters after the final clustering step, which treats the subclusters from the leaves as new samples. If None, this final clustering step is not performed and the subclusters are returned as they are. If a model is provided, the model is fit treating the subclusters as new samples and the initial data is mapped to the label of the closest subcluster. If an int is provided, the model fit is AgglomerativeClustering with n_clusters set to the int.

compute_labels : bool, default True

Whether or not to compute labels for each fit.

copy : bool, default True

Whether or not to make a copy of the given data. If set to False, the initial data will be overwritten.

Attributesroot_ : _CFNode

Root of the CFTree.

dummy_leaf_ : _CFNode

Start pointer to all the leaves.

subcluster_centers_ : ndarray,

Centroids of all subclusters read directly from the leaves.

subcluster_labels_ : ndarray,

Labels assigned to the centroids of the subclusters after they are clustered globally.

labels_ : ndarray, shape (n_samples,)

Array of labels assigned to the input data. if partial_fit is used instead of fit, they are assigned to the last batch of data.
References


• Roberto Perdisci JBirch - Java implementation of BIRCH clustering algorithm https://code.google.com/archive/p/jbirch

Examples

```python
>>> from sklearn.cluster import Birch
>>> X = [[0, 1], [0.3, 1], [-0.3, 1], [0, -1], [0.3, -1], [-0.3, -1]]
>>> brc = Birch(branching_factor=50, n_clusters=None, threshold=0.5,
... compute_labels=True)
>>> brc.fit(X)
Birch(branching_factor=50, compute_labels=True, copy=True, n_clusters=None,
... threshold=0.5)
>>> brc.predict(X)
array([0, 0, 0, 1, 1, 1])
```

Methods

- `fit(X[, y])` Build a CF Tree for the input data.
- `fit_predict(X[, y])` Performs clustering on X and returns cluster labels.
- `fit_transform(X[, y, **fit_params])` Fit to data, then transform it.
- `get_params([deep])` Get parameters for this estimator.
- `partial_fit([X, y])` Online learning.
- `predict(X)` Predict data using the centroids_ of subclusters.
- `set_params(**params)` Set the parameters of this estimator.
- `transform(X[, y])` Transform X into subcluster centroids dimension.

```python
__init__(threshold=0.5, branching_factor=50, n_clusters=3, compute_labels=True, copy=True)
```

`fit (X, y=None)`
Build a CF Tree for the input data.

Parameters:

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  Input data.

`fit_predict (X, y=None)`
Performs clustering on X and returns cluster labels.

Parameters:

- **X**: ndarray, shape (n_samples, n_features)
  Input data.

Returns:

- **y**: ndarray, shape (n_samples,)
  cluster labels

`fit_transform (X, y=None, **fit_params)`
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.
Parameters

**X** : numpy array of shape [n_samples, n_features]

Training set.

**y** : numpy array of shape [n_samples]

Target values.

**Returns**

**X_new** : numpy array of shape [n_samples, n_features_new]

Transformed array.

`get_params` *(deep=True)*

Get parameters for this estimator.

**Parameters**

**deep** : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

**params** : mapping of string to any

Parameter names mapped to their values.

`partial_fit` *(X=None, y=None)*

Online learning. Prevents rebuilding of CFTree from scratch.

**Parameters**

**X** : {array-like, sparse matrix}, shape (n_samples, n_features), None

Input data. If X is not provided, only the global clustering step is done.

`predict` *(X)*

Predict data using the centroids_ of subclusters.

Avoid computation of the row norms of X.

**Parameters**

**X** : {array-like, sparse matrix}, shape (n_samples, n_features)

Input data.

**Returns**

**labels**: ndarray, shape(n_samples):

Labelled data.

`set_params` (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self** :

`transform` *(X, y=None)*

Transform X into subcluster centroids dimension.

Each dimension represents the distance from the sample point to each cluster centroid.

**Parameters**

**X** : {array-like, sparse matrix}, shape (n_samples, n_features)

Input data.

**Returns**

**X_trans** : {array-like, sparse matrix}, shape (n_samples, n_clusters)

Transformed data.
Examples using `sklearn.cluster.Birch`

- Compare BIRCH and MiniBatchKMeans
- Comparing different clustering algorithms on toy datasets

`sklearn.cluster.DBSCAN`

class `sklearn.cluster.DBSCAN` (**eps=0.5**, **min_samples=5**, **metric='euclidean'**, **algorithm='auto'**, 
**leaf_size=30, p=None, n_jobs=1**)

Perform DBSCAN clustering from vector array or distance matrix.

DBSCAN - Density-Based Spatial Clustering of Applications with Noise. Finds core samples of high density and expands clusters from them. Good for data which contains clusters of similar density.

Read more in the User Guide.

**Parameters**

- **eps** : float, optional
  The maximum distance between two samples for them to be considered as in the same neighborhood.

- **min_samples** : int, optional
  The number of samples (or total weight) in a neighborhood for a point to be considered as a core point. This includes the point itself.

- **metric** : string, or callable
  The metric to use when calculating distance between instances in a feature array. If metric is a string or callable, it must be one of the options allowed by metrics.pairwise.calculate_distance for its metric parameter. If metric is “precomputed”, X is assumed to be a distance matrix and must be square. X may be a sparse matrix, in which case only “nonzero” elements may be considered neighbors for DBSCAN.
  New in version 0.17: metric `precomputed` to accept precomputed sparse matrix.

- **algorithm** : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
  The algorithm to be used by the NearestNeighbors module to compute pointwise distances and find nearest neighbors. See NearestNeighbors module documentation for details.

- **leaf_size** : int, optional (default = 30)
  Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

- **p** : float, optional
  The power of the Minkowski metric to be used to calculate distance between points.

- **n_jobs** : int, optional (default = 1)
  The number of parallel jobs to run. If -1, then the number of jobs is set to the number of CPU cores.

**Attributes**

- **core_sample_indices_** : array, shape = [n_core_samples]
  Indices of core samples.

- **components_** : array, shape = [n_core_samples, n_features]
Copy of each core sample found by training.

labels_ : array, shape = [n_samples]

Cluster labels for each point in the dataset given to fit(). Noisy samples are given the label -1.

Notes

See examples/cluster/plot_dbscan.py for an example.

This implementation bulk-computes all neighborhood queries, which increases the memory complexity to O(n.d) where d is the average number of neighbors, while original DBSCAN had memory complexity O(n).

Sparse neighborhoods can be precomputed using NearestNeighbors.radius_neighbors_graph with mode='distance'.

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__init__ (eps=0.5, min_samples=5, metric='euclidean', algorithm='auto', leaf_size=30, p=None, n_jobs=1)

fit (X=None, y=None, sample_weight=None)

Perform DBSCAN clustering from features or distance matrix.

Parameters

X : array or sparse (CSR) matrix of shape (n_samples, n_features), or array of shape (n_samples, n_samples)

A feature array, or an array of distances between samples if metric='precomputed'.

sample_weight : array, shape (n_samples,), optional

Weight of each sample, such that a sample with a weight of at least min_samples is by itself a core sample; a sample with negative weight may inhibit its eps-neighbor from being core. Note that weights are absolute, and default to 1.

fit_predict (X=None, y=None, sample_weight=None)

Performs clustering on X and returns cluster labels.

Parameters

X : array or sparse (CSR) matrix of shape (n_samples, n_features), or array of shape (n_samples, n_samples)

A feature array, or an array of distances between samples if metric='precomputed'.

**sample_weight**: array, shape (n_samples,), optional

Weight of each sample, such that a sample with a weight of at least \( \text{min} \text{samples} \) is by itself a core sample; a sample with negative weight may inhibit its \( \varepsilon \)-neighbor from being core. Note that weights are absolute, and default to 1.

**Returns**: ndarray, shape (n_samples,)

cluster labels

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  
  Parameter names mapped to their values.

**set_params**(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \(<\text{component}>__.<\text{parameter}>\) so that it’s possible to update each component of a nested object.

**Returns**: self

---

**Examples using sklearn.cluster.DBSCAN**

- *Comparing different clustering algorithms on toy datasets*
- *Demo of DBSCAN clustering algorithm*

---

**sklearn.cluster.FeatureAgglomeration**

```python
class sklearn.cluster.FeatureAgglomeration(n_clusters=2, affinity='euclidean', memory=Memory(cachedir=None), connectivity=None, compute_full_tree='auto', linkage='ward', pooling_func=<function mean>)
```

Agglomerate features.

Similar to AgglomerativeClustering, but recursively merges features instead of samples.

Read more in the *User Guide*.

**Parameters**

- **n_clusters**: int, default 2
  
  The number of clusters to find.

- **connectivity**: array-like or callable, optional
  
  Connectivity matrix. Defines for each feature the neighboring features following a given structure of the data. This can be a connectivity matrix itself or a callable that transforms the data into a connectivity matrix, such as derived from kneighbors_graph. Default is None, i.e, the hierarchical clustering algorithm is unstructured.

- **affinity**: string or callable, default “euclidean”
Metric used to compute the linkage. Can be “euclidean”, “l1”, “l2”, “manhattan”, “cosine”, or ‘precomputed’. If linkage is “ward”, only “euclidean” is accepted.

**memory**: Instance of joblib.Memory or string, optional
Used to cache the output of the computation of the tree. By default, no caching is done. If a string is given, it is the path to the caching directory.

**compute_full_tree**: bool or ‘auto’, optional, default “auto”
Stop early the construction of the tree at n_clusters. This is useful to decrease computation time if the number of clusters is not small compared to the number of features. This option is useful only when specifying a connectivity matrix. Note also that when varying the number of clusters and using caching, it may be advantageous to compute the full tree.

**linkage**: {“ward”, “complete”, “average”}, optional, default “ward”
Which linkage criterion to use. The linkage criterion determines which distance to use between sets of features. The algorithm will merge the pairs of cluster that minimize this criterion.
- ward minimizes the variance of the clusters being merged.
- average uses the average of the distances of each feature of the two sets.
- complete or maximum linkage uses the maximum distances between all features of the two sets.

**pooling_func**: callable, default np.mean
This combines the values of agglomerated features into a single value, and should accept an array of shape [M, N] and the keyword argument axis=1, and reduce it to an array of size [M].

**Attributes**
- **labels_**: array-like, (n_features,)
  cluster labels for each feature.
- **n_leaves_**: int
  Number of leaves in the hierarchical tree.
- **n_components_**: int
  The estimated number of connected components in the graph.
- **children_**: array-like, shape (n_nodes-1, 2)
  The children of each non-leaf node. Values less than n_features correspond to leaves of the tree which are the original samples. A node $i$ greater than or equal to n_features is a non-leaf node and has children children[i] - n_features]. Alternatively at the i-th iteration, children[i][0] and children[i][1] are merged to form node n_features + i

**Methods**

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<td>Fit to data, then transform it.</td>
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```python
__init__ (n_clusters=2, affinity='euclidean', memory=Memory(cachedir=None), connectivity=None, compute_full_tree='auto', linkage='ward', pooling_func=<function mean>)
```

```python
fit (X, y=None, **params)
Fit the hierarchical clustering on the data

Parameters:
- **X**: array-like, shape = [n_samples, n_features]
  The data

Returns:
- **self**: 

```python
fit_transform (X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters:
- **X**: numpy array of shape [n_samples, n_features]
  Training set.
- **y**: numpy array of shape [n_samples]
  Target values.

Returns:
- **X_new**: numpy array of shape [n_samples, n_features_new]
  Transformed array.
```

```python
get_params (deep=True)
Get parameters for this estimator.

Parameters:
- **deep** (default: True)
  If True, will return the parameters for this estimator and contained subobjects that are
  estimators.

Returns:
- **params**: mapping of string to any
  Parameter names mapped to their values.
```

```python
inverse_transform (Xred)
Inverse the transformation. Return a vector of size nb_features with the values of Xred assigned to each

Parameters:
- **Xred**: array-like, shape=[n_samples, n_clusters] or [n_clusters,]
  The values to be assigned to each cluster of samples

Returns:
- **X**: array, shape=[n_samples, n_features] or [n_features]
  A vector of size n_samples with the values of Xred assigned to each of the cluster of samples.
```

```python
pooling_func (a, axis=None, dtype=None, out=None, keepdims=<class numpy._globals._NoValue>)
Compute the arithmetic mean along the specified axis.

Returns the average of the array elements. The average is taken over the flattened array by default, otherwise over the specified axis. float64 intermediate and return values are used for integer inputs.
```
Parameters

**a** : array_like
Array containing numbers whose mean is desired. If *a* is not an array, a conversion is attempted.

**axis** : None or int or tuple of ints, optional
Axis or axes along which the means are computed. The default is to compute the mean of the flattened array.

If this is a tuple of ints, a mean is performed over multiple axes, instead of a single axis or all the axes as before.

**dtype** : data-type, optional
Type to use in computing the mean. For integer inputs, the default is float64; for floating point inputs, it is the same as the input dtype.

**out** : ndarray, optional
Alternate output array in which to place the result. The default is None; if provided, it must have the same shape as the expected output, but the type will be cast if necessary. See doc.ufuncs for details.

**keepdims** : bool, optional
If this is set to True, the axes which are reduced are left in the result as dimensions with size one. With this option, the result will broadcast correctly against the original arr.

If the default value is passed, then keepdims will not be passed through to the mean method of sub-classes of ndarray, however any non-default value will be. If the sub-classes sum method does not implement keepdims any exceptions will be raised.

Returns

**m** : ndarray, see dtype parameter above
If out=None, returns a new array containing the mean values, otherwise a reference to the output array is returned.

See also:

averageWeighted average

std, var, nanmean, nanstd, nanvar

Notes

The arithmetic mean is the sum of the elements along the axis divided by the number of elements.

Note that for floating-point input, the mean is computed using the same precision the input has. Depending on the input data, this can cause the results to be inaccurate, especially for float32 (see example below). Specifying a higher-precision accumulator using the dtype keyword can alleviate this issue.

Examples

```python
>>> a = np.array([[1, 2], [3, 4]])
>>> np.mean(a)
2.5
>>> np.mean(a, axis=0)
array([ 2.,  3.])
```
```python
>>> np.mean(a, axis=1)
array([ 1.5,  3.5])
```

In single precision, `mean` can be inaccurate:

```python
>>> a = np.zeros((2, 512*512), dtype=np.float32)
>>> a[0, :] = 1.0
>>> a[1, :] = 0.1
>>> np.mean(a)
0.546875
```

Computing the mean in float64 is more accurate:

```python
>>> np.mean(a, dtype=np.float64)
0.55000000074505806
```

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

```
Returns:
self:
```

**transform** (**X**)

Transform a new matrix using the built clustering

- **Parameters**

  X: array-like, shape = [n_samples, n_features] or [n_features]

  A M by N array of M observations in N dimensions or a length M array of M one-dimensional observations.

- **Returns**

  Y: array, shape = [n_samples, n_clusters] or [n_clusters]

  The pooled values for each feature cluster.

**Examples using sklearn.cluster.FeatureAgglomeration**

- Feature agglomeration
- Feature agglomeration vs. univariate selection

**sklearn.cluster.KMeans**

class **sklearn.cluster.KMeans** (**n_clusters=8, init='k-means++', n_init=10, max_iter=300, tol=0.0001, precompute_distances='auto', verbose=0, random_state=None, copy_x=True, n_jobs=1, algorithm='auto')

K-Means clustering

Read more in the *User Guide.*

- **Parameters**

  n_clusters : int, optional, default: 8

  The number of clusters to form as well as the number of centroids to generate.

  max_iter : int, default: 300

  Maximum number of iterations of the k-means algorithm for a single run.
n_init : int, default: 10

Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of n_init consecutive runs in terms of inertia.

init : {'k-means++', 'random' or an ndarray}

Method for initialization, defaults to ‘k-means++’:

‘k-means++’ : selects initial cluster centers for k-mean clustering in a smart way to speed up convergence. See section Notes in k_init for more details.

‘random’ : choose k observations (rows) at random from data for the initial centroids.

If an ndarray is passed, it should be of shape (n_clusters, n_features) and gives the initial centers.

algorithm : “auto”, “full” or “elkan”, default=“auto”

K-means algorithm to use. The classical EM-style algorithm is “full”. The “elkan” variation is more efficient by using the triangle inequality, but currently doesn’t support sparse data. “auto” chooses “elkan” for dense data and “full” for sparse data.

precompute_distances : {'auto', True, False}

Precompute distances (faster but takes more memory).

‘auto’ : do not precompute distances if n_samples * n_clusters > 12 million. This corresponds to about 100MB overhead per job using double precision.

True : always precompute distances

False : never precompute distances

tol : float, default: 1e-4

Relative tolerance with regards to inertia to declare convergence

n_jobs : int

The number of jobs to use for the computation. This works by computing each of the n_init runs in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

random_state : integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

verbose : int, default 0

Verbosity mode.

copy_x : boolean, default True

When pre-computing distances it is more numerically accurate to center the data first. If copy_x is True, then the original data is not modified. If False, the original data is modified, and put back before the function returns, but small numerical differences may be introduced by subtracting and then adding the data mean.

Attributes

cluster_centers_ : array, [n_clusters, n_features]

Coordinates of cluster centers
labels_ : 
Labels of each point

inertia_ : float
Sum of distances of samples to their closest cluster center.

See also:

MiniBatchKMeans | Alternative online implementation that does incremental updates of the centers positions using mini-batches. For large scale learning (say n_samples > 10k) MiniBatchKMeans is probably much faster than the default batch implementation.

Notes
The k-means problem is solved using Lloyd’s algorithm.
The average complexity is given by O(k n T), were n is the number of samples and T is the number of iteration.
The worst case complexity is given by O(n^(k+2/p)) with n = n_samples, p = n_features. (D. Arthur and S. Vassilvitskii, ‘How slow is the k-means method?’ SoCG2006)
In practice, the k-means algorithm is very fast (one of the fastest clustering algorithms available), but it falls in local minima. That’s why it can be useful to restart it several times.

Examples

```python
>>> from sklearn.cluster import KMeans
>>> import numpy as np

>>> X = np.array([[1, 2], [1, 4], [1, 0], ...
               [4, 2], [4, 4], [4, 0]])
>>> kmeans = KMeans(n_clusters=2, random_state=0).fit(X)
>>> kmeans.labels_
array([0, 0, 0, 1, 1, 1], dtype=int32)
>>> kmeans.predict([[0, 0], [4, 4]])
array([0, 1], dtype=int32)
>>> kmeans.cluster_centers_
array([[ 1.,  2.],
       [ 4.,  2.]])
```

Methods

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<td>Compute k-means clustering.</td>
</tr>
<tr>
<td><code>fit_predict(X[, y])</code></td>
<td>Compute cluster centers and predict cluster index for each sample.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Compute clustering and transform X to cluster-distance space.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>predict(X)</code></td>
<td>Predict the closest cluster each sample in X belongs to.</td>
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<tr>
<td><code>score(X[, y])</code></td>
<td>Opposite of the value of X on the K-means objective.</td>
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<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><code>transform(X[, y])</code></td>
<td>Transform X to a cluster-distance space.</td>
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__init__ (n_clusters=8, init='k-means++', n_init=10, max_iter=300, tol=0.0001, precompute_distances='auto', verbose=0, random_state=None, copy_x=True, n_jobs=1, algorithm='auto'))

fit (X, y=None)
Compute k-means clustering.

Parameters:
X : array-like or sparse matrix, shape=(n_samples, n_features)
Training instances to cluster.

fit_predict (X, y=None)
Compute cluster centers and predict cluster index for each sample.
Convenience method; equivalent to calling fit(X) followed by predict(X).

fit_transform (X, y=None)
Compute clustering and transform X to cluster-distance space.
Equivalent to fit(X).transform(X), but more efficiently implemented.

get_params (deep=True)
Get parameters for this estimator.

Parameters:
deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
params : mapping of string to any
Parameter names mapped to their values.

predict (X)
Predict the closest cluster each sample in X belongs to.
In the vector quantization literature, cluster_centers_ is called the code book and each value returned by predict is the index of the closest code in the code book.

Parameters:
X : {array-like, sparse matrix}, shape = [n_samples, n_features]
New data to predict.

Returns:
labels : array, shape [n_samples,]
Index of the cluster each sample belongs to.

score (X, y=None)
Opposite of the value of X on the K-means objective.

Parameters:
X : {array-like, sparse matrix}, shape = [n_samples, n_features]
New data.

Returns:
score : float
Opposite of the value of X on the K-means objective.

set_params (**params)
Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
self :
**transform** \((X, y=None)\)

Transform \(X\) to a cluster-distance space.

In the new space, each dimension is the distance to the cluster centers. Note that even if \(X\) is sparse, the array returned by `transform` will typically be dense.

**Parameters**

- **\(X\)** : [array-like, sparse matrix], \(\text{shape} = [n\_\text{samples}, n\_\text{features}]\)
  - New data to transform.

**Returns**

- **\(X\_\text{new}\)** : array, \(\text{shape} = [n\_\text{samples}, k]\)
  - \(X\) transformed in the new space.

### Examples using sklearn.cluster.KMeans

- **K-means Clustering**
- **Color Quantization using K-Means**
- **Vector Quantization Example**
- **Demonstration of k-means assumptions**
- **A demo of K-Means clustering on the handwritten digits data**
- **Selecting the number of clusters with silhouette analysis on KMeans clustering**
- **Empirical evaluation of the impact of k-means initialization**
- **Comparison of the K-Means and MiniBatchKMeans clustering algorithms**
- **Clustering text documents using k-means**

### sklearn.cluster.MiniBatchKMeans

**class sklearn.cluster.MiniBatchKMeans** (\(n\_\text{clusters}=8, \ init='k\_\text{means++}', \ max\_\text{iter}=100, \ batch\_\text{size}=100, \ verbose=0, \ compute\_\text{labels}=True, \ random\_\text{state}=None, \ tol=0.0, \ max\_\text{no}\_\text{improvement}=10, \ init\_\text{size}=None, \ n\_\text{init}=3, \ reassignment\_\text{ratio}=0.01)\)

Mini-Batch K-Means clustering

Read more in the **User Guide**.

**Parameters**

- **\(n\_\text{clusters}\)** : int, optional, default: 8
  - The number of clusters to form as well as the number of centroids to generate.

- **max\_\text{iter}\** : int, optional
  - Maximum number of iterations over the complete dataset before stopping independently of any early stopping criterion heuristics.

- **max\_\text{no}\_\text{improvement}\** : int, default: 10
  - Control early stopping based on the consecutive number of mini batches that does not yield an improvement on the smoothed inertia.
  - To disable convergence detection based on inertia, set `max_no_improvement` to `None`.

- **tol\** : float, default: 0.0
Control early stopping based on the relative center changes as measured by a smoothed, variance-normalized of the mean center squared position changes. This early stopping heuristics is closer to the one used for the batch variant of the algorithms but induces a slight computational and memory overhead over the inertia heuristic.

To disable convergence detection based on normalized center change, set tol to 0.0 (default).

**batch_size**: int, optional, default: 100

Size of the mini batches.

**init_size**: int, optional, default: 3 * batch_size

Number of samples to randomly sample for speeding up the initialization (sometimes at the expense of accuracy): the only algorithm is initialized by running a batch KMeans on a random subset of the data. This needs to be larger than n_clusters.

**init**: {'k-means++', 'random' or an ndarray}, default: ‘k-means++’

Method for initialization, defaults to ‘k-means++’:

‘k-means++’ : selects initial cluster centers for k-mean clustering in a smart way to speed up convergence. See section Notes in k_init for more details.

‘random’: choose k observations (rows) at random from data for the initial centroids.

If an ndarray is passed, it should be of shape (n_clusters, n_features) and gives the initial centers.

**n_init**: int, default=3

Number of random initializations that are tried. In contrast to KMeans, the algorithm is only run once, using the best of the n_init initializations as measured by inertia.

**compute_labels**: boolean, default=True

Compute label assignment and inertia for the complete dataset once the minibatch optimization has converged in fit.

**random_state**: integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**reassignment_ratio**: float, default: 0.01

Control the fraction of the maximum number of counts for a center to be reassigned. A higher value means that low count centers are more easily reassigned, which means that the model will take longer to converge, but should converge in a better clustering.

**verbose**: boolean, optional

Verbosity mode.

**Attributes**

- **cluster_centers_**: array, [n_clusters, n_features]
  Coordinates of cluster centers

- **labels_**: 
  Labels of each point (if compute_labels is set to True).

- **inertia_**: float
The value of the inertia criterion associated with the chosen partition (if compute_labels is set to True). The inertia is defined as the sum of square distances of samples to their nearest neighbor.

See also:

KMeans The classic implementation of the clustering method based on the Lloyd’s algorithm. It consumes the whole set of input data at each iteration.

Notes

See http://www.eecs.tufts.edu/~dsculley/papers/fastkmeans.pdf

Methods

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__init__(n_clusters=8, init='k-means++', max_iter=100, batch_size=100, verbose=0, compute_labels=True, random_state=None, tol=0.0, max_no_improvement=10, init_size=None, n_init=3, reassignment_ratio=0.01)

fit (X, y=None)

Compute the centroids on X by chunking it into mini-batches.

Parameters

X : array-like or sparse matrix, shape=(n_samples, n_features)

Training instances to cluster.

fit_predict (X, y=None)

Compute cluster centers and predict cluster index for each sample.

Convenience method; equivalent to calling fit(X) followed by predict(X).

fit_transform (X, y=None)

Compute clustering and transform X to cluster-distance space.

Equivalent to fit(X).transform(X), but more efficiently implemented.

get_params (deep=True)

Get parameters for this estimator.

Parameters

deep : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.
Returns: mapping of string to any
Parameter names mapped to their values.

partial_fit (X, y=None)
Update k means estimate on a single mini-batch X.

Parameters: X : array-like, shape = [n_samples, n_features]
Coordinates of the data points to cluster.

predict (X)
Predict the closest cluster each sample in X belongs to.
In the vector quantization literature, cluster_centers_ is called the code book and each value returned by predict is the index of the closest code in the code book.

Parameters: X : {array-like, sparse matrix}, shape = [n_samples, n_features]
New data to predict.

Returns: labels : array, shape [n_samples,]
Index of the cluster each sample belongs to.

score (X, y=None)
Opposite of the value of X on the K-means objective.

Parameters: X : {array-like, sparse matrix}, shape = [n_samples, n_features]
New data.

Returns: score : float
Opposite of the value of X on the K-means objective.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns: self

transform (X, y=None)
Transform X to a cluster-distance space.
In the new space, each dimension is the distance to the cluster centers. Note that even if X is sparse, the array returned by transform will typically be dense.

Parameters: X : {array-like, sparse matrix}, shape = [n_samples, n_features]
New data to transform.

Returns: X_new : array, shape [n_samples, k]
X transformed in the new space.

Examples using sklearn.cluster.MiniBatchKMeans

- Biclustering documents with the Spectral Co-clustering algorithm
- Compare BIRCH and MiniBatchKMeans
- Comparing different clustering algorithms on toy datasets
• Online learning of a dictionary of parts of faces
• Empirical evaluation of the impact of k-means initialization
• Comparison of the K-Means and MiniBatchKMeans clustering algorithms
• Faces dataset decompositions
• Clustering text documents using k-means

sklearn.cluster.MeanShift

class sklearn.cluster.MeanShift (bandwidth=None, seeds=None, bin_seeding=False, min_bin_freq=1, cluster_all=True, n_jobs=1)

Mean shift clustering using a flat kernel.

Mean shift clustering aims to discover “blobs” in a smooth density of samples. It is a centroid-based algorithm, which works by updating candidates for centroids to be the mean of the points within a given region. These candidates are then filtered in a post-processing stage to eliminate near-duplicates to form the final set of centroids.

Seeding is performed using a binning technique for scalability.

Read more in the User Guide.

Parameters:

bandwidth : float, optional
Bandwidth used in the RBF kernel.

If not given, the bandwidth is estimated using sklearn.cluster.estimate_bandwidth; see the documentation for that function for hints on scalability (see also the Notes, below).

seeds : array, shape=[n_samples, n_features], optional
Seeds used to initialize kernels. If not set, the seeds are calculated by clustering$get_bin_seeds with bandwidth as the grid size and default values for other parameters.

bin_seeding : boolean, optional
If true, initial kernel locations are not locations of all points, but rather the location of the discretized version of points, where points are binned onto a grid whose coarseness corresponds to the bandwidth. Setting this option to True will speed up the algorithm because fewer seeds will be initialized. default value: False Ignored if seeds argument is not None.

min_bin_freq : int, optional
To speed up the algorithm, accept only those bins with at least min_bin_freq points as seeds. If not defined, set to 1.

cluster_all : boolean, default True
If true, then all points are clustered, even those orphans that are not within any kernel. Orphans are assigned to the nearest kernel. If false, then orphans are given cluster label -1.

n_jobs : int
The number of jobs to use for the computation. This works by computing each of the n_init runs in parallel.
If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**Attributes**

**cluster_centers_** : array, [n_clusters, n_features]
Coordinates of cluster centers.

**labels_** :
Labels of each point.

**Notes**

Scalability:
Because this implementation uses a flat kernel and a Ball Tree to look up members of each kernel, the complexity will tend towards \(O(T*n*\log(n))\) in lower dimensions, with \(n\) the number of samples and \(T\) the number of points. In higher dimensions the complexity will tend towards \(O(T*n^2)\).

Scalability can be boosted by using fewer seeds, for example by using a higher value of min_bin_freq in the get_bin_seeds function.

Note that the estimate_bandwidth function is much less scalable than the mean shift algorithm and will be the bottleneck if it is used.

**References**


**Methods**

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<td><code>fit_predict(X[, y])</code></td>
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<tr>
<td><code>predict(X)</code></td>
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<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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**__init__** (bandwidth=None, seeds=None, bin_seeding=False, min_bin_freq=1, cluster_all=True, n_jobs=1)

**fit** (X, y=None)
Perform clustering.

**Parameters**

- **X** : array-like, shape=[n_samples, n_features]
Samples to cluster.

**fit_predict** (X, y=None)
Performs clustering on X and returns cluster labels.

**Parameters**

- **X** : ndarray, shape (n_samples, n_features)
Input data.

**Returns**

- **y** : ndarray, shape (n_samples,)
cluster labels

get_params (deep=True)

Get parameters for this estimator.

Parameters:
depth : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
params : mapping of string to any

Parameter names mapped to their values.

predict (X)

Predict the closest cluster each sample in X belongs to.

Parameters:
X : {array-like, sparse matrix}, shape=[n_samples, n_features]

New data to predict.

Returns:
labels : array, shape [n_samples,]

Index of the cluster each sample belongs to.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
self :

Examples using sklearn.cluster.MeanShift

- Comparing different clustering algorithms on toy datasets
- A demo of the mean-shift clustering algorithm

sklearn.cluster.SpectralClustering

class sklearn.cluster.SpectralClustering (n_clusters=8, eigen_solver=None, random_state=None, n_init=10, gamma=1.0, affinity='rbf', n_neighbors=10, eigen_tol=0.0, assign_labels='kmeans', degree=3, coef0=1, kernel_params=None, n_jobs=1)

Apply clustering to a projection to the normalized laplacian.

In practice Spectral Clustering is very useful when the structure of the individual clusters is highly non-convex or more generally when a measure of the center and spread of the cluster is not a suitable description of the complete cluster. For instance when clusters are nested circles on the 2D plan.

If affinity is the adjacency matrix of a graph, this method can be used to find normalized graph cuts.

When calling fit, an affinity matrix is constructed using either kernel function such the Gaussian (aka RBF) kernel of the euclidean distance d(X, X):

```
np.exp(-gamma * d(X, X) ** 2)
```
or a k-nearest neighbors connectivity matrix. Alternatively, using `precomputed`, a user-provided affinity matrix can be used.

Read more in the *User Guide*.

**Parameters**

- `n_clusters`: integer, optional
  - The dimension of the projection subspace.
- `affinity`: string, array-like or callable, default ‘rbf’
  - If a string, this may be one of ‘nearest_neighbors’, ‘precomputed’, ‘rbf’ or one of the kernels supported by `sklearn.metrics.pairwise_kernels`.
  - Only kernels that produce similarity scores (non-negative values that increase with similarity) should be used. This property is not checked by the clustering algorithm.
- `gamma`: float, default=1.0
  - Scaling factor of RBF, polynomial, exponential chi^2 and sigmoid affinity kernel. Ignored for `affinity='nearest_neighbors'`.
- `degree`: float, default=3
  - Degree of the polynomial kernel. Ignored by other kernels.
- `coef0`: float, default=1
  - Zero coefficient for polynomial and sigmoid kernels. Ignored by other kernels.
- `n_neighbors`: integer
  - Number of neighbors to use when constructing the affinity matrix using the nearest neighbors method. Ignored for `affinity='rbf'`.
- `eigen_solver`: {None, ‘arpack’, ‘lobpcg’, or ‘amg’}
  - The eigenvalue decomposition strategy to use. AMG requires pyamg to be installed. It can be faster on very large, sparse problems, but may also lead to instabilities.
- `random_state`: int seed, RandomState instance, or None (default)
  - A pseudo random number generator used for the initialization of the lobpcg eigen vectors decomposition when `eigen_solver == ‘amg’` and by the K-Means initialization.
- `n_init`: int, optional, default: 10
  - Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of `n_init` consecutive runs in terms of inertia.
- `eigen_tol`: float, optional, default: 0.0
  - Stopping criterion for eigendecomposition of the Laplacian matrix when using arpack eigen_solver.
- `assign_labels`: {‘kmeans’, ‘discretize’}, default: ‘kmeans’
  - The strategy to use to assign labels in the embedding space. There are two ways to assign labels after the laplacian embedding. k-means can be applied and is a popular choice. But it can also be sensitive to initialization. Discretization is another approach which is less sensitive to random initialization.
- `kernel_params`: dictionary of string to any, optional
  - Parameters (keyword arguments) and values for kernel passed as callable object. Ignored by other kernels.
**n_jobs**: int, optional (default = 1)

The number of parallel jobs to run. If -1, then the number of jobs is set to the number of CPU cores.

**Attributes**

- **affinity_matrix_**: array-like, shape (n_samples, n_samples)

  Affinity matrix used for clustering. Available only if after calling `fit`.

- **labels_**:

  Labels of each point

**Notes**

If you have an affinity matrix, such as a distance matrix, for which 0 means identical elements, and high values mean very dissimilar elements, it can be transformed in a similarity matrix that is well suited for the algorithm by applying the Gaussian (RBF, heat) kernel:

```python
np.exp(- dist_matrix ** 2 / (2. * delta ** 2))
```

Where `delta` is a free parameter representing the width of the Gaussian kernel.

Another alternative is to take a symmetric version of the k nearest neighbors connectivity matrix of the points.

If the pyamg package is installed, it is used: this greatly speeds up computation.

**References**


**Methods**

- **fit(X[, y])**

  Creates an affinity matrix for X using the selected affinity, then applies spectral clustering to this affinity matrix.

- **fit_predict(X[, y])**

  Performs clustering on X and returns cluster labels.

- **get_params([deep])**

  Get parameters for this estimator.

- **set_params(**params**)**

  Set the parameters of this estimator.

- **__init__(n_clusters=8, eigen_solver=None, random_state=None, n_init=10, gamma=1.0, affinity='rbf', n_neighbors=10, eigen_tol=0.0, assign_labels='kmeans', degree=3, coef0=1, kernel_params=None, n_jobs=1)**

  Creates an affinity matrix for X using the selected affinity, then applies spectral clustering to this affinity matrix.

  **Parameters**

  - **X**: array-like or sparse matrix, shape (n_samples, n_features)
OR, if affinity=='precomputed', a precomputed affinity matrix of shape (n_samples, n_samples)

**fit_predict** (X, y=None)
Performs clustering on X and returns cluster labels.

Parameters
- **X**: ndarray, shape (n_samples, n_features)
  Input data.
- **y**: ndarray, shape (n_samples,)
  cluster labels

**get_params** (deep=True)
Get parameters for this estimator.

Parameters
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns
- **params**: mapping of string to any
  Parameter names mapped to their values.

**set_params** (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns
- **self**:

Examples using `sklearn.cluster.SpectralClustering`

- Comparing different clustering algorithms on toy datasets

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<td>K-means clustering algorithm.</td>
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**sklearn.cluster.estimate_bandwidth**

`sklearn.cluster.estimate_bandwidth(X, quantile=0.3, n_samples=None, random_state=0, n_jobs=1)`

Estimate the bandwidth to use with the mean-shift algorithm.
That this function takes time at least quadratic in n_samples. For large datasets, it’s wise to set that parameter to a small value.

**Parameters**

- **X** : array-like, shape=[n_samples, n_features]
  Input points.
- **quantile** : float, default 0.3
  should be between [0, 1] 0.5 means that the median of all pairwise distances is used.
- **n_samples** : int, optional
  The number of samples to use. If not given, all samples are used.
- **random_state** : int or RandomState
  Pseudo-random number generator state used for random sampling.
- **n_jobs** : int, optional (default = 1)
  The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores.

**Returns**

- **bandwidth** : float
  The bandwidth parameter.

**Examples using** sklearn.cluster.estimate_bandwidth

- **Comparing different clustering algorithms on toy datasets**
- **A demo of the mean-shift clustering algorithm**

**sklearn.cluster.k_means**

**sklearn.cluster.k_means** (X, n_clusters, init='k-means++', precompute_distances='auto', n_init=10, max_iter=300, verbose=False, tol=0.0001, random_state=None, copy_x=True, n_jobs=1, algorithm='auto', return_n_iter=False)

K-means clustering algorithm.

Read more in the **User Guide**.

**Parameters**

- **X** : array-like or sparse matrix, shape (n_samples, n_features)
  The observations to cluster.
- **n_clusters** : int
  The number of clusters to form as well as the number of centroids to generate.
- **max_iter** : int, optional, default 300
  Maximum number of iterations of the k-means algorithm to run.
- **n_init** : int, optional, default: 10
  Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of n_init consecutive runs in terms of inertia.
- **init** : {‘k-means++’, ‘random’, or ndarray, or a callable}, optional
Method for initialization, default to ‘k-means++’:

‘k-means++’ : selects initial cluster centers for k-mean clustering in a smart way to speed up convergence. See section Notes in k_init for more details.

‘random’ : generate k centroids from a Gaussian with mean and variance estimated from the data.

If an ndarray is passed, it should be of shape (n_clusters, n_features) and gives the initial centers.

If a callable is passed, it should take arguments X, k and and a random state and return an initialization.

algorithm : “auto”, “full” or “elkan”, default=”auto”

K-means algorithm to use. The classical EM-style algorithm is “full”. The “elkan” variation is more efficient by using the triangle inequality, but currently doesn’t support sparse data. “auto” chooses “elkan” for dense data and “full” for sparse data.

precompute_distances : {‘auto’, True, False}

Precompute distances (faster but takes more memory).

‘auto’ : do not precompute distances if n_samples * n_clusters > 12 million. This corresponds to about 100MB overhead per job using double precision.

True : always precompute distances

False : never precompute distances

tol : float, optional

The relative increment in the results before declaring convergence.

verbose : boolean, optional

Verbosity mode.

random_state : integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

copy_x : boolean, optional

When pre-computing distances it is more numerically accurate to center the data first. If copy_x is True, then the original data is not modified. If False, the original data is modified, and put back before the function returns, but small numerical differences may be introduced by subtracting and then adding the data mean.

n_jobs : int

The number of jobs to use for the computation. This works by computing each of the n_init runs in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

return_n_iter : bool, optional

Whether or not to return the number of iterations.

Returns

centroids : float ndarray with shape (k, n_features)

Centroids found at the last iteration of k-means.
label : integer ndarray with shape (n_samples,)
label[i] is the code or index of the centroid the i’th observation is closest to.

inertia : float
The final value of the inertia criterion (sum of squared distances to the closest centroid for all observations in the training set).

best_n_iter: int :
Number of iterations corresponding to the best results. Returned only if return_n_iter is set to True.

**sklearn.cluster.ward_tree**

`sklearn.cluster.ward_tree(X, connectivity=None, n_clusters=None, return_distance=False)`
Ward clustering based on a Feature matrix.
Recursively merges the pair of clusters that minimally increases within-cluster variance.
The inertia matrix uses a Heapq-based representation.
This is the structured version, that takes into account some topological structure between samples.

Parameters:

X : array, shape (n_samples, n_features)
feature matrix representing n_samples samples to be clustered

connectivity : sparse matrix (optional).
connectivity matrix. Defines for each sample the neighboring samples following a given structure of the data. The matrix is assumed to be symmetric and only the upper triangular half is used. Default is None, i.e, the Ward algorithm is unstructured.

n_clusters : int (optional)
Stop early the construction of the tree at n_clusters. This is useful to decrease computation time if the number of clusters is not small compared to the number of samples. In this case, the complete tree is not computed, thus the ‘children’ output is of limited use, and the ‘parents’ output should rather be used. This option is valid only when specifying a connectivity matrix.

return_distance: bool (optional) :
If True, return the distance between the clusters.

Returns:
children : 2D array, shape (n_nodes-1, 2)
The children of each non-leaf node. Values less than n_samples correspond to leaves of the tree which are the original samples. A node i greater than or equal to n_samples is a non-leaf node and has children children[i - n_samples]. Alternatively at the i-th iteration, children[i][0] and children[i][1] are merged to form node n_samples + i

n_components : int
The number of connected components in the graph.

n_leaves : int
The number of leaves in the tree

parents : 1D array, shape (n_nodes, ) or None
The parent of each node. Only returned when a connectivity matrix is specified, elsewhere ‘None’ is returned.

distances : 1D array, shape (n_nodes-1,)

Only returned if return_distance is set to True (for compatibility). The distances between the centers of the nodes. distances[i] corresponds to a weighted euclidean distance between the nodes children[i, 1] and children[i, 2]. If the nodes refer to leaves of the tree, then distances[i] is their unweighted euclidean distance. Distances are updated in the following way (from scipy.hierarchy.linkage):

The new entry $d(u, v)$ is computed as follows,

$$
d(u, v) = \sqrt{\left| v \right| + \left| s \right| d(v, s)^2 + \frac{|v| + |t|}{T} d(v, t)^2 - \frac{|v|}{T} d(s, t)^2}
$$

where $u$ is the newly joined cluster consisting of clusters $s$ and $t$, $v$ is an unused cluster in the forest, $T = |v| + |s| + |t|$, and $|*|$ is the cardinality of its argument. This is also known as the incremental algorithm.

**sklearn.cluster.affinity_propagation**

```
sklearn.cluster.affinity_propagation(S, preference=None, convergence_iter=15, max_iter=200, damping=0.5, copy=True, verbose=False, return_n_iter=False)
```

Perform Affinity Propagation Clustering of data

Read more in the [User Guide](#).

**Parameters**

- `S` : array-like, shape (n_samples, n_samples)
  
  Matrix of similarities between points

- preference : array-like, shape (n_samples,) or float, optional
  
  Preferences for each point - points with larger values of preferences are more likely to be chosen as exemplars. The number of exemplars, i.e., of clusters, is influenced by the input preferences value. If the preferences are not passed as arguments, they will be set to the median of the input similarities (resulting in a moderate number of clusters). For a smaller amount of clusters, this can be set to the minimum value of the similarities.

- convergence_iter : int, optional, default: 15
  
  Number of iterations with no change in the number of estimated clusters that stops the convergence.

- max_iter : int, optional, default: 200
  
  Maximum number of iterations

- damping : float, optional, default: 0.5
  
  Damping factor between 0.5 and 1.

- copy : boolean, optional, default: True
  
  If copy is False, the affinity matrix is modified in place by the algorithm, for memory efficiency

- verbose : boolean, optional, default: False
  
  The verbosity level

- return_n_iter : bool, default False
Whether or not to return the number of iterations.

**Returns**

- **clusters**
  - *cluster_centers_indices*: array, shape (n_clusters,)
    - index of clusters centers
  - *labels*: array, shape (n_samples,)
    - cluster labels for each point
  - *n_iter*: int
    - number of iterations run. Returned only if *return_n_iter* is set to True.

**Notes**

See examples/cluster/plot_affinity_propagation.py for an example.

**References**


**Examples using sklearn.cluster.affinity_propagation**

- Visualizing the stock market structure

**sklearn.cluster.dbscan**

Perform DBSCAN clustering from vector array or distance matrix.

**Parameters**

- **X**: array or sparse (CSR) matrix of shape (n_samples, n_features), or array of shape (n_samples, n_samples)
  - A feature array, or array of distances between samples if *metric*='precomputed'.
- **eps**: float, optional
  - The maximum distance between two samples for them to be considered as in the same neighborhood.
- **min_samples**: int, optional
  - The number of samples (or total weight) in a neighborhood for a point to be considered as a core point. This includes the point itself.
- **metric**: string, or callable
  - The metric to use when calculating distance between instances in a feature array. If metric is a string or callable, it must be one of the options allowed by metrics.pairwise.pairwise_distances for its metric parameter. If metric is "precomputed", X is assumed to be a distance matrix and must be square. X may be a sparse matrix, in which case only “nonzero” elements may be considered neighbors for DBSCAN.
- **algorithm**: [‘auto’, ‘ball_tree’, ‘kd_tree’, ‘brute’], optional
The algorithm to be used by the NearestNeighbors module to compute pointwise distances and find nearest neighbors. See NearestNeighbors module documentation for details.

**leaf_size** : int, optional (default = 30)

Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

**p** : float, optional

The power of the Minkowski metric to be used to calculate distance between points.

**sample_weight** : array, shape (n_samples,), optional

Weight of each sample, such that a sample with a weight of at least min_samples is by itself a core sample; a sample with negative weight may inhibit its eps-neighbor from being core. Note that weights are absolute, and default to 1.

**n_jobs** : int, optional (default = 1)

The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores.

**Returnscore_samples** : array [n_core_samples]

Indices of core samples.

**labels** : array [n_samples]

Cluster labels for each point. Noisy samples are given the label -1.

**Notes**

See examples/cluster/plot_dbscan.py for an example.

This implementation bulk-computes all neighborhood queries, which increases the memory complexity to O(n.d) where d is the average number of neighbors, while original DBSCAN had memory complexity O(n).

Sparse neighborhoods can be precomputed using NearestNeighbors.radius_neighbors_graph with mode='distance'.

**References**


**sklearn.cluster.mean_shift**

Perform mean shift clustering of data using a flat kernel.

Read more in the User Guide.

**Parameters**

**X** : array-like, shape=[n_samples, n_features]

Input data.
**bandwidth** : float, optional

Kernel bandwidth.

If bandwidth is not given, it is determined using a heuristic based on the median of all pairwise distances. This will take quadratic time in the number of samples. The sklearn.cluster.estimate_bandwidth function can be used to do this more efficiently.

**seeds** : array-like, shape=[n_seeds, n_features] or None

Point used as initial kernel locations. If None and bin_seeding=False, each data point is used as a seed. If None and bin_seeding=True, see bin_seeding.

**bin_seeding** : boolean, default=False

If true, initial kernel locations are not locations of all points, but rather the location of the discretized version of points, where points are binned onto a grid whose coarseness corresponds to the bandwidth. Setting this option to True will speed up the algorithm because fewer seeds will be initialized. Ignored if seeds argument is not None.

**min_bin_freq** : int, default=1

To speed up the algorithm, accept only those bins with at least min_bin_freq points as seeds.

**cluster_all** : boolean, default True

If true, then all points are clustered, even those orphans that are not within any kernel. Orphans are assigned to the nearest kernel. If false, then orphans are given cluster label -1.

**max_iter** : int, default 300

Maximum number of iterations, per seed point before the clustering operation terminates (for that seed point), if has not converged yet.

**n_jobs** : int

The number of jobs to use for the computation. This works by computing each of the n_init runs in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

New in version 0.17: Parallel Execution using n_jobs.

**Returns**

cluster_centers : array, shape=[n_clusters, n_features]

Coordinates of cluster centers.

labels : array, shape=[n_samples]

Cluster labels for each point.

**Notes**

See examples/cluster/plot_mean_shift.py for an example.
Apply clustering to a projection to the normalized laplacian.

In practice Spectral Clustering is very useful when the structure of the individual clusters is highly non-convex or more generally when a measure of the center and spread of the cluster is not a suitable description of the complete cluster. For instance when clusters are nested circles on the 2D plan.

If affinity is the adjacency matrix of a graph, this method can be used to find normalized graph cuts.

Read more in the User Guide.

**Parameters**

- **affinity**: array-like or sparse matrix, shape: (n_samples, n_samples)
  - The affinity matrix describing the relationship of the samples to embed. **Must be symmetric**.
  - Possible examples:
    - adjacency matrix of a graph,
    - heat kernel of the pairwise distance matrix of the samples,
    - symmetric k-nearest neighbours connectivity matrix of the samples.

- **n_clusters**: integer, optional
  - Number of clusters to extract.

- **n_components**: integer, optional, default is n_clusters
  - Number of eigen vectors to use for the spectral embedding

- **eigen_solver**: {None, ‘arpack’, ‘lobpcg’, or ‘amg’}
  - The eigenvalue decomposition strategy to use. AMG requires pyamg to be installed. It can be faster on very large, sparse problems, but may also lead to instabilities

- **random_state**: int seed, RandomState instance, or None (default)
  - A pseudo random number generator used for the initialization of the lobpcg eigen vectors decomposition when eigen_solver == ‘amg’ and by the K-Means initialization.

- **n_init**: int, optional, default: 10
  - Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of n_init consecutive runs in terms of inertia.

- **eigen_tol**: float, optional, default: 0.0
  - Stopping criterion for eigendecomposition of the Laplacian matrix when using arpack eigen_solver.

- **assign_labels**: {‘kmeans’, ‘discretize’}, default: ‘kmeans’
  - The strategy to use to assign labels in the embedding space. There are two ways to assign labels after the laplacian embedding. k-means can be applied and is a popular choice. But it can also be sensitive to initialization. Discretization is another approach which is less sensitive to random initialization. See the ‘Multiclass spectral clustering’ paper referenced below for more details on the discretization approach.

**Returns**

- **labels**: array of integers, shape: n_samples
  - The labels of the clusters.
Notes

The graph should contain only one connect component, elsewhere the results make little sense.
This algorithm solves the normalized cut for k=2: it is a normalized spectral clustering.

References


Examples using sklearn.cluster.spectral_clustering

- Segmenting the picture of a raccoon face in regions
- Spectral clustering for image segmentation

29.3 sklearn.cluster.bicluster: Biclustering

Spectral biclustering algorithms.
Authors : Kemal Eren License: BSD 3 clause
User guide: See the Biclustering section for further details.

29.3.1 Classes

<table>
<thead>
<tr>
<th>SpectralBiclustering([n_clusters, method, ...])</th>
<th>Spectral biclustering (Kluger, 2003).</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpectralCoclustering([n_clusters, ...])</td>
<td>Spectral Co-Clustering algorithm (Dhillon, 2001).</td>
</tr>
</tbody>
</table>

sklearn.cluster.bicluster.SpectralBiclustering

class sklearn.cluster.bicluster.SpectralBiclustering (n_clusters=3,
method='bistochastic',
n_components=6, n_best=3,
svd_method='randomized',
n_svd_vecs=None,
mini_batch=False, init='k-means++', n_init=10, n_jobs=1,
random_state=None)

Spectral biclustering (Kluger, 2003).
Partitions rows and columns under the assumption that the data has an underlying checkerboard structure. For instance, if there are two row partitions and three column partitions, each row will belong to three biclusters, and each column will belong to two biclusters. The outer product of the corresponding row and column label
vectors gives this checkerboard structure.
Read more in the *User Guide*.

**Parameters**

- **n_clusters**: integer or tuple (n_row_clusters, n_column_clusters)
  The number of row and column clusters in the checkerboard structure.
- **method**: string, optional, default: ‘bistochastic’
  Method of normalizing and converting singular vectors into biclusters. May be one of ‘scale’, ‘bistochastic’, or ‘log’. The authors recommend using ‘log’. If the data is sparse, however, log normalization will not work, which is why the default is ‘bistochastic’. CAUTION: if method=’log’, the data must not be sparse.
- **n_components**: integer, optional, default: 6
  Number of singular vectors to check.
- **n_best**: integer, optional, default: 3
  Number of best singular vectors to which to project the data for clustering.
- **svd_method**: string, optional, default: ‘randomized’
  Selects the algorithm for finding singular vectors. May be ‘randomized’ or ‘arpack’. If ‘randomized’, uses sklearn.utils.extmath.randomized_svd, which may be faster for large matrices. If ‘arpack’, uses sklearn.utils.arpack.svds, which is more accurate, but possibly slower in some cases.
- **n_svd_vecs**: int, optional, default: None
  Number of vectors to use in calculating the SVD. Corresponds to ncv when svd_method=arpack and n_oversamples when svd_method is ‘randomized’.
- **mini_batch**: bool, optional, default: False
  Whether to use mini-batch k-means, which is faster but may get different results.
- **init**: {‘k-means++’, ‘random’ or an ndarray}
  Method for initialization of k-means algorithm; defaults to ‘k-means++’.
- **n_init**: int, optional, default: 10
  Number of random initializations that are tried with the k-means algorithm.
  If mini-batch k-means is used, the best initialization is chosen and the algorithm runs once. Otherwise, the algorithm is run for each initialization and the best solution chosen.
- **n_jobs**: int, optional, default: 1
  The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.
  If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.
- **random_state**: int seed, RandomState instance, or None (default)
  A pseudo random number generator used by the K-Means initialization.

**Attributes**

- **rows_**: array-like, shape (n_row_clusters, n_rows)
  Results of the clustering. rows[i, r] is True if cluster i contains row r. Available only after calling fit.
columns_ : array-like, shape (n_column_clusters, n_columns)
Results of the clustering, like rows.

row_labels_ : array-like, shape (n_rows,)
Row partition labels.

column_labels_ : array-like, shape (n_cols,)
Column partition labels.

References

- Kluger, Yuval, et. al., 2003. Spectral biclustering of microarray data: co-clustering genes and conditions.

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X)</td>
<td>Creates a biclustering for X.</td>
</tr>
<tr>
<td>get_indices(i)</td>
<td>Row and column indices of the i’th bicluster.</td>
</tr>
<tr>
<td>get_params(deep)</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>get_shape(i)</td>
<td>Shape of the i’th bicluster.</td>
</tr>
<tr>
<td>get_submatrix(i, data)</td>
<td>Returns the submatrix corresponding to bicluster i.</td>
</tr>
<tr>
<td>set_params(params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (n_clusters=3, method='bistochastic', n_components=6, n_best=3,
svd_method='randomized', n_svd_vecs=None, mini_batch=False, init='k-means++',
n_init=10, n_jobs=1, random_state=None)

biclusters_
Convenient way to get row and column indicators together.

Returns the rows_ and columns_ members.

fit (X)
Creates a biclustering for X.

get_indices (i)
Row and column indices of the i’th bicluster.

Only works if rows_ and columns_ attributes exist.

Returns
row_ind : np.array, dtype=np.intp
Indices of rows in the dataset that belong to the bicluster.

col_ind : np.array, dtype=np.intp
Indices of columns in the dataset that belong to the bicluster.

get_params (deep=True)
Get parameters for this estimator.

Parameters
deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.
Returns : mapping of string to any
Parameter names mapped to their values.

**get_shape** *(i)*
Shape of the i’th bicluster.

**Returnsshape** : (int, int)
Number of rows and columns (resp.) in the bicluster.

**get_submatrix** *(i, data)*
Returns the submatrix corresponding to bicluster i.
Works with sparse matrices. Only works if `rows_` and `columns_` attributes exist.

**set_params** (**params**)
Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returnsself** :

**sklearn.cluster.bicluster.SpectralCoclustering**

class **sklearn.cluster.bicluster.SpectralCoclustering** *(n_clusters=3, svd_method='randomized', n_svd_vecs=None, mini_batch=False, init='k-means++', n_init=10, n_jobs=1, random_state=None)*

Spectral Co-Clustering algorithm (Dhillon, 2001).
Clusters rows and columns of an array `X` to solve the relaxed normalized cut of the bipartite graph created from `X` as follows: the edge between row vertex `i` and column vertex `j` has weight `X[i, j]`.
The resulting bicluster structure is block-diagonal, since each row and each column belongs to exactly one bicluster.
Supports sparse matrices, as long as they are nonnegative.
Read more in the *User Guide*.

**Parameters**

- **n_clusters** : integer, optional, default: 3
  The number of biclusters to find.
- **svd_method** : string, optional, default: ‘randomized’
  Selects the algorithm for finding singular vectors. May be ‘randomized’ or ‘arpack’. If ‘randomized’, use `sklearn.utils.extmath.randomized_svd`, which may be faster for large matrices. If ‘arpack’, use `sklearn.utils.arpack.svds`, which is more accurate, but possibly slower in some cases.
- **n_svd_vecs** : int, optional, default: None
  Number of vectors to use in calculating the SVD. Corresponds to `ncv` when `svd_method=arpack` and `n_oversamples` when `svd_method` is ‘randomized’.
- **mini_batch** : bool, optional, default: False
  Whether to use mini-batch k-means, which is faster but may get different results.
init : {'k-means++', 'random' or an ndarray}
Method for initialization of k-means algorithm; defaults to 'k-means++'.

n_init : int, optional, default: 10
Number of random initializations that are tried with the k-means algorithm.
If mini-batch k-means is used, the best initialization is chosen and the algorithm runs once. Otherwise, the algorithm is run for each initialization and the best solution chosen.

n_jobs : int, optional, default: 1
The number of jobs to use for the computation. This works by breaking down the
pairwise matrix into n_jobs even slices and computing them in parallel.
If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which
is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for
n_jobs = -2, all CPUs but one are used.

random_state : int seed, RandomState instance, or None (default)
A pseudo random number generator used by the K-Means initialization.

Attributes
rows_ : array-like, shape (n_row_clusters, n_rows)
Results of the clustering. rows[i, r] is True if cluster i contains row r. Available only
after calling fit.
columns_ : array-like, shape (n_column_clusters, n_columns)
Results of the clustering, like rows.
row_labels_ : array-like, shape (n_rows,)
The bicluster label of each row.
column_labels_ : array-like, shape (n_cols,)
The bicluster label of each column.

References


Methods

fit(X) Creates a biclustering for X.
get_indices(i) Row and column indices of the i’th bicluster.
get_params([deep]) Get parameters for this estimator.
get_shape(i) Shape of the i’th bicluster.
get_submatrix(i, data) Returns the submatrix corresponding to bicluster i.
set_params(**params) Set the parameters of this estimator.

__init__(n_clusters=3, svd_method='randomized', n_svd_vecs=None, mini_batch=False, init='k-means++', n_init=10, n_jobs=1, random_state=None)
biclusters_
Convenient way to get row and column indicators together.
Returns the rows_ and columns_ members.

**fit**(*X*)

Create a biclustering for *X*.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)

**get_indices**(*i*)

Row and column indices of the *i*’th bicluster.

Only works if rows_ and columns_ attributes exist.

- **Returns**
  - **row_ind**: np.array, dtype=np.intp
    - Indices of rows in the dataset that belong to the bicluster.
  - **col_ind**: np.array, dtype=np.intp
    - Indices of columns in the dataset that belong to the bicluster.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**
  - **params**: mapping of string to any
    - Parameter names mapped to their values.

**get_shape**(*i*)

Shape of the *i*’th bicluster.

- **Returns**
  - **shape**: (int, int)
    - Number of rows and columns (resp.) in the bicluster.

**get_submatrix**(*i, data*)

Returns the submatrix corresponding to bicluster *i*.

Works with sparse matrices. Only works if rows_ and columns_ attributes exist.

**set_params**(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

- **Returns**
  - **self**:

### 29.4 sklearn.covariance: Covariance Estimators

The `sklearn.covariance` module includes methods and algorithms to robustly estimate the covariance of features given a set of points. The precision matrix defined as the inverse of the covariance is also estimated. Covariance estimation is closely related to the theory of Gaussian Graphical Models.

**User guide**: See the Covariance estimation section for further details.

| covariance.EmpiricalCovariance(...), maximum likelihood covariance estimator | Continued on next page |

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### 29.4.1 sklearn.covariance.EmpiricalCovariance

**class** `sklearn.covariance.EmpiricalCovariance` *(store_precision=True, assume_centered=False)*

Maximum likelihood covariance estimator

Read more in the *User Guide*.

**Parameters**

- `store_precision` : bool
  Specifies if the estimated precision is stored.
  
- `assume_centered` : bool
  If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False (default), data are centered before computation.

**Attributes**

- `covariance_` : 2D ndarray, shape (n_features, n_features)
  Estimated covariance matrix

- `precision_` : 2D ndarray, shape (n_features, n_features)
  Estimated pseudo-inverse matrix. (stored only if store_precision is True)

**Methods**

- `error_norm(comp_cov[, norm, scaling, squared])`
  Computes the Mean Squared Error between two covariance estimators.

- `fit(X[, y])`
  Fits the Maximum Likelihood Estimator covariance model according to the given training data and parameters.

- `get_params([deep])`
  Get parameters for this estimator.

- `get_precision()`
  Getter for the precision matrix.

- `mahalanobis(observations)`
  Computes the squared Mahalanobis distances of given observations.

- `score(X_test[, y])`
  Computes the log-likelihood of a Gaussian data set with `self.covariance_` as an estimator of its covariance matrix.

- `set_params(**params)`
  Set the parameters of this estimator.
__init__ (store_precision=True, assume_centered=False)

error_norm (comp_cov, norm='frobenius', scaling=True, squared=True)
Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

Parameters comp_cov : array-like, shape = [n_features, n_features]
The covariance to compare with.

norm : str
The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov - self.covariance_).

scaling : bool
If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

squared : bool
Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns The Mean Squared Error (in the sense of the Frobenius norm) between:
’self’ and ‘comp_cov’ covariance estimators.

fit (X, y=None)
Fits the Maximum Likelihood Estimator covariance model according to the given training data and parameters.

Parameters X : array-like, shape = [n_samples, n_features]
Training data, where n_samples is the number of samples and n_features is the number of features.

y : not used, present for API consistence purpose.

Returns self : object
Returns self.

get_params (deep=True)
Get parameters for this estimator.

Parameters deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns params : mapping of string to any
Parameter names mapped to their values.

get_precision ()
Getter for the precision matrix.

Returns precision_ : array-like,
The precision matrix associated to the current covariance object.

mahalanobis (observations)
Computes the squared Mahalanobis distances of given observations.
**Parameters**

- **observations**: array-like, shape = [n_observations, n_features]
  
  The observations, the Mahalanobis distances of the which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

**Returns**

- **mahalanobis_distance**: array, shape = [n_observations,]
  
  Squared Mahalanobis distances of the observations.

**score**(X_test, y=None)

Computes the log-likelihood of a Gaussian data set with `self.covariance_` as an estimator of its covariance matrix.

- **Parameters**
  - **X_test**: array-like, shape = [n_samples, n_features]
    
    Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features. X_test is assumed to be drawn from the same distribution than the data used in fit (including centering).
  
  - **y**: not used, present for API consistence purpose.

**Returns**

- **res**: float
  
  The likelihood of the data set with `self.covariance_` as an estimator of its covariance matrix.

**set_params**(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**: 

---

**Examples using sklearn.covariance.EmpiricalCovariance**

- Robust covariance estimation and Mahalanobis distances relevance

- Robust vs Empirical covariance estimate

---

### 29.4.2 sklearn.covariance.EllipticEnvelope

class sklearn.covariance.EllipticEnvelope(store_precision=True, assume_centered=False, support_fraction=None, contamination=0.1, random_state=None)

An object for detecting outliers in a Gaussian distributed dataset.

Read more in the User Guide.

- **Parameters**
  - **store_precision**: bool
    
    Specify if the estimated precision is stored.

  - **assume_centered**: Boolean
    
    If True, the support of robust location and covariance estimates is computed, and a covariance estimate is recomputed from it, without centering the data. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, the robust location and covariance are directly computed with the FastMCD algorithm without additional treatment.
support_fraction : float, 0 < support_fraction < 1
The proportion of points to be included in the support of the raw MCD estimate. Default is None, which implies that the minimum value of support_fraction will be used within the algorithm: \([n\_sample + n\_features + 1] / 2\).

contamination : float, 0. < contamination < 0.5
The amount of contamination of the data set, i.e. the proportion of outliers in the data set.

Attributes

location_ : array-like, shape (n_features,)
Estimated robust location

covariance_ : array-like, shape (n_features, n_features)
Estimated robust covariance matrix

precision_ : array-like, shape (n_features, n_features)
Estimated pseudo inverse matrix. (stored only if store_precision is True)
support_ : array-like, shape (n_samples,)
A mask of the observations that have been used to compute the robust estimates of location and shape.

See also:
EmpiricalCovariance, MinCovDet

Notes
Outlier detection from covariance estimation may break or not perform well in high-dimensional settings. In particular, one will always take care to work with \(n\_samples > n\_features ** 2\).

References

Methods

correct_covariance(data) Apply a correction to raw Minimum Covariance Determinant estimates.
decision_function(X[, raw_values]) Compute the decision function of the given observations.
error_norm(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.
fit(X[, y])
get_params([deep]) Get parameters for this estimator.
get_precision() Getter for the precision matrix.
mahalanobis(observations) Computes the squared Mahalanobis distances of given observations.
predict(X) Outlyingness of observations in X according to the fitted model.
reweight_covariance(data) Re-weight raw Minimum Covariance Determinant estimates.

Continued on next page
Table 29.24 – continued from previous page

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__(store_precision=True, assume_centered=False, support_fraction=None, contamination=0.1, random_state=None)

*correct_covariance(data)*

Apply a correction to raw Minimum Covariance Determinant estimates.

Correction using the empirical correction factor suggested by Rousseeuw and Van Driessen in [Rousseeuw1984].

Parameters:
- **data**: array-like, shape (n_samples, n_features)
  - The data matrix, with p features and n samples. The data set must be the one which was used to compute the raw estimates.

Returns:
- **covariance_corrected**: array-like, shape (n_features, n_features)
  - Corrected robust covariance estimate.

*decision_function(X, raw_values=False)*

Compute the decision function of the given observations.

Parameters:
- **X**: array-like, shape (n_samples, n_features)
- **raw_values**: bool
  - Whether or not to consider raw Mahalanobis distances as the decision function. Must be False (default) for compatibility with the others outlier detection tools.

Returns:
- **decision**: array-like, shape (n_samples, )
  - The values of the decision function for each observations. It is equal to the Mahalanobis distances if raw_values is True. By default (raw_values=True), it is equal to the cubic root of the shifted Mahalanobis distances. In that case, the threshold for being an outlier is 0, which ensures a compatibility with other outlier detection tools such as the One-Class SVM.

*error_norm(comp_cov, norm='frobenius', scaling=True, squared=True)*

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

Parameters:
- **comp_cov**: array-like, shape = [n_features, n_features]
  - The covariance to compare with.
- **norm**: str
  - The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A))) where A is the error (comp_cov -self.covariance_).
- **scaling**: bool
  - If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.
- **squared**: bool
  - Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.
Returns The Mean Squared Error (in the sense of the Frobenius norm) between:

'self' and 'comp_cov' covariance estimators.

get_params (deep=True)
Get parameters for this estimator.

Parameters deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are
    estimators.

Returns params : mapping of string to any
    Parameter names mapped to their values.

get_precision ()
Getter for the precision matrix.

Returns precision_ : array-like,
    The precision matrix associated to the current covariance object.

mahalanobis (observations)
Computes the squared Mahalanobis distances of given observations.

Parameters observations : array-like, shape = [n_observations, n_features]
    The observations, the Mahalanobis distances of the which we compute. Observations
    are assumed to be drawn from the same distribution than the data used in fit.

Returns mahalanobis_distance : array, shape = [n_observations,]
    Squared Mahalanobis distances of the observations.

predict (X)
Outlyingness of observations in X according to the fitted model.

Parameters X : array-like, shape = (n_samples, n_features)

Returns is_outliers : array, shape = (n_samples, ), dtype = bool
    For each observations, tells whether or not it should be considered as an outlier accord-
    ing to the fitted model.

threshold : float,
    The values of the less outlying point’s decision function.

reweight_covariance (data)
Re-weight raw Minimum Covariance Determinant estimates.

Re-weight observations using Rousseeuw’s method (equivalent to deleting outlying observations from the
data set before computing location and covariance estimates). [Rousseeuw1984]

Parameters data : array-like, shape (n_samples, n_features)
    The data matrix, with p features and n samples. The data set must be the one which was
    used to compute the raw estimates.

Returns location_reweighted : array-like, shape (n_features, )
    Re-weighted robust location estimate.

covariance_reweighted : array-like, shape (n_features, n_features)
    Re-weighted robust covariance estimate.
**support_reweighted**: array-like, type boolean, shape (n_samples,)

A mask of the observations that have been used to compute the re-weighted robust location and covariance estimates.

**score**(X, y, sample_weight=None)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Return**

- **score**: float
  Mean accuracy of self.predict(X) wrt. y.

**set_params**(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Return**

- **self**:

Examples using sklearn.covariance.EllipticEnvelope

- Outlier detection on a real data set
- Outlier detection with several methods.

**29.4.3 sklearn.covariance.GraphLasso**

class sklearn.covariance.GraphLasso(alpha=0.01, mode='cd', tol=0.0001, enet_tol=0.0001, max_iter=100, verbose=False, assume_centered=False)

Sparse inverse covariance estimation with an l1-penalized estimator.

Read more in the User Guide.

- **alpha**: positive float, default 0.01
  The regularization parameter: the higher alpha, the more regularization, the sparser the inverse covariance.

- **mode**: {'cd', 'lars'}, default 'cd'
  The Lasso solver to use: coordinate descent or LARS. Use LARS for very sparse underlying graphs, where p > n. Elsewhere prefer cd which is more numerically stable.

- **tol**: positive float, default 1e-4
  The tolerance to declare convergence: if the dual gap goes below this value, iterations are stopped.
enet_tol : positive float, optional

The tolerance for the elastic net solver used to calculate the descent direction. This parameter controls the accuracy of the search direction for a given column update, not of the overall parameter estimate. Only used for mode='cd'.

max_iter : integer, default 100

The maximum number of iterations.

verbose : boolean, default False

If verbose is True, the objective function and dual gap are plotted at each iteration.

assume_centered : boolean, default False

If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

Attributes

covariance_ : array-like, shape (n_features, n_features)

Estimated covariance matrix

precision_ : array-like, shape (n_features, n_features)

Estimated pseudo inverse matrix.

n_iter_ : int

Number of iterations run.

See also:

graph_lasso, GraphLassoCV

Methods

error_norm(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.

fit(X[, y])

get_params([deep]) Get parameters for this estimator.

get_precision() Getter for the precision matrix.

mahalanobis(observations) Computes the squared Mahalanobis distances of given observations.

score(X_test[, y]) Computes the log-likelihood of a Gaussian data set with self.covariance_ as an estimator of its covariance matrix.

set_params(**params) Set the parameters of this estimator.

__init__(alpha=0.01, mode='cd', tol=0.0001, enet_tol=0.0001, max_iter=100, verbose=False, assume_centered=False)

error_norm(comp_cov, norm='frobenius', scaling=True, squared=True) Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

Parameters

comp_cov : array-like, shape = [n_features, n_features]

The covariance to compare with.
norm : str
   The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov -self.covariance_).

scaling : bool
   If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

squared : bool
   Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns: The Mean Squared Error (in the sense of the Frobenius norm) between:

'self' and 'comp_cov' covariance estimators.

get_params (deep=True)
   Get parameters for this estimator.

Parameters:
depth : boolean, optional
   If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
params : mapping of string to any
   Parameter names mapped to their values.

get_precision()
   Getter for the precision matrix.

Returns:
precision_ : array-like,
   The precision matrix associated to the current covariance object.

mahalanobis(observations)
   Computes the squared Mahalanobis distances of given observations.

Parameters:
observations : array-like, shape = [n_observations, n_features]
   The observations, the Mahalanobis distances of the which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

Returns:
mahalanobis_distance : array, shape = [n_observations,
   Squared Mahalanobis distances of the observations.

score(X_test, y=None)
   Computes the log-likelihood of a Gaussian data set with self.covariance_ as an estimator of its covariance matrix.

Parameters:
X_test : array-like, shape = [n_samples, n_features]
   Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features. X_test is assumed to be drawn from the same distribution than the data used in fit (including centering).

y : not used, present for API consistence purpose.

Returns:
res : float
   The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.
**set_params(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

self

29.4.4 sklearn.covariance.GraphLassoCV

class sklearn.covariance.GraphLassoCV(alphas=4, n_refinements=4, cv=None, tol=0.0001, enet_tol=0.0001, max_iter=100, mode='cd', n_jobs=1, verbose=False, assume_centered=False)

Sparse inverse covariance w/ cross-validated choice of the 11 penalty

Read more in the User Guide.

Parameters:

alphas : integer, or list positive float, optional

If an integer is given, it fixes the number of points on the grids of alpha to be used. If a list is given, it gives the grid to be used. See the notes in the class docstring for more details.

n_refinements: strictly positive integer :

The number of times the grid is refined. Not used if explicit values of alphas are passed.

cv : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the default 3-fold cross-validation,
- integer, to specify the number of folds,
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.

For integer/None inputs KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

tol : positive float, optional

The tolerance to declare convergence: if the dual gap goes below this value, iterations are stopped.

enet_tol : positive float, optional

The tolerance for the elastic net solver used to calculate the descent direction. This parameter controls the accuracy of the search direction for a given column update, not of the overall parameter estimate. Only used for mode='cd'.

max_iter : integer, optional

Maximum number of iterations.

mode: {'cd', 'lars'} :

The Lasso solver to use: coordinate descent or LARS. Use LARS for very sparse underlying graphs, where number of features is greater than number of samples. Elsewhere prefer cd which is more numerically stable.
n_jobs : int, optional
   number of jobs to run in parallel (default 1).

verbose : boolean, optional
   If verbose is True, the objective function and duality gap are printed at each iteration.

assume_centered : Boolean
   If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

Attributes

scovariance_ : numpy.ndarray, shape (n_features, n_features)
   Estimated covariance matrix.

precision_ : numpy.ndarray, shape (n_features, n_features)
   Estimated precision matrix (inverse covariance).

alpha_ : float
   Penalization parameter selected.

cv_alphas_ : list of float
   All penalization parameters explored.

‘grid_scores’ : 2D numpy.ndarray (n_alphas, n_folds)
   Log-likelihood score on left-out data across folds.

n_iter_ : int
   Number of iterations run for the optimal alpha.

See also:

graph_lasso, GraphLasso

Notes

The search for the optimal penalization parameter (alpha) is done on an iteratively refined grid: first the cross-validated scores on a grid are computed, then a new refined grid is centered around the maximum, and so on. One of the challenges which is faced here is that the solvers can fail to converge to a well-conditioned estimate. The corresponding values of alpha then come out as missing values, but the optimum may be close to these missing values.

Methods

t_error_norm(comp_cov[, norm, scaling, squared])
   Computes the Mean Squared Error between two covariance estimators.

fit(X[, y])
   Fits the GraphLasso covariance model to X.

get_params([deep])
   Get parameters for this estimator.

get_precision()
   Getter for the precision matrix.

mahalanobis(observations)
   Computes the squared Mahalanobis distances of given observations.

Continued on next page

29.4. sklearn.covariance: Covariance Estimators
**Table 29.26 – continued from previous page**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>score(X_test[, y])</code></td>
<td>Computes the log-likelihood of a Gaussian data set with <code>self.covariance_</code> as an estimator of its covariance matrix.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(alphas=4, n_refinements=4, cv=None, enet_tol=0.0001, max_iter=100, mode='cd', n_jobs=1, verbose=False, assume_centered=False)
```

**error_norm**(comp_cov, norm='frobenius', scaling=True, squared=True)

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

- **Parameters**
  - `comp_cov` : array-like, shape = [n_features, n_features]
  - `norm` : str
    - The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): `sqrt(tr(A^t.A))` - ‘spectral’: `sqrt(max(eigenvalues(A^t.A))` where A is the error (`comp_cov - self.covariance_`).
  - `scaling` : bool
    - If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.
  - `squared` : bool
    - Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

**Returns**
The Mean Squared Error (in the sense of the Frobenius norm) between:

- ‘self’ and ‘comp_cov’ covariance estimators.

```python
fit(X, y=None)
```

Fits the GraphLasso covariance model to X.

- **Parameters**
  - `X` : ndarray, shape (n_samples, n_features)
    - Data from which to compute the covariance estimate

```python
get_params(deep=True)
```

Get parameters for this estimator.

- **Parameters**
  - `deep` : boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- `params` : mapping of string to any
  - Parameter names mapped to their values.

```python
get_precision()
```

Getter for the precision matrix.

**Returns**

- `precision_` : array-like,
  - The precision matrix associated to the current covariance object.

```python
mahalanobis(observations)
```

Computes the squared Mahalanobis distances of given observations.
Parameters

**observations** : array-like, shape = [n_observations, n_features]

The observations, the Mahalanobis distances of which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

Returns

**mahalanobis_distance** : array, shape = [n_observations,]

Squared Mahalanobis distances of the observations.

**score** *(X_test, y=None)*

Computes the log-likelihood of a Gaussian data set with *self.covariance_* as an estimator of its covariance matrix.

Parameters

**X_test** : array-like, shape = [n_samples, n_features]

Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features. X_test is assumed to be drawn from the same distribution than the data used in fit (including centering).

y : not used, present for API consistence purpose.

Returns

**res** : float

The likelihood of the data set with *self.covariance_* as an estimator of its covariance matrix.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns

**self** :

Examples using `sklearn.covariance.GraphLassoCV`

- Visualizing the stock market structure
- Sparse inverse covariance estimation

### 29.4.5 **sklearn.covariance.LedoitWolf**

**class** `sklearn.covariance.LedoitWolf` *(store_precision=True, assume_centered=False, block_size=1000)*

LedoitWolf Estimator


Read more in the User Guide.

**Parameters**

**store_precision** : bool, default=True

Specify if the estimated precision is stored.

**assume_centered** : bool, default=False

If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False (default), data are centered before computation.
block_size : int, default=1000

Size of the blocks into which the covariance matrix will be split during itsLedoit-Wolf estimation. This is purely a memory optimization and does not affect results.

Attributes

covariance_ : array-like, shape (n_features, n_features)

Estimated covariance matrix

precision_ : array-like, shape (n_features, n_features)

Estimated pseudo inverse matrix. (stored only if store_precision is True)

shrinkage_ : float, 0 <= shrinkage <= 1

Coefficient in the convex combination used for the computation of the shrunk estimate.

Notes

The regularised covariance is:

\[(1 - \text{shrinkage}) \cdot \text{cov} + \text{shrinkage} \cdot \mu \cdot \text{np.identity(n_features)}\]

where \(\mu = \text{trace(cov) / n_features}\) and shrinkage is given by the Ledoit and Wolf formula (see References)

References


Methods

error_norm(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.

fit(X[, y]) Fits the Ledoit-Wolf shrunk covariance model according to the given training data and parameters.

get_params([deep]) Get parameters for this estimator.

get_precision() Getter for the precision matrix.

mahalanobis(observations) Computes the squared Mahalanobis distances of given observations.

score(X_test[, y]) Computes the log-likelihood of a Gaussian data set with self.covariance_ as an estimator of its covariance matrix.

set_params(**params) Set the parameters of this estimator.

__init__(store_precision=True, assume_centered=False, block_size=1000)

eroerror_norm(comp_cov, norm=’frobenius’, scaling=True, squared=True)

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

Parameters

comp_cov : array-like, shape = [n_features, n_features]

The covariance to compare with.
norm : str
The type of norm used to compute the error. Available error types:
- ‘frobenius’ (default): sqrt(tr(A^t.A))
- ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov - self.covariance_).

scaling : bool
If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

squared : bool
Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns:
The Mean Squared Error (in the sense of the Frobenius norm) between:

'self' and 'comp_cov' covariance estimators.

fit(X, y=None)
Fits the Ledoit-Wolf shrunk covariance model according to the given training data and parameters.

Parameters:
X : array-like, shape = [n_samples, n_features]
Training data, where n_samples is the number of samples and n_features is the number of features.

y : not used, present for API consistence purpose.

Returns:
self : object
Returns self.

get_params(deep=True)
Get parameters for this estimator.

Parameters:
deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
params : mapping of string to any
Parameter names mapped to their values.

get_precision()
Getter for the precision matrix.

Returns:
precision_ : array-like,
The precision matrix associated to the current covariance object.

mahalanobis(observations)
Computes the squared Mahalanobis distances of given observations.

Parameters:
observations : array-like, shape = [n_observations, n_features]
The observations, the Mahalanobis distances of which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

Returns:
mahalanobis_distance : array, shape = [n_observations,
Squared Mahalanobis distances of the observations.
score(X_test, y=None)

Computes the log-likelihood of a Gaussian data set with self.covariance_ as an estimator of its covariance matrix.

Parameters:
X_test : array-like, shape = [n_samples, n_features]
    Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features. X_test is assumed to be drawn from the same distribution than the data used in fit (including centering).

y : not used, present for API consistence purpose.

Returns:
res : float
    The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.

set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
sself :

Examples using sklearn.covariance.LedoitWolf

- Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood
- Ledoit-Wolf vs OAS estimation
- Model selection with Probabilistic PCA and Factor Analysis (FA)

29.4.6 sklearn.covariance.MinCovDet

class sklearn.covariance.MinCovDet (store_precision=True, assume_centered=False, support_fraction=None, random_state=None)

Minimum Covariance Determinant (MCD): robust estimator of covariance.

The Minimum Covariance Determinant covariance estimator is to be applied on Gaussian-distributed data, but could still be relevant on data drawn from a unimodal, symmetric distribution. It is not meant to be used with multi-modal data (the algorithm used to fit a MinCovDet object is likely to fail in such a case). One should consider projection pursuit methods to deal with multi-modal datasets.

Read more in the User Guide.

Parameters:
store_precision : bool
    Specify if the estimated precision is stored.
assume_centered : Boolean
    If True, the support of the robust location and the covariance estimates is computed, and a covariance estimate is recomputed from it, without centering the data. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, the robust location and covariance are directly computed with the FastMCD algorithm without additional treatment.

support_fraction : float, 0 < support_fraction < 1
The proportion of points to be included in the support of the raw MCD estimate. Default is None, which implies that the minimum value of support_fraction will be used within the algorithm: \([n_{\text{sample}} + n_{\text{features}} + 1] / 2\)

**random_state**: integer or numpy.RandomState, optional

The random generator used. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**Attributes**

- **raw_location_**: array-like, shape (n_features,)
  
  The raw robust estimated location before correction and re-weighting.

- **raw_covariance_**: array-like, shape (n_features, n_features)
  
  The raw robust estimated covariance before correction and re-weighting.

- **raw_support_**: array-like, shape (n_samples,)

  A mask of the observations that have been used to compute the raw robust estimates of location and shape, before correction and re-weighting.

- **location_**: array-like, shape (n_features,)

  Estimated robust location

- **covariance_**: array-like, shape (n_features, n_features)

  Estimated robust covariance matrix

- **precision_**: array-like, shape (n_features, n_features)

  Estimated pseudo inverse matrix. (stored only if store_precision is True)

- **support_**: array-like, shape (n_samples,)

  A mask of the observations that have been used to compute the robust estimates of location and shape.

- **dist_**: array-like, shape (n_samples,)

  Mahalanobis distances of the training set (on which \texttt{fit} is called) observations.

**References**

[Rousseeuw1984], [Rousseeuw1999], [Butler1993]

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>correct_covariance</strong>(data)</td>
<td>Apply a correction to raw Minimum Covariance Determinant estimates.</td>
</tr>
<tr>
<td><strong>error_norm</strong>(comp_cov[, norm, scaling, squared])</td>
<td>Computes the Mean Squared Error between two covariance estimators.</td>
</tr>
<tr>
<td><strong>fit</strong>(X[, y])</td>
<td>Fits a Minimum Covariance Determinant with the FastMCD algorithm.</td>
</tr>
<tr>
<td><strong>get_params</strong>([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><strong>get_precision</strong>( )</td>
<td>Getter for the precision matrix.</td>
</tr>
<tr>
<td><strong>mahalanobis</strong>(observations)</td>
<td>Computes the squared Mahalanobis distances of given observations.</td>
</tr>
</tbody>
</table>

Continued on next page
**Table 29.28 – continued from previous page**

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>reweight_covariance(data)</code></td>
<td>Re-weight raw Minimum Covariance Determinant estimates.</td>
</tr>
<tr>
<td><code>score(X_test[, y])</code></td>
<td>Computes the log-likelihood of a Gaussian data set with <code>self.covariance_</code> as an estimator of its covariance matrix.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__ (store_precision=True, assume_centered=False, support_fraction=None, random_state=None)**

Apply a correction to raw Minimum Covariance Determinant estimates.

Correction using the empirical correction factor suggested by Rousseeuw and Van Driessen in [Rousseeuw1984].

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>data</code></td>
<td>array-like, shape (n_samples, n_features)</td>
</tr>
<tr>
<td></td>
<td>The data matrix, with p features and n samples. The data set must be the one which was used to compute the raw estimates.</td>
</tr>
</tbody>
</table>

**Returns**

<table>
<thead>
<tr>
<th>Return</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>covariance_corrected</code></td>
<td>array-like, shape (n_features, n_features)</td>
</tr>
<tr>
<td></td>
<td>Corrected robust covariance estimate.</td>
</tr>
</tbody>
</table>

**error_norm (comp_cov, norm='frobenius', scaling=True, squared=True)**

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>comp_cov</code></td>
<td>array-like, shape = [n_features, n_features]</td>
</tr>
<tr>
<td></td>
<td>The covariance to compare with.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>norm</code></td>
<td>str</td>
</tr>
<tr>
<td></td>
<td>The type of norm used to compute the error. Available error types: - 'frobenius' (default): sqrt(tr(A^t.A)) - 'spectral': sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov -self.covariance_).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>scaling</code></td>
<td>bool</td>
</tr>
<tr>
<td></td>
<td>If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>squared</code></td>
<td>bool</td>
</tr>
<tr>
<td></td>
<td>Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.</td>
</tr>
</tbody>
</table>

**Returns**

<table>
<thead>
<tr>
<th>Return</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The Mean Squared Error (in the sense of the Frobenius norm) between:</td>
</tr>
<tr>
<td></td>
<td><em>'self'</em> and <em>'comp_cov'</em> covariance estimators.</td>
</tr>
</tbody>
</table>

**fit (X, y=None)**

Fits a Minimum Covariance Determinant with the FastMCD algorithm.

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>X</code></td>
<td>array-like, shape = [n_samples, n_features]</td>
</tr>
<tr>
<td></td>
<td>Training data, where n_samples is the number of samples and n_features is the number of features.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>y</code></td>
<td>not used, present for API consistence purpose.</td>
</tr>
</tbody>
</table>
Returns `self`.

**`get_params`**

Get parameters for this estimator.

**Parameters**

- `deep`: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- `params`: mapping of string to any
  
  Parameter names mapped to their values.

**`get_precision`**

Getter for the precision matrix.

**Returns**

- `precision_`: array-like, shape (n_features, n_features)
  
  The precision matrix associated to the current covariance object.

**`mahalanobis`**

Computes the squared Mahalanobis distances of given observations.

**Parameters**

- `observations`: array-like, shape = [n_observations, n_features]
  
  The observations, the Mahalanobis distances of the which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

**Returns**

- `mahalanobis_distance`: array, shape = [n_observations,]
  
  Squared Mahalanobis distances of the observations.

**`reweight_covariance`**

Re-weight raw Minimum Covariance Determinant estimates.

Re-weight observations using Rousseeuw’s method (equivalent to deleting outlying observations from the data set before computing location and covariance estimates). [Rouseeuw1984]

**Parameters**

- `data`: array-like, shape (n_samples, n_features)
  
  The data matrix, with p features and n samples. The data set must be the one which was used to compute the raw estimates.

**Returns**

- `location_reweighted`: array-like, shape (n_features, )
  
  Re-weighted robust location estimate.

- `covariance_reweighted`: array-like, shape (n_features, n_features)
  
  Re-weighted robust covariance estimate.

- `support_reweighted`: array-like, type boolean, shape (n_samples,)
  
  A mask of the observations that have been used to compute the re-weighted robust location and covariance estimates.

**`score`**

Computes the log-likelihood of a Gaussian data set with `self.covariance_` as an estimator of its covariance matrix.

**Parameters**

- `X_test`: array-like, shape = [n_samples, n_features]
Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features. X_test is assumed to be drawn from the same distribution than the data used in fit (including centering).

**Returns** : float

The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.

```python
set_params(**params)
```

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns** : self

**Examples using sklearn.covariance.MinCovDet**

- Robust covariance estimation and Mahalanobis distances relevance
- Robust vs Empirical covariance estimate

**29.4.7 sklearn.covariance.OAS**

```python
class sklearn.covariance.OAS (store_precision=True, assume_centered=False)
```

Oracle Approximating Shrinkage Estimator

Read more in the [User Guide](#).


The formula used here does not correspond to the one given in the article. It has been taken from the Matlab program available from the authors’ webpage (http://tbayes.eecs.umich.edu/yilun/covestimation). In the original article, formula (23) states that $2/p$ is multiplied by $\text{Trace}(\text{cov}^*\text{cov})$ in both the numerator and denominator, this operation is omitted in the author’s MATLAB program because for a large $p$, the value of $2/p$ is so small that it doesn’t affect the value of the estimator.

**Parameters**

- **store_precision** : bool, default=True
  Specify if the estimated precision is stored.

- **assume_centered** : bool, default=False
  If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False (default), data are centered before computation.

**Attributes**

- **covariance_** : array-like, shape (n_features, n_features)
  Estimated covariance matrix.

- **precision_** : array-like, shape (n_features, n_features)
  Estimated pseudo inverse matrix. (stored only if store_precision is True)

- **shrinkage_** : float, 0 <= shrinkage <= 1
coefficient in the convex combination used for the computation of the shrunk estimate.

Notes

The regularised covariance is:

```
(1 - shrinkage)*cov
    + shrinkage*mu*np.identity(n_features)
```

where \( \mu = \text{trace}({\text{cov}}) / n_{\text{features}} \) and shrinkage is given by the OAS formula (see References)

References


Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>error_norm</td>
<td>Computes the Mean Squared Error between two covariance estimators.</td>
</tr>
<tr>
<td>fit(X[, y])</td>
<td>Fits the Oracle Approximating Shrinkage covariance model according to the</td>
</tr>
<tr>
<td></td>
<td>given training data and parameters.</td>
</tr>
<tr>
<td>get_params</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>get_precision</td>
<td>Getter for the precision matrix.</td>
</tr>
<tr>
<td>mahalanobis</td>
<td>Computes the squared Mahalanobis distances of given observations.</td>
</tr>
<tr>
<td>score(X_test[, y])</td>
<td>Computes the log-likelihood of a Gaussian data set with self.covariance_</td>
</tr>
<tr>
<td></td>
<td>as an estimator of its covariance matrix.</td>
</tr>
<tr>
<td>set_params</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__(store_precision=True, assume_centered=False)

error_norm(comp_cov[, norm='frobenius', scaling=True, squared=True])

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

Parameters

- **comp_cov**: array-like, shape = [n_features, n_features]
  
The covariance to compare with.

- **norm**: str
  
The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): \( \sqrt{\text{tr}(A^t.A)} \) - ‘spectral’: \( \sqrt{\text{max(eigenvalues}(A^t.A))} \) where \( A \) is the error (comp_cov - self.covariance_).

- **scaling**: bool
  
  If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

- **squared**: bool

29.4. sklearn.covariance: Covariance Estimators
Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns:
The Mean Squared Error (in the sense of the Frobenius norm) between:

\['self' and 'comp_cov' covariance estimators. :\]

fit(X, y=None)
Fits the Oracle Approximating Shrinkage covariance model according to the given training data and parameters.

Parameters:

X : array-like, shape = [n_samples, n_features]

Training data, where n_samples is the number of samples and n_features is the number of features.

y : not used, present for API consistence purpose.

Returns:

self : object

Returns self.

get_params(deep=True)
Get parameters for this estimator.

Parameters:

deep : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

params : mapping of string to any

Parameter names mapped to their values.

get_precision()
Getter for the precision matrix.

Returns:

precision_ : array-like,

The precision matrix associated to the current covariance object.

mahalanobis(observations)
Computes the squared Mahalanobis distances of given observations.

Parameters:

observations : array-like, shape = [n_observations, n_features]

The observations, the Mahalanobis distances of the which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

Returns:

mahalanobis_distance : array, shape = [n_observations,]

Squared Mahalanobis distances of the observations.

score(X_test, y=None)
Computes the log-likelihood of a Gaussian data set with self.covariance_ as an estimator of its covariance matrix.

Parameters:

X_test : array-like, shape = [n_samples, n_features]

Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features. X_test is assumed to be drawn from the same distribution than the data used in fit (including centering).

y : not used, present for API consistence purpose.

Returns:

res : float
The likelihood of the data set with `self.covariance_` as an estimator of its covariance matrix.

**set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Examples using sklearn.covariance.OAS**

- Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood
- Ledoit-Wolf vs OAS estimation

### 29.4.8 sklearn.covariance.ShrunkCovariance

**class sklearn.covariance.ShrunkCovariance**(store_precision=True, assume_centered=False, shrinkage=0.1)

Covariance estimator with shrinkage

Read more in the **User Guide**.

**Parameters**

- **store_precision** : boolean, default True
  Specify if the estimated precision is stored
- **shrinkage** : float, 0 <= shrinkage <= 1, default 0.1
  Coefficient in the convex combination used for the computation of the shrunk estimate.
- **assume_centered** : boolean, default False
  If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

**Attributes**

- **covariance_** : array-like, shape (n_features, n_features)
  Estimated covariance matrix
- **precision_** : array-like, shape (n_features, n_features)
  Estimated pseudo inverse matrix. (stored only if store_precision is True)
- ‘shrinkage‘ : float, 0 <= shrinkage <= 1
  Coefficient in the convex combination used for the computation of the shrunk estimate.

**Notes**

The regularized covariance is given by

\[(1 - shrinkage) \cdot cov\]

\*[shrinkage\*mu\*np.identity(n_features)]

where \(mu = \text{trace}(cov) / n\_features\)
Methods

### error_norm(comp_cov[, norm, scaling, squared])
Computes the Mean Squared Error between two covariance estimators.

### fit(X[, y])
Fits the shrunk covariance model according to the given training data and parameters.

### get_params([deep])
Get parameters for this estimator.

### get_precision()
Getter for the precision matrix.

### mahalanobis(observations)
Computes the squared Mahalanobis distances of given observations.

### score(X_test[, y])
Computes the log-likelihood of a Gaussian data set with `self.covariance_` as an estimator of its covariance matrix.

### set_params(**params)
Set the parameters of this estimator.

---

__init__ (store_precision=True, assume_centered=False, shrinkage=0.1)

**error_norm** (comp_cov, norm='frobenius', scaling=True, squared=True)
Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm).

- **Parameters**
  - **comp_cov**: array-like, shape = [n_features, n_features]
    - The covariance to compare with.
  - **norm**: str
    - The type of norm used to compute the error. Available error types: - 'frobenius' (default): sqrt(tr(A^t.A)) - 'spectral': sqrt(max(eigenvalues(A^t.A))) where A is the error (comp_cov - self.covariance_).
  - **scaling**: bool
    - If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.
  - **squared**: bool
    - Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

**Returns**
The Mean Squared Error (in the sense of the Frobenius norm) between:

- ‘self’ and ‘comp_cov’ covariance estimators.

**fit**(X, y=None)
Fits the shrunk covariance model according to the given training data and parameters.

- **Parameters**
  - **X**: array-like, shape = [n_samples, n_features]
    - Training data, where n_samples is the number of samples and n_features is the number of features.
  - **y**: not used, present for API consistence purpose.

**Returns**
self : object
Returns self.

**get_params**(deep=True)
Get parameters for this estimator.
Parameters
\text{deep} : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns
\text{params} : mapping of string to any
Parameter names mapped to their values.

\text{get\_precision} ()
Getter for the precision matrix.

Returns
\text{precision\_} : array-like,
The precision matrix associated to the current covariance object.

\text{mahalanobis} (\text{observations})
Computes the squared Mahalanobis distances of given observations.

Parameters
\text{observations} : array-like, shape = [n\_observations, n\_features]
The observations, the Mahalanobis distances of the which we compute. Observations are assumed to be drawn from the same distribution than the data used in fit.

Returns
\text{mahalanobis\_distance} : array, shape = [n\_observations,]
Squared Mahalanobis distances of the observations.

\text{score} (\text{X\_test}, y=\text{None})
Computes the log-likelihood of a Gaussian data set with self.covariance\_ as an estimator of its covariance matrix.

Parameters
\text{X\_test} : array-like, shape = [n\_samples, n\_features]
Test data of which we compute the likelihood, where n\_samples is the number of samples and n\_features is the number of features. X\_test is assumed to be drawn from the same distribution than the data used in fit (including centering).

\text{y} : not used, present for API consistence purpose.

Returns
\text{res} : float
The likelihood of the data set with self.covariance\_ as an estimator of its covariance matrix.

\text{set\_params} (**\text{params})
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>\_<parameter> so that it’s possible to update each component of a nested object.

Returns
self :

\text{Examples using sklearn.covariance.ShrunkCovariance}

• \text{Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood}
• \text{Model selection with Probabilistic PCA and Factor Analysis (FA)}

\text{covariance.empirical\_covariance}(\text{X}[...,]) Computes the Maximum likelihood covariance estimator
\text{covariance.ledoit\_wolf}(\text{X}, \text{assume\_centered},) Estimates the shrunk Ledoit-Wolf covariance matrix.
Table 29.31 – continued from previous page

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>covariance.shrunk_covariance</code></td>
<td>Calculates a covariance matrix shrunk on the diagonal</td>
</tr>
<tr>
<td><code>covariance.oas</code></td>
<td>Estimate covariance with the Oracle Approximating Shrinkage algorithm.</td>
</tr>
<tr>
<td><code>covariance.graph_lasso</code></td>
<td>L1-penalized covariance estimator</td>
</tr>
</tbody>
</table>

### 29.4.9 sklearn.covariance.empirical_covariance

#### sklearn.covariance.empirical_covariance

Computes the Maximum likelihood covariance estimator

**Parameters**

- `X` : ndarray, shape (n_samples, n_features)
  - Data from which to compute the covariance estimate

- `assume_centered` : Boolean
  - If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

**Returns**

- `covariance` : 2D ndarray, shape (n_features, n_features)
  - Empirical covariance (Maximum Likelihood Estimator).

#### Examples using sklearn.covariance.empirical_covariance

- Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood

### 29.4.10 sklearn.covariance.ledoit_wolf

#### sklearn.covariance.ledoit_wolf

Estimates the shrunk Ledoit-Wolf covariance matrix.

Read more in the **User Guide**.

**Parameters**

- `X` : array-like, shape (n_samples, n_features)
  - Data from which to compute the covariance estimate

- `assume_centered` : boolean, default=False
  - If True, data are not centered before computation. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

- `block_size` : int, default=1000
  - Size of the blocks into which the covariance matrix will be split. This is purely a memory optimization and does not affect results.

**Returns**

- `shrunken_cov` : array-like, shape (n_features, n_features)
  - Shrunken covariance.

- `shrinkage` : float
  - Coefficient in the convex combination used for the computation of the shrunk estimate.
Notes

The regularized (shrunk) covariance is:

$$(1 - \text{shrinkage}) \times \text{cov}$$

$$\times \text{shrinkage} \times \mu \times \text{n.p.identity(n_features)}$$

where $\mu = \text{trace(cov) / n_features}$

Examples using `sklearn.covariance.ledoit_wolf`

- Sparse inverse covariance estimation

29.4.11 `sklearn.covariance.shrunk_covariance`

`sklearn.covariance.shrunk_covariance(emp_cov, shrinkage=0.1)`

Calculates a covariance matrix shrunk on the diagonal

Read more in the `User Guide`.

Parameters:

- `emp_cov`: array-like, shape (n_features, n_features)
  
  Covariance matrix to be shrunk

- `shrinkage`: float, 0 <= shrinkage <= 1
  
  Coefficient in the convex combination used for the computation of the shrunk estimate.

Returns:

- `shrunk_cov`: array-like
  
  Shrunk covariance.

Notes

The regularized (shrunk) covariance is given by

$$(1 - \text{shrinkage}) \times \text{cov}$$

$$\times \text{shrinkage} \times \text{mu} \times \text{n.p.identity(n_features)}$$

where $\mu = \text{trace(cov) / n_features}$

29.4.12 `sklearn.covariance.oas`

`sklearn.covariance.oas(X, assume_centered=False)`

Estimate covariance with the Oracle Approximating Shrinkage algorithm.

Parameters:

- `X`: array-like, shape (n_samples, n_features)
  
  Data from which to compute the covariance estimate.

- `assume_centered`: boolean
  
  If True, data are not centered before computation. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

Returns:

- `shrunk_cov`: array-like, shape (n_features, n_features)
Shrunk covariance.

\[ \text{shrinkage} : \text{float} \]

Coefficient in the convex combination used for the computation of the shrunk estimate.

**Notes**

The regularised (shrunk) covariance is:

\[(1 - \text{shrinkage}) \cdot \text{cov} - \text{shrinkage} \cdot \mu \cdot \text{np.identity(n_features)}\]

where \(\mu = \text{trace(cov)} / \text{n_features}\)

The formula we used to implement the OAS does not correspond to the one given in the article. It has been taken from the MATLAB program available from the author's webpage (http://tbayes.eecs.umich.edu/yilun/covestimation).

### 29.4.13 sklearn.covariance.graph_lasso

**sklearn.covariance.graph_lasso**

\[
\text{sklearn.covariance.graph_lasso}(\text{emp_cov}, \alpha, \text{cov_init=none}, \text{mode=’cd’}, \text{tol=0.0001}, \text{enot_tol=0.0001}, \text{max_iter=100}, \text{verbose=False}, \text{return_costs=False}, \text{return_n_iter=False})
\]

11-penalized covariance estimator

Read more in the **User Guide**.

**Parameter**

**emp_cov** : 2D ndarray, shape (n_features, n_features)

Empirical covariance from which to compute the covariance estimate.

**alpha** : positive float

The regularization parameter: the higher alpha, the more regularization, the sparser the inverse covariance.

**cov_init** : 2D array (n_features, n_features), optional

The initial guess for the covariance.

**mode** : \{‘cd’, ‘lars’\}

The Lasso solver to use: coordinate descent or LARS. Use LARS for very sparse underlying graphs, where \(p > n\). Elsewhere prefer cd which is more numerically stable.

**tol** : positive float, optional

The tolerance to declare convergence: if the dual gap goes below this value, iterations are stopped.

**enet_tol** : positive float, optional

The tolerance for the elastic net solver used to calculate the descent direction. This parameter controls the accuracy of the search direction for a given column update, not of the overall parameter estimate. Only used for mode='cd'.

**max_iter** : integer, optional

The maximum number of iterations.
**verbose**: boolean, optional

If verbose is True, the objective function and dual gap are printed at each iteration.

**return_costs**: boolean, optional

If return_costs is True, the objective function and dual gap at each iteration are returned.

**eps**: float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**return_n_iter**: bool, optional

Whether or not to return the number of iterations.

**Returns**

**covariance**: 2D ndarray, shape (n_features, n_features)

The estimated covariance matrix.

**precision**: 2D ndarray, shape (n_features, n_features)

The estimated (sparse) precision matrix.

**costs**: list of (objective, dual_gap) pairs

The list of values of the objective function and the dual gap at each iteration. Returned only if return_costs is True.

**n_iter**: int

Number of iterations. Returned only if return_n_iter is set to True.

**See also:**

*GraphLasso, GraphLassoCV*

**Notes**

The algorithm employed to solve this problem is the GLasso algorithm, from the Friedman 2008 Biostatistics paper. It is the same algorithm as in the R glasso package.

One possible difference with the glasso R package is that the diagonal coefficients are not penalized.

### 29.5 `sklearn.model_selection`: Model Selection

**User guide**: See the Cross-validation: evaluating estimator performance, Tuning the hyper-parameters of an estimator and Learning curve sections for further details.

#### 29.5.1 Splitter Classes

<table>
<thead>
<tr>
<th>Model Selection Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>model_selection.KFold([n_splits, shuffle, ...])</code></td>
<td>K-Folds cross-validator</td>
</tr>
<tr>
<td><code>model_selection.GroupKFold([n_splits])</code></td>
<td>K-fold iterator variant with non-overlapping groups.</td>
</tr>
<tr>
<td><code>model_selection.StratifiedKFold([n_splits, ...])</code></td>
<td>Stratified K-Folds cross-validator</td>
</tr>
<tr>
<td><code>model_selection.LeaveOneGroupOut()</code></td>
<td>Leave One Group Out cross-validator</td>
</tr>
</tbody>
</table>

Continued on next page
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| model_selection.LeavePGroupsOut(n_groups) | Leave P Group(s) Out cross-validator |
| model_selection.LeaveOneOut() | Leave-One-Out cross-validator |
| model_selection.LeavePOut(p) | Leave-P-Out cross-validator |
| model_selection.ShuffleSplit([n_splits, ...]) | Random permutation cross-validator |
| model_selection.GroupShuffleSplit([...]) | Shuffle-Group(s)-Out cross-validation iterator |
| model_selection.StratifiedShuffleSplit([...]) | Stratified ShuffleSplit cross-validator |
| model_selection.PredefinedSplit(test_fold) | Predefined split cross-validator |
| model_selection.TimeSeriesSplit([n_splits]) | Time Series cross-validator |

sklearn.model_selection.KFold

class sklearn.model_selection.KFold(n_splits=3, shuffle=False, random_state=None)

K-Folds cross-validator

Provides train/test indices to split data in train/test sets. Split dataset into k consecutive folds (without shuffling by default).

Each fold is then used once as a validation while the k - 1 remaining folds form the training set.

Read more in the User Guide.

Parameters

- n_splits : int, default=3
  Number of folds. Must be at least 2.
- shuffle : boolean, optional
  Whether to shuffle the data before splitting into batches.
- random_state : None, int or RandomState
  When shuffle=True, pseudo-random number generator state used for shuffling. If None, use default numpy RNG for shuffling.

See also:

- StratifiedKFold Takes group information into account to avoid building folds with imbalanced class distributions (for binary or multiclass classification tasks).
- GroupKFold K-fold iterator variant with non-overlapping groups.

Notes

The first n_samples % n_splits folds have size n_samples // n_splits + 1, other folds have size n_samples // n_splits, where n_samples is the number of samples.

Examples

```python
>>> from sklearn.model_selection import KFold
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([1, 2, 3, 4])
>>> kf = KFold(n_splits=2)
>>> kf.get_n_splits(X)
2
>>> print(kf)
KFold(n_splits=2, random_state=None, shuffle=False)
```
```python
>>> for train_index, test_index in kf.split(X):
...     print("TRAIN: ", train_index, "TEST: ", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [2 3] TEST: [0 1]
TRAIN: [0 1] TEST: [2 3]
```

## Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>get_n_splits([X, y, groups])</code></td>
<td>Returns the number of splitting iterations in the cross-validator</td>
</tr>
<tr>
<td><code>split(X[, y, groups])</code></td>
<td>Generate indices to split data into training and test set.</td>
</tr>
</tbody>
</table>

### `__init__` (n_splits=3, shuffle=False, random_state=None)

Returns the number of splitting iterations in the cross-validator

#### Parameters

- `X` : object
  - Always ignored, exists for compatibility.

- `y` : object
  - Always ignored, exists for compatibility.

- `groups` : object
  - Always ignored, exists for compatibility.

#### Returns

- `n_splits` : int
  - Returns the number of splitting iterations in the cross-validator.

### `split(X, y=None, groups=None)`

Generate indices to split data into training and test set.

#### Parameters

- `X` : array-like, shape (n_samples, n_features)
  - Training data, where n_samples is the number of samples and n_features is the number of features.

- `y` : array-like, shape (n_samples,)
  - The target variable for supervised learning problems.

- `groups` : array-like, with shape (n_samples,), optional
  - Group labels for the samples used while splitting the dataset into train/test set.

#### Returns

- `train` : ndarray
  - The training set indices for that split.

- `test` : ndarray
  - The testing set indices for that split.
Examples using sklearn.model_selection.KFold

- Feature agglomeration vs. univariate selection
- Gradient Boosting Out-of-Bag estimates
- Cross-validation on diabetes Dataset Exercise
- Nested versus non-nested cross-validation

sklearn.model_selection.GroupKFold

class sklearn.model_selection.GroupKFold(n_splits=3)

K-fold iterator variant with non-overlapping groups.

The same group will not appear in two different folds (the number of distinct groups has to be at least equal to the number of folds).

The folds are approximately balanced in the sense that the number of distinct groups is approximately the same in each fold.

Parameters

- n_splits : int, default=3
    Number of folds. Must be at least 2.

See also:

LeaveOneGroupOut: For splitting the data according to explicit domain-specific stratification of the dataset.

Examples

```python
>>> from sklearn.model_selection import GroupKFold
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 3, 4])
>>> groups = np.array([0, 0, 2, 2])
>>> group_kfold = GroupKFold(n_splits=2)
>>> group_kfold.get_n_splits(X, y, groups)
2
>>> for train_index, test_index in group_kfold.split(X, y, groups):
...     print("TRAIN: ", train_index, "TEST: ", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print("TRAIN: ", X_train, "TEST: ", y_train, y_test)

TRAIN: [0 1] TEST: [2 3]
[[1 2]
 [3 4]] [[5 6]
 [7 8]] [1 2] [3 4] TRAIN: [2 3] TEST: [0 1]
[[5 6]
 [7 8]] [1 2] [3 4] [3 4] [1 2]
```
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>get_n_splits([X, y, groups])</code></td>
<td>Returns the number of splitting iterations in the cross-validator</td>
</tr>
<tr>
<td><code>split(X[, y, groups])</code></td>
<td>Generate indices to split data into training and test set.</td>
</tr>
</tbody>
</table>

__init__(n_splits=3)

- **get_n_splits(X=None, y=None, groups=None)**
  Returns the number of splitting iterations in the cross-validator

  - **Parameters**
    - X: object
      Always ignored, exists for compatibility.
    - y: object
      Always ignored, exists for compatibility.
    - groups: object
      Always ignored, exists for compatibility.

  - **Returns**
    - n_splits: int
      Returns the number of splitting iterations in the cross-validator.

- **split(X, y=None, groups=None)**
  Generate indices to split data into training and test set.

  - **Parameters**
    - X: array-like, shape (n_samples, n_features)
      Training data, where n_samples is the number of samples and n_features is the number of features.
    - y: array-like, shape (n_samples,)
      The target variable for supervised learning problems.
    - groups: array-like, with shape (n_samples,), optional
      Group labels for the samples used while splitting the dataset into train/test set.

  - **Returns**
    - train: ndarray
      The training set indices for that split.
    - test: ndarray
      The testing set indices for that split.

**sklearn.model_selection.StratifiedKFold**

class sklearn.model_selection.StratifiedKFold(n_splits=3, shuffle=False, random_state=None)

- Stratified K-Folds cross-validator
  Provides train/test indices to split data in train/test sets.
  This cross-validation object is a variation of KFold that returns stratified folds. The folds are made by preserving the percentage of samples for each class.

  Read more in the User Guide.
Parameters

- **n_splits**: int, default=3
  
  Number of folds. Must be at least 2.

- **shuffle**: boolean, optional
  
  Whether to shuffle each stratification of the data before splitting into batches.

- **random_state**: None, int or RandomState
  
  When shuffle=True, pseudo-random number generator state used for shuffling. If None, use default numpy RNG for shuffling.

Notes

All the folds have size \( \text{trunc}(\text{n_samples} / \text{n_splits}) \), the last one has the complementary.

Examples

```python
>>> from sklearn.model_selection import StratifiedKFold
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> skf = StratifiedKFold(n_splits=2)
>>> skf.get_n_splits(X, y)
2
>>> print(skf)
StratifiedKFold(n_splits=2, random_state=None, shuffle=False)
>>> for train_index, test_index in skf.split(X, y):
...     print("TRAIN: ", train_index, " TEST: ", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [1 3] TEST: [0 2]
TRAIN: [0 2] TEST: [1 3]
```

Methods

- **get_n_splits([X, y, groups])**

  Returns the number of splitting iterations in the cross-validator

- **split(X, y[, groups])**

  Generate indices to split data into training and test set.

__init__(n_splits=3, shuffle=False, random_state=None)

**get_n_splits(X=None, y=None, groups=None)**

Returns the number of splitting iterations in the cross-validator

Parameters

- **X**: object

  Always ignored, exists for compatibility.

- **y**: object

  Always ignored, exists for compatibility.

- **groups**: object

  Always ignored, exists for compatibility.
**Returns**

- `n_splits`: int
  
  Returns the number of splitting iterations in the cross-validator.

**split** *(X, y, groups=None)*

Generate indices to split data into training and test set.

**Parameters**

- `X`: array-like, shape (n_samples, n_features)
  
  Training data, where n_samples is the number of samples and n_features is the number of features.

  Note that providing y is sufficient to generate the splits and hence `np.zeros(n_samples)` may be used as a placeholder for X instead of actual training data.

- `y`: array-like, shape (n_samples,)
  
  The target variable for supervised learning problems. Stratification is done based on the y labels.

- `groups`: object
  
  Always ignored, exists for compatibility.

**Returns**

- `train`: ndarray
  
  The training set indices for that split.

- `test`: ndarray
  
  The testing set indices for that split.

**Examples using sklearn.model_selection.StratifiedKFold**

- Test with permutations the significance of a classification score
- Recursive feature elimination with cross-validation
- GMM covariances
- Receiver Operating Characteristic (ROC) with cross validation

**sklearn.model_selection.LeaveOneGroupOut**

**class** sklearn.model_selection.LeaveOneGroupOut

Leave One Group Out cross-validator

Provides train/test indices to split data according to a third-party provided group. This group information can be used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the groups could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

Read more in the *User Guide*.

**Examples**
```python
>>> from sklearn.model_selection import LeaveOneGroupOut
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 1, 2])
>>> groups = np.array([1, 1, 2, 2])
>>> logo = LeaveOneGroupOut()
>>> logo.get_n_splits(X, y, groups)
2
>>> print(logo)
LeaveOneGroupOut()
>>> for train_index, test_index in logo.split(X, y, groups):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print(X_train, X_test, y_train, y_test)
TRAIN: [2 3] TEST: [0 1]
[[5 6]
 [7 8]] [[1 2]
[3 4]] [1 2] [1 2]
TRAIN: [0 1] TEST: [2 3]
[[1 2]
 [3 4]] [[5 6]
[7 8]] [1 2] [1 2]
```

Methods

### get_n_splits(X, y, groups)
Returns the number of splitting iterations in the cross-validator

### split(X, y=None, groups=None)
Generate indices to split data into training and test set.

---

**_init_()**

**get_n_splits (X, y, groups)**
Returns the number of splitting iterations in the cross-validator

- **Parameters**
  - **X** : object
    
    Always ignored, exists for compatibility.
  - **y** : object
    
    Always ignored, exists for compatibility.
  - **groups** : array-like, with shape (n_samples,), optional
    
    Group labels for the samples used while splitting the dataset into train/test set.

- **Returns**
  - **n_splits** : int
    
    Returns the number of splitting iterations in the cross-validator.

**split (X, y=None, groups=None)**
Generate indices to split data into training and test set.

- **Parameters**
  - **X** : array-like, shape (n_samples, n_features)
    
    Training data, where n_samples is the number of samples and n_features is the number of features.
  - **y** : array-like, of length n_samples
The target variable for supervised learning problems.

**groups** : array-like, with shape (n_samples,), optional

Group labels for the samples used while splitting the dataset into train/test set.

**Return**

- **train** : ndarray
  - The training set indices for that split.
- **test** : ndarray
  - The testing set indices for that split.

### sklearn.model_selection.LeavePGroupsOut

class sklearn.model_selection.LeavePGroupsOut(n_groups)

**Leave P Group(s) Out cross-validator**

Provides train/test indices to split data according to a third-party provided group. This group information can be used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the groups could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

The difference between LeavePGroupsOut and LeaveOneGroupOut is that the former builds the test sets with all the samples assigned to \( p \) different values of the groups while the latter uses samples all assigned the same groups.

Read more in the User Guide.

**Parameters**

- **n_groups** : int
  - Number of groups (\( p \)) to leave out in the test split.

**See also:**

- **GroupKFold**
  - K-fold iterator variant with non-overlapping groups.

### Examples

```python
>>> from sklearn.model_selection import LeavePGroupsOut
>>> X = np.array([[1, 2], [3, 4], [5, 6]])
>>> y = np.array([1, 2, 1])
>>> groups = np.array([1, 2, 3])
>>> lpgo = LeavePGroupsOut(n_groups=2)
>>> lpgo.get_n_splits(X, y, groups)
3
>>> print(lpgo)
LeavePGroupsOut(n_groups=2)
>>> for train_index, test_index in lpgo.split(X, y, groups):
...     print("TRAIN: ", train_index, "TEST: ", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print(X_train, X_test, y_train, y_test)
TRAIN: [2] TEST: [0 1]
[[5 6]] [[1 2]]
[[3 4]] [1] [1 2]
TRAIN: [1] TEST: [0 2]
[[3 4]] [[1 2]]
```

29.5. sklearn.model_selection: Model Selection
Methods

```python
get_n_splits(X, y, groups)  # Returns the number of splitting iterations in the cross-validator

split(X[, y, groups])      # Generate indices to split data into training and test set.
```

```python
__init__(n_groups)

def get_n_splits(X, y, groups):
    # Returns the number of splitting iterations in the cross-validator
    Parameters:
    X : object
        Always ignored, exists for compatibility. np.zeros(n_samples) may be used as a placeholder.
    y : object
        Always ignored, exists for compatibility. np.zeros(n_samples) may be used as a placeholder.
    groups : array-like, with shape (n_samples,), optional
        Group labels for the samples used while splitting the dataset into train/test set.

    Returns:
    n_splits : int
        Returns the number of splitting iterations in the cross-validator.

split(X, y=None, groups=None)
# Generate indices to split data into training and test set.

Parameters:
X : array-like, shape (n_samples, n_features)
    Training data, where n_samples is the number of samples and n_features is the number of features.

y : array-like, of length n_samples
    The target variable for supervised learning problems.

groups : array-like, with shape (n_samples,), optional
    Group labels for the samples used while splitting the dataset into train/test set.

Returns:
train : ndarray
    The training set indices for that split.

test : ndarray
    The testing set indices for that split.
```
Leave-One-Out cross-validator

Provides train/test indices to split data in train/test sets. Each sample is used once as a test set (singleton) while the remaining samples form the training set.

Note: LeaveOneOut() is equivalent to KFold(n_splits=n) and LeavePOut(p=1) where n is the number of samples.

Due to the high number of test sets (which is the same as the number of samples) this cross-validation method can be very costly. For large datasets one should favor KFold, ShuffleSplit or StratifiedKFold.

Read more in the User Guide.

See also:

LeaveOneGroupOut: For splitting the data according to explicit, domain-specific stratification of the dataset.

GroupKFold: K-fold iterator variant with non-overlapping groups.

Examples

```python
>>> from sklearn.model_selection import LeaveOneOut
>>> X = np.array([[1, 2], [3, 4]])
>>> y = np.array([1, 2])
>>> loo = LeaveOneOut()
>>> loo.get_n_splits(X)
2
>>> print(loo)
LeaveOneOut()
>>> for train_index, test_index in loo.split(X):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print(X_train, X_test, y_train, y_test)
TRAIN: [1] TEST: [0]
[[3 4]] [[1 2]] [2] [1]
TRAIN: [0] TEST: [1]
[[1 2]] [[3 4]] [1] [2]
```

Methods

- `get_n_splits(X[, y, groups])`: Returns the number of splitting iterations in the cross-validator.
- `split(X[, y, groups])`: Generate indices to split data into training and test set.

```python
__init__()
get_n_splits(X, y=None, groups=None)
```

Parameters:

- `X`: array-like, shape (n_samples, n_features)
  Training data, where n_samples is the number of samples and n_features is the number
of features.

**y**: object

Always ignored, exists for compatibility.

**groups**: object

Always ignored, exists for compatibility.

**Returns**

**n_splits**: int

Returns the number of splitting iterations in the cross-validator.

**split** *(X, y=None, groups=None)*

Generate indices to split data into training and test set.

**Parameters**

**X**: array-like, shape (n_samples, n_features)

Training data, where n_samples is the number of samples and n_features is the number of features.

**y**: array-like, of length n_samples

The target variable for supervised learning problems.

**groups**: array-like, with shape (n_samples,), optional

Group labels for the samples used while splitting the dataset into train/test set.

**Returns**

**train**: ndarray

The training set indices for that split.

**test**: ndarray

The testing set indices for that split.

### sklearn.model_selection.LeavePOut

**class sklearn.model_selection.LeavePOut**(p)

Leave-P-Out cross-validator

Provides train/test indices to split data in train/test sets. This results in testing on all distinct samples of size p, while the remaining n - p samples form the training set in each iteration.

**Note**: LeavePOut(p) is NOT equivalent to KFold(n_splits=n_samples // p) which creates non-overlapping test sets.

Due to the high number of iterations which grows combinatorially with the number of samples this cross-validation method can be very costly. For large datasets one should favor KFold, StratifiedKFold or ShuffleSplit.

Read more in the *User Guide*.

**Parameters**

**p**: int

Size of the test sets.

### Examples
```python
>>> from sklearn.model_selection import LeavePOut
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 3, 4])
>>> lpo = LeavePOut(2)
>>> lpo.get_n_splits(X)
6
>>> print(lpo)
LeavePOut(p=2)

>>> for train_index, test_index in lpo.split(X):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [2 3] TEST: [0 1]
TRAIN: [1 3] TEST: [0 2]
TRAIN: [1 2] TEST: [0 3]
TRAIN: [0 3] TEST: [1 2]
TRAIN: [0 2] TEST: [1 3]
TRAIN: [0 1] TEST: [2 3]
```

### Methods

**get_n_splits(X[, y, groups])**

Returns the number of splitting iterations in the cross-validator

**split(X[, y, groups])**

Generate indices to split data into training and test set.

```python
def __init__(p)
def get_n_splits(X, y=None, groups=None)
    Returns the number of splitting iterations in the cross-validator
    Parameters
    ----------
    X : array-like, shape (n_samples, n_features)
        Training data, where n_samples is the number of samples and n_features is the number of features.
    y : object
        Always ignored, exists for compatibility.
    groups : object
        Always ignored, exists for compatibility.

def split(X, y=None, groups=None)
    Generate indices to split data into training and test set.
    Parameters
    ----------
    X : array-like, shape (n_samples, n_features)
        Training data, where n_samples is the number of samples and n_features is the number of features.
    y : array-like, of length n_samples
        The target variable for supervised learning problems.
    groups : array-like, with shape (n_samples,), optional
        Group labels for the samples used while splitting the dataset into train/test set.
```
Returnstrain : ndarray

The training set indices for that split.

test : ndarray

The testing set indices for that split.

**sklearn.model_selection.ShuffleSplit**

class sklearn.model_selection.ShuffleSplit (n_splits=10, test_size=0.1, train_size=None, random_state=None)

Random permutation cross-validator

Yields indices to split data into training and test sets.

Note: contrary to other cross-validation strategies, random splits do not guarantee that all folds will be different, although this is still very likely for sizeable datasets.

Read more in the *User Guide*.

**Parameters**

- **n_splits** : int (default 10)

  Number of re-shuffling & splitting iterations.

- **test_size** : float, int, or None, default 0.1

  If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples. If None, the value is automatically set to the complement of the train size.

- **train_size** : float, int, or None (default is None)

  If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test size.

- **random_state** : int or RandomState

  Pseudo-random number generator state used for random sampling.

**Examples**

```python
>>> from sklearn.model_selection import ShuffleSplit
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 1, 2])
>>> rs = ShuffleSplit(n_splits=3, test_size=.25, random_state=0)
>>> rs.get_n_splits(X)
3
>>> print(rs)
ShuffleSplit (n_splits=3, random_state=0, test_size=0.25, train_size=None)
>>> for train_index, test_index in rs.split(X):
...     print("TRAIN:", train_index, "TEST:", test_index)
...TRAIN: [3 1 0] TEST: [2]
TRAIN: [2 1 3] TEST: [0]
TRAIN: [0 2 1] TEST: [3]
```

```python
>>> rs = ShuffleSplit(n_splits=3, train_size=0.5, test_size=.25, random_state=0)
>>> for train_index, test_index in rs.split(X):
...
...```
Methods

get_n_splits([X, y, groups])

Returns the number of splitting iterations in the cross-validator

split(X[, y, groups])

Generate indices to split data into training and test set.

__init__(self, n_splits=10, test_size=0.1, train_size=None, random_state=None)

get_n_splits(X=None, y=None, groups=None)

Returns the number of splitting iterations in the cross-validator

Parameters

X : object
    Always ignored, exists for compatibility.

y : object
    Always ignored, exists for compatibility.

groups : object
    Always ignored, exists for compatibility.

Returns

n_splits : int
    Returns the number of splitting iterations in the cross-validator.

split(X, y=None, groups=None)

Generate indices to split data into training and test set.

Parameters

X : array-like, shape (n_samples, n_features)
    Training data, where n_samples is the number of samples and n_features is the number
    of features.

y : array-like, shape (n_samples,)
    The target variable for supervised learning problems.

groups : array-like, with shape (n_samples,), optional
    Group labels for the samples used while splitting the dataset into train/test set.

Returns

train : ndarray
    The training set indices for that split.

test : ndarray
    The testing set indices for that split.
Examples using `sklearn.model_selection.ShuffleSplit`

- **Plotting Learning Curves**
- **Scaling the regularization parameter for SVCs**

`sklearn.model_selection.GroupShuffleSplit`

class `sklearn.model_selection.GroupShuffleSplit(n_splits=5, test_size=0.2, train_size=None, random_state=None)``

Shuffle-Group(s)-Out cross-validation iterator

Provides randomized train/test indices to split data according to a third-party provided group. This group information can be used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the groups could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

The difference between LeavePGroupsOut and GroupShuffleSplit is that the former generates splits using all subsets of size \( p \) unique groups, whereas GroupShuffleSplit generates a user-determined number of random test splits, each with a user-determined fraction of unique groups.

For example, a less computationally intensive alternative to `LeavePGroupsOut(p=10)` would be `GroupShuffleSplit(test_size=10, n_splits=100)`.

Note: The parameters `test_size` and `train_size` refer to groups, and not to samples, as in `ShuffleSplit`.

**Parameters**

- `n_splits` : int (default 5)
  Number of re-shuffling & splitting iterations.
- `test_size` : float (default 0.2), int, or None
  If float, should be between 0.0 and 1.0 and represent the proportion of the groups to include in the test split. If int, represents the absolute number of test groups. If None, the value is automatically set to the complement of the train size.
- `train_size` : float, int, or None (default is None)
  If float, should be between 0.0 and 1.0 and represent the proportion of the groups to include in the train split. If int, represents the absolute number of train groups. If None, the value is automatically set to the complement of the test size.
- `random_state` : int or RandomState
  Pseudo-random number generator state used for random sampling.

**Methods**

- `get_n_splits([X, y, groups])`
  Returns the number of splitting iterations in the cross-validator

- `split(X[, y, groups])`
  Generate indices to split data into training and test set.

```
__init__(n_splits=5, test_size=0.2, train_size=None, random_state=None)
```

```
get_n_splits(X=none, y=none, groups=none)
```

Returns the number of splitting iterations in the cross-validator

**Parameters**

- `X` : object
Always ignored, exists for compatibility.

y : object
Always ignored, exists for compatibility.

groups : object
Always ignored, exists for compatibility.

Returns

sn_splits : int
Returns the number of splitting iterations in the cross-validator.

split (X, y=None, groups=None)
Generate indices to split data into training and test set.

Parameters

X : array-like, shape (n_samples, n_features)
Training data, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape (n_samples,)
The target variable for supervised learning problems.

groups : array-like, with shape (n_samples,), optional
Group labels for the samples used while splitting the dataset into train/test set.

Returns

train : ndarray
The training set indices for that split.

test : ndarray
The testing set indices for that split.

sklearn.model_selection.StratifiedShuffleSplit

class sklearn.model_selection.StratifiedShuffleSplit (n_splits=10, test_size=0.1, train_size=None, random_state=None)

Stratified ShuffleSplit cross-validator

Provides train/test indices to split data in train/test sets.

This cross-validation object is a merge of StratifiedKFold and ShuffleSplit, which returns stratified randomized folds. The folds are made by preserving the percentage of samples for each class.

Note: like the ShuffleSplit strategy, stratified random splits do not guarantee that all folds will be different, although this is still very likely for sizeable datasets.

Read more in the User Guide.

Parameters

n_splits : int (default 10)
Number of re-shuffling & splitting iterations.

test_size : float (default 0.1), int, or None
If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples. If None, the value is automatically set to the complement of the train size.

train_size : float, int, or None (default is None)
If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test size.

**random_state** : int or RandomState

Pseudo-random number generator state used for random sampling.

**Examples**

```python
>>> from sklearn.model_selection import StratifiedShuffleSplit
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> sss = StratifiedShuffleSplit(n_splits=3, test_size=0.5, random_state=0)
>>> sss.get_n_splits(X, y)
3
>>> print(sss)
StratifiedShuffleSplit(n_splits=3, random_state=0, ...)
>>> for train_index, test_index in sss.split(X, y):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [1 2] TEST: [3 0]
TRAIN: [0 2] TEST: [1 3]
TRAIN: [0 2] TEST: [3 1]
```

**Methods**

```python
get_n_splits([X, y, groups]) Returns the number of splitting iterations in the cross-validator

split(X, y[, groups]) Generate indices to split data into training and test set.
```

```python
__init__(n_splits=10, test_size=0.1, train_size=None, random_state=None)

get_n_splits(X=None, y=None, groups=None) Returns the number of splitting iterations in the cross-validator

Parameters

**X** : object

Always ignored, exists for compatibility.

**y** : object

Always ignored, exists for compatibility.

**groups** : object

Always ignored, exists for compatibility.

Returns

**n_splits** : int

Returns the number of splitting iterations in the cross-validator.

split(X, y, groups=None) Generate indices to split data into training and test set.

Parameters

**X** : array-like, shape (n_samples, n_features)
```

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Training data, where n_samples is the number of samples and n_features is the number of features.

Note that providing y is sufficient to generate the splits and hence np.zeros(n_samples) may be used as a placeholder for X instead of actual training data.

y : array-like, shape (n_samples,)

The target variable for supervised learning problems. Stratification is done based on the y labels.

groups : object

Always ignored, exists for compatibility.

Returnstrain : ndarray

The training set indices for that split.

test : ndarray

The testing set indices for that split.

Examples using sklearn.model_selection.StratifiedShuffleSplit

- RBF SVM parameters

sklearn.model_selection.PredefinedSplit

class sklearn.model_selection.PredefinedSplit (test_fold)

Predefined split cross-validator

Splits the data into training/test set folds according to a predefined scheme. Each sample can be assigned to at most one test set fold, as specified by the user through the test_fold parameter.

Read more in the User Guide.

Examples

```python
>>> from sklearn.model_selection import PredefinedSplit
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> test_fold = [0, 1, -1, 1]
>>> ps = PredefinedSplit(test_fold)
>>> ps.get_n_splits()
2
>>> print(ps)
PredefinedSplit(test_fold=array([ 0, 1, -1, 1]))
>>> for train_index, test_index in ps.split():
...    print("TRAIN:", train_index, "TEST:", test_index)
...    X_train, X_test = X[train_index], X[test_index]
...    y_train, y_test = y[train_index], y[test_index]
TRAIN: [1 2 3] TEST: [0]
TRAIN: [0 2] TEST: [1 3]
```
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>get_n_splits</code></td>
<td>Returns the number of splitting iterations in the cross-validator</td>
</tr>
<tr>
<td><code>split</code></td>
<td>Generate indices to split data into training and test set.</td>
</tr>
</tbody>
</table>

__init__(test_fold)

get_n_splits(X=None, y=None, groups=None)

Returns the number of splitting iterations in the cross-validator

**Parameters**

- **X**: object
  - Always ignored, exists for compatibility.
- **y**: object
  - Always ignored, exists for compatibility.
- **groups**: object
  - Always ignored, exists for compatibility.

**Returns**

- **n_splits**: int
  - Returns the number of splitting iterations in the cross-validator.

split(X=None, y=None, groups=None)

Generate indices to split data into training and test set.

**Parameters**

- **X**: object
  - Always ignored, exists for compatibility.
- **y**: object
  - Always ignored, exists for compatibility.
- **groups**: object
  - Always ignored, exists for compatibility.

**Returns**

- **train**: ndarray
  - The training set indices for that split.
- **test**: ndarray
  - The testing set indices for that split.

sklearn.model_selection.TimeSeriesSplit

class sklearn.model_selection.TimeSeriesSplit(n_splits=3)

Time Series cross-validator

Provides train/test indices to split time series data samples that are observed at fixed time intervals, in train/test sets. In each split, test indices must be higher than before, and thus shuffling in cross validator is inappropriate.

This cross-validation object is a variation of KFold. In the kth split, it returns first k folds as train set and the (k+1)th fold as test set.

Note that unlike standard cross-validation methods, successive training sets are supersets of those that come before them.
Parameters

**n_splits** : int, default=3

Number of splits. Must be at least 1.

Notes

The training set has size \( i \times n_samples // (n_splits + 1) + n_samples \% (n_splits + 1) \) in the \( i \)\(^{th} \) split, with a test set of size \( n_samples // (n_splits + 1) \), where \( n_samples \) is the number of samples.

Examples

```python
>>> from sklearn.model_selection import TimeSeriesSplit
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([1, 2, 3, 4])
>>> tscv = TimeSeriesSplit(n_splits=3)
>>> print(tscv)
TimeSeriesSplit(n_splits=3)
>>> for train_index, test_index in tscv.split(X):
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [0] TEST: [1]
TRAIN: [0 1] TEST: [2]
TRAIN: [0 1 2] TEST: [3]
```

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>get_n_splits(X, y, groups)</code></td>
<td>Returns the number of splitting iterations in the cross-validator</td>
</tr>
<tr>
<td><code>split(X, y, groups)</code></td>
<td>Generate indices to split data into training and test set.</td>
</tr>
</tbody>
</table>

`__init__(n_splits=3)`

`get_n_splits(X=None, y=None, groups=None)`

Returns the number of splitting iterations in the cross-validator

Parameters

**X** : object

Always ignored, exists for compatibility.

**y** : object

Always ignored, exists for compatibility.

**groups** : object

Always ignored, exists for compatibility.

**Returns**

**n_splits** : int

Returns the number of splitting iterations in the cross-validator.
split \((X, y=None, groups=None)\)
Generate indices to split data into training and test set.

**Parameters**
- **X**: array-like, shape \((n_{samples}, n_{features})\)
  - Training data, where \(n_{samples}\) is the number of samples and \(n_{features}\) is the number of features.
- **y**: array-like, shape \((n_{samples},)\)
  - Always ignored, exists for compatibility.
- **groups**: array-like, with shape \((n_{samples},)\), optional
  - Always ignored, exists for compatibility.

**Returns**
- **train**: ndarray
  - The training set indices for that split.
- **test**: ndarray
  - The testing set indices for that split.

### 29.5.2 Splitter Functions

- **model_selection.train_test_split**(*arrays, ...*)
  - Split arrays or matrices into random train and test subsets

- **model_selection.check_cv**([cv, y, classifier])
  - Input checker utility for building a cross-validator

**sklearn.model_selection.train_test_split**

Split arrays or matrices into random train and test subsets

Quick utility that wraps input validation and \(\text{next}(\text{ShuffleSplit().split}(X,y))\) and application to input data into a single call for splitting (and optionally subsampling) data in a oneliner.

Read more in the *User Guide*.

**Parameters**
- **arrays**: sequence of indexables with same length / shape[0]
  - Allowed inputs are lists, numpy arrays, scipy-sparse matrices or pandas dataframes.
- **test_size**: float, int, or None (default is None)
  - If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples. If None, the value is automatically set to the complement of the train size. If train size is also None, test size is set to 0.25.
- **train_size**: float, int, or None (default is None)
  - If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test size.
- **random_state**: int or RandomState
  - Pseudo-random number generator state used for random sampling.
- **stratify**: array-like or None (default is None)
If not None, data is split in a stratified fashion, using this as the class labels.

**Returnssplitting** : list, length=2 * len(arrays)

List containing train-test split of inputs.

New in version 0.16: If the input is sparse, the output will be a `scipy.sparse.csr_matrix`. Else, output type is the same as the input type.

**Examples**

```python
>>> import numpy as np
>>> from sklearn.model_selection import train_test_split
>>> X, y = np.arange(10).reshape((5, 2)), range(5)
>>> X
array([[0, 1],
       [2, 3],
       [4, 5],
       [6, 7],
       [8, 9]])
>>> list(y)
[0, 1, 2, 3, 4]
```

```python
>>> X_train, X_test, y_train, y_test = train_test_split(
...     X, y, test_size=0.33, random_state=42)
...
>>> X_train
array([[4, 5],
       [0, 1],
       [6, 7]])
>>> y_train
[2, 0, 3]
>>> X_test
array([[2, 3],
       [8, 9]])
>>> y_test
[1, 4]
```

**Examples using sklearn.model_selection.train_test_split**

- Faces recognition example using eigenfaces and SVMs
- Prediction Latency
- Probability calibration of classifiers
- Probability Calibration curves
- Classifier comparison
- Feature transformations with ensembles of trees
- Gradient Boosting Out-of-Bag estimates
- Partial Dependence Plots
- Comparing random forests and the multi-output meta estimator
Comparing various online solvers
Parameter estimation using grid search with cross-validation
Confusion matrix
Precision-Recall
Receiver Operating Characteristic (ROC)
Varying regularization in Multi-layer Perceptron
Restricted Boltzmann Machine features for digit classification
Using FunctionTransformer to select columns
Understanding the decision tree structure

**sklearn.model_selection.check_cv**

```python
sklearn.model_selection.check_cv(cv=3, y=None, classifier=False)
```

Input checker utility for building a cross-validator

**Parameters**

- `cv`: int, cross-validation generator or an iterable, optional
  Determines the cross-validation splitting strategy. Possible inputs for `cv` are:
  - None, to use the default 3-fold cross-validation,
  - integer, to specify the number of folds.
  - An object to be used as a cross-validation generator.
  - An iterable yielding train/test splits.
  For integer/None inputs, if classifier is True and `y` is either binary or multiclass, `StratifiedKFold` is used. In all other cases, `KFold` is used.
  Refer [User Guide](#) for the various cross-validation strategies that can be used here.

- `y`: array-like, optional
  The target variable for supervised learning problems.

- `classifier`: boolean, optional, default False
  Whether the task is a classification task, in which case stratified KFold will be used.

**Returns**

- `checked_cv`: a cross-validator instance.
  The return value is a cross-validator which generates the train/test splits via the `split` method.

### 29.5.3 Hyper-parameter optimizers

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>model_selection.GridSearchCV</code></td>
<td>Exhaustive search over specified parameter values for an estimator.</td>
</tr>
<tr>
<td><code>model_selection.RandomizedSearchCV</code></td>
<td>Randomized search on hyper parameters.</td>
</tr>
<tr>
<td><code>model_selection.ParameterGrid</code></td>
<td>Grid of parameters with a discrete number of values for each.</td>
</tr>
<tr>
<td><code>model_selection.ParameterSampler</code></td>
<td>Generator on parameters sampled from given distributions.</td>
</tr>
</tbody>
</table>
sklearn.model_selection.GridSearchCV

class sklearn.model_selection.GridSearchCV (estimator, param_grid, scoring=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, pre_dispatch='2*n_jobs', error_score='raise', return_train_score=True)

Exhaustive search over specified parameter values for an estimator.

Important members are fit, predict.

GridSearchCV implements a “fit” and a “score” method. It also implements “predict”, “predict_probas”, “decision_function”, “transform” and “inverse_transform” if they are implemented in the estimator used.

The parameters of the estimator used to apply these methods are optimized by cross-validated grid-search over a parameter grid.

Read more in the User Guide.

Parameters

estimator : estimator object.

This is assumed to implement the scikit-learn estimator interface. Either estimator needs to provide a score function, or scoring must be passed.

param_grid : dict or list of dictionaries

Dictionary with parameters names (string) as keys and lists of parameter settings to try as values, or a list of such dictionaries, in which case the grids spanned by each dictionary in the list are explored. This enables searching over any sequence of parameter settings.

scoring : string, callable or None, default=None

A string (see model evaluation documentation) or a scorer callable object / function with signature scorer (estimator, X, y). If None, the score method of the estimator is used.

fit_params : dict, optional

Parameters to pass to the fit method.

n_jobs : int, default=1

Number of jobs to run in parallel.

pre_dispatch : int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

• None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs

• An int, giving the exact number of total jobs that are spawned

• A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

iid : boolean, default=True

If True, the data is assumed to be identically distributed across the folds, and the loss minimized is the total loss per sample, and not the mean loss across the folds.

cv : int, cross-validation generator or an iterable, optional
Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the default 3-fold cross validation,
- integer, to specify the number of folds in a (Stratified)KFold,
- An object to be used as a cross-validation generator.
- An iterable yielding train, test splits.

For integer/None inputs, if the estimator is a classifier and y is either binary or multi-class, StratifiedKFold is used. In all other cases, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**refit** : boolean, default=True

Refit the best estimator with the entire dataset. If “False”, it is impossible to make predictions using this GridSearchCV instance after fitting.

**verbose** : integer

Controls the verbosity: the higher, the more messages.

**error_score** : ‘raise’ (default) or numeric

Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.

**return_train_score** : boolean, default=True

If 'False', the cv_results_ attribute will not include training scores.

**Attributes**

- cv_results_ : dict of numpy (masked) ndarrays
  
  A dict with keys as column headers and values as columns, that can be imported into a pandas DataFrame.

For instance the below given table

<table>
<thead>
<tr>
<th>param_kernel</th>
<th>param_gamma</th>
<th>param_degree</th>
<th>split0_test_score</th>
<th>...</th>
<th>rank....</th>
</tr>
</thead>
<tbody>
<tr>
<td>'poly'</td>
<td>--</td>
<td>2</td>
<td>0.8</td>
<td>...</td>
<td>2</td>
</tr>
<tr>
<td>'poly'</td>
<td>--</td>
<td>3</td>
<td>0.7</td>
<td>...</td>
<td>4</td>
</tr>
<tr>
<td>'rbf'</td>
<td>0.1</td>
<td>--</td>
<td>0.8</td>
<td>...</td>
<td>3</td>
</tr>
<tr>
<td>'rbf'</td>
<td>0.2</td>
<td>--</td>
<td>0.9</td>
<td>...</td>
<td>1</td>
</tr>
</tbody>
</table>

will be represented by a cv_results_ dict of:

```python
{
    'param_kernel': masked_array(data = ['poly', 'poly', 'rbf', 'rbf
    ...
    -> '],
    mask = [False False False False False]
    ...
    -> 'param_gamma': masked_array(data = [-- -- 0.1 0.2],
    mask = [ True True False False]
    ...
    -> 'param_degree': masked_array(data = [2.0 3.0 -- --],
    mask = [False False True True]
    ...
    -> 'split0_test_score' : [0.8, 0.7, 0.8, 0.9],
    'split1_test_score' : [0.82, 0.5, 0.7, 0.78],
    'mean_test_score' : [0.81, 0.60, 0.75, 0.82],
    'std_test_score' : [0.02, 0.01, 0.03, 0.03],
    'rank_test_score' : [2, 4, 3, 1],
```
NOTE that the key 'params' is used to store a list of parameter settings dict for all the parameter candidates.

The mean_fit_time, std_fit_time, mean_score_time and std_score_time are all in seconds.

best_estimator_: estimator

Estimator that was chosen by the search, i.e. estimator which gave highest score (or smallest loss if specified) on the left out data. Not available if refit=False.

best_score_: float

Score of best_estimator on the left out data.

best_params_: dict

Parameter setting that gave the best results on the hold out data.

best_index_: int

The index (of the cv_results_ arrays) which corresponds to the best candidate parameter setting.

The dict at search.cv_results_['params'][search.best_index_] gives the parameter setting for the best model, that gives the highest mean score (search.best_score_).

scorer_: function

Scorer function used on the held out data to choose the best parameters for the model.

n_splits_: int

The number of cross-validation splits (folds/iterations).

See also:

ParameterGrid generates all the combinations of a hyperparameter grid.
sklearn.model_selection.train_test_split utility function to split the data into a development set usable for fitting a GridSearchCV instance and an evaluation set for its final evaluation.
sklearn.metrics.make_scorer Make a scorer from a performance metric or loss function.

Notes

The parameters selected are those that maximize the score of the left out data, unless an explicit score is passed in which case it is used instead.
If \( n_{\text{jobs}} \) was set to a value higher than one, the data is copied for each point in the grid (and not \( n_{\text{jobs}} \) times). This is done for efficiency reasons if individual jobs take very little time, but may raise errors if the dataset is large and not enough memory is available. A workaround in this case is to set \( \text{pre}_\text{dispatch} \). Then, the memory is copied only \( \text{pre}_\text{dispatch} \) many times. A reasonable value for \( \text{pre}_\text{dispatch} \) is \( 2 \* n_{\text{jobs}} \).

**Examples**

```python
>>> from sklearn import svm, datasets
>>> from sklearn.model_selection import GridSearchCV

>>> iris = datasets.load_iris()

>>> parameters = {'kernel':('linear', 'rbf'), 'C':[1, 10]}

>>> svr = svm.SVC()

>>> clf = GridSearchCV(svr, parameters)

>>> clf.fit(iris.data, iris.target)
...
```

```
GridSearchCV(cv=None, error_score=..., estimator=SVC(C=1.0, cache_size=..., class_weight=..., coef0=..., decision_function_shape=None, degree=..., gamma=..., kernel='rbf', max_iter=-1, probability=False, random_state=None, shrinking=True, tol=..., verb...

>>> sorted(clf.cv_results_.keys())
...
['mean_fit_time', 'mean_score_time', 'mean_test_score', ... 'mean_train_score', 'param_C', 'param_kernel', 'params', ... 'rank_test_score', 'split0_test_score', ... 'split0_train_score', 'split1_test_score', 'split1_train_score', ... 'split2_test_score', 'split2_train_score', ...

'std_fit_time', 'std_score_time', 'std_test_score', 'std_train_score']
```

**Methods**

- `decision_function(*args, **kwargs)` Call decision_function on the estimator with the best found parameters.
- `fit(X[, y, groups])` Run fit with all sets of parameters.
- `get_params([deep])` Get parameters for this estimator.
- `inverse_transform(*args, **kwargs)` Call inverse_transform on the estimator with the best found params.
- `predict(*args, **kwargs)` Call predict on the estimator with the best found parameters.
- `predict_log_proba(*args, **kwargs)` Call predict_log_proba on the estimator with the best found parameters.
- `predict_proba(*args, **kwargs)` Call predict_proba on the estimator with the best found parameters.
- `score(X[, y])` Returns the score on the given data, if the estimator has been refit.
- `set_params(**params)` Set the parameters of this estimator.

Continued on next page
**Table 29.47 – continued from previous page**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>transform</code></td>
<td>Call transform on the estimator with the best found parameters.</td>
</tr>
<tr>
<td><code>__init__</code></td>
<td>Constructor method for GridSearchCV.</td>
</tr>
<tr>
<td><code>decision_function</code></td>
<td>Call decision_function on the estimator with the best found parameters.</td>
</tr>
<tr>
<td><code>fit</code></td>
<td>Run fit with all sets of parameters.</td>
</tr>
<tr>
<td><code>get_params</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>inverse_transform</code></td>
<td>Call inverse_transform on the estimator with the best found params.</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>Call predict on the estimator with the best found parameters.</td>
</tr>
<tr>
<td><code>predict_log_proba</code></td>
<td>Call predict_log_proba on the estimator with the best found parameters.</td>
</tr>
</tbody>
</table>

**Methods**

- **`__init__`**
  
  ```python
  def __init__(estimator, param_grid, scoring=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, pre_dispatch='2*n_jobs', error_score='raise', return_train_score=True)
  ```

- **`decision_function`**
  
  ```python
  def decision_function(*args, **kwargs)
  ```

- **`fit`**
  
  ```python
  def fit(X, y=None, groups=None)
  ```

- **`get_params`**
  
  ```python
  def get_params(deep=True)
  ```

- **`inverse_transform`**
  
  ```python
  def inverse_transform(Xt)
  ```

- **`predict`**
  
  ```python
  def predict(X)
  ```

- **`predict_log_proba`**
  
  ```python
  def predict_log_proba(X)
  ```

**Parameters**

- `X` : array-like, shape = [n_samples, n_features]
  - Training vector, where n_samples is the number of samples and n_features is the number of features.

- `y` : array-like, shape = [n_samples] or [n_samples, n_output], optional
  - Target relative to X for classification or regression; None for unsupervised learning.

- `groups` : array-like, with shape (n_samples,), optional
  - Group labels for the samples used while splitting the dataset into train/test set.

- `deep` : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- `return_train_score` : boolean, optional
  - Whether to return the scores for the training dataset as well.

**Notes**

- Only available if `refit=True` and the underlying estimator supports `decision_function`.
- Only available if the underlying estimator implements `inverse_transform` and `refit=True`.
- Only available if `refit=True` and the underlying estimator supports `predict`. 
- Only available if `refit=True` and the underlying estimator supports `predict_log_proba`. 

---

29.5. `sklearn.model_selection`: Model Selection
Parameters

X : indexable, length n_samples
Must fulfill the input assumptions of the underlying estimator.

predict_proba(*args, **kwargs)
Call predict_proba on the estimator with the best found parameters.
Only available if refit=True and the underlying estimator supports predict_proba.

Parameters

X : indexable, length n_samples
Must fulfill the input assumptions of the underlying estimator.

score(X, y=None)
Returns the score on the given data, if the estimator has been refit.
This uses the score defined by scoring where provided, and the best_estimator_.score method otherwise.

Parameters

X : array-like, shape = [n_samples, n_features]
Input data, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples] or [n_samples, n_output], optional
Target relative to X for classification or regression; None for unsupervised learning.

Returns

score : float

set_params(**params)
Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns

self : 

transform(*args, **kwargs)
Call transform on the estimator with the best found parameters.
Only available if the underlying estimator supports transform and refit=True.

Parameters

X : indexable, length n_samples
Must fulfill the input assumptions of the underlying estimator.

Examples using sklearn.model_selection.GridSearchCV

- Concatenating multiple feature extraction methods
- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Pipelining: chaining a PCA and a logistic regression
- Comparison of kernel ridge regression and SVR
- Faces recognition example using eigenfaces and SVMs
- Feature agglomeration vs. univariate selection
- Shrinkage covariance estimation: LedoitWolf vs OAS and max-likelihood
- Model selection with Probabilistic PCA and Factor Analysis (FA)
Randomized search on hyper parameters.

RandomizedSearchCV implements a “fit” and a “score” method. It also implements “predict”, “predict_proba”, “decision_function”, “transform” and “inverse_transform” if they are implemented in the estimator used.

The parameters of the estimator used to apply these methods are optimized by cross-validated search over parameter settings.

In contrast to GridSearchCV, not all parameter values are tried out, but rather a fixed number of parameter settings is sampled from the specified distributions. The number of parameter settings that are tried is given by n_iter.

If all parameters are presented as a list, sampling without replacement is performed. If at least one parameter is given as a distribution, sampling with replacement is used. It is highly recommended to use continuous distributions for continuous parameters.

Read more in the User Guide.

**Parameters**

- **estimator** : estimator object.
  A object of that type is instantiated for each grid point. This is assumed to implement the scikit-learn estimator interface. Either estimator needs to provide a score function, or scoring must be passed.

- **param_distributions** : dict
  Dictionary with parameters names (string) as keys and distributions or lists of parameters to try. Distributions must provide a rvs method for sampling (such as those from scipy.stats.distributions). If a list is given, it is sampled uniformly.

- **n_iter** : int, default=10
  Number of parameter settings that are sampled. n_iter trades off runtime vs quality of the solution.

- **scoring** : string, callable or None, default=None
A string (see model evaluation documentation) or a scorer callable object / function with signature `scorer(estimator, X, y)`. If `None`, the `score` method of the estimator is used.

**fit_params** : dict, optional
Parameters to pass to the fit method.

**n_jobs** : int, default=1
Number of jobs to run in parallel.

**pre_dispatch** : int, or string, optional
Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
- An int, giving the exact number of total jobs that are spawned
- A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

**iid** : boolean, default=True
If True, the data is assumed to be identically distributed across the folds, and the loss minimized is the total loss per sample, and not the mean loss across the folds.

**cv** : int, cross-validation generator or an iterable, optional
Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the default 3-fold cross validation,
- integer, to specify the number of folds in a (Stratified)KFold,
- An object to be used as a cross-validation generator.
- An iterable yielding train, test splits.

For integer/None inputs, if the estimator is a classifier and `y` is either binary or multiclass, `StratifiedKFold` is used. In all other cases, `KFold` is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**refit** : boolean, default=True
Refit the best estimator with the entire dataset. If “False”, it is impossible to make predictions using this RandomizedSearchCV instance after fitting.

**verbose** : integer
Controls the verbosity: the higher, the more messages.

**random_state** : int or RandomState
Pseudo random number generator state used for random uniform sampling from lists of possible values instead of scipy.stats distributions.

**error_score** : ‘raise’ (default) or numeric
Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.
return_train_score : boolean, default=True

If 'False', the cv_results_ attribute will not include training scores.

Attributes:

- cv_results_ : dict of numpy (masked) ndarrays

A dict with keys as column headers and values as columns, that can be imported into a pandas DataFrame.

For instance the below given table

<table>
<thead>
<tr>
<th>param_kernel</th>
<th>param_gamma</th>
<th>split0_test_score</th>
<th>...</th>
<th>rank_test_score</th>
</tr>
</thead>
<tbody>
<tr>
<td>'rbf'</td>
<td>0.1</td>
<td>0.8</td>
<td>...</td>
<td>2</td>
</tr>
<tr>
<td>'rbf'</td>
<td>0.2</td>
<td>0.9</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>'rbf'</td>
<td>0.3</td>
<td>0.7</td>
<td>...</td>
<td>1</td>
</tr>
</tbody>
</table>

will be represented by a cv_results_dict of:

```python
{
    'param_kernel' : masked_array(data = ['rbf', 'rbf', 'rbf'],
                               mask = False),
    'param_gamma' : masked_array(data = [0.1 0.2 0.3], mask = _False),
    'split0_test_score' : [0.8, 0.9, 0.7],
    'split1_test_score' : [0.82, 0.5, 0.7],
    'mean_test_score' : [0.81, 0.7, 0.7],
    'std_test_score' : [0.02, 0.2, 0.],
    'rank_test_score' : [3, 1, 1],
    'split0_train_score' : [0.8, 0.9, 0.7],
    'split1_train_score' : [0.82, 0.5, 0.7],
    'mean_train_score' : [0.81, 0.7, 0.7],
    'std_train_score' : [0.03, 0.03, 0.04],
    'mean_fit_time' : [0.73, 0.63, 0.43, 0.49],
    'std_fit_time' : [0.01, 0.02, 0.01, 0.01],
    'mean_score_time' : [0.007, 0.06, 0.04, 0.04],
    'std_score_time' : [0.001, 0.002, 0.003, 0.005],
    'params' : [{'kernel' : 'rbf', 'gamma' : 0.1}, ...],
}
```

NOTE that the key 'params' is used to store a list of parameter settings dict for all
the parameter candidates.

The mean_fit_time, std_fit_time, mean_score_time and std_score_time are all in seconds.

best_estimator_ : estimator

Estimator that was chosen by the search, i.e. estimator which gave highest score (or
smallest loss if specified) on the left out data. Not available if refit=False.

best_score_ : float

Score of best_estimator on the left out data.

best_params_ : dict

Parameter setting that gave the best results on the hold out data.

best_index_ : int

The index (of the cv_results_ arrays) which corresponds to the best candidate pa-
rameter setting.
The dict at `search.cv_results_['params'][search.best_index_]` gives the parameter setting for the best model, that gives the highest mean score (`search.best_score_`).

**scorer_** : function

Scorer function used on the held out data to choose the best parameters for the model.

**n_splits_** : int

The number of cross-validation splits (folds/iterations).

See also:

* **GridSearchCV** — Does exhaustive search over a grid of parameters.

* **ParameterSampler** — A generator over parameter settings, constructed from param_distributions.

Notes

The parameters selected are those that maximize the score of the held-out data, according to the scoring parameter.

If `n_jobs` was set to a value higher than one, the data is copied for each parameter setting (and not `n_jobs` times). This is done for efficiency reasons if individual jobs take very little time, but may raise errors if the dataset is large and not enough memory is available. A workaround in this case is to set `pre_dispatch`. Then, the memory is copied only `pre_dispatch` many times. A reasonable value for `pre_dispatch` is `2 * n_jobs`.

Methods

<table>
<thead>
<tr>
<th><code>decision_function</code>(*args, **kwargs)</th>
<th>Call decision_function on the estimator with the best found parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X[, y, groups])</code></td>
<td>Run fit on the estimator with randomly drawn parameters.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>inverse_transform(*args, **kwargs)</code></td>
<td>Call inverse_transform on the estimator with the best found params.</td>
</tr>
<tr>
<td><code>predict(*args, **kwargs)</code></td>
<td>Call predict on the estimator with the best found parameters.</td>
</tr>
<tr>
<td><code>predict_log_proba(*args, **kwargs)</code></td>
<td>Call predict_log_proba on the estimator with the best found parameters.</td>
</tr>
<tr>
<td><code>predict_proba(*args, **kwargs)</code></td>
<td>Call predict_proba on the estimator with the best found parameters.</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the score on the given data, if the estimator has been refit.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(*args, **kwargs)</code></td>
<td>Call transform on the estimator with the best found parameters.</td>
</tr>
</tbody>
</table>

```
__init__(estimator, param_distributions, n_iter=10, scoring=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, pre_dispatch='2*n_jobs', random_state=None, error_score='raise', return_train_score=True)

decision_function(*args, **kwargs)
```

Call decision_function on the estimator with the best found parameters.
Only available if `refit=True` and the underlying estimator supports `decision_function`.

**Parameters**

- `X`: indexable, length `n_samples`
  
  Must fulfill the input assumptions of the underlying estimator.

**fit** *(X, y=None, groups=None)*

Run fit on the estimator with randomly drawn parameters.

**Parameters**

- `X`: array-like, shape = `[n_samples, n_features]`
  
  Training vector, where `n_samples` in the number of samples and `n_features` is the num-
  ber of features.

- `y`: array-like, shape = `[n_samples]` or `[n_samples, n_output]`, optional
  
  Target relative to `X` for classification or regression; None for unsupervised learning.

- `groups`: array-like, with shape `(n_samples,)`, optional
  
  Group labels for the samples used while splitting the dataset into train/test set.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- `deep`: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are
  estimators.

**Returns**

- `params`: mapping of string to any
  
  Parameter names mapped to their values.

**inverse_transform** *(args, **kwargs)*

Call `inverse_transform` on the estimator with the best found params.

Only available if the underlying estimator implements `inverse_transform` and `refit=True`.

**Parameters**

- `Xt`: indexable, length `n_samples`
  
  Must fulfill the input assumptions of the underlying estimator.

**predict** *(args, **kwargs)*

Call `predict` on the estimator with the best found parameters.

Only available if `refit=True` and the underlying estimator supports `predict`.

**Parameters**

- `X`: indexable, length `n_samples`
  
  Must fulfill the input assumptions of the underlying estimator.

**predict_log_proba** *(args, **kwargs)*

Call `predict_log_proba` on the estimator with the best found parameters.

Only available if `refit=True` and the underlying estimator supports `predict_log_proba`.

**Parameters**

- `X`: indexable, length `n_samples`
  
  Must fulfill the input assumptions of the underlying estimator.

**predict_proba** *(args, **kwargs)*

Call `predict_proba` on the estimator with the best found parameters.

Only available if `refit=True` and the underlying estimator supports `predict_proba`.

**Parameters**

- `X`: indexable, length `n_samples`
  
  Must fulfill the input assumptions of the underlying estimator.
**score**(*X*, *y=None*)

Returns the score on the given data, if the estimator has been refit.

This uses the score defined by `scoring` where provided, and the `best_estimator_.score` method otherwise.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Input data, where n_samples is the number of samples and n_features is the number of features.

- **y**: array-like, shape = [n_samples] or [n_samples, n_output], optional
  
  Target relative to X for classification or regression; None for unsupervised learning.

**Returns**

- **score**: float

**set_params**(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**transform**(*args, **kwargs*)

Call transform on the estimator with the best found parameters.

Only available if the underlying estimator supports `transform` and `refit=True`.

**Parameters**

- **X**: indexable, length n_samples
  
  Must fulfill the input assumptions of the underlying estimator.

Examples using `sklearn.model_selection.RandomizedSearchCV`

- Comparing randomized search and grid search for hyperparameter estimation

`sklearn.model_selection.ParameterGrid`

**class** `sklearn.model_selection.ParameterGrid`(param_grid)

Grid of parameters with a discrete number of values for each.

Can be used to iterate over parameter value combinations with the Python built-in function `iter`.

Read more in the User Guide.

**Parameters**

- **param_grid**: dict of string to sequence, or sequence of such
  
  The parameter grid to explore, as a dictionary mapping estimator parameters to sequences of allowed values.

  An empty dict signifies default parameters.

  A sequence of dicts signifies a sequence of grids to search, and is useful to avoid exploring parameter combinations that make no sense or have no effect. See the examples below.

**See also**

- GridSearchCV
**Uses**class `ParameterGrid` to perform a full parallelized parameter search.

**Examples**

```python
>>> from sklearn.model_selection import ParameterGrid
>>> param_grid = {'a': [1, 2], 'b': [True, False]}
>>> list(ParameterGrid(param_grid)) == ([
...     {'a': 1, 'b': True}, {'a': 1, 'b': False},
...     {'a': 2, 'b': True}, {'a': 2, 'b': False}])
True

>>> grid = [{'kernel': ['linear']}, {'kernel': ['rbf'], 'gamma': [1, 10]}]
>>> list(ParameterGrid(grid)) == [{'kernel': 'linear'},
...     {'kernel': 'rbf', 'gamma': 1},
...     {'kernel': 'rbf', 'gamma': 10}]
True
>>> ParameterGrid(grid)[1] == {'kernel': 'rbf', 'gamma': 1}
True
```

### `sklearn.model_selection.ParameterSampler`

**class** `sklearn.model_selection.ParameterSampler`(*param_distributions*,  *n_iter*,  *random_state=None*)

Generator on parameters sampled from given distributions.

Non-deterministic iterable over random candidate combinations for hyper-parameter search. If all parameters are presented as a list, sampling without replacement is performed. If at least one parameter is given as a distribution, sampling with replacement is used. It is highly recommended to use continuous distributions for continuous parameters.

Note that before SciPy 0.16, the `scipy.stats.distributions` do not accept a custom RNG instance and always use the singleton RNG from `numpy.random`. Hence setting `random_state` will not guarantee a deterministic iteration whenever `scipy.stats` distributions are used to define the parameter search space. Deterministic behavior is however guaranteed from SciPy 0.16 onwards.

Read more in the User Guide.

**Parameters**

- **param_distributions** : dict
  Dictionary where the keys are parameters and values are distributions from which a parameter is to be sampled. Distributions either have to provide a `rvs` function to sample from them, or can be given as a list of values, where a uniform distribution is assumed.

- **n_iter** : integer
  Number of parameter settings that are produced.

- **random_state** : int or RandomState
  Pseudo random number generator state used for random uniform sampling from lists of possible values instead of `scipy.stats` distributions.

**Returns**

- **params** : dict of string to any
  Yields dictionaries mapping each estimator parameter to as sampled value.
Examples

```python
>>> from sklearn.model_selection import ParameterSampler
>>> from scipy.stats.distributions import expon
>>> import numpy as np

>>> np.random.seed(0)

>>> param_grid = {'a':[1, 2], 'b': expon()}

>>> param_list = list(ParameterSampler(param_grid, n_iter=4))

>>> rounded_list = [dict((k, round(v, 6)) for (k, v) in d.items()) for d in param_list]

>>> rounded_list == [{'b': 0.89856, 'a': 1},
...                   {'b': 0.923223, 'a': 1},
...                   {'b': 1.878964, 'a': 2},
...                   {'b': 1.038159, 'a': 2}]
True
```

.. automethod:: __init__

**model_selection.fit_grid_point** *(X, y, ...) ->* Run fit on one set of parameters.

**sklearn.model_selection.fit_grid_point**

Run fit on one set of parameters.

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>array-like, sparse matrix or list</td>
</tr>
<tr>
<td>y</td>
<td>array-like or None</td>
</tr>
<tr>
<td>estimator</td>
<td>estimator object</td>
</tr>
<tr>
<td>parameters</td>
<td>dict</td>
</tr>
<tr>
<td>train</td>
<td>ndarray, dtype int or bool</td>
</tr>
<tr>
<td>test</td>
<td>ndarray, dtype int or bool</td>
</tr>
<tr>
<td>scorer</td>
<td>callable or None</td>
</tr>
<tr>
<td>verbose</td>
<td>int</td>
</tr>
</tbody>
</table>

**sklearn.model_selection.fit_grid_point** *(X, y, estimator, parameters, train, test, scorer, verbose, error_score='raise', **fit_params)*

Run fit on one set of parameters.

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>array-like, sparse matrix or list</td>
</tr>
<tr>
<td>y</td>
<td>array-like or None</td>
</tr>
<tr>
<td>estimator</td>
<td>estimator object</td>
</tr>
<tr>
<td>parameters</td>
<td>dict</td>
</tr>
<tr>
<td>train</td>
<td>ndarray, dtype int or bool</td>
</tr>
<tr>
<td>test</td>
<td>ndarray, dtype int or bool</td>
</tr>
<tr>
<td>scorer</td>
<td>callable or None</td>
</tr>
<tr>
<td>verbose</td>
<td>int</td>
</tr>
</tbody>
</table>

**sklearn.model_selection.fit_grid_point** *(X, y, ...[, ...]) ->* Run fit on one set of parameters.
Additional parameter passed to the fit function of the estimator.

**error_score** : ‘raise’ (default) or numeric

Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.

**returns**

score : float

Score of this parameter setting on given training / test split.

parameters : dict

The parameters that have been evaluated.

n_samples_test : int

Number of test samples in this split.

### 29.5.4 Model validation

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>model_selection.cross_val_score</code> (estimator, X)</td>
<td>Evaluate a score by cross-validation</td>
</tr>
<tr>
<td><code>model_selection.cross_val_predict</code> (estimator, X)</td>
<td>Generate cross-validated estimates for each input data point</td>
</tr>
<tr>
<td><code>model_selection.permutation_test_score</code> (...)</td>
<td>Evaluate the significance of a cross-validated score with permutations</td>
</tr>
<tr>
<td><code>model_selection.learning_curve</code> (estimator, X, y)</td>
<td>Learning curve.</td>
</tr>
<tr>
<td><code>model_selection.validation_curve</code> (estimator, ...)</td>
<td>Validation curve.</td>
</tr>
</tbody>
</table>

**sklearn.model_selection.cross_val_score**

Evaluate a score by cross-validation

Read more in the [User Guide](#).

**Parameters**

- **estimator** : estimator object implementing ‘fit’
  
The object to use to fit the data.

- **X** : array-like
  
The data to fit. Can be, for example a list, or an array at least 2d.

- **y** : array-like, optional, default: None
  
The target variable to try to predict in the case of supervised learning.

- **groups** : array-like, with shape (n_samples,), optional
  
  Group labels for the samples used while splitting the dataset into train/test set.

- **scoring** : string, callable or None, optional, default: None
  
  scoring=..., cv=None, n_jobs=1, verbose=0, fit_params=None, pre_dispatch='2*n_jobs')
A string (see model evaluation documentation) or a scorer callable object / function with
signature `scorer(estimator, X, y)`.

cv : int, cross-validation generator or an iterable, optional
Determines the cross-validation splitting strategy. Possible inputs for cv are:
  • None, to use the default 3-fold cross validation,
  • integer, to specify the number of folds in a (Stratified)KFold,
  • An object to be used as a cross-validation generator.
  • An iterable yielding train, test splits.
For integer/None inputs, if the estimator is a classifier and y is either binary or multi-
class, `StratifiedKFold` is used. In all other cases, `KFold` is used.
Refer `User Guide` for the various cross-validation strategies that can be used here.

n_jobs : integer, optional
The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

verbose : integer, optional
The verbosity level.

fit_params : dict, optional
Parameters to pass to the fit method of the estimator.

pre_dispatch : int, or string, optional
Controls the number of jobs that get dispatched during parallel execution. Reducing
this number can be useful to avoid an explosion of memory consumption when more
jobs get dispatched than CPUs can process. This parameter can be:
  • None, in which case all the jobs are immediately created and spawned. Use this for
    lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the
    jobs
  • An int, giving the exact number of total jobs that are spawned
  • A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

Returnsscores : array of float, shape=(len(list(cv)),)
Array of scores of the estimator for each run of the cross validation.

See also:

`sklearn.metrics.make_scorer` Make a scorer from a performance metric or loss function.

Examples

```python
>>> from sklearn import datasets, linear_model
>>> from sklearn.model_selection import cross_val_score
diabetes = datasets.load_diabetes()
X = diabetes.data[:150]
y = diabetes.target[:150]
lasso = linear_model.Lasso()
print(cross_val_score(lasso, X, y))
[ 0.33150734 0.08022311 0.03531764]
```
Examples using `sklearn.model_selection.cross_val_score`

- Imputing missing values before building an estimator
- Model selection with Probabilistic PCA and Factor Analysis (FA)
- Cross-validation on diabetes Dataset Exercise
- Cross-validation on Digits Dataset Exercise
- Nested versus non-nested cross-validation
- Underfitting vs. Overfitting
- SVM-Anova: SVM with univariate feature selection

`sklearn.model_selection.cross_val_predict`

`sklearn.model_selection.cross_val_predict(estimator, X, y=None, groups=None, cv=None, n_jobs=1, verbose=0, fit_params=None, pre_dispatch='2*n_jobs', method='predict')`

Generate cross-validated estimates for each input data point.

Read more in the User Guide.

**Parameters**

- **estimator**: estimator object implementing ‘fit’ and ‘predict’
  - The object to use to fit the data.
- **X**: array-like
  - The data to fit. Can be, for example a list, or an array at least 2d.
- **y**: array-like, optional, default: None
  - The target variable to try to predict in the case of supervised learning.
- **groups**: array-like, with shape (n_samples,), optional
  - Group labels for the samples used while splitting the dataset into train/test set.
- **cv**: int, cross-validation generator or an iterable, optional
  - Determines the cross-validation splitting strategy. Possible inputs for cv are:
    - None, to use the default 3-fold cross validation,
    - integer, to specify the number of folds in a (Stratified)KFold,
    - An object to be used as a cross-validation generator.
    - An iterable yielding train, test splits.
  - For integer/None inputs, if the estimator is a classifier and y is either binary or multi-class, StratifiedKFold is used. In all other cases, KFold is used.
  - Refer User Guide for the various cross-validation strategies that can be used here.
- **n_jobs**: integer, optional
  - The number of CPUs to use to do the computation. -1 means ‘all CPUs’.
- **verbose**: integer, optional
  - The verbosity level.
- **fit_params**: dict, optional
Parameters to pass to the fit method of the estimator.

**pre_dispatch** : int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
- An int, giving the exact number of total jobs that are spawned
- A string, giving an expression as a function of n_jobs, as in `2*n_jobs`

**method** : string, optional, default: ‘predict’

Invokes the passed method name of the passed estimator.

**Returns**

predictions : ndarray

This is the result of calling `method`

**Examples**

```python
>>> from sklearn import datasets, linear_model
>>> from sklearn.model_selection import cross_val_predict
>>> diabetes = datasets.load_diabetes()
>>> X = diabetes.data[:150]
>>> y = diabetes.target[:150]
>>> lasso = linear_model.Lasso()
>>> y_pred = cross_val_predict(lasso, X, y)
```

**Examples using sklearn.model_selection.cross_val_predict**

- **Plotting Cross-Validated Predictions**

**sklearn.model_selection.permutation_test_score**

```python
sklearn.model_selection.permutation_test_score(estimator, X, y, groups=None, cv=None, n_permutations=100, n_jobs=1, random_state=0, verbose=0, scoring=None)
```

Evaluate the significance of a cross-validated score with permutations

Read more in the **User Guide**.

**Parameters**

- **estimator** : estimator object implementing ‘fit’

  The object to use to fit the data.

  - **X** : array-like of shape at least 2D

    The data to fit.

  - **y** : array-like

    The target variable to try to predict in the case of supervised learning.
groups : array-like, with shape (n_samples,), optional

Labels to constrain permutation within groups, i.e. y values are permuted among samples with the same group identifier. When not specified, y values are permuted among all samples.

When a grouped cross-validator is used, the group labels are also passed on to the split method of the cross-validator. The cross-validator uses them for grouping the samples while splitting the dataset into train/test set.

scoring : string, callable or None, optional, default: None

A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y).

cv : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

• None, to use the default 3-fold cross validation,

• integer, to specify the number of folds in a (Stratified)KFold,

• An object to be used as a cross-validation generator.

• An iterable yielding train, test splits.

For integer/None inputs, if the estimator is a classifier and y is either binary or multiclass, StratifiedKFold is used. In all other cases, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

n_permutations : integer, optional

Number of times to permute y.

n_jobs : integer, optional

The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

random_state : RandomState or an int seed (0 by default)

A random number generator instance to define the state of the random permutations generator.

verbose : integer, optional

The verbosity level.

Returns

score : float

The true score without permuting targets.

permutation_scores : array, shape (n_permutations,)

The scores obtained for each permutations.

pvalue : float

The returned value equals p-value if scoring returns bigger numbers for better scores (e.g., accuracy_score). If scoring is rather a loss function (i.e. when lower is better such as with mean_squared_error) then this is actually the complement of the p-value: 1 - p-value.
Notes

This function implements Test 1 in:


Examples using sklearn.model_selection.permutation_test_score

• Test with permutations the significance of a classification score

sklearn.model_selection.learning_curve

sklearn.model_selection.learning_curve(estimator, X, y, groups=None, train_sizes=array([0.1, 0.33, 0.55, 0.78, 1.]), cv=None, scoring=None, exploit_incremental_learning=False, n_jobs=1, pre_dispatch='all', verbose=0)

Learning curve.

Determines cross-validated training and test scores for different training set sizes.

A cross-validation generator splits the whole dataset k times in training and test data. Subsets of the training set with varying sizes will be used to train the estimator and a score for each training subset size and the test set will be computed. Afterwards, the scores will be averaged over all k runs for each training subset size.

Read more in the User Guide.

Parameters

estimator : object type that implements the “fit” and “predict” methods

An object of that type which is cloned for each validation.

X : array-like, shape (n_samples, n_features)

Training vector, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape (n_samples) or (n_samples, n_features), optional

Target relative to X for classification or regression; None for unsupervised learning.

groups : array-like, with shape (n_samples,), optional

Group labels for the samples used while splitting the dataset into train/test set.

train_sizes : array-like, shape (n_ticks,), dtype float or int

Relative or absolute numbers of training examples that will be used to generate the learning curve. If the dtype is float, it is regarded as a fraction of the maximum size of the training set (that is determined by the selected validation method), i.e. it has to be within (0, 1]. Otherwise it is interpreted as absolute sizes of the training sets. Note that for classification the number of samples usually have to be big enough to contain at least one sample from each class. (default: np.linspace(0.1, 1.0, 5))

cv : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

• None, to use the default 3-fold cross validation,

• integer, to specify the number of folds in a (Stratified)KFold,

• An object to be used as a cross-validation generator.
• An iterable yielding train, test splits.

For integer/None inputs, if the estimator is a classifier and \( y \) is either binary or multiclass, StratifiedKFold is used. In all other cases, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

scoring : string, callable or None, optional, default: None

A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y).

exploit_incremental_learning : boolean, optional, default: False

If the estimator supports incremental learning, this will be used to speed up fitting for different training set sizes.

n_jobs : integer, optional

Number of jobs to run in parallel (default 1).

pre_dispatch : integer or string, optional

Number of predispatched jobs for parallel execution (default is all). The option can reduce the allocated memory. The string can be an expression like ‘2*n_jobs’.

verbose : integer, optional

Controls the verbosity: the higher, the more messages.

Return

strain_sizes_abs : array, shape = (n_unique_ticks,), dtype int

Numbers of training examples that has been used to generate the learning curve. Note that the number of ticks might be less than n_ticks because duplicate entries will be removed.

train_scores : array, shape (n_ticks, n_cv_folds)

Scores on training sets.

test_scores : array, shape (n_ticks, n_cv_folds)

Scores on test set.

Notes

See examples/model_selection/plot_learning_curve.py

sklearn.model_selection.validation_curve

sklearn.model_selection.validation_curve(estimator, X, y, param_name, param_range, groups=None, cv=None, scoring=None, n_jobs=1, pre_dispatch='all', verbose=0)

Validation curve.

Determine training and test scores for varying parameter values.

Compute scores for an estimator with different values of a specified parameter. This is similar to grid search with one parameter. However, this will also compute training scores and is merely a utility for plotting the results.

Read more in the User Guide.
Parameters:

- **estimator**: object type that implements the “fit” and “predict” methods
  - An object of that type which is cloned for each validation.
- **X**: array-like, shape (n_samples, n_features)
  - Training vector, where n_samples is the number of samples and n_features is the number of features.
- **y**: array-like, shape (n_samples) or (n_samples, n_features), optional
  - Target relative to X for classification or regression; None for unsupervised learning.
- **param_name**: string
  - Name of the parameter that will be varied.
- **param_range**: array-like, shape (n_values,)
  - The values of the parameter that will be evaluated.
- **groups**: array-like, with shape (n_samples,), optional
  - Group labels for the samples used while splitting the dataset into train/test set.
- **cv**: int, cross-validation generator or an iterable, optional
  - Determines the cross-validation splitting strategy. Possible inputs for cv are:
    - None, to use the default 3-fold cross validation,
    - integer, to specify the number of folds in a (Stratified)KFold,
    - An object to be used as a cross-validation generator.
    - An iterable yielding train, test splits.
  - For integer/None inputs, if the estimator is a classifier and y is either binary or multi-class, StratifiedKFold is used. In all other cases, KFold is used.
  - Refer [User Guide](#) for the various cross-validation strategies that can be used here.
- **scoring**: string, callable or None, optional, default: None
  - A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y).
- **n_jobs**: integer, optional
  - Number of jobs to run in parallel (default 1).
- **pre_dispatch**: integer or string, optional
  - Number of predispatched jobs for parallel execution (default is all). The option can reduce the allocated memory. The string can be an expression like ‘2*n_jobs’.
- **verbose**: integer, optional
  - Controls the verbosity: the higher, the more messages.
- **Returnstrain_scores**: array, shape (n_ticks, n_cv_folds)
  - Scores on training sets.
- **test_scores**: array, shape (n_ticks, n_cv_folds)
  - Scores on test set.
Notes

See Plotting Validation Curves

Examples using sklearn.model_selection.validation_curve

• Plotting Validation Curves

29.6 sklearn.datasets: Datasets

The sklearn.datasets module includes utilities to load datasets, including methods to load and fetch popular reference datasets. It also features some artificial data generators.

User guide: See the Dataset loading utilities section for further details.

29.6.1 Loaders

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>datasets.clear_data_home(data_home)</td>
<td>Delete all the content of the data home cache.</td>
</tr>
<tr>
<td>datasets.get_data_home(data_home)</td>
<td>Return the path of the scikit-learn data dir.</td>
</tr>
<tr>
<td>datasets.fetch_20newsgroups(data_home, ...)</td>
<td>Load the filenames and data from the 20 newsgroups dataset.</td>
</tr>
<tr>
<td>datasets.fetch_20newsgroups_vectorized(...)</td>
<td>Load the 20 newsgroups dataset and transform it into tf-idf vectors.</td>
</tr>
<tr>
<td>datasets.load_boston(return_X_y)</td>
<td>Load and return the boston house-prices dataset (regression).</td>
</tr>
<tr>
<td>datasets.load_breast_cancer(return_X_y)</td>
<td>Load and return the breast cancer wisconsin dataset (classification).</td>
</tr>
<tr>
<td>datasets.load_diabetes(return_X_y)</td>
<td>Load and return the diabetes dataset (regression).</td>
</tr>
<tr>
<td>datasets.load_digits(n_class, return_X_y)</td>
<td>Load and return the digits dataset (classification).</td>
</tr>
<tr>
<td>datasets.load_files(container_path, ...)</td>
<td>Load text files with categories as subfolder names.</td>
</tr>
<tr>
<td>datasets.load_iris(return_X_y)</td>
<td>Load and return the iris dataset (classification).</td>
</tr>
<tr>
<td>datasets.fetch_lfw_pairs(subset, ...)</td>
<td>Loader for the Labeled Faces in the Wild (LFW) pairs dataset.</td>
</tr>
<tr>
<td>datasets.fetch_lfw_people(data_home, ...)</td>
<td>Loader for the Labeled Faces in the Wild (LFW) people dataset.</td>
</tr>
<tr>
<td>datasets.load_linnerud(return_X_y)</td>
<td>Load and return the linnerud dataset (multivariate regression).</td>
</tr>
<tr>
<td>datasets.mldata_filename(dataname)</td>
<td>Convert a raw name for a data set in a mldata.org filename.</td>
</tr>
<tr>
<td>datasets.fetch_mldata(dataname, ...)</td>
<td>Fetch an mldata.org data set.</td>
</tr>
<tr>
<td>datasets.fetch_olivetti_faces(data_home, ...)</td>
<td>Loader for the Olivetti faces data-set from AT&amp;T.</td>
</tr>
<tr>
<td>datasets.fetch_california_housing(...)</td>
<td>Loader for the California housing dataset from StatLib.</td>
</tr>
<tr>
<td>datasets.fetch_covtype(data_home, ...)</td>
<td>Load the covertype dataset, downloading it if necessary.</td>
</tr>
<tr>
<td>datasets.fetch_kddcup99(subset, shuffle, ...)</td>
<td>Load and return the kddcup 99 dataset (classification).</td>
</tr>
<tr>
<td>datasets.fetch_rcv1(data_home, subset)</td>
<td>Load the RCV1 multilabel dataset, downloading it if necessary.</td>
</tr>
<tr>
<td>datasets.load_mlcomp(name_or_id[, set, ...])</td>
<td>Load a datasets as downloaded from <a href="http://mlcomp.org">http://mlcomp.org</a>.</td>
</tr>
<tr>
<td>datasets.load_sample_image(image_name)</td>
<td>Load the numpy array of a single sample image.</td>
</tr>
</tbody>
</table>
## Table 29.51 – continued from previous page

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>datasets.load_sample_images()</code></td>
<td>Load sample images for image manipulation.</td>
</tr>
<tr>
<td><code>datasets.fetch_species_distributions([...])</code></td>
<td>Loader for species distribution dataset from Phillips et.</td>
</tr>
<tr>
<td><code>datasets.load_svmlight_file(f[, n_features,...])</code></td>
<td>Load datasets in the svmlight / libsvm format into sparse CSR matrix</td>
</tr>
<tr>
<td><code>datasets.load_svmlight_files(files[, ...])</code></td>
<td>Load dataset from multiple files in SVMlight format</td>
</tr>
<tr>
<td><code>datasets.dump_svmlight_file(X, y, f[, ...])</code></td>
<td>Dump the dataset in svmlight / libsvm file format.</td>
</tr>
</tbody>
</table>

### sklearn.datasets.clear_data_home

#### sklearn.datasets.clear_data_home (data_home=None)

Delete all the content of the data home cache.

### sklearn.datasets.get_data_home

#### sklearn.datasets.get_data_home (data_home=None)

Return the path of the scikit-learn data dir.

This folder is used by some large dataset loaders to avoid downloading the data several times.

By default the data dir is set to a folder named ‘scikit_learn_data’ in the user home folder.

Alternatively, it can be set by the ‘SCIKIT_LEARN_DATA’ environment variable or programmatically by giving an explicit folder path. The ‘~’ symbol is expanded to the user home folder.

If the folder does not already exist, it is automatically created.

### Examples using sklearn.datasets.get_data_home

- Out-of-core classification of text documents

### sklearn.datasets.fetch_20newsgroups

#### sklearn.datasets.fetch_20newsgroups (data_home=None, subset='train', categories=None, shuffle=True, random_state=42, remove=(), download_if_missing=True)

Load the filenames and data from the 20 newsgroups dataset.

Read more in the User Guide.

#### Parameters

- **subset**: ‘train’ or ‘test’, ‘all’, optional
  
  Select the dataset to load: ‘train’ for the training set, ‘test’ for the test set, ‘all’ for both, with shuffled ordering.

- **data_home**: optional, default: None
  
  Specify a download and cache folder for the datasets. If None, all scikit-learn data is stored in ‘~/scikit_learn_data’ subfolders.

- **categories**: None or collection of string or unicode
  
  If None (default), load all the categories. If not None, list of category names to load (other categories ignored).

- **shuffle**: bool, optional
Whether or not to shuffle the data: might be important for models that make the assumption that the samples are independent and identically distributed (i.i.d.), such as stochastic gradient descent.

**random_state**: numpy random number generator or seed integer

Used to shuffle the dataset.

**download_if_missing**: optional, True by default

If False, raise an IOError if the data is not locally available instead of trying to download the data from the source site.

**remove**: tuple

May contain any subset of (‘headers’, ‘footers’, ‘quotes’). Each of these are kinds of text that will be detected and removed from the newsgroup posts, preventing classifiers from overfitting on metadata.

‘headers’ removes newsgroup headers, ‘footers’ removes blocks at the ends of posts that look like signatures, and ‘quotes’ removes lines that appear to be quoting another post.

‘headers’ follows an exact standard; the other filters are not always correct.

**Examples using sklearn.datasets.fetch_20newsgroups**

- Feature Union with Heterogeneous Data Sources
- Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation
- Biclustering documents with the Spectral Co-clustering algorithm
- Sample pipeline for text feature extraction and evaluation
- Classification of text documents using sparse features
- Clustering text documents using k-means
- FeatureHasher and DictVectorizer Comparison

**sklearn.datasets.fetch_20newsgroups_vectorized**

*sklearn.datasets.fetch_20newsgroups_vectorized*(subset='train', remove=(), data_home=None)

Load the 20 newsgroups dataset and transform it into tf-idf vectors.

This is a convenience function; the tf-idf transformation is done using the default settings for *sklearn.feature_extraction.textVectorizer*. For more advanced usage (stopword filtering, n-gram extraction, etc.), combine fetch_20newsgroups with a custom *Vectorizer* or *CountVectorizer*.

Read more in the *User Guide*.

**Parameters**

- **subset**: ‘train’ or ‘test’, ‘all’, optional

  Select the dataset to load: ‘train’ for the training set, ‘test’ for the test set, ‘all’ for both, with shuffled ordering.

- **data_home**: optional, default: None

  Specify an download and cache folder for the datasets. If None, all scikit-learn data is stored in ‘~/scikit_learn_data’ subfolders.

- **remove**: tuple
May contain any subset of ('headers', 'footers', 'quotes'). Each of these are kinds of text that will be detected and removed from the newsgroup posts, preventing classifiers from overfitting on metadata.

‘headers’ removes newsgroup headers, ‘footers’ removes blocks at the ends of posts that look like signatures, and ‘quotes’ removes lines that appear to be quoting another post.

**Returns**

bunch : Bunch object

bunch.data: sparse matrix, shape [n_samples, n_features]  
bunch.target: array, shape [n_samples]  
bunch.target_names: list, length [n_classes]

**Examples using sklearn.datasets.fetch_20newsgroups_vectorized**

- The Johnson-Lindenstrauss bound for embedding with random projections
- Model Complexity Influence

**sklearn.datasets.load_boston**

sklearn.datasets.load_boston(return_X_y=False)

Load and return the boston house-prices dataset (regression).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>return_X_y</td>
<td>boolean, default=False. If True, returns (data, target) instead of a Bunch object. See below for more information about the data and target object. New in version 0.18.</td>
</tr>
</tbody>
</table>

**Parameters**

- return_X_y : boolean, default=False.

- If True, returns (data, target) instead of a Bunch object. See below for more information about the data and target object.

**Returns**

- data : Bunch

  Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘target’, the regression targets, and ‘DESCR’, the full description of the dataset.

  (data, target) : tuple if return_X_y is True

  New in version 0.18.

**Examples**

```python
>>> from sklearn.datasets import load_boston
>>> boston = load_boston()
>>> print(boston.data.shape)
(506, 13)
```

**Examples using sklearn.datasets.load_boston**

- Imputing missing values before building an estimator
- Plotting Cross-Validated Predictions
sklearn.datasets.load_breast_cancer

sklearn.datasets.load_breast_cancer(return_X_y=False)
Load and return the breast cancer wisconsin dataset (classification).

The breast cancer dataset is a classic and very easy binary classification dataset.

<table>
<thead>
<tr>
<th>Classes</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samples per class</td>
<td>212(M), 357(B)</td>
</tr>
<tr>
<td>Samples total</td>
<td>569</td>
</tr>
<tr>
<td>Dimensionality</td>
<td>30</td>
</tr>
<tr>
<td>Features</td>
<td>real, positive</td>
</tr>
</tbody>
</table>

Parameters

return_X_y : boolean, default=False
If True, returns (data, target) instead of a Bunch object. See below for more information about the data and target object.
New in version 0.18.

Returns

data : Bunch
Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘target’, the classification labels, ‘target_names’, the meaning of the labels, ‘feature_names’, the meaning of the features, and ‘DESCR’, the full description of the dataset.

(data, target) : tuple if return_X_y is True
New in version 0.18.

The copy of UCI ML Breast Cancer Wisconsin (Diagnostic) dataset is:
downloaded from: : 
https://goo.gl/U2Uwz2 :

Examples

Let’s say you are interested in the samples 10, 50, and 85, and want to know their class name.

```python
>>> from sklearn.datasets import load_breast_cancer
>>> data = load_breast_cancer()
>>> data.target[[10, 50, 85]]
array([0, 1, 0])
>>> list(data.target_names)
['malignant', 'benign']
```
sklearn.datasets.load_diabetes

sklearn.datasets.load_diabetes(return_X_y=False)
Load and return the diabetes dataset (regression).

<table>
<thead>
<tr>
<th>Samples total</th>
<th>442</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionality</td>
<td>10</td>
</tr>
<tr>
<td>Features</td>
<td>real, (-0.2 &lt; x &lt; 0.2)</td>
</tr>
<tr>
<td>Targets</td>
<td>integer 25 - 346</td>
</tr>
</tbody>
</table>

Read more in the User Guide.

Parameters

**return_X_y**: boolean, default=False.
If True, returns \((\text{data}, \text{target})\) instead of a Bunch object. See below for more information about the \text{data} and \text{target} object.
New in version 0.18.

Returns

**data**: Bunch
Dictionary-like object, the interesting attributes are: `data`, the data to learn and `target`, the regression target for each sample.

\((\text{data}, \text{target})\): tuple if return\_X\_y is True
New in version 0.18.

Examples using `sklearn.datasets.load_diabetes`

- Cross-validation on diabetes Dataset Exercise
- Lasso and Elastic Net
- Lasso path using LARS
- Lasso model selection: Cross-Validation / AIC / BIC
- Linear Regression Example
- Sparsity Example: Fitting only features 1 and 2

sklearn.datasets.load_digits

sklearn.datasets.load_digits(n_class=10, return_X_y=False)
Load and return the digits dataset (classification).

Each datapoint is a 8x8 image of a digit.

<table>
<thead>
<tr>
<th>Classes</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samples per class</td>
<td>~180</td>
</tr>
<tr>
<td>Samples total</td>
<td>1797</td>
</tr>
<tr>
<td>Dimensionality</td>
<td>64</td>
</tr>
<tr>
<td>Features</td>
<td>integers 0-16</td>
</tr>
</tbody>
</table>

Read more in the User Guide.

Parameters

**n_class**: integer, between 0 and 10, optional (default=10)
The number of classes to return.

**return_X_y**: boolean, default=False.
If True, returns \( (\text{data}, \text{target}) \) instead of a Bunch object. See below for more information about the \( \text{data} \) and \( \text{target} \) object.

New in version 0.18.

**Returns**

**data** : Bunch

Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘images’, the images corresponding to each sample, ‘target’, the classification labels for each sample, ‘target_names’, the meaning of the labels, and ‘DESCR’, the full description of the dataset.

\( (\text{data}, \text{target}) \) : tuple if \( \text{return\_X\_y} \) is True

New in version 0.18.

**Examples**

To load the data and visualize the images:

```python
>>> from sklearn.datasets import load_digits
>>> digits = load_digits()
>>> print(digits.data.shape)
(1797, 64)
>>> import matplotlib.pyplot as plt
>>> plt.gray()
>>> plt.matshow(digits.images[0])
>>> plt.show()
```

**Examples using sklearn.datasets.load_digits**

- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Pipelining: chaining a PCA and a logistic regression
- The Johnson-Lindenstrauss bound for embedding with random projections
- Explicit feature map approximation for RBF kernels
- Recognizing hand-written digits
- Feature agglomeration
- Various Agglomerative Clustering on a 2D embedding of digits
- A demo of K-Means clustering on the handwritten digits data
- The Digit Dataset
- Digits Classification Exercise
- Cross-validation on Digits Dataset Exercise
- Recursive feature elimination
- \( L1 \) Penalty and Sparsity in Logistic Regression
- Comparing various online solvers
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...
- Parameter estimation using grid search with cross-validation
• Plotting Learning Curves
• Plotting Validation Curves
• Comparing randomized search and grid search for hyperparameter estimation
• Kernel Density Estimation
• Compare Stochastic learning strategies for MLPClassifier
• Restricted Boltzmann Machine features for digit classification
• Label Propagation digits: Demonstrating performance
• Label Propagation digits active learning
• SVM-Anova: SVM with univariate feature selection

sklearn.datasets.load_files

sklearn.datasets.load_files(container_path, description=None, categories=None, load_content=True, shuffle=True, encoding=None, decode_error='strict', random_state=0)

Load text files with categories as subfolder names.

Individual samples are assumed to be files stored a two levels folder structure such as the following:

```
container_folder/
  category_1_folder/file_1.txt file_2.txt ... file_42.txt
  category_2_folder/file_43.txt file_44.txt ...
```

The folder names are used as supervised signal label names. The individual file names are not important.

This function does not try to extract features into a numpy array or scipy sparse matrix. In addition, if load_content is false it does not try to load the files in memory.

To use text files in a scikit-learn classification or clustering algorithm, you will need to use the sklearn.feature_extraction.text module to build a feature extraction transformer that suits your problem.

If you set load_content=True, you should also specify the encoding of the text using the ‘encoding’ parameter. For many modern text files, ‘utf-8’ will be the correct encoding. If you leave encoding equal to None, then the content will be made of bytes instead of Unicode, and you will not be able to use most functions in sklearn.feature_extraction.text.

Similar feature extractors should be built for other kind of unstructured data input such as images, audio, video, ...

Read more in the User Guide.

Parameters

container_path : string or unicode

Path to the main folder holding one subfolder per category

description: string or unicode, optional (default=None):

A paragraph describing the characteristic of the dataset: its source, reference, etc.

categories : A collection of strings or None, optional (default=None)

If None (default), load all the categories. If not None, list of category names to load (other categories ignored).

load_content : boolean, optional (default=True)
Whether to load or not the content of the different files. If true a ‘data’ attribute containing the text information is present in the data structure returned. If not, a filenames attribute gives the path to the files.

**encoding** : string or None (default is None)

If None, do not try to decode the content of the files (e.g. for images or other non-text content). If not None, encoding to use to decode text files to Unicode if load_content is True.

**decode_error** : {'strict', 'ignore', 'replace'}, optional :

Instruction on what to do if a byte sequence is given to analyze that contains characters not of the given encoding. Passed as keyword argument ‘errors’ to bytes.decode.

**shuffle** : bool, optional (default=True)

Whether or not to shuffle the data: might be important for models that make the assumption that the samples are independent and identically distributed (i.i.d.), such as stochastic gradient descent.

**random_state** : int, RandomState instance or None, optional (default=0)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

Dictionary-like object, the interesting attributes are: either data, the raw text data to learn, or ‘filenames’, the files holding it, ‘target’, the classification labels (integer index), ‘target_names’, the meaning of the labels, and ‘DESCR’, the full description of the dataset.

### sklearn.datasets.load_iris

**sklearn.datasets.load_iris**(return_X_y=False)

Load and return the iris dataset (classification).

The iris dataset is a classic and very easy multi-class classification dataset.

<table>
<thead>
<tr>
<th>Classes</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samples per class</td>
<td>50</td>
</tr>
<tr>
<td>Samples total</td>
<td>150</td>
</tr>
<tr>
<td>Dimensionality</td>
<td>4</td>
</tr>
<tr>
<td>Features</td>
<td>real, positive</td>
</tr>
</tbody>
</table>


**Parameters**

**return_X_y** : boolean, default=False.

If True, returns (data, target) instead of a Bunch object. See below for more information about the data and target object.

New in version 0.18.

**Returns**

Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘target’, the classification labels, ‘target_names’, the meaning of the labels, ‘feature_names’, the meaning of the features, and ‘DESCR’, the full description of the dataset.

(data, target) : tuple if return_X_y is True
New in version 0.18.

Examples

Let’s say you are interested in the samples 10, 25, and 50, and want to know their class name.

```python
>>> from sklearn.datasets import load_iris
>>> data = load_iris()
>>> data.target[[10, 25, 50]]
array([0, 0, 1])
>>> list(data.target_names)
['setosa', 'versicolor', 'virginica']
```

Examples using `sklearn.datasets.load_iris`

- Concatenating multiple feature extraction methods
- Plot classification probability
- K-means Clustering
- The Iris Dataset
- Incremental PCA
- PCA example with Iris Data-set
- Comparison of LDA and PCA 2D projection of Iris dataset
- Plot the decision surfaces of ensembles of trees on the iris dataset
- Plot the decision boundaries of a VotingClassifier
- SVM Exercise
- Univariate Feature Selection
- Test with permutations the significance of a classification score
- Gaussian process classification (GPC) on iris dataset
- Logistic Regression 3-class Classifier
- Path with L1- Logistic Regression
- Plot multi-class SGD on the iris dataset
- GMM covariances
- Confusion matrix
- Nested versus non-nested cross-validation
- Precision-Recall
- Receiver Operating Characteristic (ROC)
- Receiver Operating Characteristic (ROC) with cross validation
- Nearest Neighbors Classification
- Nearest Centroid Classification
- Compare Stochastic learning strategies for MLPClassifier
• Decision boundary of label propagation versus SVM on the Iris dataset
• SVM with custom kernel
• Plot different SVM classifiers in the iris dataset
• RBF SVM parameters
• Plot the decision surface of a decision tree on the iris dataset
• Understanding the decision tree structure

sklearn.datasets.fetch_lfw_pairs

`sklearn.datasets.fetch_lfw_pairs(subset='train', data_home=None, funneled=True, resize=0.5, color=False, slice_=(slice(70, 195, None), slice(78, 172, None)), download_if_missing=True)`

Loader for the Labeled Faces in the Wild (LFW) pairs dataset

This dataset is a collection of JPEG pictures of famous people collected on the internet, all details are available on the official website:

http://vis-www.cs.umass.edu/lfw/

Each picture is centered on a single face. Each pixel of each channel (color in RGB) is encoded by a float in range 0.0 - 1.0.

The task is called Face Verification: given a pair of two pictures, a binary classifier must predict whether the two images are from the same person.

In the official README.txt this task is described as the “Restricted” task. As I am not sure as to implement the “Unrestricted” variant correctly, I left it as unsupported for now.

The original images are 250 x 250 pixels, but the default slice and resize arguments reduce them to 62 x 74.

Read more in the User Guide.

**Parameters**

**subset**: optional, default: ‘train’

Select the dataset to load: ‘train’ for the development training set, ‘test’ for the development test set, and ‘10_folds’ for the official evaluation set that is meant to be used with a 10-folds cross validation.

**data_home**: optional, default: None

Specify another download and cache folder for the datasets. By default all scikit-learn data is stored in ‘~/scikit_learn_data’ subfolders.

**funneled**: boolean, optional, default: True

Download and use the funneled variant of the dataset.

**resize**: float, optional, default 0.5

Ratio used to resize the each face picture.

**color**: boolean, optional, default False

Keep the 3 RGB channels instead of averaging them to a single gray level channel. If color is True the shape of the data has one more dimension than the shape with color = False.

**slice_**: optional
Provide a custom 2D slice (height, width) to extract the ‘interesting’ part of the jpeg files and avoid use statistical correlation from the background

download_if_missing : optional, True by default

If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

Returns The data is returned as a Bunch object with the following attributes:

data : numpy array of shape (2200, 5828). Shape depends on subset.

Each row corresponds to 2 ravel’d face images of original size 62 x 47 pixels. Changing the slice_, resize or subset parameters will change the shape of the output.

pairs : numpy array of shape (2200, 2, 62, 47). Shape depends on subset.

Each row has 2 face images corresponding to same or different person from the dataset containing 5749 people. Changing the slice_, resize or subset parameters will change the shape of the output.

target : numpy array of shape (2200,). Shape depends on subset.

Labels associated to each pair of images. The two label values being different persons or the same person.

DESCR : string

Description of the Labeled Faces in the Wild (LFW) dataset.

sklearn.datasets.fetch_lfw_people

Loader for the Labeled Faces in the Wild (LFW) people dataset

This dataset is a collection of JPEG pictures of famous people collected on the internet, all details are available on the official website:

http://vis-www.cs.umass.edu/lfw/

Each picture is centered on a single face. Each pixel of each channel (color in RGB) is encoded by a float in range 0.0 - 1.0.

The task is called Face Recognition (or Identification): given the picture of a face, find the name of the person given a training set (gallery).

The original images are 250 x 250 pixels, but the default slice and resize arguments reduce them to 62 x 74.

Parameters data_home : optional, default: None

Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

funneled : boolean, optional, default: True

Download and use the funneled variant of the dataset.

resize : float, optional, default 0.5

Ratio used to resize the each face picture.
min_faces_per_person : int, optional, default None

The extracted dataset will only retain pictures of people that have at least
min_faces_per_person different pictures.

color : boolean, optional, default False

Keep the 3 RGB channels instead of averaging them to a single gray level channel. If
color is True the shape of the data has one more dimension than the shape with color =
False.

slice_ : optional

Provide a custom 2D slice (height, width) to extract the ‘interesting’ part of the jpeg
files and avoid use statistical correlation from the background

download_if_missing : optional, True by default

If False, raise a IOError if the data is not locally available instead of trying to download
the data from the source site.

Returns dataset : dict-like object with the following attributes:

dataset.data : numpy array of shape (13233, 2914)

Each row corresponds to a ravelled face image of original size 62 x 47 pixels. Changing
the slice_ or resize parameters will change the shape of the output.

dataset.images : numpy array of shape (13233, 62, 47)

Each row is a face image corresponding to one of the 5749 people in the dataset. Chang-
ing the slice_ or resize parameters will change the shape of the output.

dataset.target : numpy array of shape (13233,)

Labels associated to each face image. Those labels range from 0-5748 and correspond
to the person IDs.

dataset.DESCR : string

Description of the Labeled Faces in the Wild (LFW) dataset.

Examples using sklearn.datasets.fetch_lfw_people

- Faces recognition example using eigenfaces and SVMs

sklearn.datasets.load_linnerud

sklearn.datasets.load_linnerud(return_X_y=False)

Load and return the linnerud dataset (multivariate regression).

Samples total: 20 Dimensionality: 3 for both data and targets Features: integer Targets: integer

Parameters return_X_y : boolean, default=False.

If True, returns (data, target) instead of a Bunch object. See below for more
information about the data and target object.

New in version 0.18.

Returns data : Bunch
Dictionary-like object, the interesting attributes are: ‘data’ and ‘targets’, the two multivariate datasets, with ‘data’ corresponding to the exercise and ‘targets’ corresponding to the physiological measurements, as well as ‘feature_names’ and ‘target_names’.

\[(data, target)\] : tuple if \(\text{return\_X\_y}\) is True

New in version 0.18.

**sklearn.datasets.mldata_filename**

sklearn.datasets.mldata_filename\((\text{dataname})\)

Convert a raw name for a data set in a mldata.org filename.

**sklearn.datasets.fetch_mldata**

sklearn.datasets.fetch_mldata\((\text{dataname}, \text{target\_name}=\text{\textquote{label}}, \text{data\_name}=\text{\textquote{data}}, \text{transpose\_data}=\text{True}, \text{data\_home}=\text{None})\)

Fetch an mldata.org data set

If the file does not exist yet, it is downloaded from mldata.org.

mldata.org does not have an enforced convention for storing data or naming the columns in a data set. The default behavior of this function works well with the most common cases:

1. data values are stored in the column ‘data’, and target values in the column ‘label’
2. alternatively, the first column stores target values, and the second data values
3. the data array is stored as \(n\_features \times n\_samples\) , and thus needs to be transposed to match the sklearn standard

Keyword arguments allow to adapt these defaults to specific data sets (see parameters target\_name, data\_name, transpose\_data, and the examples below).

mldata.org data sets may have multiple columns, which are stored in the Bunch object with their original name.

- **Parameters**
  - dataname : 
    - Name of the data set on mldata.org, e.g.: “leukemia”, “Whistler Daily Snowfall”, etc.
    - The raw name is automatically converted to a mldata.org URL.
  - target\_name : optional, default: ‘label’
    - Name or index of the column containing the target values.
  - data\_name : optional, default: ‘data’
    - Name or index of the column containing the data.
  - transpose\_data : optional, default: True
    - If True, transpose the downloaded data array.
  - data\_home : optional, default: None
    - Specify another download and cache folder for the data sets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

- **Returns**
  - data : Bunch
    - Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘target’, the classification labels, ‘DESCR’, the full description of the dataset, and ‘COL\_NAMES’, the original names of the dataset columns.
Examples

Load the ‘iris’ dataset from mldata.org:

```python
>>> from sklearn.datasets.mldata import fetch_mldata
>>> import tempfile
>>> test_data_home = tempfile.mkdtemp()

>>> iris = fetch_mldata('iris', data_home=test_data_home)
>>> iris.target.shape
(150,)
>>> iris.data.shape
(150, 4)
```

Load the ‘leukemia’ dataset from mldata.org, which needs to be transposed to respects the scikit-learn axes convention:

```python
>>> leuk = fetch_mldata('leukemia', transpose_data=True, data_home=test_data_home)
>>> leuk.data.shape
(72, 7129)
```

Load an alternative ‘iris’ dataset, which has different names for the columns:

```python
>>> iris2 = fetch_mldata('datasets-UCI iris', target_name=1, data_name=0, data_home=test_data_home)
>>> iris3 = fetch_mldata('datasets-UCI iris', target_name='class', data_name='double0', data_home=test_data_home)

>>> import shutil
>>> shutil.rmtree(test_data_home)
```

Examples using `sklearn.datasets.fetch_mldata`

- Gaussian process regression (GPR) on Mauna Loa CO2 data.
- Visualization of MLP weights on MNIST

`sklearn.datasets.fetch_olivetti_faces`

`sklearn.datasets.fetch_olivetti_faces(data_home=None, shuffle=False, random_state=0, download_if_missing=True)`

Loader for the Olivetti faces data-set from AT&T.

Read more in the User Guide.

Parameters

- **data_home**: optional, default: None
  Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

- **shuffle**: boolean, optional
  If True the order of the dataset is shuffled to avoid having images of the same person grouped.
download_if_missing : optional, True by default
If False, raise an IOError if the data is not locally available instead of trying to download
the data from the source site.

random_state : optional, integer or RandomState object
The seed or the random number generator used to shuffle the data.

ReturnsAn object with the following attributes:

data : numpy array of shape (400, 4096)
Each row corresponds to a ravelled face image of original size 64 x 64 pixels.

images : numpy array of shape (400, 64, 64)
Each row is a face image corresponding to one of the 40 subjects of the dataset.

target : numpy array of shape (400,)
Labels associated to each face image. Those labels are ranging from 0-39 and corre-
spond to the Subject IDs.

DESCR : string
Description of the modified Olivetti Faces Dataset.

Notes
This dataset consists of 10 pictures each of 40 individuals. The original database was available from (now
defunct)

http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html

The version retrieved here comes in MATLAB format from the personal web page of Sam Roweis:

http://www.cs.nyu.edu/~roweis/

Examples using sklearn.datasets.fetch_olivetti_faces

- Face completion with a multi-output estimators
- Online learning of a dictionary of parts of faces
- Faces dataset decompositions
- Pixel importances with a parallel forest of trees

sklearn.datasets.fetch_california_housing

sklearn.datasets.fetch_california_housing(data_home=None, download_if_missing=True)

Loader for the California housing dataset from StatLib.

Read more in the User Guide.

Parametersdata_home : optional, default: None
Specify another download and cache folder for the datasets. By default all scikit learn
data is stored in ‘~/.scikit_learn_data’ subfolders.
download_if_missing: optional, True by default:
If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

Returns:
dataset: dict-like object with the following attributes:

dataset.data: ndarray, shape [20640, 8]
Each row corresponding to the 8 feature values in order.

dataset.target: numpy array of shape (20640,)
Each value corresponds to the average house value in units of 100,000.

dataset.feature_names: array of length 8
Array of ordered feature names used in the dataset.

dataset.DESCR: string
Description of the California housing dataset.

Notes
This dataset consists of 20,640 samples and 9 features.

Examples using sklearn.datasets.fetch_california_housing

- Partial Dependence Plots

sklearn.datasets.fetch_covtype

sklearn.datasets.fetch_covtype (data_home=None, download_if_missing=True, random_state=None, shuffle=False)
Load the covtype dataset, downloading it if necessary.

Read more in the User Guide.

Parameters:
data_home: string, optional
Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

download_if_missing: boolean, default=True
If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

random_state: int, RandomState instance or None, optional (default=None)
Random state for shuffling the dataset. If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

shuffle: bool, default=False
Whether to shuffle dataset.
Returns

- **dataset**: dict-like object with the following attributes:
  - **dataset.data**: numpy array of shape (581012, 54)
    - Each row corresponds to the 54 features in the dataset.
  - **dataset.target**: numpy array of shape (581012,)
    - Each value corresponds to one of the 7 forest covertypes with values ranging between 1 to 7.
  - **dataset.DESCR**: string
    - Description of the forest covertype dataset.

**sklearn.datasets.fetch_kddcup99**

Load and return the kddcup 99 dataset (classification).

The KDD Cup ’99 dataset was created by processing the tcpdump portions of the 1998 DARPA Intrusion Detection System (IDS) Evaluation dataset, created by MIT Lincoln Lab [1]. The artificial data was generated using a closed network and hand-injected attacks to produce a large number of different types of attack with normal activity in the background. As the initial goal was to produce a large training set for supervised learning algorithms, there is a large proportion (80.1%) of abnormal data which is unrealistic in real world, and inappropriate for unsupervised anomaly detection which aims at detecting ‘abnormal’ data, ie

1. qualitatively different from normal data.
2. in large minority among the observations.

We thus transform the KDD Data set into two different data sets: SA and SF.

- SA is obtained by simply selecting all the normal data, and a small proportion of abnormal data to gives an anomaly proportion of 1%.
- SF is obtained as in [2] by simply picking up the data whose attribute logged_in is positive, thus focusing on the intrusion attack, which gives a proportion of 0.3% of attack.
- http and smtp are two subsets of SF corresponding with third feature equal to ‘http’ (resp. to ‘smtp’)

**General KDD structure**

<table>
<thead>
<tr>
<th>Samples total</th>
<th>4898431</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionality</td>
<td>41</td>
</tr>
<tr>
<td>Features</td>
<td>discrete (int) or continuous (float)</td>
</tr>
<tr>
<td>Targets</td>
<td>str, ‘normal.’ or name of the anomaly type</td>
</tr>
</tbody>
</table>

**SA structure**

<table>
<thead>
<tr>
<th>Samples total</th>
<th>976158</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionality</td>
<td>41</td>
</tr>
<tr>
<td>Features</td>
<td>discrete (int) or continuous (float)</td>
</tr>
<tr>
<td>Targets</td>
<td>str, ‘normal.’ or name of the anomaly type</td>
</tr>
</tbody>
</table>

**SF structure**

<table>
<thead>
<tr>
<th>Samples total</th>
<th>699691</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionality</td>
<td>4</td>
</tr>
<tr>
<td>Features</td>
<td>discrete (int) or continuous (float)</td>
</tr>
<tr>
<td>Targets</td>
<td>str, ‘normal.’ or name of the anomaly type</td>
</tr>
</tbody>
</table>

**http structure**
New in version 0.18.

Parameters

subset

To return the corresponding classical subsets of kddcup 99. If None, return the entire kddcup 99 dataset.

random_state

Random state for shuffling the dataset. If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

shuffle

Whether to shuffle dataset.

percent10

Whether to load only 10 percent of the data.

download_if_missing

If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

Returns
data

Dictionary-like object, the interesting attributes are: ‘data’, the data to learn and ‘target’, the regression target for each sample.

References

[R132], [R133]

sklearn.datasets.fetch_rcv1

Load the RCV1 multilabel dataset, downloading it if necessary.

Version: RCV1-v2, vectors, full sets, topics multilabels.

Classes 103
Samples total 804414
Dimensionality 47236
Features real, between 0 and 1
Read more in the *User Guide*.

New in version 0.17.

**Parameters**

**data_home**: string, optional

Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

**subset**: string, ‘train’, ‘test’, or ‘all’, default=’all’:

Select the dataset to load: ‘train’ for the training set (23149 samples), ‘test’ for the test set (781265 samples), ‘all’ for both, with the training samples first if shuffle is False. This follows the official LYRL2004 chronological split.

**download_if_missing**: boolean, default=True

If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

**random_state**: int, RandomState instance or None, optional (default=None)

Random state for shuffling the dataset. If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**shuffle**: bool, default=False

Whether to shuffle dataset.

**Returns**

**dataset**: dict-like object with the following attributes:

**dataset.data**: scipy csr array, dtype np.float64, shape (804414, 47236)

The array has 0.16% of non zero values.

**dataset.target**: scipy csr array, dtype np.uint8, shape (804414, 103)

Each sample has a value of 1 in its categories, and 0 in others. The array has 3.15% of non zero values.

**dataset.sample_id**: numpy array, dtype np.uint32, shape (804414,)

Identification number of each sample, as ordered in dataset.data.

**dataset.target_names**: numpy array, dtype object, length (103)

Names of each target (RCV1 topics), as ordered in dataset.target.

**dataset.DESCRI**: string

Description of the RCV1 dataset.

**References**


**sklearn.datasets.load_mlcomp**

Load a datasets as downloaded from http://mlcomp.org
Parameters

- **name_or_id**: the integer id or the string name metadata of the MLComp dataset to load
- **set_**: select the portion to load: ‘train’, ‘test’ or ‘raw’
- **mlcomp_root**: the filesystem path to the root folder where MLComp datasets are stored, if mlcomp_root is None, the MLCOMP_DATASETS_HOME environment variable is looked up instead.
- **kwargs**: domain specific kwargs to be passed to the dataset loader.


Returns

Dictionary-like object, the interesting attributes are: ‘filenames’, the files holding the raw to learn, ‘target’, the classification labels (integer index), ‘target_names’, the meaning of the labels, and ‘DESCR’, the full description of the dataset.

Note on the lookup process: depending on the type of name_or_id, will choose between integer id lookup or metadata name lookup by looking at the unzipped archives and metadata file.

TODO: implement zip dataset loading too

**Examples using sklearn.datasets.load_mlcomp**

- Classification of text documents: using a MLComp dataset

**sklearn.datasets.load_sample_image**

Load the numpy array of a single sample image

Parameters

- **image_name**: {'china.jpg', 'flower.jpg'}

The name of the sample image loaded

Returns

The image as a numpy array: height x width x color

**Examples**

```python
>>> from sklearn.datasets import load_sample_image
>>> china = load_sample_image('china.jpg')
>>> china.dtype
dtype('uint8')
>>> china.shape
(427, 640, 3)
>>> flower = load_sample_image('flower.jpg')
>>> flower.dtype
dtype('uint8')
>>> flower.shape
(427, 640, 3)
```
Examples using `sklearn.datasets.load_sample_images`

- Color Quantization using K-Means

`sklearn.datasets.load_sample_images`

`sklearn.datasets.load_sample_images()`

Load sample images for image manipulation. Loads both, `china` and `flower`.

**Returns**

Bunch

Dictionary-like object with the following attributes: ‘images’, the two sample images, ‘filenames’, the file names for the images, and ‘DESCR’ the full description of the dataset.

**Examples**

To load the data and visualize the images:

```python
>>> from sklearn.datasets import load_sample_images
>>> dataset = load_sample_images()
>>> len(dataset.images)
2
>>> first_img_data = dataset.images[0]
>>> first_img_data.shape
(427, 640, 3)
>>> first_img_data.dtype
dtype('uint8')
```

`sklearn.datasets.fetch_species_distributions`

`sklearn.datasets.fetch_species_distributions(data_home=None, download_if_missing=True)`

Loader for species distribution dataset from Phillips et. al. (2006)

Read more in the User Guide.

**Parameters**

- `data_home`: optional, default: None
  
  Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

- `download_if_missing`: optional, True by default
  
  If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

**Returns**

The data is returned as a Bunch object with the following attributes:

- `coverages`: array, shape = [14, 1592, 1212]
  
  These represent the 14 features measured at each point of the map grid. The latitude/longitude values for the grid are discussed below. Missing data is represented by the value -9999.

- `train`: record array, shape = (1623,)
  
  The training points for the data. Each point has three fields:
• train[‘species’] is the species name
• train[‘dd long’] is the longitude, in degrees
• train[‘dd lat’] is the latitude, in degrees

**test** : record array, shape = (619,)

The test points for the data. Same format as the training data.

**Nx, Ny** : integers

The number of longitudes (x) and latitudes (y) in the grid

**x_left_lower_corner, y_left_lower_corner** : floats

The (x,y) position of the lower-left corner, in degrees

**grid_size** : float

The spacing between points of the grid, in degrees

**Notes**

• See examples/applications/plot_species_distribution_modeling.py for an example of using this dataset with scikit-learn

**References**


**Examples using sklearn.datasets.fetch_species_distributions**

• *Species distribution modeling*

• *Kernel Density Estimate of Species Distributions*

**sklearn.datasets.load_svmlight_file**

Load datasets in the svmlight / libsvm format into sparse CSR matrix

This format is a text-based format, with one sample per line. It does not store zero valued features hence is suitable for sparse dataset.

The first element of each line can be used to store a target variable to predict.

This format is used as the default format for both svmlight and the libsvm command line programs.

Parsing a text based source can be expensive. When working on repeatedly on the same dataset, it is recommended to wrap this loader with joblib.Memory.cache to store a memmapped backup of the CSR results of the first call and benefit from the near instantaneous loading of memmapped structures for the subsequent calls.

In case the file contains a pairwise preference constraint (known as “qid” in the svmlight format) these are ignored unless the query_id parameter is set to True. These pairwise preference constraints can be used to
constraint the combination of samples when using pairwise loss functions (as is the case in some learning to rank problems) so that only pairs with the same query_id value are considered.

This implementation is written in Cython and is reasonably fast. However, a faster API-compatible loader is also available at:

https://github.com/mblondel/svmlight-loader

**Parameters**

{str, file-like, int}

( Path to) a file to load. If a path ends in ".gz" or ".bz2", it will be uncompressed on the fly. If an integer is passed, it is assumed to be a file descriptor. A file-like or file descriptor will not be closed by this function. A file-like object must be opened in binary mode.

n_features : int or None

The number of features to use. If None, it will be inferred. This argument is useful to load several files that are subsets of a bigger sliced dataset: each subset might not have examples of every feature, hence the inferred shape might vary from one slice to another.

multilabel : boolean, optional, default False

Samples may have several labels each (see http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/multilabel.html)

zero_based : boolean or “auto”, optional, default “auto”

Whether column indices in f are zero-based (True) or one-based (False). If column indices are one-based, they are transformed to zero-based to match Python/NumPy conventions. If set to “auto”, a heuristic check is applied to determine this from the file contents. Both kinds of files occur “in the wild”, but they are unfortunately not self-identifying. Using “auto” or True should always be safe.

query_id : boolean, default False

If True, will return the query_id array for each file.

dtype : numpy data type, default np.float64

Data type of dataset to be loaded. This will be the data type of the output numpy arrays X and y.

Returns:

X : scipy.sparse matrix of shape (n_samples, n_features)

y : ndarray of shape (n_samples,), or, in the multilabel a list of
tuples of length n_samples.

query_id : array of shape (n_samples,)

query_id for each sample. Only returned when query_id is set to True.

See also:

load_svmlight_files similar function for loading multiple files in this format, enforcing
Examples

To use joblib.Memory to cache the svmlight file:

```python
from sklearn.externals.joblib import Memory
from sklearn.datasets import load_svmlight_file
mem = Memory("./mycache")

@mem.cache
def get_data():
    data = load_svmlight_file("mysvmlightfile")
    return data[0], data[1]

X, y = get_data()
```

text:

Load dataset from multiple files in SVMlight format

This function is equivalent to mapping load_svmlight_file over a list of files, except that the results are concatenated into a single, flat list and the samples vectors are constrained to all have the same number of features.

In case the file contains a pairwise preference constraint (known as “qid” in the svmlight format) these are ignored unless the query_id parameter is set to True. These pairwise preference constraints can be used to constraint the combination of samples when using pairwise loss functions (as is the case in some learning to rank problems) so that only pairs with the same query_id value are considered.

**Parameters**

- `files`: iterable over {str, file-like, int}
  
  (Paths of) files to load. If a path ends in ".gz" or ".bz2", it will be uncompressed on the fly. If an integer is passed, it is assumed to be a file descriptor. File-likes and file descriptors will not be closed by this function. File-like objects must be opened in binary mode.

- `n_features`: int or None
  
  The number of features to use. If None, it will be inferred from the maximum column index occurring in any of the files.

  This can be set to a higher value than the actual number of features in any of the input files, but setting it to a lower value will cause an exception to be raised.

- `multilabel`: boolean, optional
  
  Samples may have several labels each (see http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/multilabel.html)

- `zero_based`: boolean or “auto”, optional
  
  Whether column indices in f are zero-based (True) or one-based (False). If column indices are one-based, they are transformed to zero-based to match Python/NumPy conventions. If set to “auto”, a heuristic check is applied to determine this from the file contents. Both kinds of files occur “in the wild”, but they are unfortunately not self-identifying. Using “auto” or True should always be safe.

- `query_id`: boolean, defaults to False

Reference:

29.6. sklearn.datasets: Datasets

1273
If True, will return the query_id array for each file.

**dtype**: numpy data type, default np.float64

Data type of dataset to be loaded. This will be the data type of the output numpy arrays `X` and `y`.

**Returns**

`[X1, y1, ..., Xn, yn]` :

where each `(Xi, yi)` pair is the result from `load_svmlight_file(files[i])` : 

If `query_id` is set to True, this will return instead `[X1, y1, q1, ...

..., Xn, yn, qn] where `(Xi, yi, qi)` is the result from :

`load_svmlight_file(files[i])` :

**See also:**

`load_svmlight_file`

**Notes**

When fitting a model to a matrix `X_train` and evaluating it against a matrix `X_test`, it is essential that `X_train` and `X_test` have the same number of features (`X_train.shape[1] == X_test.shape[1]`). This may not be the case if you load the files individually with `load_svmlight_file`.

`sklearn.datasets.dump_svmlight_file`

`sklearn.datasets.dump_svmlight_file(X, y, f, zero_based=True, comment=None, query_id=None, multilabel=False)`

Dump the dataset in svmlight / libsvm file format.

This format is a text-based format, with one sample per line. It does not store zero valued features hence is suitable for sparse dataset.

The first element of each line can be used to store a target variable to predict.

**Parameters**

- `X` : {array-like, sparse matrix}, shape = `[n_samples, n_features]`
  
  Training vectors, where `n_samples` is the number of samples and `n_features` is the number of features.

- `y` : {array-like, sparse matrix}, shape = `[n_samples, n_labels]`
  
  Target values. Class labels must be an integer or float, or array-like objects of integer or float for multilabel classifications.

- `f` : string or file-like in binary mode
  
  If string, specifies the path that will contain the data. If file-like, data will be written to `f`. `f` should be opened in binary mode.

- `zero_based` : boolean, optional
  
  Whether column indices should be written zero-based (True) or one-based (False).

- `comment` : string, optional
  
  Comment to insert at the top of the file. This should be either a Unicode string, which will be encoded as UTF-8, or an ASCII byte string. If a comment is given, then it will be preceded by one that identifies the file as having been dumped by scikit-learn. Note that not all tools grok comments in SVMlight files.
query_id : array-like, shape = [n_samples]
    Array containing pairwise preference constraints (qid in svmlight format).

multilabel : boolean, optional
    Samples may have several labels each (see http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/multilabel.html)

    New in version 0.17: parameter multilabel to support multilabel datasets.

Examples using sklearn.datasets.dump_svmlight_file

- Libsvm GUI

29.6.2 Samples generator

datasets.make_blobs([n_samples, n_features, ...]) Generate isotropic Gaussian blobs for clustering.
datasets.make_classification([n_samples, ...]) Generate a random n-class classification problem.
datasets.make_circles([n_samples, shuffle, ...]) Make a large circle containing a smaller circle in 2d.
datasets.make_friedman1([n_samples, ...]) Generate the “Friedman #1” regression problem
datasets.make_friedman2([n_samples, noise, ...]) Generate the “Friedman #2” regression problem
datasets.make_friedman3([n_samples, noise, ...]) Generate the “Friedman #3” regression problem
datasets.make_gaussian_quantiles([mean, ...]) Generate isotropic Gaussian and label samples by quantile
datasets.make_hastie_10_2([n_samples, ...]) Generates data for binary classification used in Hastie et al.
datasets.make_low_rank_matrix([n_samples, ...]) Generate a mostly low rank matrix with bell-shaped singular values
datasets.make_moons([n_samples, shuffle, ...]) Make two interleaving half circles
datasets.make_multilabel_classification([...]) Generate a random multilabel classification problem.
datasets.make_multilabel_classification([...]) Generate a random multilabel classification problem.
datasets.make_regression([n_samples, ...]) Generate a random regression problem.
datasets.make_s_curve([n_samples, noise, ...]) Generate an S curve dataset.
datasets.make_sparse_coded_signal([n_samples, ...]) Generate a signal as a sparse combination of dictionary elements.
datasets.make_sparse_spd_matrix([dim, ...]) Generate a sparse symmetric definite positive matrix.
datasets.make_sparse_spd_matrix([dim, ...]) Generate a sparse symmetric definite positive matrix.
datasets.make_spd_matrix([n_dim[, random_state]]) Generate a random symmetric, positive-definite matrix.
datasets.make_swiss_roll([n_samples, noise, ...]) Generate a swiss roll dataset.
datasets.make_biclusters(shape, n_clusters) Generate an array with constant block diagonal structure for biclustering.
datasets.make_checkerboard(shape, n_clusters) Generate an array with block checkerboard structure for biclustering.

sklearn.datasets.make_blobs

sklearn.datasets.make_blobs (n_samples=100, n_features=2, centers=3, cluster_std=1.0, 
center_box=(-10.0, 10.0), shuffle=True, random_state=None)
    Generate isotropic Gaussian blobs for clustering.
Parameters

- **n_samples**: int, optional (default=100)
  
  The total number of points equally divided among clusters.

- **n_features**: int, optional (default=2)
  
  The number of features for each sample.

- **centers**: int or array of shape [n_centers, n_features], optional
  
  (default=3) The number of centers to generate, or the fixed center locations.

- **cluster_std**: float or sequence of floats, optional (default=1.0)
  
  The standard deviation of the clusters.

- **center_box**: pair of floats (min, max), optional (default=(-10.0, 10.0))
  
  The bounding box for each cluster center when centers are generated at random.

- **shuffle**: boolean, optional (default=True)
  
  Shuffle the samples.

- **random_state**: int, RandomState instance or None, optional (default=None)
  
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns

- **X**: array of shape [n_samples, n_features]
  
  The generated samples.

- **y**: array of shape [n_samples]
  
  The integer labels for cluster membership of each sample.

See also:

- **make_classification**: a more intricate variant

Examples

```python
>>> from sklearn.datasets.samples_generator import make_blobs
>>> X, y = make_blobs(n_samples=10, centers=3, n_features=2,
...                    random_state=0)
>>> print(X.shape)
(10, 2)
>>> y
array([0, 0, 1, 0, 2, 2, 2, 1, 1, 0])
```

Examples using sklearn.datasets.make_blobs

- Probability calibration of classifiers
- Probability Calibration for 3-class classification
- Normal and Shrinkage Linear Discriminant Analysis for classification
- Demo of affinity propagation clustering algorithm
Generate a random n-class classification problem.

This initially creates clusters of points normally distributed (std=1) about vertices of a 2 * class_sep-sided hypercube, and assigns an equal number of clusters to each class. It introduces interdependence between these features and adds various types of further noise to the data.

Prior to shuffling, X stacks a number of these primary “informative” features, “redundant” linear combinations of these, “repeated” duplicates of sampled features, and arbitrary noise for and remaining features.

Read more in the User Guide.

**Parameters**

- `n_samples` : int, optional (default=100)
  The number of samples.

- `n_features` : int, optional (default=20)
  The total number of features. These comprise `n_informative` informative features, `n_redundant` redundant features, `n_repeated` duplicated features and `n_features-n_informative-n_redundant- n_repeated` useless features drawn at random.

- `n_informative` : int, optional (default=2)
  The number of informative features. Each class is composed of a number of gaussian clusters each located around the vertices of a hypercube in a subspace of dimension `n_informative`. For each cluster, informative features are drawn independently from N(0, 1) and then randomly linearly combined within each cluster in order to add covariance. The clusters are then placed on the vertices of the hypercube.

- `n_redundant` : int, optional (default=2)
  The number of redundant features. These features are generated as random linear combinations of the informative features.
n_repeated : int, optional (default=0)

The number of duplicated features, drawn randomly from the informative and the redundant features.

n_classes : int, optional (default=2)

The number of classes (or labels) of the classification problem.

n_clusters_per_class : int, optional (default=2)

The number of clusters per class.

weights : list of floats or None (default=None)

The proportions of samples assigned to each class. If None, then classes are balanced. Note that if len(weights) == n_classes - 1, then the last class weight is automatically inferred. More than n_samples samples may be returned if the sum of weights exceeds 1.

flip_y : float, optional (default=0.01)

The fraction of samples whose class are randomly exchanged.

class_sep : float, optional (default=1.0)

The factor multiplying the hypercube dimension.

hypercube : boolean, optional (default=True)

If True, the clusters are put on the vertices of a hypercube. If False, the clusters are put on the vertices of a random polytope.

shift : float, array of shape [n_features] or None, optional (default=0.0)

Shift features by the specified value. If None, then features are shifted by a random value drawn in [-class_sep, class_sep].

scale : float, array of shape [n_features] or None, optional (default=1.0)

Multiply features by the specified value. If None, then features are scaled by a random value drawn in [1, 100]. Note that scaling happens after shifting.

shuffle : boolean, optional (default=True)

Shuffle the samples and the features.

random_state : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns:
X : array of shape [n_samples, n_features]

The generated samples.

y : array of shape [n_samples]

The integer labels for class membership of each sample.

See also:

make_blobs simplified variant

make_multilabel_classification unrelated generator for multilabel tasks
Notes

The algorithm is adapted from Guyon [1] and was designed to generate the “Madelon” dataset.

References

[R7]

Examples using sklearn.datasets.make_classification

- Probability Calibration curves
- Comparison of Calibration of Classifiers
- Classifier comparison
- Plot randomly generated classification dataset
- OOB Errors for Random Forests
- Feature transformations with ensembles of trees
- Feature importances with forests of trees
- Pipeline Anova SVM
- Recursive feature elimination with cross-validation
- Varying regularization in Multi-layer Perceptron
- Scaling the regularization parameter for SVCs

sklearn.datasets.make_circles

sklearn.datasets.make_circles(n_samples=100, shuffle=True, noise=None, random_state=None, factor=0.8)

Make a large circle containing a smaller circle in 2d.

A simple toy dataset to visualize clustering and classification algorithms.

Read more in the User Guide.

Parameters:
	n_samples : int, optional (default=100)

The total number of points generated.

shuffle : bool, optional (default=True)

Whether to shuffle the samples.

noise : double or None (default=None)

Standard deviation of Gaussian noise added to the data.

factor : double < 1 (default=0.8)

Scale factor between inner and outer circle.

Returns:

X : array of shape [n_samples, 2]

The generated samples.

y : array of shape [n_samples]
The integer labels (0 or 1) for class membership of each sample.

**Examples using `sklearn.datasets.make_circles`**

- Classifier comparison
- Comparing different clustering algorithms on toy datasets
- Kernel PCA
- Hashing feature transformation using Totally Random Trees
- Varying regularization in Multi-layer Perceptron
- Compare Stochastic learning strategies for MLPClassifier
- Label Propagation learning a complex structure

**`sklearn.datasets.make_friedman1`**

`sklearn.datasets.make_friedman1(n_samples=100, n_features=10, noise=0.0, random_state=None)`

Generate the “Friedman #1” regression problem

This dataset is described in Friedman [1] and Breiman [2].

Inputs $X$ are independent features uniformly distributed on the interval [0, 1]. The output $y$ is created according to the formula:

\[
    y(X) = 10 \times \sin(\pi \times X[:, 0] \times X[:, 1]) + 20 \times (X[:, 2] - 0.5)^2 + 10 \times X[:, 3] + 5 \times X[:, 4] + \text{noise} \times \mathcal{N}(0, 1).
\]

Out of the $n_{features}$ features, only 5 are actually used to compute $y$. The remaining features are independent of $y$.

The number of features has to be $\geq 5$.

Read more in the *User Guide*.

**Parameters**

- `n_samples` : int, optional (default=100)
  
  The number of samples.

- `n_features` : int, optional (default=10)
  
  The number of features. Should be at least 5.

- `noise` : float, optional (default=0.0)
  
  The standard deviation of the gaussian noise applied to the output.

- `random_state` : int, RandomState instance or None, optional (default=None)
  
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns**

- `$X$`: array of shape `[n_samples, n_features]`
  
  The input samples.

- `$y$`: array of shape `[n_samples]`
  
  The output values.
References

[R137], [R138]

sklearn.datasets.make_friedman2

Generate the “Friedman #2” regression problem

This dataset is described in Friedman [1] and Breiman [2].

Inputs \( X \) are 4 independent features uniformly distributed on the intervals:

\[
0 \leq X[:, 0] \leq 100, \\
40 \pi \leq X[:, 1] \leq 560 \pi, \\
0 \leq X[:, 2] \leq 1, \\
1 \leq X[:, 3] \leq 11.
\]

The output \( y \) is created according to the formula:

\[
y(X) = (X[:, 0] ^ 2 + (X[:, 1] \times X[:, 2] - 1 / (X[:, 1] \times X[:, 3])) ^ 2) ^ {0.5} + \text{noise} \times \text{N}(0, 1).
\]

Read more in the User Guide.

Parameters

- **n_samples** : int, optional (default=100)
  The number of samples.
- **noise** : float, optional (default=0.0)
  The standard deviation of the gaussian noise applied to the output.
- **random_state** : int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns

- **X** : array of shape [n_samples, 4]
  The input samples.
- **y** : array of shape [n_samples]
  The output values.

References

[R139], [R140]

sklearn.datasets.make_friedman3

Generate the “Friedman #3” regression problem

This dataset is described in Friedman [1] and Breiman [2].

Inputs \( X \) are 4 independent features uniformly distributed on the intervals:
The output \( y \) is created according to the formula:

\[
y(X) = \arctan\left(\frac{X[:, 1] \times X[:, 2] - 1 / (X[:, 1] \times X[:, 3])}{X[:, 0]}\right) + \text{noise} \times \mathcal{N}(0, 1).
\]

Read more in the User Guide.

**Parameters**

- \( n_{\text{samples}} \): int, optional (default=100)
  The number of samples.
- \( \text{noise} \): float, optional (default=0.0)
  The standard deviation of the gaussian noise applied to the output.
- \( \text{random\_state} \): int, RandomState instance or None, optional (default=None)
  If int, \( \text{random\_state} \) is the seed used by the random number generator; If RandomState instance, \( \text{random\_state} \) is the random number generator; If None, the random number generator is the RandomState instance used by \text{np.random}.

**Returns**

- \( X \): array of shape [\( n_{\text{samples}}, 4 \)]
  The input samples.
- \( y \): array of shape [\( n_{\text{samples}} \)]
  The output values.

**References**

[R141], [R142]

**sklearn.datasets.make_gaussian_quantiles**

The classification dataset is constructed by taking a multi-dimensional standard normal distribution and defining classes separated by nested concentric multi-dimensional spheres such that roughly equal numbers of samples are in each class (quantiles of the \( \chi^2 \) distribution).

Read more in the User Guide.

**Parameters**

- \( \text{mean} \): array of shape [\( n_{\text{features}} \)], optional (default=None)
  The mean of the multi-dimensional normal distribution. If None then use the origin (0, 0, ...).
- \( \text{cov} \): float, optional (default=1.)
  The covariance matrix will be this value times the unit matrix. This dataset only produces symmetric normal distributions.
n_samples : int, optional (default=100)
    The total number of points equally divided among classes.

n_features : int, optional (default=2)
    The number of features for each sample.

n_classes : int, optional (default=3)
    The number of classes

shuffle : boolean, optional (default=True)
    Shuffle the samples.

random_state : int, RandomState instance or None, optional (default=None)
    If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns

X : array of shape [n_samples, n_features]
    The generated samples.

y : array of shape [n_samples]
    The integer labels for quantile membership of each sample.

Notes

The dataset is from Zhu et al [1].

References

[R8]

Examples using sklearn.datasets.make_gaussian_quantiles

- Plot randomly generated classification dataset
- Multi-class AdaBoosted Decision Trees
- Two-class AdaBoost

sklearn.datasets.make_hastie_10_2

sklearn.datasets.make_hastie_10_2 (n_samples=12000, random_state=None)
Generates data for binary classification used in Hastie et al. 2009, Example 10.2.
The ten features are standard independent Gaussian and the target y is defined by:

```python
y[i] = 1 if np.sum(X[i] ** 2) > 9.34 else -1
```

Read more in the User Guide.

Parameters

n_samples : int, optional (default=12000)
    The number of samples.
random_state : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

ReturnsX : array of shape [n_samples, 10]

The input samples.
y : array of shape [n_samples]

The output values.

See also:

make_gaussian_quantiles a generalization of this dataset approach

References

[R9]

Examples using sklearn.datasets.make_hastie_10_2

- Discrete versus Real AdaBoost
- Gradient Boosting regularization

sklearn.datasets.make_low_rank_matrix

sklearn.datasets.make_low_rank_matrix (n_samples=100, n_features=100, effective_rank=10, tail_strength=0.5, random_state=None)

Generate a mostly low rank matrix with bell-shaped singular values

Most of the variance can be explained by a bell-shaped curve of width effective_rank: the low rank part of the singular values profile is:

\[(1 - \text{tail_strength}) \times \exp(-1.0 \times (i / \text{effective_rank}) ^ 2)\]

The remaining singular values’ tail is fat, decreasing as:

\[\text{tail_strength} \times \exp(-0.1 \times i / \text{effective_rank}).\]

The low rank part of the profile can be considered the structured signal part of the data while the tail can be considered the noisy part of the data that cannot be summarized by a low number of linear components (singular vectors).

This kind of singular profiles is often seen in practice, for instance:

* gray level pictures of faces
* TF-IDF vectors of text documents crawled from the web

Read more in the User Guide.

Parameters

n_samples : int, optional (default=100)

The number of samples.
**n_features**: int, optional (default=100)

The number of features.

**effective_rank**: int, optional (default=10)

The approximate number of singular vectors required to explain most of the data by linear combinations.

**tail_strength**: float between 0.0 and 1.0, optional (default=0.5)

The relative importance of the fat noisy tail of the singular values profile.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

**X**: array of shape [n_samples, n_features]

The matrix.

---

**sklearn.datasets.make_moons**

**sklearn.datasets.make_moons** (*n_samples=100, shuffle=True, noise=None, random_state=None*)

Make two interleaving half circles

A simple toy dataset to visualize clustering and classification algorithms. Read more in the User Guide.

**Parameters**

**n_samples**: int, optional (default=100)

The total number of points generated.

**shuffle**: bool, optional (default=True)

Whether to shuffle the samples.

**noise**: double or None (default=None)

Standard deviation of Gaussian noise added to the data.

**Returns**

**X**: array of shape [n_samples, 2]

The generated samples.

**y**: array of shape [n_samples]

The integer labels (0 or 1) for class membership of each sample.

---

**Examples using sklearn.datasets.make_moons**

- Classifier comparison
- Comparing different clustering algorithms on toy datasets
- Varying regularization in Multi-layer Perceptron
- Compare Stochastic learning strategies for MLPClassifier
Generate a random multilabel classification problem.

For each sample, the generative process is:

- pick the number of labels: \( n \sim \text{Poisson}(n_{\text{labels}}) \)
- \( n \) times, choose a class \( c \sim \text{Multinomial}(\text{theta}) \)
- pick the document length: \( k \sim \text{Poisson}(\text{length}) \)
- \( k \) times, choose a word: \( w \sim \text{Multinomial}(\text{theta}_c) \)

In the above process, rejection sampling is used to make sure that \( n \) is never zero or more than \( n_{\text{classes}} \), and that the document length is never zero. Likewise, we reject classes which have already been chosen.

Read more in the User Guide.

Parameters:

- **n_samples**: int, optional (default=100)
  - The number of samples.
- **n_features**: int, optional (default=20)
  - The total number of features.
- **n_classes**: int, optional (default=5)
  - The number of classes of the classification problem.
- **n_labels**: int, optional (default=2)
  - The average number of labels per instance. More precisely, the number of labels per sample is drawn from a Poisson distribution with \( n_{\text{labels}} \) as its expected value, but samples are bounded (using rejection sampling) by \( n_{\text{classes}} \), and must be nonzero if \( \text{allow_unlabeled} \) is False.
- **length**: int, optional (default=50)
  - The sum of the features (number of words if documents) is drawn from a Poisson distribution with this expected value.
- **allow_unlabeled**: bool, optional (default=True)
  - If True, some instances might not belong to any class.
- **sparse**: bool, optional (default=False)
  - If True, return a sparse feature matrix
  - New in version 0.17: parameter to allow sparse output.
- **return_indicator**: ‘dense’ (default) | ‘sparse’ | False
  - If dense return \( Y \) in the dense binary indicator format. If 'sparse' return \( Y \) in the sparse binary indicator format. False returns a list of lists of labels.
- **return_distributions**: bool, optional (default=False)
If `True`, return the prior class probability and conditional probabilities of features given classes, from which the data was drawn.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns**

**X**: array of shape [n_samples, n_features]

The generated samples.

**Y**: array or sparse CSR matrix of shape [n_samples, n_classes]

The label sets.

**p_c**: array, shape [n_classes]

The probability of each class being drawn. Only returned if `return_distributions=True`.

**p_w_c**: array, shape [n_features, n_classes]

The probability of each feature being drawn given each class. Only returned if `return_distributions=True`.

### Examples using `sklearn.datasets.make_multilabel_classification`

- [Multilabel classification](#)
- [Plot randomly generated multilabel dataset](#)

### `sklearn.datasets.make_regression`

**sklearn.datasets.make_regression**

`sklearn.datasets.make_regression(n_samples=100, n_features=100, n_informative=10, n_targets=1, bias=0.0, effective_rank=None, tail_strength=0.5, noise=0.0, shuffle=True, coef=False, random_state=None)`

Generate a random regression problem.

The input set can either be well conditioned (by default) or have a low rank-fat tail singular profile. See `make_low_rank_matrix` for more details.

The output is generated by applying a (potentially biased) random linear regression model with `n_informative` nonzero regressors to the previously generated input and some gaussian centered noise with some adjustable scale.

Read more in the User Guide.

**Parameters**

**n_samples**: int, optional (default=100)

The number of samples.

**n_features**: int, optional (default=100)

The number of features.

**n_informative**: int, optional (default=10)

The number of informative features, i.e., the number of features used to build the linear model used to generate the output.
n_targets : int, optional (default=1)
The number of regression targets, i.e., the dimension of the y output vector associated
with a sample. By default, the output is a scalar.

bias : float, optional (default=0.0)
The bias term in the underlying linear model.

effective_rank : int or None, optional (default=None)
if not None: The approximate number of singular vectors required to explain most of
the input data by linear combinations. Using this kind of singular spectrum in the
input allows the generator to reproduce the correlations often observed in practice.
if None: The input set is well conditioned, centered and gaussian with unit variance.

tail_strength : float between 0.0 and 1.0, optional (default=0.5)
The relative importance of the fat noisy tail of the singular values profile if effective_rank is not None.

noise : float, optional (default=0.0)
The standard deviation of the gaussian noise applied to the output.

shuffle : boolean, optional (default=True)
Shuffle the samples and the features.

coeff : boolean, optional (default=False)
If True, the coefficients of the underlying linear model are returned.

random_state : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState
instance, random_state is the random number generator; If None, the random number
generator is the RandomState instance used by np.random.

ReturnsX : array of shape [n_samples, n_features]
The input samples.

y : array of shape [n_samples] or [n_samples, n_targets]
The output values.

coeff : array of shape [n_features] or [n_features, n_targets], optional
The coefficient of the underlying linear model. It is returned only if coeff is True.

Examples using sklearn.datasets.make_regression

• Prediction Latency
• Lasso on dense and sparse data
• HuberRegressor vs Ridge on dataset with strong outliers
• Robust linear model estimation using RANSAC
• Plot Ridge coefficients as a function of the L2 regularization
sklearn.datasets.make_s_curve

sklearn.datasets.make_s_curve(n_samples=100, noise=0.0, random_state=None)

Generate an S curve dataset.

Read more in the User Guide.

Parameters:
- **n_samples**: int, optional (default=100)
  The number of sample points on the S curve.
- **noise**: float, optional (default=0.0)
  The standard deviation of the gaussian noise.
- **random_state**: int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns:
- **X**: array of shape [n_samples, 3]
  The points.
- **t**: array of shape [n_samples]
  The univariate position of the sample according to the main dimension of the points in the manifold.

Examples using sklearn.datasets.make_s_curve

- Comparison of Manifold Learning methods

sklearn.datasets.make_sparse_coded_signal

sklearn.datasets.make_sparse_coded_signal(n_samples, n_components, n_features, n_nonzero_coefs, random_state=None)

Generate a signal as a sparse combination of dictionary elements.

Returns a matrix Y = DX, such as D is (n_features, n_components), X is (n_components, n_samples) and each column of X has exactly n_nonzero_coefs non-zero elements.

Read more in the User Guide.

Parameters:
- **n_samples**: int
  number of samples to generate
- **n_components**: int,
  number of components in the dictionary
- **n_features**: int
  number of features of the dataset to generate
- **n_nonzero_coefs**: int
  number of active (non-zero) coefficients in each sample
- **random_state**: int or RandomState instance, optional (default=None)
  seed used by the pseudo random number generator
**Returns**

- **data**: array of shape [n_features, n_samples]
  - The encoded signal (Y).
- **dictionary**: array of shape [n_features, n_components]
  - The dictionary with normalized components (D).
- **code**: array of shape [n_components, n_samples]
  - The sparse code such that each column of this matrix has exactly n_nonzero_coefs non-zero items (X).

**Examples using `sklearn.datasets.make_sparse_coded_signal`**

- **Orthogonal Matching Pursuit**

**sklearn.datasets.make_sparse_spd_matrix**

```python
sklearn.datasets.make_sparse_spd_matrix(dim=1, alpha=0.95, norm_diag=False, smallest_coef=0.1, largest_coef=0.9, random_state=None)
```

Generate a sparse symmetric definite positive matrix.

Read more in the *User Guide*.

**Parameters**

- **dim**: integer, optional (default=1)
  - The size of the random matrix to generate.
- **alpha**: float between 0 and 1, optional (default=0.95)
  - The probability that a coefficient is zero (see notes). Larger values enforce more sparsity.
- **random_state**: int, RandomState instance or None, optional (default=None)
  - If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.
- **largest_coef**: float between 0 and 1, optional (default=0.9)
  - The value of the largest coefficient.
- **smallest_coef**: float between 0 and 1, optional (default=0.1)
  - The value of the smallest coefficient.
- **norm_diag**: boolean, optional (default=False)
  - Whether to normalize the output matrix to make the leading diagonal elements all 1

**Returns**

- **prec**: sparse matrix of shape (dim, dim)
  - The generated matrix.

**See also:**

- `make_spd_matrix`
Notes

The sparsity is actually imposed on the cholesky factor of the matrix. Thus alpha does not translate directly into the filling fraction of the matrix itself.

Examples using `sklearn.datasets.make_sparse_spd_matrix`

- **Sparse inverse covariance estimation**

`sklearn.datasets.make_sparse_uncorrelated`

`sklearn.datasets.make_sparse_uncorrelated(n_samples=100, n_features=10, random_state=None)`

Generate a random regression problem with sparse uncorrelated design

This dataset is described in Celeux et al [1]. as:

\[
X \sim N(0, 1)
\]

\[
y(X) = X[:, 0] + 2 \times X[:, 1] - 2 \times X[:, 2] - 1.5 \times X[:, 3]
\]

Only the first 4 features are informative. The remaining features are useless.

Read more in the *User Guide*.

**Parameters**

- `n_samples` : int, optional (default=100)
  
  The number of samples.

- `n_features` : int, optional (default=10)
  
  The number of features.

- `random_state` : int, RandomState instance or None, optional (default=None)
  
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns**

- `X` : array of shape [n_samples, n_features]
  
  The input samples.

- `y` : array of shape [n_samples]
  
  The output values.

**References**

[R145]

`sklearn.datasets.make_spd_matrix`

`sklearn.datasets.make_spd_matrix(n_dim, random_state=None)`

Generate a random symmetric, positive-definite matrix.

Read more in the *User Guide*.

**Parameters**

- `n_dim` : int
The matrix dimension.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

- **X** : array of shape [n_dim, n_dim]
  The random symmetric, positive-definite matrix.

See also:

- `make_sparse_spd_matrix`

**sklearn.datasets.make_swiss_roll**

`sklearn.datasets.make_swiss_roll(n_samples=100, noise=0.0, random_state=None)`  
Generate a swiss roll dataset.

Read more in the User Guide.

**Parameters**

- **n_samples** : int, optional (default=100)
  The number of sample points on the S curve.

- **noise** : float, optional (default=0.0)
  The standard deviation of the gaussian noise.

- **random_state** : int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

- **X** : array of shape [n_samples, 3]
  The points.

- **t** : array of shape [n_samples]
  The univariate position of the sample according to the main dimension of the points in the manifold.

**Notes**

The algorithm is from Marsland [1].

**References**

[R10]

**Examples using sklearn.datasets.make_swiss_roll**

- Hierarchical clustering: structured vs unstructured ward
- Swiss Roll reduction with LLE
sklearn.datasets.make_biclusters

sklearn.datasets.make_biclusters (shape, n_clusters, noise=0.0, minval=10, maxval=100, shuffle=True, random_state=None)

Generate an array with constant block diagonal structure for biclustering.

Read more in the User Guide.

Parameters:

- **shape**: iterable (n_rows, n_cols)
  The shape of the result.

- **n_clusters**: integer
  The number of biclusters.

- **noise**: float, optional (default=0.0)
  The standard deviation of the gaussian noise.

- **minval**: int, optional (default=10)
  Minimum value of a bicluster.

- **maxval**: int, optional (default=100)
  Maximum value of a bicluster.

- **shuffle**: boolean, optional (default=True)
  Shuffle the samples.

- **random_state**: int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns:

- **X**: array of shape shape
  The generated array.

- **rows**: array of shape (n_clusters, X.shape[0],)
  The indicators for cluster membership of each row.

- **cols**: array of shape (n_clusters, X.shape[1],)
  The indicators for cluster membership of each column.

See also:

- make_checkerboard

References:

[R5]

Examples using sklearn.datasets.make_biclusters

- A demo of the Spectral Co-Clustering algorithm
sklearn.datasets.make_checkerboard

sklearn.datasets.make_checkerboard(shape, n_clusters, noise=0.0, minval=10, maxval=100, shuffle=True, random_state=None)

Generate an array with block checkerboard structure for biclustering.

Parameters:
- **shape**: iterable (n_rows, n_cols)
  The shape of the result.
- **n_clusters**: integer or iterable (n_row_clusters, n_column_clusters)
  The number of row and column clusters.
- **noise**: float, optional (default=0.0)
  The standard deviation of the gaussian noise.
- **minval**: int, optional (default=10)
  Minimum value of a bicluster.
- **maxval**: int, optional (default=100)
  Maximum value of a bicluster.
- **shuffle**: boolean, optional (default=True)
  Shuffle the samples.
- **random_state**: int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns:
- **X**: array of shape shape
  The generated array.
- **rows**: array of shape (n_clusters, X.shape[0],)
  The indicators for cluster membership of each row.
- **cols**: array of shape (n_clusters, X.shape[1],)
  The indicators for cluster membership of each column.

See also:
- `make_biclusters`

References:

[R6]

Examples using sklearn.datasets.make_checkerboard

- A demo of the Spectral Biclustering algorithm
The `sklearn.decomposition` module includes matrix decomposition algorithms, including among others PCA, NMF or ICA. Most of the algorithms of this module can be regarded as dimensionality reduction techniques.

**User guide:** See the *Decomposing signals in components (matrix factorization problems)* section for further details.

### 29.7 sklearn.decomposition: Matrix Decomposition

The `sklearn.decomposition` module includes matrix decomposition algorithms, including among others PCA, NMF or ICA. Most of the algorithms of this module can be regarded as dimensionality reduction techniques.

**User guide:** See the *Decomposing signals in components (matrix factorization problems)* section for further details.

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#### 29.7.1 sklearn.decomposition.PCA

**class** `sklearn.decomposition.PCA(n_components=None, copy=True, whiten=False, svd_solver='auto', tol=0.0, iterated_power='auto', random_state=None)`

- **Principal component analysis (PCA)**
- Linear dimensionality reduction using Singular Value Decomposition of the data to project it to a lower dimensional space.
- It uses the LAPACK implementation of the full SVD or a randomized truncated SVD by the method of Halko et al. 2009, depending on the shape of the input data and the number of components to extract.
- It can also use the scipy.sparse.linalg ARPACK implementation of the truncated SVD.
- Notice that this class does not support sparse input. See `TruncatedSVD` for an alternative with sparse data.

**Parameters**

- `n_components`: int, float, None or string
  
  Number of components to keep. if `n_components` is not set all components are kept:
  
  ```python
  n_components == \min(n_samples, n_features)
  ```

  if `n_components == 'mle'` and `svd_solver == 'full'`, Minka’s MLE is used to guess the dimension if `0 < n_components < 1` and `svd_solver == 'full'`, select the number of components such that the amount of variance that needs to be explained is greater than
the percentage specified by n_components cannot be equal to n_features for svd_solver == ‘arpack’.

copy : bool (default True)

If False, data passed to fit are overwritten and running fit(X).transform(X) will not yield the expected results, use fit_transform(X) instead.

whiten : bool, optional (default False)

When True (False by default) the components_ vectors are multiplied by the square root of n_samples and then divided by the singular values to ensure uncorrelated outputs with unit component-wise variances.

Whitening will remove some information from the transformed signal (the relative variance scales of the components) but can sometime improve the predictive accuracy of the downstream estimators by making their data respect some hard-wired assumptions.


auto : the solver is selected by a default policy based on X.shape and n_components:

if the input data is larger than 500x500 and the number of components to extract is lower than 80% of the smallest dimension of the data, then the more efficient ‘randomized’ method is enabled. Otherwise the exact full SVD is computed and optionally truncated afterwards.

full : run exact full SVD calling the standard LAPACK solver via scipy.linalg.svd and select the components by postprocessing

arpack : run SVD truncated to n_components calling ARPACK solver via scipy.sparse.linalg.svds. It requires strictly 0 < n_components < X.shape[1]

randomized : run randomized SVD by the method of Halko et al.

New in version 0.18.0.

tol : float >= 0, optional (default .0)

Tolerance for singular values computed by svd_solver == ‘arpack’.

New in version 0.18.0.

iterated_power : int >= 0, or ‘auto’, (default ‘auto’)

Number of iterations for the power method computed by svd_solver == ‘randomized’.

New in version 0.18.0.

random_state : int or RandomState instance or None (default None)

Pseudo Random Number generator seed control. If None, use the numpy.random singleton. Used by svd_solver == ‘arpack’ or ‘randomized’.

New in version 0.18.0.

Attributescomponents_ : array, [n_components, n_features]

Principal axes in feature space, representing the directions of maximum variance in the data. The components are sorted by explained_variance_.

explained_variance_ : array, [n_components]

The amount of variance explained by each of the selected components.

New in version 0.18.

explained_variance_ratio_ : array, [n_components]
Percentage of variance explained by each of the selected components.

If n_components is not set then all components are stored and the sum of explained variances is equal to 1.0.

mean_ : array, [n_features]

Per-feature empirical mean, estimated from the training set.

Equal to X.mean(axis=1).

n_components_ : int

The estimated number of components. When n_components is set to ‘mle’ or a number between 0 and 1 (with svd_solver == ‘full’) this number is estimated from input data. Otherwise it equals the parameter n_components, or n_features if n_components is None.

noise_variance_ : float

The estimated noise covariance following the Probabilistic PCA model from Tipping and Bishop 1999. See “Pattern Recognition and Machine Learning” by C. Bishop, 12.2.1 p. 574 or http://www.miketipping.com/papers/met-mppca.pdf. It is required to computed the estimated data covariance and score samples.

See also:

KernelPCA, SparsePCA, TruncatedSVD, IncrementalPCA

References

For n_components == ‘mle’, this class uses the method of Thomas P. Minka: Automatic Choice of Dimensionality for PCA. NIPS 2000: 598-604


For svd_solver == ‘arpack’, refer to scipy.sparse.linalg.svds.


Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import PCA
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> pca = PCA(n_components=2)
>>> pca.fit(X)
PCA(copy=True, iterated_power='auto', n_components=2, random_state=None,
    svd_solver='auto', tol=0.0, whiten=False)
>>> print(pca.explained_variance_ratio_)
[ 0.99244... 0.00755...]

>>> pca = PCA(n_components=2, svd_solver='full')
>>> pca.fit(X)
PCA(copy=True, iterated_power='auto', n_components=2, random_state=None,
    svd_solver='full', tol=0.0, whiten=False)
```
svd_solver='full', tol=0.0, whiten=False)

>>> print(pca.explained_variance_ratio_)
[ 0.99244... 0.00755...]

>>> pca = PCA(n_components=1, svd_solver='arpack')
>>> pca.fit(X)
PCA(copy=True, iterated_power='auto', n_components=1, random_state=None,
    svd_solver='arpack', tol=0.0, whiten=False)

Methods

**fit**(X[, y])  
Fit the model with X.

**fit_transform**(X[, y])  
Fit the model with X and apply the dimensionality reduction on X.

**get_covariance**()  
Compute data covariance with the generative model.

**get_params**(deep)  
Get parameters for this estimator.

**get_precision**()  
Compute data precision matrix with the generative model.

**inverse_transform**(X[, y])  
Transform data back to its original space.

**score**(X[, y])  
Return the average log-likelihood of all samples.

**score_samples**(X)  
Return the log-likelihood of each sample.

**set_params**(**\*\*params)  
Set the parameters of this estimator.

**transform**(X[, y])  
Apply dimensionality reduction to X.

**init**(n_components=None, copy=True, whiten=False, svd_solver='auto', tol=0.0, iterated_power='auto', random_state=None)

**fit**(X, y=None)
Fit the model with X.

**Parameters**
X: array-like, shape (n_samples, n_features) :
Training data, where n_samples in the number of samples and n_features is the number of features.

**Returns**
self : object
Returns the instance itself.

**fit_transform**(X, y=None)
Fit the model with X and apply the dimensionality reduction on X.

**Parameters**
X : array-like, shape (n_samples, n_features)
Training data, where n_samples is the number of samples and n_features is the number of features.

**Returns**
X_new : array-like, shape (n_samples, n_components)

**get_covariance**()
Compute data covariance with the generative model.

cov = components_.T * S**2 * components_ + sigma2 * eye(n_features)
where S**2 contains the explained variances, and sigma2 contains the noise variances.
**Returns**
\[ \text{scov} \] : array, shape=(n_features, n_features)
Estimated covariance of data.

**get_params** *(deep=True)*
Get parameters for this estimator.

**Parameters**
\[ \text{deep} \] : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
\[ \text{params} \] : mapping of string to any
Parameter names mapped to their values.

**get_precision** *
Compute data precision matrix with the generative model.

Equals the inverse of the covariance but computed with the matrix inversion lemma for efficiency.

**Returns**
\[ \text{precision} \] : array, shape=(n_features, n_features)
Estimated precision of data.

**inverse_transform** *(X, y=None)*
Transform data back to its original space.

In other words, return an input \( X_{\text{original}} \) whose transform would be \( X \).

**Parameters**
\[ X \] : array-like, shape (n_samples, n_components)
New data, where \( n_{\text{samples}} \) is the number of samples and \( n_{\text{components}} \) is the number of components.

**Returns**
\[ X_{\text{original}} \] array-like, shape (n_samples, n_features) :

**Notes**
If whitening is enabled, inverse_transform will compute the exact inverse operation, which includes reversing whitening.

**score** *(X, y=None)*
Return the average log-likelihood of all samples.


**Parameters**
\[ X \] : array, shape (n_samples, n_features) :
The data.

**Returns**
\[ \text{ll} \] : float :
Average log-likelihood of the samples under the current model

**score_samples** *(X)*
Return the log-likelihood of each sample.


**Parameters**
\[ X \] : array, shape (n_samples, n_features) :
The data.
Returns ll: array, shape (n_samples,)

Log-likelihood of each sample under the current model

set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

transform(X, y=None)

Apply dimensionality reduction to X.

X is projected on the first principal components previously extracted from a training set.

Parameters:

X : array-like, shape (n_samples, n_features)

New data, where n_samples is the number of samples and n_features is the number of features.

Returns:

X_new : array-like, shape (n_samples, n_components)

Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import IncrementalPCA
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> ipca = IncrementalPCA(n_components=2, batch_size=3)
>>> ipca.fit(X)
IncrementalPCA(batch_size=3, copy=True, n_components=2, whiten=False)
>>> ipca.transform(X)
```

Examples using `sklearn.decomposition.PCA`

- Concatenating multiple feature extraction methods
- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Pipelining: chaining a PCA and a logistic regression
- Explicit feature map approximation for RBF kernels
- Multilabel classification
- Faces recognition example using eigenfaces and SVMs
- A demo of K-Means clustering on the handwritten digits data
- The Iris Dataset
- Faces dataset decompositions
- Blind source separation using FastICA
- FastICA on 2D point clouds
- Incremental PCA
- Kernel PCA
29.7.2 sklearn.decomposition.IncrementalPCA

class sklearn.decomposition.IncrementalPCA(n_components=None, whiten=False, copy=True, batch_size=None)

Incremental principal components analysis (IPCA).

Linear dimensionality reduction using Singular Value Decomposition of centered data, keeping only the most significant singular vectors to project the data to a lower dimensional space.

Depending on the size of the input data, this algorithm can be much more memory efficient than a PCA.

This algorithm has constant memory complexity, on the order of batch_size, enabling use of np.memmap files without loading the entire file into memory.

The computational overhead of each SVD is $O(\text{batch}_\text{size} \times \text{n}\_\text{features} \times 2)$, but only $2 \times \text{batch}_\text{size}$ samples remain in memory at a time. There will be $\frac{\text{n}\_\text{samples}}{\text{batch}_\text{size}}$ SVD computations to get the principal components, versus 1 large SVD of complexity $O(\text{n}\_\text{samples} \times \text{n}\_\text{features} \times 2)$ for PCA.

Read more in the User Guide.

Parameters

- **n_components** : int or None, (default=None)
  
  Number of components to keep. If n_components is None, then n_components is set to min(n_samples, n_features).

- **batch_size** : int or None, (default=None)
  
  The number of samples to use for each batch. Only used when calling fit. If batch_size is None, then batch_size is inferred from the data and set to 5 + n_features, to provide a balance between approximation accuracy and memory consumption.

- **copy** : bool, (default=True)
  
  If False, X will be overwritten. copy=False can be used to save memory but is unsafe for general use.

- **whiten** : bool, optional
  
  When True (False by default) the components_ vectors are divided by n_samples times components_ to ensure uncorrelated outputs with unit component-wise variances.

  Whitening will remove some information from the transformed signal (the relative variance scales of the components) but can sometimes improve the predictive accuracy of the downstream estimators by making data respect some hard-wired assumptions.

Attributes

- **components_** : array, shape (n_components, n_features)
Components with maximum variance.

**explained_variance_**: array, shape (n_components,)

Variance explained by each of the selected components.

**explained_variance_ratio_**: array, shape (n_components,)

Percentage of variance explained by each of the selected components. If all components are stored, the sum of explained variances is equal to 1.0

**mean_**: array, shape (n_features,)

Per-feature empirical mean, aggregate over calls to `partial_fit`.

**var_**: array, shape (n_features,)

Per-feature empirical variance, aggregate over calls to `partial_fit`.

**noise_variance_**: float


**n_components_**: int

The estimated number of components. Relevant when `n_components=None`.

**n_samples_seen_**: int

The number of samples processed by the estimator. Will be reset on new calls to fit, but increments across `partial_fit` calls.

See also:

*PCA, RandomizedPCA, KernelPCA, SparsePCA, TruncatedSVD*

Notes


We have specifically abstained from an optimization used by authors of both papers, a QR decomposition used in specific situations to reduce the algorithmic complexity of the SVD. The source for this technique is Matrix Computations, Third Edition, G. Holub and C. Van Loan, Chapter 5, section 5.4.4, pp 252-253. This technique has been omitted because it is advantageous only when decomposing a matrix with `n_samples` (rows) >= 5/3 * `n_features` (columns), and hurts the readability of the implemented algorithm. This would be a good opportunity for future optimization, if it is deemed necessary.

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---

**init** (n_components=None, whiten=False, copy=True, batch_size=None)

**fit** (X=None)

Fit the model with X, using minibatches of size batch_size.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  - Training data, where n_samples is the number of samples and n_features is the number of features.
- **y**: Passthrough for "Pipeline" compatibility.

**Returns**

- **self**: object
  - Returns the instance itself.

**fit_transform** (X=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_covariance()**

Compute data covariance with the generative model.

\[
\text{cov} = \text{components}_-.T \ast \text{S**2} \ast \text{components}_- + \text{sigma2} \ast \text{eye(n_features)}
\]

where S**2 contains the explained variances, and sigma2 contains the noise variances.

**Returns**

- **cov**: array, shape=(n_features, n_features)
  - Estimated covariance of data.

**get_params** (deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

`params` : mapping of string to any
Parameter names mapped to their values.

### get_precision()

Compute data precision matrix with the generative model.

Equals the inverse of the covariance but computed with the matrix inversion lemma for efficiency.

**Returns**

`precision` : array, shape=(n_features, n_features)
Estimated precision of data.

### inverse_transform(X, y=None)

Transform data back to its original space.

In other words, return an input X_original whose transform would be X.

**Parameters**

`X` : array-like, shape (n_samples, n_components)
New data, where n_samples is the number of samples and n_components is the number of components.

**Returns**

`X_original` array-like, shape (n_samples, n_features)

### Notes

If whitening is enabled, inverse_transform will compute the exact inverse operation, which includes reversing whitening.

### partial_fit(X, y=None, check_input=True)

Incremental fit with X. All of X is processed as a single batch.

**Parameters**

`X` : array-like, shape (n_samples, n_features)
Training data, where n_samples is the number of samples and n_features is the number of features.

**Returns**

`self` : object
Returns the instance itself.

### set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

`self` :

### transform(X, y=None)

Apply dimensionality reduction to X.

X is projected on the first principal components previously extracted from a training set.

**Parameters**

`X` : array-like, shape (n_samples, n_features)
New data, where n_samples is the number of samples and n_features is the number of features.

**Returns**

`X_new` : array-like, shape (n_samples, n_components)
Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import IncrementalPCA
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> ipca = IncrementalPCA(n_components=2, batch_size=3)
>>> ipca.fit(X)
IncrementalPCA(batch_size=3, copy=True, n_components=2, whiten=False)
>>> ipca.transform(X)
```

Examples using `sklearn.decomposition.IncrementalPCA`

- Incremental PCA

29.7.3 `sklearn.decompositionProjectedGradientNMF`

class `sklearn.decomposition.ProjectedGradientNMF(*args, **kwargs)`

Non-Negative Matrix Factorization (NMF)

Find two non-negative matrices (W, H) whose product approximates the non-negative matrix X. This factorization can be used for example for dimensionality reduction, source separation or topic extraction.

The objective function is:

\[
0.5 \times ||X - WH||_{\text{Fro}}^2 \\
+ \alpha \times \text{l1_ratio} \times ||\text{vec}(W)||_{\text{1}} \\
+ \alpha \times \text{l1_ratio} \times ||\text{vec}(H)||_{\text{1}} \\
+ 0.5 \times \alpha \times (1 - \text{l1_ratio}) \times ||W||_{\text{Fro}}^2 \\
+ 0.5 \times \alpha \times (1 - \text{l1_ratio}) \times ||H||_{\text{Fro}}^2
\]

Where:

\[
||A||_{\text{Fro}}^2 = \sum_{i,j} A_{ij}^2 \quad \text{(Frobenius norm)}
\]

\[
||\text{vec}(A)||_{\text{1}} = \sum_{i,j} \text{abs}(A_{ij}) \quad \text{(Elementwise L1 norm)}
\]

The objective function is minimized with an alternating minimization of W and H.

Read more in the User Guide.

Parameters:

- `n_components` : int or None
  Number of components, if `n_components` is not set all features are kept.

- `init` : ‘random’ | ‘nndsvd’ | ‘nndsvda’ | ‘nndsvdar’ | ‘custom’
  Method used to initialize the procedure. Default: ‘nndsvdar’ if `n_components` < `n_features`, otherwise random. Valid options:

  - ‘random’: non-negative random matrices, scaled with \(\sqrt{\text{X}.\text{mean()}}\)
    / `n_components`

  - ‘nndsvd’: Nonnegative Double Singular Value Decomposition (NNDSDV)
    initialization (better for sparseness)

  - ‘nndsvda’: NNDSDV with zeros filled with the average of X (better when sparsity is not desired)
•‘nndsvdar’: NNDSVD with zeros filled with small random values (generally faster, less accurate alternative to NNDSVDa for when sparsity is not desired)

•‘custom’: use custom matrices W and H

**solver** : ‘pg’ I ‘cd’

Numerical solver to use: ‘pg’ is a Projected Gradient solver (deprecated). ‘cd’ is a Coordinate Descent solver (recommended).

New in version 0.17: Coordinate Descent solver.

Changed in version 0.17: Deprecated Projected Gradient solver.

**tol** : double, default: 1e-4

Tolerance value used in stopping conditions.

**max_iter** : integer, default: 200

Number of iterations to compute.

**random_state** : integer seed, RandomState instance, or None (default)

Random number generator seed control.

**alpha** : double, default: 0.

Constant that multiplies the regularization terms. Set it to zero to have no regularization.

New in version 0.17: alpha used in the Coordinate Descent solver.

**l1_ratio** : double, default: 0.

The regularization mixing parameter, with 0 <= l1_ratio <= 1. For l1_ratio = 0 the penalty is an elementwise L2 penalty (aka Frobenius Norm). For l1_ratio = 1 it is an elementwise L1 penalty. For 0 < l1_ratio < 1, the penalty is a combination of L1 and L2.

New in version 0.17: Regularization parameter l1_ratio used in the Coordinate Descent solver.

**shuffle** : boolean, default: False

If true, randomize the order of coordinates in the CD solver.

New in version 0.17: shuffle parameter used in the Coordinate Descent solver.

**nls_max_iter** : integer, default: 2000

Number of iterations in NLS subproblem. Used only in the deprecated ‘pg’ solver.

Changed in version 0.17: Deprecated Projected Gradient solver. Use Coordinate Descent solver instead.

**sparseness** : ‘data’ I ‘components’ I None, default: None

Where to enforce sparsity in the model. Used only in the deprecated ‘pg’ solver.

Changed in version 0.17: Deprecated Projected Gradient solver. Use Coordinate Descent solver instead.

**beta** : double, default: 1

Degree of sparseness, if sparseness is not None. Larger values mean more sparseness.

Used only in the deprecated ‘pg’ solver.
eta : double, default: 0.1

Degree of correctness to maintain, if sparsity is not None. Smaller values mean larger error. Used only in the deprecated ‘pg’ solver.

Attributes

components_ : array, [n_components, n_features]

Non-negative components of the data.

reconstruction_err_ : number

Frobenius norm of the matrix difference between the training data and the reconstructed data from the fit produced by the model. \|X - WH\|_2

n_iter_ : int

Actual number of iterations.

References


Examples

```python
>>> import numpy as np
>>> X = np.array([[1,1], [2, 1], [3, 1.2], [4, 1], [5, 0.8], [6, 1]])
>>> from sklearn.decomposition import NMF
>>> model = NMF(n_components=2, init='random', random_state=0)
>>> model.fit(X)
NMF(alpha=0.0, beta=1, eta=0.1, init='random', l1_ratio=0.0, max_iter=200,
    n_components=2, nls_max_iter=2000, random_state=0, shuffle=False,
    solver='cd', sparseness=None, tol=0.0001, verbose=0)
```

```python
>>> model.components_
array([[ 2.09783018,  0.30560234],
        [ 2.13443044,  2.13171694]])
>>> model.reconstruction_err_
0.00115993...
```

Methods

fit(X[, y])  
Learn a NMF model for the data X.
### fit_transform(X, y, W, H)
Learn a NMF model for the data X and returns the transformed data.

### get_params([deep])
Get parameters for this estimator.

### inverse_transform(W)
Transform data back to its original space.

### set_params(**params)
Set the parameters of this estimator.

### transform(X)
Transform the data X according to the fitted NMF model.

---

**__init__ (args, **kwargs)**

DEPRECATED: It will be removed in release 0.19. Use NMF instead. 'pg' solver is still available until release 0.19.

**fit(X, y=None, **params)**

Learn a NMF model for the data X.

**Parameters**
- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  Data matrix to be decomposed

**Returns**
- **self**:

**fit_transform(X, y=None, W=None, H=None)**

Learn a NMF model for the data X and returns the transformed data.

This is more efficient than calling fit followed by transform.

**Parameters**
- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  Data matrix to be decomposed
- **W**: array-like, shape (n_samples, n_components)
  If init='custom', it is used as initial guess for the solution.
- **H**: array-like, shape (n_components, n_features)
  If init='custom', it is used as initial guess for the solution.

**Returns**
- **W**: array, shape (n_samples, n_components)
  Transformed data.

**get_params (deep=True)**

Get parameters for this estimator.

**Parameters**
- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params** : mapping of string to any
  Parameter names mapped to their values.

**inverse_transform(W)**

Transform data back to its original space.

**Parameters**
- **W**: {array-like, sparse matrix}, shape (n_samples, n_components)
  Transformed data matrix

**Returns**
- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  Data matrix of original shape

.. versionadded:: 0.18:
**set_params** (**params**)  
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Return** self:

**transform** (**X**)  
Transform the data X according to the fitted NMF model

**Parameters**

**X**: {array-like, sparse matrix}, shape (n_samples, n_features):
- Data matrix to be transformed by the model

**Returns**

**W**: array, shape (n_samples, n_components):
- Transformed data

### 29.7.4 sklearn.decomposition.KernelPCA

class sklearn.decomposition.KernelPCA (n_components=None, kernel='linear', gamma=None, degree=3, coef0=1, kernel_params=None, alpha=1.0, fit_inverse_transform=False, eigen_solver='auto', tol=0, max_iter=None, remove_zero_eig=False, random_state=None, copy_X=True, n_jobs=1)

Kernel Principal component analysis (KPCA)  
Non-linear dimensionality reduction through the use of kernels (see Pairwise metrics, Affinities and Kernels).  
Read more in the User Guide.

**Parameters**

**n_components** : int, default=None  
- Number of components. If None, all non-zero components are kept.

**kernel** : “linear” | “poly” | “rbf” | “sigmoid” | “cosine” | “precomputed”  
- Kernel. Default=“linear”.

**degree** : int, default=3  
- Degree for poly kernels. Ignored by other kernels.

**gamma** : float, default=1/n_features  
- Kernel coefficient for rbf and poly kernels. Ignored by other kernels.

**coef0** : float, default=1  
- Independent term in poly and sigmoid kernels. Ignored by other kernels.

**kernel_params** : mapping of string to any, default=None  
- Parameters (keyword arguments) and values for kernel passed as callable object. Ignored by other kernels.

**alpha** : int, default=1.0  
- Hyperparameter of the ridge regression that learns the inverse transform (when fit_inverse_transform=True).

**fit_inverse_transform** : bool, default=False
Learn the inverse transform for non-precomputed kernels. (i.e. learn to find the pre-image of a point)

**eigen_solver**: string [‘auto’,’dense’,’arpack’], default=’auto’

Select eigensolver to use. If n_components is much less than the number of training samples, arpack may be more efficient than the dense eigensolver.

**tol**: float, default=0

Convergence tolerance for arpack. If 0, optimal value will be chosen by arpack.

**max_iter**: int, default=None

Maximum number of iterations for arpack. If None, optimal value will be chosen by arpack.

**remove_zero_eig**: boolean, default=False

If True, then all components with zero eigenvalues are removed, so that the number of components in the output may be < n_components (and sometimes even zero due to numerical instability). When n_components is None, this parameter is ignored and components with zero eigenvalues are removed regardless.

**random_state**: int seed, RandomState instance, or None, default=None

A pseudo random number generator used for the initialization of the residuals when eigen_solver == ‘arpack’.

New in version 0.18.

**n_jobs**: int, default=1

The number of parallel jobs to run. If -1, then the number of jobs is set to the number of CPU cores.

New in version 0.18.

**copy_X**: boolean, default=True

If True, input X is copied and stored by the model in the X_fit_ attribute. If no further changes will be done to X, setting copy_X=False saves memory by storing a reference.

New in version 0.18.

**Attributes**

**lambdas_**: array, (n_components,)

Eigenvalues of the centered kernel matrix in decreasing order. If n_components and remove_zero_eig are not set, then all values are stored.

**alphas_**: array, (n_samples, n_components)

Eigenvectors of the centered kernel matrix. If n_components and remove_zero_eig are not set, then all components are stored.

**dual_coef_**: array, (n_samples, n_features)

Inverse transform matrix. Set if fit_inverse_transform is True.

**X_transformed_fit_**: array, (n_samples, n_components)

Projection of the fitted data on the kernel principal components.

**X_fit_**: (n_samples, n_features)

The data used to fit the model. If copy_X=False, then X_fit_ is a reference. This attribute is used for the calls to transform.
### References


### Methods

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<td>Fit the model from data in X.</td>
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<td><code>fit_transform(X[, y])</code></td>
<td>Fit the model from data in X and transform X.</td>
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<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<td><code>inverse_transform(X)</code></td>
<td>Transform X back to original space.</td>
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<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><code>transform(X)</code></td>
<td>Transform X.</td>
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```python
__init__(n_components=None, kernel='linear', gamma=None, degree=3, coef0=1, kernel_params=None, alpha=1.0, fit_inverse_transform=False, eigen_solver='auto', tol=0, max_iter=None, remove_zero_eig=False, random_state=None, copy_X=True, n_jobs=1)
```

**fit** *(X, y=None)*  
Fit the model from data in X.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)  
  Training vector, where n_samples is the number of samples and n_features is the number of features.

**Returns**

- **self**: object
  Returns the instance itself.

```python
fit_transform(X, y=None, **params)
```

Fit the model from data in X and transform X.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)  
  Training vector, where n_samples is the number of samples and n_features is the number of features.

**Returns**

- **X_new**: array-like, shape (n_samples, n_components)  
  Transformed data.

```python
get_params(deep=True)
```

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any  
  Parameter names mapped to their values.

```python
inverse_transform(X)
```

Transform X back to original space.

**Parameters**

- **X**: array-like, shape (n_samples, n_components)  
  Transformed data.

**Returns**

- **X_new**: array-like, shape (n_samples, n_features)  
  Transformed data.
References


**set_params (**)params**
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns **self**: 

**transform (X)**
Transform X.

Parameters **X**: array-like, shape `(n_samples, n_features)`: 

Returns **X_new**: array-like, shape `(n_samples, n_components)`: 

Examples using `sklearn.decomposition.KernelPCA`

- Kernel PCA

29.7.5 `sklearn.decomposition.FactorAnalysis`

class `sklearn.decomposition.FactorAnalysis` (**n_components=None**, **tol=0.01**, **copy=True**, **max_iter=1000**, **noise_variance_init=None**, **svd_method=’randomized’**, **iterated_power=3**, **random_state=0**)

Factor Analysis (FA)
A simple linear generative model with Gaussian latent variables.

The observations are assumed to be caused by a linear transformation of lower dimensional latent factors and added Gaussian noise. Without loss of generality the factors are distributed according to a Gaussian with zero mean and unit covariance. The noise is also zero mean and has an arbitrary diagonal covariance matrix.

If we would restrict the model further, by assuming that the Gaussian noise is even isotropic (all diagonal entries are the same) we would obtain PPCA.

FactorAnalysis performs a maximum likelihood estimate of the so-called loading matrix, the transformation of the latent variables to the observed ones, using expectation-maximization (EM).

Read more in the User Guide.

Parameters **n_components** : int | None

Dimensionality of latent space, the number of components of `X` that are obtained after `transform`. If None, `n_components` is set to the number of features.

**tol** : float

Stopping tolerance for EM algorithm.

**copy** : bool

Whether to make a copy of `X`. If `False`, the input `X` gets overwritten during fitting.

**max_iter** : int

Maximum number of iterations.
noise_variance_init : None | array, shape=(n_features,)

The initial guess of the noise variance for each feature. If None, it defaults to np.ones(n_features)

svd_method : {'lapack', 'randomized'}

Which SVD method to use. If ‘lapack’ use standard SVD from scipy.linalg, if ‘randomized’ use fast randomized_svd function. Defaults to ‘randomized’. For most applications ‘randomized’ will be sufficiently precise while providing significant speed gains. Accuracy can also be improved by setting higher values for iterated_power. If this is not sufficient, for maximum precision you should choose ‘lapack’.

iterated_power : int, optional

Number of iterations for the power method. 3 by default. Only used if svd_method equals ‘randomized’

random_state : int or RandomState

Pseudo number generator state used for random sampling. Only used if svd_method equals ‘randomized’

Attributes

components_ : array, [n_components, n_features]

Components with maximum variance.

loglike_ : list, [n_iterations]

The log likelihood at each iteration.

noise_variance_ : array, shape=(n_features,)

The estimated noise variance for each feature.

n_iter_ : int

Number of iterations run.

See also:

PCA
Principal component analysis is also a latent linear variable model which however assumes equal noise variance for each feature. This extra assumption makes probabilistic PCA faster as it can be computed in closed form.

FastICA
Independent component analysis, a latent variable model with non-Gaussian latent variables.

References

Methods

fit(X[, y])
Fit the FactorAnalysis model to X using EM

fit_transform(X[, y])
Fit to data, then transform it.

get_covariance()
Compute data covariance with the FactorAnalysis model.

get_params([deep])
Get parameters for this estimator.

get_precision()
Compute data precision matrix with the FactorAnalysis model.

score(X[, y])
Compute the average log-likelihood of the samples

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<td>Compute the log-likelihood of each sample</td>
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<td><code>set_params()</code></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><code>transform(X)</code></td>
<td>Apply dimensionality reduction to X using the model.</td>
</tr>
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</table>

```
__init__(n_components=None, tol=0.01, copy=True, max_iter=1000, noise_variance_init=None, svd_method='randomized', iterated_power=3, random_state=0)
```

```
fit(X, y=None)
```

- Fits the FactorAnalysis model to X using EM
- Parameters:
  - `X`: array-like, shape (n_samples, n_features)
    - Training data.
- Returns: self

```
fit_transform(X, y=None, **fit_params)
```

- Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.
- Parameters:
  - `X`: numpy array of shape [n_samples, n_features]
    - Training set.
  - `y`: numpy array of shape [n_samples]
    - Target values.
- Returns:
  - `X_new`: numpy array of shape [n_samples, n_features_new]
    - Transformed array.

```
get_covariance()
```

- Compute data covariance with the FactorAnalysis model.
- Parameters:
  - `X`: array-like, shape (n_samples, n_features)
    - Training data.
- Returns:
  - `cov`: array, shape (n_features, n_features)
    - Estimated covariance of data.

```
get_params(deep=True)
```

- Get parameters for this estimator.
- Parameters:
  - `deep`: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.
- Returns:
  - `params`: mapping of string to any
    - Parameter names mapped to their values.

```
get_precision()
```

- Compute data precision matrix with the FactorAnalysis model.
- Returns:
  - `precision`: array, shape (n_features, n_features)
    - Estimated precision of data.

```
score(X, y=None)
```

- Compute the average log-likelihood of the samples
- Parameters:
  - `X`: array, shape (n_samples, n_features)
  - `y`: array-like, shape (n_samples, n_features)
    - Target values.
The data

Returns: float :
Average log-likelihood of the samples under the current model

score_samples (X)
Compute the log-likelihood of each sample

Parameters:
X: array, shape (n_samples, n_features) :
The data

Returns:
array, shape (n_samples,) :
Log-likelihood of each sample under the current model

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns: self :

transform (X)
Apply dimensionality reduction to X using the model.

Compute the expected mean of the latent variables. See Barber, 21.2.33 (or Bishop, 12.66).

Parameters:
X: array-like, shape (n_samples, n_features)
Training data.

Returns:
X_new: array-like, shape (n_samples, n_components)
The latent variables of X.

Examples using sklearn.decomposition.FactorAnalysis

- Faces dataset decompositions
- Model selection with Probabilistic PCA and Factor Analysis (FA)

29.7.6 sklearn.decomposition.FastICA

class sklearn.decomposition.FastICA (n_components=None, algorithm='parallel', whiten=True, fun='logcosh', fun_args=None, max_iter=200, tol=0.0001, w_init=None, random_state=None)

FastICA: a fast algorithm for Independent Component Analysis.

Read more in the User Guide.

Parameters:
- n_components : int, optional
  Number of components to use. If none is passed, all are used.
- algorithm : {‘parallel’, ‘deflation’}
  Apply parallel or deflational algorithm for FastICA.
- whiten : boolean, optional
If whiten is false, the data is already considered to be whitened, and no whitening is performed.

**fun** : string or function, optional. Default: ‘logcosh’

The functional form of the G function used in the approximation to neg-entropy. Could be either ‘logcosh’, ‘exp’, or ‘cube’. You can also provide your own function. It should return a tuple containing the value of the function, and of its derivative, in the point. Example:

```python
def my_g(x): return x ** 3, 3 * x ** 2
```

**fun_args** : dictionary, optional

Arguments to send to the functional form. If empty and if fun=’logcosh’, fun_args will take value {‘alpha’ : 1.0}.

**max_iter** : int, optional

Maximum number of iterations during fit.

**tol** : float, optional

Tolerance on update at each iteration.

**w_init** : None or an (n_components, n_components) ndarray

The mixing matrix to be used to initialize the algorithm.

**random_state** : int or RandomState

Pseudo number generator state used for random sampling.

**Attributes**

**components_** : 2D array, shape (n_components, n_features)

The unmixing matrix.

**mixing_** : array, shape (n_features, n_components)

The mixing matrix.

**n_iter_** : int

If the algorithm is “deflation”, n_iter is the maximum number of iterations run across all components. Else they are just the number of iterations taken to converge.

**Notes**


**Methods**

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<td><code>fit(X[, y])</code></td>
<td>Fit the model to X.</td>
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<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit the model and recover the sources from X.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>inverse_transform(X[, copy])</code></td>
<td>Transform the sources back to the mixed data (apply mixing matrix).</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><code>transform(X[, y, copy])</code></td>
<td>Recover the sources from X (apply the unmixing matrix).</td>
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```python
__init__(n_components=None, algorithm='parallel', whiten=True, fun='logcosh', fun_args=None, max_iter=200, tol=0.0001, w_init=None, random_state=None)
```

- **fit** (*X*, *y=None*)
  - Fit the model to *X*.
  - **Parameters**
    - `X`: array-like, shape (n_samples, n_features)
      - Training data, where n_samples is the number of samples and n_features is the number of features.
  - **Returns**
    - `self`

- **fit_transform** (*X*, *y=None*)
  - Fit the model and recover the sources from *X*.
  - **Parameters**
    - `X`: array-like, shape (n_samples, n_features)
      - Training data, where n_samples is the number of samples and n_features is the number of features.
  - **Returns**
    - `X_new`: array-like, shape (n_samples, n_components)

```python
get_params(deep=True)
```

- **get_params** (*deep=True*)
  - Get parameters for this estimator.
  - **Parameters**
    - `deep` : boolean, optional
      - If True, will return the parameters for this estimator and contained subobjects that are estimators.
  - **Returns**
    - `params`: mapping of string to any
      - Parameter names mapped to their values.

```python
inverse_transform(X, copy=True)
```

- **inverse_transform** (*X*, *copy=True*)
  - Transform the sources back to the mixed data (apply mixing matrix).
  - **Parameters**
    - `X`: array-like, shape (n_samples, n_components)
      - Sources, where n_samples is the number of samples and n_components is the number of components.
    - `copy` : bool (optional)
      - If False, data passed to fit are overwritten. Defaults to True.
  - **Returns**
    - `X_new`: array-like, shape (n_samples, n_features)

```python
set_params(**params)
```

- **set_params** (***params*)
  - Set the parameters of this estimator.
  - The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.
  - **Returns**
    - `self`

```python
transform(X, y=None, copy=True)
```

- **transform** (*X*, *y=None, copy=True*)
  - Recover the sources from *X* (apply the unmixing matrix).
Parameters

\(X\) : array-like, shape (n_samples, n_features)

Data to transform, where n_samples is the number of samples and n_features is the number of features.

\(copy\) : bool (optional)

If False, data passed to fit are overwritten. Defaults to True.

Returns

\(X_{\text{new}}\) : array-like, shape (n_samples, n_components)

Examples using `sklearn.decomposition.FastICA`

- Faces dataset decompositions
- Blind source separation using FastICA
- FastICA on 2D point clouds

29.7.7 `sklearn.decomposition.TruncatedSVD`

class `sklearn.decomposition.TruncatedSVD` (n_components=2, algorithm='randomized', n_iter=5, random_state=None, tol=0.0)

Dimensionality reduction using truncated SVD (aka LSA).

This transformer performs linear dimensionality reduction by means of truncated singular value decomposition (SVD). Contrary to PCA, this estimator does not center the data before computing the singular value decomposition. This means it can work with scipy.sparse matrices efficiently.

In particular, truncated SVD works on term count/tf-idf matrices as returned by the vectorizers in `sklearn.feature_extraction.text`. In that context, it is known as latent semantic analysis (LSA).

This estimator supports two algorithms: a fast randomized SVD solver, and a “naive” algorithm that uses ARPACK as an eigensolver on \((X * X.T)\) or \((X.T * X)\), whichever is more efficient.

Read more in the User Guide.

Parameters

\(n_{\text{components}}\) : int, default = 2

Desired dimensionality of output data. Must be strictly less than the number of features. The default value is useful for visualisation. For LSA, a value of 100 is recommended.

\(algorithm\) : string, default = “randomized”

SVD solver to use. Either “arpack” for the ARPACK wrapper in SciPy (scipy.sparse.linalg.svds), or “randomized” for the randomized algorithm due to Halko (2009).

\(n_{\text{iter}}\) : int, optional (default 5)

Number of iterations for randomized SVD solver. Not used by ARPACK. The default is larger than the default in \(\text{randomized}_\text{svd}\) to handle sparse matrices that may have large slowly decaying spectrum.

\(random_{\text{state}}\) : int or RandomState, optional

(Seed for) pseudo-random number generator. If not given, the numpy.random singleton is used.

\(tol\) : float, optional

Tolerance for ARPACK. 0 means machine precision. Ignored by randomized SVD solver.
Attributes

- `components_` : array, shape (n_components, n_features)
- `explained_variance_ratio_` : array, [n_components]
  - Percentage of variance explained by each of the selected components.
- `explained_variance_` : array, [n_components]
  - The variance of the training samples transformed by a projection to each component.

See also:

- `PCA`
- `RandomizedPCA`

Notes

SVD suffers from a problem called “sign indeterminancy”, which means the sign of the `components_` and the output from transform depend on the algorithm and random state. To work around this, fit instances of this class to data once, then keep the instance around to do transformations.

References

Finding structure with randomness: Stochastic algorithms for constructing approximate matrix decompositions

Examples

```python
>>> from sklearn.decomposition import TruncatedSVD
>>> from sklearn.random_projection import sparse_random_matrix
>>> X = sparse_random_matrix(100, 100, density=0.01, random_state=42)
>>> svd = TruncatedSVD(n_components=5, n_iter=7, random_state=42)
>>> svd.fit(X)
TruncatedSVD(algorithm='randomized', n_components=5, n_iter=7,
             random_state=42, tol=0.0)
>>> print(svd.explained_variance_ratio_)
[ 0.0606... 0.0584... 0.0497... 0.0434... 0.0372...]
>>> print(svd.explained_variance_ratio_.sum())
0.249...
```

Methods

- `fit(X[, y])` : Fit LSI model on training data X.
- `fit_transform(X[, y])` : Fit LSI model to X and perform dimensionality reduction on X.
- `get_params([deep])` : Get parameters for this estimator.
- `inverse_transform(X)` : Transform X back to its original space.
- `set_params(**params)` : Set the parameters of this estimator.
- `transform(X)` : Perform dimensionality reduction on X.

```
__init__(n_components=2, algorithm='randomized', n_iter=5, random_state=None, tol=0.0)
fit(X, y=None)
```

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Fit LSI model on training data X.

**Parameters**
- X : {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training data.

**Returns**
- self : object
  - Returns the transformer object.

**fit_transform**(X, y=None)
Fit LSI model to X and perform dimensionality reduction on X.

**Parameters**
- X : {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training data.

**Returns**
- X_new : array, shape (n_samples, n_components)
  - Reduced version of X. This will always be a dense array.

**get_params**(deep=True)
Get parameters for this estimator.

**Parameters**
- deep : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- params : mapping of string to any
  - Parameter names mapped to their values.

**inverse_transform**(X)
Transform X back to its original space.

Returns an array X_original whose transform would be X.

**Parameters**
- X : array-like, shape (n_samples, n_components)
  - New data.

**Returns**
- X_original : array, shape (n_samples, n_features)
  - Note that this is always a dense array.

**set_params**(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- self : 

**transform**(X)
Perform dimensionality reduction on X.

**Parameters**
- X : {array-like, sparse matrix}, shape (n_samples, n_features)
  - New data.

**Returns**
- X_new : array, shape (n_samples, n_components)
  - Reduced version of X. This will always be a dense array.
Examples using `sklearn.decomposition.TruncatedSVD`

- Feature Union with Heterogeneous Data Sources
- Hashing feature transformation using Totally Random Trees
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...
- Clustering text documents using k-means

### 29.7.8 sklearn.decomposition.NMF

**class** `sklearn.decomposition.NMF(n_components=None, init=None, solver='cd', tol=0.0001, max_iter=200, random_state=None, alpha=0.0, l1_ratio=0.0, verbose=0, shuffle=False, nls_max_iter=2000, sparseness=None, beta=1, eta=0.1)`

Non-Negative Matrix Factorization (NMF)

Find two non-negative matrices (W, H) whose product approximates the non-negative matrix X. This factorization can be used for example for dimensionality reduction, source separation or topic extraction.

The objective function is:

\[
0.5 \times ||X - WH||_Fro^2 \\
+ \alpha \times l1\_ratio \times ||vec(W)||_1 \\
+ \alpha \times l1\_ratio \times ||vec(H)||_1 \\
+ 0.5 \times \alpha \times (1 - l1\_ratio) \times ||W||_Fro^2 \\
+ 0.5 \times \alpha \times (1 - l1\_ratio) \times ||H||_Fro^2
\]

Where:

\[
||A||_Fro^2 = \sum_{i,j} A_{ij}^2 \text{ (Frobenius norm)} \\
||vec(A)||_1 = \sum_{i,j} abs(A_{ij}) \text{ (Elementwise L1 norm)}
\]

The objective function is minimized with an alternating minimization of W and H.

Read more in the User Guide.

**Parameters**

- `n_components` : int or None
  
  Number of components, if `n_components` is not set all features are kept.

- `init` : ‘random’ | ‘nndsvd’ | ‘nndsvda’ | ‘nndsvdar’ | ‘custom’
  
  Method used to initialize the procedure. Default: ‘nndsvdar’ if `n_components` < `n_features`, otherwise random. Valid options:

  - ‘random’: **non-negative random matrices, scaled with**: \( \sqrt{X.\text{mean}()} / n\_\text{components} \)
  
  - ‘nndsvd’: **Nonnegative Double Singular Value Decomposition (NNDSVD)** initialization (better for sparseness)
  
  - ‘nndsvda’: **NNDSVD with zeros filled with the average of X** (better when sparsity is not desired)
  
  - ‘nndsvdar’: **NNDSVD with zeros filled with small random values** (generally faster, less accurate alternative to NNDSVDa for when sparsity is not desired)
  
  - ‘custom’: use custom matrices W and H

- `solver` : ‘pg’ | ‘cd’
Numerical solver to use: ‘pg’ is a Projected Gradient solver (deprecated). ‘cd’ is a Coordinate Descent solver (recommended).

New in version 0.17: Coordinate Descent solver.

Changed in version 0.17: Deprecated Projected Gradient solver.

tol : double, default: 1e-4

Tolerance value used in stopping conditions.

max_iter : integer, default: 200

Number of iterations to compute.

random_state : integer seed, RandomState instance, or None (default)

Random number generator seed control.

alpha : double, default: 0.

Constant that multiplies the regularization terms. Set it to zero to have no regularization.

New in version 0.17: alpha used in the Coordinate Descent solver.

l1_ratio : double, default: 0.

The regularization mixing parameter, with $0 \leq \text{l1\_ratio} \leq 1$. For $\text{l1\_ratio} = 0$ the penalty is an elementwise L2 penalty (aka Frobenius Norm). For $\text{l1\_ratio} = 1$ it is an elementwise L1 penalty. For $0 < \text{l1\_ratio} < 1$, the penalty is a combination of L1 and L2.

New in version 0.17: Regularization parameter l1\_ratio used in the Coordinate Descent solver.

shuffle : boolean, default: False

If true, randomize the order of coordinates in the CD solver.

New in version 0.17: shuffle parameter used in the Coordinate Descent solver.

nls_max_iter : integer, default: 2000

Number of iterations in NLS subproblem. Used only in the deprecated ‘pg’ solver.

Changed in version 0.17: Deprecated Projected Gradient solver. Use Coordinate Descent solver instead.

sparseness : ‘data’ | ‘components’ | None, default: None

Where to enforce sparsity in the model. Used only in the deprecated ‘pg’ solver.

Changed in version 0.17: Deprecated Projected Gradient solver. Use Coordinate Descent solver instead.

beta : double, default: 1

Degree of sparseness, if sparseness is not None. Larger values mean more sparseness. Used only in the deprecated ‘pg’ solver.

Changed in version 0.17: Deprecated Projected Gradient solver. Use Coordinate Descent solver instead.

eta : double, default: 0.1

Degree of correctness to maintain, if sparsity is not None. Smaller values mean larger error. Used only in the deprecated ‘pg’ solver.
Changed in version 0.17: Deprecate Projected Gradient solver. Use Coordinate Descent solver instead.

**Attributes**

- `components_`: array, [n_components, n_features]
  Non-negative components of the data.

- `reconstruction_err_`: number
  Frobenius norm of the matrix difference between the training data and the reconstructed data from the fit produced by the model. $||X - WH||_2$

- `n_iter_`: int
  Actual number of iterations.

**References**


**Examples**

```python
>>> import numpy as np
>>> X = np.array([[1,1], [2,1], [3,1.2], [4,1], [5,0.8], [6,1]])
>>> from sklearn.decomposition import NMF
>>> model = NMF(n_components=2, init='random', random_state=0)
>>> model.fit(X)
NMF(alpha=0.0, beta=1, eta=0.1, init='random', l1_ratio=0.0, max_iter=200, n_components=2, nls_max_iter=2000, random_state=0, shuffle=False, solver='cd', sparseness=None, tol=0.0001, verbose=0)
>>> model.components_
array([[ 2.09783018,  0.30560234],
         [ 2.13443044,  2.13171694]])
>>> model.reconstruction_err_
0.00115993...
```

**Methods**

- `fit(X, y)`: Learn a NMF model for the data X.
- `fit_transform(X, y, W, H)`: Learn a NMF model for the data X and returns the transformed data.
- `get_params([deep])`: Get parameters for this estimator.
- `inverse_transform(W)`: Transform data back to its original space.
- `set_params(**params)`: Set the parameters of this estimator.
- `transform(X)`: Transform the data X according to the fitted NMF model.
__init__(n_components=None, init=None, solver='cd', tol=0.0001, max_iter=200, random_state=None, alpha=0.0, l1_ratio=0.0, verbose=0, shuffle=False, nls_max_iter=2000, sparseness=None, beta=1, eta=0.1)

fit(X, y=None, **params)
Learn a NMF model for the data X.

Parameters:
- X: {array-like, sparse matrix}, shape (n_samples, n_features):
  Data matrix to be decomposed

Returns:
- self:

fit_transform(X, y=None, W=None, H=None)
Learn a NMF model for the data X and returns the transformed data.
This is more efficient than calling fit followed by transform.

Parameters:
- X: {array-like, sparse matrix}, shape (n_samples, n_features):
  Data matrix to be decomposed
- W: array-like, shape (n_samples, n_components):
  If init='custom', it is used as initial guess for the solution.
- H: array-like, shape (n_components, n_features):
  If init='custom', it is used as initial guess for the solution.

Returns:
- W: array, shape (n_samples, n_components):
  Transformed data.

get_params(deep=True)
Get parameters for this estimator.

Parameters:
- deep: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params: mapping of string to any
  Parameter names mapped to their values.

inverse_transform(W)
Transform data back to its original space.

Parameters:
- W: {array-like, sparse matrix}, shape (n_samples, n_components):
  Transformed data matrix

Returns:
- X: {array-like, sparse matrix}, shape (n_samples, n_features):
  Data matrix of original shape

.. versionadded:: 0.18:

set_params(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
- self:
transform\((X)\)
Transform the data \(X\) according to the fitted NMF model

**Parameters**
- **X**: array-like, sparse matrix, shape (n_samples, n_features)
  Data matrix to be transformed by the model

**Returns**
- **W**: array, shape (n_samples, n_components)
  Transformed data

**Examples using sklearn.decomposition.NMF**
- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation
- Faces dataset decompositions

29.7.9 sklearn.decomposition.SparsePCA

**class sklearn.decomposition.SparsePCA**

Sparse Principal Components Analysis (SparsePCA)
Finds the set of sparse components that can optimally reconstruct the data. The amount of sparseness is controllable by the coefficient of the L1 penalty, given by the parameter alpha.

Read more in the User Guide.

**Parameters**
- **n_components**: int,
  Number of sparse atoms to extract.
- **alpha**: float,
  Sparsity controlling parameter. Higher values lead to sparser components.
- **ridge_alpha**: float,
  Amount of ridge shrinkage to apply in order to improve conditioning when calling the transform method.
- **max_iter**: int,
  Maximum number of iterations to perform.
- **tol**: float,
  Tolerance for the stopping condition.
- **method**: {'lars', 'cd'}
  lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.
- **n_jobs**: int,
  Number of parallel jobs to run.
- **U_init**: array of shape (n_samples, n_components),
Initial values for the loadings for warm restart scenarios.

\( V_{\text{init}} \) : array of shape \( (n_{\text{components}}, n_{\text{features}}) \),

- Initial values for the components for warm restart scenarios.

\( \text{verbose} \) :

- Degree of verbosity of the printed output.

\( \text{random\_state} \) : int or RandomState

- Pseudo number generator state used for random sampling.

**Attributes**

- \( \text{components\_} \) : array, \([n_{\text{components}}, n_{\text{features}}]\)

- Sparse components extracted from the data.

- \( \text{error\_} \) : array

- Vector of errors at each iteration.

- \( n_{\text{iter\_}} \) : int

- Number of iterations run.

*See also:* 

- PCA, MiniBatchSparsePCA, DictionaryLearning

**Methods**

- **fit**(*X*, *y*)

  Fit the model from data in X.

- **fit_transform**(*X*, *y*)

  Fit to data, then transform it.

- **get_params**(*[deep]*)

  Get parameters for this estimator.

- **set_params**(*\**params*)

  Set the parameters of this estimator.

- **transform**(*X*, *ridge\_alpha*)

  Least Squares projection of the data onto the sparse components.

**__init__**

\( n_{\text{components}}=\text{None}, \ \alpha=1, \ \text{ridge\_alpha}=0.01, \ \text{max\_iter}=1000, \ \text{tol}=1e-08, \ \text{method}='\text{lars}', \ \text{n\_jobs}=1, \ \text{U\_init}=\text{None}, \ \text{V\_init}=\text{None}, \ \text{verbose}=\text{False}, \ \text{random\_state}=\text{None} \)

**fit** (*X*, *y*=\text{None})

- Fit the model from data in X.

  **Parameters**

  - \( X \) : array-like, shape \((n_{\text{samples}}, n_{\text{features}})\)

    Training vector, where \( n_{\text{samples}} \) in the number of samples and \( n_{\text{features}} \) is the number of features.

  **Returns**

  - \( \text{self} \) : object

    Returns the instance itself.

**fit_transform** (*X*, *y*=\text{None}, **fit\_params*)

- Fit to data, then transform it.

  Fits transformer to \( X \) and \( y \) with optional parameters fit\_params and returns a transformed version of \( X \).

  **Parameters**

  - \( X \) : numpy array of shape \([n_{\text{samples}}, n_{\text{features}}]\)

    Training set.
y : numpy array of shape [n_samples]

    Target values.

Returns

X_new : numpy array of shape [n_samples, n_features_new]

    Transformed array.

get_params (deep=True)

    Get parameters for this estimator.

Parameters

    deep : boolean, optional

        If True, will return the parameters for this estimator and contained subobjects that are
        estimators.

Returns

    params : mapping of string to any

        Parameter names mapped to their values.

set_params (**params)

    Set the parameters of this estimator.

    The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
    parameters of the form <component>__<parameter> so that it’s possible to update each component
    of a nested object.

Returns

    self

transform (X, ridge_alpha=None)

    Least Squares projection of the data onto the sparse components.

    To avoid instability issues in case the system is under-determined, regularization can be applied (Ridge
    regression) via the ridge_alpha parameter.

    Note that Sparse PCA components orthogonality is not enforced as in PCA hence one cannot use a simple
    linear projection.

Parameters

    X: array of shape (n_samples, n_features)

        Test data to be transformed, must have the same number of features as the data used to
        train the model.

    ridge_alpha: float, default: 0.01

        Amount of ridge shrinkage to apply in order to improve conditioning.

Returns

    X_new array, shape (n_samples, n_components)

        Transformed data.

29.7.10 sklearn.decomposition.MiniBatchSparsePCA

class sklearn.decomposition.MiniBatchSparsePCA (n_components=None, alpha=1, ridge_alpha=0.01, n_iter=100, callback=None, batch_size=3, verbose=False, shuffle=True, n_jobs=1, method='lars', random_state=None)

    Mini-batch Sparse Principal Components Analysis

    Finds the set of sparse components that can optimally reconstruct the data. The amount of sparseness is controllable by the coefficient of the L1 penalty, given by the parameter alpha.

    Read more in the User Guide.
Parameters

- **n_components**: int,
  - number of sparse atoms to extract
- **alpha**: int,
  - Sparsity controlling parameter. Higher values lead to sparser components.
- **ridge_alpha**: float,
  - Amount of ridge shrinkage to apply in order to improve conditioning when calling the transform method.
- **n_iter**: int,
  - number of iterations to perform for each mini batch
- **callback**: callable,
  - callable that gets invoked every five iterations
- **batch_size**: int,
  - the number of features to take in each mini batch
- **verbose**: :
  - degree of output the procedure will print
- **shuffle**: boolean,
  - whether to shuffle the data before splitting it in batches
- **n_jobs**: int,
  - number of parallel jobs to run, or -1 to autodetect.
- **method**: {'lars', 'cd'}
  - lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.
- **random_state**: int or RandomState
  - Pseudo number generator state used for random sampling.

Attributes

- **components_**: array, [n_components, n_features]
  - Sparse components extracted from the data.
- **error_**: array
  - Vector of errors at each iteration.
- **n_iter_**: int
  - Number of iterations run.

See also:

- **PCA**, **SparsePCA**, **DictionaryLearning**

Methods

- **fit(X[, y])**
  - Fit the model from data in X.

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<td>Fit to data, then transform it.</td>
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<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><code>transform(X[, ridge_alpha])</code></td>
<td>Least Squares projection of the data onto the sparse components.</td>
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</table>

```python
__init__ (n_components=None, alpha=1, ridge_alpha=0.01, n_iter=100, callback=None, batch_size=3, verbose=False, shuffle=True, n_jobs=1, method='lars', random_state=None)
```

```python
fit (X, y=None)
Fit the model from data in X.
```

**Parameters**
- **X**: array-like, shape (n_samples, n_features)
  - Training vector, where n_samples is the number of samples and n_features is the number of features.

**Returns**
- **self**: object
  - Returns the instance itself.

```python
fit_transform (X, y=None, **fit_params)
Fit to data, then transform it.
```

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**
- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

```python
get_params (deep=True)
Get parameters for this estimator.
```

**Parameters**
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params**: mapping of string to any
  - Parameter names mapped to their values.

```python
set_params (**params)
Set the parameters of this estimator.
```

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**
- **self**: object

```python
transform (X, ridge_alpha=None)
Least Squares projection of the data onto the sparse components.
```

29.7. `sklearn.decomposition`: Matrix Decomposition
To avoid instability issues in case the system is under-determined, regularization can be applied (Ridge regression) via the `ridge_alpha` parameter.

Note that Sparse PCA components orthogonality is not enforced as in PCA hence one cannot use a simple linear projection.

**Parameters**

`X`: array of shape (n_samples, n_features) :

Test data to be transformed, must have the same number of features as the data used to train the model.

`ridge_alpha`: float, default: 0.01 :

Amount of ridge shrinkage to apply in order to improve conditioning.

**Returns**

`X_new` array, shape (n_samples, n_components) :

Transformed data.

**Examples using sklearn.decomposition.MiniBatchSparsePCA**

- Faces dataset decompositions

**29.7.11 sklearn.decomposition.SparseCoder**

```python
class sklearn.decomposition.SparseCoder(dictionary, transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, split_sign=False, n_jobs=1)
```

Sparse coding

Finds a sparse representation of data against a fixed, precomputed dictionary.

Each row of the result is the solution to a sparse coding problem. The goal is to find a sparse array `code` such that:

\[
X \approx \text{code} \times \text{dictionary}
\]

Read more in the **User Guide**.

**Parameters**

`dictionary` : array, [n_components, n_features]

The dictionary atoms used for sparse coding. Lines are assumed to be normalized to unit norm.


Algorithm used to transform the data: lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary \* X'

`transform_n_nonzero_coefs` : int, 0.1 * n_features by default

Number of nonzero coefficients to target in each column of the solution. This is only used by `algorithm='lars'` and `algorithm='omp'` and is overridden by `alpha` in the `omp` case.

`transform_alpha` : float, 1. by default
If `algorithm='lasso_lars'` or `algorithm='lasso_cd'`, `alpha` is the penalty applied to the L1 norm. If `algorithm='threshold'`, `alpha` is the absolute value of the threshold below which coefficients will be squashed to zero. If `algorithm='omp'`, `alpha` is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides `n_nonzero_coefs`.

**split_sign** : bool, False by default

Whether to split the sparse feature vector into the concatenation of its negative part and its positive part. This can improve the performance of downstream classifiers.

**n_jobs** : int,

number of parallel jobs to run

**Attributes**

`components_` : array, [n_components, n_features]

The unchanged dictionary atoms

**See also:**

`DictionaryLearning`, `MiniBatchDictionaryLearning`, `SparsePCA`, `MiniBatchSparsePCA`, `sparse_encode`

**Methods**

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<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X[, y])</code></td>
<td>Encode the data as a sparse combination of the dictionary atoms.</td>
</tr>
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**__init__**(dictionary, `transform_algorithm='omp'`, `transform_n_nonzero_coefs=None`, `transform_alpha=None`, `split_sign=False`, `n_jobs=1`)

**fit**(X, y=None)

Do nothing and return the estimator unchanged

This method is just there to implement the usual API and hence work in pipelines.

**fit_transform**(X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- `X` : numpy array of shape [n_samples, n_features]
  
  Training set.

- `y` : numpy array of shape [n_samples]
  
  Target values.

**Returns**

- `X_new` : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- `deep` : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**:

- **params**: mapping of string to any
  Parameter names mapped to their values.

**set_params(**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**:

- **self**

**transform(**X, y=None)**

Encode the data as a sparse combination of the dictionary atoms.

Coding method is determined by the object parameter `transform_algorithm`.

**Parameters**:

- **X**: array of shape (n_samples, n_features)
  Test data to be transformed, must have the same number of features as the data used to train the model.

**Returns**:

- **X_new**: array, shape (n_samples, n_components)
  Transformed data

**Examples using** sklearn.decomposition.SparseCoder

- Sparse coding with a precomputed dictionary

29.7.12 sklearn.decomposition.DictionaryLearning

**class** sklearn.decomposition.DictionaryLearning(**n_components=None, alpha=1, max_iter=1000, tol=1e-08, fit_algorithm='lars', transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, n_jobs=1, code_init=None, dict_init=None, verbose=False, split_sign=False, random_state=None)**

Dictionary learning

Finds a dictionary (a set of atoms) that can best be used to represent data using a sparse code.

Solves the optimization problem:

$\{U^*, V^*\} = \arg \min_{U, V} 0.5 \| Y - U V \|_2^2 + \alpha \| U \|_1$

with $\| V_k \|_2 = 1$ for all $0 \leq k < n \_components$

Read more in the *User Guide*.

**Parameters**:

- **n_components**: int,
  number of dictionary elements to extract
alpha : float,
    sparsity controlling parameter
max_iter : int,
    maximum number of iterations to perform
tol : float,
    tolerance for numerical error
fit_algorithm : {'lars', 'cd'}
    lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

New in version 0.17: cd coordinate descent method to improve speed.

transform_algorithm : {'lasso_lars', 'lasso_cd', 'lars', 'omp', 'threshold'}
    Algorithm used to transform the data lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary \ast X'

New in version 0.17: lasso_cd coordinate descent method to improve speed.

transform_n_nonzero_coefs : int, 0.1 * n_features by default
    Number of nonzero coefficients to target in each column of the solution. This is only used by algorithm='lars' and algorithm='omp' and is overridden by alpha in the omp case.

transform_alpha : float, 1. by default
    If algorithm='lasso_lars' or algorithm='lasso_cd', alpha is the penalty applied to the L1 norm. If algorithm='threshold', alpha is the absolute value of the threshold below which coefficients will be squashed to zero. If algorithm='omp', alpha is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides n_nonzero_coefs.

split_sign : bool, False by default
    Whether to split the sparse feature vector into the concatenation of its negative part and its positive part. This can improve the performance of downstream classifiers.

n_jobs : int,
    number of parallel jobs to run
code_init : array of shape (n_samples, n_components),
    initial value for the code, for warm restart
dict_init : array of shape (n_components, n_features),
    initial values for the dictionary, for warm restart
verbose : :
    degree of verbosity of the printed output
random_state : int or RandomState
Pseudo number generator state used for random sampling.

**Attributes**

- `components_`: array, [n_components, n_features]
  - dictionary atoms extracted from the data
- `error_`: array
  - vector of errors at each iteration
- `n_iter_`: int
  - Number of iterations run.

**See also:**

`SparseCoder`, `MiniBatchDictionaryLearning`, `SparsePCA`, `MiniBatchSparsePCA`

**Notes**

**References:**


**Methods**

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<td>Fit the model from data in X.</td>
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</tr>
<tr>
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<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X[, y])</code></td>
<td>Encode the data as a sparse combination of the dictionary atoms.</td>
</tr>
</tbody>
</table>

**__init__**(n_components=None, alpha=1, max_iter=1000, tol=1e-08, fit_algorithm='lars', transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, n_jobs=1, code_init=None, dict_init=None, verbose=False, split_sign=False, random_state=None)

`fit`(X, y=None)

Fit the model from data in X.

**Parameters**

- `X`: array-like, shape (n_samples, n_features)
  - Training vector, where n_samples in the number of samples and n_features is the number of features.

**Returns**

- `self`: object
  - Returns the object itself

`fit_transform`(X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- `X`: numpy array of shape [n_samples, n_features]
  - Training set.
y : numpy array of shape [n_samples]
    Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
    Transformed array.

get_params (deep=True)
    Get parameters for this estimator.

    Parameters deep : boolean, optional
        If True, will return the parameters for this estimator and contained subobjects that are
        estimators.

    Returns params : mapping of string to any
        Parameter names mapped to their values.

set_params (**params)
    Set the parameters of this estimator.

        The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
        parameters of the form <component>__<parameter> so that it’s possible to update each component
        of a nested object.

    Returns self :

transform (X, y=None)
    Encode the data as a sparse combination of the dictionary atoms.

        Coding method is determined by the object parameter transform_algorithm.

    Parameters X : array of shape (n_samples, n_features)
        Test data to be transformed, must have the same number of features as the data used to
        train the model.

    Returns X_new : array, shape (n_samples, n_components)
        Transformed data
\[(U^*, V^*) = \arg\min_{(U, V)} 0.5 \| Y - U V \|_2^2 + \alpha \| U \|_1 \]

with \( \| V_k \|_2 = 1 \) for all \( 0 \leq k < n\_components \)

Read more in the *User Guide*.

**Parameters**

- `components` : int,
  number of dictionary elements to extract

- `alpha` : float,
  sparsity controlling parameter

- `n_iter` : int,
  total number of iterations to perform

- `fit_algorithm` : {‘lars’, ‘cd’}
  lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

  Algorithm used to transform the data. lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse.omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary \(* X^\top\)

- `transform_n_nonzero_coefs` : int, 0.1 * n_features by default
  Number of nonzero coefficients to target in each column of the solution. This is only used by `algorithm='lars'` and `algorithm='omp'` and is overridden by `alpha` in the `omp` case.

- `transform_alpha` : float, 1. by default
  If `algorithm='lasso_lars'` or `algorithm='lasso_cd'`, `alpha` is the penalty applied to the L1 norm. If `algorithm='threshold'`, `alpha` is the absolute value of the threshold below which coefficients will be squashed to zero. If `algorithm='omp'`, `alpha` is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides `n_nonzero_coefs`.

- `split_sign` : bool, False by default
  Whether to split the sparse feature vector into the concatenation of its negative part and its positive part. This can improve the performance of downstream classifiers.

- `n_jobs` : int,
  number of parallel jobs to run

- `dict_init` : array of shape (n_components, n_features),
  initial value of the dictionary for warm restart scenarios

- `verbose` :
  degree of verbosity of the printed output

- `batch_size` : int,
number of samples in each mini-batch

shuffle : bool,
whether to shuffle the samples before forming batches

random_state : int or RandomState
Pseudo number generator state used for random sampling.

Attributes

components_ : array, [n_components, n_features]
components extracted from the data

inner_stats_ : tuple of (A, B) ndarrays
Internal sufficient statistics that are kept by the algorithm. Keeping them is useful in online settings, to avoid loosing the history of the evolution, but they shouldn’t have any use for the end user. A (n_components, n_components) is the dictionary covariance matrix. B (n_features, n_components) is the data approximation matrix

n_iter_ : int
Number of iterations run.

See also:

SparseCoder, DictionaryLearning, SparsePCA, MiniBatchSparsePCA

Notes

References:


Methods

fit(X[, y])
Fit the model from data in X.

fit_transform(X[, y])
Fit to data, then transform it.

get_params([deep])
Get parameters for this estimator.

partial_fit(X[, y, iter_offset])
Updates the model using the data in X as a mini-batch.

set_params(**params)
Set the parameters of this estimator.

transform(X[, y])
Encode the data as a sparse combination of the dictionary atoms.

__init__(n_components=None, alpha=1, n_iter=1000, fit_algorithm='lars', n_jobs=1, batch_size=3, shuffle=True, dict_init=None, transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, verbose=False, split_sign=False, random_state=None)

fit(X, y=None)
Fit the model from data in X.

Parameters:

X: array-like, shape (n_samples, n_features) :
Training vector, where n_samples in the number of samples and n_features is the number of features.
Returns self: object

Returns the instance itself.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**partial_fit** *(X, y=None, iter_offset=None)*

Updates the model using the data in X as a mini-batch.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  - Training vector, where n_samples in the number of samples and n_features is the number of features.
- **iter_offset**: integer, optional
  - The number of iteration on data batches that has been performed before this call to partial_fit. This is optional: if no number is passed, the memory of the object is used.

**Returns**

- **self**: object
  - Returns the instance itself.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: object

**transform** *(X, y=None)*

Encode the data as a sparse combination of the dictionary atoms.

Coding method is determined by the object parameter `transform_algorithm`.

**Parameters**

- **X**: array of shape (n_samples, n_features)
  - Test data to be transformed, must have the same number of features as the data used to train the model.
Returns $X_{\text{new}}$: array, shape (n_samples, n_components)

Transformed data

Examples using sklearn.decomposition.MiniBatchDictionaryLearning

- Faces dataset decompositions
- Image denoising using dictionary learning

29.7.14 sklearn.decomposition.LatentDirichletAllocation

class sklearn.decomposition.LatentDirichletAllocation(n_topics=10,
           doc_topic_prior=None,
           topic_word_prior=None,
           learning_method=None,
           learning_decay=0.7,
           learning_offset=10.0,
           max_iter=10, batch_size=128,
           evaluate_every=-1,
           total_samples=1000000.0,
           perp_tol=0.1,
           mean_change_tol=0.001,
           max_doc_update_iter=100,
           n_jobs=1, verbose=0, random_state=None)

Latent Dirichlet Allocation with online variational Bayes algorithm

New in version 0.17.

Read more in the User Guide.

Parameters

- **n_topics**: int, optional (default=10)
  Number of topics.

- **doc_topic_prior**: float, optional (default=None)
  Prior of document topic distribution $\theta$. If the value is None, defaults to $1 / n\_topics$.
  In the literature, this is called $\alpha$.

- **topic_word_prior**: float, optional (default=None)
  Prior of topic word distribution $\beta$. If the value is None, defaults to $1 / n\_topics$. In the literature, this is called $\eta$.

- **learning_method**: 'batch' | 'online', default='online'
  Method used to update $\_\text{component}$. Only used in fit method. In general, if the data size is large, the online update will be much faster than the batch update. The default learning method is going to be changed to 'batch' in the 0.20 release. Valid options:

  ```
  'batch': Batch variational Bayes method. Use all training data in each EM update.
  Old `components_` will be overwritten in each iteration.
  'online': Online variational Bayes method. In each EM update, use mini-batch of training data to update the `components_`
  ```

29.7. sklearn.decomposition: Matrix Decomposition
variable incrementally. The learning rate is controlled by the 
``learning_decay`` and the ``learning_offset`` parameters.

**learning_decay**: float, optional (default=0.7)

   It is a parameter that controls learning rate in the online learning method. The value 
   should be set between (0.5, 1.0] to guarantee asymptotic convergence. When the value 
   is 0.0 and batch_size is n_samples, the update method is same as batch learning. In 
   the literature, this is called kappa.

**learning_offset**: float, optional (default=10.)

   A (positive) parameter that downweights early iterations in online learning. It should 
   be greater than 1.0. In the literature, this is called tau_0.

**max_iter**: integer, optional (default=10)

   The maximum number of iterations.

**total_samples**: int, optional (default=1e6)

   Total number of documents. Only used in the partial_fit method.

**batch_size**: int, optional (default=128)

   Number of documents to use in each EM iteration. Only used in online learning.

**evaluate_every**: int optional (default=0)

   How often to evaluate perplexity. Only used in fit method. set it to 0 or negative number 
   to not evaluate perplexity in training at all. Evaluating perplexity can help you check 
   convergence in training process, but it will also increase total training time. Evaluating 
   perplexity in every iteration might increase training time up to two-fold.

**perp_tol**: float, optional (default=1e-1)

   Perplexity tolerance in batch learning. Only used when evaluate_every is greater 
   than 0.

**mean_change_tol**: float, optional (default=1e-3)

   Stopping tolerance for updating document topic distribution in E-step.

**max_doc_update_iter**: int (default=100)

   Max number of iterations for updating document topic distribution in the E-step.

**n_jobs**: int, optional (default=1)

   The number of jobs to use in the E-step. If -1, all CPUs are used. For n_jobs below 
   -1, (n_CPUs + 1 + n_jobs) are used.

**verbose**: int, optional (default=0)

   Verbosity level.

**random_state**: int or RandomState instance or None, optional (default=None)

   Pseudo-random number generator seed control.

**Attributes**

   **components_**: array, [n_topics, n_features]

   Topic word distribution. components_[i, j] represents word j in topic i.

   **n_batch_iter_**: int
Number of iterations of the EM step.

\[ n_{\text{iter}} \]: int

Number of passes over the dataset.

References


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\[ \text{__init__} (n_{\text{topics}}=10, \; \text{doc_topic_prior}=\text{None}, \; \text{topic_word_prior}=\text{None}, \; \text{learning_method}=\text{None}, \; \text{learning_decay}=0.7, \; \text{learning_offset}=10.0, \; \text{max_iter}=10, \; \text{batch_size}=128, \; \text{evaluate_every}=-1, \; \text{total_samples}=1000000.0, \; \text{perp_tol}=0.1, \; \text{mean_change_tol}=0.001, \; \text{max_doc_update_iter}=100, \; \text{n_jobs}=1, \; \text{verbose}=0, \; \text{random_state}=\text{None}) \]

fit (X, y=None)

Learn model for the data X with variational Bayes method.

When learning_method is ‘online’, use mini-batch update. Otherwise, use batch update.

**Parameters**

- X : array-like or sparse matrix, shape=(n_samples, n_features)
  Document word matrix.

**Returns**

- self :

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- X : numpy array of shape [n_samples, n_features]
  Training set.

- y : numpy array of shape [n_samples]
  Target values.

**Returns**

- X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

**partial_fit** *(X, y=None)*

Online VB with Mini-Batch update.

**Parameters**

- **X** : array-like or sparse matrix, shape=(n_samples, n_features)

  Document word matrix.

**Returns**

- **self** :

**perplexity** *(X, doc_topic_distr=None, sub_sampling=False)*

Calculate approximate perplexity for data X.

Perplexity is defined as exp(-1. * log-likelihood per word)

**Parameters**

- **X** : array-like or sparse matrix, [n_samples, n_features]

  Document word matrix.

- **doc_topic_distr** : None or array, shape=(n_samples, n_topics)

  Document topic distribution. If it is None, it will be generated by applying transform on X.

**Returns**

- **score** : float

  Perplexity score.

**score** *(X, y=None)*

Calculate approximate log-likelihood as score.

**Parameters**

- **X** : array-like or sparse matrix, shape=(n_samples, n_features)

  Document word matrix.

**Returns**

- **score** : float

  Use approximate bound as score.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :

**transform** *(X)*

Transform data X according to the fitted model.

**Parameters**

- **X** : array-like or sparse matrix, shape=(n_samples, n_features)

  Document word matrix.
Returns `doc_topic_distr`: shape=(n_samples, n_topics)

Document topic distribution for X.

Examples using `sklearn.decomposition.LatentDirichletAllocation`

- **Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation**

  - `decomposition.fastica(X[, n_components, ...])` Perform Fast Independent Component Analysis.
  - `decomposition.dict_learning(X, n_components, ...)` Solves a dictionary learning matrix factorization problem.
  - `decomposition.dict_learning_online(X[, n_components, ...])` Solves a dictionary learning matrix factorization problem online.
  - `decomposition.sparse_encode(X, dictionary[, ...])` Sparse coding

29.7.15 `sklearn.decomposition.fastica`

`sklearn.decomposition.fastica(X, n_components=None, algorithm='parallel', whiten=True, fun='logcosh', fun_args=None, max_iter=200, tol=0.0001, w_init=None, random_state=None, return_X_mean=False, compute_sources=True, return_n_iter=False)`

Perform Fast Independent Component Analysis.

Read more in the *User Guide*.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  Training vector, where n_samples is the number of samples and n_features is the number of features.

- **n_components**: int, optional
  Number of components to extract. If None no dimension reduction is performed.

- **algorithm**: {'parallel', 'deflation'}, optional
  Apply a parallel or deflational FASTICA algorithm.

- **whiten**: boolean, optional
  If True perform an initial whitening of the data. If False, the data is assumed to have already been preprocessed: it should be centered, normed and white. Otherwise you will get incorrect results. In this case the parameter n_components will be ignored.

- **fun**: string or function, optional. Default: ‘logcosh’
  The functional form of the G function used in the approximation to neg-entropy. Could be either ‘logcosh’, ‘exp’, or ‘cube’. You can also provide your own function. It should return a tuple containing the value of the function, and of its derivative, in the point. Example:
  ```python
  def my_g(x): return x ** 3, 3 * x ** 2
  ```

- **fun_args**: dictionary, optional
  Arguments to send to the functional form. If empty or None and if fun='logcosh', fun_args will take value {'alpha': 1.0}.

- **max_iter**: int, optional

### 29.7. sklearn.decomposition: Matrix Decomposition

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Maximum number of iterations to perform.

tol: float, optional

A positive scalar giving the tolerance at which the un-mixing matrix is considered to have converged.

w_init: (n_components, n_components) array, optional

Initial un-mixing array of dimension (n.comp,n.comp). If None (default) then an array of normal r.v.’s is used.

random_state: int or RandomState

Pseudo number generator state used for random sampling.

return_X_mean: bool, optional

If True, X_mean is returned too.

compute_sources: bool, optional

If False, sources are not computed, but only the rotation matrix. This can save memory when working with big data. Defaults to True.

return_n_iter: bool, optional

Whether or not to return the number of iterations.

Returns:

K: array, shape (n_components, n_features) | None.

If whiten is ‘True’, K is the pre-whitening matrix that projects data onto the first n_components principal components. If whiten is ‘False’, K is ‘None’.

W: array, shape (n_components, n_components)

Estimated un-mixing matrix. The mixing matrix can be obtained by:

\[
\begin{align*}
    w &= \text{np.dot}(\hat{W}, K.T) \\
    A &= w.T \ast (w \ast w.T).I
\end{align*}
\]

S: array, shape (n_samples, n_components) | None

Estimated source matrix

X_mean: array, shape (n_features, )

The mean over features. Returned only if return_X_mean is True.

n_iter: int

If the algorithm is “deflation”, n_iter is the maximum number of iterations run across all components. Else they are just the number of iterations taken to converge. This is returned only when return_n_iter is set to True.

Notes

The data matrix X is considered to be a linear combination of non-Gaussian (independent) components i.e. \( X = AS \) where columns of S contain the independent components and A is a linear mixing matrix. In short ICA attempts to un-mix' the data by estimating an un-mixing matrix W where ‘\( S = WKX. \)’

This implementation was originally made for data of shape [n_features, n_samples]. Now the input is transposed before the algorithm is applied. This makes it slightly faster for Fortran-ordered input.

29.7.16 sklearn.decomposition.dict_learning

sklearn.decomposition.dict_learning(X, n_components, alpha, max_iter=100, tol=1e-08, method='lars', n_jobs=1, dict_init=None, code_init=None, callback=None, verbose=False, random_state=None, return_n_iter=False)

Solves a dictionary learning matrix factorization problem.

Finds the best dictionary and the corresponding sparse code for approximating the data matrix X by solving:

\[(U^{\ast}, V^{\ast}) = \arg\min_{(U,V)} 0.5 \| X - UV \|_2^2 + \alpha \| U \|_1 \]

with \( \| V_k \|_2 = 1 \) for all \( 0 \leq k < n\_components \)

where V is the dictionary and U is the sparse code.

Read more in the User Guide.

Parameters:

- **X**: array of shape (n_samples, n_features)
  - Data matrix.

- **n_components**: int
  - Number of dictionary atoms to extract.

- **alpha**: int
  - Sparsity controlling parameter.

- **max_iter**: int
  - Maximum number of iterations to perform.

- **tol**: float
  - Tolerance for the stopping condition.

- **method**: {'lars', 'cd'}
  - lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

- **n_jobs**: int
  - Number of parallel jobs to run, or -1 to autodetect.

- **dict_init**: array of shape (n_components, n_features),
  - Initial value for the dictionary for warm restart scenarios.

- **code_init**: array of shape (n_samples, n_components),
  - Initial value for the sparse code for warm restart scenarios.

- **callback**: Callable that gets invoked every five iterations.

- **verbose**: boolean
  - Degree of output the procedure will print.
random_state : int or RandomState
    Pseudo number generator state used for random sampling.

return_n_iter : bool
    Whether or not to return the number of iterations.

Returns
code : array of shape (n_samples, n_components)
    The sparse code factor in the matrix factorization.

dictionary : array of shape (n_components, n_features),
    The dictionary factor in the matrix factorization.

errors : array
    Vector of errors at each iteration.

n_iter : int
    Number of iterations run. Returned only if return_n_iter is set to True.

See also:

dict_learning_online, DictionaryLearning, MiniBatchDictionaryLearning,
SparsePCA, MiniBatchSparsePCA

29.7.17 sklearn.decomposition.dict_learning_online

Solves a dictionary learning matrix factorization problem online.
Finds the best dictionary and the corresponding sparse code for approximating the data matrix X by solving:

\[(U^*, V^*) = \arg\min_{(U,V)} 0.5 \| X - U V \|_2^2 + \alpha \| U \|_1 \]

with \( \| V_k \|_2 = 1 \) for all \( 0 \leq k < n\_components \)

where V is the dictionary and U is the sparse code. This is accomplished by repeatedly iterating over mini-batches by slicing the input data.

Read more in the User Guide.

Parameters

X: array of shape (n_samples, n_features)
    Data matrix.

n_components : int,
    Number of dictionary atoms to extract.

alpha : float,
    Sparsity controlling parameter.

n_iter : int,
Number of iterations to perform.

**return_code** : boolean,

Whether to also return the code U or just the dictionary V.

**dict_init** : array of shape (n_components, n_features),

Initial value for the dictionary for warm restart scenarios.

**callback** :

Callable that gets invoked every five iterations.

**batch_size** : int,

The number of samples to take in each batch.

**verbose** :

Degree of output the procedure will print.

**shuffle** : boolean,

Whether to shuffle the data before splitting it in batches.

**n_jobs** : int,

Number of parallel jobs to run, or -1 to autodetect.

**method** : {'lars', 'cd'}

lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

**iter_offset** : int, default 0

Number of previous iterations completed on the dictionary used for initialization.

**random_state** : int or RandomState

Pseudo number generator state used for random sampling.

**return_inner_stats** : boolean, optional

Return the inner statistics A (dictionary covariance) and B (data approximation). Useful to restart the algorithm in an online setting. If return_inner_stats is True, return_code is ignored

**inner_stats** : tuple of (A, B) ndarrays

Inner sufficient statistics that are kept by the algorithm. Passing them at initialization is useful in online settings, to avoid loosing the history of the evolution. A (n_components, n_components) is the dictionary covariance matrix. B (n_features, n_components) is the data approximation matrix

**return_n_iter** : bool

Whether or not to return the number of iterations.

**Returns**

**code** : array of shape (n_samples, n_components),
the sparse code (only returned if return_code=True)

**dictionary** : array of shape (n_components, n_features),
the solutions to the dictionary learning problem
**n_iter**: int

Number of iterations run. Returned only if *return_n_iter* is set to *True*.

See also:

dict_learning, DictionaryLearning, MiniBatchDictionaryLearning, SparsePCA, MiniBatchSparsePCA

### 29.7.18 sklearn.decomposition.sparse_encode

```python
sklearn.decomposition.sparse_encode(X, dictionary, gram=None, cov=None, algorithm='lasso_lars', n_nonzero_coefs=None, alpha=None, copy_cov=True, init=None, max_iter=1000, n_jobs=1, check_input=True, verbose=0)
```

Sparse coding

Each row of the result is the solution to a sparse coding problem. The goal is to find a sparse array *code* such that:

\[ X \approx \text{code} \cdot \text{dictionary} \]

Read more in the User Guide.

**Parameters**

- **X**: array of shape (*n_samples*, *n_features*)
  
  Data matrix

- **dictionary**: array of shape (*n_components*, *n_features*)
  
  The dictionary matrix against which to solve the sparse coding of the data. Some of the algorithms assume normalized rows for meaningful output.

- **gram**: array, shape=(*n_components*, *n_components*)
  
  Precomputed Gram matrix, dictionary * dictionary'

- **cov**: array, shape=(*n_components*, *n_samples*)
  
  Precomputed covariance, dictionary' * X

- **algorithm**: {'lasso_lars', 'lasso_cd', 'lars', 'omp', 'threshold'}
  
  lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary' * X'

- **n_nonzero_coefs**: int, 0.1 * *n_features* by default
  
  Number of nonzero coefficients to target in each column of the solution. This is only used by *algorithm='lars'* and *algorithm='omp'* and is overridden by *alpha* in the *omp* case.

- **alpha**: float, 1. by default
  
  If *algorithm='lasso_lars'* or *algorithm='lasso_cd'* , *alpha* is the penalty applied to the L1 norm. If *algorithm='threshold'* , *alpha* is the absolute value of the threshold below which coefficients will be squashed to zero. If *algorithm='omp'* , *alpha* is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides *n_nonzero_coefs*. 
init: array of shape (n_samples, n_components) :
    Initialization value of the sparse codes. Only used if algorithm=’lasso_cd’.

max_iter: int, 1000 by default :
    Maximum number of iterations to perform if algorithm=’lasso_cd’.

copy_cov: boolean, optional :
    Whether to copy the precomputed covariance matrix; if False, it may be overwritten.

n_jobs: int, optional :
    Number of parallel jobs to run.

check_input: boolean, optional :
    If False, the input arrays X and dictionary will not be checked.

verbose : int, optional
    Controls the verbosity; the higher, the more messages. Defaults to 0.

Returns
code: array of shape (n_samples, n_components) :
    The sparse codes

See also:
    sklearn.linear_model.lars_path, sklearn.linear_model.orthogonal_mp,
    sklearn.linear_model.Lasso, SparseCoder

29.8 sklearn.dummy: Dummy estimators

User guide: See the Model evaluation: quantifying the quality of predictions section for further details.

| dummy.DummyClassifier([strategy, ...]) | DummyClassifier is a classifier that makes predictions using simple rules. |
| dummy.DummyRegressor([strategy, constant, ...]) | DummyRegressor is a regressor that makes predictions using simple rules. |

29.8.1 sklearn.dummy.DummyClassifier

class sklearn.dummy.DummyClassifier (strategy=’stratified’, random_state=None, constant=None)
    DummyClassifier is a classifier that makes predictions using simple rules.
    This classifier is useful as a simple baseline to compare with other (real) classifiers. Do not use it for real problems.
    Read more in the User Guide.

Parameters
strategy : str, default=’stratified’
    Strategy to use to generate predictions.
    • “stratified”: generates predictions by respecting the training set’s class distribution.
    • “most_frequent”: always predicts the most frequent label in the training set.
    • “prior”: always predicts the class that maximizes the class prior (like “most_frequent”) and predict_proba returns the class prior.
• “uniform”: generates predictions uniformly at random.

• “constant”: always predicts a constant label that is provided by the user. This is useful for metrics that evaluate a non-majority class

New in version 0.17: Dummy Classifier now supports prior fitting strategy using parameter `prior`.

**random_state**: int seed, RandomState instance, or None (default)

The seed of the pseudo random number generator to use.

**constant**: int or str or array of shape = [n_outputs]

The explicit constant as predicted by the “constant” strategy. This parameter is useful only for the “constant” strategy.

**Attributes**

`classes_`: array or list of array of shape = [n_classes]

Class labels for each output.

`n_classes_`: array or list of array of shape = [n_classes]

Number of label for each output.

`class_prior_`: array or list of array of shape = [n_classes]

Probability of each class for each output.

`n_outputs_`: int,

Number of outputs.

`outputs_2d_`: bool,

True if the output at fit is 2d, else false.

`sparse_output_`: bool,

True if the array returned from predict is to be in sparse CSC format. Is automatically set to True if the input y is passed in sparse format.

**Methods**

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit the random classifier.</td>
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<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>predict(X)</code></td>
<td>Perform classification on test vectors X.</td>
</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>Return log probability estimates for the test vectors X.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Return probability estimates for the test vectors X.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
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</table>

**__init__ (strategy='stratified', random_state=None, constant=None)**

**fit (X, y, sample_weight=None)**

Fit the random classifier.

**Parameters**

`X`: {array-like, sparse matrix}, shape = [n_samples, n_features]

Training vectors, where n_samples is the number of samples and n_features is the number of features.
y : array-like, shape = [n_samples] or [n_samples, n_outputs]
    Target values.

sample_weight : array-like of shape = [n_samples], optional
    Sample weights.

Returns self:
    object
    Returns self.

get_params (deep=True)
    Get parameters for this estimator.

    Parameters:
    deep : boolean, optional
        If True, will return the parameters for this estimator and contained
        subobjects that are estimators.

    Returns:
    params : mapping of string to any
        Parameter names mapped to their values.

predict (X)
    Perform classification on test vectors X.

    Parameters:
    X : {array-like, sparse matrix}, shape = [n_samples, n_features]
        Input vectors, where n_samples is the number of samples and
        n_features is the number of features.

    Returns:
    y : array, shape = [n_samples] or [n_samples, n_outputs]
        Predicted target values for X.

predict_log_proba (X)
    Return log probability estimates for the test vectors X.

    Parameters:
    X : {array-like, sparse matrix}, shape = [n_samples, n_features]
        Input vectors, where n_samples is the number of samples and
        n_features is the number of features.

    Returns:
    P : array-like or list of array-like of shape = [n_samples, n_classes]
        Returns the log probability of the sample for each class in the model, where
        classes are ordered arithmetically for each output.

predict_proba (X)
    Return probability estimates for the test vectors X.

    Parameters:
    X : {array-like, sparse matrix}, shape = [n_samples, n_features]
        Input vectors, where n_samples is the number of samples and
        n_features is the number of features.

    Returns:
    P : array-like or list of array-like of shape = [n_samples, n_classes]
        Returns the probability of the sample for each class in the model, where
        classes are ordered arithmetically, for each output.

score (X, y, sample_weight=None)
    Returns the mean accuracy on the given test data and labels.

    In multi-label classification, this is the subset accuracy which is a harsh metric since you require for
    each sample that each label set be correctly predicted.
Parameters

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns

- **score**: float
  Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns

- **self**: 

29.8.2 sklearn.dummy.DummyRegressor

**class** sklearn.dummy.DummyRegressor (**strategy=**'mean', **constant=None**, **quantile=None**)

DummyRegressor is a regressor that makes predictions using simple rules.

This regressor is useful as a simple baseline to compare with other (real) regressors. Do not use it for real problems.

Read more in the User Guide.

Parameters

- **strategy**: str
  Strategy to use to generate predictions.
  - “mean”: always predicts the mean of the training set
  - “median”: always predicts the median of the training set
  - “quantile”: always predicts a specified quantile of the training set, provided with the quantile parameter.
  - “constant”: always predicts a constant value that is provided by the user.

- **constant**: int or float or array of shape = [n_outputs]
  The explicit constant as predicted by the “constant” strategy. This parameter is useful only for the “constant” strategy.

- **quantile**: float in [0.0, 1.0]
  The quantile to predict using the “quantile” strategy. A quantile of 0.5 corresponds to the median, while 0.0 to the minimum and 1.0 to the maximum.

Attributes

- **constant_**: float or array of shape [n_outputs]
  Mean or median or quantile of the training targets or constant value given by the user.

- **n_outputs_**: int
  Number of outputs.

- **outputs_2d_**: bool
True if the output at fit is 2d, else false.

Methods

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<tbody>
<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit the random regressor.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Perform classification on test vectors X.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
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</tbody>
</table>

__init__ (strategy='mean', constant=None, quantile=None)

fit (X, y, sample_weight=None)
Fit the random regressor.

Parameters

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

- **y**: array-like, shape = [n_samples] or [n_samples, n_outputs]
  Target values.

- **sample_weight**: array-like of shape = [n_samples], optional
  Sample weights.

Returns

- **self**: object
  Returns self.

get_params (deep=True)
Get parameters for this estimator.

Parameters

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

- **params**: mapping of string to any
  Parameter names mapped to their values.

predict (X)
Perform classification on test vectors X.

Parameters

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  Input vectors, where n_samples is the number of samples and n_features is the number of features.

Returns

- **y**: array, shape = [n_samples] or [n_samples, n_outputs]
  Predicted target values for X.

score (X, y, sample_weight=None)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ($(y_{true} - y_{pred})**2).sum()$ and $v$ is the residual sum of squares ($(y_{true} - y_{true.mean})**2).sum()$. Best possible score
is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

**Parameters**
- X : array-like, shape = (n_samples, n_features)
  Test samples.
- y : array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- sample_weight : array-like, shape = [n_samples], optional
  Sample weights.

**Returns**
- score : float
  R^2 of self.predict(X) wrt. y.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- self :

---

### 29.9 sklearn.ensemble: Ensemble Methods

The `sklearn.ensemble` module includes ensemble-based methods for classification, regression and anomaly detection.

**User guide:** See the Ensemble methods section for further details.

- `ensemble.BaggingClassifier((base_estimator, ...))` A Bagging classifier.
- `ensemble.BaggingRegressor((base_estimator, ...))` A Bagging regressor.
- `ensemble.GradientBoostingClassifier((loss, ...))` Gradient Boosting for classification.
- `ensemble.GradientBoostingRegressor((loss, ...))` Gradient Boosting for regression.
- `ensemble.IsolationForest((n_estimators, ...))` Isolation Forest Algorithm
- `ensemble.RandomForestClassifier((...))` A random forest classifier.
- `ensemble.RandomTreesEmbedding((...))` A random forest classifier.
29.9.1 sklearn.ensemble.AdaBoostClassifier

class sklearn.ensemble.AdaBoostClassifier (base_estimator=None, n_estimators=50, learning_rate=1.0, algorithm='SAMME.R', random_state=None)

An AdaBoost classifier.

An AdaBoost [1] classifier is a meta-estimator that begins by fitting a classifier on the original dataset and then fits additional copies of the classifier on the same dataset but where the weights of incorrectly classified instances are adjusted such that subsequent classifiers focus more on difficult cases.

This class implements the algorithm known as AdaBoost-SAMME [2].

Read more in the User Guide.

Parameters

base_estimator : object, optional (default=DecisionTreeClassifier)
    The base estimator from which the boosted ensemble is built. Support for sample weighting is required, as well as proper classes_ and n_classes_ attributes.

n_estimators : integer, optional (default=50)
    The maximum number of estimators at which boosting is terminated. In case of perfect fit, the learning procedure is stopped early.

learning_rate : float, optional (default=1.)
    Learning rate shrinks the contribution of each classifier by learning_rate. There is a trade-off between learning_rate and n_estimators.

algorithm : {'SAMME', 'SAMME.R'}, optional (default='SAMME.R')
    If 'SAMME.R' then use the SAMME.R real boosting algorithm. base_estimator must support calculation of class probabilities. If 'SAMME' then use the SAMME discrete boosting algorithm. The SAMME.R algorithm typically converges faster than SAMME, achieving a lower test error with fewer boosting iterations.

random_state : int, RandomState instance or None, optional (default=None)
    If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Attributes

estimators_ : list of classifiers
    The collection of fitted sub-estimators.

classes_ : array of shape = [n_classes]
    The classes labels.

n_classes_ : int
    The number of classes.

estimator_weights_ : array of floats
    Weights for each estimator in the boosted ensemble.

estimator_errors_ : array of floats
    Classification error for each estimator in the boosted ensemble.

feature_importances_ : array of shape = [n_features]
    The feature importances if supported by the base_estimator.
See also:

AdaBoostRegressor, GradientBoostingClassifier, DecisionTreeClassifier

References

[R11], [R12]

Methods

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<td>Compute the decision function of X.</td>
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<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Build a boosted classifier from the training set (X, y).</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
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<td>Predict class probabilities for X.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params([**params])</td>
<td>Set the parameters of this estimator.</td>
</tr>
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</tr>
<tr>
<td>staged_predict(X)</td>
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</tr>
<tr>
<td>staged_score(X, y[, sample_weight])</td>
<td>Return staged scores for X, y.</td>
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</table>

__init__ (base_estimator=None, n_estimators=50, learning_rate=1.0, algorithm='SAMME.R', random_state=None)

Compute the decision function of X.

**Parameters**

- `X` : array-like, sparse matrix of shape = [n_samples, n_features]

  The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

**Returns**

- `score` : array, shape = [n_samples, k]

  The decision function of the input samples. The order of outputs is the same as that of the classes_ attribute. Binary classification is a special cases with k == 1, otherwise k=n_classes. For binary classification, values closer to -1 or 1 mean more like the first or second class in classes_, respectively.

feature_importances_

Return the feature importances (the higher, the more important the feature).

**Returns**

- `feature_importances_` : array, shape = [n_features]

fit (X, y, sample_weight=None)

Build a boosted classifier from the training set (X, y).

**Parameters**

- `X` : array-like, sparse matrix of shape = [n_samples, n_features]

  The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.
y : array-like of shape = [n_samples]
The target values (class labels).

**sample_weight** : array-like of shape = [n_samples], optional
Sample weights. If None, the sample weights are initialized to \(1 / n_{\text{samples}}\).

**Return** : object
Returns self.

**get_params**(deep=True)
Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  Parameter names mapped to their values.

**predict**(X)
Predict classes for X.
The predicted class of an input sample is computed as the weighted mean prediction of the classifiers in the ensemble.

**Parameters**

- **X** : {array-like, sparse matrix} of shape = [n_samples, n_features]
The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

**Return**

- **y** : array of shape = [n_samples]
The predicted classes.

**predict_log_proba**(X)
Predict class log-probabilities for X.
The predicted class log-probabilities of an input sample is computed as the weighted mean predicted class log-probabilities of the classifiers in the ensemble.

**Parameters**

- **X** : {array-like, sparse matrix} of shape = [n_samples, n_features]
The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

**Return**

- **p** : array of shape = [n_samples]
The class probabilities of the input samples. The order of outputs is the same of that of the \(\text{classes}_.\) attribute.

**predict_proba**(X)
Predict class probabilities for X.
The predicted class probabilities of an input sample is computed as the weighted mean predicted class probabilities of the classifiers in the ensemble.

**Parameters**

- **X** : {array-like, sparse matrix} of shape = [n_samples, n_features]
The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

**Return**

- **p** : array of shape = [n_samples]
The class probabilities of the input samples. The order of outputs is the same of that of the classes_ attribute.

**score** (*X*, *y*, *sample_weight=None*)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  - Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**

- **score**: float
  - Mean accuracy of self.predict(*X*) wrt. *y*.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**

**staged_decision_function** (*X*)

Compute decision function of *X* for each boosting iteration.

This method allows monitoring (i.e. determine error on testing set) after each boosting iteration.

**Parameters**

- **X**: {array-like, sparse matrix} of shape = [n_samples, n_features]
  - The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

**Returns**

- **score**: generator of array, shape = [n_samples, k]
  - The decision function of the input samples. The order of outputs is the same of that of the classes_ attribute. Binary classification is a special cases with k == 1, otherwise k==n_classes. For binary classification, values closer to -1 or 1 mean more like the first or second class in classes_, respectively.

**staged_predict** (*X*)

Return staged predictions for *X*.

The predicted class of an input sample is computed as the weighted mean prediction of the classifiers in the ensemble.

This generator method yields the ensemble prediction after each iteration of boosting and therefore allows monitoring, such as to determine the prediction on a test set after each boost.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  - The input samples.

**Returns**

- **y**: generator of array, shape = [n_samples]
  - The predicted classes.
**staged_predict_proba** *(X)*
Predict class probabilities for X.

The predicted class probabilities of an input sample is computed as the weighted mean predicted class probabilities of the classifiers in the ensemble.

This generator method yields the ensemble predicted class probabilities after each iteration of boosting and therefore allows monitoring, such as to determine the predicted class probabilities on a test set after each boost.

**Parameters**

- **X** : array-like, sparse matrix of shape = [n_samples, n_features]
  The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

**Returns**

- **p** : generator of array, shape = [n_samples]
  The class probabilities of the input samples. The order of outputs is the same of that of the `classes_` attribute.

**staged_score** *(X, y, sample_weight=None)*
Return staged scores for X, y.

This generator method yields the ensemble score after each iteration of boosting and therefore allows monitoring, such as to determine the score on a test set after each boost.

**Parameters**

- **X** : array-like, sparse matrix of shape = [n_samples, n_features]
  The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

- **y** : array-like, shape = [n_samples]
  Labels for X.

- **sample_weight** : array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **z** : float

**Examples using sklearn.ensemble.AdaBoostClassifier**

- Classifier comparison
- Discrete versus Real AdaBoost
- Multi-class AdaBoosted Decision Trees
- Two-class AdaBoost
- Plot the decision surfaces of ensembles of trees on the iris dataset

**29.9.2 sklearn.ensemble.AdaBoostRegressor**

class sklearn.ensemble.AdaBoostRegressor *(base_estimator=None, n_estimators=50, learning_rate=1.0, loss='linear', random_state=None)*

An AdaBoost regressor.

An AdaBoost [1] regressor is a meta-estimator that begins by fitting a regressor on the original dataset and then fits additional copies of the regressor on the same dataset but where the weights of instances are adjusted according to the error of the current prediction. As such, subsequent regressors focus more on difficult cases.
This class implements the algorithm known as AdaBoost.R2 [2].

Read more in the User Guide.

**Parameters**

**base_estimator** : object, optional (default=DecisionTreeRegressor)

The base estimator from which the boosted ensemble is built. Support for sample weighting is required.

**n_estimators** : integer, optional (default=50)

The maximum number of estimators at which boosting is terminated. In case of perfect fit, the learning procedure is stopped early.

**learning_rate** : float, optional (default=1.)

Learning rate shrinks the contribution of each regressor by learning_rate. There is a trade-off between learning_rate and n_estimators.

**loss** : {'linear', 'square', 'exponential'}, optional (default='linear')

The loss function to use when updating the weights after each boosting iteration.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Attributes**

**estimators_** : list of classifiers

The collection of fitted sub-estimators.

**estimator_weights_** : array of floats

Weights for each estimator in the boosted ensemble.

**estimator_errors_** : array of floats

Regression error for each estimator in the boosted ensemble.

**feature_importances_** : array of shape = [n_features]

The feature importances if supported by the base_estimator.

See also:

*AdaBoostClassifier, GradientBoostingRegressor, DecisionTreeRegressor*

References

[R13], [R14]

Methods

*fit(X, y[, sample_weight])*

Build a boosted regressor from the training set (X, y).

*get_params([deep])*

Get parameters for this estimator.

*predict(X)*

Predict regression value for X.

*score(X, y[, sample_weight])*

Returns the coefficient of determination $R^2$ of the prediction.

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<td>Set the parameters of this estimator.</td>
</tr>
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<td><code>staged_predict(X)</code></td>
<td>Return staged predictions for X.</td>
</tr>
<tr>
<td><code>staged_score(X, y[, sample_weight])</code></td>
<td>Return staged scores for X, y.</td>
</tr>
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</table>

```python
__init__(base_estimator=None, n_estimators=50, learning_rate=1.0, loss='linear', random_state=None)
```

`feature_importances_`

Return the feature importances (the higher, the more important the feature).

Returns `feature_importances_` : array, shape = [n_features]

`fit(X, y, sample_weight=None)`

Build a boosted regressor from the training set (X, y).

Parameters

- `X` : {array-like, sparse matrix} of shape = [n_samples, n_features]
  The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.
- `y` : array-like of shape = [n_samples]
  The target values (real numbers).
- `sample_weight` : array-like of shape = [n_samples], optional
  Sample weights. If None, the sample weights are initialized to 1 / n_samples.

Returns `self`

`get_params(deep=True)`

Get parameters for this estimator.

Parameters

- `deep` : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns `params` : mapping of string to any

Parameter names mapped to their values.

`predict(X)`

Predict regression value for X.

The predicted regression value of an input sample is computed as the weighted median prediction of the classifiers in the ensemble.

Parameters

- `X` : {array-like, sparse matrix} of shape = [n_samples, n_features]
  The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.

Returns `y` : array of shape = [n_samples]

The predicted regression values.

`score(X, y, sample_weight=None)`

Returns the coefficient of determination R^2 of the prediction.
The coefficient $R^2$ is defined as $1 - \frac{u}{v}$, where $u$ is the regression sum of squares $((y_{\text{true}} - y_{\text{pred}})^2 \sum$) and $v$ is the residual sum of squares $((y_{\text{true}} - y_{\text{true,mean}})^2 \sum)$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**
- $X$: array-like, shape = (n_samples, n_features)
  Test samples.
- $y$: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for $X$.
- `sample_weight`: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**
- $score$: float
  $R^2$ of self.predict($X$) wrt. $y$.

**set_params**( **params**)
  Set the parameters of this estimator.

  The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

  **Returns**
  - `self`

**staged_predict**( $X$)
  Return staged predictions for $X$.

  The predicted regression value of an input sample is computed as the weighted median prediction of the classifiers in the ensemble.

  This generator method yields the ensemble prediction after each iteration of boosting and therefore allows monitoring, such as to determine the prediction on a test set after each boost.

  **Parameters**
  - $X$: {array-like, sparse matrix} of shape = [n_samples, n_features]
    The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.
  - `sample_weight`: array-like, shape = [n_samples], optional
    Sample weights.

  **Returns**
  - `y`: generator of array, shape = [n_samples]
    The predicted regression values.

**staged_score**( $X$, $y$, sample_weight=None)
  Return staged scores for $X$, $y$.

  This generator method yields the ensemble score after each iteration of boosting and therefore allows monitoring, such as to determine the score on a test set after each boost.

  **Parameters**
  - $X$: {array-like, sparse matrix} of shape = [n_samples, n_features]
    The training input samples. Sparse matrix can be CSC, CSR, COO, DOK, or LIL. DOK and LIL are converted to CSR.
  - $y$: array-like, shape = [n_samples]
    Labels for $X$.
  - `sample_weight`: array-like, shape = [n_samples], optional
    Sample weights.
A Bagging classifier is an ensemble meta-estimator that fits base classifiers each on random subsets of the original dataset and then aggregate their individual predictions (either by voting or by averaging) to form a final prediction. Such a meta-estimator can typically be used as a way to reduce the variance of a black-box estimator (e.g., a decision tree), by introducing randomization into its construction procedure and then making an ensemble out of it.

This algorithm encompasses several works from the literature. When random subsets of the dataset are drawn as random subsets of the samples, then this algorithm is known as Pasting [R151]. If samples are drawn with replacement, then the method is known as Bagging [R152]. When random subsets of the dataset are drawn as random subsets of the features, then the method is known as Random Subspaces [R153]. Finally, when base estimators are built on subsets of both samples and features, then the method is known as Random Patches [R154].

Read more in the User Guide.

**Parameters**

- **base_estimator** : object or None, optional (default=None)
  The base estimator to fit on random subsets of the dataset. If None, then the base estimator is a decision tree.

- **n_estimators** : int, optional (default=10)
  The number of base estimators in the ensemble.

- **max_samples** : int or float, optional (default=1.0)
  The number of samples to draw from X to train each base estimator.
  - If int, then draw `max_samples` samples.
  - If float, then draw `max_samples` * X.shape[0] samples.

- **max_features** : int or float, optional (default=1.0)
  The number of features to draw from X to train each base estimator.
  - If int, then draw `max_features` features.
  - If float, then draw `max_features` * X.shape[1] features.

- **bootstrap** : boolean, optional (default=True)
  Whether samples are drawn with replacement.

- **bootstrap_features** : boolean, optional (default=False)
  Whether features are drawn with replacement.
**oob_score** : bool
Whether to use out-of-bag samples to estimate the generalization error.

**warm_start** : bool, optional (default=False)
When set to True, reuse the solution of the previous call to fit and add more estimators
to the ensemble, otherwise, just fit a whole new ensemble.

New in version 0.17: warm_start constructor parameter.

**n_jobs** : int, optional (default=1)
The number of jobs to run in parallel for both fit and predict. If -1, then the number of
jobs is set to the number of cores.

**random_state** : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState
instance, random_state is the random number generator; If None, the random number
generator is the RandomState instance used by np.random.

**verbose** : int, optional (default=0)
Controls the verbosity of the building process.

**Attributes**

**base_estimator_** : estimator
The base estimator from which the ensemble is grown.

**estimators_** : list of estimators
The collection of fitted base estimators.

**estimators_samples_** : list of arrays
The subset of drawn samples (i.e., the in-bag samples) for each base estimator. Each
subset is defined by a boolean mask.

**estimators_features_** : list of arrays
The subset of drawn features for each base estimator.

**classes_** : array of shape = [n_classes]
The classes labels.

**n_classes_** : int or list
The number of classes.

**oob_score_** : float
Score of the training dataset obtained using an out-of-bag estimate.

**oob_decision_function_** : array of shape = [n_samples, n_classes]
Decision function computed with out-of-bag estimate on the training set. If
n_estimators is small it might be possible that a data point was never left out during
the bootstrap. In this case, oob_decision_function_ might contain NaN.

**References**

[R151], [R152], [R153], [R154]
## Methods

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<td>Average of the decision functions of the base classifiers.</td>
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<tr>
<td>fit</td>
<td>Build a Bagging ensemble of estimators from the training set (X, y).</td>
</tr>
<tr>
<td>get_params</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict</td>
<td>Predict class for X.</td>
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<tr>
<td>predict_log_proba</td>
<td>Predict class log-probabilities for X.</td>
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<tr>
<td>predict_proba</td>
<td>Predict class probabilities for X.</td>
</tr>
<tr>
<td>score</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params</td>
<td>Set the parameters of this estimator.</td>
</tr>
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</table>

**__init__**

```python
(base_estimator=None, n_estimators=10, max_samples=1.0, max_features=1.0, bootstrap=True, bootstrap_features=False, oob_score=False, warm_start=False, n_jobs=1, random_state=None, verbose=0)
```

**decision_function**

Average of the decision functions of the base classifiers.

**Parameters**

- **X**: array-like, sparse matrix of shape = [n_samples, n_features]

  The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

- **Return**

  `score`: array, shape = [n_samples, k]

  The decision function of the input samples. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`. Regression and binary classification are special cases with `k == 1`, otherwise `k == n_classes`.

###_estimators_samples_

The subset of drawn samples for each base estimator.

Returns a dynamically generated list of boolean masks identifying the samples used for for fitting each member of the ensemble, i.e., the in-bag samples.

*Note:* the list is re-created at each call to the property in order to reduce the object memory footprint by not storing the sampling data. Thus fetching the property may be slower than expected.

**fit**

Build a Bagging ensemble of estimators from the training set (X, y).

**Parameters**

- **X**: array-like, sparse matrix of shape = [n_samples, n_features]

  The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

- **y**: array-like, shape = [n_samples]

  The target values (class labels in classification, real numbers in regression).

- **sample_weight**: array-like, shape = [n_samples] or None

  Sample weights. If None, then samples are equally weighted. Note that this is supported only if the base estimator supports sample weighting.

**Return**

- **self**: object

  Returns self.
get_params (deep=True)
Get parameters for this estimator.

Parameters deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns params : mapping of string to any
Parameter names mapped to their values.

predict (X)
Predict class for X.

The predicted class of an input sample is computed as the class with the highest mean predicted probability. If base estimators do not implement a predict_proba method, then it resorts to voting.

Parameters X : {array-like, sparse matrix} of shape = [n_samples, n_features]
The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

Returns y : array of shape = [n_samples]
The predicted classes.

predict_log_proba (X)
Predict class log-probabilities for X.

The predicted class log-probabilities of an input sample is computed as the log of the mean predicted class probabilities of the base estimators in the ensemble.

Parameters X : {array-like, sparse matrix} of shape = [n_samples, n_features]
The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

Returns p : array of shape = [n_samples, n_classes]
The class log-probabilities of the input samples. The order of the classes corresponds to that in the attribute classes_.

predict_proba (X)
Predict class probabilities for X.

The predicted class probabilities of an input sample is computed as the mean predicted class probabilities of the base estimators in the ensemble. If base estimators do not implement a predict_proba method, then it resorts to voting and the predicted class probabilities of an input sample represents the proportion of estimators predicting each class.

Parameters X : {array-like, sparse matrix} of shape = [n_samples, n_features]
The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

Returns p : array of shape = [n_samples, n_classes]
The class probabilities of the input samples. The order of the classes corresponds to that in the attribute classes_.

score (X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.
Parameters

\( \textbf{X} \) : array-like, shape = (n_samples, n_features)
Test samples.

\( y \) : array-like, shape = (n_samples) or (n_samples, n_outputs)
True labels for X.

\( \textbf{sample\_weight} \) : array-like, shape = [n_samples], optional
Sample weights.

Returns

\( \textbf{score} \) : float
Mean accuracy of self.predict(X) wrt. y.

\( \textbf{set\_params} (**\text{params}) \)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \(<\text{component}>__<\text{parameter}>\) so that it’s possible to update each component of a nested object.

Returns

\( \textbf{self} \)

29.9.4 sklearn.ensemble.BaggingRegressor

class sklearn.ensemble.BaggingRegressor (base_estimator=None, n_estimators=10, max_samples=1.0, max_features=1.0, bootstrap=True, bootstrap_features=False, oob_score=False, warm_start=False, n_jobs=1, random_state=None, verbose=0)

A Bagging regressor.

A Bagging regressor is an ensemble meta-estimator that fits base regressors each on random subsets of the original dataset and then aggregate their individual predictions (either by voting or by averaging) to form a final prediction. Such a meta-estimator can typically be used as a way to reduce the variance of a black-box estimator (e.g., a decision tree), by introducing randomization into its construction procedure and then making an ensemble out of it.

This algorithm encompasses several works from the literature. When random subsets of the dataset are drawn as random subsets of the samples, then this algorithm is known as Pasting [R15]. If samples are drawn with replacement, then the method is known as Bagging [R16]. When random subsets of the dataset are drawn as random subsets of the features, then the method is known as Random Subspaces [R17]. Finally, when base estimators are built on subsets of both samples and features, then the method is known as Random Patches [R18].

Read more in the User Guide.

Parameters

\( \textbf{base\_estimator} \) : object or None, optional (default=None)
The base estimator to fit on random subsets of the dataset. If None, then the base estimator is a decision tree.

\( \textbf{n\_estimators} \) : int, optional (default=10)
The number of base estimators in the ensemble.

\( \textbf{max\_samples} \) : int or float, optional (default=1.0)
The number of samples to draw from X to train each base estimator.
- If int, then draw \( \text{max\_samples} \) samples.
• If float, then draw max_samples * X.shape[0] samples.

max_features : int or float, optional (default=1.0)
The number of features to draw from X to train each base estimator.
• If int, then draw max_features features.
• If float, then draw max_features * X.shape[1] features.

bootstrap : boolean, optional (default=True)
Whether samples are drawn with replacement.

bootstrap_features : boolean, optional (default=False)
Whether features are drawn with replacement.

oob_score : bool
Whether to use out-of-bag samples to estimate the generalization error.

warm_start : bool, optional (default=False)
When set to True, reuse the solution of the previous call to fit and add more estimators
to the ensemble, otherwise, just fit a whole new ensemble.

n_jobs : int, optional (default=1)
The number of jobs to run in parallel for both fit and predict. If -1, then the number of
jobs is set to the number of cores.

random_state : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState
instance, random_state is the random number generator; If None, the random number
generator is the RandomState instance used by np.random.

verbose : int, optional (default=0)
Controls the verbosity of the building process.

Attributes:
estimators_ : list of estimators
The collection of fitted sub-estimators.
estimators_samples_ : list of arrays
The subset of drawn samples (i.e., the in-bag samples) for each base estimator. Each
subset is defined by a boolean mask.
estimators_features_ : list of arrays
The subset of drawn features for each base estimator.
oob_score_ : float
Score of the training dataset obtained using an out-of-bag estimate.
oob_prediction_ : array of shape = [n_samples]
Prediction computed with out-of-bag estimate on the training set. If n_estimators is
small it might be possible that a data point was never left out during the bootstrap. In
this case, oob_prediction_ might contain NaN.
References

[R15], [R16], [R17], [R18]

Methods

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Build a Bagging ensemble of estimators from the training set (X, y).</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict regression target for X.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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```python
__init__ (base_estimator=None, n_estimators=10, max_samples=1.0, max_features=1.0, bootstrap=True, bootstrap_features=False, oob_score=False, warm_start=False, n_jobs=1, random_state=None, verbose=0)
```

**estimators_samples_**

The subset of drawn samples for each base estimator.

Returns a dynamically generated list of boolean masks identifying the samples used for fitting each member of the ensemble, i.e., the in-bag samples.

Note: the list is re-created at each call to the property in order to reduce the object memory footprint by not storing the sampling data. Thus fetching the property may be slower than expected.

```python
fit (X, y, sample_weight=None)
```

Build a Bagging ensemble of estimators from the training set (X, y).

**Parameters**

- `X` : {array-like, sparse matrix} of shape = [n_samples, n_features]
  - The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.
- `y` : array-like, shape = [n_samples]
  - The target values (class labels in classification, real numbers in regression).
- `sample_weight` : array-like, shape = [n_samples] or None
  - Sample weights. If None, then samples are equally weighted. Note that this is supported only if the base estimator supports sample weighting.

**Returns**

- `self` : object
  - Returns self.

```python
get_params (deep=True)
```

Get parameters for this estimator.

**Parameters**

- `deep` : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- `params` : mapping of string to any
Parameter names mapped to their values.

**predict** (*X*)

Predict regression target for *X*.

The predicted regression target of an input sample is computed as the mean predicted regression targets of the estimators in the ensemble.

**Parameters**

*`X`*: {array-like, sparse matrix} of shape = [n_samples, n_features]

The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

**Returns**

*`y`*: array of shape = [n_samples]

The predicted values.

**score** (*X*, *y*, *sample_weight=None*)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of *y*, disregarding the input features, would get a R^2 score of 0.0.

**Parameters**

*`X`*: array-like, shape = (n_samples, n_features)

Test samples.

*`y`*: array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for *X*.

*`sample_weight`*: array-like, shape = [n_samples], optional

Sample weights.

**Returns**

*`score`*: float

R^2 of self.predict(*X*) wrt. *y*.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

*`self`*:

Examples using `sklearn.ensemble.BaggingRegressor`

- Single estimator versus bagging: bias-variance decomposition

29.9.5 `sklearn.ensemble.IsolationForest`

**class** `sklearn.ensemble.IsolationForest` (*n_estimators=100, max_samples='auto', contamination=0.1, max_features=1.0, bootstrap=False, n_jobs=1, random_state=None, verbose=0)

Isolation Forest Algorithm

Return the anomaly score of each sample using the IsolationForest algorithm
The IsolationForest ‘isolates’ observations by randomly selecting a feature and then randomly selecting a split value between the maximum and minimum values of the selected feature.

Since recursive partitioning can be represented by a tree structure, the number of splittings required to isolate a sample is equivalent to the path length from the root node to the terminating node.

This path length, averaged over a forest of such random trees, is a measure of normality and our decision function.

Random partitioning produces noticeably shorter paths for anomalies. Hence, when a forest of random trees collectively produce shorter path lengths for particular samples, they are highly likely to be anomalies.

Read more in the User Guide.

New in version 0.18.

**Parameters**

- `n_estimators` : int, optional (default=100)
  The number of base estimators in the ensemble.

- `max_samples` : int or float, optional (default=”auto”)
  The number of samples to draw from X to train each base estimator.
  - If int, then draw `max_samples` samples.
  - If float, then draw `max_samples * X.shape[0]` samples.
  - If “auto”, then `max_samples=min(256, n_samples)`.
  If `max_samples` is larger than the number of samples provided, all samples will be used for all trees (no sampling).

- `contamination` : float in (0., 0.5), optional (default=0.1)
  The amount of contamination of the data set, i.e. the proportion of outliers in the data set. Used when fitting to define the threshold on the decision function.

- `max_features` : int or float, optional (default=1.0)
  The number of features to draw from X to train each base estimator.
  - If int, then draw `max_features` features.
  - If float, then draw `max_features * X.shape[1]` features.

- `bootstrap` : boolean, optional (default=False)
  If True, individual trees are fit on random subsets of the training data sampled with replacement. If False, sampling without replacement is performed.

- `n_jobs` : integer, optional (default=1)
  The number of jobs to run in parallel for both `fit` and `predict`. If -1, then the number of jobs is set to the number of cores.

- `random_state` : int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

- `verbose` : int, optional (default=0)
  Controls the verbosity of the tree building process.

**Attributes**

- `estimators_` : list of DecisionTreeClassifier

29.9. sklearn.ensemble: Ensemble Methods
The collection of fitted sub-estimators.

**estimators_samples_** : list of arrays

The subset of drawn samples (i.e., the in-bag samples) for each base estimator.

**max_samples_** : integer

The actual number of samples

**References**

[R21], [R22]

**Methods**

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<th>Description</th>
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<td>decision_function(X)</td>
<td>Average anomaly score of X of the base classifiers.</td>
</tr>
<tr>
<td>fit(X, y, sample_weight)</td>
<td>Fit estimator.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict if a particular sample is an outlier or not.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
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</table>

**__init__** (n_estimators=100, max_samples=‘auto’, contamination=0.1, max_features=1.0, bootstrap=False, n_jobs=1, random_state=None, verbose=0)

**decision_function**(X)
Average anomaly score of X of the base classifiers.

The anomaly score of an input sample is computed as the mean anomaly score of the trees in the forest. The measure of normality of an observation given a tree is the depth of the leaf containing this observation, which is equivalent to the number of splittings required to isolate this point. In case of several observations n_left in the leaf, the average path length of a n_left samples isolation tree is added.

**Parameters**

- **X**: (array-like, sparse matrix), shape (n_samples, n_features)
  - The training input samples. Sparse matrices are accepted only if they are supported by the base estimator.

**Returnsscores**: array of shape (n_samples,)

- The anomaly score of the input samples. The lower, the more abnormal.

**estimators_samples_**

The subset of drawn samples for each base estimator.

Returns a dynamically generated list of boolean masks identifying the samples used for for fitting each member of the ensemble, i.e., the in-bag samples.

Note: the list is re-created at each call to the property in order to reduce the object memory footprint by not storing the sampling data. Thus fetching the property may be slower than expected.

**fit**(X, y=None, sample_weight=None)

Fit estimator.

**Parameters**

- **X**: array-like or sparse matrix, shape (n_samples, n_features)
  - The input samples. Use `dtype=np.float32` for maximum efficiency. Sparse matrices are also supported, use sparse `csc_matrix` for maximum efficiency.
Returns self:

    object

Returns self.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

    deep : boolean, optional
        If True, will return the parameters for this estimator and contained subobjects that are
        estimators.

Returns:

    params : mapping of string to any
        Parameter names mapped to their values.

def predict (X)

Predict if a particular sample is an outlier or not.

Parameters:

    X : array-like or sparse matrix, shape (n_samples, n_features)
        The input samples. Internally, it will be converted to dtype=np.float32 and if a
        sparse matrix is provided to a sparse csr_matrix.

Returns:

    is_inlier : array, shape (n_samples,)
        For each observations, tells whether or not (+1 or -1) it should be considered as an inlier
        according to the fitted model.

def set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns:

    self : object

Examples using sklearn.ensemble.IsolationForest

- Outlier detection with several methods.
- IsolationForest example

29.9.6 sklearn.ensemble.RandomTreesEmbedding

class sklearn.ensemble.RandomTreesEmbedding (n_estimators=10, max_depth=5,
min_samples_split=2, min_samples_leaf=1,
min_weight_fraction_leaf=0.0,
max_leaf_nodes=None, min_impurity_split=1e-07, sparse_output=True, n_jobs=1,
random_state=None, verbose=0, warm_start=False)

An ensemble of totally random trees.

An unsupervised transformation of a dataset to a high-dimensional sparse representation. A datapoint is coded
according to which leaf of each tree it is sorted into. Using a one-hot encoding of the leaves, this leads to a
binary coding with as many ones as there are trees in the forest.
The dimensionality of the resulting representation is \( n_{\text{out}} \leq n_{\text{estimators}} \times \text{max}_\text{leaf}_\text{nodes} \). If \( \text{max}_\text{leaf}_\text{nodes} == \text{None} \), the number of leaf nodes is at most \( n_{\text{estimators}} \times 2^\text{max}_\text{depth} \).

Read more in the *User Guide*.

**Parameters**

- **n_estimators**: integer, optional (default=10)
  
  Number of trees in the forest.
  
- **max_depth**: integer, optional (default=5)
  
  The maximum depth of each tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than \( \text{min}_\text{samples}_\text{split} \) samples.
  
- **min_samples_split**: int, float, optional (default=2)
  
  The minimum number of samples required to split an internal node:
  
  - If int, then consider \( \text{min}_\text{samples}_\text{split} \) as the minimum number.
  
  - If float, then \( \text{min}_\text{samples}_\text{split} \) is a percentage and \( \text{cei}(\text{min}_\text{samples}_\text{split} \times n_{\text{samples}}) \) is the minimum number of samples for each split.
  
  Changed in version 0.18: Added float values for percentages.
  
- **min_samples_leaf**: int, float, optional (default=1)
  
  The minimum number of samples required to be at a leaf node:
  
  - If int, then consider \( \text{min}_\text{samples}_\text{leaf} \) as the minimum number.
  
  - If float, then \( \text{min}_\text{samples}_\text{leaf} \) is a percentage and \( \text{cei}(\text{min}_\text{samples}_\text{leaf} \times n_{\text{samples}}) \) is the minimum number of samples for each node.
  
  Changed in version 0.18: Added float values for percentages.
  
- **min_weight_fraction_leaf**: float, optional (default=0.)
  
  The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.
  
- **max_leaf_nodes**: int or None, optional (default=None)
  
  Grow trees with \( \text{max}_\text{leaf}_\text{nodes} \) in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.
  
- **min_impurity_split**: float, optional (default=1e-7)
  
  Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.
  
  New in version 0.18.
  
- **sparse_output**: bool, optional (default=True)
  
  Whether or not to return a sparse CSR matrix, as default behavior, or to return a dense array compatible with dense pipeline operators.
  
- **n_jobs**: integer, optional (default=1)
  
  The number of jobs to run in parallel for both fit and predict. If -1, then the number of jobs is set to the number of cores.
  
- **random_state**: int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**verbose**: int, optional (default=0)
Controls the verbosity of the tree building process.

**warm_start**: bool, optional (default=False)
When set to True, reuse the solution of the previous call to fit and add more estimators to the ensemble, otherwise, just fit a whole new forest.

**Attributes**

**estimators_**: list of DecisionTreeClassifier
The collection of fitted sub-estimators.

**References**

[R25], [R26]

**Methods**

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<td>Apply trees in the forest to X, return leaf indices.</td>
</tr>
<tr>
<td>decision_path(X)</td>
<td>Return the decision path in the forest</td>
</tr>
<tr>
<td>fit(X[, y, sample_weight])</td>
<td>Fit estimator.</td>
</tr>
<tr>
<td>fit_transform(X[, y, sample_weight])</td>
<td>Fit estimator and transform dataset.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform dataset.</td>
</tr>
</tbody>
</table>

**__init__**(n_estimators=10, max_depth=5, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_leaf_nodes=None, min_impurity_split=1e-07, sparse_output=True, n_jobs=1, random_state=None, verbose=0, warm_start=False)

**apply**(X)
Apply trees in the forest to X, return leaf indices.

**Parameters**

**X** : array-like or sparse matrix, shape = [n_samples, n_features]
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

**Returns**

**X_leaves** : array_like, shape = [n_samples, n_estimators]
For each datapoint x in X and for each tree in the forest, return the index of the leaf x ends up in.

**decision_path**(X)
Return the decision path in the forest

New in version 0.18.

**Parameters**

**X** : array-like or sparse matrix, shape = [n_samples, n_features]
The input samples. Internally, its dtype will be converted to dtype=np.float32. If a sparse matrix is provided, it will be converted into a sparse csr_matrix.

**Returns**

**indicator** : sparse csr array, shape = [n_samples, n_nodes]
Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

**n_nodes_ptr** : array of size (n_estimators + 1, )

The columns from indicator[n_nodes_ptr[i]:n_nodes_ptr[i+1]] gives the indicator value for the i-th estimator.

**feature_importances_**

Return the feature importances (the higher, the more important the feature).

**Returns**

feature_importances_ : array, shape = [n_features]

**fit** *(X, y=None, sample_weight=None)*

Fit estimator.

**Parameters**

- **X** : array-like or sparse matrix, shape=(n_samples, n_features)
  - The input samples. Use dtype=np.float32 for maximum efficiency. Sparse matrices are also supported, use sparse csc_matrix for maximum efficiency.

  **Returns**

  - **self** : object

**fit_transform** *(X, y=None, sample_weight=None)*

Fit estimator and transform dataset.

**Parameters**

- **X** : array-like or sparse matrix, shape=(n_samples, n_features)
  - Input data used to build forests. Use dtype=np.float32 for maximum efficiency.

**Returns**

- **X_transformed** : sparse matrix, shape=(n_samples, n_out)
  - Transformed dataset.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

  **Returns**

  - **params** : mapping of string to any
    - Parameter names mapped to their values.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :

**transform** *(X)*

Transform dataset.

**Parameters**

- **X** : array-like or sparse matrix, shape=(n_samples, n_features)
  - Input data to be transformed. Use dtype=np.float32 for maximum efficiency. Sparse matrices are also supported, use sparse csr_matrix for maximum efficiency.
**Returns**

\[
\text{X\_transformed} : \text{sparse matrix, shape=(n\_samples, n\_out)}
\]

Transformed dataset.

**Examples using sklearn.ensemble.RandomTreesEmbedding**

- Feature transformations with ensembles of trees
- Hashing feature transformation using Totally Random Trees
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...

**29.9.7 sklearn.ensemble.VotingClassifier**

class sklearn.ensemble.VotingClassifier(estimators, voting='hard', weights=None, n_jobs=1)

Soft Voting/Majority Rule classifier for unfitted estimators.

New in version 0.17.

Read more in the User Guide.

**Parametersestimators**: list of (string, estimator) tuples

Invoking the fit method on the VotingClassifier will fit clones of those original estimators that will be stored in the class attribute `self.estimators_`.

**voting**: str, {'hard', 'soft'} (default='hard')

If 'hard', uses predicted class labels for majority rule voting. Else if 'soft', predicts the class label based on the argmax of the sums of the predicted probabilities, which is recommended for an ensemble of well-calibrated classifiers.

**weights**: array-like, shape = [n\_classifiers], optional (default='None')

Sequence of weights (float or int) to weight the occurrences of predicted class labels (hard voting) or class probabilities before averaging (soft voting). Uses uniform weights if None.

**n\_jobs**: int, optional (default=1)

The number of jobs to run in parallel for fit. If -1, then the number of jobs is set to the number of cores.

**Attributes**

**estimators_**: list of classifiers

The collection of fitted sub-estimators.

**classes_**: array-like, shape = [n\_predictions]

The classes labels.

**Examples**

```python
>>> import numpy as np
>>> from sklearn.linear_model import LogisticRegression
>>> from sklearn.naive_bayes import GaussianNB
>>> from sklearn.ensemble import RandomForestClassifier, VotingClassifier
>>> clf1 = LogisticRegression(random_state=1)
>>> clf2 = RandomForestClassifier(random_state=1)
>>> clf3 = GaussianNB()
```
```
>>> X = np.array([[1, 1], [2, 2], [3, 3]])
>>> y = np.array([1, 2, 3])
>>> eclf1 = VotingClassifier(estimators=[
          ('lr', clf1), ('rf', clf2), ('gnb', clf3)], voting='hard')
>>> eclf1 = eclf1.fit(X, y)
>>> print(eclf1.predict(X))
[1 2 3]
>>> eclf2 = VotingClassifier(estimators=[
          ('lr', clf1), ('rf', clf2), ('gnb', clf3)], voting='soft')
>>> eclf2 = eclf2.fit(X, y)
>>> print(eclf2.predict(X))
[1 1 1]
>>> eclf3 = VotingClassifier(estimators=[
          ('lr', clf1), ('rf', clf2), ('gnb', clf3)], voting='soft', weights=[2, 1, 1])
>>> eclf3 = eclf3.fit(X, y)
>>> print(eclf3.predict(X))
[1 1 1]
```
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- X : numpy array of shape [n_samples, n_features]
  - Training set.
- y : numpy array of shape [n_samples]
  - Target values.

**Returns**
- X_new : numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params**(deep=True)
Return estimator parameter names for GridSearch support

**predict**(X)
Predict class labels for X.

**Parameters**
- X : {array-like, sparse matrix}, shape = [n_samples, n_features]
  - Training vectors, where n_samples is the number of samples and n_features is the number of features.

**Returns**
- maj : array-like, shape = [n_samples]
  - Predicted class labels.

**predict_proba**(X)
Compute probabilities of possible outcomes for samples in X.

**Parameters**
- X : {array-like, sparse matrix}, shape = [n_samples, n_features]
  - Training vectors, where n_samples is the number of samples and n_features is the number of features.

**Returns**
- avg : array-like, shape = [n_samples, n_classes]
  - Weighted average probability for each class per sample.

**score**(X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**
- X : array-like, shape = (n_samples, n_features)
  - Test samples.
- y : array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True labels for X.
- sample_weight : array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**
- score : float
  - Mean accuracy of self.predict(X) wrt. y.

**set_params**(**params)**
Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns: self:

**transform** *(X)*

Return class labels or probabilities for X for each estimator.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

**Returns**

If `voting='soft'`: :

array-like = [n_classifiers, n_samples, n_classes]Class probabilities calculated by each classifier.

If `voting='hard'`: :

array-like = [n_samples, n_classifiers]Class labels predicted by each classifier.

**Examples using sklearn.ensemble.VotingClassifier**

- Plot the decision boundaries of a VotingClassifier
- Plot class probabilities calculated by the VotingClassifier

**29.9.8 partial dependence**

Partial dependence plots for tree ensembles.

```
ensemble.partial_dependence.partial_dependence(...) Partial dependence of target_variables.
ensemble.partial_dependence.plot_partial_dependence(...) Partial dependence plots for features.
```

```
sklearn.ensemble.partial_dependence.partial_dependence(gbrt, target_variables, grid=None, X=None, percentiles=(0.05, 0.95), grid_resolution=100)
```

Partial dependence of target_variables.

Partial dependence plots show the dependence between the joint values of the target_variables and the function represented by the gbrt.

Read more in the User Guide.

**Parameters**

- **gbrt**: BaseGradientBoosting
  
  A fitted gradient boosting model.

- **target_variables**: array-like, dtype=int
  
  The target features for which the partial dependency should be computed (size should be smaller than 3 for visual renderings).
grid: array-like, shape=(n_points, len(target_variables))

The grid of target_variables values for which the partial dependency should be evaluated (either grid or X must be specified).

X: array-like, shape=(n_samples, n_features)

The data on which gbrt was trained. It is used to generate a grid for the target_variables. The grid comprises grid_resolution equally spaced points between the two percentiles.

percentiles: (low, high), default=(0.05, 0.95)

The lower and upper percentile used create the extreme values for the grid. Only if X is not None.

grid_resolution: int, default=100

The number of equally spaced points on the grid.

Returns:
pdp: array, shape=(n_classes, n_points)

The partial dependence function evaluated on the grid. For regression and binary classification n_classes==1.

axes: seq of ndarray or None

The axes with which the grid has been created or None if the grid has been given.

Examples

```python
>>> samples = [[0, 0, 2], [1, 0, 0]]
>>> labels = [0, 1]
>>> from sklearn.ensemble import GradientBoostingClassifier
>>> gb = GradientBoostingClassifier(random_state=0).fit(samples, labels)
>>> kwargs = dict(X=samples, percentiles=(0, 1), grid_resolution=2)
>>> partial_dependence(gb, [0], **kwargs)
(array([[ 0.52,..., 4.52...]])
```

sklearn.ensemble.partial_dependence.plot_partial_dependence

sklearn.ensemble.partial_dependence.plot_partial_dependence(gbrt, X, features, feature_names=None, label=None, n_cols=3, grid_resolution=100, percentiles=(0.05, 0.95), n_jobs=1, verbose=0, ax=None, line_kw=None, contour_kw=None, **fig_kw)

Partial dependence plots for features.

The len(features) plots are arranged in a grid with n_cols columns. Two-way partial dependence plots are plotted as contour plots.

Read more in the User Guide.
**Parameters**

- **gb**rt : BaseGradientBoosting
  A fitted gradient boosting model.

- **X** : array-like, shape=(n_samples, n_features)
  The data on which gb**rt** was trained.

- **features** : seq of ints, strings, or tuples of ints or strings
  If seq[i] is an int or a tuple with one int value, a one-way PDP is created; if seq[i] is a tuple of two ints, a two-way PDP is created. If feature_names is specified and seq[i] is an int, seq[i] must be < len(feature_names). If seq[i] is a string, feature_names must be specified, and seq[i] must be in feature_names.

- **feature_names** : seq of str
  Name of each feature; feature_names[i] holds the name of the feature with index i.

- **label** : object
  The class label for which the PDPs should be computed. Only if gb**rt** is a multi-class model. Must be in gb**rt**.classes_.

- **n_cols** : int
  The number of columns in the grid plot (default: 3).

- **percentiles** : (low, high), default=(0.05, 0.95)
  The lower and upper percentile used to create the extreme values for the PDP axes.

- **grid_resolution** : int, default=100
  The number of equally spaced points on the axes.

- **n_jobs** : int
  The number of CPUs to use to compute the PDs. -1 means ‘all CPUs’. Defaults to 1.

- **verbose** : int
  Verbose output during PD computations. Defaults to 0.

- **ax** : Matplotlib axis object, default None
  An axis object onto which the plots will be drawn.

- **line_kw** : dict
  Dct with keywords passed to the matplotlib.pyplot.plot call. For one-way partial dependence plots.

- **contour_kw** : dict
  Dct with keywords passed to the matplotlib.pyplot.plot call. For two-way partial dependence plots.

- **fig_kw** : dict
  Dct with keywords passed to the figure() call. Note that all keywords not recognized above will be automatically included here.

**Returns**

- **fig** : figure
  The Matplotlib Figure object.

- **axs** : seq of Axis objects
A seq of Axis objects, one for each subplot.

Examples

```python
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.ensemble import GradientBoostingRegressor
>>> X, y = make_friedman1()
>>> clf = GradientBoostingRegressor(n_estimators=10).fit(X, y)
>>> fig, axs = plot_partial_dependence(clf, X, [0, (0, 1)])
```

Examples using `sklearn.ensemble.partial_dependence.plot_partial_dependence`

- Partial Dependence Plots

29.10 sklearn.exceptions: Exceptions and warnings

The `sklearn.exceptions` module includes all custom warnings and error classes used across scikit-learn.

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<td>Exception class to raise if estimator is used before fitting.</td>
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<td>Warning class used to notify the user of any change in the behavior.</td>
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<td><code>exceptions.DataConversionWarning</code></td>
<td>Warning used to notify implicit data conversions happening in the code.</td>
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<td><code>exceptions.DataDimensionalityWarning</code></td>
<td>Custom warning to notify potential issues with data dimensionality.</td>
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<td><code>exceptions.EfficiencyWarning</code></td>
<td>Warning used to notify the user of inefficient computation.</td>
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<td>Warning class used if there is an error while fitting the estimator.</td>
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<td><code>exceptions.NonBLASDotWarning</code></td>
<td>Warning used when the dot operation does not use BLAS.</td>
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<td><code>exceptions.UndefinedMetricWarning</code></td>
<td>Warning used when the metric is invalid.</td>
</tr>
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29.10.1 sklearn.exceptions.NotFittedError

Class `sklearn.exceptions.NotFittedError`  
Exception class to raise if estimator is used before fitting.

This class inherits from both ValueError and AttributeError to help with exception handling and backward compatibility.

Examples

```python
>>> from sklearn.svm import LinearSVC
>>> from sklearn.exceptions import NotFittedError
>>> try:
...     LinearSVC().predict([[1, 2], [2, 3], [3, 4]])
... except NotFittedError as e:
```
... print(repr(e))
...
NotFittedError('This LinearSVC instance is not fitted yet',)

Changed in version 0.18: Moved from sklearn.utils.validation.

29.10.2 sklearn.exceptions.ChangedBehaviorWarning
class sklearn.exceptions.ChangedBehaviorWarning
Warning class used to notify the user of any change in the behavior.
    Changed in version 0.18: Moved from sklearn.base.

29.10.3 sklearn.exceptions.ConvergenceWarning
class sklearn.exceptions.ConvergenceWarning
Custom warning to capture convergence problems
    Changed in version 0.18: Moved from sklearn.utils.

Examples using sklearn.exceptions.ConvergenceWarning

• Sparse recovery: feature selection for sparse linear models

29.10.4 sklearn.exceptions.DataConversionWarning
class sklearn.exceptions.DataConversionWarning
Warning used to notify implicit data conversions happening in the code.
    This warning occurs when some input data needs to be converted or interpreted in a way that may not match the user’s expectations.

For example, this warning may occur when the user

• passes an integer array to a function which expects float input and will convert the input
• requests a non-copying operation, but a copy is required to meet the implementation’s data-type expectations;
• passes an input whose shape can be interpreted ambiguously.

Changed in version 0.18: Moved from sklearn.utils.validation.

29.10.5 sklearn.exceptions.DataDimensionalityWarning
class sklearn.exceptions.DataDimensionalityWarning
Custom warning to notify potential issues with data dimensionality.

For example, in random projection, this warning is raised when the number of components, which quantifies the dimensionality of the target projection space, is higher than the number of features, which quantifies the dimensionality of the original source space, to imply that the dimensionality of the problem will not be reduced.

Changed in version 0.18: Moved from sklearn.utils.
29.10.6 sklearn.exceptions.EfficiencyWarning

class sklearn.exceptions.EfficiencyWarning
    Warning used to notify the user of inefficient computation.

    This warning notifies the user that the efficiency may not be optimal due to some reason which may be included
    as a part of the warning message. This may be subclassed into a more specific Warning class.

    New in version 0.18.

29.10.7 sklearn.exceptions.FitFailedWarning

class sklearn.exceptions.FitFailedWarning
    Warning class used if there is an error while fitting the estimator.

    This Warning is used in meta estimators GridSearchCV and RandomizedSearchCV and the cross-validation
    helper function cross_val_score to warn when there is an error while fitting the estimator.

    Examples
    >>> from sklearn.model_selection import GridSearchCV
    >>> from sklearn.svm import LinearSVC
    >>> from sklearn.exceptions import FitFailedWarning
    >>> import warnings
    >>> warnings.simplefilter('always', FitFailedWarning)
    >>>
    >>> gs = GridSearchCV(LinearSVC(), {'C': [-1, -2]}, error_score=0)
    >>> X, y = [[1, 2], [3, 4], [5, 6], [7, 8], [8, 9]], [0, 0, 0, 1, 1]
    >>> with warnings.catch_warnings(record=True) as w:
    ...     try:
    ...         gs.fit(X, y)
    ...     except ValueError:
    ...         pass
    ...     print(repr(w[-1].message))
    ...
    FitFailedWarning("Classifier fit failed. The score on this train-test
    partition for these parameters will be set to 0.000000. Details:
    \nValueError('Penalty term must be positive; got (C=-2)',)",)

    Changed in version 0.18: Moved from sklearn.cross_validation.

29.10.8 sklearn.exceptions.NonBLASDotWarning

class sklearn.exceptions.NonBLASDotWarning
    Warning used when the dot operation does not use BLAS.

    This warning is used to notify the user that BLAS was not used for dot operation and hence the efficiency may
    be affected.

    Changed in version 0.18: Moved from sklearn.util.validation, extends EfficiencyWarning.

29.10.9 sklearn.exceptions.UndefinedMetricWarning

class sklearn.exceptions.UndefinedMetricWarning
    Warning used when the metric is invalid
29.11 sklearn.feature_extraction: Feature Extraction

The `sklearn.feature_extraction` module deals with feature extraction from raw data. It currently includes methods to extract features from text and images.

**User guide:** See the *Feature extraction* section for further details.

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<td><code>DictVectorizer()</code></td>
<td>Transforms lists of feature-value mappings to vectors.</td>
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<td><code>FeatureHasher()</code></td>
<td>Implements feature hashing, aka the hashing trick.</td>
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### 29.11.1 sklearn.feature_extraction.DictVectorizer

**class sklearn.feature_extraction.DictVectorizer**

```python
dictVectorizer = DictVectorizer(dtype=numpy.float64, separator='=', sparse=True, sort=True)
```

Transforms lists of feature-value mappings to vectors.

This transformer turns lists of mappings (dict-like objects) of feature names to feature values into Numpy arrays or scipy.sparse matrices for use with scikit-learn estimators.

When feature values are strings, this transformer will do a binary one-hot (aka one-of-K) coding: one boolean-valued feature is constructed for each of the possible string values that the feature can take on. For instance, a feature “f” that can take on the values “ham” and “spam” will become two features in the output, one signifying “f=ham”, the other “f=spam”.

However, note that this transformer will only do a binary one-hot encoding when feature values are of type string. If categorical features are represented as numeric values such as int, the DictVectorizer can be followed by OneHotEncoder to complete binary one-hot encoding.

Features that do not occur in a sample (mapping) will have a zero value in the resulting array/matrix.

Read more in the *User Guide*.

**Parameters**

- `dtype` : callable, optional
  - The type of feature values. Passed to Numpy array/scipy.sparse matrix constructors as the dtype argument.

- `separator` : string, optional
  - Separator string used when constructing new features for one-hot coding.

- `sparse` : boolean, optional.
  - Whether transform should produce scipy.sparse matrices. True by default.

- `sort` : boolean, optional.
  - Whether `feature_names_` and `vocabulary_` should be sorted when fitting. True by default.

**Attributes**

- `vocabulary_` : dict
  - A dictionary mapping feature names to feature indices.

- `feature_names_` : list
  - A list of length n_features containing the feature names (e.g., “f=ham” and “f=spam”).
See also:

**FeatureHasher** performs vectorization using only a hash function.

**sklearn.preprocessing.OneHotEncoder** handles nominal/categorical features encoded as columns of integers.

### Examples

```python
>>> from sklearn.feature_extraction import DictVectorizer
>>> v = DictVectorizer(sparse=False)
>>> D = [{'foo': 1, 'bar': 2}, {'foo': 3, 'baz': 1}]
>>> X = v.fit_transform(D)
>>> X
array([[ 2., 0., 1.],
       [ 0., 1., 3.]])
>>> v.inverse_transform(X) == [{'bar': 2.0, 'foo': 1.0}, {'baz': 1.0, 'foo': 3.0}]
True
>>> v.transform({'foo': 4, 'unseen_feature': 3})
array([[ 0., 0., 4.]])
```

### Methods

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<td><code>fit(X[, y])</code></td>
<td>Learn a list of feature name -&gt; indices mappings.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Learn a list of feature name -&gt; indices mappings and transform X.</td>
</tr>
<tr>
<td><code>get_feature_names()</code></td>
<td>Returns a list of feature names, ordered by their indices.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>inverse_transform(X[, dict_type])</code></td>
<td>Transform array or sparse matrix X back to feature mappings.</td>
</tr>
<tr>
<td><code>restrict(support[, indices])</code></td>
<td>Restrict the features to those in support using feature selection.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X[, y])</code></td>
<td>Transform feature-&gt;value dicts to array or sparse matrix.</td>
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#### `DictVectorizer`

**`__init__`**

dtype=<type ‘numpy.float64’>, separator=‘=’, sparse=True, sort=True)

**`fit`**(X, y=None)

Learn a list of feature name -> indices mappings.

**Parameters**

- X : Mapping or iterable over Mappings
  - Dict(s) or Mapping(s) from feature names (arbitrary Python objects) to feature values (strings or convertible to dtype).
  - y : (ignored)

**Returns**

self :

**`fit_transform`**(X, y=None)

Learn a list of feature name -> indices mappings and transform X.

Like `fit(X)` followed by `transform(X)`, but does not require materializing X in memory.
Parameters

- **X**: Mapping or iterable over Mappings
  Dict(s) or Mapping(s) from feature names (arbitrary Python objects) to feature values (strings or convertible to dtype).

- **y**: (ignored)

Returns

- **Xa**: {array, sparse matrix}
  Feature vectors; always 2-d.

**get_feature_names()**

Returns a list of feature names, ordered by their indices.
If one-of-K coding is applied to categorical features, this will include the constructed feature names but not the original ones.

**get_params**(deep=True)

Get parameters for this estimator.

- **Parameters**
  - **deep**: boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**
  - **params**: mapping of string to any
    Parameter names mapped to their values.

**inverse_transform**(X, dict_type=<type 'dict'>)

Transform array or sparse matrix X back to feature mappings.
X must have been produced by this DictVectorizer’s transform or fit_transform method; it may only have passed through transformers that preserve the number of features and their order.
In the case of one-hot/one-of-K coding, the constructed feature names and values are returned rather than the original ones.

- **Parameters**
  - **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
    Sample matrix.

  - **dict_type**: callable, optional
    Constructor for feature mappings. Must conform to the collections.Mapping API.

- **Returns**
  - **D**: list of dict_type objects, length = n_samples
    Feature mappings for the samples in X.

**restrict**(support, indices=False)

Restrict the features to those in support using feature selection.
This function modifies the estimator in-place.

- **Parameters**
  - **support**: array-like
    Boolean mask or list of indices (as returned by the get_support member of feature selectors).

  - **indices**: boolean, optional
    Whether support is a list of indices.

- **Returns**
  - **self**: 
Examples

```python
>>> from sklearn.feature_extraction import DictVectorizer
>>> from sklearn.feature_selection import SelectKBest, chi2

>>> v = DictVectorizer()
>>> D = [{'foo': 1, 'bar': 2}, {'foo': 3, 'baz': 1}]
>>> X = v.fit_transform(D)
>>> support = SelectKBest(chi2, k=2).fit(X, [0, 1])
>>> v.get_feature_names()
['bar', 'baz', 'foo']
>>> v.restrict(support.get_support())
DictVectorizer(dtype=..., separator='=', sort=True, sparse=True)
>>> v.get_feature_names()
['bar', 'foo']
```

```python
set_params(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:
```
transform(X, y=None)
Transform feature->value dicts to array or sparse matrix.

Named features not encountered during fit or fit_transform will be silently ignored.

Parameters

X : Mapping or iterable over Mappings, length = n_samples
Dict(s) or Mapping(s) from feature names (arbitrary Python objects) to feature values (strings or convertible to dtype).

y : (ignored)

Returns

Xa : {array, sparse matrix}
Feature vectors; always 2-d.
```

Examples using sklearn.feature_extraction.DictVectorizer

- Feature Union with Heterogeneous Data Sources
- FeatureHasher and DictVectorizer Comparison

29.11.2 sklearn.feature_extraction.FeatureHasher

class sklearn.feature_extraction.FeatureHasher(n_features=1048576, input_type='dict', dtype=<type 'numpy.float64'>, non_negative=False)

Implements feature hashing, aka the hashing trick.

This class turns sequences of symbolic feature names (strings) into scipy.sparse matrices, using a hash function to compute the matrix column corresponding to a name. The hash function employed is the signed 32-bit version of MurmurHash3.
Feature names of type byte string are used as-is. Unicode strings are converted to UTF-8 first, but no Unicode normalization is done. Feature values must be (finite) numbers.

This class is a low-memory alternative to DictVectorizer and CountVectorizer, intended for large-scale (online) learning and situations where memory is tight, e.g. when running prediction code on embedded devices.

Read more in the User Guide.

**Parameters**

- `n_features`: integer, optional
  The number of features (columns) in the output matrices. Small numbers of features are likely to cause hash collisions, but large numbers will cause larger coefficient dimensions in linear learners.

- `dtype`: numpy type, optional, default np.float64
  The type of feature values. Passed to scipy.sparse matrix constructors as the dtype argument. Do not set this to bool, np.boolean or any unsigned integer type.

- `input_type`: string, optional, default “dict”
  Either “dict” (the default) to accept dictionaries over (feature_name, value); “pair” to accept pairs of (feature_name, value); or “string” to accept single strings. feature_name should be a string, while value should be a number. In the case of “string”, a value of 1 is implied. The feature_name is hashed to find the appropriate column for the feature. The value’s sign might be flipped in the output (but see non_negative, below).

- `non_negative`: boolean, optional, default False
  Whether output matrices should contain non-negative values only; effectively calls abs on the matrix prior to returning it. When True, output values can be interpreted as frequencies. When False, output values will have expected value zero.

**See also:**

- `DictVectorizer` - vectorizes string-valued features using a hash table.
- `sklearn.preprocessing.OneHotEncoder` - handles nominal/categorical features encoded as columns of integers.

**Examples**

```python
>>> from sklearn.feature_extraction import FeatureHasher
>>> h = FeatureHasher(n_features=10)
>>> D = [{'dog': 1, 'cat':2, 'elephant':4},{'dog': 2, 'run': 5}]
>>> f = h.transform(D)
>>> f.toarray()
array([[ 0.,  0., -4., -1.,  0.,  0.,  0.,  0.,  2.],
       [ 0.,  0.,  0., -2., -5.,  0.,  0.,  0.,  0.]])
```

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><code>fit(X, y)</code></td>
<td>No-op</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td><code>get_params()</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>set_params()</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>
Table 29.86 – continued from previous page

**transform**((raw_X[, y]))

Transform a sequence of instances to a scipy.sparse matrix.

---

__init__(n_features=1048576, input_type='dict', dtype=<type 'numpy.float64'>, non_negative=False)

**fit**(X=None, y=None)

No-op.

This method doesn’t do anything. It exists purely for compatibility with the scikit-learn transformer API.

   Returnsself : FeatureHasher

**fit_transform**(X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

   ParametersX : numpy array of shape [n_samples, n_features]

   Training set.

   y : numpy array of shape [n_samples]

   Target values.

   ReturnsX_new : numpy array of shape [n_samples, n_features_new]

   Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.

   Parametersdeep : boolean, optional

   If True, will return the parameters for this estimator and contained subobjects that are estimators.

   Returnsparams : mapping of string to any

   Parameter names mapped to their values.

**set_params**(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form *<component>__<parameter>* so that it’s possible to update each component of a nested object.

   Returnsself :

**transform**(raw_X, y=None)

Transform a sequence of instances to a scipy.sparse matrix.

   Parametersraw_X : iterable over iterable over raw features, length = n_samples

   Samples. Each sample must be iterable an (e.g., a list or tuple) containing/generating feature names (and optionally values, see the input_type constructor argument) which will be hashed. raw_X need not support the len function, so it can be the result of a generator; n_samples is determined on the fly.

   y : (ignored)

   ReturnsX : scipy.sparse matrix, shape = (n_samples, self.n_features)
Feature matrix, for use with estimators or further transformers.

Examples using `sklearn.feature_extraction.FeatureHasher`

- `FeatureHasher and DictVectorizer Comparison`

### 29.11.3 From images

The `sklearn.feature_extraction.image` submodule gathers utilities to extract features from images.

```python
feature_extraction.image.img_to_graph(img, mask=None, return_as='sp.coo_matrix', dtype=None)
```

Graph of the pixel-to-pixel gradient connections

Edges are weighted with the gradient values.

Read more in the User Guide.

**Parameters**

- `img` : ndarray, 2D or 3D
  2D or 3D image
- `mask` : ndarray of boolean, optional
  An optional mask of the image, to consider only part of the pixels.
- `return_as` : np.ndarray or a sparse matrix class, optional
  The class to use to build the returned adjacency matrix.
- `dtype` : None or dtype, optional
  The data of the returned sparse matrix. By default it is the dtype of `img`

**Notes**

For scikit-learn versions 0.14.1 and prior, `return_as=np.ndarray` was handled by returning a dense `np.matrix` instance. Going forward, `np.ndarray` returns an `np.ndarray`, as expected.

For compatibility, user code relying on this method should wrap its calls in `np.asarray` to avoid type issues.
Graph of the pixel-to-pixel connections
Edges exist if 2 voxels are connected.

**Parameters**

- **n_x**: int
  - Dimension in x axis
- **n_y**: int
  - Dimension in y axis
- **n_z**: int, optional, default 1
  - Dimension in z axis
- **mask**: ndarray of booleans, optional
  - An optional mask of the image, to consider only part of the pixels.
- **return_as**: np.ndarray or a sparse matrix class, optional
  - The class to use to build the returned adjacency matrix.
- **dtype**: dtype, optional, default int
  - The data of the returned sparse matrix. By default it is int

**Notes**

For scikit-learn versions 0.14.1 and prior, return_as=np.ndarray was handled by returning a dense np.matrix instance. Going forward, np.ndarray returns an np.ndarray, as expected.

For compatibility, user code relying on this method should wrap its calls in np.asarray to avoid type issues.

Reshape a 2D image into a collection of patches
The resulting patches are allocated in a dedicated array.

**Parameters**

- **image**: array, shape = (image_height, image_width) or (image_height, image_width, n_channels)
  - The original image data. For color images, the last dimension specifies the channel: a RGB image would have n_channels=3.
- **patch_size**: tuple of ints (patch_height, patch_width)
  - The dimensions of one patch
- **max_patches**: integer or float, optional default is None

- **random_state**: None
  - Random number generator seed or None (to use the singleton instance).
The maximum number of patches to extract. If max_patches is a float between 0 and 1, it is taken to be a proportion of the total number of patches.

**random_state**: int or RandomState

Pseudo number generator state used for random sampling to use if max_patches is not None.

**Returns**

patches : array, shape = (n_patches, patch_height, patch_width) or (n_patches, patch_height, patch_width, n_channels) The collection of patches extracted from the image, where n_patches is either max_patches or the total number of patches that can be extracted.

**Examples**

```python
>>> from sklearn.feature_extraction import image
>>> one_image = np.arange(16).reshape((4, 4))
>>> one_image
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11],
       [12, 13, 14, 15]])
>>> patches = image.extract_patches_2d(one_image, (2, 2))
>>> print(patches.shape)
(9, 2, 2)
>>> patches[0]
array([[ 0,  1],
       [ 4,  5]])
>>> patches[1]
array([[ 1,  2],
       [ 5,  6]])
>>> patches[8]
array([[10, 11],
       [14, 15]])
```

**Examples using sklearn.feature_extraction.image.extract_patches_2d**

- Online learning of a dictionary of parts of faces
- Image denoising using dictionary learning

**sklearn.feature_extraction.image.reconstruct_from_patches_2d**

Reconstruct the image from all of its patches.

Patches are assumed to overlap and the image is constructed by filling in the patches from left to right, top to bottom, averaging the overlapping regions.

Read more in the User Guide.

**Parameters**

patches : array, shape = (n_patches, patch_height, patch_width) or
(n_patches, patch_height, patch_width, n_channels) The complete set of patches. If the patches contain colour information, channels are indexed along the last dimension: RGB patches would have n_channels=3.

**image_size**: tuple of ints (image_height, image_width) or (image_height, image_width, n_channels) the size of the image that will be reconstructed

**Returns**

**image**: array, shape = image_size

the reconstructed image

**Examples using** sklearn.feature_extraction.image.reconstruct_from_patches_2d

- *Image denoising using dictionary learning*

**sklearn.feature_extraction.image.PatchExtractor**

class sklearn.feature_extraction.image.PatchExtractor (patch_size=None, max_patches=None, random_state=None)

Extracts patches from a collection of images

Read more in the *User Guide*.

**Parameters**

**patch_size**: tuple of ints (patch_height, patch_width)

the dimensions of one patch

**max_patches**: integer or float, optional default is None

The maximum number of patches per image to extract. If max_patches is a float in (0, 1), it is taken to mean a proportion of the total number of patches.

**random_state**: int or RandomState

Pseudo number generator state used for random sampling.

**Methods**

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<tr>
<th>Method</th>
<th>Description</th>
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<tr>
<td>fit(X[, y])</td>
<td>Do nothing and return the estimator unchanged</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transforms the image samples in X into a matrix of patch data.</td>
</tr>
</tbody>
</table>

**__init__**(patch_size=None, max_patches=None, random_state=None)

**fit**(X, y=None)

Do nothing and return the estimator unchanged

This method is just there to implement the usual API and hence work in pipelines.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

**deep**: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**  
params : mapping of string to any  
Parameter names mapped to their values.

**set_params(**params)**  
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**  
self:

**transform(X)**  
Transforms the image samples in X into a matrix of patch data.

**Parameters**  
X : array, shape = (n_samples, image_height, image_width) or  
(n_samples, image_height, image_width, n_channels) Array of images from which to extract patches. For color images, the last dimension specifies the channel: a RGB image would have n_channels=3.

**Returns**  
patches: array, shape = (n_patches, patch_height, patch_width) or  
(n_patches, patch_height, patch_width, n_channels) The collection of patches extracted from the images, where n_patches is either n_samples * max_patches or the total number of patches that can be extracted.

### 29.11.4 From text

The `sklearn.feature_extraction.text` submodule gathers utilities to build feature vectors from text documents.

- `feature_extraction.text.CountVectorizer`  
  Convert a collection of text documents to a matrix of token counts

- `feature_extraction.text.HashingVectorizer`  
  Convert a collection of text documents to a matrix of token occurrences

- `feature_extraction.text.TfidfTransformer`  
  Transform a count matrix to a normalized tf or tf-idf representation

- `feature_extraction.text.TfidfVectorizer`  
  Convert a collection of raw documents to a matrix of TF-IDF features.
Convert a collection of text documents to a matrix of token counts.

This implementation produces a sparse representation of the counts using scipy.sparse.coo_matrix.

If you do not provide an a-priori dictionary and you do not use an analyzer that does some kind of feature selection then the number of features will be equal to the vocabulary size found by analyzing the data.

Read more in the User Guide.

**Parameters**

- **input**: string {'filename', 'file', 'content'}
  - If 'filename', the sequence passed as an argument to fit is expected to be a list of filenames that need reading to fetch the raw content to analyze.
  - If 'file', the sequence items must have a 'read' method (file-like object) that is called to fetch the bytes in memory.
  - Otherwise the input is expected to be the sequence strings or bytes items are expected to be analyzed directly.

- **encoding**: string, 'utf-8' by default.
  - If bytes or files are given to analyze, this encoding is used to decode.

- **decode_error**: {'strict', 'ignore', 'replace'}
  - Instruction on what to do if a byte sequence is given to analyze that contains characters not of the given encoding. By default, it is 'strict', meaning that a UnicodeDecodeError will be raised. Other values are 'ignore' and 'replace'.

- **strip_accents**: {'ascii', 'unicode', None}
  - Remove accents during the preprocessing step. ‘ascii’ is a fast method that only works on characters that have an direct ASCII mapping. ‘unicode’ is a slightly slower method that works on any characters. None (default) does nothing.

- **analyzer**: string, {'word', 'char', 'char_wb'} or callable
  - Whether the feature should be made of word or character n-grams. Option ‘char_wb’ creates character n-grams only from text inside word boundaries.
  - If a callable is passed it is used to extract the sequence of features out of the raw, unprocessed input.

- **preprocessor**: callable or None (default)
  - Override the preprocessing (string transformation) stage while preserving the tokenizing and n-grams generation steps.
tokenizer : callable or None (default)

Override the string tokenization step while preserving the preprocessing and n-grams generation steps. Only applies if analyzer == 'word'.

ngram_range : tuple (min_n, max_n)

The lower and upper boundary of the range of n-values for different n-grams to be extracted. All values of n such that min_n <= n <= max_n will be used.

stop_words : string {'english'}, list, or None (default)

If 'english', a built-in stop word list for English is used.

If a list, that list is assumed to contain stop words, all of which will be removed from the resulting tokens. Only applies if analyzer == 'word'.

If None, no stop words will be used. max_df can be set to a value in the range [0.7, 1.0) to automatically detect and filter stop words based on intra corpus document frequency of terms.

lowercase : boolean, True by default

Convert all characters to lowercase before tokenizing.

token_pattern : string

Regular expression denoting what constitutes a “token”, only used if analyzer == 'word'. The default regexp select tokens of 2 or more alphanumeric characters (punctuation is completely ignored and always treated as a token separator).

max_df : float in range [0.0, 1.0] or int, default=1.0

When building the vocabulary ignore terms that have a document frequency strictly higher than the given threshold (corpus-specific stop words). If float, the parameter represents a proportion of documents, integer absolute counts. This parameter is ignored if vocabulary is not None.

min_df : float in range [0.0, 1.0] or int, default=1

When building the vocabulary ignore terms that have a document frequency strictly lower than the given threshold. This value is also called cut-off in the literature. If float, the parameter represents a proportion of documents, integer absolute counts. This parameter is ignored if vocabulary is not None.

max_features : int or None, default=None

If not None, build a vocabulary that only consider the top max_features ordered by term frequency across the corpus.

This parameter is ignored if vocabulary is not None.

vocabulary : Mapping or iterable, optional

Either a Mapping (e.g., a dict) where keys are terms and values are indices in the feature matrix, or an iterable over terms. If not given, a vocabulary is determined from the input documents. Indices in the mapping should not be repeated and should not have any gap between 0 and the largest index.

binary : boolean, default=False

If True, all non zero counts are set to 1. This is useful for discrete probabilistic models that model binary events rather than integer counts.

dtype : type, optional
Type of the matrix returned by fit_transform() or transform().

**Attributes**
- `vocabulary_` : dict
  A mapping of terms to feature indices.
- `stop_words_` : set
  Terms that were ignored because they either:
  - occurred in too many documents (max_df)
  - occurred in too few documents (min_df)
  - were cut off by feature selection (max_features).
  This is only available if no vocabulary was given.

See also:
- HashingVectorizer, TfidfVectorizer

**Notes**

The `stop_words_` attribute can get large and increase the model size when pickling. This attribute is provided only for introspection and can be safely removed using delattr or set to None before pickling.

**Methods**

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<tr>
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<td><code>build_analyzer()</code></td>
<td>Return a callable that handles preprocessing and tokenization</td>
</tr>
<tr>
<td><code>build_preprocessor()</code></td>
<td>Return a function to preprocess the text before tokenization</td>
</tr>
<tr>
<td><code>build_tokenizer()</code></td>
<td>Return a function that splits a string into a sequence of tokens</td>
</tr>
<tr>
<td><code>decode(doc)</code></td>
<td>Decode the input into a string of unicode symbols</td>
</tr>
<tr>
<td><code>fit(raw_documents[, y])</code></td>
<td>Learn a vocabulary dictionary of all tokens in the raw documents.</td>
</tr>
<tr>
<td><code>fit_transform(raw_documents[, y])</code></td>
<td>Learn the vocabulary dictionary and return term-document matrix.</td>
</tr>
<tr>
<td><code>get_feature_names()</code></td>
<td>Array mapping from feature integer indices to feature name</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>get_stop_words()</code></td>
<td>Build or fetch the effective stop words list</td>
</tr>
<tr>
<td><code>inverse_transform(X)</code></td>
<td>Return terms per document with nonzero entries in X.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(raw_documents)</code></td>
<td>Transform documents to document-term matrix.</td>
</tr>
</tbody>
</table>

__init__ (input=u'content', encoding=u'utf-8', decode_error=u'strict', strip_accents=None, lowercase=True, preprocessor=None, tokenizer=None, stop_words=None, token_pattern=u'(?u)\b\w\w+\b', ngram_range=(1, 1), analyzer=u'word', max_df=1.0, min_df=1, max_features=None, vocabulary=None, binary=False, dtype=<type 'numpy.int64'>)

`build_analyzer()`

Return a callable that handles preprocessing and tokenization
build_preprocessor()  
    Return a function to preprocess the text before tokenization

build_tokenizer()  
    Return a function that splits a string into a sequence of tokens

dedecode(doc)  
    Decode the input into a string of unicode symbols
    The decoding strategy depends on the vectorizer parameters.

fit(raw_documents, y=None)  
    Learn a vocabulary dictionary of all tokens in the raw documents.

        Parameters
        raw_documents : iterable
            An iterable which yields either str, unicode or file objects.

        Returns
        self : 

fit_transform(raw_documents, y=None)  
    Learn the vocabulary dictionary and return term-document matrix.
    This is equivalent to fit followed by transform, but more efficiently implemented.

        Parameters
        raw_documents : iterable
            An iterable which yields either str, unicode or file objects.

        Returns
        X : array, [n_samples, n_features]

gget_feature_names()  
    Array mapping from feature integer indices to feature name

gget_params(deep=True)  
    Get parameters for this estimator.

        Parameters
        deep : boolean, optional
            If True, will return the parameters for this estimator and contained subobjects that are
            estimators.

        Returns
        params : mapping of string to any
            Parameter names mapped to their values.

gget_stop_words()  
    Build or fetch the effective stop words list

inverse_transform(X)  
    Return terms per document with nonzero entries in X.

        Parameters
        X : {array, sparse matrix}, shape = [n_samples, n_features]

        Returns
        X_inv : list of arrays, len = n_samples
            List of arrays of terms.

set_params(**params)  
    Set the parameters of this estimator.
    The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
    parameters of the form <component>__<parameter> so that it’s possible to update each component
    of a nested object.
Returns: 
transform (raw_documents)
Transform documents to document-term matrix.

Extract token counts out of raw text documents using the vocabulary fitted with fit or the one provided to the constructor.

Parameters:
raw_documents : iterable
An iterable which yields either str, unicode or file objects.

Returns:
X : sparse matrix, [n_samples, n_features]

Examples using sklearn.feature_extraction.text.CountVectorizer

• Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation
• Sample pipeline for text feature extraction and evaluation

sklearn.feature_extraction.text.HashingVectorizer

class sklearn.feature_extraction.text.HashingVectorizer (input=u'content', encoding=u'utf-8', decode_error=u'strict', strip_accents=None, lowercase=True, preprocessor=None, tokenizer=None, stop_words=None, token_pattern=u'(?u)\b\w\w+\b', ngram_range=(1, 1), analyzer=u'word', n_features=1048576, binary=False, norm=u'l2', non_negative=False, dtype=<type 'numpy.float64'>)

Convert a collection of text documents to a matrix of token occurrences

It turns a collection of text documents into a scipy.sparse matrix holding token occurrence counts (or binary occurrence information), possibly normalized as token frequencies if norm='l1' or projected on the euclidean unit sphere if norm='l2'.

This text vectorizer implementation uses the hashing trick to find the token string name to feature integer index mapping.

This strategy has several advantages:

• it is very low memory scalable to large datasets as there is no need to store a vocabulary dictionary in memory
• it is fast to pickle and un-pickle as it holds no state besides the constructor parameters
• it can be used in a streaming (partial fit) or parallel pipeline as there is no state computed during fit.

There are also a couple of cons (vs using a CountVectorizer with an in-memory vocabulary):
• there is no way to compute the inverse transform (from feature indices to string feature names) which can be a problem when trying to introspect which features are most important to a model.

• there can be collisions: distinct tokens can be mapped to the same feature index. However in practice this is rarely an issue if n_features is large enough (e.g. $2^{**} 18$ for text classification problems).

• no IDF weighting as this would render the transformer stateful.

The hash function employed is the signed 32-bit version of Murmurhash3.

Read more in the User Guide.

Parameters:

- **input** : string {'filename', 'file', 'content'}

  If 'filename', the sequence passed as an argument to fit is expected to be a list of filenames that need reading to fetch the raw content to analyze.

  If 'file', the sequence items must have a 'read' method (file-like object) that is called to fetch the bytes in memory.

  Otherwise the input is expected to be the sequence strings or bytes items are expected to be analyzed directly.

- **encoding** : string, default='utf-8'

  If bytes or files are given to analyze, this encoding is used to decode.

- **decode_error** : {'strict', 'ignore', 'replace'}

  Instruction on what to do if a byte sequence is given to analyze that contains characters not of the given encoding. By default, it is 'strict', meaning that a UnicodeDecodeError will be raised. Other values are 'ignore' and 'replace'.

- **strip_accents** : {'ascii', 'unicode', None}

  Remove accents during the preprocessing step. ‘ascii’ is a fast method that only works on characters that have an direct ASCII mapping. ‘unicode’ is a slightly slower method that works on any characters. None (default) does nothing.

- **analyzer** : string, {'word', 'char', 'char_wb'} or callable

  Whether the feature should be made of word or character n-grams. Option 'char_wb' creates character n-grams only from text inside word boundaries.

  If a callable is passed it is used to extract the sequence of features out of the raw, unprocessed input.

- **preprocessor** : callable or None (default)

  Override the preprocessing (string transformation) stage while preserving the tokenizing and n-grams generation steps.

- **tokenizer** : callable or None (default)

  Override the string tokenization step while preserving the preprocessing and n-grams generation steps. Only applies if analyzer == 'word'.

- **ngram_range** : tuple (min_n, max_n), default=(1, 1)

  The lower and upper boundary of the range of n-values for different n-grams to be extracted. All values of n such that min_n <= n <= max_n will be used.

- **stop_words** : string {'english'}, list, or None (default)

  If 'english', a built-in stop word list for English is used.
If a list, that list is assumed to contain stop words, all of which will be removed from the resulting tokens. Only applies if `analyzer == 'word'`.

**lowercase** : boolean, default=True

Convert all characters to lowercase before tokenizing.

**token_pattern** : string

Regular expression denoting what constitutes a “token”, only used if `analyzer == 'word'`. The default regexp selects tokens of 2 or more alphanumeric characters (punctuation is completely ignored and always treated as a token separator).

**n_features** : integer, default=(2 ** 20)

The number of features (columns) in the output matrices. Small numbers of features are likely to cause hash collisions, but large numbers will cause larger coefficient dimensions in linear learners.

**norm** : ‘l1’, ‘l2’ or None, optional

Norm used to normalize term vectors. None for no normalization.

**binary** : boolean, default=False.

If True, all non zero counts are set to 1. This is useful for discrete probabilistic models that model binary events rather than integer counts.

**dtype** : type, optional

Type of the matrix returned by fit_transform() or transform().

**non_negative** : boolean, default=False

Whether output matrices should contain non-negative values only; effectively calls abs on the matrix prior to returning it. When True, output values can be interpreted as frequencies. When False, output values will have expected value zero.

See also:

`CountVectorizer`, `TfidfVectorizer`

Methods

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<td><code>decode(doc)</code></td>
<td>Decode the input into a string of unicode symbols</td>
</tr>
<tr>
<td><code>fit(X[, y])</code></td>
<td>Does nothing: this transformer is stateless</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Transform a sequence of documents to a document-term matrix.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>get_stop_words()</code></td>
<td>Build or fetch the effective stop words list</td>
</tr>
<tr>
<td><code>partial_fit(X[, y])</code></td>
<td>Does nothing: this transformer is stateless</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>
Table 29.91 – continued from previous page

transform(X[, y])  Transform a sequence of documents to a document-term matrix.

**transform**

.. _transform:

```
__init__(input=u'content', encoding=u'utf-8', decode_error=u'strict', strip_accents=None,
lowercase=True, preprocessor=None, tokenizer=None, stop_words=None, token_pattern=u'(?u)\b\w\w+\b', ngram_range=(1, 1), analyzer=u'word',
n_features=1048576, binary=False, norm=u'l2', non_negative=False, dtype=<type 'numpy.float64'>)
```

**build_analyzer()**

Return a callable that handles preprocessing and tokenization

**build_preprocessor()**

Return a function to preprocess the text before tokenization

**build_tokenizer()**

Return a function that splits a string into a sequence of tokens

**decode**

```
decode(doc)
```

Decode the input into a string of unicode symbols

The decoding strategy depends on the vectorizer parameters.

**fit**

```
fit(X, y=None)
```

Does nothing: this transformer is stateless.

**fit_transform**

```
fit_transform(X, y=None)
```

Transform a sequence of documents to a document-term matrix.

**Parameters**

- **X**: iterable over raw text documents, length = n_samples
  
  Samples. Each sample must be a text document (either bytes or unicode strings, file
  name or file object depending on the constructor argument) which will be tokenized and
  hashed.

- **y**: (ignored)

**Returns**

- **X**: scipy.sparse matrix, shape = (n_samples, self.n_features)
  

**get_params**

```
get_params(deep=True)
```

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are
  estimators.

**Returns**

- **params**: mapping of string to any

  Parameter names mapped to their values.

**get_stop_words()**

Build or fetch the effective stop words list

**partial_fit**

```
partial_fit(X, y=None)
```

Does nothing: this transformer is stateless.

This method is just there to mark the fact that this transformer can work in a streaming setup.
set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns: self

transform (X, y=None)

Transform a sequence of documents to a document-term matrix.

Parameters:

X : iterable over raw text documents, length = n_samples

Samples. Each sample must be a text document (either bytes or unicode strings, file name or file object depending on the constructor argument) which will be tokenized and hashed.

y : (ignored)

Returns:

X : scipy.sparse matrix, shape = (n_samples, self.n_features)


Examples using sklearn.feature_extraction.text.HashingVectorizer

- Out-of-core classification of text documents
- Classification of text documents using sparse features
- Clustering text documents using k-means

sklearn.feature_extraction.text.TfidfTransformer
class sklearn.feature_extraction.text.TfidfTransformer (norm='l2', use_idf=True, smooth_idf=True, sublinear_tf=False)

Transform a count matrix to a normalized tf or tf-idf representation

Tf means term-frequency while tf-idf means term-frequency times inverse document-frequency. This is a common term weighting scheme in information retrieval, that has also found good use in document classification.

The goal of using tf-idf instead of the raw frequencies of occurrence of a token in a given document is to scale down the impact of tokens that occur very frequently in a given corpus and that are hence empirically less informative than features that occur in a small fraction of the training corpus.

The formula that is used to compute the tf-idf of term t is tf-idf(d, t) = tf(t) * idf(d, t), and the idf is computed as idf(d, t) = log [ n / df(d, t) ] + 1 (if smooth_idf=False), where n is the total number of documents and df(d, t) is the document frequency; the document frequency is the number of documents d that contain term t. The effect of adding “1” to the idf in the equation above is that terms with zero idf, i.e., terms that occur in all documents in a training set, will not be entirely ignored. (Note that the idf formula above differs from the standard textbook notation that defines the idf as idf(d, t) = log [ n / (df(d, t) + 1) ].

If smooth_idf=True (the default), the constant “1” is added to the numerator and denominator of the idf as if an extra document was seen containing every term in the collection exactly once, which prevents zero divisions: idf(d, t) = log [ (1 + n) / (1 + df(d, t)) ] + 1.

Furthermore, the formulas used to compute tf and idf depend on parameter settings that correspond to the SMART notation used in IR as follows:
Tf is “n” (natural) by default, “l” (logarithmic) when `sublinear_tf=True`. Idf is “t” when `use_idf` is given, “n” (none) otherwise. Normalization is “c” (cosine) when `norm='l2'`, “n” (none) when `norm=None`.

Read more in the `User Guide`.

**Parameters**

- `norm` : ‘l1’, ‘l2’ or None, optional
  
  Norm used to normalize term vectors. None for no normalization.

- `use_idf` : boolean, default=True
  
  Enable inverse-document-frequency reweighting.

- `smooth_idf` : boolean, default=True
  
  Smooth idf weights by adding one to document frequencies, as if an extra document was seen containing every term in the collection exactly once. Prevents zero divisions.

- `sublinear_tf` : boolean, default=False
  
  Apply sublinear tf scaling, i.e. replace tf with 1 + log(tf).

**References**

[Yates2011], [MRS2008]

**Methods**

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<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
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<td><code>get_params([deep])</code></td>
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<tr>
<td><code>transform(X[, copy])</code></td>
<td>Transform a count matrix to a tf or tf-idf representation</td>
</tr>
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</table>

**__init__** (`norm='l2'`, `use_idf=True`, `smooth_idf=True`, `sublinear_tf=False`)

**fit** (`X, y=None`)  

Learn the idf vector (global term weights)

**Parameters**

- `X` : sparse matrix, [n_samples, n_features]
  
  a matrix of term/token counts

**fit_transform** (`X, y=None, **fit_params`)  

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- `X` : numpy array of shape [n_samples, n_features]
  
  Training set.

- `y` : numpy array of shape [n_samples]
  
  Target values.

**Returns**

- `X_new` : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.
get_params (deep=True)
Get parameters for this estimator.

Parameters:
- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params** : mapping of string to any
  
  Parameter names mapped to their values.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
- **self** :

transform (X, copy=True)
Transform a count matrix to a tf or tf-idf representation

Parameters:
- **X** : sparse matrix, [n_samples, n_features]
  
  a matrix of term/token counts

- **copy** : boolean, default True
  
  Whether to copy X and operate on the copy or perform in-place operations.

Returns:
- **vectors** : sparse matrix, [n_samples, n_features]

Examples using sklearn.feature_extraction.text.TfidfTransformer

- Sample pipeline for text feature extraction and evaluation
- Clustering text documents using k-means
sklearn.feature_extraction.text.TfidfVectorizer

class sklearn.feature_extraction.text.TfidfVectorizer(input=u'content', encoding=u'utf-8', decode_error=u'strict', strip_accents=None, lowercase=True, preprocessor=None, tokenizer=None, analyzer=u'word', stop_words=None, token_pattern=u'(?u)\b\w\w+\b', ngram_range=(1, 1), max_df=1.0, min_df=1, max_features=None, vocabulary=None, binary=False, dtype=<type 'numpy.int64'>, norm=u'l2', use_idf=True, smooth_idf=True, sublinear_tf=False)

Convert a collection of raw documents to a matrix of TF-IDF features.

Equivalent to CountVectorizer followed by TfidfTransformer.

Read more in the User Guide.

Parameters:

input : string {'filename', 'file', 'content'}

If ‘filename’, the sequence passed as an argument to fit is expected to be a list of filenames that need reading to fetch the raw content to analyze.

If ‘file’, the sequence items must have a ‘read’ method (file-like object) that is called to fetch the bytes in memory.

Otherwise the input is expected to be the sequence strings or bytes items are expected to be analyzed directly.

coding : string, ‘utf-8’ by default.

If bytes or files are given to analyze, this encoding is used to decode.

de code_error : {'strict', 'ignore', 'replace'}

Instruction on what to do if a byte sequence is given to analyze that contains characters not of the given encoding. By default, it is ‘strict’, meaning that a UnicodeDecodeError will be raised. Other values are ‘ignore’ and ‘replace’.

strip_accents : {'ascii', 'unicode', None}

Remove accents during the preprocessing step. ‘ascii’ is a fast method that only works on characters that have an direct ASCII mapping. ‘unicode’ is a slightly slower method that works on any characters. None (default) does nothing.

analyzer : string, {'word', 'char'} or callable

Whether the feature should be made of word or character n-grams.

If a callable is passed it is used to extract the sequence of features out of the raw, unprocessed input.

preprocessor : callable or None (default)

Override the preprocessing (string transformation) stage while preserving the tokenizing and n-grams generation steps.
tokenizer : callable or None (default)

Override the string tokenization step while preserving the preprocessing and n-grams generation steps. Only applies if `analyzer == 'word'`.

ngram_range : tuple (min_n, max_n)

The lower and upper boundary of the range of n-values for different n-grams to be extracted. All values of n such that min_n <= n <= max_n will be used.

stop_words : string {'english'}, list, or None (default)

If a string, it is passed to _check_stop_list and the appropriate stop list is returned. ‘english’ is currently the only supported string value.

If a list, that list is assumed to contain stop words, all of which will be removed from the resulting tokens. Only applies if `analyzer == 'word'`.

If None, no stop words will be used. max_df can be set to a value in the range [0.7, 1.0) to automatically detect and filter stop words based on intra corpus document frequency of terms.

lowercase : boolean, default True

Convert all characters to lowercase before tokenizing.

token_pattern : string

Regular expression denoting what constitutes a “token”, only used if `analyzer == 'word'`. The default regexp selects tokens of 2 or more alphanumeric characters (punctuation is completely ignored and always treated as a token separator).

max_df : float in range [0.0, 1.0] or int, default=1.0

When building the vocabulary ignore terms that have a document frequency strictly higher than the given threshold (corpus-specific stop words). If float, the parameter represents a proportion of documents, integer absolute counts. This parameter is ignored if vocabulary is not None.

min_df : float in range [0.0, 1.0] or int, default=1

When building the vocabulary ignore terms that have a document frequency strictly lower than the given threshold. This value is also called cut-off in the literature. If float, the parameter represents a proportion of documents, integer absolute counts. This parameter is ignored if vocabulary is not None.

max_features : int or None, default=None

If not None, build a vocabulary that only consider the top max_features ordered by term frequency across the corpus.

This parameter is ignored if vocabulary is not None.

vocabulary : Mapping or iterable, optional

Either a Mapping (e.g., a dict) where keys are terms and values are indices in the feature matrix, or an iterable over terms. If not given, a vocabulary is determined from the input documents.

binary : boolean, default=False

If True, all non-zero term counts are set to 1. This does not mean outputs will have only 0/1 values, only that the tf term in tf-idf is binary. (Set idf and normalization to False to get 0/1 outputs.)
**dtype**: type, optional
Type of the matrix returned by fit_transform() or transform().

**norm**: ‘l1’, ‘l2’ or None, optional
Norm used to normalize term vectors. None for no normalization.

**use_idf**: boolean, default=True
Enable inverse-document-frequency reweighting.

**smooth_idf**: boolean, default=True
Smooth idf weights by adding one to document frequencies, as if an extra document was seen containing every term in the collection exactly once. Prevents zero divisions.

**sublinear_tf**: boolean, default=False
Apply sublinear tf scaling, i.e. replace tf with 1 + log(tf).

**Attributes**

**vocabulary_**: dict
A mapping of terms to feature indices.

**idf_**: array, shape = [n_features], or None
The learned idf vector (global term weights) when use_idf is set to True, None otherwise.

**stop_words_**: set
Terms that were ignored because they either:
• occurred in too many documents (max_df)
• occurred in too few documents (min_df)
• were cut off by feature selection (max_features).
This is only available if no vocabulary was given.

**See also:**

*CountVectorizer* Tokenize the documents and count the occurrences of token and return them as a sparse matrix

*TfidfTransformer* Apply Term Frequency Inverse Document Frequency normalization to a sparse matrix of occurrence counts.

**Notes**

The stop_words_ attribute can get large and increase the model size when pickling. This attribute is provided only for introspection and can be safely removed using delattr or set to None before pickling.

**Methods**

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<tr>
<th>Method</th>
<th>Description</th>
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<td>Return a callable that handles preprocessing and tokenization</td>
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<td>build_preprocessor()</td>
<td>Return a function to preprocess the text before tokenization</td>
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<tr>
<td>get_feature_names()</td>
<td>Array mapping from feature integer indices to feature name</td>
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<td>get_params([deep])</td>
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<td>get_stop_words()</td>
<td>Build or fetch the effective stop words list</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Return terms per document with nonzero entries in X.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(raw_documents[, copy])</td>
<td>Transform documents to document-term matrix.</td>
</tr>
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#### __init__

```python
__init__(input=u'content', encoding=u'utf-8', decode_error=u'strict', strip_accents=None, lowercase=True, preprocessor=None, tokenizer=None, analyzer=u'word', stop_words=None, token_pattern=u'(?u)\b\w\w+\b', ngram_range=(1, 1), max_df=1.0, min_df=1, max_features=None, vocabulary=None, binary=False, dtype=<type 'numpy.int64'>, norm=u'l2', use_idf=True, smooth_idf=True, sublinear_tf=False)
```

- **build_analyzer()**
  - Return a callable that handles preprocessing and tokenization

- **build_preprocessor()**
  - Return a function to preprocess the text before tokenization

- **build_tokenizer()**
  - Return a function that splits a string into a sequence of tokens

- **decode(doc)**
  - Decode the input into a string of unicode symbols
  - The decoding strategy depends on the vectorizer parameters.

- **fit(raw_documents, y=None)**
  - Learn vocabulary and idf from training set.

  **Parameters**
  - **raw_documents**: iterable
    - An iterable which yields either str, unicode or file objects
  
  **Returns**
  - **self**: TfidfVectorizer

- **fit_transform(raw_documents, y=None)**
  - Learn vocabulary and idf, return term-document matrix.
  - This is equivalent to fit followed by transform, but more efficiently implemented.

  **Parameters**
  - **raw_documents**: iterable
    - An iterable which yields either str, unicode or file objects

  **Returns**
  - **X**: sparse matrix, [n_samples, n_features]

- **get_feature_names()**
  - Array mapping from feature integer indices to feature name

- **get_params(deep=True)**
  - Get parameters for this estimator.
Parameters

deep : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns

params : mapping of string to any

Parameter names mapped to their values.

get_stop_words()

Build or fetch the effective stop words list

inverse_transform(X)

Return terms per document with nonzero entries in X.

Parameters

X : {array, sparse matrix}, shape = [n_samples, n_features]

Returns

X_inv : list of arrays, len = n_samples

List of arrays of terms.

set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns

self :

transform(raw_documents, copy=True)

Transform documents to document-term matrix.

Uses the vocabulary and document frequencies (df) learned by fit (or fit_transform).

Parameters

raw_documents : iterable

an iterable which yields either str, unicode or file objects

copy : boolean, default True

Whether to copy X and operate on the copy or perform in-place operations.

Returns

X : sparse matrix, [n_samples, n_features]


Examples using sklearn.feature_extraction.text.TfidfVectorizer

- Feature Union with Heterogeneous Data Sources
- Topic extraction with Non-negative Matrix Factorization and Latent Dirichlet Allocation
- Biclustering documents with the Spectral Co-clustering algorithm
- Classification of text documents using sparse features
- Clustering text documents using k-means
- Classification of text documents: using a MLComp dataset
29.12 sklearn.feature_selection: Feature Selection

The `sklearn.feature_selection` module implements feature selection algorithms. It currently includes univariate filter selection methods and the recursive feature elimination algorithm.

**User guide:** See the Feature selection section for further details.

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### 29.12.1 sklearn.feature_selection.GenericUnivariateSelect

**Class** `sklearn.feature_selection.GenericUnivariateSelect` *(score_func=<function f_classif>, mode='percentile', param=1e-05)*

Univariate feature selector with configurable strategy.

Read more in the User Guide.

**Parameters**

- `score_func` : callable
  Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues). For modes ‘percentile’ or ‘kbest’ it can return a single array scores.

  Feature selection mode.

- `param` : float or int depending on the feature selection mode
  Parameter of the corresponding mode.

**Attributes**

- `scores_` : array-like, shape=(n_features,)
  Scores of features.

- `pvalues_` : array-like, shape=(n_features,)
  p-values of feature scores, None if `score_func` returned scores only.

See also:

- `f_classif` : ANOVA F-value between label/feature for classification tasks.
**mutual_info_classif** Mutual information for a discrete target.

**chi2** Chi-squared stats of non-negative features for classification tasks.

**f_regression** F-value between label/feature for regression tasks.

**mutual_info_regression** Mutual information for a continuous target.

**SelectPercentile** Select features based on percentile of the highest scores.

**SelectKBest** Select features based on the k highest scores.

**SelectFpr** Select features based on a false positive rate test.

**SelectFdr** Select features based on an estimated false discovery rate.

**SelectFwe** Select features based on family-wise error rate.

### Methods

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<td>Run score function on (X, y) and get the appropriate features.</td>
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<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
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<tr>
<td><code>get_params([deep])</code></td>
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<td><code>get_support([indices])</code></td>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Reduce X to the selected features.</td>
</tr>
</tbody>
</table>

```python
__init__(score_func=<function f_classif>, mode='percentile', param=1e-05)
```

**fit(X, y)**

Run score function on (X, y) and get the appropriate features.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - The training input samples.
- **y**: array-like, shape = [n_samples]
  - The target values (class labels in classification, real numbers in regression).

**Returns**

- **self** : object
  - Returns self.

**fit_transform(X, y=None, **fit_params)**

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.
get_params (deep=True)
Get parameters for this estimator.

Parameters:
- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params** : mapping of string to any
  
  Parameter names mapped to their values.

get_support (indices=False)
Get a mask, or integer index, of the features selected

Parameters:
- **indices** : boolean (default False)
  
  If True, the return value will be an array of integers, rather than a boolean mask.

Returns:
- **support** : array
  
  An index that selects the retained features from a feature vector. If indices is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If indices is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

inverse_transform(X)
Reverse the transformation operation

Parameters:
- **X** : array of shape [n_samples, n_selected_features]
  
  The input samples.

Returns:
- **X_r** : array of shape [n_samples, n_original_features]
  
  X with columns of zeros inserted where features would have been removed by transform.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form **<component>__<parameter>** so that it’s possible to update each component of a nested object.

Returns:
- **self**

transform(X)
Reduce X to the selected features.

Parameters:
- **X** : array of shape [n_samples, n_features]
  
  The input samples.

Returns:
- **X_r** : array of shape [n_samples, n_selected_features]
  
  The input samples with only the selected features.

29.12.2 sklearn.feature_selection.SelectPercentile

class sklearn.feature_selection.SelectPercentile(score_func=<function f_classif>, percentile=10)
Select features according to a percentile of the highest scores.

Read more in the User Guide.
Parameters

score_func : callable
   Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues) or a single array with scores. Default is f_classif (see below “See also”). The default function only works with classification tasks.

percentile : int, optional, default=10
   Percent of features to keep.

Attributes

scores_ : array-like, shape=(n_features,)
   Scores of features.

cpvalues_ : array-like, shape=(n_features,)
   p-values of feature scores, None if score_func returned only scores.

See also:

f_classif  ANOVA F-value between label/feature for classification tasks.
mutable_info_classif  Mutual information for a discrete target.
chi2  Chi-squared stats of non-negative features for classification tasks.
f_regression  F-value between label/feature for regression tasks.
mutable_info_regression  Mutual information for a continuous target.
SelectKBest  Select features based on the k highest scores.
SelectFpr  Select features based on a false positive rate test.
SelectFdr  Select features based on an estimated false discovery rate.
SelectFwe  Select features based on family-wise error rate.
GenericUnivariateSelect  Univariate feature selector with configurable mode.

Notes

Ties between features with equal scores will be broken in an unspecified way.

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<td>get_support([indices])</td>
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<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Reduce X to the selected features.</td>
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</table>

__init__ (score_func=<function f_classif>, percentile=10)

fit (X, y)
   Run score function on (X, y) and get the appropriate features.
scikit-learn user guide, Release 0.18.2

Parameters

**X** : array-like, shape = [n_samples, n_features]

The training input samples.

**y** : array-like, shape = [n_samples]

The target values (class labels in classification, real numbers in regression).

Returns

**self** : object

Returns self.

**fit_transform**(X, y=None, **fit_params)**

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

**X** : numpy array of shape [n_samples, n_features]

Training set.

**y** : numpy array of shape [n_samples]

Target values.

Returns

**X_new** : numpy array of shape [n_samples, n_features_new]

Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.

Parameters

**deep** : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

**params** : mapping of string to any

Parameter names mapped to their values.

**get_support**(indices=False)

Get a mask, or integer index, of the features selected

Parameters

**indices** : boolean (default False)

If True, the return value will be an array of integers, rather than a boolean mask.

Returns

**support** : array

An index that selects the retained features from a feature vector. If indices is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If indices is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

**inverse_transform**(X)

Reverse the transformation operation

Parameters

**X** : array of shape [n_samples, n_selected_features]

The input samples.

Returns

**X_r** : array of shape [n_samples, n_original_features]

X with columns of zeros inserted where features would have been removed by transform.
**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Return**:

**transform**(X)

Reduce X to the selected features.

**Parameters**

- **X**: array of shape [n_samples, n_features]
  
The input samples.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  
The input samples with only the selected features.

**Examples using** `sklearn.feature_selection.SelectPercentile`

- Feature agglomeration vs. univariate selection
- Univariate Feature Selection
- SVM-Anova: SVM with univariate feature selection

### 29.12.3 `sklearn.feature_selection.SelectKBest`

**class** `sklearn.feature_selection.SelectKBest(score_func=<function f_classif>, k=10)`

Select features according to the k highest scores.

**Read more in the** User Guide.

**Parameters**

- **score_func**: callable
  
  Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues) or a single array with scores. Default is f_classif (see below “See also”). The default function only works with classification tasks.

- **k**: int or “all”, optional, default=10
  
  Number of top features to select. The “all” option bypasses selection, for use in a parameter search.

**Attributes**

- **scores_**: array-like, shape=(n_features,)
  
  Scores of features.

- **pvalues_**: array-like, shape=(n_features,)
  
  p-values of feature scores, None if score_func returned only scores.

**See also**:

- *f_classif* ANOVA F-value between label/feature for classification tasks.
- *mutual_info_classif* Mutual information for a discrete target.
- *chi2* Chi-squared stats of non-negative features for classification tasks.
- *f_regression* F-value between label/feature for regression tasks.
**mutual_info_regression** Mutual information for a continuous target.

**SelectPercentile** Select features based on percentile of the highest scores.

**SelectFpr** Select features based on a false positive rate test.

**SelectFdr** Select features based on an estimated false discovery rate.

**SelectFwe** Select features based on family-wise error rate.

**GenericUnivariateSelect** Univariate feature selector with configurable mode.

## Notes

Ties between features with equal scores will be broken in an unspecified way.

## Methods

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<td><code>inverse_transform(X)</code></td>
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<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Reduce X to the selected features.</td>
</tr>
</tbody>
</table>

**__init__** *(score_func=<function f_classif>, k=10)*

**fit**(X, y)
Run score function on (X, y) and get the appropriate features.

**Parameters**
- **X** : array-like, shape = [n_samples, n_features]
  The training input samples.
- **y** : array-like, shape = [n_samples]
  The target values (class labels in classification, real numbers in regression).

**Returns**
- **self** : object
  Returns self.

**fit_transform**(X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- **X** : numpy array of shape [n_samples, n_features]
  Training set.
- **y** : numpy array of shape [n_samples]
  Target values.

**Returns**
- **X_new** : numpy array of shape [n_samples, n_features_new]
  Transformed array.
get_params (deep=True)
Get parameters for this estimator.

Parameters:

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

- **params**: mapping of string to any
  Parameter names mapped to their values.

get_support (indices=False)
Get a mask, or integer index, of the features selected

Parameters:

- **indices**: boolean (default False)
  If True, the return value will be an array of integers, rather than a boolean mask.

Returns:

- **support**: array
  An index that selects the retained features from a feature vector. If `indices` is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If `indices` is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

inverse_transform (X)
Reverse the transformation operation

Parameters:

- **X**: array of shape [n_samples, n_selected_features]
  The input samples.

Returns:

- **X_r**: array of shape [n_samples, n_original_features]
  X with columns of zeros inserted where features would have been removed by transform.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

- **self**: object

transform (X)
Reduce X to the selected features.

Parameters:

- **X**: array of shape [n_samples, n_features]
  The input samples.

Returns:

- **X_r**: array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

Examples using sklearn.feature_selection.SelectKBest

- Concatenating multiple feature extraction methods
- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Pipeline Anova SVM
• Classification of text documents using sparse features

29.12.4 sklearn.feature_selection.SelectFpr

class sklearn.feature_selection.SelectFpr(score_func=<function f_classif>, alpha=0.05)

Filter: Select the pvalues below alpha based on a FPR test.
FPR test stands for False Positive Rate test. It controls the total amount of false detections.
Read more in the User Guide.

Parameters

score_func : callable
Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues).
Default is f_classif (see below “See also”). The default function only works with classification tasks.

alpha : float, optional
The highest p-value for features to be kept.

Attributes

scores_ : array-like, shape=(n_features,)
Scores of features.

pvalues_ : array-like, shape=(n_features,)
p-values of feature scores.

See also:

f_classif: ANOVA F-value between label/feature for classification tasks.
chi2: Chi-squared stats of non-negative features for classification tasks.

mutual_info_classif
f_regression: F-value between label/feature for regression tasks.
mutual_info_regression: Mutual information between features and the target.
SelectPercentile: Select features based on percentile of the highest scores.
SelectKBest: Select features based on the k highest scores.
SelectFdr: Select features based on an estimated false discovery rate.
SelectFwe: Select features based on family-wise error rate.
GenericUnivariateSelect: Univariate feature selector with configurable mode.

Methods

fit(X, y) Run score function on (X, y) and get the appropriate features.

fit_transform(X[, y]) Fit to data, then transform it.

get_params([deep]) Get parameters for this estimator.

get_support([indices]) Get a mask, or integer index, of the features selected

inverse_transform(X) Reverse the transformation operation

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<tr>
<td><strong>transform</strong></td>
<td>Reduce X to the selected features.</td>
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</table>

```python
__init__(score_func=<function f_classif>, alpha=0.05)
```

**fit** *(X, y)*
Run score function on *(X, y)* and get the appropriate features.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]
  The training input samples.
- **y**: array-like, shape = [n_samples]
  The target values (class labels in classification, real numbers in regression).

**Returns**
- **self**: object
  Returns self.

```python
fit_transform(X, y=None, **fit_params)
```
Fit to data, then transform it.
Fits transformer to X and y with optional parameters **fit_params** and returns a transformed version of X.

**Parameters**
- **X**: numpy array of shape [n_samples, n_features]
  Training set.
- **y**: numpy array of shape [n_samples]
  Target values.

**Returns**
- **X_new**: numpy array of shape [n_samples, n_features_new]
  Transformed array.

```python
get_params(deep=True)
```
Get parameters for this estimator.

**Parameters**
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params**: mapping of string to any
  Parameter names mapped to their values.

```python
get_support(indices=False)
```
Get a mask, or integer index, of the features selected

**Parameters**
- **indices**: boolean (default False)
  If True, the return value will be an array of integers, rather than a boolean mask.

**Returns**
- **support**: array
  An index that selects the retained features from a feature vector. If *indices* is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If *indices* is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

```python
inverse_transform(X)
```
Reverse the transformation operation.
**Parameters**

X : array of shape [n_samples, n_selected_features]

The input samples.

**Returns**

X_r : array of shape [n_samples, n_original_features]

X with columns of zeros inserted where features would have been removed by transform.

```
set_params(**params)
```

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

```
transform(X)
```

Reduce X to the selected features.

**Parameters**

X : array of shape [n_samples, n_features]

The input samples.

**Returns**

X_r : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

### 29.12.5 sklearn.feature_selection.SelectFdr

class sklearn.feature_selection.SelectFdr (score_func=<function f_classif>, alpha=0.05)

Filter: Select the p-values for an estimated false discovery rate

This uses the Benjamini-Hochberg procedure. alpha is an upper bound on the expected false discovery rate.

Read more in the User Guide.

**Parameters**

score_func : callable

Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues). Default is f_classif (see below “See also”). The default function only works with classification tasks.

alpha : float, optional

The highest uncorrected p-value for features to keep.

**Attributes**

scores_ : array-like, shape=(n_features,)

Scores of features.

pvalues_ : array-like, shape=(n_features,)

p-values of feature scores.

See also:

f_classif: ANOVA F-value between label/feature for classification tasks.

mutual_info_classif: Mutual information for a discrete target.

chi2: Chi-squared stats of non-negative features for classification tasks.

f_regression: F-value between label/feature for regression tasks.
mutual_info_regression Mutual information for a continuous target.

SelectPercentile Select features based on percentile of the highest scores.

SelectKBest Select features based on the k highest scores.

SelectFpr Select features based on a false positive rate test.

SelectFwe Select features based on family-wise error rate.

GenericUnivariateSelect Univariate feature selector with configurable mode.

References

https://en.wikipedia.org/wiki/False_discovery_rate

Methods

**init**(score_func=<function f_classif>, alpha=0.05)

**fit**(X, y)
  Run score function on (X, y) and get the appropriate features.

**fit_transform**(X[, y], **fit_params)
  Fit to data, then transform it.

get_params([deep])
  Get parameters for this estimator.

get_support([indices])
  Get a mask, or integer index, of the features selected

inverse_transform(X)
  Reverse the transformation operation

set_params(**params)
  Set the parameters of this estimator.

transform(X)
  Reduce X to the selected features.

**fit**(X, y)
  Run score function on (X, y) and get the appropriate features.

  **Parameters**

  X : array-like, shape = [n_samples, n_features]
      The training input samples.
  y : array-like, shape = [n_samples]
      The target values (class labels in classification, real numbers in regression).

  **Returns**

  self : object
      Returns self.

**fit_transform**(X, y=None, **fit_params)
  Fit to data, then transform it.

  Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

  **Parameters**

  X : numpy array of shape [n_samples, n_features]
      Training set.
  y : numpy array of shape [n_samples]
      Target values.

  **Returns**

  X_new : numpy array of shape [n_samples, n_features_new]
      Transformed array.
**get_params** *(deep=True)*
Get parameters for this estimator.

**Parameters**
-
- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
-
- **params** : mapping of string to any
  Parameter names mapped to their values.

**get_support** *(indices=False)*
Get a mask, or integer index, of the features selected

**Parameters**
-
- **indices** : boolean (default False)
  If True, the return value will be an array of integers, rather than a boolean mask.

**Returns**
-
- **support** : array
  An index that selects the retained features from a feature vector. If indices is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If indices is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

**inverse_transform** *(X)*
Reverse the transformation operation

**Parameters**
-
- **X** : array of shape [n_samples, n_selected_features]
  The input samples.

**Returns**
-
- **X_r** : array of shape [n_samples, n_original_features]
  X with columns of zeros inserted where features would have been removed by transform.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
-
- **self** :

**transform** *(X)*
Reduce X to the selected features.

**Parameters**
-
- **X** : array of shape [n_samples, n_features]
  The input samples.

**Returns**
-
- **X_r** : array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

### 29.12.6 sklearn.feature_selection.SelectFromModel

**class** `sklearn.feature_selection.SelectFromModel` *(estimator, threshold=None, prefit=False)*
Meta-transformer for selecting features based on importance weights.

New in version 0.17.
**Parametersestimator**: object

The base estimator from which the transformer is built. This can be both a fitted (if `prefit` is set to `True`) or a non-fitted estimator.

**threshold**: string, float, optional, default=None

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if the estimator has a parameter penalty set to l1, either explicitly or implicitly (e.g, Lasso), the threshold used is 1e-5. Otherwise, “mean” is used by default.

**prefit**: bool, default=False

Whether a prefit model is expected to be passed into the constructor directly or not. If True, `transform` must be called directly and SelectFromModel cannot be used with `cross_val_score`, `GridSearchCV` and similar utilities that clone the estimator. Otherwise train the model using `fit` and then `transform` to do feature selection.

**Attributes'estimator_': an estimator**

The base estimator from which the transformer is built. This is stored only when a non-fitted estimator is passed to the `SelectFromModel`, i.e when prefit is False.

‘threshold_': float

The threshold value used for feature selection.

**Methods**

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<td>Fit the SelectFromModel meta-transformer.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>get_support([indices])</code></td>
<td>Get a mask, or integer index, of the features selected</td>
</tr>
<tr>
<td><code>inverse_transform(X)</code></td>
<td>Reverse the transformation operation</td>
</tr>
<tr>
<td><code>partial_fit(X[, y])</code></td>
<td>Fit the SelectFromModel meta-transformer only once.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Reduce X to the selected features.</td>
</tr>
</tbody>
</table>

**init**(estimator, threshold=None, prefit=False)

Fit the SelectFromModel meta-transformer.

**Parameters**

- `X` : array-like of shape (n_samples, n_features)

  The training input samples.

- `y` : array-like, shape (n_samples,)

  The target values (integers that correspond to classes in classification, real numbers in regression).

**Returns**

- `self`: object

  Returns self.

---

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**fit_transform** (*X, y=None, **fit_params*)
Fit to data, then transform it.
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- **X**: numpy array of shape [n_samples, n_features]
  Training set.
- **y**: numpy array of shape [n_samples]
  Target values.

**Returns**
- **X_new**: numpy array of shape [n_samples, n_features_new]
  Transformed array.

**get_params** (*deep=True*)
Get parameters for this estimator.

**Parameters**
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params**: mapping of string to any
  Parameter names mapped to their values.

**get_support** (*indices=False*)
Get a mask, or integer index, of the features selected

**Parameters**
- **indices**: boolean (default False)
  If True, the return value will be an array of integers, rather than a boolean mask.

**Returns**
- **support**: array
  An index that selects the retained features from a feature vector. If indices is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If indices is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

**inverse_transform** (*X*)
Reverse the transformation operation

**Parameters**
- **X**: array of shape [n_samples, n_selected_features]
  The input samples.

**Returns**
- **X_r**: array of shape [n_samples, n_original_features]
  X with columns of zeros inserted where features would have been removed by transform.

**partial_fit** (*X, y=None, **fit_params*)
Fit the SelectFromModel meta-transformer only once.

**Parameters**
- **X**: array-like of shape (n_samples, n_features)
  The training input samples.
- **y**: array-like, shape (n_samples,)
  The target values (integers that correspond to classes in classification, real numbers in regression).

****fit_params**: Other estimator specific parameters
Returns self.

**set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns self.

**transform(X)

Reduce X to the selected features.

Parameters:

- **X**: array of shape [n_samples, n_features]
The input samples.

Returns:

- **X_r**: array of shape [n_samples, n_selected_features]
The input samples with only the selected features.

Examples using **sklearn.feature_selection.SelectFromModel**

- Feature selection using SelectFromModel and LassoCV

29.12.7 **sklearn.feature_selection.SelectFwe**

class sklearn.feature_selection.SelectFwe(score_func=<function f_classif>, alpha=0.05)

Filter: Select the p-values corresponding to Family-wise error rate

Read more in the User Guide.

Parameters:

- **score_func**: callable
  Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues). Default is f_classif (see below “See also”). The default function only works with classification tasks.

- **alpha**: float, optional
  The highest uncorrected p-value for features to keep.

Attributes:

- **scores_**: array-like, shape=(n_features,)
  Scores of features.

- **pvalues_**: array-like, shape=(n_features,)
  p-values of feature scores.

See also:

- **f_classif**: ANOVA F-value between label/feature for classification tasks.
- **chi2**: Chi-squared stats of non-negative features for classification tasks.
- **f_regression**: F-value between label/feature for regression tasks.
- **SelectPercentile**: Select features based on percentile of the highest scores.
- **SelectKBest**: Select features based on the k highest scores.
**SelectFpr** Select features based on a false positive rate test.

**SelectFdr** Select features based on an estimated false discovery rate.

**GenericUnivariateSelect** Univariate feature selector with configurable mode.

### Methods

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<td>Run score function on (X, y) and get the appropriate features.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>get_support([indices])</code></td>
<td>Get a mask, or integer index, of the features selected</td>
</tr>
<tr>
<td><code>inverse_transform(X)</code></td>
<td>Reverse the transformation operation</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Reduce X to the selected features.</td>
</tr>
</tbody>
</table>

```python
__init__(score_func=<function f_classif>, alpha=0.05)
```

**fit** (X, y)

Run score function on (X, y) and get the appropriate features.

**Parameters**

- **X** : array-like, shape = [n_samples, n_features]
  The training input samples.
- **y** : array-like, shape = [n_samples]
  The target values (class labels in classification, real numbers in regression).

**Returns**

- **self** : object
  Returns self.

**fit_transform** (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X** : numpy array of shape [n_samples, n_features]
  Training set.
- **y** : numpy array of shape [n_samples]
  Target values.

**Returns**

- **X_new** : numpy array of shape [n_samples, n_features_new]
  Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  Parameter names mapped to their values.
get_support( indices=False)
Get a mask, or integer index, of the features selected

**Parameters**
- **indices** : boolean (default False)
  If True, the return value will be an array of integers, rather than a boolean mask.

**Return**
- **support** : array
  An index that selects the retained features from a feature vector. If indices is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If indices is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

inverse_transform(X)
Reverse the transformation operation

**Parameters**
- **X** : array of shape [n_samples, n_selected_features]
  The input samples.

**Returns**
- **X_r** : array of shape [n_samples, n_original_features]
  X with columns of zeros inserted where features would have been removed by transform.

set_params(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- **self** :

transform(X)
Reduce X to the selected features.

**Parameters**
- **X** : array of shape [n_samples, n_features]
  The input samples.

**Returns**
- **X_r** : array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

### 29.12.8 sklearn.feature_selection.RFE

class sklearn.feature_selection.RFE(estimator, n_features_to_select=None, step=1, verbose=0)
Feature ranking with recursive feature elimination.

Given an external estimator that assigns weights to features (e.g., the coefficients of a linear model), the goal of recursive feature elimination (RFE) is to select features by recursively considering smaller and smaller sets of features. First, the estimator is trained on the initial set of features and weights are assigned to each one of them. Then, features whose absolute weights are the smallest are pruned from the current set features. That procedure is recursively repeated on the pruned set until the desired number of features to select is eventually reached.

Read more in the **User Guide**.

**Parameters**
- **estimator** : object
  A supervised learning estimator with a fit method that updates a coef_ attribute that holds the fitted parameters. Important features must correspond to high absolute values in the coef_ array.
For instance, this is the case for most supervised learning algorithms such as Support Vector Classifiers and Generalized Linear Models from the `svm` and `linear_model` modules.

**n_features_to_select**: int or None (default=None)

The number of features to select. If None, half of the features are selected.

**step**: int or float, optional (default=1)

If greater than or equal to 1, then step corresponds to the (integer) number of features to remove at each iteration. If within (0.0, 1.0), then step corresponds to the percentage (rounded down) of features to remove at each iteration.

**verbose**: int, default=0

Controls verbosity of output.

**Attributes**

- **n_features_**: int
  The number of selected features.

- **support_**: array of shape [n_features]
  The mask of selected features.

- **ranking_**: array of shape [n_features]
  The feature ranking, such that ranking_[i] corresponds to the ranking position of the i-th feature. Selected (i.e., estimated best) features are assigned rank 1.

- **estimator_**: object
  The external estimator fit on the reduced dataset.

**References**

[R27]

**Examples**

The following example shows how to retrieve the 5 right informative features in the Friedman #1 dataset.

```python
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.feature_selection import RFE
>>> from sklearn.svm import SVR

>>> X, y = make_friedman1(n_samples=50, n_features=10, random_state=0)
>>> estimator = SVR(kernel="linear")
>>> selector = RFE(estimator, 5, step=1)
>>> selector = selector.fit(X, y)
>>> selector.support_
array([True, True, True, True, True,
       False, False, False, False, False], dtype=bool)
>>> selector.ranking_
array([1, 1, 1, 1, 1, 6, 4, 3, 2, 5])
```

**Methods**

29.12. sklearn.feature_selection: Feature Selection
**decision_function**(\*args, \*\*kwargs)

**fit**(X, y)

Fit the RFE model and then the underlying estimator on the selected features.

**fit_transform**(X[, y])

Fit to data, then transform it.

**get_params**(deep)

Get parameters for this estimator.

**get_support**(indices)

Get a mask, or integer index, of the features selected.

**inverse_transform**(X)

Reverse the transformation operation.

**predict**(\*args, \*\*kwargs)

Reduce X to the selected features and then predict using the underlying estimator.

**predict_log_proba**(\*args, \*\*kwargs)

**predict_proba**(\*args, \*\*kwargs)

**score**(\*args, \*\*kwargs)

Reduce X to the selected features and then return the score of the underlying estimator.

**set_params**(\*\*params)

Set the parameters of this estimator.

**transform**(X)

Reduce X to the selected features.

---

**init**(estimator, n_features_to_select=\None\, step=\d1\, verbose=\d0\)

**fit**(X, y)

Fit the RFE model and then the underlying estimator on the selected features.

**Parameters**

- **X**: \{array-like, sparse matrix\}, shape = \[n_samples, n_features\]
  - The training input samples.
- **y**: array-like, shape = \[n_samples\]
  - The target values.

**fit_transform**(X, y=\None\, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape \[n_samples, n_features\]
  - Training set.
- **y**: numpy array of shape \[n_samples\]
  - Target values.

**Returns**

- **X_new**: numpy array of shape \[n_samples, n_features_new\]
  - Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**get_support**(indices=False)

Get a mask, or integer index, of the features selected.
**Parameters**

- `indices` : boolean (default False)
  
  If True, the return value will be an array of integers, rather than a boolean mask.

- `support` : array
  
  An index that selects the retained features from a feature vector. If `indices` is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If `indices` is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

**inverse_transform** *(X)*

Reverse the transformation operation

- `Parameters` : `X` : array of shape [n_samples, n_selected_features]
  
  The input samples.

- `Returns` : `X_r` : array of shape [n_samples, n_original_features]
  
  `X` with columns of zeros inserted where features would have been removed by `transform`.

**predict** *(args, **kwargs)*

Reduce X to the selected features and then predict using the underlying estimator.

- `Parameters` : `X` : array of shape [n_samples, n_features]
  
  The input samples.

- `Returns` : `y` : array of shape [n_samples]
  
  The predicted target values.

**score** *(args, **kwargs)*

Reduce X to the selected features and then return the score of the underlying estimator.

- `Parameters` : `X` : array of shape [n_samples, n_features]
  
  The input samples.

- `y` : array of shape [n_samples]
  
  The target values.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

- `Returns` : `self`

**transform** *(X)*

Reduce X to the selected features.

- `Parameters` : `X` : array of shape [n_samples, n_features]
  
  The input samples.

- `Returns` : `X_r` : array of shape [n_samples, n_selected_features]
  
  The input samples with only the selected features.
Examples using sklearn.feature_selection.RFE

• Recursive feature elimination

29.12.9 sklearn.feature_selection.RFECV

class sklearn.feature_selection.RFECV(estimator, step=1, cv=None, scoring=None, verbose=0, n_jobs=1)

Feature ranking with recursive feature elimination and cross-validated selection of the best number of features.

Read more in the User Guide.

Parameters:
estimator : object

A supervised learning estimator with a fit method that updates a coef_ attribute that holds the fitted parameters. Important features must correspond to high absolute values in the coef_ array.

For instance, this is the case for most supervised learning algorithms such as Support Vector Classifiers and Generalized Linear Models from the svm and linear_model modules.

step : int or float, optional (default=1)

If greater than or equal to 1, then step corresponds to the (integer) number of features to remove at each iteration. If within (0.0, 1.0), then step corresponds to the percentage (rounded down) of features to remove at each iteration.

cv : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

• None, to use the default 3-fold cross-validation,
• integer, to specify the number of folds.
• An object to be used as a cross-validation generator.
• An iterable yielding train/test splits.

For integer/None inputs, if y is binary or multiclass, sklearn.model_selection.StratifiedKFold is used. If the estimator is a classifier and if y is neither binary nor multiclass, sklearn.model_selection.KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

scoring : string, callable or None, optional, default: None

A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y).

verbose : int, default=0

Controls verbosity of output.

n_jobs : int, default 1

Number of cores to run in parallel while fitting across folds. Defaults to 1 core. If n_jobs=-1, then number of jobs is set to number of cores.

Attributes:

n_features_ : int

The number of selected features with cross-validation.
**support_**: array of shape [n_features]

The mask of selected features.

**ranking_**: array of shape [n_features]

The feature ranking, such that ranking_[i] corresponds to the ranking position of the i-th feature. Selected (i.e., estimated best) features are assigned rank 1.

**grid_scores_**: array of shape [n_subsets_of_features]

The cross-validation scores such that grid_scores_[i] corresponds to the CV score of the i-th subset of features.

**estimator_**: object

The external estimator fit on the reduced dataset.

**Notes**

The size of grid_scores_ is equal to ceil((n_features - 1) / step) + 1, where step is the number of features removed at each iteration.

**References**

[R28]

**Examples**

The following example shows how to retrieve the a-priori not known 5 informative features in the Friedman #1 dataset.

```python
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.feature_selection import RFECV
>>> from sklearn.svm import SVR

>>> X, y = make_friedman1(n_samples=50, n_features=10, random_state=0)
>>> estimator = SVR(kernel="linear")
>>> selector = RFECV(estimator, step=1, cv=5)
>>> selector = selector.fit(X, y)

>>> selector.support_
array([True, True, True, True, True, False, False, False, False, False], dtype=bool)

>>> selector.ranking_
array([1, 1, 1, 1, 1, 6, 4, 3, 2, 5])
```

**Methods**

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<td>Fit the RFE model and automatically tune the number of selected features.</td>
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<td>Get parameters for this estimator.</td>
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- `get_support(indices)` Get a mask, or integer index, of the features selected
- `inverse_transform(X)` Reverse the transformation operation
- `predict(*args, **kwargs)` Reduce X to the selected features and then predict using the underlying estimator.
- `predict_log_proba(*args, **kwargs)`
- `predict_proba(*args, **kwargs)`
- `score(*args, **kwargs)` Reduce X to the selected features and then return the score of the underlying estimator.
- `set_params(**params)` Set the parameters of this estimator.
- `transform(X)` Reduce X to the selected features.

```
__init__(estimator, step=1, cv=None, scoring=None, verbose=0, n_jobs=1)
```

**fit**(X, y)

Fit the RFE model and automatically tune the number of selected features.

**Parameters**
- `X` : {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Training vector, where `n_samples` is the number of samples and `n_features` is the total number of features.
- `y` : array-like, shape = [n_samples]
  
  Target values (integers for classification, real numbers for regression).

```
fit_transform(X, y=None, **fit_params)
```

Fit to data, then transform it.

Fits transformer to X and y with optional parameters `fit_params` and returns a transformed version of X.

**Parameters**
- `X` : numpy array of shape [n_samples, n_features]
  
  Training set.
- `y` : numpy array of shape [n_samples]
  
  Target values.

**Returns**
- `X_new` : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

```
get_params(deep=True)
```

Get parameters for this estimator.

**Parameters**
- `deep` : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- `params` : mapping of string to any
  
  Parameter names mapped to their values.

```
get_support(indices=False)
```

Get a mask, or integer index, of the features selected

**Parameters**
- `indices` : boolean (default False)
  
  If True, the return value will be an array of integers, rather than a boolean mask.

**Returns**
- `support` : array
An index that selects the retained features from a feature vector. If `indices` is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If `indices` is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

**inverse_transform** *(X)*

Reverse the transformation operation

**Parameters**

- **X** : array of shape [n_samples, n_selected_features]
  
  The input samples.

**Returns**

- **X_r** : array of shape [n_samples, n_original_features]
  
  X with columns of zeros inserted where features would have been removed by *transform*.

**predict** *(args, **kwargs)*

Reduce X to the selected features and then predict using the underlying estimator.

**Parameters**

- **X** : array of shape [n_samples, n_features]
  
  The input samples.

**Returns**

- **y** : array of shape [n_samples]
  
  The predicted target values.

**score** *(args, **kwargs)*

Reduce X to the selected features and then return the score of the underlying estimator.

**Parameters**

- **X** : array of shape [n_samples, n_features]
  
  The input samples.

- **y** : array of shape [n_samples]
  
  The target values.

**set_params** *(**params)*

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :

**transform** *(X)*

Reduce X to the selected features.

**Parameters**

- **X** : array of shape [n_samples, n_features]
  
  The input samples.

**Returns**

- **X_r** : array of shape [n_samples, n_selected_features]
  
  The input samples with only the selected features.

---

**Examples using sklearn.feature_selection.RFECV**

- *Recursive feature elimination with cross-validation*
29.12.10 sklearn.feature_selection.VarianceThreshold

class sklearn.feature_selection.VarianceThreshold(threshold=0.0)

Feature selector that removes all low-variance features.

This feature selection algorithm looks only at the features (X), not the desired outputs (y), and can thus be used for unsupervised learning.

Read more in the User Guide.

Parameters:
- **threshold**: float, optional
  Features with a training-set variance lower than this threshold will be removed. The default is to keep all features with non-zero variance, i.e. remove the features that have the same value in all samples.

Attributes:
- **variances_**: array, shape (n_features,)
  Variances of individual features.

Examples

The following dataset has integer features, two of which are the same in every sample. These are removed with the default setting for threshold:

```python
>>> X = [[0, 2, 0, 3], [0, 1, 4, 3], [0, 1, 1, 3]]
>>> selector = VarianceThreshold()
>>> selector.fit_transform(X)
array([[2, 0],
       [1, 4],
       [1, 1]])
```

Methods

- **fit(X, y)**
  Learn empirical variances from X.
- **fit_transform(X, y)**
  Fit to data, then transform it.
- **get_params([deep])**
  Get parameters for this estimator.
- **get_support([indices])**
  Get a mask, or integer index, of the features selected
- **inverse_transform(X)**
  Reverse the transformation operation
- **set_params(**params)**
  Set the parameters of this estimator.
- **transform(X)**
  Reduce X to the selected features.

__init__(threshold=0.0)

fit (X, y=None)

Learn empirical variances from X.

Parameters:
- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  Sample vectors from which to compute variances.
- **y**: any
  Ignored. This parameter exists only for compatibility with sklearn.pipeline.Pipeline.

Returns:
- **self** :
**fit_transform** (*X*, *y=None, **fit_params*)
Fit to data, then transform it.
Fits transformer to *X* and *y* with optional parameters *fit_params* and returns a transformed version of *X*.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]  
  Training set.
- **y**: numpy array of shape [n_samples]  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]  
  Transformed array.

**get_params** (*deep=True*)
Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

**get_support** (*indices=False*)
Get a mask, or integer index, of the features selected

**Parameters**

- **indices**: boolean (default False)
  If True, the return value will be an array of integers, rather than a boolean mask.

**Returns**

- **support**: array
  An index that selects the retained features from a feature vector. If *indices* is False, this is a boolean array of shape [# input features], in which an element is True iff its corresponding feature is selected for retention. If *indices* is True, this is an integer array of shape [# output features] whose values are indices into the input feature vector.

**inverse_transform** (*X*)
Reverse the transformation operation

**Parameters**

- **X**: array of shape [n_samples, n_selected_features]  
  The input samples.

**Returns**

- **X_r**: array of shape [n_samples, n_original_features]  
  *X* with columns of zeros inserted where features would have been removed by transform.

**set_params** (**params**)  
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 
  
**transform** (*X*)
Reduce *X* to the selected features.
Parameters

\( X \) : array of shape [n_samples, n_features]

The input samples.

Returns

\( X_r \) : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

---

**feature_selection.chi2**

```python
sklearn.feature_selection.chi2(X, y)
```

Compute chi-squared stats between each non-negative feature and class.

This score can be used to select the n_features features with the highest values for the test chi-squared statistic from X, which must contain only non-negative features such as booleans or frequencies (e.g., term counts in document classification), relative to the classes.

Recall that the chi-square test measures dependence between stochastic variables, so using this function “weeds out” the features that are the most likely to be independent of class and therefore irrelevant for classification.

Read more in the User Guide.

**Parameters**

- \( X \) : {array-like, sparse matrix}, shape = (n_samples, n_features_in)
  - Sample vectors.

- \( y \) : array-like, shape = (n_samples,)
  - Target vector (class labels).

**Returns**

- \( \text{chi2} \) : array, shape = (n_features,)
  - chi2 statistics of each feature.

- \( \text{pval} \) : array, shape = (n_features,)
  - p-values of each feature.

**See also:**

- **f_classif** ANOVA F-value between label/feature for classification tasks.
- **f_regression** F-value between label/feature for regression tasks.

**Notes**

Complexity of this algorithm is \( O(n_{classes} \times n_{features}) \).
Examples using `sklearn.feature_selection.chi2`

- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Classification of text documents using sparse features

29.12.12 `sklearn.feature_selection.f_classif`

`sklearn.feature_selection.f_classif(X, y)`

Compute the ANOVA F-value for the provided sample.

Read more in the User Guide.

Parameters:
- **X**: {array-like, sparse matrix} shape = [n_samples, n_features]
  The set of regressors that will be tested sequentially.
- **y**: array of shape(n_samples)
  The data matrix.

Returns:
- **F**: array, shape = [n_features,]
  The set of F values.
- **pval**: array, shape = [n_features,]
  The set of p-values.

See also:
- `chi2` Chi-squared stats of non-negative features for classification tasks.
- `f_regression` F-value between label/feature for regression tasks.

Examples using `sklearn.feature_selection.f_classif`

- Univariate Feature Selection
- SVM-Anova: SVM with univariate feature selection

29.12.13 `sklearn.feature_selection.f_regression`

`sklearn.feature_selection.f_regression(X, y, center=True)`

Univariate linear regression tests.

Quick linear model for testing the effect of a single regressor, sequentially for many regressors.

This is done in 2 steps:

1. The cross correlation between each regressor and the target is computed, that is, 
   \[ \frac{(X[:, i] - \text{mean}(X[:, i])) \cdot (y - \text{mean}_y)}{(\text{std}(X[:, i]) \cdot \text{std}(y))} \]

2. It is converted to an F score then to a p-value.

Read more in the User Guide.

Parameters:
- **X**: {array-like, sparse matrix} shape = (n_samples, n_features)
  The set of regressors that will be tested sequentially.
- **y**: array of shape(n_samples).
The data matrix

**center** : True, bool,
If true, X and y will be centered.

**Returns**

- **F** : array, shape=(n_features,)
  F values of features.
- **pval** : array, shape=(n_features,)
  p-values of F-scores.

**See also:**

- **f_classif** ANOVA F-value between label/feature for classification tasks.
- **chi2** Chi-squared stats of non-negative features for classification tasks.

### Examples using sklearn.feature_selection.f_regression

- Feature agglomeration vs. univariate selection
- Pipeline Anova SVM
- Comparison of F-test and mutual information
- Sparse recovery: feature selection for sparse linear models

### 29.12.14 sklearn.feature_selection.mutual_info_classif

**sklearn.feature_selection.mutual_info_classif** *(X, y, discrete_features='auto',
  n_neighbors=3, copy=True, random_state=None)*

Estimate mutual information for a discrete target variable.

Mutual information (MI) [R169] between two random variables is a non-negative value, which measures the dependency between the variables. It is equal to zero if and only if two random variables are independent, and higher values mean higher dependency.

The function relies on nonparametric methods based on entropy estimation from k-nearest neighbors distances as described in [R170] and [R171]. Both methods are based on the idea originally proposed in [R172].

It can be used for univariate features selection, read more in the User Guide.

**Parameters**

- **X** : array_like or sparse matrix, shape (n_samples, n_features)
  Feature matrix.
- **y** : array_like, shape (n_samples,)
  Target vector.
- **discrete_features** : {'auto', bool, array_like}, default 'auto'
  If bool, then determines whether to consider all features discrete or continuous. If array, then it should be either a boolean mask with shape (n_features,) or array with indices of discrete features. If ‘auto’, it is assigned to False for dense X and to True for sparse X.
- **n_neighbors** : int, default 3
  Number of neighbors to use for MI estimation for continuous variables, see [R170] and [R171]. Higher values reduce variance of the estimation, but could introduce a bias.
**copy** : bool, default True

Whether to make a copy of the given data. If set to False, the initial data will be overwritten.

**random_state** : int seed, RandomState instance or None, default None

The seed of the pseudo random number generator for adding small noise to continuous variables in order to remove repeated values.

**Returns**

*smi* : ndarray, shape (n_features,)

Estimated mutual information between each feature and the target.

**Notes**

1. The term “discrete features” is used instead of naming them “categorical”, because it describes the essence more accurately. For example, pixel intensities of an image are discrete features (but hardly categorical) and you will get better results if mark them as such. Also note, that treating a continuous variable as discrete and vice versa will usually give incorrect results, so be attentive about that.

2. True mutual information can’t be negative. If its estimate turns out to be negative, it is replaced by zero.

**References**

[R169], [R170], [R171], [R172]

---

**29.12.15 sklearn.feature_selection.mutual_info_regression**

*sklearn.feature_selection.mutual_info_regression*(X, y, discrete_features='auto', n_neighbors=3, copy=True, random_state=None)

Estimate mutual information for a continuous target variable.

Mutual information (MI) [R29] between two random variables is a non-negative value, which measures the dependency between the variables. It is equal to zero if and only if two random variables are independent, and higher values mean higher dependency.

The function relies on nonparametric methods based on entropy estimation from k-nearest neighbors distances as described in [R30] and [R31]. Both methods are based on the idea originally proposed in [R32].

It can be used for univariate features selection, read more in the User Guide.

**Parameters**

*X* : array_like or sparse matrix, shape (n_samples, n_features)

Feature matrix.

*y* : array_like, shape (n_samples,)

Target vector.

*discrete_features* : ['auto', bool, array_like], default 'auto'

If bool, then determines whether to consider all features discrete or continuous. If array, then it should be either a boolean mask with shape (n_features,) or array with indices of discrete features. If ‘auto’, it is assigned to False for dense X and to True for sparse X.

*n_neighbors* : int, default 3
Number of neighbors to use for MI estimation for continuous variables, see [R30] and [R31]. Higher values reduce variance of the estimation, but could introduce a bias.

**copy**: bool, default True

Whether to make a copy of the given data. If set to False, the initial data will be overwritten.

**random_state**: int seed, RandomState instance or None, default None

The seed of the pseudo random number generator for adding small noise to continuous variables in order to remove repeated values.

**Returns**

- **mi**: ndarray, shape (n_features,)
  
  Estimated mutual information between each feature and the target.

**Notes**

1. The term “discrete features” is used instead of naming them “categorical”, because it describes the essence more accurately. For example, pixel intensities of an image are discrete features (but hardly categorical) and you will get better results if mark them as such. Also note, that treating a continuous variable as discrete and vice versa will usually give incorrect results, so be attentive about that.

2. True mutual information can’t be negative. If its estimate turns out to be negative, it is replaced by zero.

**References**

[R29], [R30], [R31], [R32]

**Examples using sklearn.feature_selection.mutual_info_regression**

- Comparison of F-test and mutual information

**29.13 sklearn.gaussian_process: Gaussian Processes**

The sklearn.gaussian_process module implements Gaussian Process based regression and classification.

**User guide**: See the Gaussian Processes section for further details.

- Gaussian process regression (GPR).
- Gaussian process classification (GPC) based on Laplace approximation.
29.13.1 sklearn.gaussian_process.GaussianProcessRegressor

class sklearn.gaussian_process.GaussianProcessRegressor(
    kernel=None, alpha=1e-10,
    optimizer='fmin_l_bfgs_b',
    n_restarts_optimizer=0,
    normalize_y=False,
    copy_X_train=True,
    random_state=None
)

Gaussian process regression (GPR).

The implementation is based on Algorithm 2.1 of Gaussian Processes for Machine Learning (GPML) by Ras-mussen and Williams.

In addition to standard scikit-learn estimator API, GaussianProcessRegressor:

• allows prediction without prior fitting (based on the GP prior)
• provides an additional method sample_y(X), which evaluates samples drawn from the GPR (prior or posterior) at given inputs
• exposes a method log_marginal_likelihood(theta), which can be used externally for other ways of selecting hyperparameters, e.g., via Markov chain Monte Carlo.

Read more in the User Guide.

New in version 0.18.

Parameters:

- kernel : kernel object
  The kernel specifying the covariance function of the GP. If None is passed, the kernel “1.0 * RBF(1.0)” is used as default. Note that the kernel’s hyperparameters are optimized during fitting.

- alpha : float or array-like, optional (default: 1e-10)
  Value added to the diagonal of the kernel matrix during fitting. Larger values correspond to increased noise level in the observations and reduce potential numerical issue during fitting. If an array is passed, it must have the same number of entries as the data used for fitting and is used as datapoint-dependent noise level. Note that this is equivalent to adding a WhiteKernel with c=alpha. Allowing to specify the noise level directly as a parameter is mainly for convenience and for consistency with Ridge.

- optimizer : string or callable, optional (default: “fmin_l_bfgs_b”)
  Can either be one of the internally supported optimizers for optimizing the kernel’s parameters, specified by a string, or an externally defined optimizer passed as a callable. If a callable is passed, it must have the signature:

```python
def optimizer(obj_func, initial_theta, bounds):
    # * 'obj_func' is the objective function to be maximized,
    # which
    #  takes the hyperparameters theta as parameter and an
    #  optional flag eval_gradient, which determines if the
    #  gradient is returned additionally to the function value
    # * 'initial_theta': the initial value for theta, which can
    #  be
    #  used by local optimizers
    # * 'bounds': the bounds on the values of theta
    ....
    # Returned are the best found hyperparameters theta and
    # the corresponding value of the target function.
    return theta_opt, func_min
```

29.13. sklearn.gaussian_process: Gaussian Processes 1445
Per default, the ‘fmin_l_bfgs_b’ algorithm from scipy.optimize is used. If None is passed, the kernel’s parameters are kept fixed. Available internal optimizers are:

```
'fmin_l_bfgs_b'
```

**n_restarts_optimizer** : int, optional (default: 0)

The number of restarts of the optimizer for finding the kernel’s parameters which maximize the log-marginal likelihood. The first run of the optimizer is performed from the kernel’s initial parameters, the remaining ones (if any) from thetas sampled log-uniform randomly from the space of allowed theta-values. If greater than 0, all bounds must be finite. Note that n_restarts_optimizer == 0 implies that one run is performed.

**normalize_y** : boolean, optional (default: False)

Whether the target values y are normalized, i.e., the mean of the observed target values become zero. This parameter should be set to True if the target values’ mean is expected to differ considerably from zero. When enabled, the normalization effectively modifies the GP’s prior based on the data, which contradicts the likelihood principle; normalization is thus disabled per default.

**copy_X_train** : bool, optional (default: True)

If True, a persistent copy of the training data is stored in the object. Otherwise, just a reference to the training data is stored, which might cause predictions to change if the data is modified externally.

**random_state** : integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**Attributes**

**X_train_** : array-like, shape = (n_samples, n_features)

Feature values in training data (also required for prediction)

**y_train_** : array-like, shape = (n_samples, [n_output_dims])

Target values in training data (also required for prediction)

**kernel_** : kernel object

The kernel used for prediction. The structure of the kernel is the same as the one passed as parameter but with optimized hyperparameters

**L_** : array-like, shape = (n_samples, n_samples)

Lower-triangular Cholesky decomposition of the kernel in X_train_

**alpha_** : array-like, shape = (n_samples,)

Dual coefficients of training data points in kernel space

**log_marginal_likelihood_value_** : float

The log-marginal-likelihood of self.kernel_.theta

**Methods**

```python
fit(X, y)
```

Fit Gaussian process regression model

Continued on next page
get_params([deep])

Get parameters for this estimator.

log_marginal_likelihood([theta, eval_gradient])

Returns log-marginal likelihood of theta for training data.

predict(X[, return_std, return_cov])

Predict using the Gaussian process regression model.

sample_y(X[, n_samples, random_state])

Draw samples from Gaussian process and evaluate at X.

score(X, y[, sample_weight])

Returns the coefficient of determination $R^2$ of the prediction.

set_params(**params)

Set the parameters of this estimator.

__init__(kernel=None, alpha=1e-10, optimizer='fmin_l_bfgs_b', n_restarts_optimizer=0, normalize_y=False, copy_X_train=True, random_state=None)

fit (X, y)

Fit Gaussian process regression model.

Parameters:

- **X**: array-like, shape = (n_samples, n_features)
  - Training data
- **y**: array-like, shape = (n_samples, [n_output_dims])
  - Target values

Returns **self**: returns an instance of self.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns **params**: mapping of string to any

Parameter names mapped to their values.

log_marginal_likelihood (theta=None, eval_gradient=False)

Returns log-marginal likelihood of theta for training data.

Parameters:

- **theta**: array-like, shape = (n_kernel_params,) or None
  - Kernel hyperparameters for which the log-marginal likelihood is evaluated. If None, the precomputed log_marginal_likelihood of self.kernel_.theta is returned.
- **eval_gradient**: bool, default: False
  - If True, the gradient of the log-marginal likelihood with respect to the kernel hyperparameters at position theta is returned additionally. If True, theta must not be None.

Returns **log_likelihood**: float

Log-marginal likelihood of theta for training data.

log_likelihood_gradient : array, shape = (n_kernel_params,), optional

Gradient of the log-marginal likelihood with respect to the kernel hyperparameters at position theta. Only returned when eval_gradient is True.

predict (X, return_std=False, return_cov=False)

Predict using the Gaussian process regression model.
We can also predict based on an unfitted model by using the GP prior. In addition to the mean of the predictive distribution, also its standard deviation (return_std=True) or covariance (return_cov=True). Note that at most one of the two can be requested.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  - Query points where the GP is evaluated
- **return_std**: bool, default: False
  - If True, the standard-deviation of the predictive distribution at the query points is returned along with the mean.
- **return_cov**: bool, default: False
  - If True, the covariance of the joint predictive distribution at the query points is returned along with the mean

**Returns**

- **y_mean**: array, shape = (n_samples, [n_output_dims])
  - Mean of predictive distribution a query points
- **y_std**: array, shape = (n_samples,), optional
  - Standard deviation of predictive distribution at query points. Only returned when return_std is True.
- **y_cov**: array, shape = (n_samples, n_samples), optional
  - Covariance of joint predictive distribution a query points. Only returned when return_cov is True.

**sample_y** (X, n_samples=1, random_state=0)

- Draw samples from Gaussian process and evaluate at X.
  - **Parameters**
    - **X**: array-like, shape = (n_samples_X, n_features)
      - Query points where the GP samples are evaluated
    - **n_samples**: int, default: 1
      - The number of samples drawn from the Gaussian process
    - **random_state**: RandomState or an int seed (0 by default)
      - A random number generator instance
  - **Returns**
    - **y_samples**: array, shape = (n_samples_X, [n_output_dims], n_samples)
      - Values of n_samples samples drawn from Gaussian process and evaluated at query points.

**score** (X, y, sample_weight=None)

- Returns the coefficient of determination R^2 of the prediction.
  - The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.
  - **Parameters**
    - **X**: array-like, shape = (n_samples, n_features)
      - Test samples.
    - **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
      - True values for X.
**sample_weight** : array-like, shape = [n_samples], optional

Sample weights.

**Returns**

**score** : float

R^2 of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self** :

---

**Examples using sklearn.gaussian_process.GaussianProcessRegressor**

- Comparison of kernel ridge and Gaussian process regression
- Gaussian process regression (GPR) on Mauna Loa CO2 data.
- Gaussian process regression (GPR) with noise-level estimation
- Gaussian Processes regression: basic introductory example
- Illustration of prior and posterior Gaussian process for different kernels

**29.13.2 sklearn.gaussian_process.GaussianProcessClassifier**

```python
class sklearn.gaussian_process.GaussianProcessClassifier (kernel=None, optimizer='fmin_l_bfgs_b', n_restarts_optimizer=0, max_iter_predict=100, warm_start=False, copy_X_train=True, random_state=None, multi_class='one_vs_rest', n_jobs=1)
```

Gaussian process classification (GPC) based on Laplace approximation.

The implementation is based on Algorithm 3.1, 3.2, and 5.1 of Gaussian Processes for Machine Learning (GPML) by Rasmussen and Williams.

Internally, the Laplace approximation is used for approximating the non-Gaussian posterior by a Gaussian.

Currently, the implementation is restricted to using the logistic link function. For multi-class classification, several binary one-versus rest classifiers are fitted. Note that this class thus does not implement a true multi-class Laplace approximation.

**Parameters**

- **kernel** : kernel object

  The kernel specifying the covariance function of the GP. If None is passed, the kernel “1.0 * RBF(1.0)” is used as default. Note that the kernel’s hyperparameters are optimized during fitting.

- **optimizer** : string or callable, optional (default: “fmin_l_bfgs_b”)
Can either be one of the internally supported optimizers for optimizing the kernel’s parameters, specified by a string, or an externally defined optimizer passed as a callable. If a callable is passed, it must have the signature:

```python
def optimizer(obj_func, initial_theta, bounds):
    # * 'obj_func' is the objective function to be maximized,
    #   which
    #   takes the hyperparameters theta as parameter and an
    #   optional flag eval_gradient, which determines if the
    #   gradient is returned additionally to the function value
    # * 'initial_theta': the initial value for theta, which can
    #   be
    #   used by local optimizers
    # * 'bounds': the bounds on the values of theta

    # Returned are the best found hyperparameters theta and
    # the corresponding value of the target function.
    return theta_opt, func_min
```

Per default, the ‘fmin_l_bfgs_b’ algorithm from scipy.optimize is used. If None is passed, the kernel’s parameters are kept fixed. Available internal optimizers are:

`'fmin_l_bfgs_b'`

**n_restarts_optimizer**: int, optional (default: 0)

The number of restarts of the optimizer for finding the kernel’s parameters which maximize the log-marginal likelihood. The first run of the optimizer is performed from the kernel’s initial parameters, the remaining ones (if any) from thetas sampled log-uniform randomly from the space of allowed theta-values. If greater than 0, all bounds must be finite. Note that n_restarts_optimizer=0 implies that one run is performed.

**max_iter_predict**: int, optional (default: 100)

The maximum number of iterations in Newton’s method for approximating the posterior during predict. Smaller values will reduce computation time at the cost of worse results.

**warm_start**: bool, optional (default: False)

If warm-starts are enabled, the solution of the last Newton iteration on the Laplace approximation of the posterior mode is used as initialization for the next call of _posterior_mode(). This can speed up convergence when _posterior_mode is called several times on similar problems as in hyperparameter optimization.

**copy_X_train**: bool, optional (default: True)

If True, a persistent copy of the training data is stored in the object. Otherwise, just a reference to the training data is stored, which might cause predictions to change if the data is modified externally.

**random_state**: integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**multi_class**: string, default: “one_vs_rest”

Specifies how multi-class classification problems are handled. Supported are “one_vs_rest” and “one_vs_one”. In “one_vs_rest”, one binary Gaussian process classifier is fitted for each class, which is trained to separate this class from the rest. In “one_vs_one”, one binary Gaussian process classifier is fitted for each pair of classes,
which is trained to separate these two classes. The predictions of these binary predictors are combined into multi-class predictions. Note that “one-vs-one” does not support predicting probability estimates.

**n_jobs** : int, optional, default: 1

The number of jobs to use for the computation. If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**Attributes**

**kernel_** : kernel object

The kernel used for prediction. In case of binary classification, the structure of the kernel is the same as the one passed as parameter but with optimized hyperparameters. In case of multi-class classification, a CompoundKernel is returned which consists of the different kernels used in the one-versus-rest classifiers.

**log_marginal_likelihood_value_** : float

The log-marginal-likelihood of \( \text{self.kernel_.theta} \)

**classes_** : array-like, shape = (n_classes,)

Unique class labels.

**n_classes_** : int

The number of classes in the training data

.. versionadded:: 0.18 :

**Methods**

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<td>set_params(**params)</td>
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**__init__** (kernel=None, optimizer='fmin_l_bfgs_b', n_restarts_optimizer=0, max_iter_predict=100, warm_start=False, copy_X_train=True, random_state=None, multi_class='one_vs_rest', n_jobs=1)

Fit Gaussian process classification model

**Parameters**

**X** : array-like, shape = (n_samples, n_features)

Training data

**y** : array-like, shape = (n_samples,)

Target values, must be binary

**Returns**

**self** : returns an instance of self.
get_params (deep=True)
Get parameters for this estimator.

Parameters:
- deep: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params: mapping of string to any
  Parameter names mapped to their values.

log_marginal_likelihood (theta=None, eval_gradient=False)
Returns log-marginal likelihood of theta for training data.
In the case of multi-class classification, the mean log-marginal likelihood of the one-versus-rest classifiers are returned.

Parameters:
- theta: array-like, shape = (n_kernel_params,) or none
  Kernel hyperparameters for which the log-marginal likelihood is evaluated. In the case of multi-class classification, theta may be the hyperparameters of the compound kernel or of an individual kernel. In the latter case, all individual kernel get assigned the same theta values. If None, the precomputed log_marginal_likelihood of self.kernel_.theta is returned.

- eval_gradient: bool, default: False
  If True, the gradient of the log-marginal likelihood with respect to the kernel hyperparameters at position theta is returned additionally. Note that gradient computation is not supported for non-binary classification. If True, theta must not be None.

Returns:
- log_likelihood: float
  Log-marginal likelihood of theta for training data.

  log_likelihood_gradient: array, shape = (n_kernel_params,), optional
  Gradient of the log-marginal likelihood with respect to the kernel hyperparameters at position theta. Only returned when eval_gradient is True.

predict (X)
Perform classification on an array of test vectors X.

Parameters:
- X: array-like, shape = (n_samples, n_features)

Returns:
- C: array, shape = (n_samples,)
  Predicted target values for X, values are from classes_

predict_proba (X)
Return probability estimates for the test vector X.

Parameters:
- X: array-like, shape = (n_samples, n_features)

Returns:
- C: array-like, shape = (n_samples, n_classes)
  Returns the probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute classes_.

score (X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.
In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.
Parameters:

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns:

- **score**: float
  Mean accuracy of self.predict(X) wrt. y.

**set_params(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns:**self**

Examples using `sklearn.gaussian_process.GaussianProcessClassifier`

- Plot classification probability
- Classifier comparison
- Probabilistic predictions with Gaussian process classification (GPC)
- Gaussian process classification (GPC) on iris dataset
- Iso-probability lines for Gaussian Processes classification (GPC)
- Illustration of Gaussian process classification (GPC) on the XOR dataset

Kernels:

- `gaussian_process.kernels.Kernel` Base class for all kernels.
- `gaussian_process.kernels.Sum(k1, k2)` Sum-kernel k1 + k2 of two kernels k1 and k2.
- `gaussian_process.kernels.Product(k1, k2)` Product-kernel k1 * k2 of two kernels k1 and k2.
- `gaussian_process.kernels.Exponentiation(...)` Exponentiate kernel by given exponent.
- `gaussian_process.kernels.ConstantKernel(...)` Constant kernel.
- `gaussian_process.kernels.WhiteKernel(...)` White kernel.
- `gaussian_process.kernels.RBF([length_scale, ...])` Radial-basis function kernel (aka squared-exponential kernel).
- `gaussian_process.kernels.Matern([...])` Matern kernel.
- `gaussian_process.kernels.RationalQuadratic([...])` Rational Quadratic kernel.
- `gaussian_process.kernels.PairwiseKernel([...])` Wrapper for kernels in sklearn.metrics.pairwise.
- `gaussian_process.kernels.CompoundKernel([Kernel])` Which is composed of a set of other kernels.
- `gaussian_process.kernels.Hyperparameter` A kernel hyperparameter’s specification in form of a namedtuple.
29.13.3 sklearn.gaussian_process.kernels.Kernel

class sklearn.gaussian_process.kernels.Kernel
    Base class for all kernels.
    New in version 0.18.

    Methods

    clone_with_theta(theta) Returns a clone of self with given hyperparameters theta.
    diag(X) Returns the diagonal of the kernel k(X, X).
    get_params([deep]) Get parameters of this kernel.
    is_stationary() Returns whether the kernel is stationary.
    set_params(**params) Set the parameters of this kernel.

    __init__() x.__init__(...) initializes x; see help(type(x)) for signature

    bounds
        Returns the log-transformed bounds on the theta.
        Returns bounds : array, shape (n_dims, 2)
        The log-transformed bounds on the kernel's hyperparameters theta

    clone_with_theta(theta)
        Returns a clone of self with given hyperparameters theta.

    diag(X)
        Returns the diagonal of the kernel k(X, X).
        The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently since only the diagonal is evaluated.
        Parameters X : array, shape (n_samples_X, n_features)
            Left argument of the returned kernel k(X, Y)
        Returns K_diag : array, shape (n_samples_X,)
            Diagonal of kernel k(X, X)

    get_params (deep=True)
        Get parameters of this kernel.
        Parameters deep: boolean, optional :
            If True, will return the parameters for this estimator and contained subobjects that are estimators.
        Returns params : mapping of string to any
            Parameter names mapped to their values.

    hyperparameters
        Returns a list of all hyperparameter specifications.

    is_stationary()
        Returns whether the kernel is stationary.
n_dims
Returns the number of non-fixed hyperparameters of the kernel.

set_params(**params)
Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form
<component>__<parameter> so that it’s possible to update each component of a nested object.

Returnsself:

theta
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representa-
tion of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales
naturally live on a log-scale.

Returnstheta : array, shape (n_dims,)

The non-fixed, log-transformed hyperparameters of the kernel

29.13.4 sklearn.gaussian_process.kernels.Sum

class sklearn.gaussian_process.kernels.Sum(k1, k2)
Sum-kernel k1 + k2 of two kernels k1 and k2.

The resulting kernel is defined as k_sum(X, Y) = k1(X, Y) + k2(X, Y)

New in version 0.18.

Parameters k1 : Kernel object
The first base-kernel of the sum-kernel

k2 : Kernel object
The second base-kernel of the sum-kernel

Methods

clone_with_theta(theta)
Returns a clone of self with given hyperparameters theta.

diag(X)
Returns the diagonal of the kernel k(X, X).

get_params([deep])
Get parameters of this kernel.

is_stationary()
Returns whether the kernel is stationary.

set_params(**params)
Set the parameters of this kernel.

__init__(k1, k2)

bounds
Returns the log-transformed bounds on the theta.

Returnsbounds : array, shape (n_dims, 2)

The log-transformed bounds on the kernel’s hyperparameters theta

clone_with_theta(theta)
Returns a clone of self with given hyperparameters theta.
\textbf{diag}(X)  
Returns the diagonal of the kernel \( k(X, X) \).

The result of this method is identical to \texttt{np.diag(self(X))}; however, it can be evaluated more efficiently since only the diagonal is evaluated.

\textbf{Parameters}  
\( X \) : array, shape (n_samples_X, n_features)  
Left argument of the returned kernel \( k(X, Y) \)

\textbf{Returns}  
\( K_{\text{diag}} \) : array, shape (n_samples_X,)  
Diagonal of kernel \( k(X, X) \)

def get_params(deep=True)  
Get parameters of this kernel.

\textbf{Parameters}  
\texttt{deep}: boolean, optional  
If True, will return the parameters for this estimator and contained subobjects that are estimators.

\textbf{Returns}  
\texttt{params} : mapping of string to any  
Parameter names mapped to their values.

\textbf{hyperparameters}  
Returns a list of all hyperparameter.

\textbf{is_stationary}()  
Returns whether the kernel is stationary.

\textbf{n_dims}  
Returns the number of non-fixed hyperparameters of the kernel.

\textbf{set_params(**params)}  
Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

\textbf{Returns}  
\texttt{self}  

\textbf{theta}  
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

\textbf{Returns}  
\texttt{theta} : array, shape (n_dims,)  
The non-fixed, log-transformed hyperparameters of the kernel

\textbf{29.13.5 \texttt{sklearn.gaussian_process.kernels.Product}}

class \texttt{sklearn.gaussian_process.kernels.Product}(k1, k2)  
Product-kernel \( k1 \ast k2 \) of two kernels \( k1 \) and \( k2 \).

The resulting kernel is defined as \( k_{\text{prod}}(X, Y) = k1(X, Y) \ast k2(X, Y) \)

New in version 0.18.

\textbf{Parameters}  
\texttt{k1} : Kernel object
The first base-kernel of the product-kernel

\( k2 \) : Kernel object

The second base-kernel of the product-kernel

### Methods

<table>
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<tr>
<th>Method</th>
<th>Description</th>
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<tr>
<td>clone_with_theta(( \theta ))</td>
<td>Returns a clone of self with given hyperparameters ( \theta ).</td>
</tr>
<tr>
<td>diag(( X ))</td>
<td>Returns the diagonal of the kernel ( k(X, X) ).</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

\[ \text{__init__}(k1, k2) \]

#### bounds

Returns the log-transformed bounds on the \( \theta \).

**Returns**

**bounds** : array, shape (\( n_{\text{dims}} \), 2)

The log-transformed bounds on the kernel’s hyperparameters \( \theta \)

#### clone_with_theta(\( \theta \))

Returns a clone of self with given hyperparameters \( \theta \).

#### diag(\( X \))

Returns the diagonal of the kernel \( k(X, X) \).

The result of this method is identical to np.diag(self(\( X \))); however, it can be evaluated more efficiently since only the diagonal is evaluated.

**Parameters**

\( X \) : array, shape (\( n_{\text{samples}\_X} \), \( n_{\text{features}} \))

Left argument of the returned kernel \( k(X, Y) \)

**Returns**

**K\_diag** : array, shape (\( n_{\text{samples}\_X} \),)

Diagonal of kernel \( k(X, X) \)

#### get_params(\( deep=True \))

Get parameters of this kernel.

**Parameters**

\( deep \) : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

**params** : mapping of string to any

Parameter names mapped to their values.

#### hyperparameters

Returns a list of all hyperparameter.

#### is_stationary()

Returns whether the kernel is stationary.

#### n_dims

Returns the number of non-fixed hyperparameters of the kernel.
**set_params(**\texttt{params})**

Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form <\texttt{component}>>\texttt{__}<\texttt{parameter}> so that it's possible to update each component of a nested object.

Return self:

**theta**

Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

Returns theta : array, shape (n_dims,)

The non-fixed, log-transformed hyperparameters of the kernel

29.13.6 sklearn.gaussian_process.kernels.Exponentiation

class sklearn.gaussian_process.kernels.Exponentiation (kernel, exponent)

Exponentiate kernel by given exponent.

The resulting kernel is defined as $k_{\text{exp}}(X, Y) = k(X, Y) ** \text{exponent}$

New in version 0.18.

Parameters kernel : Kernel object

The base kernel

exponent : float

The exponent for the base kernel

Methods

<table>
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<tbody>
<tr>
<td>clone_with_theta(theta)</td>
<td>Returns a clone of self with given hyperparameters theta.</td>
</tr>
<tr>
<td>diag(X)</td>
<td>Returns the diagonal of the kernel $k(X, X)$.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

__init__ (kernel, exponent)

bounds

Returns the log-transformed bounds on the theta.

Returns bounds : array, shape (n_dims, 2)

The log-transformed bounds on the kernel’s hyperparameters theta

clone_with_theta(theta)

Returns a clone of self with given hyperparameters theta.

diag(X)

Returns the diagonal of the kernel $k(X, X)$. |
The result of this method is identical to \( np.diag(self(X)) \); however, it can be evaluated more efficiently since only the diagonal is evaluated.

**Parameters**
- \( X \): array, shape (n_samples_X, n_features)
  - Left argument of the returned kernel \( k(X, Y) \)

**Returns**
- \( K_{\text{diag}} \): array, shape (n_samples_X,)
  - Diagonal of kernel \( k(X, X) \)

**get_params** (deep=True)
Get parameters of this kernel.

**Parameters**
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- \( \text{params} \): mapping of string to any
  - Parameter names mapped to their values.

**hyperparameters**
Returns a list of all hyperparameter.

**is_stationary**()
Returns whether the kernel is stationary.

**n_dims**
Returns the number of non-fixed hyperparameters of the kernel.

**set_params** (**params**)
Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- \(self\)

**theta**
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

**Returns**
- \( \text{theta} \): array, shape (n_dims,)
  - The non-fixed, log-transformed hyperparameters of the kernel

### 29.13.7 sklearn.gaussian_process.kernels.ConstantKernel

**class** sklearn.gaussian_process.kernels.ConstantKernel(**constant_value=1.0,**
\[constant_value\_bounds\] = (1e-05, 100000.0))

Constant kernel.

Can be used as part of a product-kernel where it scales the magnitude of the other factor (kernel) or as part of a sum-kernel, where it modifies the mean of the Gaussian process.

\( k(x_1, x_2) = \text{constant\_value} \) for all \( x_1, x_2 \)

New in version 0.18.
**Parameters**

- **constant_value**: float, default: 1.0
  
The constant value which defines the covariance: \( k(x_1, x_2) = \text{constant\_value} \)

- **constant_value_bounds**: pair of floats >= 0, default: (1e-5, 1e5)
  
The lower and upper bound on constant_value

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>clone_with_theta((\theta))</td>
<td>Returns a clone of self with given hyperparameters (\theta).</td>
</tr>
<tr>
<td>diag((X))</td>
<td>Returns the diagonal of the kernel (k(X, X)).</td>
</tr>
<tr>
<td>get_params(([\text{deep}]))</td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params((^\star^\text{params}))</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

**__init__** (constant\_value=1.0, constant\_value\_bounds=(1e-05, 100000.0))

**bounds**

Returns the log-transformed bounds on the \(\theta\).

**Returns**

- **bounds**: array, shape (n\_dims, 2)
  
The log-transformed bounds on the kernel’s hyperparameters \(\theta\)

**clone_with_theta(\(\theta\))**

Returns a clone of self with given hyperparameters \(\theta\).

**diag(\(X\))**

Returns the diagonal of the kernel \(k(X, X)\).

The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently since only the diagonal is evaluated.

**Parameters**

- **X**: array, shape (n\_samples\_X, n\_features)
  
  Left argument of the returned kernel \(k(X, Y)\)

**Returns**

- **K\_diag**: array, shape (n\_samples\_X,)
  
  Diagonal of kernel \(k(X, X)\)

**get_params(\(\text{deep}=\text{True}\))**

Get parameters of this kernel.

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  
  Parameter names mapped to their values.

**hyperparameters**

Returns a list of all hyperparameter specifications.

**is_stationary()**

Returns whether the kernel is stationary.
n_dims
Returns the number of non-fixed hyperparameters of the kernel.

set_params (**params)
Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form
<component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

theta
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation
of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales
naturally live on a log-scale.

Returns theta: array, shape (n_dims,)

The non-fixed, log-transformed hyperparameters of the kernel

Examples using sklearn.gaussian_process.kernels.ConstantKernel

- Iso-probability lines for Gaussian Processes classification (GPC)
- Gaussian Processes regression: basic introductory example
- Illustration of prior and posterior Gaussian process for different kernels

29.13.8 sklearn.gaussian_process.kernels.WhiteKernel
class sklearn.gaussian_process.kernels.WhiteKernel (noise_level=1.0,
noise_level_bounds=(1e-05, 100000.0))

White kernel.

The main use-case of this kernel is as part of a sum-kernel where it explains the noise-component of the signal.
Tuning its parameter corresponds to estimating the noise-level.
k(x_1, x_2) = noise_level if x_1 == x_2 else 0

New in version 0.18.

Parameters noise_level : float, default: 1.0
Parameter controlling the noise level

noise_level_bounds : pair of floats >= 0, default: (1e-5, 1e5)
The lower and upper bound on noise_level

Methods

clone_with_theta(theta) Returns a clone of self with given hyperparameters theta.
diag(X) Returns the diagonal of the kernel k(X, X).
get_params([deep]) Get parameters of this kernel.
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<table>
<thead>
<tr>
<th>Method</th>
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<tbody>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params()</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

__init__(noise_level=1.0, noise_level_bounds=(1e-05, 100000.0))

bounds

Returns the log-transformed bounds on the theta.

Returns bounds: array, shape (n_dims, 2)

The log-transformed bounds on the kernel’s hyperparameters theta

clonewith_theta(theta)

Returns a clone of self with given hyperparameters theta.

diag(X)

Returns the diagonal of the kernel k(X, X).

The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently since only the diagonal is evaluated.

Parameters X: array, shape (n_samples_X, n_features)
Left argument of the returned kernel k(X, Y)

Returns K_diag: array, shape (n_samples_X,)
Diagonal of kernel k(X, X)

get_params(deep=True)

Get parameters of this kernel.

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns params: mapping of string to any
Parameter names mapped to their values.

hyperparameters

Returns a list of all hyperparameter specifications.

is_stationary()

Returns whether the kernel is stationary.

n_dims

Returns the number of non-fixed hyperparameters of the kernel.

set_params(**params)

Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form <component>_<parameter> so that it’s possible to update each component of a nested object.

Returns self:

theta

Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.
Returnstheta : array, shape (n_dims,)

The non-fixed, log-transformed hyperparameters of the kernel

Examples using sklearn.gaussian_process.kernels.WhiteKernel

- Comparison of kernel ridge and Gaussian process regression
- Gaussian process regression (GPR) on Mauna Loa CO2 data.
- Gaussian process regression (GPR) with noise-level estimation

29.13.9 sklearn.gaussian_process.kernels.RBF

class sklearn.gaussian_process.kernels.RBF (length_scale=1.0, length_scale_bounds=(1e-05, 100000.0))

Radial-basis function kernel (aka squared-exponential kernel).

The RBF kernel is a stationary kernel. It is also known as the “squared exponential” kernel. It is parameterized by a length-scale parameter length_scale>0, which can either be a scalar (isotropic variant of the kernel) or a vector with the same number of dimensions as the inputs X (anisotropic variant of the kernel). The kernel is given by:
k(x_i, x_j) = exp(-1 / 2 d(x_i / length_scale, x_j / length_scale)^2)

This kernel is infinitely differentiable, which implies that GPs with this kernel as covariance function have mean square derivatives of all orders, and are thus very smooth.

New in version 0.18.

Parameters

length_scale : float or array with shape (n_features,), default: 1.0

The length scale of the kernel. If a float, an isotropic kernel is used. If an array, an anisotropic kernel is used where each dimension of l defines the length-scale of the respective feature dimension.

length_scale_bounds : pair of floats >= 0, default: (1e-5, 1e5)

The lower and upper bound on length_scale

Methods

clone_with_theta(theta) Returns a clone of self with given hyperparameters theta.
diag(X) Returns the diagonal of the kernel k(X, X).
get_params([deep]) Get parameters of this kernel.
is_stationary() Returns whether the kernel is stationary.
set_params(**params) Set the parameters of this kernel.

__init__ (length_scale=1.0, length_scale_bounds=(1e-05, 100000.0))

bounds

Returns the log-transformed bounds on the theta.

Returns

bounds : array, shape (n_dims, 2)

The log-transformed bounds on the kernel’s hyperparameters theta
clone_with_theta \( (\theta) \)
Returns a clone of self with given hyperparameters \( \theta \).

diag \( (X) \)
Returns the diagonal of the kernel \( k(X, X) \).
The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently since only the diagonal is evaluated.

Parameters

- **X** : array, shape (n_samples_X, n_features)
  Left argument of the returned kernel \( k(X, Y) \)

Returns

- **K_diag** : array, shape (n_samples_X,)
  Diagonal of kernel \( k(X, X) \)

get_params \( (deep=True) \)
Get parameters of this kernel.

Parameters

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

- **params** : mapping of string to any
  Parameter names mapped to their values.

hyperparameters
Returns a list of all hyperparameter specifications.

is_stationary()
Returns whether the kernel is stationary.

n_dims
Returns the number of non-fixed hyperparameters of the kernel.

set_params (**params)
Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns

- **self** :

theta
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

Returns

- **theta** : array, shape (n_dims,)
  The non-fixed, log-transformed hyperparameters of the kernel

Examples using `sklearn.gaussian_process.kernels.RBF`

- Plot classification probability
- Classifier comparison
- Probabilistic predictions with Gaussian process classification (GPC)
• Gaussian process classification (GPC) on iris dataset
• Illustration of Gaussian process classification (GPC) on the XOR dataset
• Gaussian process regression (GPR) on Mauna Loa CO2 data.
• Gaussian process regression (GPR) with noise-level estimation
• Gaussian Processes regression: basic introductory example
• Illustration of prior and posterior Gaussian process for different kernels

29.13.10 sklearn.gaussian_process.kernels.Matern

class sklearn.gaussian_process.kernels.Matern

Matern kernel.

The class of Matern kernels is a generalization of the RBF and the absolute exponential kernel parameterized by an additional parameter nu. The smaller nu, the less smooth the approximated function is. For nu=\infty, the kernel becomes equivalent to the RBF kernel and for nu=0.5 to the absolute exponential kernel. Important intermediate values are nu=1.5 (once differentiable functions) and nu=2.5 (twice differentiable functions).

See Rasmussen and Williams 2006, pp84 for details regarding the different variants of the Matern kernel.

New in version 0.18.

Parameters

length_scale : float or array with shape (n_features,), default: 1.0

The length scale of the kernel. If a float, an isotropic kernel is used. If an array, an anisotropic kernel is used where each dimension of \( l \) defines the length-scale of the respective feature dimension.

length_scale_bounds : pair of floats >= 0, default: (1e-5, 1e5)

The lower and upper bound on length_scale

nu : float, default: 1.5 :

The parameter nu controlling the smoothness of the learned function. The smaller nu, the less smooth the approximated function is. For nu=\infty, the kernel becomes equivalent to the RBF kernel and for nu=0.5 to the absolute exponential kernel. Important intermediate values are nu=1.5 (once differentiable functions) and nu=2.5 (twice differentiable functions). Note that values of nu not in \([0.5, 1.5, 2.5, \infty]\) incur a considerably higher computational cost (appr. 10 times higher) since they require to evaluate the modified Bessel function. Furthermore, in contrast to l, nu is kept fixed to its initial value and not optimized.

Methods

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<td>clone_with_theta(theta)</td>
<td>Returns a clone of self with given hyperparameters theta.</td>
</tr>
<tr>
<td>diag(X)</td>
<td>Returns the diagonal of the kernel ( k(X, X) ).</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params({}</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

    __init__ (length_scale=1.0, length_scale_bounds=(1e-05, 100000.0), nu=1.5)

29.13. sklearn.gaussian_process: Gaussian Processes
bounds
Returns the log-transformed bounds on the theta.

Returns bounds: array, shape (n_dims, 2)
The log-transformed bounds on the kernel’s hyperparameters theta

clonewith_theta(theta)
Returns a clone of self with given hyperparameters theta.

diag(X)
Returns the diagonal of the kernel k(X, X).
The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently
since only the diagonal is evaluated.

Parameters X: array, shape (n_samples_X, n_features)
Left argument of the returned kernel k(X, Y)

Returns K_diag: array, shape (n_samples_X,)
Diagonal of kernel k(X, X)

get_params (deep=True)
Get parameters of this kernel.

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns params: mapping of string to any
Parameter names mapped to their values.

hyperparameters
Returns a list of all hyperparameter specifications.

is_stationary()
Returns whether the kernel is stationary.

n_dims
Returns the number of non-fixed hyperparameters of the kernel.

set_params (**params)
Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form
<component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

theta
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representa-
tion of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales
naturally live on a log-scale.

Returns theta: array, shape (n_dims,)
The non-fixed, log-transformed hyperparameters of the kernel
Examples using sklearn.gaussian_process.kernels.Matern

- Illustration of prior and posterior Gaussian process for different kernels

29.13.11 sklearn.gaussian_process.kernels.RationalQuadratic

class sklearn.gaussian_process.kernels.RationalQuadratic (length_scale=1.0, alpha=1.0, length_scale_bounds=(1e-05, 100000.0), alpha_bounds=(1e-05, 100000.0))

Rational Quadratic kernel.

The RationalQuadratic kernel can be seen as a scale mixture (an infinite sum) of RBF kernels with different characteristic length-scales. It is parameterized by a length-scale parameter length_scale>0 and a scale mixture parameter alpha>0. Only the isotropic variant where length_scale is a scalar is supported at the moment. The kernel given by:

\[ k(x_i, x_j) = (1 + \frac{d(x_i, x_j)^2}{2 \alpha \cdot \text{length_scale}^2})^{-\alpha} \]

New in version 0.18.

Parameters

- **length_scale** : float > 0, default: 1.0
  The length scale of the kernel.
- **alpha** : float > 0, default: 1.0
  Scale mixture parameter
- **length_scale_bounds** : pair of floats >= 0, default: (1e-5, 1e5)
  The lower and upper bound on length_scale
- **alpha_bounds** : pair of floats >= 0, default: (1e-5, 1e5)
  The lower and upper bound on alpha

Methods

- **clone_with_theta**(theta) Returns a clone of self with given hyperparameters theta.
- **diag**(X) Returns the diagonal of the kernel k(X, X).
- **get_params**(deep) Get parameters of this kernel.
- **is_stationary**( ) Returns whether the kernel is stationary.
- **set_params**(params) Set the parameters of this kernel.

__init__ (length_scale=1.0, alpha=1.0, length_scale_bounds=(1e-05, 100000.0), alpha_bounds=(1e-05, 100000.0))

bounds

Returns the log-transformed bounds on the theta.

Returns bounds: array, shape (n_dims, 2)

The log-transformed bounds on the kernel’s hyperparameters theta
clone_with_theta(theta)
Returns a clone of self with given hyperparameters theta.

diag(X)
Returns the diagonal of the kernel k(X, X).
The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently since only the diagonal is evaluated.

Parameters
X : array, shape (n_samples_X, n_features)
Left argument of the returned kernel k(X, Y)

Returns
K_diag : array, shape (n_samples_X,)
Diagonal of kernel k(X, X)

get_params(deep=True)
Get parameters of this kernel.

Parameters
deep: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns
params : mapping of string to any
Parameter names mapped to their values.

hyperparameters
Returns a list of all hyperparameter specifications.

is_stationary()
Returns whether the kernel is stationary.

n_dims
Returns the number of non-fixed hyperparameters of the kernel.

set_params(**params)
Set the parameters of this kernel.
The method works on simple kernels as well as on nested kernels. The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns
self:

theta
Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

Returns
theta : array, shape (n_dims,)
The non-fixed, log-transformed hyperparameters of the kernel

Examples using sklearn.gaussian_process.kernels.RationalQuadratic

• Gaussian process regression (GPR) on Mauna Loa CO2 data.
• Illustration of prior and posterior Gaussian process for different kernels
Exp-Sine-Squared kernel.

The ExpSineSquared kernel allows modeling periodic functions. It is parameterized by a length-scale parameter length_scale > 0 and a periodicity parameter periodicity > 0. Only the isotropic variant where l is a scalar is supported at the moment. The kernel given by:

\[ k(x_i, x_j) = \exp(-2 \sin(\pi / \text{periodicity} \times d(x_i, x_j)) / \text{length_scale})^2 \]

New in version 0.18.

**Parameters**

- **length_scale**: float > 0, default: 1.0
  
The length scale of the kernel.

- **periodicity**: float > 0, default: 1.0
  
The periodicity of the kernel.

- **length_scale_bounds**: pair of floats >= 0, default: (1e-5, 1e5)
  
The lower and upper bound on length_scale

- **periodicity_bounds**: pair of floats >= 0, default: (1e-5, 1e5)
  
The lower and upper bound on periodicity

**Methods**

- **clone_with_theta**(theta) Returns a clone of self with given hyperparameters theta.

- **diag**(X) Returns the diagonal of the kernel k(X, X).

- **get_params**(deep) Get parameters of this kernel.

- **is_stationary**() Returns whether the kernel is stationary.

- **set_params**(**
  \*\*params) Set the parameters of this kernel.

**__init__** (length_scale=1.0, periodicity=1.0, length_scale_bounds=(1e-05, 100000.0), periodicity_bounds=(1e-05, 100000.0))

**bounds**

Returns the log-transformed bounds on the theta.

**Returns**

- **bounds**: array, shape (n_dims, 2)
  
The log-transformed bounds on the kernel’s hyperparameters theta

- **clone_with_theta**(theta) Returns a clone of self with given hyperparameters theta.

- **diag**(X) Returns the diagonal of the kernel k(X, X).

  The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently
since only the diagonal is evaluated.

**Parameters**

- **X**: array, shape (n_samples_X, n_features)
  
  Left argument of the returned kernel k(X, Y)

- **Returns**
  
  **K_diag**: array, shape (n_samples_X,)
  
  Diagonal of kernel k(X, X)

**get_params** *(deep=True)*

Get parameters of this kernel.

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  
  Parameter names mapped to their values.

**hyperparameters**

Returns a list of all hyperparameter specifications.

**is_stationary**

Returns whether the kernel is stationary.

**n_dims**

Returns the number of non-fixed hyperparameters of the kernel.

**set_params** *(**params)*

Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

- **self**: .

**theta**

Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

**Returns**

- **theta**: array, shape (n_dims,)
  
  The non-fixed, log-transformed hyperparameters of the kernel.

**Examples using** *sklearn.gaussian_process.kernels.ExpSineSquared*

- *Comparison of kernel ridge and Gaussian process regression*
- *Gaussian process regression (GPR) on Mauna Loa CO2 data.*
- *Illustration of prior and posterior Gaussian process for different kernels*

**29.13.13 sklearn.gaussian_process.kernels.DotProduct**

**class** *sklearn.gaussian_process.kernels.DotProduct*(**sigma_0=1.0,  
  sigm...0_bounds=(1e-05, 100000.0))

Dot-Product kernel.
The DotProduct kernel is non-stationary and can be obtained from linear regression by putting N(0, 1) priors on the coefficients of \( x_d \) (\( d = 1, \ldots, D \)) and a prior of N(0, \( \sigma_0^2 \)) on the bias. The DotProduct kernel is invariant to a rotation of the coordinates about the origin, but not translations. It is parameterized by a parameter \( \sigma_0^2 \). For \( \sigma_0^2 = 0 \), the kernel is called the homogeneous linear kernel, otherwise it is inhomogeneous. The kernel is given by

\[
k(x_i, x_j) = \sigma_0^2 + x_i \cdot x_j
\]

The DotProduct kernel is commonly combined with exponentiation.

New in version 0.18.

**Parameters**

- **\( \sigma_0 \)**: float \( \geq 0 \), default: 1.0
  - Parameter controlling the inhomogenity of the kernel. If \( \sigma_0=0 \), the kernel is homogenous.

- **\( \sigma_0 \textunderscore \text{bounds} \)**: pair of floats \( \geq 0 \), default: (1e-5, 1e5)
  - The lower and upper bound on \( \sigma_0 \)

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>clone_with_theta(theta)</td>
<td>Returns a clone of self with given hyperparameters theta.</td>
</tr>
<tr>
<td>diag(X)</td>
<td>Returns the diagonal of the kernel ( k(X, X) ).</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

**__init__** (\( \sigma_0=1.0, \sigma_0 \textunderscore \text{bounds}=(1e-05, 100000.0) \))

**bounds**

Returns the log-transformed bounds on the theta.

<table>
<thead>
<tr>
<th>Returnsbounds</th>
<th>array, shape (n_dims, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The log-transformed bounds on the kernel’s hyperparameters theta</td>
</tr>
</tbody>
</table>

**clone_with_theta(theta)**

Returns a clone of self with given hyperparameters theta.

**diag(X)**

Returns the diagonal of the kernel \( k(X, X) \).

<table>
<thead>
<tr>
<th>ParametersX</th>
<th>array, shape (n_samples_X, n_features)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left argument of the returned kernel ( k(X, Y) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ReturnsK _diag</th>
<th>array, shape (n_samples_X,)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diagonal of kernel ( k(X, X) )</td>
</tr>
</tbody>
</table>

**get_params (deep=True)**

Get parameters of this kernel.

<table>
<thead>
<tr>
<th>Parametersdeep</th>
<th>boolean, optional</th>
</tr>
</thead>
</table>
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

`params` : mapping of string to any
Parameter names mapped to their values.

**hyperparameters**

Returns a list of all hyperparameter specifications.

**is_stationary** ()

Returns whether the kernel is stationary.

**n_dims**

Returns the number of non-fixed hyperparameters of the kernel.

**set_params** (**params**)

Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

`self`

**theta**

Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

**Returns**

`theta` : array, shape (n_dims,)
The non-fixed, log-transformed hyperparameters of the kernel

**Examples using** sklearn.gaussian_process.kernels.DotProduct

- *Iso-probability lines for Gaussian Processes classification (GPC)*
- *Illustration of Gaussian process classification (GPC) on the XOR dataset*
- *Illustration of prior and posterior Gaussian process for different kernels*

**29.13.14 sklearn.gaussian_process.kernels.PairwiseKernel**

class sklearn.gaussian_process.kernels.PairwiseKernel (gamma=1.0, gamma_bounds=(1e-05, 100000.0), metric='linear', pairwise_kernels_kwars=None)

Wrapper for kernels in sklearn.metrics.pairwise.

A thin wrapper around the functionality of the kernels in sklearn.metrics.pairwise.

**Note:** Evaluation of eval_gradient is not analytic but numeric and all kernels support only isotropic distances. The parameter gamma is considered to be a hyperparameter and may be optimized. The other kernel parameters are set directly at initialization and are kept fixed.

New in version 0.18.

**Parameters**

`gamma` : float >= 0, default: 1.0:
Parameter gamma of the pairwise kernel specified by metric
**gamma_bounds**: pair of floats >= 0, default: (1e-5, 1e5)

The lower and upper bound on gamma

**metric**: string, or callable, default: “linear”

The metric to use when calculating kernel between instances in a feature array. If metric is a string, it must be one of the metrics in pairwise.PAIRWISE_KERNEL_FUNCTIONS. If metric is “precomputed”, X is assumed to be a kernel matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from X as input and return a value indicating the distance between them.

**pairwise_kernels_kwargs**: dict, default: None

All entries of this dict (if any) are passed as keyword arguments to the pairwise kernel function.

### Methods

<table>
<thead>
<tr>
<th>Method</th>
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</tr>
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<tbody>
<tr>
<td>clone_with_theta(theta)</td>
<td>Returns a clone of self with given hyperparameters theta.</td>
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<tr>
<td>diag(X)</td>
<td>Returns the diagonal of the kernel k(X, X).</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td>is_stationary()</td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

#### __init__(gamma=1.0, gamma_bounds=(1e-05, 100000.0), metric='linear', pairwise_kernels_kwargs=None)

**bounds**

Returns the log-transformed bounds on the theta.

**Returns bounds**: array, shape (n_dims, 2)

The log-transformed bounds on the kernel’s hyperparameters theta

**clone_with_theta (theta)**

Returns a clone of self with given hyperparameters theta.

**diag (X)**

Returns the diagonal of the kernel k(X, X).

The result of this method is identical to np.diag(self(X)); however, it can be evaluated more efficiently since only the diagonal is evaluated.

**Parameters X**: array, shape (n_samples_X, n_features)

Left argument of the returned kernel k(X, Y)

**Returns K_diag**: array, shape (n_samples_X,)

Diagonal of kernel k(X, X)

**get_params (deep=True)**

Get parameters of this kernel.

**Parameters deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.
**Returns**

- **params**: mapping of string to any
  
  Parameter names mapped to their values.

**hyperparameters**

- Returns a list of all hyperparameter specifications.

**is_stationary**()

- Returns whether the kernel is stationary.

**n_dims**

- Returns the number of non-fixed hyperparameters of the kernel.

**set_params(**params**)

- Set the parameters of this kernel.

  The method works on simple kernels as well as on nested kernels. The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

  Returns self:

**theta**

- Returns the (flattened, log-transformed) non-fixed hyperparameters.

  Note that theta are typically the log-transformed values of the kernel's hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

  Returns theta: array, shape (n_dims,)

  The non-fixed, log-transformed hyperparameters of the kernel

### 29.13.15 sklearn.gaussian_process.kernels.CompoundKernel

**class** sklearn.gaussian_process.kernels.CompoundKernel(**kernels**)

- Kernel which is composed of a set of other kernels.

  New in version 0.18.

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>clone_with_theta(theta)</code></td>
<td>Returns a clone of self with given hyperparameters theta.</td>
</tr>
<tr>
<td><code>diag(X)</code></td>
<td>Returns the diagonal of the kernel k(X, X).</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters of this kernel.</td>
</tr>
<tr>
<td><code>is_stationary()</code></td>
<td>Returns whether the kernel is stationary.</td>
</tr>
<tr>
<td>`set_params(**params)</td>
<td>Set the parameters of this kernel.</td>
</tr>
</tbody>
</table>

**__init__**(**kernels**)

**bounds**

- Returns the log-transformed bounds on the theta.

  Returns bounds: array, shape (n_dims, 2)

  The log-transformed bounds on the kernel’s hyperparameters theta

**clone_with_theta**(**theta**)

- Returns a clone of self with given hyperparameters theta.
**diag**(*X*)

Returns the diagonal of the kernel \(k(X, X)\).

The result of this method is identical to `np.diag(self(X))`; however, it can be evaluated more efficiently since only the diagonal is evaluated.

**Parameters**

- **X** : array, shape (n_samples_X, n_features)
  
  Left argument of the returned kernel \(k(X, Y)\)

**Returns**

- **K_diag** : array, shape (n_samples_X, n_kernels)
  
  Diagonal of kernel \(k(X, X)\)

**get_params** *(deep=True)*

Get parameters of this kernel.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

**hyperparameters**

Returns a list of all hyperparameter specifications.

**is_stationary**

Returns whether the kernel is stationary.

**n_dims**

Returns the number of non-fixed hyperparameters of the kernel.

**set_params** *(**params)*

Set the parameters of this kernel.

The method works on simple kernels as well as on nested kernels. The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self** :

**theta**

Returns the (flattened, log-transformed) non-fixed hyperparameters.

Note that theta are typically the log-transformed values of the kernel’s hyperparameters as this representation of the search space is more amenable for hyperparameter search, as hyperparameters like length-scales naturally live on a log-scale.

**Returns**

- **theta** : array, shape (n_dims,)
  
  The non-fixed, log-transformed hyperparameters of the kernel

**29.13.16 sklearn.gaussian_process.kernels.Hyperparameter**

class **sklearn.gaussian_process.kernels.Hyperparameter**

A kernel hyperparameter’s specification in form of a namedtuple.

New in version 0.18.

**Attributes**

- **name** : string
  
  The name of the hyperparameter. Note that a kernel using a hyperparameter with name “x” must have the attributes `self.x` and `self.x_bounds`
**value_type**: string

The type of the hyperparameter. Currently, only “numeric” hyperparameters are supported.

**bounds**: pair of floats >= 0 or “fixed”

The lower and upper bound on the parameter. If n_elements>1, a pair of 1d array with n_elements each may be given alternatively. If the string “fixed” is passed as bounds, the hyperparameter’s value cannot be changed.

**n_elements**: int, default=1

The number of elements of the hyperparameter value. Defaults to 1, which corresponds to a scalar hyperparameter. n_elements > 1 corresponds to a hyperparameter which is vector-valued, such as, e.g., anisotropic length-scales.

**fixed**: bool, default: None

Whether the value of this hyperparameter is fixed, i.e., cannot be changed during hyperparameter tuning. If None is passed, the “fixed” is derived based on the given bounds.

### Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>count</strong>(...)</td>
<td></td>
</tr>
<tr>
<td><strong>index</strong>(...)</td>
<td>Raises ValueError if the value is not present.</td>
</tr>
</tbody>
</table>

```
__init__(...)  
x.__init__(...) initializes x; see help(type(x)) for signature
```

**bounds**  
Alias for field number 2

**count** (value) → integer – return number of occurrences of value

**fixed**  
Alias for field number 4

```
index(value[, start, ...]) → integer – return first index of value.  
 Raises ValueError if the value is not present.
```

**n_elements**  
Alias for field number 3

**name**  
Alias for field number 0

**value_type**  
Alias for field number 1

---

### 29.14 sklearn.isotonic: Isotonic regression

**User guide**: See the Isotonic regression section for further details.

```
isotonic.IsotonicRegression([y_min, y_max, ...])
```

---

Chapter 29. API Reference
Isotonic regression model.

The isotonic regression optimization problem is defined by:

\[
\min \sum w_i (y[i] - y_[i]) ^ 2 \\
\text{subject to } y_[i] \leq y_[j] \text{ whenever } X[i] \leq X[j] \\
\text{and } \min(y_) = y_min, \max(y_) = y_max
\]

where:

- \(y[i]\) are inputs (real numbers)
- \(y_[i]\) are fitted
- \(X\) specifies the order. If \(X\) is non-decreasing then \(y_\) is non-decreasing.
- \(w[i]\) are optional strictly positive weights (default to 1.0)

Read more in the User Guide.

**Parameters**

- **y_min**: optional, default: None
  - If not None, set the lowest value of the fit to y_min.
- **y_max**: optional, default: None
  - If not None, set the highest value of the fit to y_max.
- **increasing**: boolean or string, optional, default: True
  - If boolean, whether or not to fit the isotonic regression with y increasing or decreasing.
  - The string value “auto” determines whether y should increase or decrease based on the Spearman correlation estimate’s sign.
- **out_of_bounds**: string, optional, default: “nan”
  - The out_of_bounds parameter handles how x-values outside of the training domain are handled. When set to “nan”, predicted y-values will be NaN. When set to “clip”, predicted y-values will be set to the value corresponding to the nearest train interval endpoint. When set to “raise”, allow interp1d to throw ValueError.

**Attributes**

- **X_min_**: float
  - Minimum value of input array \(X_\) for left bound.
- **X_max_**: float
  - Maximum value of input array \(X_\) for right bound.
- **f_**: function
  - The stepwise interpolating function that covers the domain \(X_\).

**Notes**

Ties are broken using the secondary method from Leeuw, 1977.
References


Isotone Optimization in R: Pool-Adjacent-Violators Algorithm (PAVA) and Active Set Methods Leeuw, Hornik, Mair Journal of Statistical Software 2009

Correctness of Kruskal’s algorithms for monotone regression with ties Leeuw, Psychometrica, 1977

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(T)</td>
<td>Predict new data by linear interpolation.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(T)</td>
<td>Transform new data by linear interpolation.</td>
</tr>
</tbody>
</table>

**init__(y_min=None, y_max=None, increasing=True, out_of_bounds='nan')**

X_  
DEPRECATED: Attribute X_ is deprecated in version 0.18 and will be removed in version 0.20.

**fit (X, y, sample_weight=None)**
Fit the model using X, y as training data.

**Parameters**

- **X**: array-like, shape=(n_samples,)
  Training data.

- **y**: array-like, shape=(n_samples,)
  Training target.

- **sample_weight**: array-like, shape=(n_samples,), optional, default: None
  Weights. If set to None, all weights will be set to 1 (equal weights).

**Returns**

- **self**: object
  Returns an instance of self.

**Notes**

X is stored for future use, as transform needs X to interpolate new input data.

**fit_transform (X, y=None, **fit_params)**
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  Training set.
y : numpy array of shape [n_samples]
    Target values.

Returns
X_new : numpy array of shape [n_samples, n_features_new]
    Transformed array.

get_params (deep=True)
Get parameters for this estimator.

Parameters
deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are
    estimators.

Returns
params : mapping of string to any
    Parameter names mapped to their values.

predict (T)
Predict new data by linear interpolation.

Parameters
T : array-like, shape=(n_samples,)
    Data to transform.

Returns
T_ : array, shape=(n_samples,)
    Transformed data.

score (X, y, sample_weight=None)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares
$((y_{true} - y_{pred})^2).sum()$ and $v$ is the residual sum of squares
$((y_{true} - y_{true}.mean())^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always
predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters
X : array-like, shape = (n_samples, n_features)
    Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)
    True values for X.

sample_weight : array-like, shape = [n_samples], optional
    Sample weights.

Returns
score : float
    $R^2$ of self.predict(X) wrt. y.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns
self

transform (T)
Transform new data by linear interpolation

Parameters
T : array-like, shape=(n_samples,)

Data to transform.

**Returns**

- `T_`: array, shape=(n_samples,)
  
The transformed data

- `y_`
  
  DEPRECATED: Attribute `y_` is deprecated in version 0.18 and will be removed in version 0.20.

### Examples using `sklearn.isotonic.IsotonicRegression`

- **Isotonic Regression**

  * `isotonic.isotonic_regression(y[, ...])` Solve the isotonic regression model:

  * `isotonic.check_increasing(x, y)` Determine whether `y` is monotonically correlated with `x`.

### 29.14.2 `sklearn.isotonic.isotonic_regression`

**`sklearn.isotonic.isotonic_regression`**

- `sklearn.isotonic.isotonic_regression(y, sample_weight=None, y_min=None, y_max=None, increasing=True)` Solve the isotonic regression model:

  ```
  \[
  \text{min} \ \sum_{i} w[i] \ (y[i] - y_\text{[i]}) \ 1
  \text{subject to } y_\text{[min]} = y_\text{[1]} \leq y_\text{[2]} \ldots \leq y_\text{[n]} = y_\text{[max]} \\
  \]
  ```

  where:

  - `y[i]` are inputs (real numbers)
  - `y_[i]` are fitted
  - `w[i]` are optional strictly positive weights (default to 1.0)

  Read more in the *User Guide*.

**Parameters**

- `y`: iterable of floating-point values
  
  The data.

- `sample_weight`: iterable of floating-point values, optional, default: None
  
  Weights on each point of the regression. If None, weight is set to 1 (equal weights).

- `y_min`: optional, default: None
  
  If not None, set the lowest value of the fit to `y_min`.

- `y_max`: optional, default: None
  
  If not None, set the highest value of the fit to `y_max`.

- `increasing`: boolean, optional, default: True
  
  Whether to compute `y_` is increasing (if set to True) or decreasing (if set to False)

**Returns**

- `y_`: list of floating-point values
  
  Isotonic fit of `y`. 
29.14.3 `sklearn.isotonic.check_increasing`

`sklearn.isotonic.check_increasing(x, y)`

Determine whether y is monotonically correlated with x.

y is found increasing or decreasing with respect to x based on a Spearman correlation test.

**Parameters**

- `x`: array-like, shape=(n_samples,)
  - Training data.
- `y`: array-like, shape=(n_samples,)
  - Training target.

**Returns**

- `'increasing_bool'`: boolean
  - Whether the relationship is increasing or decreasing.

**Notes**

The Spearman correlation coefficient is estimated from the data, and the sign of the resulting estimate is used as the result.

In the event that the 95% confidence interval based on Fisher transform spans zero, a warning is raised.

**References**


29.15 `sklearn.kernel_approximation` Kernel Approximation

The `sklearn.kernel_approximation` module implements several approximate kernel feature maps based on Fourier transforms.

**User guide:** See the *Kernel Approximation* section for further details.

| `kernel_approximation.AdditiveChi2Sampler` | Approximate feature map for additive chi2 kernel. |
| `kernel_approximation.Nystroem` | Approximate a kernel map using a subset of the training data. |
| `kernel_approximation.RBFSampler` | Approximates feature map of an RBF kernel by Monte Carlo approximation of its Fourier transform. |
| `kernel_approximation.SkewedChi2Sampler` | Approximates feature map of the “skewed chi-squared” kernel by Monte Carlo approximation of its Fourier transform. |
29.15.1 sklearn.kernel_approximation.AdditiveChi2Sampler

class sklearn.kernel_approximation.AdditiveChi2Sampler(sample_steps=2, sample_interval=None)

Approximate feature map for additive chi2 kernel.

Uses sampling the fourier transform of the kernel characteristic at regular intervals.

Since the kernel that is to be approximated is additive, the components of the input vectors can be treated separately. Each entry in the original space is transformed into 2*sample_steps+1 features, where sample_steps is a parameter of the method. Typical values of sample_steps include 1, 2 and 3.

Optimal choices for the sampling interval for certain data ranges can be computed (see the reference). The default values should be reasonable.

Read more in the User Guide.

Parameters

- sample_steps : int, optional
  Gives the number of (complex) sampling points.

- sample_interval : float, optional
  Sampling interval. Must be specified when sample_steps not in {1,2,3}.

See also:

SkewedChi2Sampler A Fourier-approximation to a non-additive variant of the chi squared kernel.

sklearn.metrics.pairwise.chi2_kernel The exact chi squared kernel.

sklearn.metrics.pairwise.additive_chi2_kernel The exact additive chi squared kernel.

Notes

This estimator approximates a slightly different version of the additive chi squared kernel than metric.additive_chi2 computes.

References

See “Efficient additive kernels via explicit feature maps” A. Vedaldi and A. Zisserman, Pattern Analysis and Machine Intelligence, 2011

Methods

- fit(X[, y]) Set parameters.
- fit_transform(X[, y]) Fit to data, then transform it.
- get_params([deep]) Get parameters for this estimator.
- set_params(**params) Set the parameters of this estimator.
- transform(X[, y]) Apply approximate feature map to X.

__init__(sample_steps=2, sample_interval=None)

fit(X, y=None)
Set parameters.
**fit_transform** (*X, y=None, **fit_params*)

Fit to data, then transform it.

Fits transformer to *X* and *y* with optional parameters *fit_params* and returns a transformed version of *X*.

**Parameters**

* *X*: numpy array of shape [n_samples, n_features]
  
  Training set.

  * *y*: numpy array of shape [n_samples]
    
    Target values.

**Returns**

*X_new*: numpy array of shape [n_samples, n_features_new]

Transformed array.

**get_params** (*deep=True*)

Get parameters for this estimator.

**Parameters**

* *deep*: boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

* *params*: mapping of string to any

  Parameter names mapped to their values.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

* *self*:

**transform** (*X, y=None*)

Apply approximate feature map to *X*.

**Parameters**

* *X*: {array-like, sparse matrix}, shape = (n_samples, n_features)

**Returns**

*X_new*: {array, sparse matrix}, shape = (n_samples, n_features * (2*sample_steps + 1))

Whether the return value is an array of sparse matrix depends on the type of the input *X*.

### 29.15.2 sklearn.kernel_approximation.Nystroem

**class** *sklearn.kernel_approximation.Nystroem* (*kernel='rbf', gamma=None, coef0=1, degree=3, kernel_params=None, n_components=100, random_state=None*)

Approximate a kernel map using a subset of the training data.

Constructs an approximate feature map for an arbitrary kernel using a subset of the data as basis.

Read more in the *User Guide*.

**Parameters**

* *kernel*: string or callable, default="rbf"

  Kernel map to be approximated. A callable should accept two arguments and the keyword arguments passed to this object as kernel_params, and should return a floating point number.
**n_components** : int

Number of features to construct. How many data points will be used to construct the mapping.

**gamma** : float, default=None

Gamma parameter for the RBF, polynomial, exponential chi2 and sigmoid kernels. Interpretation of the default value is left to the kernel; see the documentation for sklearn.metrics.pairwise. Ignored by other kernels.

**degree** : float, default=3

Degree of the polynomial kernel. Ignored by other kernels.

**coef0** : float, default=1

Zero coefficient for polynomial and sigmoid kernels. Ignored by other kernels.

**kernel_params** : mapping of string to any, optional

Additional parameters (keyword arguments) for kernel function passed as callable object.

**random_state** : {int, RandomState}, optional

If int, random_state is the seed used by the random number generator; if RandomState instance, random_state is the random number generator.

**Attributes**

**components_** : array, shape (n_components, n_features)

Subset of training points used to construct the feature map.

**component_indices_** : array, shape (n_components)

Indices of components_ in the training set.

**normalization_** : array, shape (n_components, n_components)

Normalization matrix needed for embedding. Square root of the kernel matrix on components_.

**See also:**

**RBFSampler** An approximation to the RBF kernel using random Fourier features.

**sklearn.metrics.pairwise.kernel_metrics** List of built-in kernels.

**References**

• Williams, C.K.I. and Seeger, M. “Using the Nyström method to speed up kernel machines”, Advances in neural information processing systems 2001


**Methods**

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<tr>
<td><code>transform(X)</code></td>
<td>Apply feature map to X.</td>
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__init__ (kernel='rbf', gamma=None, coef0=1, degree=3, kernel_params=None, n_components=100, random_state=None)

fit (X, y=None)
Fit estimator to data.

Samples a subset of training points, computes kernel on these and computes normalization matrix.

**Parameters**
- \(X\) : array-like, shape=(n_samples, n_features)
  - Training data.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to \(X\) and \(y\) with optional parameters fit_params and returns a transformed version of \(X\).

**Parameters**
- \(X\) : numpy array of shape [n_samples, n_features]
  - Training set.
- \(y\) : numpy array of shape [n_samples]
  - Target values.

**Returns**
- \(X_{\text{new}}\) : numpy array of shape [n_samples, n_features_new]
  - Transformed array.

get_params (deep=True)
Get parameters for this estimator.

**Parameters**
- \(\text{deep}\) : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- \(\text{params}\) : mapping of string to any
  - Parameter names mapped to their values.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- self

transform (X)
Apply feature map to \(X\).

Computes an approximate feature map using the kernel between some training points and \(X\).

**Parameters**
- \(X\) : array-like, shape=(n_samples, n_features)
  - Data to transform.

**Returns**
- \(X_{\text{transformed}}\) : array, shape=(n_samples, n_components)
  - Transformed data.
Examples using sklearn.kernel_approximation.Nystroem

- Explicit feature map approximation for RBF kernels

29.15.3 sklearn.kernel_approximation.RBFSampler

class sklearn.kernel_approximation.RBFSampler(gamma=1.0, n_components=100, random_state=None)

Approximates feature map of an RBF kernel by Monte Carlo approximation of its Fourier transform.

It implements a variant of Random Kitchen Sinks.[1]

Read more in the User Guide.

Parameters

gamma : float

Parameter of RBF kernel: \( \exp(-\gamma \cdot x^2) \)

n_components : int

Number of Monte Carlo samples per original feature. Equals the dimensionality of the computed feature space.

random_state : {int, RandomState}, optional

If int, random_state is the seed used by the random number generator; if RandomState instance, random_state is the random number generator.

Notes

See “Random Features for Large-Scale Kernel Machines” by A. Rahimi and Benjamin Recht.


Methods

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<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X[, y])</td>
<td>Apply the approximate feature map to X.</td>
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</table>

__init__(gamma=1.0, n_components=100, random_state=None)

fit (X[, y=None])

Fit the model with X.

Samples random projection according to n_features.

Parameters

X : {array-like, sparse matrix}, shape (n_samples, n_features)

Training data, where n_samples is the number of samples and n_features is the number of features.

Returns

self : object
Returns the transformer.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters** *

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns** *

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters** *

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns** *

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**set_params** *(**params)*

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns** *

- **self**: 

**transform** *(X, y=None)*

Apply the approximate feature map to X.

**Parameters** *

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  - New data, where n_samples in the number of samples and n_features is the number of features.

**Returns** *

- **X_new**: array-like, shape (n_samples, n_components)

**Examples using sklearn.kernel_approximation.RBFSampler**

- Explicit feature map approximation for RBF kernels

**29.15.4 sklearn.kernel_approximation.SkewedChi2Sampler**

class sklearn.kernel_approximation.SkewedChi2Sampler *(skewedness=1.0, n_components=100, random_state=None)*

Approximates feature map of the “skewed chi-squared” kernel by Monte Carlo approximation of its Fourier transform.

Read more in the User Guide.
Parameters:

- **skewedness**: float
  
  “skewedness” parameter of the kernel. Needs to be cross-validated.

- **n_components**: int
  
  Number of Monte Carlo samples per original feature. Equals the dimensionality of the computed feature space.

- **random_state**: {int, RandomState}, optional
  
  If int, random_state is the seed used by the random number generator; if RandomState instance, random_state is the random number generator.

See also:

- **AdditiveChi2Sampler**
  A different approach for approximating an additive variant of the chi squared kernel.

- **sklearn.metrics.pairwise.chi2_kernel**
  The exact chi squared kernel.

**References**

See “Random Fourier Approximations for Skewed Multiplicative Histogram Kernels” by Fuxin Li, Catalin Ionescu and Cristian Sminchisescu.

**Methods**

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</tr>
<tr>
<td><code>transform(X[, y])</code></td>
<td>Apply the approximate feature map to X.</td>
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**__init__** (skewedness=1.0, n_components=100, random_state=None)

**fit**(X, y=None)

  Fit the model with X.

  Samples random projection according to n_features.

  **Parameters**
  
  X : array-like, shape (n_samples, n_features)

  Training data, where n_samples in the number of samples and n_features is the number of features.

  **Returns**
  
  self : object

  Returns the transformer.

**fit_transform**(X, y=None, **fit_params)

  Fit to data, then transform it.

  Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

  **Parameters**
  
  X : numpy array of shape [n_samples, n_features]

  Training set.
y: numpy array of shape [n_samples]
    Target values.

Returns X_new: numpy array of shape [n_samples, n_features_new]
    Transformed array.

get_params (deep=True)
    Get parameters for this estimator.

Parameters:
    deep: boolean, optional
        If True, will return the parameters for this estimator and contained subobjects that are
        estimators.

Returns:
    param: mapping of string to any
        Parameter names mapped to their values.

set_params (**params)
    Set the parameters of this estimator.

    The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
    parameters of the form <component>__<parameter> so that it's possible to update each component
    of a nested object.

Returns:
    self:

transform (X, y=None)
    Apply the approximate feature map to X.

Parameters:
    X: array-like, shape (n_samples, n_features)
        New data, where n_samples in the number of samples and n_features is the number of
        features.

Returns:
    X_new: array-like, shape (n_samples, n_components)

29.16 sklearn.kernel_ridge Kernel Ridge Regression

Module sklearn.kernel_ridge implements kernel ridge regression.

User guide: See the Kernel ridge regression section for further details.


29.16.1 sklearn.kernel_ridge.KernelRidge

class sklearn.kernel_ridge.KernelRidge (alpha=1, kernel='linear', gamma=None, degree=3, coe0=1, kernel_params=None)
    Kernel ridge regression.

    Kernel ridge regression (KRR) combines ridge regression (linear least squares with l2-norm regulariza-
tion) with the kernel trick. It thus learns a linear function in the space induced by the respective kernel and the data. For
    non-linear kernels, this corresponds to a non-linear function in the original space.

    The form of the model learned by KRR is identical to support vector regression (SVR). However, different loss
    functions are used: KRR uses squared error loss while support vector regression uses epsilon-insensitive loss,
    both combined with l2 regularization. In contrast to SVR, fitting a KRR model can be done in closed-form and
is typically faster for medium-sized datasets. On the other hand, the learned model is non-sparse and thus slower than SVR, which learns a sparse model for epsilon > 0, at prediction-time.

This estimator has built-in support for multi-variate regression (i.e., when y is a 2d-array of shape \([n_{\text{samples}}, n_{\text{targets}}]\)).

Read more in the *User Guide*.

**Parameters**

- **alpha**: \([\text{float, array-like}], \text{shape} = [n_{\text{targets}}]\)
  
  Small positive values of alpha improve the conditioning of the problem and reduce the variance of the estimates. Alpha corresponds to \((2 + C)^{-1}\) in other linear models such as LogisticRegression or LinearSVC. If an array is passed, penalties are assumed to be specific to the targets. Hence they must correspond in number.

- **kernel**: \(\text{string or callable, default=}'linear'\)
  
  Kernel mapping used internally. A callable should accept two arguments and the keyword arguments passed to this object as kernel_params, and should return a floating point number.

- **gamma**: float, default=None
  
  Gamma parameter for the RBF, laplacian, polynomial, exponential chi2 and sigmoid kernels. Interpretation of the default value is left to the kernel; see the documentation for sklearn.metrics.pairwise. Ignored by other kernels.

- **degree**: float, default=3
  
  Degree of the polynomial kernel. Ignored by other kernels.

- **coef0**: float, default=1
  
  Zero coefficient for polynomial and sigmoid kernels. Ignored by other kernels.

- **kernel_params**: mapping of string to any, optional
  
  Additional parameters (keyword arguments) for kernel function passed as callable object.

**Attributes**

- **dual_coef_**: array, shape = [n_samples] or [n_samples, n_targets]
  
  Representation of weight vector(s) in kernel space

- **X_fit_**: \([\text{array-like, sparse matrix}], \text{shape} = [n_{\text{samples}}, n_{\text{features}}]\)
  
  Training data, which is also required for prediction

**See also:**

- **Ridge**: Linear ridge regression.

- **SVR**: Support Vector Regression implemented using libsvm.

**References**

Examples

```python
>>> from sklearn.kernel_ridge import KernelRidge
>>> import numpy as np

>>> n_samples, n_features = 10, 5
>>> rng = np.random.RandomState(0)
>>> y = rng.randn(n_samples)
>>> X = rng.randn(n_samples, n_features)
>>> clf = KernelRidge(alpha=1.0)
>>> clf.fit(X, y)
```

Methods

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<td>Get parameters for this estimator.</td>
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<td>predict(X)</td>
<td>Predict using the kernel ridge model</td>
</tr>
<tr>
<td>score(X, y, sample_weight)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(alpha=1, kernel='linear', gamma=None, degree=3, coef0=1, kernel_params=None)
```
Returns C : array, shape = [n_samples] or [n_samples, n_targets]
Returns predicted values.

score (X, y, sample_weight=None)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})**2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true.mean()})**2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters X : array-like, shape = (n_samples, n_features)
Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)
True values for X.

sample_weight : array-like, shape = [n_samples], optional
Sample weights.

Returns score : float
$R^2$ of self.predict(X) wrt. y.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

Examples using sklearn.kernel_ridge.KernelRidge

- Comparison of kernel ridge regression and SVR
- Comparison of kernel ridge and Gaussian process regression

29.17 sklearn.discriminant_analysis: Discriminant Analysis

Linear Discriminant Analysis and Quadratic Discriminant Analysis

User guide: See the Linear and Quadratic Discriminant Analysis section for further details.

discriminant_analysis.LinearDiscriminantAnalysis

discriminant_analysis.QuadraticDiscriminantAnalysis
29.17.1 sklearn.discriminant_analysis.LinearDiscriminantAnalysis

class sklearn.discriminant_analysis.LinearDiscriminantAnalysis (solver='svd',
    shrinkage=None,
    priors=None,
    n_components=None,
    store_covariance=False,
    tol=0.0001)

Linear Discriminant Analysis

A classifier with a linear decision boundary, generated by fitting class conditional densities to the data and using Bayes’ rule.

The model fits a Gaussian density to each class, assuming that all classes share the same covariance matrix.

The fitted model can also be used to reduce the dimensionality of the input by projecting it to the most discriminative directions.

New in version 0.17: LinearDiscriminantAnalysis.

Read more in the User Guide.

Parameters

solver : string, optional
    Solver to use, possible values:
    • 'svd': Singular value decomposition (default). Does not compute the covariance matrix, therefore this solver is recommended for data with a large number of features.
    • 'lsqr': Least squares solution, can be combined with shrinkage.
    • 'eigen': Eigenvalue decomposition, can be combined with shrinkage.

shrinkage : string or float, optional
    Shrinkage parameter, possible values:
    • None: no shrinkage (default).
    • 'auto': automatic shrinkage using the Ledoit-Wolf lemma.
    • float between 0 and 1: fixed shrinkage parameter.

    Note that shrinkage works only with 'lsqr' and 'eigen' solvers.

priors : array, optional, shape (n_classes,)
    Class priors.

n_components : int, optional
    Number of components (< n_classes - 1) for dimensionality reduction.

store_covariance : bool, optional
    Additionally compute class covariance matrix (default False).

    New in version 0.17.

tol : float, optional
    Threshold used for rank estimation in SVD solver.

    New in version 0.17.

Attributes

coef_ : array, shape (n_features,) or (n_classes, n_features)
Weight vector(s).

**intercept_** : array, shape (n_features,)
Intercept term.

**covariance_** : array-like, shape (n_features, n_features)
Covariance matrix (shared by all classes).

**explained_variance_ratio_** : array, shape (n_components,)
Percentage of variance explained by each of the selected components. If
n_components is not set then all components are stored and the sum of explained
variances is equal to 1.0. Only available when eigen or svd solver is used.

**means_** : array-like, shape (n_classes, n_features)
Class means.

**priors_** : array-like, shape (n_classes,)
Class priors (sum to 1).

**scalings_** : array-like, shape (rank, n_classes - 1)
Scaling of the features in the space spanned by the class centroids.

**xbar_** : array-like, shape (n_features,)
Overall mean.

**classes_** : array-like, shape (n_classes,)
Unique class labels.

See also:

```
sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis
```

Notes

The default solver is ‘svd’. It can perform both classification and transform, and it does not rely on the calculation of the covariance matrix. This can be an advantage in situations where the number of features is large. However, the ‘svd’ solver cannot be used with shrinkage.

The ‘lsqr’ solver is an efficient algorithm that only works for classification. It supports shrinkage.

The ‘eigen’ solver is based on the optimization of the between class scatter to within class scatter ratio. It can be used for both classification and transform, and it supports shrinkage. However, the ‘eigen’ solver needs to compute the covariance matrix, so it might not be suitable for situations with a high number of features.

Examples

```python
>>> import numpy as np
>>> from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = LinearDiscriminantAnalysis()
>>> clf.fit(X, y)
```
LinearDiscriminantAnalysis(n_components=None, priors=None, shrinkage=None, solver='svd', store_covariance=False, tol=0.0001)

>>> print(clf.predict([[-0.8, -1]]))
[1]

Methods

decision_function(X) Predict confidence scores for samples.

fit(X, y[, store_covariance, tol]) Fit LinearDiscriminantAnalysis model according to the given training data and parameters.

fit_transform(X[, y]) Fit to data, then transform it.

get_params([deep]) Get parameters for this estimator.

predict(X) Predict class labels for samples in X.

predict_log_proba(X) Estimate log probability.

predict_proba(X) Estimate probability.

score(X, y[, sample_weight]) Returns the mean accuracy on the given test data and labels.

set_params(**params) Set the parameters of this estimator.

transform(X) Project data to maximize class separation.

__init__(solver='svd', shrinkage=None, priors=None, n_components=None, store_covariance=False, tol=0.0001)

decision_function(X)

Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

Parameters:

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returns:

array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes):

Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

fit(X, y, store_covariance=None, tol=None)

Fit LinearDiscriminantAnalysis model according to the given training data and parameters.

Changed in version 0.17: Deprecated store_covariance have been moved to main constructor.

Changed in version 0.17: Deprecated tol have been moved to main constructor.

Parameters:

X : array-like, shape (n_samples, n_features)

Training data.

y : array, shape (n_samples,)

Target values.

fit_transform(X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.
Parameters

X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns

X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

get_params (deep=True)
Get parameters for this estimator.

Parameters

depth : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns

params : mapping of string to any
Parameter names mapped to their values.

predict (X)
Predict class labels for samples in X.

Parameters

X : {array-like, sparse matrix}, shape = [n_samples, n_features]
Samples.

Returns

C : array, shape = [n_samples]
Predicted class label per sample.

predict_log_proba (X)
Estimate log probability.

Parameters

X : array-like, shape (n_samples, n_features)
Input data.

Returns

C : array, shape (n_samples, n_classes)
Estimated log probabilities.

predict_proba (X)
Estimate probability.

Parameters

X : array-like, shape (n_samples, n_features)
Input data.

Returns

C : array, shape (n_samples, n_classes)
Estimated probabilities.

score (X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each
sample that each label set be correctly predicted.

Parameters

X : array-like, shape = (n_samples, n_features)
Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)
True labels for X.

**sample_weight** : array-like, shape [n_samples], optional

Sample weights.

**Returns**

**score** : float

Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self** :

**transform** (**X**)

Project data to maximize class separation.

**Parameters**

**X** : array-like, shape (n_samples, n_features)

Input data.

**Returns**

**X_new** : array, shape (n_samples, n_components)

Transformed data.

**Examples using sklearn.discriminant_analysis.LinearDiscriminantAnalysis**

- Normal and Shrinkage Linear Discriminant Analysis for classification
- Linear and Quadratic Discriminant Analysis with confidence ellipsoid
- Comparison of LDA and PCA 2D projection of Iris dataset
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...

**29.17.2 sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis**

**class**

```python
sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis(priors=None,
reg_param=0.0,
store_covariances=False,
tol=0.0001)
```

Quadratic Discriminant Analysis

A classifier with a quadratic decision boundary, generated by fitting class conditional densities to the data and using Bayes’ rule.

The model fits a Gaussian density to each class.

New in version 0.17: *QuadraticDiscriminantAnalysis*

Read more in the *User Guide*.

**Parameters**

**priors** : array, optional, shape = [n_classes]

Priors on classes

**reg_param** : float, optional
Regularizes the covariance estimate as \((1 - \text{reg\_param}) \cdot \Sigma + \text{reg\_param} \cdot \text{n\_eye(n\_features)}\)

**Attributes**

- **covariances_**: list of array-like, shape = [n_features, n_features]
  Covariance matrices of each class.
- **means_**: array-like, shape = [n_classes, n_features]
  Class means.
- **priors_**: array-like, shape = [n_classes]
  Class priors (sum to 1).
- **rotations_**: list of arrays
  For each class \(k\) an array of shape \([n\_features, n\_k]\), with \(n\_k = \min(n\_features, \text{number of elements in class } k)\) It is the rotation of the Gaussian distribution, i.e. its principal axis.
- **scalings_**: list of arrays
  For each class \(k\) an array of shape \([n\_k]\). It contains the scaling of the Gaussian distributions along its principal axes, i.e. the variance in the rotated coordinate system.
- **store_covariances**: boolean
  If True the covariance matrices are computed and stored in the `self.covariances_` attribute.
  New in version 0.17.
- **tol**: float, optional, default 1.0e-4
  Threshold used for rank estimation.
  New in version 0.17.

**See also:**

`sklearn.discriminant_analysis.LinearDiscriminantAnalysis` Linear Discriminant Analysis

**Examples**

```python
>>> from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = QuadraticDiscriminantAnalysis()
>>> clf.fit(X, y)
... QuadraticDiscriminantAnalysis(priors=None, reg_param=0.0,
    store_covariances=False, tol=0.0001)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

**Methods**
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<th>Description</th>
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<td><code>decision_function(X)</code></td>
<td>Apply decision function to an array of samples.</td>
</tr>
<tr>
<td><code>fit(X, y[, store_covariances, tol])</code></td>
<td>Fit the model according to the given training data and parameters.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>predict(X)</code></td>
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</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>Return posterior probabilities of classification.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Return posterior probabilities of classification.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**`__init__`** (priors=None, reg_param=0.0, store_covariances=False, tol=0.0001)

**`decision_function(X)`**
Apply decision function to an array of samples.

Parameters:
- `X`: array-like, shape = [n_samples, n_features]
  - Array of samples (test vectors).

Returns:
- `C`: array, shape = [n_samples, n_classes] or [n_samples,]
  - Decision function values related to each class, per sample. In the two-class case, the shape is [n_samples,], giving the log likelihood ratio of the positive class.

**`fit(X, y[, store_covariances, tol])`**
Fit the model according to the given training data and parameters.

Changed in version 0.17: Deprecated `store_covariances` have been moved to main constructor.

Changed in version 0.17: Deprecated `tol` have been moved to main constructor.

Parameters:
- `X`: array-like, shape = [n_samples, n_features]
  - Training vector, where n_samples in the number of samples and n_features is the number of features.
- `y`: array, shape = [n_samples]
  - Target values (integers)

**`get_params(deep=True)`**
Get parameters for this estimator.

Parameters:
- `deep`: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- `params`: mapping of string to any
  - Parameter names mapped to their values.

**`predict(X)`**
Perform classification on an array of test vectors X.

The predicted class C for each sample in X is returned.

Parameters:
- `X`: array-like, shape = [n_samples, n_features]

Returns:
- `C`: array, shape = [n_samples]
predict_log_proba \((X)\)

Return posterior probabilities of classification.

**Parameters**

- \(X\) : array-like, shape = \([n\_samples, n\_features]\)
  
  Array of samples/test vectors.

**Returns**

- \(C\) : array, shape = \([n\_samples, n\_classes]\)
  
  Posterior log-probabilities of classification per class.

predict_proba \((X)\)

Return posterior probabilities of classification.

**Parameters**

- \(X\) : array-like, shape = \([n\_samples, n\_features]\)
  
  Array of samples/test vectors.

**Returns**

- \(C\) : array, shape = \([n\_samples, n\_classes]\)
  
  Posterior probabilities of classification per class.

score \((X, y, sample\_weight=None)\)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- \(X\) : array-like, shape = \((n\_samples, n\_features)\)
  
  Test samples.

- \(y\) : array-like, shape = \((n\_samples)\) or \((n\_samples, n\_outputs)\)
  
  True labels for \(X\).

- \(sample\_weight\) : array-like, shape = \([n\_samples]\), optional
  
  Sample weights.

**Returns**

- \(score\) : float
  
  Mean accuracy of self.predict(X) wrt. \(y\).

set_params (**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \(<component>\__<parameter>\) so that it’s possible to update each component of a nested object.

**Returns**

- \(self\) : 

Examples using `sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis`

- *Classifier comparison*

- *Linear and Quadratic Discriminant Analysis with confidence ellipsoid*
29.18 sklearn.linear_model: Generalized Linear Models

The sklearn.linear_model module implements generalized linear models. It includes Ridge regression, Bayesian Regression, Lasso and Elastic Net estimators computed with Least Angle Regression and coordinate descent. It also implements Stochastic Gradient Descent related algorithms.

User guide: See the Generalized Linear Models section for further details.

```python
linear_model.ARDRegression([n_iter, tol, ...]) Bayesian ARD regression.
linear_model.BayesianRidge([n_iter, tol, ...]) Bayesian ridge regression
linear_model.ElasticNet([alpha, l1_ratio, ...]) Linear regression with combined L1 and L2 priors as regularizer.
linear_model.ElasticNetCV([l1_ratio, eps, ...]) Elastic Net model with iterative fitting along a regularization path
linear_model.HuberRegressor([epsilon, ...]) Linear regression model that is robust to outliers.
linear_model.Lars([fit_intercept, verbose, ...]) Least Angle Regression model a.k.a.
linear_model.LarsCV([fit_intercept, ...]) Cross-validated Least Angle Regression model
linear_model.Lasso([alpha, fit_intercept, ...]) Linear Model trained with L1 prior as regularizer (aka the Lasso)
linear_model.LassoCV([eps, n_alphas, ...]) Lasso linear model with iterative fitting along a regularization path
linear_model.LassoLars([alpha, ...]) Lasso model fit with Least Angle Regression a.k.a.
linear_model.LassoLarsCV([fit_intercept, ...]) Cross-validated Lasso, using the LARS algorithm
linear_model.LassoLarsIC([criterion, ...]) Lasso model fit with Lars using BIC or AIC for model selection
linear_model.LinearRegression(...) Ordinary least squares Linear Regression.
linear_model.LogisticRegression([penalty, ...]) Logistic Regression (aka logit, MaxEnt) classifier.
linear_model.LogisticRegressionCV([Cs, ...]) Logistic Regression CV (aka logit, MaxEnt) classifier.
linear_model.MultiTaskLasso([alpha, ...]) Multi-task Lasso model trained with L1/L2 mixed-norm as regularizer
linear_model.MultiTaskElasticNet([alpha, ...]) Multi-task ElasticNet model trained with L1/L2 mixed-norm as regularizer
linear_model.OrthogonalMatchingPursuit([...]) Orthogonal Matching Pursuit model (OMP)
linear_model.OrthogonalMatchingPursuitCV([...]) Orthogonal Matching Pursuit model (OMP)
linear_model.PassiveAggressiveClassifier([...]) Passive Aggressive Classifier
linear_model.PassiveAggressiveRegressor([...]) Passive Aggressive Regressor
linear_model.Perceptron([penalty, alpha, ...]) Read more in the User Guide.
linear_model.RandomizedLasso([alpha, ...]) Randomized Lasso.
linear_model.RandomizedLogisticRegression([...]) Randomized Logistic Regression
linear_model.RANSACRegressor([...]) RANSAC (RANdom SAmple Consensus) algorithm.
linear_model.Ridge([alpha, fit_intercept, ...]) Linear least squares with L2 regularization.
linear_model.RidgeClassifier([alpha, ...]) Classifier using Ridge regression.
linear_model.RidgeClassifierCV([alphas, ...]) Ridge classifier with built-in cross-validation.
linear_model.RidgeCV([alphas, ...]) Ridge regression with built-in cross-validation.
linear_model.SGDClassifier([loss, penalty, ...]) Linear classifiers (SVM, logistic regression, a.o.) with SGD training.
```
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<table>
<thead>
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<th>Method</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><code>linear_model.SGDRegressor([loss, penalty, ...])</code></td>
<td>Linear model fitted by minimizing a regularized empirical loss with SGD</td>
</tr>
</tbody>
</table>

### 29.18.1 `sklearn.linear_model.ARDRegression`

**class** `sklearn.linear_model.ARDRegression`(n_iter=300, tol=0.001, alpha_1=1e-06, alpha_2=1e-06, lambda_1=1e-06, lambda_2=1e-06, compute_score=False, threshold_lambda=10000.0, fit_intercept=True, normalize=False, copy_X=True, verbose=False)

Bayesian ARD regression.

Fit the weights of a regression model, using an ARD prior. The weights of the regression model are assumed to be in Gaussian distributions. Also estimate the parameters lambda (precisions of the distributions of the weights) and alpha (precision of the distribution of the noise). The estimation is done by an iterative procedures (Evidence Maximization)

Read more in the [User Guide](https://scikit-learn.org/stable/).

**Parameters**
- `n_iter` : int, optional
  Maximum number of iterations. Default is 300
- `tol` : float, optional
  Stop the algorithm if w has converged. Default is 1.e-3.
- `alpha_1` : float, optional
  Hyper-parameter: shape parameter for the Gamma distribution prior over the alpha parameter. Default is 1.e-6.
- `alpha_2` : float, optional
  Hyper-parameter: inverse scale parameter (rate parameter) for the Gamma distribution prior over the alpha parameter. Default is 1.e-6.
- `lambda_1` : float, optional
  Hyper-parameter: shape parameter for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.
- `lambda_2` : float, optional
  Hyper-parameter: inverse scale parameter (rate parameter) for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.
- `compute_score` : boolean, optional
  If True, compute the objective function at each step of the model. Default is False.
- `threshold_lambda` : float, optional
  threshold for removing (pruning) weights with high precision from the computation. Default is 1.e+4.
- `fit_intercept` : boolean, optional
  whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered). Default is True.
normalize : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

copy_X : boolean, optional, default True.

If True, X will be copied; else, it may be overwritten.

verbose : boolean, optional, default False

Verbose mode when fitting the model.

Attributes

coef_ : array, shape = (n_features)

Coefficients of the regression model (mean of distribution)

alpha_ : float

estimated precision of the noise.

lambda_ : array, shape = (n_features)

estimated precisions of the weights.

sigma_ : array, shape = (n_features, n_features)

estimated variance-covariance matrix of the weights

scores_ : float

if computed, value of the objective function (to be maximized)

Notes

See examples/linear_model/plot_ard.py for an example.

Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.ARDRegression()
>>> clf.fit([[0,0], [1, 1], [2, 2]], [0, 1, 2])
... ARDRegression(alpha_1=1e-06, alpha_2=1e-06, compute_score=False,
    copy_X=True, fit_intercept=True, lambda_1=1e-06, lambda_2=1e-06,
    n_iter=300, normalize=False, threshold_lambda=10000.0, tol=0.001,
    verbose=False)
>>> clf.predict([[1, 1]])
array([[ 1.])
```

Methods

decision_function(*args, **kwargs) DEPRECATED: and will be removed in 0.19.

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<table>
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<tr>
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<tr>
<td>fit(X, y)</td>
<td>Fit the ARDRegression model according to the given training data and parameters.</td>
</tr>
<tr>
<td>get_params()</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (n_iter=300, tol=0.001, alpha_1=1e-06, alpha_2=1e-06, lambda_1=1e-06, lambda_2=1e-06, compute_score=False, threshold_lambda=10000.0, fit_intercept=True, normalize=False, copy_X=True, verbose=False)

decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:
- X : {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

Returns:
- C : array, shape = (n_samples,)
  Returns predicted values.

fit (X, y)
Fit the ARDRegression model according to the given training data and parameters.

Iterative procedure to maximize the evidence

Parameters:
- X : array-like, shape = [n_samples, n_features]
  Training vector, where n_samples in the number of samples and n_features is the number of features.
- y : array, shape = [n_samples]
  Target values (integers)

Returns:
- self : returns an instance of self.

get_params (deep=True)
Get parameters for this estimator.

Parameters:
- deep : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params : mapping of string to any
  Parameter names mapped to their values.

predict (X)
Predict using the linear model

Parameters:
- X : {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

Returns:
- C : array, shape = (n_samples,)
  Returns predicted values.
score(X, y, sample_weight=None)
Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

Parameters:
- X : array-like, shape = (n_samples, n_features)
  Test samples.
- y : array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- sample_weight : array-like, shape = [n_samples], optional
  Sample weights.

Returns:
score : float
  R^2 of self.predict(X) wrt. y.

set_params(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns:
self :
**alpha_2** : float, optional

Hyper-parameter : inverse scale parameter (rate parameter) for the Gamma distribution prior over the alpha parameter. Default is 1.e-6.

**lambda_1** : float, optional

Hyper-parameter : shape parameter for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.

**lambda_2** : float, optional

Hyper-parameter : inverse scale parameter (rate parameter) for the Gamma distribution prior over the lambda parameter. Default is 1.e-6

**compute_score** : boolean, optional

If True, compute the objective function at each step of the model. Default is False

**fit_intercept** : boolean, optional

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered). Default is True.

**normalize** : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when **fit_intercept** is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**verbose** : boolean, optional, default False

Verbose mode when fitting the model.

**Attributes**

**coef_** : array, shape = (n_features)

Coefficients of the regression model (mean of distribution)

**alpha_** : float

Estimated precision of the noise.

**lambda_** : float

Estimated precision of the weights.

**scores_** : float

If computed, value of the objective function (to be maximized)

**Notes**

See examples/linear_model/plot_bayesian_ridge.py for an example.
Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.BayesianRidge()
>>> clf.fit([[0,0], [1, 1], [2, 2]], [0, 1, 2])
... BayesianRidge(alpha_1=1e-06, alpha_2=1e-06, compute_score=False,
    copy_X=True, fit_intercept=True, lambda_1=1e-06, lambda_2=1e-06,
    n_iter=300, normalize=False, tol=0.001, verbose=False)
>>> clf.predict([[1, 1]])
array([1.])
```

Methods

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>decision_function</td>
<td>DEPRECATED: and will be removed in 0.19.</td>
</tr>
<tr>
<td>fit(X, y)</td>
<td>Fit the model</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```
__init__ (n_iter=300, tol=0.001, alpha_1=1e-06, alpha_2=1e-06, lambda_1=1e-06, lambda_2=1e-06, compute_score=False, fit_intercept=True, normalize=False, copy_X=True, verbose=False)

decision_function (*args, **kwargs) DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

Returns

- C : array, shape = (n_samples,)
  - Returns predicted values.

fit (X, y)

Fit the model

Parameters

- **X**: numpy array of shape [n_samples,n_features]
  - Training data
- **y**: numpy array of shape [n_samples]
  - Target values

Returns

- self: returns an instance of self.

generate_random_samples (n_samples=1, n_features=2, random_state=None) DEPRECATED: Use sklearn.datasets.make_regression.

Get parameters for this estimator.

Parameters

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.
```
Returns

**params** : mapping of string to any
Parameter names mapped to their values.

**predict** *(X)*
Predict using the linear model

**Parameters**

**X** : {array-like, sparse matrix}, shape = (n_samples, n_features)
Samples.

**Returns**

**C** : array, shape = (n_samples,)
Returns predicted values.

**score** *(X, y, sample_weight=None)*
Returns the coefficient of determination \( R^2 \) of the prediction.

The coefficient \( R^2 \) is defined as \( 1 - \frac{u}{v} \), where \( u \) is the regression sum of squares \( \sum (y_{true} - y_{pred})^2 \) and \( v \) is the residual sum of squares \( \sum (y_{true} - y_{true.mean})^2 \). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \( y \), disregarding the input features, would get a \( R^2 \) score of 0.0.

**Parameters**

**X** : array-like, shape = (n_samples, n_features)
Test samples.

**y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
True values for \( X \).

**sample_weight** : array-like, shape = [n_samples], optional
Sample weights.

**Returns**

**score** : float
\( R^2 \) of self.predict(X) wrt. \( y \).

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

self

**Examples using sklearn.linear_model.BayesianRidge**

- Feature agglomeration vs. univariate selection
- Bayesian Ridge Regression

**29.18.3 sklearn.linear_model.ElasticNet**

class sklearn.linear_model.ElasticNet(\( \alpha=1.0, \ l1_ratio=0.5, \ fit_intercept=True, \ normalize=False, \ precompute=False, \ max_iter=1000, \ copy_X=True, \ tol=0.0001, \ warm_start=False, \ positive=False, \ random_state=None, \ selection=’cyclic’ \)

Linear regression with combined L1 and L2 priors as regularizer.

Minimizes the objective function:
\[
\frac{1}{2n_{\text{samples}}} \|y - Xw\|_2^2 + \alpha \cdot l1_{\text{ratio}} \|w\|_1 + 0.5 \cdot \alpha \cdot (1 - l1_{\text{ratio}}) \|w\|_2^2
\]

If you are interested in controlling the L1 and L2 penalty separately, keep in mind that this is equivalent to:

\[
a \cdot L1 + b \cdot L2
\]

where:

\[
\alpha = a + b \quad \text{and} \quad l1_{\text{ratio}} = \frac{a}{a + b}
\]

The parameter \(l1_{\text{ratio}}\) corresponds to alpha in the glmnet R package while alpha corresponds to the lambda parameter in glmnet. Specifically, \(l1_{\text{ratio}} = 1\) is the lasso penalty. Currently, \(l1_{\text{ratio}} \leq 0.01\) is not reliable, unless you supply your own sequence of alpha.

Read more in the User Guide.

**Parameters**

- **alpha** : float, optional
  
  Constant that multiplies the penalty terms. Defaults to 1.0. See the notes for the exact mathematical meaning of this parameter. “alpha = 0” is equivalent to an ordinary least square, solved by the `LinearRegression` object. For numerical reasons, using `alpha = 0` with the Lasso object is not advised. Given this, you should use the `LinearRegression` object.

- **l1_ratio** : float
  
  The ElasticNet mixing parameter, with \(0 \leq l1_{\text{ratio}} \leq 1\). For \(l1_{\text{ratio}} = 0\) the penalty is an L2 penalty. For \(l1_{\text{ratio}} = 1\) it is an L1 penalty. For \(0 < l1_{\text{ratio}} < 1\), the penalty is a combination of L1 and L2.

- **fit_intercept** : bool
  
  Whether the intercept should be estimated or not. If `False`, the data is assumed to be already centered.

- **normalize** : boolean, optional, default `False`
  
  If `True`, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to `False`. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

- **precompute** : `True` | `False` | array-like
  
  Whether to use a precomputed Gram matrix to speed up calculations. The Gram matrix can also be passed as argument. For sparse input this option is always `True` to preserve sparsity.

- **max_iter** : int, optional
  
  The maximum number of iterations

- **copy_X** : boolean, optional, default `True`
  
  If `True`, X will be copied; else, it may be overwritten.

- **tol** : float, optional
  
  Controls the precision of the solution. If you pass a number, it is the absolute tolerance. If you pass `None` (the default), it is set to `\sqrt{\epsilon}`.
The tolerance for the optimization: if the updates are smaller than tol, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

**warm_start** : bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

**positive** : bool, optional

When set to True, forces the coefficients to be positive.

**selection** : str, default ‘cyclic’

If set to ‘random’, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to ‘random’) often leads to significantly faster convergence especially when tol is higher than 1e-4.

**random_state** : int, RandomState instance, or None (default)

The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to ‘random’.

**Attributes**

**coef_** : array, shape (n_features,) | (n_targets, n_features)

parameter vector (w in the cost function formula)

**sparse_coef_** : scipy.sparse matrix, shape (n_features, 1) | (n_targets, n_features)

**sparse_coef_** is a readonly property derived from coef_

**intercept_** : float | array, shape (n_targets,)

independent term in decision function.

**n_iter_** : array-like, shape (n_targets,)

number of iterations run by the coordinate descent solver to reach the specified tolerance.

See also:

**SGDRegressor** implements elastic net regression with incremental training.

**SGDClassifier** implements logistic regression with elastic net penalty

(SGDClassifier(loss="log", penalty="elasticnet").

**Notes**

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

**Methods**

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<th>Method</th>
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<td>decision_function(*args, **kwargs)</td>
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<tr>
<td>fit(X, y[, check_input])</td>
<td>Fit model with coordinate descent.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
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<tr>
<td>path(X, y[, l1_ratio, eps, n_alphas, ...])</td>
<td>Compute elastic net path with coordinate descent</td>
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<tr>
<td>predict(X)</td>
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<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (alpha=1.0, l1_ratio=0.5, fit_intercept=True, normalize=False, precompute=False, max_iter=1000, copy_X=True, tol=0.0001, warm_start=False, positive=False, random_state=None, selection='cyclic')

decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19

Decision function of the linear model

Parameters

X : numpy array or scipy.sparse matrix of shape (n_samples, n_features)

Returns

T : array, shape (n_samples,)

The predicted decision function

fit (X, y, check_input=True)

Fit model with coordinate descent.

Parameters

X : ndarray or scipy.sparse matrix, (n_samples, n_features)

Data

y : ndarray, shape (n_samples,) or (n_samples, n_targets)

Target

check_input : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

Notes

Coordinate descent is an algorithm that considers each column of data at a time hence it will automatically convert the X input as a Fortran-contiguous numpy array if necessary.

To avoid memory re-allocation it is advised to allocate the initial data in memory directly using that format.

get_params (deep=True)

Get parameters for this estimator.

Parameters

depth : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

params : mapping of string to any

Parameter names mapped to their values.

static path (X, y, l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, check_input=True, **params)

Compute elastic net path with coordinate descent

The elastic net optimization function varies for mono and multi-outputs.

For mono-output tasks it is:
\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||y - Xw||^2_2
+ \alpha \times l1_{\text{ratio}} \times ||w||_1
+ 0.5 \times \alpha \times (1 - l1_{\text{ratio}}) \times ||w||^2_2
\]

For multi-output tasks it is:

\[
(\frac{1}{(2 \times n_{\text{samples}})}) \times ||Y - XW||^{\text{Fro}}_2
+ \alpha \times l1_{\text{ratio}} \times ||W||_{21}
+ 0.5 \times \alpha \times (1 - l1_{\text{ratio}}) \times ||W||^{\text{Fro}}_2
\]

Where:

\[
||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the *User Guide*.

**Parameters**

- **X** : array-like, shape (n_samples, n_features)
  - Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If `y` is mono-output then `X` can be sparse.

- **y** : ndarray, shape (n_samples,) or (n_samples, n_outputs)
  - Target values

- **l1_ratio** : float, optional
  - float between 0 and 1 passed to elastic net (scaling between l1 and l2 penalties).
  - `l1_ratio=1` corresponds to the Lasso

- **eps** : float
  - Length of the path. `eps=1e-3` means that `alpha_min / alpha_max = 1e-3`

- **n_alphas** : int, optional
  - Number of alphas along the regularization path

- **alphas** : ndarray, optional
  - List of alphas where to compute the models. If None alphas are set automatically

- **precompute** : True | False | ‘auto’ | array-like
  - Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

- **Xy** : array-like, optional
  - `Xy = np.dot(X.T, y)` that can be precomputed. It is useful only when the Gram matrix is precomputed.

- **copy_X** : boolean, optional, default True
  - If True, `X` will be copied; else, it may be overwritten.

- **coef_init** : array, shape (n_features,) | None
  - The initial values of the coefficients.

- **verbose** : bool or integer
  - Amount of verbosity.
params : kwargs
    keyword arguments passed to the coordinate descent solver.

return_n_iter : bool
    whether to return the number of iterations or not.

positive : bool, default False
    If set to True, forces coefficients to be positive.

check_input : bool, default True
    Skip input validation checks, including the Gram matrix when provided assuming there
    are handled by the caller when check_input=False.

Returns:
    alphas : array, shape (n_alphas,)
        The alphas along the path where models are computed.

coeffs : array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
    Coefficients along the path.

dual_gaps : array, shape (n_alphas,)
    The dual gaps at the end of the optimization for each alpha.

n_iters : array-like, shape (n_alphas,)
    The number of iterations taken by the coordinate descent optimizer to reach the specified
    tolerance for each alpha. (Is returned when return_n_iter is set to True).

See also:

MultiTaskElasticNet, MultiTaskElasticNetCV, ElasticNet, ElasticNetCV

Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

predict (X)
    Predict using the linear model

    Parameters:
    X : {array-like, sparse matrix}, shape = (n_samples, n_features)
        Samples.

    Returns:
    C : array, shape = (n_samples,)
        Returns predicted values.

score (X, y, sample_weight=None)
    Returns the coefficient of determination R^2 of the prediction.

    The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) **
    2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score
    is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always
    predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

    Parameters:
    X : array-like, shape = (n_samples, n_features)
        Test samples.
    y : array-like, shape = (n_samples) or (n_samples, n_outputs)
True values for $X$.

```python
sample_weight : array-like, shape = [n_samples], optional
    Sample weights.
```

Returns

```python
score : float
    $R^2$ of self.predict(X) wrt. y.
```

```python
set_params(**params)
    Set the parameters of this estimator.
```

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns

```python
self :
```

```python
sparse_coef_ :
    sparse representation of the fitted coef_
```

**Examples using sklearn.linear_model.ElasticNet**

- Lasso and Elastic Net for Sparse Signals
- Train error vs Test error

**29.18.4 sklearn.linear_model.HuberRegressor**

```python
class sklearn.linear_model.HuberRegressor(epsilon=1.35, max_iter=100, alpha=0.0001, warm_start=False, fit_intercept=True, tol=1e-05)
```

Linear regression model that is robust to outliers.

The Huber Regressor optimizes the squared loss for the samples where $|y - X'w| / \sigma < \epsilon$, and the absolute loss for the samples where $|y - X'w| / \sigma > \epsilon$, where $w$ and $\sigma$ are parameters to be optimized. The parameter $\sigma$ makes sure that if $y$ is scaled up or down by a certain factor, one does not need to rescale $\epsilon$ to achieve the same robustness. Note that this does not take into account the fact that the different features of $X$ may be of different scales.

This makes sure that the loss function is not heavily influenced by the outliers while not completely ignoring their effect.

Read more in the User Guide

New in version 0.18.

```python
Parameter epsilon : float, greater than 1.0, default 1.35
    The parameter epsilon controls the number of samples that should be classified as outliers. The smaller the epsilon, the more robust it is to outliers.
```

```python
max_iter : int, default 100
    Maximum number of iterations that scipy.optimize.fmin_l_bfgs_b should run for.
```

```python
alpha : float, default 0.0001
    Regularization parameter.
```

```python
warm_start : bool, default False
```

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This is useful if the stored attributes of a previously used model has to be reused. If set to False, then the coefficients will be rewritten for every call to fit.

**fit_intercept**: bool, default True

Whether or not to fit the intercept. This can be set to False if the data is already centered around the origin.

**tol**: float, default 1e-5

The iteration will stop when \( \max(|\text{proj } g_i| \ i = 1, \ldots, n) \leq \text{tol} \) where pg_i is the i-th component of the projected gradient.

**Attributes**

**coef_**: array, shape (n_features,)

Features got by optimizing the Huber loss.

**intercept_**: float

Bias.

**scale_**: float

The value by which \(|y - X'w - c|\) is scaled down.

**n_iter_**: int

Number of iterations that fmin_l_bfgs_b has run for. Not available if SciPy version is 0.9 and below.

**outliers_**: array, shape (n_samples,)

A boolean mask which is set to True where the samples are identified as outliers.

**References**

[R33], [R34]

**Methods**

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<th>Description</th>
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<tbody>
<tr>
<td>decision_function(*args, **kwargs)</td>
<td>DEPRECATED: and will be removed in 0.19.</td>
</tr>
<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit the model according to the given training data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination ( R^2 ) of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__** *(epsilon=1.35, max_iter=100, alpha=0.0001, warm_start=False, fit_intercept=True, tol=1e-05)*

**decision_function** (*args, **kwargs*)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

**Parameters**

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.
Returns $C$, array, shape = (n_samples,)

Returns predicted values.

**fit** ($X$, $y$, *sample_weight=None*)

Fit the model according to the given training data.

**Parameters**

- $X$: array-like, shape (n_samples, n_features)
  Training vector, where n_samples in the number of samples and n_features is the number of features.
- $y$: array-like, shape (n_samples,)
  Target vector relative to $X$.
- *sample_weight*: array-like, shape (n_samples,)
  Weight given to each sample.

Returns $self$.

**get_params** (**deep=True**)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns $params$, mapping of string to any

Parameter names mapped to their values.

**predict** ($X$)

Predict using the linear model

**Parameters**

- $X$: {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

Returns $C$, array, shape = (n_samples,)

Returns predicted values.

**score** ($X$, $y$, *sample_weight=None*)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares (($y_{true} - y_{pred}$) ** 2).sum() and $v$ is the residual sum of squares (($y_{true} - y_{true}.mean()$) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- $X$: array-like, shape = (n_samples, n_features)
  Test samples.
- $y$: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for $X$.
- *sample_weight*: array-like, shape = [n_samples], optional
  Sample weights.

Returns $score$, float
R^2 of self.predict(X) wrt. y.

**set_params (**)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns:

self:

**Examples using sklearn.linear_model.HuberRegressor**

- HuberRegressor vs Ridge on dataset with strong outliers
- Robust linear estimator fitting

**29.18.5 sklearn.linear_model.Lars**

**class sklearn.linear_model.Lars (fit_intercept=True, verbose=False, normalize=True, precompute='auto', n_nonzero_coefs=500, eps=2.220446049250313e-16, copy_X=True, fit_path=True, positive=False)**

Least Angle Regression model a.k.a. LAR

Read more in the [User Guide](#).

**Parameters**

- `n_nonzero_coefs` : int, optional
  Target number of non-zero coefficients. Use np.inf for no limit.

- `fit_intercept` : boolean
  Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- `positive` : boolean (default=False)
  Restrict coefficients to be >= 0. Be aware that you might want to remove fit_intercept which is set True by default.

- `verbose` : boolean or integer, optional
  Sets the verbosity amount

- `normalize` : boolean, optional, default False
  If True, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with `normalize=True`.

- `precompute` : True | False | `auto` | array-like
  Whether to use a precomputed Gram matrix to speed up calculations. If set to `auto`, let us decide. The Gram matrix can also be passed as argument.

- `copy_X` : boolean, optional, default True
  If True, X will be copied; else, it may be overwritten.

- `eps` : float, optional
The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the tol parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

**fit_path**: boolean

If True the full path is stored in the coef_path_ attribute. If you compute the solution for a large problem or many targets, setting fit_path to False will lead to a speedup, especially with a small alpha.

**Attributes**

- **alphas_**: array, shape (n_alphas + 1, | list of n_targets such arrays
  
  Maximum of covariances (in absolute value) at each iteration. n_alphas is either n_nonzero_coefs or n_features, whichever is smaller.

- **active_**: list, length = n_alphas | list of n_targets such lists
  
  Indices of active variables at the end of the path.

- **coef_path_**: array, shape (n_features, n_alphas + 1) | list of n_targets such arrays
  
  The varying values of the coefficients along the path. It is not present if the fit_path parameter is False.

- **coef_**: array, shape (n_features,) or (n_targets, n_features)
  
  Parameter vector (w in the formulation formula).

- **intercept_**: float | array, shape (n_targets,)
  
  Independent term in decision function.

- **n_iter_**: array-like or int
  
  The number of iterations taken by lars_path to find the grid of alphas for each target.

See also:

- lars_path, LarsCV, sklearn.decomposition.sparse_encode

Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.Lars(n_nonzero_coefs=1)
>>> clf.fit([[[-1, 1], [0, 0], [1, 1]], [-1.1111, 0, -1.1111]])
... Lars(copy_X=True, eps=..., fit_intercept=True, fit_path=True,
    n_nonzero_coefs=1, normalize=True, positive=False, precompute='auto',
    verbose=False)
>>> print(clf.coef_)
[ 0. -1.11...]
```

Methods

- **decision_function** *(\*args, **kwargs)*
  
  DEPRECATED: and will be removed in 0.19.

- **fit**(X, y[, Xy])
  
  Fit the model using X, y as training data.

- **get_params**(deep)
  
  Get parameters for this estimator.
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<th>Method</th>
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<tr>
<td><code>predict(X)</code></td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

`__init__` (fit_intercept=True, verbose=False, normalize=True, precompute='auto', n_nonzero_coefs=500, eps=2.2204460492503131e-16, copy_X=True, fit_path=True, positive=False)

`decision_function(*args, **kwargs)`
DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

- **Parameters**
  - `X`: {array-like, sparse matrix}, shape = (n_samples, n_features)
    - Samples.
  - `y`: array-like, shape = (n_samples,)
    - Returns predicted values.

`fit(X, y, Xy=None)`
Fit the model using X, y as training data.

- **Parameters**
  - `X`: array-like, shape (n_samples, n_features)
    - Training data.
  - `y`: array-like, shape (n_samples,) or (n_samples, n_targets)
    - Target values.
  - `Xy`: array-like, shape (n_samples,) or (n_samples, n_targets), optional
    - Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.

- **Returns**
  - `self`: object
    - returns an instance of self.

`get_params(deep=True)`
Get parameters for this estimator.

- **Parameters**
  - `deep`: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**
  - `params`: mapping of string to any
    - Parameter names mapped to their values.

`predict(X)`
Predict using the linear model

- **Parameters**
  - `X`: {array-like, sparse matrix}, shape = (n_samples, n_features)
    - Samples.

- **Returns**
  - `C`: array, shape = (n_samples,)
    - Returns predicted values.
score \((X, y, sample\_weight=\text{None})\)

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y_{\text{true}} - y_{\text{pred}})^2).\sum()\) and \(v\) is the residual sum of squares \(((y_{\text{true}} - y_{\text{true.mean}})^2).\sum()\). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \(y\), disregarding the input features, would get a \(R^2\) score of 0.0.

Parameters

- \(X\) : array-like, shape \((n\_samples, n\_features)\)
  Test samples.
- \(y\) : array-like, shape \((n\_samples)\) or \((n\_samples, n\_outputs)\)
  True values for \(X\).
- \(sample\_weight\) : array-like, shape \([n\_samples]\), optional
  Sample weights.

Returns

- \(score\) : float
  \(R^2\) of self.predict(X) wrt. \(y\).

set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns

- \(self\) :

29.18.6 sklearn.linear_model.Lasso

class sklearn.linear_model.Lasso(alpha=1.0, fit_intercept=True, normalize=False, precompute=False, copy_X=True, max_iter=1000, tol=0.0001, warm_start=False, positive=False, random_state=None, selection='cyclic')

Linear Model trained with L1 prior as regularizer (aka the Lasso)

The optimization objective for Lasso is:

\[
(1 / (2 * n\_samples)) * \|y - Xw\|^2_2 + \alpha * \|w\|_1
\]

Technically the Lasso model is optimizing the same objective function as the Elastic Net with \(l1\_ratio=1.0\) (no L2 penalty).

Read more in the User Guide.

Parameters

- \(alpha\) : float, optional
  Constant that multiplies the L1 term. Defaults to 1.0. \(alpha = 0\) is equivalent to an ordinary least square, solved by the LinearRegression object. For numerical reasons, using \(alpha = 0\) with the Lasso object is not advised. Given this, you should use the LinearRegression object.

- \(fit\_intercept\) : boolean
  whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- \(normalize\) : boolean, optional, default False
If `True`, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to `False`. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

**copy_X**: boolean, optional, default `True`

If `True`, X will be copied; else, it may be overwritten.

**precompute**: `True` | `False` | array-like, default=`False`

Whether to use a precomputed Gram matrix to speed up calculations. If set to `auto` let us decide. The Gram matrix can also be passed as argument. For sparse input this option is always `True` to preserve sparsity.

**max_iter**: int, optional

The maximum number of iterations

**tol**: float, optional

The tolerance for the optimization: if the updates are smaller than `tol`, the optimization code checks the dual gap for optimality and continues until it is smaller than `tol`.

**warm_start**: bool, optional

When set to `True`, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

**positive**: bool, optional

When set to `True`, forces the coefficients to be positive.

**selection**: str, default `cyclic`

If set to `random`, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to `random`) often leads to significantly faster convergence especially when `tol` is higher than `1e-4`.

**random_state**: int, RandomState instance, or None (default)

The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to `random`.

**Attributes**

- `coef_`: array, shape (n_features,) | (n_targets, n_features)
  parameter vector (w in the cost function formula)

- `sparse_coef_`: scipy.sparse matrix, shape (n_features, 1) | (n_targets, n_features)
  `sparse_coef_` is a readonly property derived from `coef_`

- `intercept_`: float | array, shape (n_targets,)
  independent term in decision function.

- `n_iter_`: int | array-like, shape (n_targets,)
  number of iterations run by the coordinate descent solver to reach the specified tolerance.

**See also:**

- `lars_path`
- `lasso_path`
- `LassoLars`
- `LassoCV`
- `LassoLarsCV`
- `sklearn.decomposition.sparse_encode`
Notes

The algorithm used to fit the model is coordinate descent.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.Lasso(alpha=0.1)
>>> clf.fit([[0,0], [1, 1], [2, 2]], [0, 1, 2])
Lasso(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000,
     normalize=False, positive=False, precompute=False, random_state=None,
     selection='cyclic', tol=0.0001, warm_start=False)
>>> print(clf.coef_)
[ 0.85 0.  ]
>>> print(clf.intercept_)
0.15
```

Methods

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<td>(alpha=1.0, fit_intercept=True, normalize=False, precompute=False, copy_X=True, max_iter=1000, tol=0.0001, warm_start=False, positive=False, random_state=None, selection='cyclic')</td>
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<td>decision_function</td>
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<td>fit</td>
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<td>set_params</td>
<td>([*params]) Set the parameters of this estimator.</td>
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Decision function of the linear model

Parameters:

- **X**: numpy array or scipy.sparse matrix of shape (n_samples, n_features)

Returns:

- **T**: array, shape (n_samples,)

The predicted decision function

Fit model with coordinate descent.

Parameters:

- **X**: ndarray or scipy.sparse matrix, (n_samples, n_features)

Data

- **y**: ndarray, shape (n_samples,) or (n_samples, n_targets)

Target
**check_input** : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Notes**

Coordinate descent is an algorithm that considers each column of data at a time hence it will automatically convert the X input as a Fortran-contiguous numpy array if necessary.

To avoid memory re-allocation it is advised to allocate the initial data in memory directly using that format.

**get_params**(deep=True)

Get parameters for this estimator.

Parameters

**deep** : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

**params** : mapping of string to any

Parameter names mapped to their values.

**static** **path**(X, y, l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, check_input=True, **params)

Compute elastic net path with coordinate descent

The elastic net optimization function varies for mono and multi-outputs.

For mono-output tasks it is:

\[
\frac{1}{2n_{\text{samples}}} ||y - Xw||^2_2 + \alpha l_1 \text{ratio} ||w||_1 + \frac{1}{2} \alpha (1 - l_1 \text{ratio}) ||w||^2_2
\]

For multi-output tasks it is:

\[
\frac{1}{2n_{\text{samples}}} ||Y - XW||_\text{Fro}^2 + \alpha l_1 \text{ratio} ||W||_{21} + \frac{1}{2} \alpha (1 - l_1 \text{ratio}) ||W||_\text{Fro}^2
\]

Where:

\[
||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

**X** : {array-like}, shape (n_samples, n_features)

Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output then X can be sparse.

**y** : ndarray, shape (n_samples,) or (n_samples, n_outputs)

Target values

**l1_ratio** : float, optional
float between 0 and 1 passed to elastic net (scaling between l1 and l2 penalties). l1_ratio=1 corresponds to the Lasso

eps : float
   Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3

n_alphas : int, optional
   Number of alphas along the regularization path

alphas : ndarray, optional
   List of alphas where to compute the models. If None alphas are set automatically

precompute : True | False | ‘auto’ | array-like
   Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto'
   let us decide. The Gram matrix can also be passed as argument.

Xy : array-like, optional
   Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is
   precomputed.

copy_X : boolean, optional, default True
   If True, X will be copied; else, it may be overwritten.

coeff_init : array, shape (n_features, ) | None
   The initial values of the coefficients.

verbose : bool or integer
   Amount of verbosity.

params : kwargs
   keyword arguments passed to the coordinate descent solver.

return_n_iter : bool
   whether to return the number of iterations or not.

positive : bool, default False
   If set to True, forces coefficients to be positive.

check_input : bool, default True
   Skip input validation checks, including the Gram matrix when provided assuming there
   are handled by the caller when check_input=False.

Returns

alphas : array, shape (n_alphas,)
   The alphas along the path where models are computed.

coefs : array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
   Coefficients along the path.

dual_gaps : array, shape (n_alphas,)
   The dual gaps at the end of the optimization for each alpha.

n_iters : array-like, shape (n_alphas,)
   The number of iterations taken by the coordinate descent optimizer to reach the specified
   tolerance for each alpha. (Is returned when return_n_iter is set to True).
Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

**predict** (*X*)
Predict using the linear model

**Parameters**
- *X*: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**
- *C*: array, shape = (n_samples,)
  - Returns predicted values.

**score** (*X*, *y*, *sample_weight=None*)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - \frac{u}{v})$, where $u$ is the regression sum of squares $\sum((y_{true} - y_{pred})^2)$ and $v$ is the residual sum of squares $\sum((y_{true} - y_{true.mean})^2)$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of *y*, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**
- *X*: array-like, shape = (n_samples, n_features)
  - Test samples.

- *y*: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True values for *X*.

- *sample_weight*: array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**
- *score*: float
  - $R^2$ of self.predict(*X*) wrt. *y*.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**
- *self*: 

**sparse_coef_**
spare representation of the fitted coef_

**Examples using** **sklearn.linear_model.Lasso**

- *Compressive sensing: tomography reconstruction with L1 prior (Lasso)*
- *Cross-validation on diabetes Dataset Exercise*
- *Lasso on dense and sparse data*
- *Lasso and Elastic Net for Sparse Signals*
Joint feature selection with multi-task Lasso

29.18.7 sklearn.linear_model.LassoLars

class sklearn.linear_model.LassoLars (alpha=1.0, fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, copy_X=True, fit_path=True, positive=False)

Lasso model fit with Least Angle Regression a.k.a. Lars

It is a Linear Model trained with an L1 prior as regularizer.

The optimization objective for Lasso is:

\[(1 / (2 \times n\_samples)) \times \| y - Xw \|^2_2 + \alpha \times \| w \|_1\]

Read more in the User Guide.

**Parameters**

**alpha** : float

Constant that multiplies the penalty term. Defaults to 1.0. alpha = 0 is equivalent to an ordinary least square, solved by LinearRegression. For numerical reasons, using alpha = 0 with the LassoLars object is not advised and you should prefer the LinearRegression object.

**fit_intercept** : boolean

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**positive** : boolean (default=False)

Restrict coefficients to be >= 0. Be aware that you might want to remove fit_intercept which is set True by default. Under the positive restriction the model coefficients will not converge to the ordinary-least-squares solution for small values of alpha. Only coefficients up to the smallest alpha value (alphas_[alphas_ > 0.].min() when fit_path=True) reached by the stepwise Lars-Lasso algorithm are typically in congruence with the solution of the coordinate descent Lasso estimator.

**verbose** : boolean or integer, optional

Sets the verbosity amount

**normalize** : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**precompute** : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto', let us decide. The Gram matrix can also be passed as argument.

**max_iter** : integer, optional
Maximum number of iterations to perform.

**eps**: float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the `tol` parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

**fit_path**: boolean

If `True` the full path is stored in the `coef_path_` attribute. If you compute the solution for a large problem or many targets, setting `fit_path` to `False` will lead to a speedup, especially with a small alpha.

**Attributes**

- **alphas_**: array, shape (n_alphas + 1) | list of n_targets such arrays
  
  Maximum of covariances (in absolute value) at each iteration. `n_alphas` is either `max_iter`, `n_features`, or the number of nodes in the path with correlation greater than `alpha`, whichever is smaller.

- **active_**: list, length = n_alphas | list of n_targets such lists
  
  Indices of active variables at the end of the path.

- **coef_path_**: array, shape (n_features, n_alphas + 1) or list
  
  If a list is passed it’s expected to be one of n_targets such arrays. The varying values of the coefficients along the path. It is not present if the `fit_path` parameter is `False`.

- **coef_**: array, shape (n_features,) or (n_targets, n_features)
  
  Parameter vector (w in the formulation formula).

- **intercept_**: float | array, shape (n_targets,)
  
  Independent term in decision function.

- **n_iter_**: array-like or int.
  
  The number of iterations taken by `lars_path` to find the grid of alphas for each target.

**See also:**

- `lars_path`, `lasso_path`, `Lasso`, `LassoCV`, `LassoLarsCV`, `sklearn.decomposition.sparse_encode`

**Examples**

```python
>>> from sklearn import linear_model
>>> clf = linear_model.LassoLars(alpha=0.01)
>>> clf.fit([[-1, 1], [0, 0], [1, 1]], [-1, 0, -1])
... LassoLars(alpha=0.01, copy_X=True, eps=..., fit_intercept=True,
  fit_path=True, max_iter=500, normalize=True, positive=False,
  precompute='auto', verbose=False)
>>> print(clf.coef_)
[ 0. -0.963257...]
```

**Methods**
**decision_function**(*args, **kwargs) DEPRECATED: and will be removed in 0.19.

**fit**(X, y[, Xy]) Fit the model using X, y as training data.

**get_params**([deep]) Get parameters for this estimator.

**predict**(X) Predict using the linear model

**score**(X, y[, sample_weight]) Returns the coefficient of determination $R^2$ of the prediction.

**set_params**(**params) Set the parameters of this estimator.

__init__ (alpha=1.0, fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, copy_X=True, fit_path=True, positive=False)

**decision_function** (*args, **kwargs) DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.
- **C**: array, shape = (n_samples,)
  - Returns predicted values.

**fit**(X, y, Xy=None)

Fit the model using X, y as training data.

Parameters:
- **X**: array-like, shape (n_samples, n_features)
  - Training data.
- **y**: array-like, shape (n_samples,) or (n_samples, n_targets)
  - Target values.
- **Xy**: array-like, shape (n_samples,) or (n_samples, n_targets), optional
  - $Xy = np.dot(X.T, y)$ that can be precomputed. It is useful only when the Gram matrix is precomputed.

Returns:
- **self**: object
  - returns an instance of self.

**get_params**(deep=True)

Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params**: mapping of string to any
  - Parameter names mapped to their values.

**predict**(X)

Predict using the linear model

Parameters:
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.
Returns\( C \) : array, shape = (n_samples,)

Returns predicted values.

\[ \text{score}(X, y, \text{sample\_weight}=\text{None}) \]

Returns the coefficient of determination \( R^2 \) of the prediction.

The coefficient \( R^2 \) is defined as \( (1 - \frac{u}{v}) \), where \( u \) is the regression sum of squares \( ((y_{\text{true}} - y_{\text{pred}}) ^2) \) and \( v \) is the residual sum of squares \( ((y_{\text{true}} - y_{\text{true\_mean}}) ^2) \). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \( y \), disregarding the input features, would get a \( R^2 \) score of 0.0.

\[ \text{Parameters} \]

\( X \) : array-like, shape = (n_samples, n_features)

Test samples.

\( y \) : array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for \( X \).

\( \text{sample\_weight} \) : array-like, shape = [n_samples], optional

Sample weights.

\[ \text{Returns} \]

\( \text{score} \) : float

\( R^2 \) of self.predict(X) wrt. \( y \).

\[ \text{set\_params}(**\text{params}) \]

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \( \text{component\_name}__\text{parameter} \) so that it’s possible to update each component of a nested object.

\[ \text{Returns} \]

\( \text{self} \) :

### 29.18.8 sklearn.linear_model.LinearRegression

\[ \text{class} \] sklearn.linear_model.LinearRegression\( (\text{fit\_intercept}=\text{True}, \quad \text{normalize}=\text{False}, \quad \text{copy\_X}=\text{True}, \quad \text{n\_jobs}=1) \)

Ordinary least squares Linear Regression.

\[ \text{Parameters} \]

\( \text{fit\_intercept} \) : boolean, optional

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

\( \text{normalize} \) : boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when \( \text{fit\_intercept} \) is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

\( \text{copy\_X} \) : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

\( \text{n\_jobs} \) : int, optional, default 1
The number of jobs to use for the computation. If -1 all CPUs are used. This will only provide speedup for n_targets > 1 and sufficient large problems.

Attributes

**coef_** : array, shape (n_features,) or (n_targets, n_features)

Estimated coefficients for the linear regression problem. If multiple targets are passed during the fit (y 2D), this is a 2D array of shape (n_targets, n_features), while if only one target is passed, this is a 1D array of length n_features.

**residues_** : array, shape (n_targets,) or (1,) or empty

Sum of residuals. Squared Euclidean 2-norm for each target passed during the fit. If the linear regression problem is under-determined (the number of linearly independent rows of the training matrix is less than its number of linearly independent columns), this is an empty array. If the target vector passed during the fit is 1-dimensional, this is a (1,) shape array.

New in version 0.18.

**intercept_** : array

Independent term in the linear model.

Notes

From the implementation point of view, this is just plain Ordinary Least Squares (scipy.linalg.lstsq) wrapped as a predictor object.

Methods

```
decision_function(*args,**kwargs) DEPRECATED: and will be removed in 0.19.
fit(X, y[, sample_weight]) Fit linear model.
get_params([deep]) Get parameters for this estimator.
predict(X) Predict using the linear model
score(X, y[, sample_weight]) Returns the coefficient of determination R^2 of the prediction.
set_params(**params) Set the parameters of this estimator.

__init__ (fit_intercept=True, normalize=False, copy_X=True, n_jobs=1)
```

decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

**Parameters**

- **X** : {array-like, sparse matrix}, shape = (n_samples, n_features)
  
  Samples.

- **y** : array, shape = (n_samples,)
  
  Returns predicted values.

- **sample_weight**

  Fit linear model.

  **Parameters**

  - **X** : numpy array or sparse matrix of shape [n_samples,n_features]

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<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit linear model.</td>
<td></td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
<td></td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
<td></td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
<td></td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
<td></td>
</tr>
</tbody>
</table>

---

__init__ (fit_intercept=True, normalize=False, copy_X=True, n_jobs=1)
Training data

\( y \) : numpy array of shape [n_samples, n_targets]

Target values

**sample_weight** : numpy array of shape [n_samples]

Individual weights for each sample

New in version 0.17: parameter **sample_weight** support to LinearRegression.

**Returns**

self : returns an instance of self.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

deep : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

params : mapping of string to any

Parameter names mapped to their values.

**predict**(X)

Predict using the linear model

**Parameters**

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

**Returns**

C : array, shape = (n_samples,)

Returns predicted values.

**residues_**

DEPRECATED: residues_ is deprecated and will be removed in 0.19

Get the residues of the fitted model.

**score**(X, y, **sample_weight**=None)

Returns the coefficient of determination \( R^2 \) of the prediction.

The coefficient \( R^2 \) is defined as \( 1 - \frac{u}{v} \), where \( u \) is the regression sum of squares \( \sum(y_{true} - y_{pred})^2 \) and \( v \) is the residual sum of squares \( \sum((y_{true} - \bar{y}_{true})^2) \). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \( y \), disregarding the input features, would get a \( R^2 \) score of 0.0.

**Parameters**

X : array-like, shape = (n_samples, n_features)

Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for X.

**sample_weight** : array-like, shape = [n_samples], optional

Sample weights.

**Returns**

score : float

\( R^2 \) of self.predict(X) wrt. \( y \).
**set_params** (**params**)  
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns** self:

Examples using `sklearn.linear_model.LinearRegression`

- Plotting Cross-Validated Predictions
- Isotonic Regression
- Face completion with a multi-output estimators
- Automatic Relevance Determination Regression (ARD)
- Bayesian Ridge Regression
- Logistic function
- Linear Regression Example
- Sparsity Example: Fitting only features 1 and 2
- Ordinary Least Squares and Ridge Regression Variance
- Robust linear model estimation using RANSAC
- Robust linear estimator fitting
- Theil-Sen Regression
- Underfitting vs. Overfitting

**29.18.9 sklearn.linear_model.LogisticRegression**

```python
class sklearn.linear_model.LogisticRegression (penalty=‘l2’, dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, class_weight=None, random_state=None, solver=‘liblinear’, max_iter=100, multi_class=‘ovr’, verbose=0, warm_start=False, n_jobs=1)
```

Logistic Regression (aka logit, MaxEnt) classifier.

In the multiclass case, the training algorithm uses the one-vs-rest (OvR) scheme if the ‘multi_class’ option is set to ‘ovr’, and uses the cross-entropy loss if the ‘multi_class’ option is set to ‘multinomial’. (Currently the ‘multinomial’ option is supported only by the ‘lbfgs’, ‘sag’ and ‘newton-cg’ solvers.)

This class implements regularized logistic regression using the ‘liblinear’ library, ‘newton-cg’, ‘sag’ and ‘lbfgs’ solvers. It can handle both dense and sparse input. Use C-ordered arrays or CSR matrices containing 64-bit floats for optimal performance; any other input format will be converted (and copied).

The ‘newton-cg’, ‘sag’, and ‘lbfgs’ solvers support only L2 regularization with primal formulation. The ‘liblinear’ solver supports both L1 and L2 regularization, with a dual formulation only for the L2 penalty.

Read more in the User Guide.

**Parameters**

`penalty` : str, ‘l1’ or ‘l2’, default: ‘l2’
Used to specify the norm used in the penalization. The ‘newton-cg’, ‘sag’ and ‘lbfgs’ solvers support only l2 penalties.

**dual** : bool, default: False

Dual or primal formulation. Dual formulation is only implemented for l2 penalty with liblinear solver. Prefer dual=False when n_samples > n_features.

**C** : float, default: 1.0

Inverse of regularization strength; must be a positive float. Like in support vector machines, smaller values specify stronger regularization.

**fit_intercept** : bool, default: True

Specifies if a constant (a.k.a. bias or intercept) should be added to the decision function.

**intercept_scaling** : float, default: 1

Useful only when the solver ‘liblinear’ is used and self.fit_intercept is set to True. In this case, x becomes [x, self.intercept_scaling], i.e. a “synthetic” feature with constant value equal to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic_feature_weight.

Note! the synthetic feature weight is subject to l1/l2 regularization as all other features. To lessen the effect of regularization on synthetic feature weight (and therefore on the intercept) intercept_scaling has to be increased.

**class_weight** : dict or ‘balanced’, default: None

Weights associated with classes in the form {class_label: weight}. If not given, all classes are supposed to have weight one.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as n_samples / (n_classes * np.bincount(y)).

Note that these weights will be multiplied with sample_weight (passed through the fit method) if sample_weight is specified.

New in version 0.17: class_weight=’balanced’ instead of deprecated class_weight=’auto’.

**max_iter** : int, default: 100

Useful only for the newton-cg, sag and lbfgs solvers. Maximum number of iterations taken for the solvers to converge.

**random_state** : int seed, RandomState instance, default: None

The seed of the pseudo random number generator to use when shuffling the data. Used only in solvers ‘sag’ and ‘liblinear’.


Algorithm to use in the optimization problem.

•For small datasets, ‘liblinear’ is a good choice, whereas ‘sag’ is faster for large ones.

•For multiclass problems, only ‘newton-cg’, ‘sag’ and ‘lbfgs’ handle multinomial loss; ‘liblinear’ is limited to one-versus-rest schemes.

•‘newton-cg’, ‘lbfgs’ and ‘sag’ only handle L2 penalty.
Note that ‘sag’ fast convergence is only guaranteed on features with approximately the same scale. You can preprocess the data with a scaler from sklearn.preprocessing.

New in version 0.17: Stochastic Average Gradient descent solver.

tol : float, default: 1e-4

Tolerance for stopping criteria.

multi_class : str, {'ovr', 'multinomial'}, default: ‘ovr’

Multiclass option can be either ‘ovr’ or ‘multinomial’. If the option chosen is ‘ovr’, then a binary problem is fit for each label. Else the loss minimised is the multinomial loss fit across the entire probability distribution. Works only for the ‘newton-cg’, ‘sag’ and ‘lbfgs’ solver.

New in version 0.18: Stochastic Average Gradient descent solver for ‘multinomial’ case.

verbose : int, default: 0

For the liblinear and lbfgs solvers set verbose to any positive number for verbosity.

warm_start : bool, default: False

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution. Useless for liblinear solver.

New in version 0.17: warm_start to support lbfgs, newton-cg, sag solvers.

n_jobs : int, default: 1

Number of CPU cores used during the cross-validation loop. If given a value of -1, all cores are used.

Attributes

coef_ : array, shape (n_classes, n_features)

Coefficient of the features in the decision function.

intercept_ : array, shape (n_classes,)

Intercept (a.k.a. bias) added to the decision function. If fit_intercept is set to False, the intercept is set to zero.

n_iter_ : array, shape (n_classes,) or (1,)

Actual number of iterations for all classes. If binary or multinomial, it returns only 1 element. For liblinear solver, only the maximum number of iteration across all classes is given.

See also:

SGDClassifier incrementally trained logistic regression (when given the parameter loss="log").

sklearn.svm.LinearSVC learns SVM models using the same algorithm.

Notes

The underlying C implementation uses a random number generator to select features when fitting the model. It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a smaller tol parameter.

Predict output may not match that of standalone liblinear in certain cases. See differences from liblinear in the narrative documentation.
References

LIBLINEAR – A Library for Large Linear Classification [1]
http://www.csie.ntu.edu.tw/~cjlin/liblinear/

SAG – Mark Schmidt, Nicolas Le Roux, and Francis Bach
Minimizing Finite Sums with the Stochastic Average Gradient [2]
https://hal.inria.fr/hal-00860051/document


Methods

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`__init__(penalty='l2', dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, class_weight=None, random_state=None, solver='liblinear', max_iter=100, multi_class='ovr', verbose=0, warm_start=False, n_jobs=1)`

decision_function (X)

Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

Parameters

- **X**: array-like, sparse matrix, shape = (n_samples, n_features)

  Samples.

- **Returns**: array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes)

  Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

densify()

Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

Returns: estimator:

fit (X, y, sample_weight=None)

Fit the model according to the given training data.
Parameters

X : {array-like, sparse matrix}, shape (n_samples, n_features)
   Training vector, where n_samples is the number of samples and n_features is the number
   of features.

y : array-like, shape (n_samples,)
   Target vector relative to X.

sample_weight : array-like, shape (n_samples,) optional
   Array of weights that are assigned to individual samples. If not provided, then each
   sample is given unit weight.

   New in version 0.17: sample_weight support to LogisticRegression.

Returns

self : object
   Returns self.

fit_transform (X, y=None, **fit_params)
   Fit to data, then transform it.

   Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

   Parameters

   X : numpy array of shape [n_samples, n_features]
      Training set.

   y : numpy array of shape [n_samples]
      Target values.

   Returns

   X_new : numpy array of shape [n_samples, n_features_new]
      Transformed array.

get_params (deep=True)
   Get parameters for this estimator.

   Parameters

   deep : boolean, optional
      If True, will return the parameters for this estimator and contained subobjects that are
      estimators.

   Returns

   params : mapping of string to any
      Parameter names mapped to their values.

predict (X)
   Predict class labels for samples in X.

   Parameters

   X : {array-like, sparse matrix}, shape = [n_samples, n_features]
      Samples.

   Returns

   C : array, shape = [n_samples]
      Predicted class label per sample.

predict_log_proba (X)
   Log of probability estimates.

   The returned estimates for all classes are ordered by the label of classes.

   Parameters

   X : array-like, shape = [n_samples, n_features]

   Returns

   T : array-like, shape = [n_samples, n_classes]
Returns the log-probability of the sample for each class in the model, where classes are ordered as they are in self.classes_.

**predict_proba** (*X*)

Probability estimates.

The returned estimates for all classes are ordered by the label of classes.

For a multi_class problem, if multi_class is set to be "multinomial" the softmax function is used to find the predicted probability of each class. Else use a one-vs-rest approach, i.e calculate the probability of each class assuming it to be positive using the logistic function. and normalize these values across all the classes.

**Parameters**

- *X*: array-like, shape = [n_samples, n_features]

**Returns**

- *T*: array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered as they are in self.classes_.

**score** (*X, y, sample_weight=None*)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- *X*: array-like, shape = (n_samples, n_features)
- *y*: array-like, shape = (n_samples) or (n_samples, n_outputs)

**Returns**

- *score*: float

Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)  

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

self: estimator

**sparsify** ()

Convert coefficient matrix to sparse format.

Converts the coef_ member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The intercept_ member is not converted.

**Returns**

self: estimator
Notes

For non-sparse models, i.e. when there are not many zeros in `coef_`, this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with `(coef_ == 0).sum()`, must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the `partial_fit` method (if any) will not work until you call `densify`.

`transform(*args, **kwargs)`

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use `SelectFromModel` instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

**Parameters**

- `X` : array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.

- `threshold` [string, float or None, optional (default=None)]
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**

- `X_r` : array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

Examples using `sklearn.linear_model.LogisticRegression`

- Pipelining: chaining a PCA and a logistic regression
- Probability Calibration curves
- Comparison of Calibration of Classifiers
- Plot classification probability
- Feature transformations with ensembles of trees
- Plot class probabilities calculated by the VotingClassifier
- Digits Classification Exercise
- Logistic Regression 3-class Classifier
- Logistic function
- L1 Penalty and Sparsity in Logistic Regression
- Plot multinomial and One-vs-Rest Logistic Regression
- Path with L1- Logistic Regression
- Comparing various online solvers
- Restricted Boltzmann Machine features for digit classification
29.18.10 sklearn.linear_model.MultiTaskLasso

```
class sklearn.linear_model.MultiTaskLasso(alpha=1.0, fit_intercept=True, normalize=False, copy_X=True, max_iter=1000, tol=0.0001, warm_start=False, random_state=None, selection='cyclic')
```

Multi-task Lasso model trained with L1/L2 mixed-norm as regularizer

The optimization objective for Lasso is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||Y - XW||_2^\text{Fro} + \alpha \times ||W||_{21}
\]

Where:

\[
||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

- **alpha** : float, optional
  
  Constant that multiplies the L1/L2 term. Defaults to 1.0

- **fit_intercept** : boolean
  
  Whether to calculate the intercept for this model. If set to False, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **normalize** : boolean, optional, default False
  
  If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

- **copy_X** : boolean, optional, default True
  
  If True, X will be copied; else, it may be overwritten.

- **max_iter** : int, optional
  
  The maximum number of iterations

- **tol** : float, optional
  
  The tolerance for the optimization: if the updates are smaller than tol, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

- **warm_start** : bool, optional
  
  When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

- **selection** : str, default ‘cyclic’
  
  If set to ‘random’, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to ‘random’) often leads to significantly faster convergence especially when tol is higher than 1e-4

- **random_state** : int, RandomState instance, or None (default)
The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to ‘random’.

**Attributes**

- **coef_**: array, shape (n_tasks, n_features)
  - parameter vector (W in the cost function formula)
- **intercept_**: array, shape (n_tasks,)
  - independent term in decision function.
- **n_iter_**: int
  - number of iterations run by the coordinate descent solver to reach the specified tolerance.

**See also:**

*Lasso*, *MultiTaskElasticNet*

**Notes**

The algorithm used to fit the model is coordinate descent.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

**Examples**

```python
>>> from sklearn import linear_model
>>> clf = linear_model.MultiTaskLasso(alpha=0.1)
>>> clf.fit([[0,0], [1, 1], [2, 2]], [[0, 0], [1, 1], [2, 2]])
MultiTaskLasso(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000,
               normalize=False, random_state=None, selection='cyclic', tol=0.0001,
               warm_start=False)
>>> print(clf.coef_)
[[ 0.89393398  0.        ]
 [ 0.89393398  0.        ]
>>> print(clf.intercept_)
[ 0.10606602  0.10606602]
```

**Methods**

- **decision_function**(**\*args**, **\*kwargs**) DEPRECATED: and will be removed in 0.19
  - Fit MultiTaskLasso model with coordinate descent
- **get_params**(**deep**) Get parameters for this estimator.
- **path**(**X**, **y**, **l1_ratio**, **eps**, **n_alphas**, **...)** Compute elastic net path with coordinate descent
- **predict**(**X**) Predict using the linear model
- **score**(**X**, **y**, **sample_weight**) Returns the coefficient of determination $R^2$ of the prediction.
- **set_params**(**\*\*params**) Set the parameters of this estimator.

```python
>>> clf = linear_model.MultiTaskLasso(alpha=1.0, fit_intercept=True, normalize=False, copy_X=True, max_iter=1000, tol=0.0001, warm_start=False, random_state=None, selection='cyclic')
```
**decision_function** (*args, **kwargs)

DEPRECATED: and will be removed in 0.19

Decision function of the linear model

**Parameters**

- **X**: numpy array or scipy.sparse matrix of shape (n_samples, n_features)
- **Returns**
  - **T**: array, shape (n_samples,)
    The predicted decision function

**fit** (*X, y*)

Fit MultiTaskLasso model with coordinate descent

**Parameters**

- **X**: ndarray, shape (n_samples, n_features)
  Data
- **y**: ndarray, shape (n_samples, n_tasks)
  Target

**Notes**

Coordinate descent is an algorithm that considers each column of data at a time hence it will automatically convert the X input as a Fortran-contiguous numpy array if necessary.

To avoid memory re-allocation it is advised to allocate the initial data in memory directly using that format.

**get_params** (*deep=True*)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

**path** (*X, y, l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, check_input=True, **params*)

Compute elastic net path with coordinate descent

The elastic net optimization function varies for mono and multi-outputs.

For mono-output tasks it is:

\[
\frac{1}{2 \times n_{\text{samples}}} \times ||y - Xw||^2_2 \\
+ \alpha \times l1_{\text{ratio}} \times ||w||_1 \\
+ 0.5 \times \alpha \times (1 - l1_{\text{ratio}}) \times ||w||^2_2
\]

For multi-output tasks it is:

\[
(\frac{1}{2 \times n_{\text{samples}}}) \times ||Y - XW||_{\text{Fro}}^2 \\
+ \alpha \times l1_{\text{ratio}} \times ||W||_{21} \\
+ 0.5 \times \alpha \times (1 - l1_{\text{ratio}}) \times ||W||_{\text{Fro}}^2
\]

Where:
\[ ||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2} \]

i.e. the sum of norm of each row.

Read more in the User Guide.

Parameters:
- **X**: {array-like}, shape (n_samples, n_features)
  Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If y is mono-output then X can be sparse.
- **y**: ndarray, shape (n_samples,) or (n_samples, n_outputs)
  Target values
- **l1_ratio**: float, optional
  float between 0 and 1 passed to elastic net (scaling between l1 and l2 penalties). l1_ratio=1 corresponds to the Lasso
- **eps**: float
  Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3
- **n_alphas**: int, optional
  Number of alphas along the regularization path
- **alphas**: ndarray, optional
  List of alphas where to compute the models. If None alphas are set automatically
- **precompute**: True | False | 'auto' | array-like
  Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto' let us decide. The Gram matrix can also be passed as argument.
- **Xy**: array-like, optional
  Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.
- **copy_X**: boolean, optional, default True
  If True, X will be copied; else, it may be overwritten.
- **coef_init**: array, shape (n_features,) | None
  The initial values of the coefficients.
- **verbose**: bool or integer
  Amount of verbosity.
- **params**: kwargs
  keyword arguments passed to the coordinate descent solver.
- **return_n_iter**: bool
  whether to return the number of iterations or not.
- **positive**: bool, default False
  If set to True, forces coefficients to be positive.
- **check_input**: bool, default True
Skip input validation checks, including the Gram matrix when provided assuming there are handled by the caller when check_input=False.

**Returns**
- **alphas**: array, shape (n_alphas,)
The alphas along the path where models are computed.
- **coeffs**: array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
  Coefficients along the path.
- **dual_gaps**: array, shape (n_alphas,)
The dual gaps at the end of the optimization for each alpha.
- **n_iters**: array-like, shape (n_alphas,)
The number of iterations taken by the coordinate descent optimizer to reach the specified tolerance for each alpha. (Is returned when return_n_iter is set to True).

See also:

**Notes**
See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

**predict** *(X)*
Predict using the linear model

**Parameters**
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

**Returns**
- **C**: array, shape = (n_samples,)
  Returns predicted values.

**score** *(X, y, sample_weight=None)*
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $\sum((y_{true} - y_{pred})**2)$ and $v$ is the residual sum of squares $\sum((y_{true} - y_{true.mean})**2)$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**
- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for $X$.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**
- **score**: float
  $R^2$ of self.predict(X) wrt. $y$. 

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

self :

**sparse_coef_**

sparse representation of the fitted coef_

Examples using `sklearn.linear_model.MultiTaskLasso`

• Joint feature selection with multi-task Lasso

29.18.11 sklearn.linear_model.MultiTaskElasticNet

class sklearn.linear_model.MultiTaskElasticNet (alpha=1.0, l1_ratio=0.5, 
fit_intercept=True, normalize=False, 
copy_X=True, max_iter=1000, tol=0.0001, 
warm_start=False, random_state=None, 
selection='cyclic')

Multi-task ElasticNet model trained with L1/L2 mixed-norm as regularizer

The optimization objective for MultiTaskElasticNet is:

\[
(1 / (2 * n_samples)) * ||Y - XW||^2_{Fro} + alpha * l1_ratio * ||W||^1_2 + 0.5 * alpha * (1 - l1_ratio) * ||W||^2_{Fro}
\]

Where:

\[
||W||^1_2 = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

- **alpha** : float, optional
  
  Constant that multiplies the L1/L2 term. Defaults to 1.0

- **l1_ratio** : float

  The ElasticNet mixing parameter, with 0 < l1_ratio <= 1. For l1_ratio = 0 the penalty is an L1/L2 penalty. For l1_ratio = 1 it is an L1 penalty. For 0 < l1_ratio < 1, the penalty is a combination of L1/L2 and L2.

- **fit_intercept** : boolean

  whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **normalize** : boolean, optional, default False

  If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However,
if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

copy_X : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

max_iter : int, optional

The maximum number of iterations

tol : float, optional

The tolerance for the optimization: if the updates are smaller than `tol`, the optimization code checks the dual gap for optimality and continues until it is smaller than `tol`.

warm_start : bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

selection : str, default ‘cyclic’

If set to ‘random’, a random coefficient is updated every iteration rather than looping over features sequentially by default. This (setting to ‘random’) often leads to significantly faster convergence especially when `tol` is higher than 1e-4.

random_state : int, RandomState instance, or None (default)

The seed of the pseudo random number generator that selects a random feature to update. Useful only when selection is set to ‘random’.

Attributes

intercept_ : array, shape (n_tasks,)

Independent term in decision function.

coeff_ : array, shape (n_tasks, n_features)

Parameter vector (W in the cost function formula). If a 1D y is passed in at fit (non multi-task usage), coef_ is then a 1D array

n_iter_ : int

number of iterations run by the coordinate descent solver to reach the specified tolerance.

See also:

`ElasticNet`, `MultiTaskLasso`

Notes

The algorithm used to fit the model is coordinate descent.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

Examples
```python
>>> from sklearn import linear_model
>>> clf = linear_model.MultiTaskElasticNet(alpha=0.1)
>>> clf.fit([[0,0], [1, 1], [2, 2]], [[0, 0], [1, 1], [2, 2]])
... MultiTaskElasticNet(alpha=0.1, copy_X=True, fit_intercept=True,
                   l1_ratio=0.5, max_iter=1000, normalize=False,
                   random_state=None, selection='cyclic', tol=0.0001,
                   warm_start=False)
>>> print(clf.coef_)
[[ 0.45663524  0.45612256]
 [ 0.45663524  0.45612256]]
>>> print(clf.intercept_)
[ 0.0872422  0.0872422]
```

## Methods

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<th>Method Name</th>
<th>Description</th>
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<tr>
<td><code>decision_function(*args, **kwargs)</code></td>
<td>DEPRECATED: and will be removed in 0.19&lt;br&gt;Decision function of the linear model</td>
</tr>
<tr>
<td><code>fit(X, y)</code></td>
<td>Fit MultiTaskLasso model with coordinate descent</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>path(X, y[, l1_ratio, eps, n_alphas, ...])</code></td>
<td>Compute elastic net path with coordinate descent</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

### `__init__`(alpha=1.0, l1_ratio=0.5, fit_intercept=True, normalize=False, copy_X=True, max_iter=1000, tol=0.0001, warm_start=False, random_state=None, selection='cyclic')

#### `decision_function(*args, **kwargs)`

DEPRECATED: and will be removed in 0.19

Decision function of the linear model

**Parameters**
- **X**: numpy array or scipy.sparse matrix of shape (n_samples, n_features)

**Returns**
- **T**: array, shape (n_samples,)

  The predicted decision function

#### `fit(X, y)`

Fit MultiTaskLasso model with coordinate descent

**Parameters**
- **X**: ndarray, shape (n_samples, n_features)
  - Data
- **y**: ndarray, shape (n_samples, n_tasks)
  - Target

### Notes

Coordinate descent is an algorithm that considers each column of data at a time hence it will automatically convert the X input as a Fortran-contiguous numpy array if necessary.

To avoid memory re-allocation it is advised to allocate the initial data in memory directly using that format.
get_params (deep=True)
Get parameters for this estimator.

Parameters

*deep* : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns

*params* : mapping of string to any
Parameter names mapped to their values.

path (X, y, l1_ratio=0.5, eps=0.001, n_alphas=100, alphas=\(\text{None}\), precompute='auto', Xy=\(\text{None}\),
copy_X=True, coef_init=\(\text{None}\), verbose=False, return_n_iter=False, positive=False,
check_input=True, **params)
Compute elastic net path with coordinate descent
The elastic net optimization function varies for mono and multi-outputs.
For mono-output tasks it is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times \| y - Xw \|^2_2 \quad + \quad \alpha \times l1_{\text{ratio}} \times \| w \|_1 \\
+ \quad 0.5 \times \alpha \times (1 - l1_{\text{ratio}}) \times \| w \|^2_2
\]

For multi-output tasks it is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times \| Y - XW \|^\text{Fro}_2 \\
+ \quad \alpha \times l1_{\text{ratio}} \times \| W \|_{21} \\
+ \quad 0.5 \times \alpha \times (1 - l1_{\text{ratio}}) \times \| W \|^\text{Fro}_2
\]

Where:

\[ \| W \|_{21} = \sum_i \sqrt{\sum_j w_{ij}^2} \]
i.e. the sum of norm of each row.
Read more in the User Guide.

Parameters

*X* : {array-like}, shape (n_samples, n_features)
Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory
duplication. If \(y\) is mono-output then \(X\) can be sparse.

*\(y\)* : ndarray, shape (n_samples,) or (n_samples, n_outputs)
Target values

*l1\_ratio* : float, optional
float between 0 and 1 passed to elastic net (scaling between l1 and l2 penalties).
\(l1_{\text{ratio}}=1\) corresponds to the Lasso

*\(\varepsilon\)* : float
Length of the path. \(\varepsilon=1e-3\) means that \(\alpha_{\text{min}} / \alpha_{\text{max}} = 1e-3\)

*n\_alphas* : int, optional
Number of alphas along the regularization path

*alphas* : ndarray, optional
List of alphas where to compute the models. If None alphas are set automatically

*precompute* : True | False | ‘auto’ | array-like
Whether to use a precomputed Gram matrix to speed up calculations. If set to 'auto', let us decide. The Gram matrix can also be passed as argument.

**Xy**: array-like, optional

Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.

**copy_X**: boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**coef_init**: array, shape (n_features,) | None

The initial values of the coefficients.

**verbose**: bool or integer

Amount of verbosity.

**params**: kwargs

Keyword arguments passed to the coordinate descent solver.

**return_n_iter**: bool

Whether to return the number of iterations or not.

**positive**: bool, default False

If set to True, forces coefficients to be positive.

**check_input**: bool, default True

Skip input validation checks, including the Gram matrix when provided assuming there are handled by the caller when check_input=False.

**Returns**

alpha**: array, shape (n_alphas,)

The alphas along the path where models are computed.

**coefs**: array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)

Coefficients along the path.

**dual_gaps**: array, shape (n_alphas,)

The dual gaps at the end of the optimization for each alpha.

**n_iters**: array-like, shape (n_alphas,)

The number of iterations taken by the coordinate descent optimizer to reach the specified tolerance for each alpha. (Is returned when return_n_iter is set to True).

See also:

MultiTaskElasticNet, MultiTaskElasticNetCV, ElasticNet, ElasticNetCV

**Notes**

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

**predict (X)**

Predict using the linear model

**Parameters**

**X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
Samples.

**Returns**

- **C**: array, shape = (n_samples,)
  
  Returns predicted values.

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true.mean()})^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  
  Test samples.

- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  
  True values for X.

- **sample_weight**: array-like, shape = [n_samples], optional
  
  Sample weights.

**Returns**

- **score**: float
  
  $R^2$ of self.predict(X) wrt. $y$.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**: self

**sparse_coef_**

Sparse representation of the fitted `coef_`

---

### 29.18.12 sklearn.linear_model.OrthogonalMatchingPursuit

**class** `sklearn.linear_model.OrthogonalMatchingPursuit` *(n_nonzero_coefs=None, tol=None, fit_intercept=True, normalize=True, precompute='auto')*

Orthogonal Matching Pursuit model (OMP)

**Parameters**

- **n_nonzero_coefs**: int, optional
  
  Desired number of non-zero entries in the solution. If None (by default) this value is set to 10% of $n_{features}$.

- **tol**: float, optional
  
  Maximum norm of the residual. If not None, overrides $n_{nonzero_{coefs}}$.

- **fit_intercept**: boolean, optional
  
  Whether to calculate the intercept for this model. If set to False, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **normalize**: boolean, optional, default False

---

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If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

precompute: {True, False, ‘auto’}, default ‘auto’

Whether to use a precomputed Gram and Xy matrix to speed up calculations. Improves performance when n_targets or n_samples is very large. Note that if you already have such matrices, you can pass them directly to the fit method.

Read more in the \ref{User Guide <omp>}: :

Attributes

- coef_: array, shape (n_features,) or (n_features, n_targets)
  parameter vector (w in the formula)

- intercept_: float or array, shape (n_targets,)
  independent term in decision function.

- n_iter_: int or array-like
  Number of active features across every target.

See also:

- orthogonal_mp
- orthogonal_mp_gram
- lars_path
- Lars
- LassoLars
- decomposition.sparse_encode

Notes


Methods

- decision_function(*args, **kwargs)
  DEPRECATED: and will be removed in 0.19.

- fit(X, y)
  Fit the model using X, y as training data.

- get_params([deep])
  Get parameters for this estimator.

- predict(X)
  Predict using the linear model

- score(X, y[, sample_weight])
  Returns the coefficient of determination R^2 of the prediction.

- set_params(**params)
  Set the parameters of this estimator.

- __init__(n_nonzero_coefs=None, tol=None, fit_intercept=True, normalize=True, precompute='auto')

- decision_function(*args, **kwargs)
  DEPRECATED: and will be removed in 0.19.
Decision function of the linear model.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**

- **C**: array, shape = (n_samples,)
  - Returns predicted values.

**fit** *(X, y)*

Fit the model using X, y as training data.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  - Training data.
- **y**: array-like, shape (n_samples,) or (n_samples, n_targets)
  - Target values.

**Returns**

- **self**: object
  - returns an instance of self.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**predict** *(X)*

Predict using the linear model

**Parameters**

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**

- **C**: array, shape = (n_samples,)
  - Returns predicted values.

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y\_true - y\_pred)^2).sum()$ and $v$ is the residual sum of squares $((y\_true - y\_true.mean())^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  - Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**

- **score**: float
R^2 of self.predict(X) wrt. y.

**set_params (**)params**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns: self

Examples using `sklearn.linear_model.OrthogonalMatchingPursuit`

- Orthogonal Matching Pursuit

### 29.18.13 sklearn.linear_model.PassiveAggressiveClassifier

**class sklearn.linear_model.PassiveAggressiveClassifier**

```python
C=1.0, fit_intercept=True,
n_iter=5, shuffle=True,
verbose=0, loss='hinge',
n_jobs=1, random_state=None,
warm_start=False,
class_weight=None)
```

Passive Aggressive Classifier

Read more in the User Guide.

**Parameters**

- **C** : float
  - Maximum step size (regularization). Defaults to 1.0.

- **fit_intercept** : bool, default=False
  - Whether the intercept should be estimated or not. If False, the data is assumed to be already centered.

- **n_iter** : int, optional
  - The number of passes over the training data (aka epochs). Defaults to 5.

- **shuffle** : bool, default=True
  - Whether or not the training data should be shuffled after each epoch.

- **random_state** : int seed, RandomState instance, or None (default)
  - The seed of the pseudo random number generator to use when shuffling the data.

- **verbose** : integer, optional
  - The verbosity level

- **n_jobs** : integer, optional
  - The number of CPUs to use to do the OVA (One Versus All, for multi-class problems) computation. -1 means ‘all CPUs’. Defaults to 1.

- **loss** : string, optional
  - The loss function to be used: hinge: equivalent to PA-I in the reference paper. squared_hinge: equivalent to PA-II in the reference paper.

- **warm_start** : bool, optional
When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

**class_weight**: dict, {class_label: weight} or “balanced” or None, optional

Preset for the class_weight fit parameter.

Weights associated with classes. If not given, all classes are supposed to have weight one.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as $n_{samples} / (n_{classes} * np.bincount(y))$

New in version 0.17: parameter class_weight to automatically weight samples.

**Attributes**

- **coef_**: array, shape = [1, n_features] if n_classes == 2 else [n_classes, n_features]
  
  Weights assigned to the features.

- **intercept_**: array, shape = [1] if n_classes == 2 else [n_classes]
  
  Constants in decision function.

**See also:**

`SGDClassifier`, `Perceptron`

**References**


**Methods**

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**__init__** (C=1.0, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, loss='hinge', n_jobs=1, random_state=None, warm_start=False, class_weight=None)

**decision_function** *(X)*

Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

**Parameters**

- **X** : array-like, sparse matrix, shape = (n_samples, n_features)
  
  Samples.

**Returns**

array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes) :
Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

**densify()**
Convert coefficient matrix to dense array format.

Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

**Returns**: estimator

**fit(X, y, coef_init=None, intercept_init=None)**
Fit linear model with Passive Aggressive algorithm.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  Training data
- **y**: numpy array of shape [n_samples]
  Target values
- **coef_init**: array, shape = [n_classes, n_features]
  The initial coefficients to warm-start the optimization.
- **intercept_init**: array, shape = [n_classes]
  The initial intercept to warm-start the optimization.

**Returns**: returns an instance of self.

**get_params(deep=True)**
Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**: params
  Parameter names mapped to their values.

**partial_fit(X, y, classes=None)**
Fit linear model with Passive Aggressive algorithm.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  Subset of the training data
- **y**: numpy array of shape [n_samples]
  Subset of the target values
- **classes**: array, shape = [n_classes]
  Classes across all calls to partial_fit. Can be obtained by via np.unique(y_all), where y_all is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in classes.

**Returns**: returns an instance of self.

**predict(X)**
Predict class labels for samples in X.
**Parameters**

\( X \) : {array-like, sparse matrix}, shape = [n_samples, n_features]

Samples.

**Returns**

\( C \) : array, shape = [n_samples]

Predicted class label per sample.

**score** \((X, y, \text{sample_weight=None})\)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

\( X \) : array-like, shape = (n_samples, n_features)

Test samples.

\( y \) : array-like, shape = (n_samples) or (n_samples, n_outputs)

True labels for X.

**sample_weight** : array-like, shape = [n_samples], optional

Sample weights.

**Returnsscore** : float

Mean accuracy of self.predict(X) wrt. y.

**sparsify**

Convert coefficient matrix to sparse format.

Converts the \( \text{coef} \_ \) member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The \( \text{intercept} \_ \) member is not converted.

**Returnsself** : estimator

**Notes**

For non-sparse models, i.e. when there are not many zeros in \( \text{coef} \_ \), this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with \((\text{coef} \_ == 0).\text{sum}()\), must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

**Examples using** `sklearn.linear_model.PassiveAggressiveClassifier`

- Out-of-core classification of text documents
- Comparing various online solvers
- Classification of text documents using sparse features
Passive Aggressive Regressor

Read more in the User Guide.

Parameters

- **C**: float
  - Maximum step size (regularization). Defaults to 1.0.
- **epsilon**: float
  - If the difference between the current prediction and the correct label is below this threshold, the model is not updated.
- **fit_intercept**: bool
  - Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.
- **n_iter**: int, optional
  - The number of passes over the training data (aka epochs). Defaults to 5.
- **shuffle**: bool, default=True
  - Whether or not the training data should be shuffled after each epoch.
- **random_state**: int seed, RandomState instance, or None (default)
  - The seed of the pseudo random number generator to use when shuffling the data.
- **verbose**: integer, optional
  - The verbosity level
- **loss**: string, optional
  - The loss function to be used: epsilon_insensitive: equivalent to PA-I in the reference paper. squared_epsilon_insensitive: equivalent to PA-II in the reference paper.
- **warm_start**: bool, optional
  - When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

Attributes

- **coef_**: array, shape = [1, n_features] if n_classes == 2 else [n_classes, n_features]
  - Weights assigned to the features.
- **intercept_**: array, shape = [1] if n_classes == 2 else [n_classes]
  - Constants in decision function.

See also:

- **SGDRegressor**
References


Methods

decision_function(*args, **kwargs) DEPRECATED: and will be removed in 0.19.
densify() Convert coefficient matrix to dense array format.
fit(X, y[, coef_init, intercept_init]) Fit linear model with Passive Aggressive algorithm.
get_params([deep]) Get parameters for this estimator.
partial_fit(X, y) Fit linear model with Passive Aggressive algorithm.
predict(X) Predict using the linear model
score(X, y[, sample_weight]) Returns the coefficient of determination $R^2$ of the prediction.
set_params(*args, **kwargs)
sparseify() Convert coefficient matrix to sparse format.

__init__(C=1.0, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, loss='epsilon_insensitive', epsilon=0.1, random_state=None, warm_start=False)
decision_function(*args, **kwargs) DEPRECATED: and will be removed in 0.19.
Predict using the linear model

Parameters:

- X : {array-like, sparse matrix}, shape (n_samples, n_features)

Returns:

- array, shape (n_samples,) Predicted target values per element in X.

densify()
Convert coefficient matrix to dense array format.

Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

Returns:

self: estimator

fit(X, y[, coef_init=None, intercept_init=None]) Fit linear model with Passive Aggressive algorithm.

Parameters:

- X : {array-like, sparse matrix}, shape = [n_samples, n_features]

  Training data

- y : numpy array of shape [n_samples]

  Target values

- coef_init : array, shape = [n_features]

  The initial coefficients to warm-start the optimization.

- intercept_init : array, shape = [1]

  The initial intercept to warm-start the optimization.
Returns `self`: returns an instance of `self`.

`get_params` *(deep=True)*
Get parameters for this estimator.

**Parameters**
- `deep` : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- `params` : mapping of string to any
  Parameter names mapped to their values.

`partial_fit` *(X, y)*
Fit linear model with Passive Aggressive algorithm.

**Parameters**
- `X` : {array-like, sparse matrix}, shape = [n_samples, n_features]
  Subset of training data
- `y` : numpy array of shape [n_samples]
  Subset of target values

**Returns**
- `self`: returns an instance of `self`.

`predict` *(X)*
Predict using the linear model

**Parameters**
- `X` : {array-like, sparse matrix}, shape (n_samples, n_features)

**Returns**
- `array`, shape (n_samples,)
  Predicted target values per element in X.

`score` *(X, y, sample_weight=None)*
Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

**Parameters**
- `X` : array-like, shape = (n_samples, n_features)
  Test samples.
- `y` : array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- `sample_weight` : array-like, shape = [n_samples], optional
  Sample weights.

**Returns**
- `score` : float
  R^2 of self.predict(X) wrt. y.

`sparseify` ()
Convert coefficient matrix to sparse format.

Converts the `coef_` member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The `intercept_` member is not converted.
Returns self: estimator

Notes

For non-sparse models, i.e. when there are not many zeros in coef_, this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with (coef_ == 0).sum(), must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

29.18.15 sklearn.linear_model.Perceptron

class sklearn.linear_model.Perceptron(penalty=None, alpha=0.0001, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, eta0=1.0, n_jobs=1, random_state=0, class_weight=None, warm_start=False)

Read more in the User Guide.

Parameters

penalty : None, ‘l2’ or ‘l1’ or ‘elasticnet’

The penalty (aka regularization term) to be used. Defaults to None.

alpha : float

Constant that multiplies the regularization term if regularization is used. Defaults to 0.0001

fit_intercept : bool

Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.

n_iter : int, optional

The number of passes over the training data (aka epochs). Defaults to 5.

shuffle : bool, optional, default True

Whether or not the training data should be shuffled after each epoch.

random_state : int seed, RandomState instance, or None (default)

The seed of the pseudo random number generator to use when shuffling the data.

verbose : integer, optional

The verbosity level

n_jobs : integer, optional

The number of CPUs to use to do the OVA (One Versus All, for multi-class problems) computation. -1 means ‘all CPUs’. Defaults to 1.

eta0 : double

Constant by which the updates are multiplied. Defaults to 1.

class_weight : dict, {class_label: weight} or “balanced” or None, optional
Preset for the class_weight fit parameter.

Weights associated with classes. If not given, all classes are supposed to have weight one.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as $n_{samples} / (n_{classes} \times np.bincount(y))$

**warm_start** : bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

Attributes

**coef_** : array, shape = [1, n_features] if n_classes == 2 else [n_classes, n_features]

Weights assigned to the features.

**intercept_** : array, shape = [1] if n_classes == 2 else [n_classes]

Constants in decision function.

See also:

*SGDClassifier*

Notes

Perceptron and SGDClassifier share the same underlying implementation. In fact, Perceptron() is equivalent to SGDClassifier(loss="perceptron", eta0=1, learning_rate="constant", penalty=None).

References


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<tr>
<td>decision_function(X)</td>
<td>Predict confidence scores for samples.</td>
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<td>predict(X)</td>
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<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
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__init__ (penalty=None, alpha=0.0001, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, eta0=1.0, n_jobs=1, random_state=0, class_weight=None, warm_start=False)

decision_function(X)
Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**

- array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes):
  - Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

**densify()**

Convert coefficient matrix to dense array format.

Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

**Return**

- self: estimator

**fit** *(X, y, coef_init=None, intercept_init=None, sample_weight=None)*

Fit linear model with Stochastic Gradient Descent.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training data

- **y**: numpy array, shape (n_samples,)
  - Target values

- **coef_init**: array, shape (n_classes, n_features)
  - The initial coefficients to warm-start the optimization.

- **intercept_init**: array, shape (n_classes,)
  - The initial intercept to warm-start the optimization.

- **sample_weight**: array-like, shape (n_samples,), optional
  - Weights applied to individual samples. If not provided, uniform weights are assumed. These weights will be multiplied with class_weight (passed through the constructor) if class_weight is specified

**Return**

- self: returns an instance of self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.

- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.
Parameters:

**deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

**params**: mapping of string to any

Parameter names mapped to their values.

**partial_fit** *(X, y, classes=\text{None}, sample_weight=\text{None})*

Fit linear model with Stochastic Gradient Descent.

**Parameters**

**X**: {array-like, sparse matrix}, shape (n_samples, n_features)

Subset of the training data

**y**: numpy array, shape (n_samples,)

Subset of the target values

**classes**: array, shape (n_classes,)

Classes across all calls to partial_fit. Can be obtained by via `np.unique(y_all)`, where y_all is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in *classes*.

**sample_weight**: array-like, shape (n_samples,), optional

Weights applied to individual samples. If not provided, uniform weights are assumed.

**Returns**

**self**: returns an instance of self.

**predict** *(X)*

Predict class labels for samples in *X*.

**Parameters**

**X**: {array-like, sparse matrix}, shape = [n_samples, n_features]

Samples.

**Returns**

**C**: array, shape = [n_samples]

Predicted class label per sample.

**score** *(X, y, sample_weight=\text{None})*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

**X**: array-like, shape = (n_samples, n_features)

Test samples.

**y**: array-like, shape = (n_samples) or (n_samples, n_outputs)

True labels for *X*.

**sample_weight**: array-like, shape = [n_samples], optional

Sample weights.

**Returns**

**score**: float

Mean accuracy of self.predict(*X*) wrt. *y*. 
sparsify()
Convert coefficient matrix to sparse format.

Converts the coef_ member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The intercept_ member is not converted.

Returns: estimator :

Notes

For non-sparse models, i.e. when there are not many zeros in coef_, this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with (coef_ == 0).sum(), must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

transform(*args, **kwargs)
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses coef_ or feature_importances_ to determine the most important features. For models with a coef_ for each class, the absolute sum over the classes is used.

Parameters

X : array or scipy sparse matrix of shape [n_samples, n_features]
The input samples.

threshold : string, float or None, optional (default=None)
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

Returns

X_r : array of shape [n_samples, n_selected_features]
The input samples with only the selected features.

Examples using sklearn.linear_model.Perceptron

- Out-of-core classification of text documents
- Comparing various online solvers
- Classification of text documents using sparse features
Randomized Lasso.

Randomized Lasso works by subsampling the training data and computing a Lasso estimate where the penalty of a random subset of coefficients has been scaled. By performing this double randomization several times, the method assigns high scores to features that are repeatedly selected across randomizations. This is known as stability selection. In short, features selected more often are considered good features.

Read more in the User Guide.

**Parameters**

- **alpha**: float, ‘aic’, or ‘bic’, optional
  The regularization parameter alpha parameter in the Lasso. Warning: this is not the alpha parameter in the stability selection article which is scaling.

- **scaling**: float, optional
  The s parameter used to randomly scale the penalty of different features (See User Guide for details). Should be between 0 and 1.

- **sample_fraction**: float, optional
  The fraction of samples to be used in each randomized design. Should be between 0 and 1. If 1, all samples are used.

- **n_resampling**: int, optional
  Number of randomized models.

- **selection_threshold**: float, optional
  The score above which features should be selected.

- **fit_intercept**: boolean, optional
  Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **verbose**: boolean or integer, optional
  Sets the verbosity amount.

- **normalize**: boolean, optional, default False
  If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learned more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

- **precompute**: True | False | ‘auto’
  Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.
max_iter : integer, optional

Maximum number of iterations to perform in the Lars algorithm.

eps : float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the ‘tol’ parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

n_jobs : integer, optional

Number of CPUs to use during the resampling. If ‘-1’, use all the CPUs

random_state : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

pre_dispatch : int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

• None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs

• An int, giving the exact number of total jobs that are spawned

• A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

memory : Instance of joblib.Memory or string

Used for internal caching. By default, no caching is done. If a string is given, it is the path to the caching directory.

Attributes

scores_ : array, shape = [n_features]

Feature scores between 0 and 1.

all_scores_ : array, shape = [n_features, n_reg_parameter]

Feature scores between 0 and 1 for all values of the regularization parameter. The reference article suggests scores_ is the max of all_scores_.

See also:

RandomizedLogisticRegression, Lasso, ElasticNet

Notes

See examples/linear_model/plot_sparse_recovery.py for an example.

References

Examples

```python
>>> from sklearn.linear_model import RandomizedLasso
>>> randomized_lasso = RandomizedLasso()
```

Methods

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<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
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<td><code>inverse_transform(X)</code></td>
<td>Transform a new matrix using the selected features.</td>
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<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><code>transform(X)</code></td>
<td>Transform a new matrix using the selected features.</td>
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```__init__``` (alpha='aic', scaling=0.5, sample_fraction=0.75, n_resampling=200, selection_threshold=0.25, fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, random_state=None, n_jobs=1, pre_dispatch='3*n_jobs', memory=Memory(cachedir=None))

```fit(X, y)```
Fit the model using X, y as training data.

- **Parameters**
  - `X` : array-like, shape = [n_samples, n_features]
    Training data.
  - `y` : array-like, shape = [n_samples]
    Target values.

- **Returns**
  - `self` : object
    Returns an instance of self.

```fit_transform(X, y=None, **fit_params)```
Fit to data, then transform it.
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

- **Parameters**
  - `X` : numpy array of shape [n_samples, n_features]
    Training set.
  - `y` : numpy array of shape [n_samples]
    Target values.

- **Returns**
  - `X_new` : numpy array of shape [n_samples, n_features_new]
    Transformed array.

```get_params(deep=True)```
Get parameters for this estimator.

- **Parameters**
  - `deep` : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.
**Returns**

`params`: mapping of string to any
Parameter names mapped to their values.

**get_support** *(indices=False)*
Return a mask, or list, of the features/indices selected.

**inverse_transform** *(X)*
Transform a new matrix using the selected features

**set_params** *(**params)*
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Return**

self:

**transform** *(X)*
Transform a new matrix using the selected features

---

**Examples using sklearn.linear_model.RandomizedLasso**

- Sparse recovery: feature selection for sparse linear models

---

**29.18.17 sklearn.linear_model.RandomizedLogisticRegression**

```python
class sklearn.linear_model.RandomizedLogisticRegression(C=1, scaling=0.5, sample_fraction=0.75, n_resampling=200, selection_threshold=0.25, tol=0.001, fit_intercept=True, normalize=True, random_state=None, n_jobs=1, pre_dispatch='3*n_jobs', memory=Memory(cachedir=None))
```

Randomized Logistic Regression

Randomized Logistic Regression works by subsampling the training data and fitting a L1-penalized LogisticRegression model where the penalty of a random subset of coefficients has been scaled. By performing this double randomization several times, the method assigns high scores to features that are repeatedly selected across randomizations. This is known as stability selection. In short, features selected more often are considered good features.


**Parameters**

- **C**: float, optional, default=1
  The regularization parameter C in the LogisticRegression.

- **scaling**: float, optional, default=0.5
  The s parameter used to randomly scale the penalty of different features (See User Guide for details). Should be between 0 and 1.

- **sample_fraction**: float, optional, default=0.75
  The fraction of samples used to calculate the importance of features. If smaller than 1.0, this randomizer will draw a subset of samples only.
The fraction of samples to be used in each randomized design. Should be between 0 and 1. If 1, all samples are used.

**n_resampling**: int, optional, default=200

Number of randomized models.

**selection_threshold**: float, optional, default=0.25

The score above which features should be selected.

**fit_intercept**: boolean, optional, default=True

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**verbose**: boolean or integer, optional

Sets the verbosity amount

**normalize**: boolean, optional, default=False

If True, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

**tol**: float, optional, default=1e-3

tolerance for stopping criteria of LogisticRegression

**n_jobs**: integer, optional

Number of CPUs to use during the resampling. If `-1`, use all the CPUs

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**pre_dispatch**: int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
- An int, giving the exact number of total jobs that are spawned
- A string, giving an expression as a function of n_jobs, as in `2*n_jobs`

**memory**: Instance of joblib.Memory or string

Used for internal caching. By default, no caching is done. If a string is given, it is the path to the caching directory.

**Attributes**

- **scores_**: array, shape = [n_features]

  Feature scores between 0 and 1.

- **all_scores_**: array, shape = [n_features, n_reg_parameter]
Feature scores between 0 and 1 for all values of the regularization parameter. The reference article suggests `scores_` is the max of `all_scores_`.

**See also:**

`RandomizedLasso`, `LogisticRegression`

**Notes**

See examples/linear_model/plot_sparse_recovery.py for an example.

**References**


**Examples**

```python
>>> from sklearn.linear_model import RandomizedLogisticRegression

>>> randomized_logistic = RandomizedLogisticRegression()
```

**Methods**

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<td><code>inverse_transform(X)</code></td>
<td>Transform a new matrix using the selected features</td>
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<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Transform a new matrix using the selected features</td>
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</table>

**__init__**(C=1, scaling=0.5, sample_fraction=0.75, n_resampling=200, selection_threshold=0.25, tol=0.001, fit_intercept=True, verbose=False, normalize=True, random_state=None, n_jobs=1, pre_dispatch='3*n_jobs', memory=Memory(cachedir=None))

**fit**(X, y)

Fit the model using X, y as training data.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training data.
- **y**: array-like, shape = [n_samples]
  - Target values.

**Returns**

- **self**: object
  - Returns an instance of self.

**fit_transform**(X, y=None, **fit_params)

Fit to data, then transform it.
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**get_support** *(indices=False)*

Return a mask, or list, of the features/indices selected.

**inverse_transform** *(X)*

Transform a new matrix using the selected features

**set_params** *(**params)*

Set the parameters of this estimator.

- The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 
  - Transform a new matrix using the selected features

---

### 29.18.18 sklearn.linear_model.RANSACRegressor

**class sklearn.linear_model.RANSACRegressor** *(base_estimator=None, min_samples=None, residual_threshold=None, is_data_valid=None, is_model_valid=None, max_trials=100, stop_n_inliers=inf, stop_score=inf, stop_probability=0.99, residual_metric=None, loss='absolute_loss', random_state=None)*

RANSAC (RANdom SAmple Consensus) algorithm.

RANSAC is an iterative algorithm for the robust estimation of parameters from a subset of inliers from the complete data set. More information can be found in the general documentation of linear models.

A detailed description of the algorithm can be found in the documentation of the `linear_model` sub-package.

Read more in the *User Guide*.

**Parameters**

- **base_estimator**: object, optional
  - Base estimator object which implements the following methods:
• `fit(X, y)`: Fit model to given training data and target values.

• `score(X, y)`: Returns the mean accuracy on the given test data, which is used for the stop criterion defined by `stop_score`. Additionally, the score is used to decide which of two equally large consensus sets is chosen as the better one.

If `base_estimator` is None, then `base_estimator=sklearn.linear_model.LinearRegression()` is used for target values of dtype float.

Note that the current implementation only supports regression estimators.

**min_samples**: int (>= 1) or float ([0, 1]), optional

Minimum number of samples chosen randomly from original data. Treated as an absolute number of samples for `min_samples` >= 1, treated as a relative number `ceil(min_samples * X.shape[0])` for `min_samples` < 1. This is typically chosen as the minimal number of samples necessary to estimate the given `base_estimator`. By default a `sklearn.linear_model.LinearRegression()` estimator is assumed and `min_samples` is chosen as `X.shape[1] + 1`.

**residual_threshold**: float, optional

Maximum residual for a data sample to be classified as an inlier. By default the threshold is chosen as the MAD (median absolute deviation) of the target values y.

**is_data_valid**: callable, optional

This function is called with the randomly selected data before the model is fitted to it: `is_data_valid(X, y)`. If its return value is False the current randomly chosen sub-sample is skipped.

**is_model_valid**: callable, optional

This function is called with the estimated model and the randomly selected data: `is_model_valid(model, X, y)`. If its return value is False the current randomly chosen sub-sample is skipped. Rejecting samples with this function is computationally costlier than with `is_data_valid`. `is_model_valid` should therefore only be used if the estimated model is needed for making the rejection decision.

**max_trials**: int, optional

Maximum number of iterations for random sample selection.

**stop_n_inliers**: int, optional

Stop iteration if at least this number of inliers are found.

**stop_score**: float, optional

Stop iteration if score is greater equal than this threshold.

**stop_probability**: float in range [0, 1], optional

RANSAC iteration stops if at least one outlier-free set of the training data is sampled in RANSAC. This requires to generate at least N samples (iterations):

\[ N \geq \log(1 - \text{probability}) / \log(1 - e^{\text{stop} \times \text{m}}) \]

where the probability (confidence) is typically set to high value such as 0.99 (the default) and m is the current fraction of inliers w.r.t. the total number of samples.

**residual_metric**: callable, optional

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Metric to reduce the dimensionality of the residuals to 1 for multi-dimensional target values \( y \). By default the sum of absolute differences is used:

```
lambda dy: np.sum(np.abs(dy), axis=1)
```

NOTE: residual_metric is deprecated from 0.18 and will be removed in 0.20 Use loss instead.

**loss** : string, callable, optional, default "absolute_loss"

String inputs, "absolute_loss" and "squared_loss" are supported which find the absolute loss and squared loss per sample respectively.

If **loss** is a callable, then it should be a function that takes two arrays as inputs, the true and predicted value and returns a 1-D array with the \( i \)th value of the array corresponding to the loss on \( X[i] \).

If the loss on a sample is greater than the residual_threshold, then this sample is classified as an outlier.

**random_state** : integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**Attributes**

**estimator_** : object

Best fitted model (copy of the base_estimator object).

**n_trials_** : int

Number of random selection trials until one of the stop criteria is met. It is always <= max_trials.

**inlier_mask_** : bool array of shape [n_samples]

Boolean mask of inliers classified as True.

**References**

[R35], [R36], [R37]

**Methods**

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit estimator using RANSAC algorithm.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict using the estimated model.</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the score of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(base_estimator=None, min_samples=None, residual_threshold=None, is_data_valid=None, is_model_valid=None, max_trials=100, stop_n_inliers=inf, stop_score=inf, stop_probability=0.99, residual_metric=None, loss='absolute_loss', random_state=None)
```

**fit** (X, y, sample_weight=None)

Fit estimator using RANSAC algorithm.
Parameters:

- **X**: array-like or sparse matrix, shape [n_samples, n_features]
  
  Training data.

- **y**: array-like, shape = [n_samples] or [n_samples, n_targets]
  
  Target values.

- **sample_weight**: array-like, shape = [n_samples]
  
  Individual weights for each sample raises error if sample_weight is passed and
  base_estimator fit method does not support it.

Raises: ValueError

- If no valid consensus set could be found. This occurs if `is_data_valid` and
  `is_model_valid` return False for all `max_trials` randomly chosen sub-samples.

**get_params**(deep=True)

Get parameters for this estimator.

Parameters:

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are
  estimators.

Returns:

- **params**: mapping of string to any
  
  Parameter names mapped to their values.

**predict**(X)

Predict using the estimated model.

This is a wrapper for `estimator_.predict(X)`.

Parameters:

- **X**: numpy array of shape [n_samples, n_features]

Returns:

- **y**: array, shape = [n_samples] or [n_samples, n_targets]
  
  Returns predicted values.

**score**(X, y)

Returns the score of the prediction.

This is a wrapper for `estimator_.score(X, y)`.

Parameters:

- **X**: numpy array or sparse matrix of shape [n_samples, n_features]
  
  Training data.

- **y**: array, shape = [n_samples] or [n_samples, n_targets]
  
  Target values.

Returns:

- **score**: float
  
  Score of the prediction.

**set_params**(**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
parameters of the form `<component>__<parameter>` so that it’s possible to update each component of
a nested object.

Returns:

- **self**:
Examples using `sklearn.linear_model.RANSACRegressor`

- Robust linear model estimation using RANSAC
- Robust linear estimator fitting
- Theil-Sen Regression

29.18.19 `sklearn.linear_model.Ridge`

class `sklearn.linear_model.Ridge`(
    `alpha=1.0`, `fit_intercept=True`, `normalize=False`,
    `copy_X=True`, `max_iter=None`, `tol=0.001`, `solver='auto'`,
    `random_state=None`
)

Linear least squares with l2 regularization.

This model solves a regression model where the loss function is the linear least squares function and regularization is given by the l2-norm. Also known as Ridge Regression or Tikhonov regularization. This estimator has built-in support for multi-variate regression (i.e., when y is a 2d-array of shape [n_samples, n_targets]).

Read more in the User Guide.

**Parameters**

- `alpha`: {float, array-like}, shape (n_targets)
  Regularization strength; must be a positive float. Regularization improves the conditioning of the problem and reduces the variance of the estimates. Larger values specify stronger regularization. Alpha corresponds to $C^{-1}$ in other linear models such as LogisticRegression or LinearSVC. If an array is passed, penalties are assumed to be specific to the targets. Hence they must correspond in number.

- `copy_X`: boolean, optional, default True
  If True, X will be copied; else, it may be overwritten.

- `fit_intercept`: boolean
  Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- `max_iter`: int, optional
  Maximum number of iterations for conjugate gradient solver. For ‘sparse_cg’ and ‘lsqr’ solvers, the default value is determined by scipy.sparse.linalg. For ‘sag’ solver, the default value is 1000.

- `normalize`: boolean, optional, default False
  If True, the regressors X will be normalized before regression. This parameter is ignored when `fit_intercept` is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use `preprocessing.StandardScaler` before calling `fit` on an estimator with `normalize=False`.

  Solver to use in the computational routines:
  * ‘auto’ chooses the solver automatically based on the type of data.
  * ‘svd’ uses a Singular Value Decomposition of X to compute the Ridge coefficients. More stable for singular matrices than ‘cholesky’.
• 'cholesky' uses the standard scipy.linalg.solve function to obtain a closed-form solution.

• 'sparse_cg' uses the conjugate gradient solver as found in scipy.sparse.linalg.cg. As an iterative algorithm, this solver is more appropriate than 'cholesky' for large-scale data (possibility to set tol and max_iter).

• 'lsqr' uses the dedicated regularized least-squares routine scipy.sparse.linalg.lsqr. It is the fastest but may not be available in old scipy versions. It also uses an iterative procedure.

• 'sag' uses a Stochastic Average Gradient descent. It also uses an iterative procedure, and is often faster than other solvers when both n_samples and n_features are large. Note that 'sag' fast convergence is only guaranteed on features with approximately the same scale. You can preprocess the data with a scaler from sklearn.preprocessing.

All last four solvers support both dense and sparse data. However, only 'sag' supports sparse input when fit_intercept is True.

New in version 0.17: Stochastic Average Gradient descent solver.

tol : float
   Precision of the solution.

random_state : int seed, RandomState instance, or None (default)
   The seed of the pseudo random number generator to use when shuffling the data. Used only in 'sag' solver.

   New in version 0.17: random_state to support Stochastic Average Gradient.

Attributes

• coef_ : array, shape (n_features,) or (n_targets, n_features)
   Weight vector(s).

   • intercept_ : float | array, shape = (n_targets,)
      Independent term in decision function. Set to 0.0 if fit_intercept = False.

   • n_iter_ : array or None, shape (n_targets,)
      Actual number of iterations for each target. Available only for sag and lsqr solvers. Other solvers will return None.

   New in version 0.17.

See also:

RidgeClassifier, RidgeCV, sklearn.kernel_ridge.KernelRidge

Examples

```python
>>> from sklearn.linear_model import Ridge
>>> import numpy as np
>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)
>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)
>>> clf = Ridge(alpha=1.0)
>>> clf.fit(X, y)
Ridge(alpha=1.0, copy_X=True, fit_intercept=True, max_iter=None,
      normalize=False, random_state=None, solver='auto', tol=0.001)
```
Methods

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__init__ (alpha=1.0, fit_intercept=True, normalize=False, copy_X=True, max_iter=None, tol=0.001, solver='auto', random_state=None)

decision_function (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

Parameters:

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returns:

C : array, shape = (n_samples,)

Returns predicted values.

fit (X, y, sample_weight=None)

Fit Ridge regression model

Parameters:

X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training data

y : array-like, shape = [n_samples] or [n_samples, n_targets]

Target values

sample_weight : float or numpy array of shape [n_samples]

Individual weights for each sample

Returns:

self : returns an instance of self.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

depth : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

params : mapping of string to any

Parameter names mapped to their values.

predict (X)

Predict using the linear model

Parameters:

X : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returns:

C : array, shape = (n_samples,)
Returns predicted values.

score ($X$, $y$, sample_weight=None)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred}) ** 2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true.mean}) ** 2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters $X$ : array-like, shape = (n_samples, n_features)

Test samples.

$y$ : array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for $X$.

sample_weight : array-like, shape = [n_samples], optional

Sample weights.

Returns $score$ : float

$R^2$ of self.predict($X$) wrt. $y$.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form $<component>__<parameter>$ so that it’s possible to update each component of a nested object.

Returns $self$ :

Examples using sklearn.linear_model.Ridge

- Prediction Latency
- Compressive sensing: tomography reconstruction with L1 prior (Lasso)
- HuberRegressor vs Ridge on dataset with strong outliers
- Ordinary Least Squares and Ridge Regression Variance
- Polynomial interpolation
- Plot Ridge coefficients as a function of the L2 regularization
- Plot Ridge coefficients as a function of the regularization

29.18.20 sklearn.linear_model.RidgeClassifier

class sklearn.linear_model.RidgeClassifier ($alpha=1.0$, fit_intercept=True, normalize=False, copy_X=True, max_iter=None, tol=0.001, class_weight=None, solver='auto', random_state=None)

Classifier using Ridge regression.

Read more in the User Guide.

Parameters $alpha$ : float
Regularization strength; must be a positive float. Regularization improves the conditioning of the problem and reduces the variance of the estimates. Larger values specify stronger regularization. Alpha corresponds to $C^{-1}$ in other linear models such as LogisticRegression or LinearSVC.

**class_weight**: dict or 'balanced', optional

Weights associated with classes in the form {class_label: weight}. If not given, all classes are supposed to have weight one.

The "balanced" mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as $n_{samples} / (n_{classes} * \text{np.bincount}(y))$

**copy_X**: boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**fit_intercept**: boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**max_iter**: int, optional

Maximum number of iterations for conjugate gradient solver. The default value is determined by scipy.sparse.linalg.

**normalize**: boolean, optional, default False

If True, the regressors X will be normalized before regression. This parameter is ignored when fit_intercept is set to False. When the regressors are normalized, note that this makes the hyperparameters learnt more robust and almost independent of the number of samples. The same property is not valid for standardized data. However, if you wish to standardize, please use preprocessing.StandardScaler before calling fit on an estimator with normalize=False.

**solver**: {'auto', 'svd', 'cholesky', 'lsqr', 'sparse_cg', 'sag'}

Solver to use in the computational routines:

- 'auto' chooses the solver automatically based on the type of data.
- 'svd' uses a Singular Value Decomposition of X to compute the Ridge coefficients. More stable for singular matrices than 'cholesky'.
- 'cholesky' uses the standard scipy.linalg.solve function to obtain a closed-form solution.
- 'sparse_cg' uses the conjugate gradient solver as found in scipy.sparse.linalg.cg. As an iterative algorithm, this solver is more appropriate than 'cholesky' for large-scale data (possibility to set tol and max_iter).
- 'lsqr' uses the dedicated regularized least-squares routine scipy.sparse.linalg.lsqr. It is the fastest but may not be available in old scipy versions. It also uses an iterative procedure.
- 'sag' uses a Stochastic Average Gradient descent. It also uses an iterative procedure, and is faster than other solvers when both n_samples and n_features are large.

New in version 0.17: Stochastic Average Gradient descent solver.

**tol**: float

Precision of the solution.
random_state : int seed, RandomState instance, or None (default)
The seed of the pseudo random number generator to use when shuffling the data. Used in ‘sag’ solver.

Attributes

coef_ : array, shape (n_features,) or (n_classes, n_features)
Weight vector(s).

intercept_ : float | array, shape = (n_targets,)
Independent term in decision function. Set to 0.0 if fit_intercept = False.
n_iter_ : array or None, shape (n_targets,)
Actual number of iterations for each target. Available only for sag and lsqr solvers. Other solvers will return None.

See also:
Ridge, RidgeClassifierCV

Notes

For multi-class classification, n_class classifiers are trained in a one-versus-all approach. Concretely, this is implemented by taking advantage of the multi-variate response support in Ridge.

Methods

decision_function(X)
Predict confidence scores for samples.

fit(X, y[, sample_weight])
Fit Ridge regression model.

get_params([deep])
Get parameters for this estimator.

predict(X)
Predict class labels for samples in X.

score(X, y[, sample_weight])
Returns the mean accuracy on the given test data and labels.

set_params(**params)
Set the parameters of this estimator.

__init__(alpha=1.0, fit_intercept=True, normalize=False, copy_X=True, max_iter=None, tol=0.001,
class_weight=None, solver='auto', random_state=None)

decision_function(X)
Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

Parameters

X : array-like, sparse matrix, shape = (n_samples, n_features)
Samples.

Returns

array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes) :
Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

fit(X, y, sample_weight=None)
Fit Ridge regression model.

Parameters

X : array-like, sparse matrix, shape = [n_samples,n_features]
Training data
y : array-like, shape = [n_samples]
   Target values

   sample_weight : float or numpy array of shape (n_samples,)
   Sample weight.
   New in version 0.17: sample_weight support to Classifier.

   Returns self : returns an instance of self.

get_params (deep=True)
Get parameters for this estimator.

   Parameters deep : boolean, optional
   If True, will return the parameters for this estimator and contained subobjects that are
   estimators.

   Returns params : mapping of string to any
   Parameter names mapped to their values.

predict (X)
Predict class labels for samples in X.

   Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]
   Samples.

   Returns C : array, shape = [n_samples]
   Predicted class label per sample.

score (X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.
   In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each
   sample that each label set be correctly predicted.

   Parameters X : array-like, shape = (n_samples, n_features)
   Test samples.
   y : array-like, shape = (n_samples) or (n_samples, n_outputs)
   True labels for X.
   sample_weight : array-like, shape = [n_samples], optional
   Sample weights.

   Returns score : float
   Mean accuracy of self.predict(X) wrt. y.

set_params (**params)
Set the parameters of this estimator.

   The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
   parameters of the form <component>__<parameter> so that it’s possible to update each component
   of a nested object.

   Returns self :
Examples using `sklearn.linear_model.RidgeClassifier`

- Classification of text documents using sparse features

**29.18.21 sklearn.linear_model.SGDClassifier**

class sklearn.linear_model.SGDClassifier(loss='hinge', penalty='l2', alpha=0.0001, l1_ratio=0.15, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, epsilon=0.1, n_jobs=1, random_state=None, learning_rate='optimal', eta0=0.0, power_t=0.5, class_weight=None, warm_start=False, average=False)

Linear classifiers (SVM, logistic regression, a.o.) with SGD training.

This estimator implements regularized linear models with stochastic gradient descent (SGD) learning: the gradient of the loss is estimated each sample at a time and the model is updated along the way with a decreasing strength schedule (aka learning rate). SGD allows minibatch (online/out-of-core) learning, see the partial_fit method. For best results using the default learning rate schedule, the data should have zero mean and unit variance.

This implementation works with data represented as dense or sparse arrays of floating point values for the features. The model it fits can be controlled with the loss parameter; by default, it fits a linear support vector machine (SVM).

The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using either the squared euclidean norm L2 or the absolute norm L1 or a combination of both (Elastic Net). If the parameter update crosses the 0.0 value because of the regularizer, the update is truncated to 0.0 to allow for learning sparse models and achieve online feature selection.

Read more in the User Guide.

- Parameters
    
    The loss function to be used. Defaults to ‘hinge’, which gives a linear SVM. The ‘log’ loss gives logistic regression, a probabilistic classifier. ‘modified_huber’ is another smooth loss that brings tolerance to outliers as well as probability estimates. ‘squared_hinge’ is like hinge but is quadratically penalized. ‘perceptron’ is the linear loss used by the perceptron algorithm. The other losses are designed for regression but can be useful in classification as well; see SGDRegressor for a description.
  
    
    The penalty (aka regularization term) to be used. Defaults to ‘l2’ which is the standard regularizer for linear SVM models. ‘l1’ and ‘elasticnet’ might bring sparsity to the model (feature selection) not achievable with ‘l2’.
  
  - `alpha`: float
    
    Constant that multiplies the regularization term. Defaults to 0.0001 Also used to compute learning_rate when set to ‘optimal’.
  
  - `l1_ratio`: float
    
    The Elastic Net mixing parameter, with 0 <= l1_ratio <= 1. l1_ratio=0 corresponds to L2 penalty, l1_ratio=1 to L1. Defaults to 0.15.
  
  - `fit_intercept`: bool
Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.

**n_iter :** int, optional

The number of passes over the training data (aka epochs). The number of iterations is set to 1 if using partial_fit. Defaults to 5.

**shuffle :** bool, optional

Whether or not the training data should be shuffled after each epoch. Defaults to True.

**random_state :** int seed, RandomState instance, or None (default)

The seed of the pseudo random number generator to use when shuffling the data.

**verbose :** integer, optional

The verbosity level

**epsilon :** float

Epsilon in the epsilon-insensitive loss functions; only if loss is ‘huber’, ‘epsilon_insensitive’, or ‘squared_epsilon_insensitive’. For ‘huber’, determines the threshold at which it becomes less important to get the prediction exactly right. For epsilon-insensitive, any differences between the current prediction and the correct label are ignored if they are less than this threshold.

**n_jobs :** integer, optional

The number of CPUs to use to do the OVA (One Versus All, for multi-class problems) computation. -1 means ‘all CPUs’. Defaults to 1.

**learning_rate :** string, optional

The learning rate schedule:

- ‘constant’: eta = eta0
- ‘optimal’: eta = 1.0 / (alpha * (t + t0)) [default]
- ‘invscaling’: eta = eta0 / pow(t, power_t)

where t0 is chosen by a heuristic proposed by Leon Bottou.

**eta0 :** double

The initial learning rate for the ‘constant’ or ‘invscaling’ schedules. The default value is 0.0 as eta0 is not used by the default schedule ‘optimal’.

**power_t :** double

The exponent for inverse scaling learning rate [default 0.5].

**class_weight :** dict, {class_label: weight} or “balanced” or None, optional

Preset for the class_weight fit parameter.

Weights associated with classes. If not given, all classes are supposed to have weight one.

The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as \( \frac{n_{samples}}{n_{classes} * \text{np.bincount}(y)} \)

**warm_start :** bool, optional
When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

average: bool or int, optional

When set to True, computes the averaged SGD weights and stores the result in the coef_ attribute. If set to an int greater than 1, averaging will begin once the total number of samples seen reaches average. So average=10 will begin averaging after seeing 10 samples.

Attributes

coeff_: array, shape (1, n_features) if n_classes == 2 else (n_classes, n_features)

Weights assigned to the features.

intercept_: array, shape (1,) if n_classes == 2 else (n_classes,)

Constants in decision function.

See also:

LinearSVC, LogisticRegression, Perceptron

Examples

```python
>>> import numpy as np
>>> from sklearn import linear_model
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> y = np.array([1, 1, 2, 2])
>>> clf = linear_model.SGDClassifier()
>>> clf.fit(X, y)
...
SGDClassifier(alpha=0.0001, average=False, class_weight=None, epsilon=0.1,
eta0=0.0, fit_intercept=True, l1_ratio=0.15,
learning_rate='optimal', loss='hinge', n_iter=5, n_jobs=1,
penalty='l2', power_t=0.5, random_state=None, shuffle=True,
verbose=0, warm_start=False)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

Methods

decision_function(X) Predict confidence scores for samples.
densify() Convert coefficient matrix to dense array format.
fit(X, y[, coef_init, intercept_init, ...]) Fit linear model with Stochastic Gradient Descent.
fit_transform(X[, y]) Fit to data, then transform it.
get_params([deep]) Get parameters for this estimator.
partial_fit(X, y[, classes, sample_weight]) Fit linear model with Stochastic Gradient Descent.
predict(X) Predict class labels for samples in X.
score(X, y[, sample_weight]) Returns the mean accuracy on the given test data and labels.
set_params(**params) Set the parameters of this estimator.
sparsefy() Convert coefficient matrix to sparse format.
transform(**params) DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.
__init__(loss='hinge', penalty='l2', alpha=0.0001, l1_ratio=0.15, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, epsilon=0.1, n_jobs=1, random_state=None, learning_rate='optimal', eta0=0.0, power_t=0.5, class_weight=None, warm_start=False, average=False)

decision_function(X)
Predict confidence scores for samples.
The confidence score for a sample is the signed distance of that sample to the hyperplane.

Parameters
X : {array-like, sparse matrix}, shape = (n_samples, n_features)
Samples.

Returns
array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes):
Confidence scores per (sample, class) combination. In the binary case, confidence score
for self.classes_[1] where >0 means this class would be predicted.

densify()
Convert coefficient matrix to dense array format.
Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is
required for fitting, so calling this method is only required on models that have previously been sparsified;
otherwise, it is a no-op.

Returns
self: estimator

fit(X, y, coef_init=None, intercept_init=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters
X : {array-like, sparse matrix}, shape (n_samples, n_features)
Training data
y : numpy array, shape (n_samples,)
Target values
coef_init : array, shape (n_classes, n_features)
The initial coefficients to warm-start the optimization.
intercept_init : array, shape (n_classes,)
The initial intercept to warm-start the optimization.
sample_weight : array-like, shape (n_samples,), optional
Weights applied to individual samples. If not provided, uniform weights are assumed.
These weights will be multiplied with class_weight (passed through the constructor) if
class_weight is specified

Returns
self : returns an instance of self.

fit_transform(X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters
X : numpy array of shape [n_samples, n_features]
Training set.
y : numpy array of shape [n_samples]
Target values.
Returns $X_{\text{new}}$ : numpy array of shape [n_samples, n_features_new]

Transformed array.

get_params (deep=True)
Get parameters for this estimator.

Parameters:
- deep : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params : mapping of string to any
  Parameter names mapped to their values.

partial_fit (X, y, classes=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters:
- X : {array-like, sparse matrix}, shape (n_samples, n_features)
  Subset of the training data
- y : numpy array, shape (n_samples,)
  Subset of the target values
- classes : array, shape (n_classes,)
  Classes across all calls to partial_fit. Can be obtained by via np.unique(y_all), where y_all is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in classes.
- sample_weight : array-like, shape (n_samples,), optional
  Weights applied to individual samples. If not provided, uniform weights are assumed.

Returns:
- self

predict (X)
Predict class labels for samples in X.

Parameters:
- X : {array-like, sparse matrix}, shape = [n_samples, n_features]
  Samples.

Returns:
- C : array, shape = [n_samples]
  Predicted class label per sample.

predict_log_proba
Log of probability estimates.

This method is only available for log loss and modified Huber loss.

When loss="modified_huber", probability estimates may be hard zeros and ones, so taking the logarithm is not possible.

See predict_proba for details.

Parameters:
- X : array-like, shape (n_samples, n_features)

Returns:
- T : array-like, shape (n_samples, n_classes)
  Returns the log-probability of the sample for each class in the model, where classes are ordered as they are in self.classes_.

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predict_proba

Probability estimates.

This method is only available for log loss and modified Huber loss.

Multiclass probability estimates are derived from binary (one-vs.-rest) estimates by simple normalization, as recommended by Zadrozny and Elkan.

Binary probability estimates for loss=”modified_huber” are given by (clip(decision_function(X), -1, 1) + 1) / 2. For other loss functions it is necessary to perform proper probability calibration by wrapping the classifier with sklearn.calibration.CalibratedClassifierCV instead.

Parameters:
- X : {array-like, sparse matrix}, shape (n_samples, n_features)

Returns:
- array, shape (n_samples, n_classes)

Returns the probability of the sample for each class in the model, where classes are ordered as they are in self.classes_.

References


The justification for the formula in the loss=”modified_huber” case is in the appendix B in: http://jmlr.csail.mit.edu/papers/volume2/zhang02c/zhang02c.pdf

score (X, y, sample_weight=None)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters:
- X : array-like, shape = (n_samples, n_features)
- y : array-like, shape = (n_samples) or (n_samples, n_outputs)

Returns:
- score : float

Mean accuracy of self.predict(X) wrt. y.

sparsify()

Convert coefficient matrix to sparse format.

Converts the coef_ member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The intercept_ member is not converted.

Returns:
- self: estimator
Notes

For non-sparse models, i.e. when there are not many zeros in `coef_`, this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with `(coef_ == 0).sum()`, must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

```python
transform(*args, **kwargs)
```

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

Parameters

- **X** : array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.

- **threshold** [string, float or None, optional (default=None)]
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

Returns

- **X_r** : array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

Examples using `sklearn.linear_model.SGDClassifier`

- Model Complexity Influence
- Out-of-core classification of text documents
- Comparing various online solvers
- Plot multi-class SGD on the iris dataset
- SGD: Maximum margin separating hyperplane
- SGD: Weighted samples
- Sample pipeline for text feature extraction and evaluation
- Classification of text documents using sparse features
- Classification of text documents: using a MLComp dataset
Linear model fitted by minimizing a regularized empirical loss with SGD

SGD stands for Stochastic Gradient Descent: the gradient of the loss is estimated each sample at a time and the model is updated along the way with a decreasing strength schedule (aka learning rate).

The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using either the squared euclidean norm L2 or the absolute norm L1 or a combination of both (Elastic Net). If the parameter update crosses the 0.0 value because of the regularizer, the update is truncated to 0.0 to allow for learning sparse models and achieve online feature selection.

This implementation works with data represented as dense numpy arrays of floating point values for the features.

Read more in the User Guide.

**Parameters**

- **loss**: str, ‘squared_loss’, ‘huber’, ‘epsilon_insensitive’, or ‘squared_epsilon_insensitive’
  
  The loss function to be used. Defaults to ‘squared_loss’ which refers to the ordinary least squares fit. ‘huber’ modifies ‘squared_loss’ to focus less on getting outliers correct by switching from squared to linear loss past a distance of epsilon. ‘epsilon_insensitive’ ignores errors less than epsilon and is linear past that; this is the loss function used in SVR. ‘squared_epsilon_insensitive’ is the same but becomes squared loss past a tolerance of epsilon.

- **penalty**: str, ‘none’, ‘l2’, ‘l1’, or ‘elasticnet’
  
  The penalty (aka regularization term) to be used. Defaults to ‘l2’ which is the standard regularizer for linear SVM models. ‘l1’ and ‘elasticnet’ might bring sparsity to the model (feature selection) not achievable with ‘l2’.

- **alpha**: float
  
  Constant that multiplies the regularization term. Defaults to 0.0001 Also used to compute learning_rate when set to ‘optimal’.

- **l1_ratio**: float
  
  The Elastic Net mixing parameter, with 0 <= l1_ratio <= 1. l1_ratio=0 corresponds to L2 penalty, l1_ratio=1 to L1. Defaults to 0.15.

- **fit_intercept**: bool
  
  Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.

- **n_iter**: int, optional
  
  The number of passes over the training data (aka epochs). The number of iterations is set to 1 if using partial_fit. Defaults to 5.

- **shuffle**: bool, optional
  
  Whether or not the training data should be shuffled after each epoch. Defaults to True.

- **random_state**: int seed, RandomState instance, or None (default)
  
  The seed of the pseudo random number generator to use when shuffling the data.
verbose : integer, optional

The verbosity level.

epsilon : float

Epsilon in the epsilon-insensitive loss functions; only if loss is ‘huber’, ‘epsilon_insensitive’, or ‘squared_epsilon_insensitive’. For ‘huber’, determines the threshold at which it becomes less important to get the prediction exactly right. For epsilon-insensitive, any differences between the current prediction and the correct label are ignored if they are less than this threshold.

learning_rate : string, optional

The learning rate schedule:

• ‘constant’: \eta = \eta_0
• ‘optimal’: \eta = 1.0 / (alpha * (t + t0)) [default]
• ‘invscaling’: \eta = \eta_0 / pow(t, power_t)

where t0 is chosen by a heuristic proposed by Leon Bottou.

eta0 : double, optional

The initial learning rate [default 0.01].

power_t : double, optional

The exponent for inverse scaling learning rate [default 0.25].

warm_start : bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

average : bool or int, optional

When set to True, computes the averaged SGD weights and stores the result in the coef_ attribute. If set to an int greater than 1, averaging will begin once the total number of samples seen reaches average. So average=10 will begin averaging after seeing 10 samples.

Attributes

coef_ : array, shape (n_features,)

Weights assigned to the features.

intercept_ : array, shape (1,)

The intercept term.

average_coef_ : array, shape (n_features,)

Averaged weights assigned to the features.

average_intercept_ : array, shape (1,)

The averaged intercept term.

See also:

Ridge, ElasticNet, Lasso, SVR
Examples

```python
>>> import numpy as np
>>> from sklearn import linear_model
>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)
>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)
>>> clf = linear_model.SGDRegressor()
>>> clf.fit(X, y)
```

Methods

- `decision_function(*args, **kwargs)` DEPRECATED: and will be removed in 0.19.
- `densify()` Convert coefficient matrix to dense array format.
- `fit(X, y[, coef_init, intercept_init, ...])` Fit linear model with Stochastic Gradient Descent.
- `fit_transform(X[, y])` Fit to data, then transform it.
- `get_params([deep])` Get parameters for this estimator.
- `partial_fit(X, y[, sample_weight])` Fit linear model with Stochastic Gradient Descent.
- `predict(X)` Predict using the linear model
- `score(X, y[, sample_weight])` Returns the coefficient of determination R^2 of the prediction.
- `set_params(**kwargs)` DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.
- `sparsify()` Convert coefficient matrix to sparse format.
- `transform(*args, **kwargs)` DEPRECATED: and will be removed in 0.19.

```
>>> __init__(loss='squared_loss', penalty='l2', alpha=0.0001, l1_ratio=0.15, fit_intercept=True, n_iter=5, shuffle=True, verbose=0, epsilon=0.1, random_state=None, learning_rate='invscaling', eta0=0.01, power_t=0.25, warm_start=False, average=False)
```

```
>>> decision_function(*args, **kwargs)
DEPRECATED: and will be removed in 0.19.
```

```
>>> Predict using the linear model

Parameters

X : {array-like, sparse matrix}, shape (n_samples, n_features)

Returns

array, shape (n_samples,)

Predicted target values per element in X.
```

```
>>> densify()
Convert coefficient matrix to dense array format.

Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

Returns

self : estimator
```
**fit** *(X, y, coef_init=None, intercept_init=None, sample_weight=None)*

Fit linear model with Stochastic Gradient Descent.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training data
- **y**: numpy array, shape (n_samples,)
  - Target values
- **coef_init**: array, shape (n_features,)
  - The initial coefficients to warm-start the optimization.
- **intercept_init**: array, shape (1,)
  - The initial intercept to warm-start the optimization.
- **sample_weight**: array-like, shape (n_samples,), optional
  - Weights applied to individual samples (1. for unweighted).

**Returns**

- **self**: returns an instance of self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape `[n_samples, n_features]`
  - Training set.
- **y**: numpy array of shape `[n_samples]`
  - Target values.

**Returns**

- **X_new**: numpy array of shape `[n_samples, n_features_new]`
  - Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**partial_fit** *(X, y, sample_weight=None)*

Fit linear model with Stochastic Gradient Descent.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  - Subset of training data
- **y**: numpy array of shape (n_samples,)
  - Subset of target values
- **sample_weight**: array-like, shape (n_samples,), optional
  - Weights applied to individual samples. If not provided, uniform weights are assumed.

**Returns**

- **self**: returns an instance of self.
predict(X)
Predict using the linear model

Parameters:
- X : {array-like, sparse matrix}, shape (n_samples, n_features)

Returns:
- array, shape (n_samples,)
  Predicted target values per element in X.

score(X, y, sample_weight=None)
Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

Parameters:
- X : array-like, shape (n_samples, n_features)
  Test samples.
- y : array-like, shape (n_samples) or (n_samples, n_outputs)
  True values for X.
- sample_weight : array-like, shape [n_samples], optional
  Sample weights.

Returns:
- score : float
  R^2 of self.predict(X) wrt. y.

sparsify()
Convert coefficient matrix to sparse format.

Converts the coef_ member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The intercept_ member is not converted.

Returns:
- self : estimator

Notes

For non-sparse models, i.e. when there are not many zeros in coef_, this may actually increase memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with (coef_ == 0).sum(), must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

transform(*args, **kwargs)
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses coef_ or feature_importances_ to determine the most important features. For models with a coef_ for each class, the absolute sum over the classes is used.

Parameters:
- X : array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.
threshold[string, float or None, optional (default=None)] The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

Returns

X_r : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

Examples using sklearn.linear_model.SGDRegressor

- Prediction Latency

29.18.23 sklearn.linear_model.TheilSenRegressor
class sklearn.linear_model.TheilSenRegressor (fit_intercept=True, copy_X=True,
max_subpopulation=10000.0, n_subsamples=None, max_iter=300,
tol=0.001, random_state=None, n_jobs=1, verbose=False)

Theil-Sen Estimator: robust multivariate regression model.

The algorithm calculates least square solutions on subsets with size n_subsamples of the samples in X. Any value of n_subsamples between the number of features and samples leads to an estimator with a compromise between robustness and efficiency. Since the number of least square solutions is “n_samples choose n_subsamples”, it can be extremely large and can therefore be limited with max_subpopulation. If this limit is reached, the subsets are chosen randomly. In a final step, the spatial median (or L1 median) is calculated of all least square solutions.

Read more in the User Guide.

Parameters

fit_intercept : boolean, optional, default True

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations.

copy_X : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

max_subpopulation : int, optional, default 1e4

Instead of computing with a set of cardinality ‘n choose k’, where n is the number of samples and k is the number of subsamples (at least number of features), consider only a stochastic subpopulation of a given maximal size if ‘n choose k’ is larger than max_subpopulation. For other than small problem sizes this parameter will determine memory usage and runtime if n_subsamples is not changed.

n_subsamples : int, optional, default None

Number of samples to calculate the parameters. This is at least the number of features (plus 1 if fit_intercept=True) and the number of samples as a maximum. A lower number leads to a higher breakdown point and a low efficiency while a high number leads to a low breakdown point and a high efficiency. If None, take the minimum number of subsamples leading to maximal robustness. If n_subsamples is set to n_samples, Theil-Sen is identical to least squares.

max_iter : int, optional, default 300
Maximum number of iterations for the calculation of spatial median.

tol : float, optional, default 1.e-3
    Tolerance when calculating spatial median.

random_state : RandomState or an int seed, optional, default None
    A random number generator instance to define the state of the random permutations generator.

n_jobs : integer, optional, default 1
    Number of CPUs to use during the cross validation. If -1, use all the CPUs.

verbose : boolean, optional, default False
    Verbose mode when fitting the model.

Attributes

coef_ : array, shape = (n_features)
    Coefficients of the regression model (median of distribution).

intercept_ : float
    Estimated intercept of regression model.

breakdown_ : float
    Approximated breakdown point.

n_iter_ : int
    Number of iterations needed for the spatial median.

n_subpopulation_ : int
    Number of combinations taken into account from ‘n choose k’, where n is the number of samples and k is the number of subsamples.

References

• Theil-Sen Estimators in a Multiple Linear Regression Model, 2009 Xin Dang, Hanxiang Peng, Xueqin Wang and Heping Zhang http://home.olemiss.edu/~xdang/papers/MTSE.pdf

Methods

---

decision_function(*args, **kwargs) DEPRECATED: and will be removed in 0.19.

fit(X, y)
    Fit linear model.

get_params([deep])
    Get parameters for this estimator.

predict(X)
    Predict using the linear model

score(X, y[, sample_weight])
    Returns the coefficient of determination R^2 of the prediction.

set_params(**params)
    Set the parameters of this estimator.

---

__init__ (fit_intercept=True, copy_X=True, max_subpopulation=10000.0, n_subsamples=None, max_iter=300, tol=0.001, random_state=None, n_jobs=1, verbose=False)

decision_function (*args, **kwargs)
    DEPRECATED: and will be removed in 0.19.
Decision function of the linear model.

**Parameters**

- **X** : {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**

- **C** : array, shape = (n_samples,)
  - Returns predicted values.

**fit**(X, y)
Fit linear model.

- **Parameters**
  - **X** : numpy array of shape [n_samples, n_features]
    - Training data
  - **y** : numpy array of shape [n_samples]
    - Target values

**Returns**

- **self** : returns an instance of self.

**get_params**(deep=True)
Get parameters for this estimator.

- **Parameters**
  - **deep** : boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  - Parameter names mapped to their values.

**predict**(X)
Predict using the linear model

- **Parameters**
  - **X** : {array-like, sparse matrix}, shape = (n_samples, n_features)
    - Samples.

**Returns**

- **C** : array, shape = (n_samples,)
  - Returns predicted values.

**score**(X, y, sample_weight=None)
Returns the coefficient of determination $R^2$ of the prediction.

- The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ($(y\_true - y\_pred) ** 2).sum()$ and $v$ is the residual sum of squares ($(y\_true - y\_true.mean()) ** 2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

- **Parameters**
  - **X** : array-like, shape = (n_samples, n_features)
    - Test samples.
  - **y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
    - True values for X.
  - **sample_weight** : array-like, shape = [n_samples], optional
    - Sample weights.

**Returns**

- **score** : float
  - $R^2$ of self.predict(X) wrt. y.
set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns self:

Examples using sklearn.linear_model.TheilSenRegressor

- Robust linear estimator fitting
- Theil-Sen Regression

```
linear_model.lars_path(X, y, Xy=None, Gram=None, max_iter=500, alpha_min=0,
method='lar', copy_X=True, eps=2.2204460492503131e-16, copy_Gram=True,
verbose=0, return_path=True, return_n_iter=False, positive=False)
```

Compute Least Angle Regression or Lasso path using LARS algorithm [1]

```
linear_model.lasso_path(X, y, eps=...)  Compute Lasso path with coordinate descent
linear_model.lasso_stability_path(X, y) Stability path based on randomized Lasso estimates
linear_model.logistic_regression_path(X, y) Compute a Logistic Regression model for a list of regularization parameters.
linear_model.orthogonal_mp(X, y) Orthogonal Matching Pursuit (OMP)
linear_model.orthogonal_mp_gram(Gram, Xy) Gram Orthogonal Matching Pursuit (OMP)
```

29.18.24 sklearn.linear_model.lars_path

```
sklearn.linear_model.lars_path(X, y, Xy=None, Gram=None, max_iter=500, alpha_min=0,
method='lar', copy_X=True, eps=2.2204460492503131e-16, copy_Gram=True, verbose=0,
return_path=True, return_n_iter=False, positive=False)
```

Compute Least Angle Regression or Lasso path using LARS algorithm [1]

The optimization objective for the case method='lasso' is:

\[
(1 / (2 * n_samples)) * ||y - Xw||^2_2 + alpha * ||w||_1
\]

in the case of method='lars', the objective function is only known in the form of an implicit equation (see discussion in [1])

Read more in the User Guide.

Parameters

- **X**: array, shape: (n_samples, n_features)
  Input data.
- **y**: array, shape: (n_samples)
  Input targets.
- **positive**: boolean (default=False)
  Restrict coefficients to be >= 0. When using this option together with method ‘lasso’ the model coefficients will not converge to the ordinary-least-squares solution for small values of alpha (neither will they when using method ‘lar’ ..). Only coefficients up to the smallest alpha value (alphas_[alphas_ > 0.].min() when fit_path=True)
reached by the stepwise Lars-Lasso algorithm are typically in congruence with the solution of the coordinate descent lasso_path function.

**max_iter** : integer, optional (default=500)

Maximum number of iterations to perform, set to infinity for no limit.

**Gram** : None, ‘auto’, array, shape: (n_features, n_features), optional

Precomputed Gram matrix ($X^T \times X$), if 'auto', the Gram matrix is precomputed from the given X, if there are more samples than features.

**alpha_min** : float, optional (default=0)

Minimum correlation along the path. It corresponds to the regularization parameter alpha parameter in the Lasso.

**method** : {'lar', ‘lasso’}, optional (default='lar')

Specifies the returned model. Select ‘lar’ for Least Angle Regression, ‘lasso’ for the Lasso.

**eps** : float, optional (default=‘np.finfo(np.float).eps’)

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**copy_X** : bool, optional (default=True)

If False, X is overwritten.

**copy_Gram** : bool, optional (default=True)

If False, Gram is overwritten.

**verbose** : int (default=0)

Controls output verbosity.

**return_path** : bool, optional (default=True)

If return_path=True returns the entire path, else returns only the last point of the path.

**return_n_iter** : bool, optional (default=False)

Whether to return the number of iterations.

**Returns**

**alphas** : array, shape: [n_alphas + 1]

Maximum of covariances (in absolute value) at each iteration. n_alphas is either max_iter, n_features or the number of nodes in the path with alpha >= alpha_min, whichever is smaller.

**active** : array, shape [n_alphas]

Indices of active variables at the end of the path.

**coefs** : array, shape (n_features, n_alphas + 1)

Coefficients along the path.

**n_iter** : int

Number of iterations run. Returned only if return_n_iter is set to True.

See also:

lasso_path, LassoLars, Lars, LassoLarsCV, LarsCV, sklearn.decomposition.sparse_encode
Examples using `sklearn.linear_model.lars_path`

- Lasso path using LARS

### 29.18.25 `sklearn.linear_model.lasso_path`

```python
sklearn.linear_model.lasso_path(X, y, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, copy_X=True, coef_init=None, verbose=False, return_n_iter=False, positive=False, **params)
```

Compute Lasso path with coordinate descent

The Lasso optimization function varies for mono and multi-outputs.

For mono-output tasks it is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||y - Xw||^2_2 + \alpha \times ||w||_1
\]

For multi-output tasks it is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||Y - XW||^2_{\text{Fro}} + \alpha \times ||W||_{21}
\]

Where:

\[
||W||_{21} = \sum_i \sqrt{\sum_j w_{ij}^2}
\]

i.e. the sum of norm of each row.

Read more in the User Guide.

**Parameters**

- `X` : array-like, sparse matrix, shape (n_samples, n_features)
  
  Training data. Pass directly as Fortran-contiguous data to avoid unnecessary memory duplication. If `y` is mono-output then `X` can be sparse.

- `y` : ndarray, shape (n_samples,), or (n_samples, n_outputs)
  
  Target values

- `eps` : float, optional
  
  Length of the path. `eps=1e-3` means that `alpha_min / alpha_max = 1e-3`

- `n_alphas` : int, optional
  
  Number of alphas along the regularization path

- `alphas` : ndarray, optional
  
  List of alphas where to compute the models. If `None` alphas are set automatically

- `precompute` : True | False | ‘auto’ | array-like
  
  Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

- `Xy` : array-like, optional
Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.

**copy_X**: boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**coef_init**: array, shape (n_features, ) | None

The initial values of the coefficients.

**verbose**: bool or integer

Amount of verbosity.

**params**: kwargs

keyword arguments passed to the coordinate descent solver.

**positive**: bool, default False

If set to True, forces coefficients to be positive.

**return_n_iter**: bool

whether to return the number of iterations or not.

**Returns**

- **alphas**: array, shape (n_alphas,)
  The alphas along the path where models are computed.

- **coefs**: array, shape (n_features, n_alphas) or (n_outputs, n_features, n_alphas)
  Coefficients along the path.

- **dual_gaps**: array, shape (n_alphas,)
  The dual gaps at the end of the optimization for each alpha.

- **n_iters**: array-like, shape (n_alphas,)
  The number of iterations taken by the coordinate descent optimizer to reach the specified tolerance for each alpha.

**See also:**

- *lars_path*, *Lasso*, *LassoLars*, *LassoCV*, *LassoLarsCV*, *sklearn.decomposition.sparse_encode*

**Notes**

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a Fortran-contiguous numpy array.

Note that in certain cases, the Lars solver may be significantly faster to implement this functionality. In particular, linear interpolation can be used to retrieve model coefficients between the values output by lars_path.

**Examples**

Comparing lasso_path and lars_path with interpolation:
```python
>>> X = np.array([[1, 2, 3.1], [2.3, 5.4, 4.3]]).T
>>> y = np.array([1, 2, 3.1])
>>> # Use lasso_path to compute a coefficient path
>>> _, coef_path, _ = lasso_path(X, y, alphas=[5., 1., .5])
>>> print(coef_path)
[[ 0. 0.  0.46874778]
 [ 0.2159048 0.4425765 0.23689075]]
```

```python
>>> # Now use lars_path and 1D linear interpolation to compute the
>>> # same path
>>> from sklearn.linear_model import lars_path
>>> alphas, active, coef_path_lars = lars_path(X, y, method='lasso')
>>> from scipy import interpolate
>>> coef_path_continuous = interpolate.interp1d(alphas[::-1],
...     coef_path_lars[:, ::-1])
>>> print(coef_path_continuous([5., 1., .5]))
[[ 0. 0. 0.46915237]
 [ 0.2159048 0.4425765 0.23668876]]
```

Examples using `sklearn.linear_model.lasso_path`

- Lasso and Elastic Net

### 29.18.26 sklearn.linear_model.lasso_stability_path

`sklearn.linear_model.lasso_stability_path(X, y, scaling=0.5, random_state=None, n_resampling=200, n_grid=100, sample_fraction=0.75, eps=8.8817841970012523e-16, n_jobs=1, verbose=False)`

Stability path based on randomized Lasso estimates. Read more in the User Guide.

**Parameters**

- `X`: array-like, shape = [n_samples, n_features]
  training data.
- `y`: array-like, shape = [n_samples]
  target values.
- `scaling`: float, optional, default=0.5
  The alpha parameter in the stability selection article used to randomly scale the features. Should be between 0 and 1.
- `random_state`: integer or numpy.random.RandomState, optional
  The generator used to randomize the design.
- `n_resampling`: int, optional, default=200
  Number of randomized models.
- `n_grid`: int, optional, default=100
  Number of grid points. The path is linearly interpolated on a grid between 0 and 1 before computing the scores.
sample_fraction : float, optional, default=0.75

The fraction of samples to be used in each randomized design. Should be between 0 and 1. If 1, all samples are used.

eps : float, optional

Smallest value of \( \alpha / \alpha_{\text{max}} \) considered

n_jobs : integer, optional

Number of CPUs to use during the resampling. If `-1`, use all the CPUs

verbose : boolean or integer, optional

Sets the verbosity amount

Returns:

alphas_grid : array, shape ~ [n_grid]

The grid points between 0 and 1: \( \alpha / \alpha_{\text{max}} \)

scores_path : array, shape = [n_features, n_grid]

The scores for each feature along the path.

Notes

See examples/linear_model/plot_sparse_recovery.py for an example.

Examples using `sklearn.linear_model.lasso_stability_path`

- Sparse recovery: feature selection for sparse linear models

29.18.27 `sklearn.linear_model.logistic_regression_path`

```
sklearn.linear_model.logistic_regression_path(X, y, pos_class=None, Cs=10, fit_intercept=True, max_iter=100, tol=0.0001, solver='lbfgs', coef=None, copy=False, class_weight=None, dual=False, penalty='l2', intercept_scaling=1.0, multi_class='ovr', random_state=None, check_input=True, max_squared_sum=None, sample_weight=None)
```

Compute a Logistic Regression model for a list of regularization parameters.

This is an implementation that uses the result of the previous model to speed up computations along the set of solutions, making it faster than sequentially calling LogisticRegression for the different parameters. Note that there will be no speedup with liblinear solver, since it does not handle warm-starting.

Read more in the User Guide.

Parameters:

- **X** : array-like or sparse matrix, shape (n_samples, n_features)
  
  Input data.

- **y** : array-like, shape (n_samples,)
  
  Input data, target values.
Cs : int | array-like, shape (n_cs,)

List of values for the regularization parameter or integer specifying the number of regularization parameters that should be used. In this case, the parameters will be chosen in a logarithmic scale between 1e-4 and 1e4.

pos_class : int, None

The class with respect to which we perform a one-vs-all fit. If None, then it is assumed that the given problem is binary.

fit_intercept : bool

Whether to fit an intercept for the model. In this case the shape of the returned array is (n_cs, n_features + 1).

max_iter : int

Maximum number of iterations for the solver.

tol : float

Stopping criterion. For the newton-cg and lbfgs solvers, the iteration will stop when \( \max(|g_i| : i = 1, \ldots, n) \leq \text{tol} \) where \( g_i \) is the \( i \)-th component of the gradient.

verbose : int

For the liblinear and lbfgs solvers set verbose to any positive number for verbosity.

solver : {'lbfgs', 'newton-cg', 'liblinear', 'sag'}

Numerical solver to use.

coeff : array-like, shape (n_features,), default None

Initialization value for coefficients of logistic regression. Useless for liblinear solver.

copy : bool, default False

Whether or not to produce a copy of the data. A copy is not required anymore. This parameter is deprecated and will be removed in 0.19.

class_weight : dict or ‘balanced’, optional

Weights associated with classes in the form \{class_label: weight\}. If not given, all classes are supposed to have weight one.

The “balanced” mode uses the values of \( y \) to automatically adjust weights inversely proportional to class frequencies in the input data as \( \frac{n_{\text{samples}}}{n_{\text{classes}} \ast \text{np.bincount}(y)} \).

Note that these weights will be multiplied with sample_weight (passed through the fit method) if sample_weight is specified.

dual : bool

Dual or primal formulation. Dual formulation is only implemented for l2 penalty with liblinear solver. Prefer dual=False when n_samples > n_features.

penalty : str, ‘l1’ or ‘l2’

Used to specify the norm used in the penalization. The ‘newton-cg’, ‘sag’ and ‘lbfgs’ solvers support only l2 penalties.

intercept_scaling : float, default 1.
Useful only when the solver 'liblinear' is used and self.fit_intercept is set to True. In this case, x becomes \([x, self.intercept_scaling]\), i.e. a “synthetic” feature with constant value equal to intercept_scaling is appended to the instance vector. The intercept becomes \(\text{intercept} \times \text{synthetic} \times \text{feature} \times \text{weight}\).

Note! the synthetic feature weight is subject to l1/l2 regularization as all other features. To lessen the effect of regularization on synthetic feature weight (and therefore on the intercept) intercept_scaling has to be increased.

**multi_class** : str, {'ovr', 'multinomial'}

Multiclass option can be either ‘ovr’ or ‘multinomial’. If the option chosen is ‘ovr’, then a binary problem is fit for each label. Else the loss minimised is the multinomial loss fit across the entire probability distribution. Works only for the ‘lbfgs’ and ‘newton-cg’ solvers.

**random_state** : int seed, RandomState instance, or None (default)

The seed of the pseudo random number generator to use when shuffling the data. Used only in solvers ‘sag’ and ‘liblinear’.

**check_input** : bool, default True

If False, the input arrays X and y will not be checked.

**max_squared_sum** : float, default None

Maximum squared sum of \(X\) over samples. Used only in SAG solver. If None, it will be computed, going through all the samples. The value should be precomputed to speed up cross validation.

**sample_weight** : array-like, shape(n_samples,) optional

Array of weights that are assigned to individual samples. If not provided, then each sample is given unit weight.

**Returns**

**coefs** : ndarray, shape (n_cs, n_features) or (n_cs, n_features + 1)

List of coefficients for the Logistic Regression model. If fit_intercept is set to True then the second dimension will be \(n \times \text{features} + 1\), where the last item represents the intercept.

**Cs** : ndarray

Grid of Cs used for cross-validation.

**n_iter** : array, shape (n_cs,)

Actual number of iteration for each Cs.

Notes

You might get slightly different results with the solver liblinear than with the others since this uses LIBLINEAR which penalizes the intercept.

**29.18.28 sklearn.linear_model.orthogonal_mp**

sklearn.linear_model.orthogonal_mp(X, y, n_nonzero_coefs=None, tol=None, precompute=False, copy_X=True, return_path=False, return_n_iter=False)

Orthogonal Matching Pursuit (OMP)
Solves n_targets Orthogonal Matching Pursuit problems. An instance of the problem has the form:

When parametrized by the number of non-zero coefficients using \( n_{\text{nonzero coefs}} \): argmin \( ||y - X\gamma||^2 \)
subject to \( ||\gamma||_0 \leq n_{\text{nonzero coefs}} \)

When parametrized by error using the parameter \( tol \): argmin \( ||\gamma||_0 \) subject to \( ||y - X\gamma||^2 \leq tol \)

Read more in the User Guide.

**Parameters**

\( X \) : array, shape (n_samples, n_features)

Input data. Columns are assumed to have unit norm.

\( y \) : array, shape (n_samples,) or (n_samples, n_targets)

Input targets

\( n_{\text{nonzero coefs}} \) : int

Desired number of non-zero entries in the solution. If None (by default) this value is set to 10% of n_features.

\( tol \) : float

Maximum norm of the residual. If not None, overrides \( n_{\text{nonzero coefs}} \).

\( \text{precompute} \) : {True, False, ‘auto’},

Whether to perform precomputations. Improves performance when n_targets or n_samples is very large.

\( \text{copy}_X \) : bool, optional

Whether the design matrix \( X \) must be copied by the algorithm. A false value is only helpful if \( X \) is already Fortran-ordered, otherwise a copy is made anyway.

\( \text{return}_\text{path} \) : bool, optional. Default: False

Whether to return every value of the nonzero coefficients along the forward path. Useful for cross-validation.

\( \text{return}_n_{\text{iter}} \) : bool, optional default False

Whether or not to return the number of iterations.

**Returns**

\( \text{coef} \) : array, shape (n_features,) or (n_features, n_targets)

Coefficients of the OMP solution. If \( \text{return}_\text{path}=True \), this contains the whole coefficient path. In this case its shape is (n_features, n_features) or (n_features, n_targets, n_features) and iterating over the last axis yields coefficients in increasing order of active features.

\( \text{n}_\text{iters} \) : array-like or int

Number of active features across every target. Returned only if \( \text{return}_n_{\text{iter}} \) is set to True.

**See also:**

OrthogonalMatchingPursuit, orthogonal_mp_gram, lars_path, decomposition.sparse_encode
Notes


29.18.29 sklearn.linear_model.orthogonal_mp_gram

sklearn.linear_model.orthogonal_mp_gram(Gram, Xy, n_nonzero_coefs=None, tol=None, norms_squared=None, copy_Gram=True, copy_Xy=True, return_path=False, return_n_iter=False)

Gram Orthogonal Matching Pursuit (OMP)

Solves n_targets Orthogonal Matching Pursuit problems using only the Gram matrix X.T * X and the product X.T * y.

Read more in the User Guide.

Parameters

Gram : array, shape (n_features, n_features)
    Gram matrix of the input data: X.T * X

Xy : array, shape (n_features,) or (n_features, n_targets)
    Input targets multiplied by X: X.T * y

n_nonzero_coefs : int
    Desired number of non-zero entries in the solution. If None (by default) this value is set to 10% of n_features.

 tol : float
    Maximum norm of the residual. If not None, overrides n_nonzero_coefs.

norms_squared : array-like, shape (n_targets,)
    Squared L2 norms of the lines of y. Required if tol is not None.

copy_Gram : bool, optional
    Whether the gram matrix must be copied by the algorithm. A false value is only helpful if it is already Fortran-ordered, otherwise a copy is made anyway.

copy_Xy : bool, optional
    Whether the covariance vector Xy must be copied by the algorithm. If False, it may be overwritten.

return_path : bool, optional. Default: False
    Whether to return every value of the nonzero coefficients along the forward path. Useful for cross-validation.

return_n_iter : bool, optional default False
    Whether or not to return the number of iterations.

Returns

coef : array, shape (n_features,) or (n_features, n_targets)
Coefficients of the OMP solution. If `return_path=True`, this contains the whole coefficient path. In this case its shape is (n_features, n_features) or (n_features, n_targets, n_features) and iterating over the last axis yields coefficients in increasing order of active features.

**n_iters** : array-like or int
Number of active features across every target. Returned only if `return_n_iter` is set to True.

See also:

`OrthogonalMatchingPursuit`, `orthogonal_mp`, `lars_path`, `decomposition.sparse_encode`

Notes


29.19 **sklearn.manifold: Manifold Learning**

The `sklearn.manifold` module implements data embedding techniques.

**User guide:** See the *Manifold learning* section for further details.

| sklearn.manifold.LocallyLinearEmbedding(...) | Locally Linear Embedding |
| sklearn.manifold.Isomap(...) | Isomap Embedding |
| sklearn.manifold.MDS(...) | Multidimensional scaling |
| sklearn.manifold.TSNE(...) | t-distributed Stochastic Neighbor Embedding. |

29.19.1 **sklearn.manifold.LocallyLinearEmbedding**

class `sklearn.manifold.LocallyLinearEmbedding`(n_neighbors=5, n_components=2, reg=0.001, eigen_solver='auto', tol=1e-06, max_iter=100, method='standard', hessian_tol=0.0001, modified_tol=1e-12, neighbors_algorithm='auto', random_state=None, n_jobs=1)

Locally Linear Embedding

Read more in the *User Guide.*

**Parameters**

- **n_neighbors** : integer
  number of neighbors to consider for each point.

- **n_components** : integer
  number of coordinates for the manifold
reg : float
    regularization constant, multiplies the trace of the local covariance matrix of the
distances.

eigen_solver : string, {'auto', 'arpack', 'dense'}
    auto : algorithm will attempt to choose the best method for input data
              arpack[use arnoldi iteration in shift-invert mode.] For this method, M may be a dense
              matrix, sparse matrix, or general linear operator. Warning: ARPACK can be unstable
              for some problems. It is best to try several random seeds in order to check results.
              dense[use standard dense matrix operations for the eigenvalue] decomposition. For this
              method, M must be an array or matrix type. This method should be avoided for large
              problems.

tol : float, optional
    Tolerance for ‘arpack’ method Not used if eigen_solver==’dense’.

max_iter : integer
    maximum number of iterations for the arpack solver. Not used if
    eigen_solver==’dense’.

method : string (‘standard’, ‘hessian’, ‘modified’ or ‘ltsa’)
    standard[use the standard locally linear embedding algorithm. see] reference [1]
    hessian[use the Hessian eigenmap method. This method requires] n_neighbors >
    n_components * (1 + (n_components + 1) / 2 see reference [2]
    modified[use the modified locally linear embedding algorithm.] see reference [3]
    ltsa[use local tangent space alignment algorithm] see reference [4]

hessian_tol : float, optional
    Tolerance for Hessian eigenmapping method. Only used if method == 'hessian'

modified_tol : float, optional
    Tolerance for modified LLE method. Only used if method == 'modified'

neighbors_algorithm : string ['auto' 'brute' 'kd_tree' 'ball_tree']
    algorithm to use for nearest neighbors search, passed to neighbors.NearestNeighbors
    instance

random_state: numpy.RandomState or int, optional :
    The generator or seed used to determine the starting vector for arpack iterations. De-
defaults to numpy.random.

n_jobs : int, optional (default = 1)
    The number of parallel jobs to run. If -1, then the number of jobs is set to the number
    of CPU cores.

Attributes
embedding_vectors_ : array-like, shape [n_components, n_samples]
    Stores the embedding vectors

reconstruction_error_ : float
    Reconstruction error associated with embedding_vectors_
nbrs_: NearestNeighbors object

Stores nearest neighbors instance, including BallTree or KDtree if applicable.

References

[R42], [R43], [R44], [R45]

Methods

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<td>Compute the embedding vectors for data X</td>
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<tr>
<td>fit_transform(X[, y])</td>
<td>Compute the embedding vectors for data X and transform X.</td>
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<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
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<td>set_params(**params)</td>
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</tr>
<tr>
<td>transform(X)</td>
<td>Transform new points into embedding space.</td>
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__init__ (n_neighbors=5, n_components=2, reg=0.001, eigen_solver='auto', tol=1e-06, max_iter=100, method='standard', hessian_tol=0.0001, modified_tol=1e-12, neighbors_algorithm='auto', random_state=None, n_jobs=1)

fit (X, y=None)
Compute the embedding vectors for data X

Parameters

X : array-like of shape [n_samples, n_features]
training set.

Returns

self : returns an instance of self.

fit_transform (X, y=None)
Compute the embedding vectors for data X and transform X.

Parameters

X : array-like of shape [n_samples, n_features]
training set.

Returns

X_new: array-like, shape (n_samples, n_components):

get_params (deep=True)
Get parameters for this estimator.

Parameters

deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

params : mapping of string to any
Parameter names mapped to their values.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns

self :
transform \((X)\)

Transform new points into embedding space.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]

**Returns**
- **X_new**: array, shape = [n_samples, n_components]

**Notes**

Because of scaling performed by this method, it is discouraged to use it together with methods that are not scale-invariant (like SVMs)

**Examples using sklearn.manifold.LocallyLinearEmbedding**

- Visualizing the stock market structure
- Comparison of Manifold Learning methods
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...
- Manifold Learning methods on a severed sphere

### 29.19.2 sklearn.manifold.Isomap

**class** sklearn.manifold.Isomap

\((n\_neighbors=5, \ n\_components=2, \ eigen\_solver='auto', \ tol=0,\)
\(\ max\_iter=None, \ path\_method='auto', \ neighbors\_algorithm='auto',\)
\(\ n\_jobs=1)\)

Isomap Embedding

Non-linear dimensionality reduction through Isometric Mapping

Read more in the *User Guide*.

**Parameters**

- **n_neighbors**: integer
  
  number of neighbors to consider for each point.

- **n_components**: integer
  
  number of coordinates for the manifold

- **eigen_solver**: ['auto', 'arpack', 'dense']
  
  'auto': Attempt to choose the most efficient solver for the given problem.
  
  'arpack': Use Arnoldi decomposition to find the eigenvalues and eigenvectors.
  
  'dense': Use a direct solver (i.e. LAPACK) for the eigenvalue decomposition.

- **tol**: float
  
  Convergence tolerance passed to arpack or lobpcg. not used if eigen_solver == 'dense'.

- **max_iter**: integer
  
  Maximum number of iterations for the arpack solver. not used if eigen_solver == 'dense'.

- **path_method**: string ['auto', 'FW', 'D']
Method to use in finding shortest path.

`auto` : attempt to choose the best algorithm automatically.

`FW` : Floyd-Warshall algorithm.

`D` : Dijkstra’s algorithm.

**neighbors_algorithm** : string [‘auto’|’brute’|’kd_tree’|’ball_tree’]

Algorithm to use for nearest neighbors search, passed to neighbors.NearestNeighbors instance.

**n_jobs** : int, optional (default = 1)

The number of parallel jobs to run. If `-1`, then the number of jobs is set to the number of CPU cores.

**Attributes**

**embedding_** : array-like, shape (n_samples, n_components)

Stores the embedding vectors.

**kernel_pca_** : object

KernelPCA object used to implement the embedding.

**training_data_** : array-like, shape (n_samples, n_features)

Stores the training data.

**nbrs_** : sklearn.neighbors.NearestNeighbors instance

Stores nearest neighbors instance, including BallTree or KDTree if applicable.

**dist_matrix_** : array-like, shape (n_samples, n_samples)

Stores the geodesic distance matrix of training data.

**References**

[R41]

**Methods**

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</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
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<td>reconstruction_error()</td>
<td>Compute the reconstruction error for the embedding.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform X.</td>
</tr>
</tbody>
</table>

**__init__** (n_neighbors=5, n_components=2, eigen_solver=’auto’, tol=0, max_iter=None, path_method=’auto’, neighbors_algorithm=’auto’, n_jobs=1)

**fit** (X, y=None)

Compute the embedding vectors for data X

**Parameters**

**X** : {array-like, sparse matrix, BallTree, KDTree, NearestNeighbors}

Sample data, shape = (n_samples, n_features), in the form of a numpy array, precomputed tree, or NearestNeighbors object.
Returns `self` : returns an instance of self.

**`fit_transform`** *(X, y=None)*
Fit the model from data in X and transform X.

Parameters

- **X**: {array-like, sparse matrix, BallTree, KDTree}
  Training vector, where n_samples is the number of samples and n_features is the number of features.

Returns

- **X_new**: array-like, shape (n_samples, n_components)
  The embedding of X.

**`get_params`** *(deep=True)*
Get parameters for this estimator.

Parameters

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

- **params** : mapping of string to any
  Parameter names mapped to their values.

**`reconstruction_error`** ()
Compute the reconstruction error for the embedding.

Returns

- **reconstruction_error** : float
  The cost function of an isomap embedding is
  \[
  E = \text{frobenius}
  \text{norm} \left( K(D) - K(D_{\text{fit}}) \right) / \text{n}\_\text{samples}
  \]
  Where D is the matrix of distances for the input data X, D_{\text{fit}} is the matrix of distances for the output embedding X_{\text{fit}}, and K is the isomap kernel:
  \[
  K(D) = -0.5 \times (I - 1/n\_\text{samples}) \times D^2 \times (I - 1/n\_\text{samples})
  \]

**`set_params`** *(**params**)*
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns `self` :

**`transform`** *(X)*
Transform X.

This is implemented by linking the points X into the graph of geodesic distances of the training data. First the n_neighbors nearest neighbors of X are found in the training data, and from these the shortest geodesic distances from each point in X to each point in the training data are computed in order to construct the kernel. The embedding of X is the projection of this kernel onto the embedding vectors of the training set.

Parameters

- **X**: array-like, shape (n_samples, n_features)

Returns

- **X_new**: array-like, shape (n_samples, n_components) :
Examples using `sklearn.manifold.Isomap`

- Comparison of Manifold Learning methods
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...
- Manifold Learning methods on a severed sphere

29.19.3 `sklearn.manifold.MDS`

class `sklearn.manifold.MDS`(n_components=2, metric=True, n_init=4, max_iter=300, verbose=0, eps=0.001, n_jobs=1, random_state=None, dissimilarity='euclidean')

Multidimensional scaling

Read more in the User Guide.

**Parameters**

- **metric**: boolean, optional, default: True
  
  Compute metric or nonmetric SMACOF (Scaling by Majorizing a Complicated Function) algorithm

- **n_components**: int, optional, default: 2
  
  Number of dimension in which to immerse the similarities overridden if initial array is provided.

- **n_init**: int, optional, default: 4
  
  Number of time the smacof algorithm will be run with different initialisation. The final results will be the best output of the n_init consecutive runs in terms of stress.

- **max_iter**: int, optional, default: 300
  
  Maximum number of iterations of the SMACOF algorithm for a single run

- **verbose**: int, optional, default: 0
  
  Level of verbosity

- **eps**: float, optional, default: 1e-6
  
  Relative tolerance w.r.t stress to declare converge

- **n_jobs**: int, optional, default: 1
  
  The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.

  If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

- **random_state**: integer or numpy.RandomState, optional
  
  The generator used to initialize the centers. If an integer is given, it fixes the seed.

  Defaults to the global numpy random number generator.

- **dissimilarity**: string
  
  Which dissimilarity measure to use. Supported are ‘euclidean’ and ‘precomputed’.

**Attributes**

- **embedding_**: array-like, shape [n_components, n_samples]
  
  Stores the position of the dataset in the embedding space

- **stress_**: float
The final value of the stress (sum of squared distance of the disparities and the distances for all constrained points)

References


“Nonmetric multidimensional scaling: a numerical method” Kruskal, J. Psychometrika, 29 (1964)

“Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis” Kruskal, J. Psychometrika, 29, (1964)

Methods

**fit**

```
fit(X[, y, init])
```

Computes the position of the points in the embedding space

**fit_transform**

```
fit_transform(X[, y, init])
```

Fit the data from X, and returns the embedded coordinates

**get_params**

```
get_params([deep])
```

Get parameters for this estimator.

**set_params**

```
set_params(**params)
```

Set the parameters of this estimator.

```python
__init__ (n_components=2, metric=True, n_init=4, max_iter=300, verbose=0, eps=0.001, n_jobs=1,
random_state=None, dissimilarity='euclidean')
```

**fit**

```
fit(X, y=None, init=None)
```

Computes the position of the points in the embedding space

**Parameters**

- **X** : array, shape=[n_samples, n_features], or [n_samples, n_samples] if dissimilarity='precomputed'
  
  Input data.

- **init** : {None or ndarray, shape (n_samples,)}, optional
  
  If None, randomly chooses the initial configuration if ndarray, initialize the SMACOF algorithm with this array.

**fit_transform**

```
fit_transform(X, y=None, init=None)
```

Fit the data from X, and returns the embedded coordinates

**Parameters**

- **X** : array, shape=[n_samples, n_features], or [n_samples, n_samples] if dissimilarity='precomputed'
  
  Input data.

- **init** : {None or ndarray, shape (n_samples,)}, optional
  
  If None, randomly chooses the initial configuration if ndarray, initialize the SMACOF algorithm with this array.

**get_params**

```
get_params(deep=True)
```

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.
Returns

**params** : mapping of string to any

Parameter names mapped to their values.

set_params (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns

self:

Examples using sklearn.manifold.MDS

- Comparison of Manifold Learning methods
- Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...
- Manifold Learning methods on a severed sphere
- Multi-dimensional scaling

29.19.4 sklearn.manifold.SpectralEmbedding

class sklearn.manifold.SpectralEmbedding (n_components=2, affinity='nearest_neighbors',
                                           eigen_solver=None, random_state=None,
                                           n_neighbors=None, n_jobs=1)

Spectral embedding for non-linear dimensionality reduction.

Forms an affinity matrix given by the specified function and applies spectral decomposition to the corresponding graph laplacian. The resulting transformation is given by the value of the eigenvectors for each data point.

Read more in the User Guide.

Parameters

**n_components** : integer, default: 2

The dimension of the projected subspace.

eigen_solver : {None, ‘arpack’, ‘lobpcg’, or ‘amg’}

The eigenvalue decomposition strategy to use. AMG requires pyamg to be installed. It can be faster on very large, sparse problems, but may also lead to instabilities.

random_state : int seed, RandomState instance, or None, default

A pseudo random number generator used for the initialization of the lobpcg eigenvectors decomposition when eigen_solver == ‘amg’.

affinity : string or callable, default

How to construct the affinity matrix.

- ‘nearest_neighbors’ : construct affinity matrix by knn graph
- ‘rbf’ : construct affinity matrix by rbf kernel
- ‘precomputed’ : interpret X as precomputed affinity matrix
- callable : use passed in function as affinity the function takes in data matrix (n_samples, n_features) and return affinity matrix (n_samples, n_samples).

gamma : float, optional, default
Kernel coefficient for rbf kernel.

**n_neighbors** : int, default

Number of nearest neighbors for nearest_neighbors graph building.

**n_jobs** : int, optional (default = 1)

The number of parallel jobs to run. If -1, then the number of jobs is set to the number of CPU cores.

**Attributes**
- **embedding_** : array, shape = (n_samples, n_components)
  
  Spectral embedding of the training matrix.

- **affinity_matrix_** : array, shape = (n_samples, n_samples)
  
  Affinity matrix constructed from samples or precomputed.

**References**


**Methods**

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<td><code>fit(X[, y])</code></td>
<td>Fit the model from data in X.</td>
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<td><code>fit_transform(X[, y])</code></td>
<td>Fit the model from data in X and transform X.</td>
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<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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**__init__**(n_components=2, affinity='nearest_neighbors', gamma=None, random_state=None, eigen_solver=None, n_neighbors=None, n_jobs=1)

**fit**(X, y=None)

Fit the model from data in X.

**Parameters**
- **X** : array-like, shape (n_samples, n_features)
  
  Training vector, where n_samples is the number of samples and n_features is the number of features.

  If affinity is “precomputed” X : array-like, shape (n_samples, n_samples), Interpret X as precomputed adjacency graph computed from samples.

**Returns**
- **self** : object
  
  Returns the instance itself.

**fit_transform**(X, y=None)

Fit the model from data in X and transform X.

**Parameters**
- **X** : array-like, shape (n_samples, n_features) :
Training vector, where n_samples is the number of samples and n_features is the number of features.

If affinity is “precomputed” X : array-like, shape (n_samples, n_samples), Interpret X as precomputed adjacency graph computed from samples.

ReturnsX_new: array-like, shape (n_samples, n_components):

generate (deep=True)
Get parameters for this estimator.

Parametersdeep : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returnsparams : mapping of string to any
  Parameter names mapped to their values.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returnsself :

Examples using sklearn.manifold.SpectralEmbedding

• Various Agglomerative Clustering on a 2D embedding of digits
• Comparison of Manifold Learning methods
• Manifold learning on handwriten digits: Locally Linear Embedding, Isomap...
• Manifold Learning methods on a severed sphere

29.19.5 sklearn.manifold.TSNE
class sklearn.manifold.TSNE (n_components=2, perplexity=30.0, early_exaggeration=4.0, learning_rate=1000.0, n_iter=1000, n_iter_without_progress=30, min_grad_norm=1e-07, metric='euclidean', init='random', verbose=0, random_state=None, method='barnes_hut', angle=0.5)
t-distributed Stochastic Neighbor Embedding.

t-SNE [1] is a tool to visualize high-dimensional data. It converts similarities between data points to joint probabilities and tries to minimize the Kullback-Leibler divergence between the joint probabilities of the low-dimensional embedding and the high-dimensional data. t-SNE has a cost function that is not convex, i.e. with different initializations we can get different results.

It is highly recommended to use another dimensionality reduction method (e.g. PCA for dense data or TruncatedSVD for sparse data) to reduce the number of dimensions to a reasonable amount (e.g. 50) if the number of features is very high. This will suppress some noise and speed up the computation of pairwise distances between samples. For more tips see Laurens van der Maaten’s FAQ [2].

Read more in the User Guide.

Parametersn_components : int, optional (default: 2)
  Dimension of the embedded space.
**perplexity**: float, optional (default: 30)

The perplexity is related to the number of nearest neighbors that is used in other manifold learning algorithms. Larger datasets usually require a larger perplexity. Consider selecting a value between 5 and 50. The choice is not extremely critical since t-SNE is quite insensitive to this parameter.

**early_exaggeration**: float, optional (default: 4.0)

Controls how tight natural clusters in the original space are in the embedded space and how much space will be between them. For larger values, the space between natural clusters will be larger in the embedded space. Again, the choice of this parameter is not very critical. If the cost function increases during initial optimization, the early exaggeration factor or the learning rate might be too high.

**learning_rate**: float, optional (default: 1000)

The learning rate can be a critical parameter. It should be between 100 and 1000. If the cost function increases during initial optimization, the early exaggeration factor or the learning rate might be too high. If the cost function gets stuck in a bad local minimum increasing the learning rate helps sometimes.

**n_iter**: int, optional (default: 1000)

Maximum number of iterations for the optimization. Should be at least 200.

**n_iter_without_progress**: int, optional (default: 30)

Only used if method='exact' Maximum number of iterations without progress before we abort the optimization. If method='barnes_hut' this parameter is fixed to a value of 30 and cannot be changed.

New in version 0.17: parameter n_iter_without_progress to control stopping criteria.

**min_grad_norm**: float, optional (default: 1e-7)

Only used if method='exact' If the gradient norm is below this threshold, the optimization will be aborted. If method='barnes_hut' this parameter is fixed to a value of 1e-3 and cannot be changed.

**metric**: string or callable, optional

The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by scipy.spatial.distance.pdist for its metric parameter, or a metric listed in pairwise.PAIRWISE_DISTANCE_FUNCTIONS. If metric is “precomputed”, X is assumed to be a distance matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from X as input and return a value indicating the distance between them. The default is “euclidean” which is interpreted as squared euclidean distance.

**init**: string or numpy array, optional (default: “random”)

Initialization of embedding. Possible options are ‘random’, ‘pca’, and a numpy array of shape (n_samples, n_components). PCA initialization cannot be used with precomputed distances and is usually more globally stable than random initialization.

**verbose**: int, optional (default: 0)

Verbosity level.

**random_state**: int or RandomState instance or None (default)
Pseudo Random Number generator seed control. If None, use the numpy.random singleton. Note that different initializations might result in different local minima of the cost function.

**method** : string (default: ‘barnes_hut’)

By default the gradient calculation algorithm uses Barnes-Hut approximation running in O(NlogN) time. method='exact' will run on the slower, but exact, algorithm in O(N^2) time. The exact algorithm should be used when nearest-neighbor errors need to be better than 3%. However, the exact method cannot scale to millions of examples.

New in version 0.17: Approximate optimization method via the Barnes-Hut.

**angle** : float (default: 0.5)

Only used if method=’barnes_hut’ This is the trade-off between speed and accuracy for Barnes-Hut T-SNE. ‘angle’ is the angular size (referred to as theta in [3]) of a distant node as measured from a point. If this size is below ‘angle’ then it is used as a summary node of all points contained within it. This method is not very sensitive to changes in this parameter in the range of 0.2 - 0.8. Angle less than 0.2 has quickly increasing computation time and angle greater 0.8 has quickly increasing error.

**Attributes**

- **embedding_** : array-like, shape (n_samples, n_components)
  Stores the embedding vectors.

- **kl_divergence_** : float
  Kullback-Leibler divergence after optimization.

**References**


**Examples**

```python
>>> import numpy as np
>>> from sklearn.manifold import TSNE
>>> X = np.array([[0, 0, 0], [0, 1, 1], [1, 0, 1], [1, 1, 1]])
>>> model = TSNE(n_components=2, random_state=0)
>>> np.set_printoptions(suppress=True)
>>> model.fit_transform(X)
array([[ 0.00017599,  0.00003993],
       [ 0.00009891,  0.00021913],
       [ 0.00018554, -0.00009357],
       [ 0.00009528, -0.00001407]])
```

**Methods**
**scikit-learn user guide, Release 0.18.2**

---

### Methods

- **`fit(X[, y])`** Fit X into an embedded space.
- **`fit_transform(X[, y])`** Fit X into an embedded space and return that transformed output.
- **`get_params([deep])`** Get parameters for this estimator.
- **`set_params(**params)`** Set the parameters of this estimator.

---

```python
__init__(n_components=2, perplexity=30.0, early_exaggeration=4.0, learning_rate=1000.0, n_iter=1000, n_iter_without_progress=30, min_grad_norm=1e-07, metric='euclidean', init='random', verbose=0, random_state=None, method='barnes_hut', angle=0.5)
```

- **`fit(X, y=None)`** Fit X into an embedded space.
  - **Parameters**
    - `X` : array, shape (n_samples, n_features) or (n_samples, n_samples)
      - If the metric is ‘precomputed’ X must be a square distance matrix. Otherwise it contains a sample per row. If the method is ‘exact’, X may be a sparse matrix of type ‘csr’, ‘csc’ or ‘coo’.

- **`fit_transform(X, y=None)`** Fit X into an embedded space and return that transformed output.
  - **Parameters**
    - `X` : array, shape (n_samples, n_features) or (n_samples, n_samples)
      - If the metric is ‘precomputed’ X must be a square distance matrix. Otherwise it contains a sample per row.
  - **Returns**
    - `X_new` : array, shape (n_samples, n_components)
      - Embedding of the training data in low-dimensional space.

- **`get_params(deep=True)`** Get parameters for this estimator.
  - **Parameters**
    - `deep` : boolean, optional
      - If True, will return the parameters for this estimator and contained subobjects that are estimators.
  - **Returns**
    - `params` : mapping of string to any
      - Parameter names mapped to their values.

- **`set_params(**params)`** Set the parameters of this estimator.
  - The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.
  - **Returns**
    - `self` :

---

### Examples using `sklearn.manifold.TSNE`

- **Comparison of Manifold Learning methods**
- **Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...**
- **Manifold Learning methods on a severed sphere**

---

29.19. `sklearn.manifold`: Manifold Learning 1619
Perform a Locally Linear Embedding analysis on the data.

```python
manifold.locally_linear_embedding(X, ...]
```

Project the sample on the first eigenvectors of the graph Laplacian.

```python
manifold.spectral_embedding(adjacency[, ...])
```

## 29.19.6 sklearn.manifold.locally_linear_embedding

```python
sklearn.manifold.locally_linear_embedding(X, n_neighbors, n_components, reg=0.001,
   eigen_solver='auto', tol=1e-06, max_iter=100,
   method='standard', hessian_tol=0.0001,
   modified_tol=1e-12, random_state=None,
   n_jobs=1)
```

Perform a Locally Linear Embedding analysis on the data.

Read more in the User Guide.

**Parameters**

- **X** : {array-like, sparse matrix, BallTree, KDTree, NearestNeighbors}
  - Sample data, shape = (n_samples, n_features), in the form of a numpy array, sparse array, precomputed tree, or NearestNeighbors object.

- **n_neighbors** : integer
  - number of neighbors to consider for each point.

- **n_components** : integer
  - number of coordinates for the manifold.

- **reg** : float
  - regularization constant, multiplies the trace of the local covariance matrix of the distances.

- **eigen_solver** : string, {‘auto’, ‘arpack’, ‘dense’}
  - auto : algorithm will attempt to choose the best method for input data
  - arpack [use arnoldi iteration in shift-invert mode.] For this method, M may be a dense matrix, sparse matrix, or general linear operator. Warning: ARPACK can be unstable for some problems. It is best to try several random seeds in order to check results.
  - dense [use standard dense matrix operations for the eigenvalue decomposition. For this method, M must be an array or matrix type. This method should be avoided for large problems.]

- **tol** : float, optional
  - Tolerance for ‘arpack’ method Not used if eigen_solver==’dense’.

- **max_iter** : integer
  - maximum number of iterations for the arpack solver.

  - standard [use the standard locally linear embedding algorithm.] see reference [R46]
  - hessian [use the Hessian eigenmap method. This method requires] n_neighbors > n_components * (1 + (n_components + 1) / 2. see reference [R47]
  - modified [use the modified locally linear embedding algorithm.] see reference [R48]
ltsa [use local tangent space alignment algorithm] see reference [R49]

hessian_tol : float, optional
Tolerance for Hessian eigenmapping method. Only used if method == ‘hessian’

modified_tol : float, optional
Tolerance for modified LLE method. Only used if method == ‘modified’

random_state: numpy.RandomState or int, optional :
The generator or seed used to determine the starting vector for arpack iterations. Defaults to numpy.random.

n_jobs : int, optional (default = 1)
The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores.

ReturnsY : array-like, shape [n_samples, n_components]
Embedding vectors.
squared_error : float
Reconstruction error for the embedding vectors. Equivalent to norm(Y - W Y, 'fro')**2, where W are the reconstruction weights.

References
[R46], [R47], [R48], [R49]

Examples using sklearn.manifold.locally_linear_embedding

• Swiss Roll reduction with LLE

29.19.7 sklearn.manifold.spectral_embedding

sklearn.manifold.spectral_embedding(adjacency, n_components=8, eigen_solver=None, random_state=None, eigen_tol=0.0, norm_laplacian=True, drop_first=True)
Project the sample on the first eigenvectors of the graph Laplacian.

The adjacency matrix is used to compute a normalized graph Laplacian whose spectrum (especially the eigenvectors associated to the smallest eigenvalues) has an interpretation in terms of minimal number of cuts necessary to split the graph into comparably sized components.

This embedding can also ‘work’ even if the adjacency variable is not strictly the adjacency matrix of a graph but more generally an affinity or similarity matrix between samples (for instance the heat kernel of a euclidean distance matrix or a k-NN matrix).

However care must taken to always make the affinity matrix symmetric so that the eigenvector decomposition works as expected.
Read more in the User Guide.

Parameters
adjacency : array-like or sparse matrix, shape: (n_samples, n_samples)
The adjacency matrix of the graph to embed.
n_components : integer, optional, default 8

The dimension of the projection subspace.

eigen_solver : {None, ‘arpack’, ‘lobpcg’, or ‘amg’}, default None

The eigenvalue decomposition strategy to use. AMG requires pyamg to be installed. It can be faster on very large, sparse problems, but may also lead to instabilities.

random_state : int seed, RandomState instance, or None (default)

A pseudo random number generator used for the initialization of the lobpcg eigenvectors decomposition when eigen_solver == ‘amg’. By default, arpack is used.

eigen_tol : float, optional, default=0.0

Stopping criterion for eigendecomposition of the Laplacian matrix when using arpack eigen_solver.

drop_first : bool, optional, default=True

Whether to drop the first eigenvector. For spectral embedding, this should be True as the first eigenvector should be constant vector for connected graph, but for spectral clustering, this should be kept as False to retain the first eigenvector.

norm_laplacian : bool, optional, default=True

If True, then compute normalized Laplacian.

Returnsembedding : array, shape=(n_samples, n_components)

The reduced samples.

Notes

Spectral embedding is most useful when the graph has one connected component. If there graph has many components, the first few eigenvectors will simply uncover the connected components of the graph.

References

•https://en.wikipedia.org/wiki/LOBPCG

•Toward the Optimal Preconditioned Eigensolver: Locally Optimal Block Preconditioned Conjugate Gradient Method Andrew V. Knyazev http://dx.doi.org/10.1137%2FS1064827500366124

29.20 sklearn.metrics: Metrics

See the Model evaluation: quantifying the quality of predictions section and the Pairwise metrics, Affinities and Kernels section of the user guide for further details. The sklearn.metrics module includes score functions, performance metrics and pairwise metrics and distance computations.

29.20.1 Model Selection Interface

See the The scoring parameter: defining model evaluation rules section of the user guide for further details.

metrics.make_scorer(score_func[, ...]) Make a scorer from a performance metric or loss function.
sklearn.metrics.make_scorer

sklearn.metrics.make_scorer(score_func, greater_is_better=True, needs_proba=False, needs_threshold=False, **kwargs)

Make a scorer from a performance metric or loss function.

This factory function wraps scoring functions for use in GridSearchCV and cross_val_score. It takes a score function, such as accuracy_score, mean_squared_error, adjusted_rand_index or average_precision and returns a callable that scores an estimator’s output.

Read more in the User Guide.

Parameters:
score_func : callable,
	Score function (or loss function) with signature score_func(y, y_pred, **kwargs).

greater_is_better : boolean, default=True
	Whether score_func is a score function (default), meaning high is good, or a loss function, meaning low is good. In the latter case, the scorer object will sign-flip the outcome of the score_func.

needs_proba : boolean, default=False
	Whether score_func requires predict_proba to get probability estimates out of a classifier.

needs_threshold : boolean, default=False
	Whether score_func takes a continuous decision certainty. This only works for binary classification using estimators that have either a decision_function or predict_proba method.

For example average_precision or the area under the roc curve can not be computed using discrete predictions alone.

**kwargs : additional arguments
	Additional parameters to be passed to score_func.

Returns:
scorer : callable
	Callable object that returns a scalar score; greater is better.

Examples

```python
>>> from sklearn.metrics import fbeta_score, make_scorer
>>> ftwo_scorer = make_scorer(fbeta_score, beta=2)
>>> ftwo_scorer
make_scorer(fbeta_score, beta=2)
>>> from sklearn.model_selection import GridSearchCV
>>> from sklearn.svm import LinearSVC
>>> grid = GridSearchCV(LinearSVC(), param_grid={'C': [1, 10]},
... scoring=ftwo_scorer)
```
29.20.2 Classification metrics

See the Classification metrics section of the user guide for further details.

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<td>metrics.accuracy_score(y_true, y_pred[, ...])</td>
<td>Accuracy classification score.</td>
</tr>
<tr>
<td>metrics.auc(x, y[, reorder])</td>
<td>Compute Area Under the Curve (AUC) using the trapezoidal rule.</td>
</tr>
<tr>
<td>metrics.average_precision_score(y_true, y_score)</td>
<td>Compute average precision (AP) from prediction scores.</td>
</tr>
<tr>
<td>metrics.brier_score_loss(y_true, y_prob[, ...])</td>
<td>Compute the Brier score.</td>
</tr>
<tr>
<td>metrics.classification_report(y_true, y_pred)</td>
<td>Build a text report showing the main classification metrics.</td>
</tr>
<tr>
<td>metrics.cohen_kappa_score(y1, y2[, labels, ...])</td>
<td>Cohen's kappa: a statistic that measures inter-annotator agreement.</td>
</tr>
<tr>
<td>metrics.confusion_matrix(y_true, y_pred[, ...])</td>
<td>Compute confusion matrix to evaluate the accuracy of a classification.</td>
</tr>
<tr>
<td>metrics.f1_score(y_true, y_pred[, labels, ...])</td>
<td>Compute the F1 score, also known as balanced F-score or F-measure.</td>
</tr>
<tr>
<td>metrics.fbeta_score(y_true, y_pred, beta[, ...])</td>
<td>Compute the F-beta score.</td>
</tr>
<tr>
<td>metrics.hamming_loss(y_true, y_pred[, ...])</td>
<td>Compute the average Hamming loss.</td>
</tr>
<tr>
<td>metrics.hinge_loss(y_true, pred_decision[, ...])</td>
<td>Average hinge loss (non-regularized).</td>
</tr>
<tr>
<td>metrics.jaccard_similarity_score(y_true, y_pred)</td>
<td>Jaccard similarity coefficient score.</td>
</tr>
<tr>
<td>metrics.log_loss(y_true, y_pred[, eps, ...])</td>
<td>Log loss, aka logistic loss or cross-entropy loss.</td>
</tr>
<tr>
<td>metrics.matthews_corrcoef(y_true, y_pred[, ...])</td>
<td>Compute the Matthews correlation coefficient (MCC) for binary classes.</td>
</tr>
<tr>
<td>metrics.precision_recall_curve(y_true, ...)</td>
<td>Compute precision-recall pairs for different probability thresholds.</td>
</tr>
<tr>
<td>metrics.precision_recall_fscore_support(...)</td>
<td>Compute precision, recall, F-measure and support for each class.</td>
</tr>
<tr>
<td>metrics.recall_score(y_true, y_pred[, ...])</td>
<td>Compute the precision.</td>
</tr>
<tr>
<td>metrics.roc_auc_score(y_true, y_pred[, ...])</td>
<td>Compute Area Under the Curve (AUC) from prediction scores.</td>
</tr>
<tr>
<td>metrics.roc_auc_score(y_true, y_score[, ...])</td>
<td>Compute Receiver operating characteristic (ROC).</td>
</tr>
<tr>
<td>metrics.zero_one_loss(y_true, y_pred[, ...])</td>
<td>Zero-one classification loss.</td>
</tr>
</tbody>
</table>

**sklearn.metrics.accuracy_score**

sklearn.metrics.accuracy_score(y_true, y_pred, normalize=True, sample_weight=None)

Accuracy classification score.

In multilabel classification, this function computes subset accuracy: the set of labels predicted for a sample must exactly match the corresponding set of labels in y_true.

Read more in the User Guide.

Parameters:

- **y_true** : 1d array-like, or label indicator array / sparse matrix
Ground truth (correct) labels.

\texttt{y\_pred} : 1d array-like, or label indicator array / sparse matrix

Predicted labels, as returned by a classifier.

\texttt{normalize} : bool, optional (default=True)

If \texttt{False}, return the number of correctly classified samples. Otherwise, return the fraction of correctly classified samples.

\texttt{sample\_weight} : array-like of shape = [n\_samples], optional

Sample weights.

Returnsscore : float

If \texttt{normalize = True}, return the correctly classified samples (float), else it returns the number of correctly classified samples (int).

The best performance is 1 with \texttt{normalize = True} and the number of samples with \texttt{normalize = False}.

See also:

\texttt{jaccard\_similarity\_score, hamming\_loss, zero\_one\_loss}

Notes

In binary and multiclass classification, this function is equal to the \texttt{jaccard\_similarity\_score} function.

Examples

```python
>>> import numpy as np
>>> from sklearn.metrics import accuracy_score
>>> y_pred = [0, 2, 1, 3]
>>> y_true = [0, 1, 2, 3]
>>> accuracy_score(y_true, y_pred)
0.5
>>> accuracy_score(y_true, y_pred, normalize=False)
2
```

In the multilabel case with binary label indicators:

```python
>>> accuracy_score(np.array([[0, 1], [1, 1]]), np.ones((2, 2))) 0.5
```

Examples using \texttt{sklearn.metrics.accuracy\_score}

- \texttt{Multi-class AdaBoosted Decision Trees}
- \texttt{Probabilistic predictions with Gaussian process classification (GPC)}
- \texttt{Classification of text documents using sparse features}
sklearn.metrics.auc

sklearn.metrics.auc(x, y, reorder=False)
Compute Area Under the Curve (AUC) using the trapezoidal rule.

This is a general function, given points on a curve. For computing the area under the ROC-curve, see roc_auc_score.

Parameters
- `x`: array, shape = [n]
  x coordinates.
- `y`: array, shape = [n]
  y coordinates.
- `reorder`: boolean, optional (default=False)
  If True, assume that the curve is ascending in the case of ties, as for an ROC curve. If the curve is non-ascending, the result will be wrong.

Returns
- `auc`: float

See also:
- `roc_auc_score` Computes the area under the ROC curve
- `precision_recall_curve` Compute precision-recall pairs for different probability thresholds

Examples

```python
>>> import numpy as np
>>> from sklearn import metrics
>>> y = np.array([1, 1, 2, 2])
>>> pred = np.array([0.1, 0.4, 0.35, 0.8])
>>> fpr, tpr, thresholds = metrics.roc_curve(y, pred, pos_label=2)
>>> metrics.auc(fpr, tpr)
0.75
```

Examples using sklearn.metrics.auc

- Species distribution modeling
- Sparse recovery: feature selection for sparse linear models
- Receiver Operating Characteristic (ROC)
- Receiver Operating Characteristic (ROC) with cross validation

sklearn.metrics.average_precision_score

sklearn.metrics.average_precision_score(y_true, y_score, average='macro', sample_weight=None)
Compute average precision (AP) from prediction scores.

This score corresponds to the area under the precision-recall curve.

Note: this implementation is restricted to the binary classification task or multilabel classification task.

**Parameters**

- `y_true` : array, shape = [n_samples] or [n_samples, n_classes]
  True binary labels in binary label indicators.

- `y_score` : array, shape = [n_samples] or [n_samples, n_classes]
  Target scores, can either be probability estimates of the positive class, confidence values, or non-thresholded measure of decisions (as returned by “decision_function” on some classifiers).

- `average` : string, [None, ‘micro’, ‘macro’ (default), ‘samples’, ‘weighted’]
  If `None`, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:
  - ‘micro’: Calculate metrics globally by considering each element of the label indicator matrix as a label.
  - ‘macro’: Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.
  - ‘weighted’: Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label).
  - ‘samples’: Calculate metrics for each instance, and find their average.

- `sample_weight` : array-like of shape = [n_samples], optional
  Sample weights.

**Returns**

- `average_precision` : float

See also:

- [roc_auc_score](https://scikit-learn.org/stable/modules/generated/sklearn.metrics.roc_auc_score.html) Area under the ROC curve

**References**

[R52]

**Examples**

```python
>>> import numpy as np
>>> from sklearn.metrics import average_precision_score
>>> y_true = np.array([0, 0, 1, 1])
>>> y_scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> average_precision_score(y_true, y_scores)
0.79...
```

**Examples using sklearn.metrics.average_precision_score**

- Precision-Recall
sklearn.metrics.brier_score_loss

The smaller the Brier score, the better, hence the naming with “loss”.

Across all items in a set N predictions, the Brier score measures the mean squared difference between (1) the predicted probability assigned to the possible outcomes for item i, and (2) the actual outcome. Therefore, the lower the Brier score is for a set of predictions, the better the predictions are calibrated. Note that the Brier score always takes on a value between zero and one, since this is the largest possible difference between a predicted probability (which must be between zero and one) and the actual outcome (which can take on values of only 0 and 1).

The Brier score is appropriate for binary and categorical outcomes that can be structured as true or false, but is inappropriate for ordinal variables which can take on three or more values (this is because the Brier score assumes that all possible outcomes are equivalently “distant” from one another). Which label is considered to be the positive label is controlled via the parameter pos_label, which defaults to 1.

Read more in the User Guide.

Parameters

- **y_true** : array, shape (n_samples,)
  True targets.

- **y_prob** : array, shape (n_samples,)
  Probabilities of the positive class.

- **sample_weight** : array-like of shape = [n_samples], optional
  Sample weights.

- **pos_label** : int or str, default=None
  Label of the positive class. If None, the maximum label is used as positive class

Return

- **score** : float
  Brier score

References

[R53]

Examples

```python
>>> import numpy as np
>>> from sklearn.metrics import brier_score_loss
>>> y_true = np.array([0, 1, 1, 0])
>>> y_true_categorical = np.array(['spam', 'ham', 'ham', 'spam'])
>>> y_prob = np.array([0.1, 0.9, 0.8, 0.3])
>>> brier_score_loss(y_true, y_prob)
0.037...
>>> brier_score_loss(y_true, 1-y_prob, pos_label=0)
0.037...
>>> brier_score_loss(y_true_categorical, y_prob, pos_label="ham")
0.037...
```
Examples using `sklearn.metrics.brier_score_loss`

- Probability calibration of classifiers
- Probability Calibration curves

`sklearn.metrics.classification_report`

`sklearn.metrics.classification_report` (y_true, y_pred, labels=None, target_names=None, sample_weight=None, digits=2)

Build a text report showing the main classification metrics

Read more in the User Guide.

**Parameters**
- **y_true**: 1d array-like, or label indicator array / sparse matrix
  Ground truth (correct) target values.
- **y_pred**: 1d array-like, or label indicator array / sparse matrix
  Estimated targets as returned by a classifier.
- **labels**: array, shape = [n_labels]
  Optional list of label indices to include in the report.
- **target_names**: list of strings
  Optional display names matching the labels (same order).
- **sample_weight**: array-like of shape = [n_samples], optional
  Sample weights.
- **digits**: int
  Number of digits for formatting output floating point values

**Returns**
- **report**: string
  Text summary of the precision, recall, F1 score for each class.

**Examples**

```python
>>> from sklearn.metrics import classification_report
>>> y_true = [0, 1, 2, 2, 2]
>>> y_pred = [0, 0, 2, 2, 1]
>>> target_names = ['class 0', 'class 1', 'class 2']
>>> print(classification_report(y_true, y_pred, target_names=target_names))
                precision    recall  f1-score support
   class 0       0.50      1.00    0.67        1
   class 1       0.00      0.00    0.00        1
   class 2       1.00      0.67    0.80        3
        avg / total 0.70      0.60    0.61        5
```
Examples using `sklearn.metrics.classification_report`

- Feature Union with Heterogeneous Data Sources
- Faces recognition example using eigenfaces and SVMs
- Recognizing hand-written digits
- Parameter estimation using grid search with cross-validation
- Restricted Boltzmann Machine features for digit classification
- Label Propagation digits: Demonstrating performance
- Label Propagation digits active learning
- Classification of text documents using sparse features
- Classification of text documents: using a MLComp dataset

`sklearn.metrics.cohen_kappa_score`

`sklearn.metrics.cohen_kappa_score(y1, y2, labels=None, weights=None)`

Cohen’s kappa: a statistic that measures inter-annotator agreement.

This function computes Cohen’s kappa [R199], a score that expresses the level of agreement between two annotators on a classification problem. It is defined as

$$\kappa = (p_o - p_e) / (1 - p_e)$$

where $p_o$ is the empirical probability of agreement on the label assigned to any sample (the observed agreement ratio), and $p_e$ is the expected agreement when both annotators assign labels randomly. $p_e$ is estimated using a per-annotator empirical prior over the class labels [R200].

Read more in the `User Guide`.

**Parameters**

- `y1` : array, shape = [n_samples]
  
  Labels assigned by the first annotator.

- `y2` : array, shape = [n_samples]
  
  Labels assigned by the second annotator. The kappa statistic is symmetric, so swapping $y1$ and $y2$ doesn’t change the value.

- `labels` : array, shape = [n_classes], optional
  
  List of labels to index the matrix. This may be used to select a subset of labels. If None, all labels that appear at least once in $y1$ or $y2$ are used.

- `weights` : str, optional
  
  List of weighting type to calculate the score. None means no weighted; “linear” means linear weighted; “quadratic” means quadratic weighted.

**Returns**

- `kappa` : float
  
  The kappa statistic, which is a number between -1 and 1. The maximum value means complete agreement; zero or lower means chance agreement.

**References**

[R199], [R200], [R201]
sklearn.metrics.confusion_matrix

`sklearn.metrics.confusion_matrix(y_true, y_pred, labels=None, sample_weight=None)`

Compute confusion matrix to evaluate the accuracy of a classification.

By definition a confusion matrix $C$ is such that $C_{i,j}$ is equal to the number of observations known to be in group $i$ but predicted to be in group $j$.

Thus in binary classification, the count of true negatives is $C_{0,0}$, false negatives is $C_{1,0}$, true positives is $C_{1,1}$ and false positives is $C_{0,1}$.

Read more in the User Guide.

Parameters

- `y_true` : array, shape = [n_samples]
  Ground truth (correct) target values.
- `y_pred` : array, shape = [n_samples]
  Estimated targets as returned by a classifier.
- `labels` : array, shape = [n_classes], optional
  List of labels to index the matrix. This may be used to reorder or select a subset of labels. If none is given, those that appear at least once in `y_true` or `y_pred` are used in sorted order.
- `sample_weight` : array-like of shape = [n_samples], optional
  Sample weights.

Returns

- `C` : array, shape = [n_classes, n_classes]
  Confusion matrix.

References

[R55]

Examples

```python
>>> from sklearn.metrics import confusion_matrix
>>> y_true = [2, 0, 2, 2, 0, 1]
>>> y_pred = [0, 0, 2, 2, 0, 2]
>>> confusion_matrix(y_true, y_pred)
array([[2, 0, 0],
       [0, 0, 1],
       [1, 0, 2]])
```

```python
>>> y_true = ["cat", "ant", "cat", "cat", "ant", "bird"]
>>> y_pred = ["ant", "ant", "cat", "cat", "ant", "cat"]
>>> confusion_matrix(y_true, y_pred, labels=["ant", "bird", "cat"])
array([[2, 0, 0],
       [0, 0, 1],
       [1, 0, 2]])
```
Examples using sklearn.metrics.confusion_matrix

- Faces recognition example using eigenfaces and SVMs
- Recognizing hand-written digits
- Confusion matrix
- Label Propagation digits: Demonstrating performance
- Label Propagation digits active learning
- Classification of text documents using sparse features
- Classification of text documents: using a MLComp dataset

sklearn.metrics.f1_score

sklearn.metrics.f1_score(y_true, y_pred, labels=None, pos_label=1, average='binary', sample_weight=None)
Compute the F1 score, also known as balanced F-score or F-measure
The F1 score can be interpreted as a weighted average of the precision and recall, where an F1 score reaches its best value at 1 and worst score at 0. The relative contribution of precision and recall to the F1 score are equal. The formula for the F1 score is:

\[
F1 = \frac{2 \times (precision \times recall)}{precision + recall}
\]

In the multi-class and multi-label case, this is the weighted average of the F1 score of each class.

Read more in the User Guide.

Parameters

- **y_true**: 1d array-like, or label indicator array / sparse matrix
  Ground truth (correct) target values.
- **y_pred**: 1d array-like, or label indicator array / sparse matrix
  Estimated targets as returned by a classifier.
- **labels**: list, optional
  The set of labels to include when average != 'binary', and their order if average is None. Labels present in the data can be excluded, for example to calculate a multiclass average ignoring a majority negative class, while labels not present in the data will result in 0 components in a macro average. For multilabel targets, labels are column indices. By default, all labels in y_true and y_pred are used in sorted order.
  Changed in version 0.17: parameter labels improved for multiclass problem.
- **pos_label**: str or int, 1 by default
  The class to report if average='binary' and the data is binary. If the data are multiclass or multilabel, this will be ignored; setting labels=[pos_label] and average != 'binary' will report scores for that label only.
- **average**: string, [None, ‘binary’ (default), ‘micro’, ‘macro’, ‘samples’, ‘weighted’]
  This parameter is required for multiclass/multilabel targets. If None, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:
'binary': Only report results for the class specified by pos_label. This is applicable only if targets \((y_{\text{true}}, y_{\text{pred}})\) are binary.

'micro': Calculate metrics globally by counting the total true positives, false negatives and false positives.

'macro': Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.

'weighted': Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label). This alters ‘macro’ to account for label imbalance; it can result in an F-score that is not between precision and recall.

'samples': Calculate metrics for each instance, and find their average (only meaningful for multilabel classification where this differs from accuracy_score).

sample_weight : array-like of shape = [n_samples], optional
    Sample weights.

Returns f1_score : float or array of float, shape = [n_unique_labels]
    F1 score of the positive class in binary classification or weighted average of the F1 scores of each class for the multiclass task.

References

[R56]

Examples

```python
>>> from sklearn.metrics import f1_score
>>> y_true = [0, 1, 2, 0, 1, 2]
>>> y_pred = [0, 2, 1, 0, 0, 1]
>>> f1_score(y_true, y_pred, average='macro')
0.26...
>>> f1_score(y_true, y_pred, average='micro')
0.33...
>>> f1_score(y_true, y_pred, average='weighted')
0.26...
>>> f1_score(y_true, y_pred, average=None)
array([ 0.8, 0., 0.])
```

Examples using sklearn.metrics.f1_score

- Probability Calibration curves

sklearn.metrics.fbeta_score

sklearn.metrics.fbeta_score(y_true, y_pred, beta, labels=None, pos_label=1, average='binary', sample_weight=None)

Compute the \(F_{\beta}\) score

The \(F_{\beta}\) score is the weighted harmonic mean of precision and recall, reaching its optimal value at 1 and its worst value at 0.
The \textit{beta} parameter determines the weight of precision in the combined score. \textit{beta} < 1 lends more weight to precision, while \textit{beta} > 1 favors recall (\textit{beta} \to 0 considers only precision, \textit{beta} \to \infty only recall).

Read more in the \textit{User Guide}.

\textbf{Parameters}

\texttt{y_true} : 1d array-like, or label indicator array / sparse matrix

Ground truth (correct) target values.

\texttt{y_pred} : 1d array-like, or label indicator array / sparse matrix

Estimated targets as returned by a classifier.

\texttt{beta} : float

Weight of precision in harmonic mean.

\texttt{labels} : list, optional

The set of labels to include when \texttt{average} != \textquote{binary}, and their order if \texttt{average} is None. Labels present in the data can be excluded, for example to calculate a multiclass average ignoring a majority negative class, while labels not present in the data will result in 0 components in a macro average. For multilabel targets, labels are column indices. By default, all labels in \texttt{y_true} and \texttt{y_pred} are used in sorted order.

Changed in version 0.17: parameter \texttt{labels} improved for multiclass problem.

\texttt{pos_label} : str or int, 1 by default

The class to report if \texttt{average=\textquote{binary}} and the data is binary. If the data are multiclass or multilabel, this will be ignored; setting \texttt{labels=[pos_label]} and \texttt{average} != \textquote{binary} will report scores for that label only.

\texttt{average} : string, [None, \textquote{binary} (default), \textquote{micro}, \textquote{macro}, \textquote{samples}, \textquote{weighted}]

This parameter is required for multiclass/multilabel targets. If None, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:

\textquote{binary}: Only report results for the class specified by \texttt{pos_label}. This is applicable only if targets (\texttt{y_true, y_pred}) are binary.

\textquote{micro}: Calculate metrics globally by counting the total true positives, false negatives and false positives.

\textquote{macro}: Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.

\textquote{weighted}: Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label). This alters \textquote{macro} to account for label imbalance; it can result in an F-score that is not between precision and recall.

\textquote{samples}: Calculate metrics for each instance, and find their average (only meaningful for multilabel classification where this differs from \textit{accuracy_score}).

\texttt{sample_weight} : array-like of shape = \texttt{[n_samples]}, optional

Sample weights.

\textbf{Returns}

\texttt{fbeta_score} : float (if average is not None) or array of float, shape = \texttt{[n_unique_labels]}

F-beta score of the positive class in binary classification or weighted average of the F-beta score of each class for the multiclass task.
References

[R206], [R207]

Examples

```python
>>> from sklearn.metrics import fbeta_score
>>> y_true = [0, 1, 2, 0, 1, 2]
>>> y_pred = [0, 2, 1, 0, 0, 1]
>>> fbeta_score(y_true, y_pred, average='macro', beta=0.5)
... 0.23...
>>> fbeta_score(y_true, y_pred, average='micro', beta=0.5)
... 0.33...
>>> fbeta_score(y_true, y_pred, average='weighted', beta=0.5)
... 0.23...
>>> fbeta_score(y_true, y_pred, average=None, beta=0.5)
... array([ 0.71..., 0., 0. ])
```

sklearn.metrics.hamming_loss

sklearn.metrics.hamming_loss(y_true, y_pred, labels=None, sample_weight=None, classes=None)

Compute the average Hamming loss.

The Hamming loss is the fraction of labels that are incorrectly predicted.

Read more in the User Guide.

Parameters

- **y_true** : 1d array-like, or label indicator array / sparse matrix
  Ground truth (correct) labels.

- **y_pred** : 1d array-like, or label indicator array / sparse matrix
  Predicted labels, as returned by a classifier.

- **labels** : array, shape = [n_labels], optional (default=None)
  Integer array of labels. If not provided, labels will be inferred from y_true and y_pred.
  New in version 0.18.

- **sample_weight** : array-like of shape = [n_samples], optional
  Sample weights.
  New in version 0.18.

- **classes** : array, shape = [n_labels], optional
  (deprecated) Integer array of labels. This parameter has been renamed to labels in version 0.18 and will be removed in 0.20.

Returns

- **loss** : float or int,
  Return the average Hamming loss between element of y_true and y_pred.
See also:

accuracy_score, jaccard_similarity_score, zero_one_loss

Notes

In multiclass classification, the Hamming loss correspond to the Hamming distance between y_true and y_pred which is equivalent to the subset zero_one_loss function.

In multilabel classification, the Hamming loss is different from the subset zero-one loss. The zero-one loss considers the entire set of labels for a given sample incorrect if it does entirely match the true set of labels. Hamming loss is more forgiving in that it penalizes the individual labels.

The Hamming loss is upperbounded by the subset zero-one loss. When normalized over samples, the Hamming loss is always between 0 and 1.

References

[R57], [R58]

Examples

```python
>>> from sklearn.metrics import hamming_loss
>>> y_pred = [1, 2, 3, 4]
>>> y_true = [2, 2, 3, 4]
>>> hamming_loss(y_true, y_pred)
0.25
```

In the multilabel case with binary label indicators:

```python
>>> hamming_loss(np.array([[0, 1], [1, 1]]), np.zeros((2, 2)))
0.75
```

Examples using sklearn.metrics.hamming_loss

- Model Complexity Influence

sklearn.metrics.hinge_loss

sklearn.metrics.hinge_loss (y_true, pred_decision, labels=None, sample_weight=None)

Average hinge loss (non-regularized)

In binary class case, assuming labels in y_true are encoded with +1 and -1, when a prediction mistake is made, margin = y_true * pred_decision is always negative (since the signs disagree), implying 1 -margin is always greater than 1. The cumulated hinge loss is therefore an upper bound of the number of mistakes made by the classifier.

In multiclass case, the function expects that either all the labels are included in y_true or an optional labels argument is provided which contains all the labels. The multilabel margin is calculated according to Crammer-Singer’s method. As in the binary case, the cumulated hinge loss is an upper bound of the number of mistakes made by the classifier.

Read more in the User Guide.
Parameters

- **y_true**: array, shape = [n_samples]
  True target, consisting of integers of two values. The positive label must be greater than the negative label.

- **pred_decision**: array, shape = [n_samples] or [n_samples, n_classes]
  Predicted decisions, as output by decision_function (floats).

- **labels**: array, optional, default None
  Contains all the labels for the problem. Used in multiclass hinge loss.

- **sample_weight**: array-like of shape = [n_samples], optional
  Sample weights.

Returns

- **loss**: float

References

[R212], [R213], [R214]

Examples

```python
>>> from sklearn import svm
>>> from sklearn.metrics import hinge_loss
>>> X = [[0], [1]]
>>> y = [-1, 1]
>>> est = svm.LinearSVC(random_state=0)
>>> est.fit(X, y)
LinearSVC(C=1.0, class_weight=None, dual=True, fit_intercept=True,
         intercept_scaling=1, loss='squared_hinge', max_iter=1000,
         multi_class='ovr', penalty='l2', random_state=0, tol=0.0001,
         verbose=0)
>>> pred_decision = est.decision_function([[-2], [3], [0.5]])
>>> pred_decision
array([-2.18..., 2.36..., 0.09...])
>>> hinge_loss([-1, 1, 1], pred_decision)
0.30...
```

In the multiclass case:

```python
>>> X = np.array([[0], [1], [2], [3]])
>>> Y = np.array([0, 1, 2, 3])
>>> labels = np.array([0, 1, 2, 3])
>>> est = svm.LinearSVC()
>>> est.fit(X, Y)
LinearSVC(C=1.0, class_weight=None, dual=True, fit_intercept=True,
         intercept_scaling=1, loss='squared_hinge', max_iter=1000,
         multi_class='ovr', penalty='l2', random_state=None, tol=0.0001,
         verbose=0)
>>> pred_decision = est.decision_function([[-1], [2], [3]])
>>> y_true = [0, 2, 3]
>>> hinge_loss(y_true, pred_decision, labels)
0.56...
```
sklearn.metrics.jaccard_similarity_score

sklearn.metrics.jaccard_similarity_score(y_true, y_pred, normalize=True, sample_weight=None)

Jaccard similarity coefficient score

The Jaccard index [1], or Jaccard similarity coefficient, defined as the size of the intersection divided by the size of the union of two label sets, is used to compare set of predicted labels for a sample to the corresponding set of labels in y_true.

Read more in the User Guide.

Parameters

- **y_true**: 1d array-like, or label indicator array / sparse matrix
  - Ground truth (correct) labels.
- **y_pred**: 1d array-like, or label indicator array / sparse matrix
  - Predicted labels, as returned by a classifier.
- **normalize**: bool, optional (default=True)
  - If False, return the sum of the Jaccard similarity coefficient over the sample set. Otherwise, return the average of Jaccard similarity coefficient.
- **sample_weight**: array-like of shape = [n_samples], optional
  - Sample weights.

Returns

- **score**: float
  - If normalize == True, return the average Jaccard similarity coefficient, else it returns the sum of the Jaccard similarity coefficient over the sample set.
  - The best performance is 1 with normalize == True and the number of samples with normalize == False.

See also:

accuracy_score, hamming_loss, zero_one_loss

Notes

In binary and multiclass classification, this function is equivalent to the accuracy_score. It differs in the multilabel classification problem.

References

[R216]

Examples

```python
>>> import numpy as np
>>> from sklearn.metrics import jaccard_similarity_score
>>> y_pred = [0, 2, 1, 3]
>>> y_true = [0, 1, 2, 3]
>>> jaccard_similarity_score(y_true, y_pred)
0.5
```
In the multilabel case with binary label indicators:

```python
>>> jaccard_similarity_score(np.array([[0, 1], [1, 1]]), np.ones((2, 2)))
0.75
```

### sklearn.metrics.log_loss

**sklearn.metrics.log_loss** *(y_true, y_pred, epsilon=1e-15, normalize=True, sample_weight=None, labels=None)*  

Log loss, aka logistic loss or cross-entropy loss. This is the loss function used in (multinomial) logistic regression and extensions of it such as neural networks, defined as the negative log-likelihood of the true labels given a probabilistic classifier’s predictions. The log loss is only defined for two or more labels. For a single sample with true label yt in {0,1} and estimated probability yp that yt = 1, the log loss is

\[-\log P(y_t|y_p) = -(y_t \log(y_p) + (1 - y_t) \log(1 - y_p))\]

Read more in the User Guide.

**Parameters**

- **y_true**: array-like or label indicator matrix  
  Ground truth (correct) labels for n_samples samples.

- **y_pred**: array-like of float, shape = (n_samples, n_classes) or (n_samples,)  
  Predicted probabilities, as returned by a classifier’s predict_proba method. If y_pred.shape = (n_samples,) the probabilities provided are assumed to be that of the positive class. The labels in y_pred are assumed to be ordered alphabetically, as done by preprocessing.LabelBinarizer.

- **epsilon**: float  
  Log loss is undefined for p=0 or p=1, so probabilities are clipped to max(epsilon, min(1 - epsilon, p)).

- **normalize**: bool, optional (default=True)  
  If true, return the mean loss per sample. Otherwise, return the sum of the per-sample losses.

- **sample_weight**: array-like of shape = [n_samples], optional  
  Sample weights.

- **labels**: array-like, optional (default=None)  
  If not provided, labels will be inferred from y_true. If labels is None and y_pred has shape (n_samples,) the labels are assumed to be binary and are inferred from y_true ...

**Returns**

- **loss**: float

**Notes**

The logarithm used is the natural logarithm (base-e).
References


Examples

```python
>>> log_loss(['spam', 'ham', 'ham', 'spam'],
... [[.1, .9], [.9, .1], [.8, .2], [.35, .65]])
0.21616...
```

Examples using `sklearn.metrics.log_loss`

- Probability Calibration for 3-class classification
- Probabilistic predictions with Gaussian process classification (GPC)

`sklearn.metrics.matthews_corrcoef`

`sklearn.metrics.matthews_corrcoef (y_true, y_pred, sample_weight=None)`

Compute the Matthews correlation coefficient (MCC) for binary classes

The Matthews correlation coefficient is used in machine learning as a measure of the quality of binary (two-class) classifications. It takes into account true and false positives and negatives and is generally regarded as a balanced measure which can be used even if the classes are of very different sizes. The MCC is in essence a correlation coefficient value between -1 and +1. A coefficient of +1 represents a perfect prediction, 0 an average random prediction and -1 an inverse prediction. The statistic is also known as the phi coefficient. [source: Wikipedia]

Only in the binary case does this relate to information about true and false positives and negatives. See references below.

Read more in the User Guide.

Parameters

- `y_true` : array, shape = [n_samples]
  Ground truth (correct) target values.
- `y_pred` : array, shape = [n_samples]
  Estimated targets as returned by a classifier.
- `sample_weight` : array-like of shape = [n_samples], default None
  Sample weights.

Returns

- `mcc` : float
  The Matthews correlation coefficient (+1 represents a perfect prediction, 0 an average random prediction and -1 an inverse prediction).

References

[R218], [R219]
Examples

```python
>>> from sklearn.metrics import matthews_corrcoef
>>> y_true = [+1, +1, +1, -1]
>>> y_pred = [+1, -1, +1, +1]
>>> matthews_corrcoef(y_true, y_pred)
-0.33...
```

**sklearn.metrics.precision_recall_curve**

`sklearn.metrics.precision_recall_curve(y_true, probas_pred, pos_label=None, sample_weight=None)`

Compute precision-recall pairs for different probability thresholds

Note: this implementation is restricted to the binary classification task.

The precision is the ratio $tp / (tp + fp)$ where $tp$ is the number of true positives and $fp$ the number of false positives. The precision is intuitively the ability of the classifier not to label as positive a sample that is negative.

The recall is the ratio $tp / (tp + fn)$ where $tp$ is the number of true positives and $fn$ the number of false negatives. The recall is intuitively the ability of the classifier to find all the positive samples.

The last precision and recall values are 1. and 0. respectively and do not have a corresponding threshold. This ensures that the graph starts on the x axis.

Read more in the *User Guide*.

**Parameters**

- **y_true** : array, shape = [n_samples]
  
  True targets of binary classification in range {-1, 1} or {0, 1}.

- **probas_pred** : array, shape = [n_samples]
  
  Estimated probabilities or decision function.

- **pos_label** : int or str, default=None
  
  The label of the positive class.

- **sample_weight** : array-like of shape = [n_samples], optional
  
  Sample weights.

**Returns**

- **precision** : array, shape = [n_thresholds + 1]
  
  Precision values such that element i is the precision of predictions with score $\geq$ thresholds[i] and the last element is 1.

- **recall** : array, shape = [n_thresholds + 1]
  
  Decreasing recall values such that element i is the recall of predictions with score $\geq$ thresholds[i] and the last element is 0.

- **thresholds** : array, shape = [n_thresholds <= len(np.unique(probas_pred))]
  
  Increasing thresholds on the decision function used to compute precision and recall.
Examples

```python
>>> import numpy as np
>>> from sklearn.metrics import precision_recall_curve

>>> y_true = np.array([0, 0, 1, 1])
>>> y_scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> precision, recall, thresholds = precision_recall_curve(y_true, y_scores)

>>> precision
array([0.66..., 0.5 , 1. , 1.])
>>> recall
array([1. , 0.5, 0.5, 0.])
>>> thresholds
array([0.35, 0.4 , 0.8 ])
```

Examples using `sklearn.metrics.precision_recall_curve`

- Sparse recovery: feature selection for sparse linear models
- Precision-Recall

`sklearn.metrics.precision_recall_fscore_support`

`sklearn.metrics.precision_recall_fscore_support` computes precision, recall, F-measure and support for each class.

The precision is the ratio $\frac{tp}{(tp + fp)}$ where $tp$ is the number of true positives and $fp$ the number of false positives. The precision is intuitively the ability of the classifier not to label as positive a sample that is negative.

The recall is the ratio $\frac{tp}{(tp + fn)}$ where $tp$ is the number of true positives and $fn$ the number of false negatives. The recall is intuitively the ability of the classifier to find all the positive samples.

The F-beta score can be interpreted as a weighted harmonic mean of the precision and recall, where an F-beta score reaches its best value at 1 and worst score at 0.

The F-beta score weights recall more than precision by a factor of $\beta$. $\beta = 1.0$ means recall and precision are equally important.

The support is the number of occurrences of each class in $y_{true}$. $\beta = 1.0$ means recall and precision are equally important.

If `pos_label` is `None` and in binary classification, this function returns the average precision, recall and F-measure if `average` is one of 'micro', 'macro', 'weighted' or 'samples'.

Read more in the User Guide.

**Parameters**

- `y_true` : 1d array-like, or label indicator array / sparse matrix
  Ground truth (correct) target values.
- `y_pred` : 1d array-like, or label indicator array / sparse matrix
  Estimated targets as returned by a classifier.
- `beta` : float, 1.0 by default
The strength of recall versus precision in the F-score.

**labels**: list, optional

The set of labels to include when `average != 'binary'`, and their order if `average is None`. Labels present in the data can be excluded, for example to calculate a multiclass average ignoring a majority negative class, while labels not present in the data will result in 0 components in a macro average. For multilabel targets, labels are column indices. By default, all labels in `y_true` and `y_pred` are used in sorted order.

**pos_label**: str or int, 1 by default

The class to report if `average='binary'` and the data is binary. If the data are multiclass or multilabel, this will be ignored; setting `labels=[pos_label]` and `average != 'binary'` will report scores for that label only.

**average**: string, [None (default), 'binary', 'micro', 'macro', 'samples', 'weighted']

If `None`, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:

- 'binary': Only report results for the class specified by `pos_label`. This is applicable only if targets (`y_{true,pred}`) are binary.
- 'micro': Calculate metrics globally by counting the total true positives, false negatives and false positives.
- 'macro': Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.
- 'weighted': Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label). This alters 'macro' to account for label imbalance; it can result in an F-score that is not between precision and recall.
- 'samples': Calculate metrics for each instance, and find their average (only meaningful for multilabel classification where this differs from `accuracy_score`).

**warn_for**: tuple or set, for internal use

This determines which warnings will be made in the case that this function is being used to return only one of its metrics.

**sample_weight**: array-like of shape = `[n_samples]`, optional

Sample weights.

Returns

- **precision**: float (if average is not None) or array of float, shape = `[n_unique_labels]`
- **recall**: float (if average is not None) or array of float, shape = `[n_unique_labels]`
- **fbeta_score**: float (if average is not None) or array of float, shape = `[n_unique_labels]`
- **support**: int (if average is not None) or array of int, shape = `[n_unique_labels]`

The number of occurrences of each label in `y_true`.

**References**

[R220], [R221], [R222]
Examples

```python
>>> from sklearn.metrics import precision_recall_fscore_support
>>> y_true = np.array(['cat', 'dog', 'pig', 'cat', 'dog', 'pig'])
>>> y_pred = np.array(['cat', 'pig', 'dog', 'cat', 'cat', 'dog'])
>>> precision_recall_fscore_support(y_true, y_pred, average='macro')
(0.22..., 0.33..., 0.26..., None)
>>> precision_recall_fscore_support(y_true, y_pred, average='micro')
(0.33..., 0.33..., 0.33..., None)
>>> precision_recall_fscore_support(y_true, y_pred, average='weighted')
(0.22..., 0.33..., 0.26..., None)
```

It is possible to compute per-label precisions, recalls, F1-scores and supports instead of averaging: >>> precision_recall_fscore_support(y_true, y_pred, average=None, ... labels=['pig', 'dog', 'cat']) ... # doctest: +ELLIPSIS,+NORMALIZE_WHITESPACE

```python
array([ 0., 0., 1.], array([ 0., 0., 0.8]), array([2, 2, 2]))
```

`sklearn.metrics.precision_score`

`sklearn.metrics.precision_score(y_true, y_pred, labels=None, pos_label=1, average='binary', sample_weight=None)`

Compute the precision

The precision is the ratio $tp / (tp + fp)$ where $tp$ is the number of true positives and $fp$ the number of false positives. The precision is intuitively the ability of the classifier not to label as positive a sample that is negative.

The best value is 1 and the worst value is 0.

Read more in the `User Guide`.

Parameters:

- `y_true`: 1d array-like, or label indicator array / sparse matrix
  - Ground truth (correct) target values.

- `y_pred`: 1d array-like, or label indicator array / sparse matrix
  - Estimated targets as returned by a classifier.

- `labels`: list, optional
  - The set of labels to include when average != 'binary', and their order if average is None. Labels present in the data can be excluded, for example to calculate a multiclass average ignoring a majority negative class, while labels not present in the data will result in 0 components in a macro average. For multilabel targets, labels are column indices. By default, all labels in y_true and y_pred are used in sorted order.

- `pos_label`: str or int, 1 by default
  - The class to report if average='binary' and the data is binary. If the data are multiclass or multilabel, this will be ignored; setting labels=[pos_label] and average != 'binary' will report scores for that label only.

This parameter is required for multiclass/multilabel targets. If None, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:

'binary': Only report results for the class specified by pos_label. This is applicable only if targets (y_{true,pred}) are binary.

'micro': Calculate metrics globally by counting the total true positives, false negatives and false positives.

'macro': Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.

'weighted': Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label). This alters ‘macro’ to account for label imbalance; it can result in an F-score that is not between precision and recall.

'samples': Calculate metrics for each instance, and find their average (only meaningful for multilabel classification where this differs from accuracy_score).

**sample_weight**: array-like of shape = [n_samples], optional
Sample weights.

**Returns**
precision : float (if average is not None) or array of float, shape = [n_unique_labels]
Precision of the positive class in binary classification or weighted average of the precision of each class for the multiclass task.

**Examples**

```python
>>> from sklearn.metrics import precision_score
>>> y_true = [0, 1, 2, 0, 1, 2]
>>> y_pred = [0, 2, 1, 0, 0, 1]
>>> precision_score(y_true, y_pred, average='macro')
0.22...
>>> precision_score(y_true, y_pred, average='micro')
0.33...
>>> precision_score(y_true, y_pred, average='weighted')
...
0.22...
>>> precision_score(y_true, y_pred, average=None)
array([ 0.66..., 0., 0.])
```

**Examples using sklearn.metrics.precision_score**

- Probability Calibration curves

**sklearn.metrics.recall_score**

```python
sklearn.metrics.recall_score(y_true, y_pred, labels=None, pos_label=1, average='binary', sample_weight=None)
```

Compute the recall

The recall is the ratio $tp / (tp + fn)$ where $tp$ is the number of true positives and $fn$ the number of false negatives. The recall is intuitively the ability of the classifier to find all the positive samples.
The best value is 1 and the worst value is 0.
Read more in the User Guide.

**Parameters**

- **y_true**: 1d array-like, or label indicator array / sparse matrix
  Ground truth (correct) target values.
- **y_pred**: 1d array-like, or label indicator array / sparse matrix
  Estimated targets as returned by a classifier.
- **labels**: list, optional
  The set of labels to include when `average != 'binary'`, and their order if `average is None`. Labels present in the data can be excluded, for example to calculate a multiclass average ignoring a majority negative class, while labels not present in the data will result in 0 components in a macro average. For multilabel targets, labels are column indices. By default, all labels in `y_true` and `y_pred` are used in sorted order.

Changed in version 0.17: parameter `labels` improved for multiclass problem.

- **pos_label**: str or int, 1 by default
  The class to report if `average='binary'` and the data is binary. If the data are multiclass or multilabel, this will be ignored; setting `labels=[pos_label]` and `average != 'binary'` will report scores for that label only.

- **average**: string, [None, 'binary' (default), 'micro', 'macro', 'samples', 'weighted']
  This parameter is required for multiclass/multilabel targets. If None, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:

  - `binary`: Only report results for the class specified by `pos_label`. This is applicable only if targets (`y_{true,pred}`) are binary.
  - `micro`: Calculate metrics globally by counting the total true positives, false negatives and false positives.
  - `macro`: Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.
  - `weighted`: Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label). This alters ‘macro’ to account for label imbalance; it can result in an F-score that is not between precision and recall.
  - `samples`: Calculate metrics for each instance, and find their average (only meaningful for multilabel classification where this differs from accuracy_score).

- **sample_weight**: array-like of shape = [n_samples], optional
  Sample weights.

**Returns**

- **recall**: float (if average is not None) or array of float, shape = [n_unique_labels]
  Recall of the positive class in binary classification or weighted average of the recall of each class for the multiclass task.
Examples

```python
>>> from sklearn.metrics import recall_score
>>> y_true = [0, 1, 2, 0, 1, 2]
>>> y_pred = [0, 2, 1, 0, 0, 1]
>>> recall_score(y_true, y_pred, average='macro')
0.33...
>>> recall_score(y_true, y_pred, average='micro')
0.33...
>>> recall_score(y_true, y_pred, average='weighted')
0.33...
>>> recall_score(y_true, y_pred, average=None)
array([ 1.,  0.,  0.])
```

Examples using `sklearn.metrics.recall_score`

- **Probability Calibration curves**

`sklearn.metrics.roc_auc_score`

`sklearn.metrics.roc_auc_score(y_true, y_score, average='macro', sample_weight=None)`

Compute Area Under the Curve (AUC) from prediction scores

Note: this implementation is restricted to the binary classification task or multilabel classification task in label indicator format.

Read more in the User Guide.

Parameters:
- `y_true`: array, shape = [n_samples] or [n_samples, n_classes]
  True binary labels in binary label indicators.
- `y_score`: array, shape = [n_samples] or [n_samples, n_classes]
  Target scores, can either be probability estimates of the positive class, confidence values, or non-thresholded measures of decisions (as returned by “decision_function” on some classifiers).
- `average`: string, [None, ‘micro’, ‘macro’ (default), ‘samples’, ‘weighted’]
  If None, the scores for each class are returned. Otherwise, this determines the type of averaging performed on the data:
  - ‘micro’: Calculate metrics globally by considering each element of the label indicator matrix as a label.
  - ‘macro’: Calculate metrics for each label, and find their unweighted mean. This does not take label imbalance into account.
  - ‘weighted’: Calculate metrics for each label, and find their average, weighted by support (the number of true instances for each label).
  - ‘samples’: Calculate metrics for each instance, and find their average.
- `sample_weight`: array-like of shape = [n_samples], optional
  Sample weights.

Returns:
- `auc`: float

29.20. `sklearn.metrics`: Metrics
See also:

average_precision_score Area under the precision-recall curve
roc_curve Compute Receiver operating characteristic (ROC)

References

[R224]

Examples

```python
>>> import numpy as np
>>> from sklearn.metrics import roc_auc_score
>>> y_true = np.array([0, 0, 1, 1])
>>> y_scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> roc_auc_score(y_true, y_scores)
0.75
```

sklearn.metrics.roc_curve

sklearn.metrics.roc_curve(y_true, y_score, pos_label=None, sample_weight=None, drop_intermediate=True)

Compute Receiver operating characteristic (ROC)

Note: this implementation is restricted to the binary classification task.

Read more in the `User Guide`

**Parameters**

- **y_true** : array, shape = [n_samples]
  
  True binary labels in range {0, 1} or {-1, 1}. If labels are not binary, pos_label should be explicitly given.

- **y_score** : array, shape = [n_samples]
  
  Target scores, can either be probability estimates of the positive class, confidence values, or non-thresholded measure of decisions (as returned by “decision_function” on some classifiers).

- **pos_label** : int or str, default=None
  
  Label considered as positive and others are considered negative.

- **sample_weight** : array-like of shape = [n_samples], optional
  
  Sample weights.

- **drop_intermediate** : boolean, optional (default=True)
  
  Whether to drop some suboptimal thresholds which would not appear on a plotted ROC curve. This is useful in order to create lighter ROC curves.

New in version 0.17: parameter drop_intermediate.

**Returns**

- **fpr** : array, shape = [>2]
  
  Increasing false positive rates such that element i is the false positive rate of predictions with score >= thresholds[i].
tpr: array, shape = [2]
Increasing true positive rates such that element i is the true positive rate of predictions
with score >= thresholds[i].

thresholds: array, shape = [n_thresholds]
Decreasing thresholds on the decision function used to compute fpr and tpr. thresholds[0]
represents no instances being predicted and is arbitrarily set to max(y_score) + 1.

See also:

roc_auc_score: Compute Area Under the Curve (AUC) from prediction scores

Notes
Since the thresholds are sorted from low to high values, they are reversed upon returning
them to ensure they correspond to both fpr and tpr, which are sorted in reversed order
during their calculation.

Examples

```python
>>> import numpy as np
>>> from sklearn import metrics
>>> y = np.array([1, 1, 2, 2])
>>> scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> fpr, tpr, thresholds = metrics.roc_curve(y, scores, pos_label=2)
>>> fpr
array([ 0. , 0.5, 0.5, 1. ])
>>> tpr
array([ 0.5, 0.5, 1. , 1. ])
>>> thresholds
array([ 0.8 , 0.4 , 0.35, 0.1 ])
```

Examples using sklearn.metrics.roc_curve

- *Species distribution modeling*
- *Feature transformations with ensembles of trees*
- *Receiver Operating Characteristic (ROC)*
- *Receiver Operating Characteristic (ROC) with cross validation*

sklearn.metrics.zero_one_loss

sklearn.metrics.zero_one_loss(y_true, y_pred, normalize=True, sample_weight=None)
Zero-one classification loss.
If normalize is True, return the fraction of misclassifications (float), else it returns the number of misclassifications (int). The best performance is 0.

Read more in the User Guide.

**Parameters**

- `y_true` : 1d array-like, or label indicator array / sparse matrix
  
  Ground truth (correct) labels.

- `y_pred` : 1d array-like, or label indicator array / sparse matrix
  
  Predicted labels, as returned by a classifier.

- `normalize` : bool, optional (default=True)
  
  If False, return the number of misclassifications. Otherwise, return the fraction of misclassifications.

- `sample_weight` : array-like of shape = [n_samples], optional
  
  Sample weights.

**Returns**

- `loss` : float or int,
  
  If normalize == True, return the fraction of misclassifications (float), else it returns the number of misclassifications (int).

**See also:**

accuracy_score, hamming_loss, jaccard_similarity_score

**Notes**

In multilabel classification, the zero_one_loss function corresponds to the subset zero-one loss: for each sample, the entire set of labels must be correctly predicted, otherwise the loss for that sample is equal to one.

**Examples**

```python
>>> from sklearn.metrics import zero_one_loss
>>> y_pred = [1, 2, 3, 4]
>>> y_true = [2, 2, 3, 4]
>>> zero_one_loss(y_true, y_pred)
0.25
>>> zero_one_loss(y_true, y_pred, normalize=False)
1
```

In the multilabel case with binary label indicators:

```python
>>> zero_one_loss(np.array([[0, 1], [1, 1]]), np.ones((2, 2)))
0.5
```

**Examples using sklearn.metrics.zero_one_loss**

- Discrete versus Real AdaBoost
29.20.3 Regression metrics

See the Regression metrics section of the user guide for further details.

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**sklearn.metrics.explained_variance_score**

`sklearn.metrics.explained_variance_score(y_true, y_pred, sample_weight=None, multioutput='uniform_average')`

Explained variance regression score function

Best possible score is 1.0, lower values are worse.

Read more in the User Guide.

**Parameters**

- `y_true` : array-like of shape (n_samples) or (n_samples, n_outputs)
  
  Ground truth (correct) target values.

- `y_pred` : array-like of shape (n_samples) or (n_samples, n_outputs)
  
  Estimated target values.

- `sample_weight` : array-like of shape (n_samples), optional
  
  Sample weights.

- `multioutput` : string in ['raw_values', 'uniform_average', 'variance_weighted'] or array-like of shape (n_outputs)
  
  Defines aggregating of multiple output scores. Array-like value defines weights used to average scores.

  - ‘raw_values’ : Returns a full set of scores in case of multioutput input.
  - ‘uniform_average’ : Scores of all outputs are averaged with uniform weight.
  - ‘variance_weighted’ : Scores of all outputs are averaged, weighted by the variances of each individual output.

**Return**

- `score` : float or ndarray of floats
  
  The explained variance or ndarray if ‘multioutput’ is ‘raw_values’.

**Notes**

This is not a symmetric function.
Examples

```python
>>> from sklearn.metrics import explained_variance_score
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> explained_variance_score(y_true, y_pred)
0.957...
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
>>> explained_variance_score(y_true, y_pred, multioutput='uniform_average')
... 0.983...
```

**sklearn.metrics.mean_absolute_error**

`sklearn.metrics.mean_absolute_error(y_true, y_pred, sample_weight=None, multioutput='uniform_average')`

Mean absolute error regression loss

Read more in the User Guide.

**Parameters**

- `y_true` : array-like of shape = (n_samples) or (n_samples, n_outputs)
  Ground truth (correct) target values.
- `y_pred` : array-like of shape = (n_samples) or (n_samples, n_outputs)
  Estimated target values.
- `sample_weight` : array-like of shape = (n_samples), optional
  Sample weights.
- `multioutput` : string in ['raw_values', 'uniform_average']
  or array-like of shape (n_outputs) Defines aggregating of multiple output values. Array-like value defines weights used to average errors.
  - `'raw_values'` :Returns a full set of errors in case of multioutput input.
  - `'uniform_average'` :Errors of all outputs are averaged with uniform weight.

**Returns**

- `loss` : float or ndarray of floats
  If multioutput is ‘raw_values’, then mean absolute error is returned for each output separately. If multioutput is ‘uniform_average’ or an ndarray of weights, then the weighted average of all output errors is returned.

MAE output is non-negative floating point. The best value is 0.0.

Examples

```python
>>> from sklearn.metrics import mean_absolute_error
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> mean_absolute_error(y_true, y_pred)
0.5
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
```
>>> mean_absolute_error(y_true, y_pred)
0.75
>>> mean_absolute_error(y_true, y_pred, multioutput='raw_values')
array([ 0.5, 1. ])
>>> mean_absolute_error(y_true, y_pred, multioutput=[0.3, 0.7])
...
0.849...

sklearn.metrics.mean_squared_error

sklearn.metrics.

sklearn.metrics.mean_squared_error(y_true, y_pred, sample_weight=None, multioutput='uniform_average')

Mean squared error regression loss

Read more in the User Guide.

Parameters:

y_true : array-like of shape = (n_samples) or (n_samples, n_outputs)
Ground truth (correct) target values.
y_pred : array-like of shape = (n_samples) or (n_samples, n_outputs)
Estimated target values.
sample_weight : array-like of shape = (n_samples), optional
Sample weights.
multioutput : string in ['raw_values', 'uniform_average']
or array-like of shape (n_outputs) Defines aggregating of multiple output values. Array-like value defines weights used to average errors.
'raw_values' : Returns a full set of errors in case of multioutput input.
'uniform_average' : Errors of all outputs are averaged with uniform weight.

Returns:

loss : float or ndarray of floats
A non-negative floating point value (the best value is 0.0), or an array of floating point values, one for each individual target.

Examples

>>> from sklearn.metrics import mean_squared_error
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> mean_squared_error(y_true, y_pred)
0.375
>>> y_true = [[0.5, 1],[-1, 1],[7, -6]]
>>> y_pred = [[0, 2],[-1, 2],[8, -5]]
>>> mean_squared_error(y_true, y_pred)
0.708...
>>> mean_squared_error(y_true, y_pred, multioutput='raw_values')
...
array([ 0.416..., 1. ])
>>> mean_squared_error(y_true, y_pred, multioutput=[0.3, 0.7])
...
0.824...
Examples using `sklearn.metrics.mean_squared_error`

- Model Complexity Influence
- Gradient Boosting regression
- Plot Ridge coefficients as a function of the L2 regularization
- Robust linear estimator fitting

`sklearn.metrics.median_absolute_error`

`sklearn.metrics.median_absolute_error(y_true, y_pred)`
Median absolute error regression loss

Read more in the User Guide.

**Parameters**

- `y_true` : array-like of shape = (n_samples)
  Ground truth (correct) target values.
- `y_pred` : array-like of shape = (n_samples)
  Estimated target values.

**Returns**

- `loss` : float
  A positive floating point value (the best value is 0.0).

**Examples**

```python
>>> from sklearn.metrics import median_absolute_error
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> median_absolute_error(y_true, y_pred)
0.5
```

`sklearn.metrics.r2_score`

`sklearn.metrics.r2_score(y_true, y_pred, sample_weight=None, multioutput=None)`
R^2 (coefficient of determination) regression score function.

Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

Read more in the User Guide.

**Parameters**

- `y_true` : array-like of shape = (n_samples) or (n_samples, n_outputs)
  Ground truth (correct) target values.
- `y_pred` : array-like of shape = (n_samples) or (n_samples, n_outputs)
  Estimated target values.
- `sample_weight` : array-like of shape = (n_samples), optional
  Sample weights.
**multioutput** : string in ['raw_values', 'uniform_average', 'variance_weighted'] or None or array-like of shape (n_outputs)

Defines aggregating of multiple output scores. Array-like value defines weights used to average scores. Default value corresponds to ‘variance_weighted’, this behaviour is deprecated since version 0.17 and will be changed to ‘uniform_average’ starting from 0.19.

‘raw_values’ : Returns a full set of scores in case of multioutput input.
‘uniform_average’ : Scores of all outputs are averaged with uniform weight.
‘variance_weighted’ : Scores of all outputs are averaged, weighted by the variances of each individual output.

**Returns** : float or ndarray of floats

The $R^2$ score or ndarray of scores if ‘multioutput’ is ‘raw_values’.

**Notes**

This is not a symmetric function.

Unlike most other scores, $R^2$ score may be negative (it need not actually be the square of a quantity $R$).

**References**

[R60]

**Examples**

```python
>>> from sklearn.metrics import r2_score
>>> y_true = [3, -0.5, 2, 7]
>>> y_pred = [2.5, 0.0, 2, 8]
>>> r2_score(y_true, y_pred)
0.948...
>>> y_true = [[0.5, 1], [-1, 1], [7, -6]]
>>> y_pred = [[0, 2], [-1, 2], [8, -5]]
>>> r2_score(y_true, y_pred, multioutput='variance_weighted')
0.938...
>>> y_true = [1,2,3]
>>> y_pred = [1,2,3]
>>> r2_score(y_true, y_pred)
1.0
>>> y_true = [1,2,3]
>>> y_pred = [2,2,2]
>>> r2_score(y_true, y_pred)
0.0
>>> y_true = [1,2,3]
>>> y_pred = [3,2,1]
>>> r2_score(y_true, y_pred)
-3.0
```
Examples using `sklearn.metrics.r2_score`

- Lasso and Elastic Net for Sparse Signals

## 29.20.4 Multilabel ranking metrics

See the Multilabel ranking metrics section of the user guide for further details.

### sklearn.metrics.coverage_error

**sklearn.metrics.coverage_error(y_true, y_score[, ...])** Coverage error measure

Compute how far we need to go through the ranked scores to cover all true labels. The best value is equal to the average number of labels in `y_true` per sample.

Ties in `y_scores` are broken by giving maximal rank that would have been assigned to all tied values.

Read more in the User Guide.

**Parameters**

- **y_true**: array, shape = [n_samples, n_labels]
  True binary labels in binary indicator format.
- **y_score**: array, shape = [n_samples, n_labels]
  Target scores, can either be probability estimates of the positive class, confidence values, or non-thresholded measure of decisions (as returned by “decision_function” on some classifiers).
- **sample_weight**: array-like of shape = [n_samples], optional
  Sample weights.

**Returns**

- **coverage_error**: float

**References**

[R204]

### sklearn.metrics.label_ranking_average_precision_score

**sklearn.metrics.label_ranking_average_precision_score(y_true, y_score)** Compute ranking-based average precision

Label ranking average precision (LRAP) is the average over each ground truth label assigned to each sample, of the ratio of true vs. total labels with lower score.

This metric is used in multilabel ranking problem, where the goal is to give better rank to the labels associated to each sample.

The obtained score is always strictly greater than 0 and the best value is 1.
Parameters

- **y_true**: array or sparse matrix, shape = [n_samples, n_labels]
  True binary labels in binary indicator format.

- **y_score**: array, shape = [n_samples, n_labels]
  Target scores, can either be probability estimates of the positive class, confidence values, or non-thresholded measure of decisions (as returned by “decision_function” on some classifiers).

Returns

- **score**: float

Examples

```python
>>> import numpy as np
>>> from sklearn.metrics import label_ranking_average_precision_score
>>> y_true = np.array([[1, 0, 0], [0, 0, 1]])
>>> y_score = np.array([[0.75, 0.5, 1], [1, 0.2, 0.1]])
>>> label_ranking_average_precision_score(y_true, y_score)
0.416...
```

**sklearn.metrics.label_ranking_loss**

*sklearn.metrics.label_ranking_loss*(y_true, y_score, sample_weight=None)

Compute Ranking loss measure

1. Compute the average number of label pairs that are incorrectly ordered given y_score weighted by the size of the label set and the number of labels not in the label set.
2. This is similar to the error set size, but weighted by the number of relevant and irrelevant labels. The best performance is achieved with a ranking loss of zero.

Read more in the *User Guide*.

New in version 0.17: A function *label_ranking_loss*

Parameters

- **y_true**: array or sparse matrix, shape = [n_samples, n_labels]
  True binary labels in binary indicator format.

- **y_score**: array, shape = [n_samples, n_labels]
  Target scores, can either be probability estimates of the positive class, confidence values, or non-thresholded measure of decisions (as returned by “decision_function” on some classifiers).

- **sample_weight**: array-like of shape = [n_samples], optional
  Sample weights.

Returns

- **loss**: float

References

[R217]
29.20.5 Clustering metrics

See the Clustering performance evaluation section of the user guide for further details. The sklearn.metrics.cluster submodule contains evaluation metrics for cluster analysis results. There are two forms of evaluation:

- supervised, which uses a ground truth class values for each sample.
- unsupervised, which does not and measures the ‘quality’ of the model itself.

- `metrics.adjusted_mutual_info_score(...)` Adjusted Mutual Information between two clusterings.
- `metrics.adjusted_rand_score(labels_true, ...)` Rand index adjusted for chance.
- `metrics.calinski_harabaz_score(X, labels)` Compute the Calinski and Harabaz score.
- `metrics.completeness_score(labels_true, ...)` Completeness metric of a cluster labeling given a ground truth.
- `metrics.fowlkes_mallows_score(labels_true, ...)` Measure the similarity of two clusterings of a set of points.
- `metrics.homogeneity_completeness_v_measure(...)` Compute the homogeneity and completeness and V-Measure scores at once.
- `metrics.homogeneity_score(labels_true, ...)` Homogeneity metric of a cluster labeling given a ground truth.
- `metrics.mutual_info_score(labels_true, ...)` Mutual Information between two clusterings.
- `metrics.normalized_mutual_info_score(...)` Normalized Mutual Information between two clusterings.
- `metrics.silhouette_score(X, labels[, ...])` Compute the mean Silhouette Coefficient of all samples.
- `metrics.silhouette_samples(X, labels[, metric])` Compute the Silhouette Coefficient for each sample.
- `metrics.v_measure_score(labels_true, labels_pred)` V-measure cluster labeling given a ground truth.

sklearn.metrics.adjusted_mutual_info_score

`sklearn.metrics.adjusted_mutual_info_score(labels_true, labels_pred)`

Adjusted Mutual Information between two clusterings.

Adjusted Mutual Information (AMI) is an adjustment of the Mutual Information (MI) score to account for chance. It accounts for the fact that the MI is generally higher for two clusterings with a larger number of clusters, regardless of whether there is actually more information shared. For two clusterings $U$ and $V$, the AMI is given as:

\[
AMI(U, V) = \frac{MI(U, V) - E(MI(U, V))}{\max(H(U), H(V)) - E(MI(U, V))}
\]

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching label_true with label_pred will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

Be mindful that this function is an order of magnitude slower than other metrics, such as the Adjusted Rand Index.

Read more in the User Guide.

Parameters:

- `labels_true`: int array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

- `labels_pred`: array, shape = [n_samples]
A clustering of the data into disjoint subsets.

**Returnsami: float(upperlimited by 1.0)**

The AMI returns a value of 1 when the two partitions are identical (ie perfectly matched). Random partitions (independent labellings) have an expected AMI around 0 on average hence can be negative.

See also:

- `adjusted_rand_score`: Adjusted Rand Index
- `mutual_information_score`: Mutual Information (not adjusted for chance)

**References**

[R50], [R51]

**Examples**

Perfect labelings are both homogeneous and complete, hence have score 1.0:

```python
>>> from sklearn.metrics.cluster import adjusted_mutual_info_score
>>> adjusted_mutual_info_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> adjusted_mutual_info_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

If classes members are completely split across different clusters, the assignment is totally in-complete, hence the AMI is null:

```python
>>> adjusted_mutual_info_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

**Examples using sklearn.metrics.adjusted_mutual_info_score**

- Adjustment for chance in clustering performance evaluation
- Demo of affinity propagation clustering algorithm
- Demo of DBSCAN clustering algorithm
- A demo of K-Means clustering on the handwritten digits data

**sklearn.metrics.adjusted_rand_score**

The Rand Index computes a similarity measure between two clusterings by considering all pairs of samples and counting pairs that are assigned in the same or different clusters in the predicted and true clusterings.

The raw RI score is then “adjusted for chance” into the ARI score using the following scheme:
ARI = (RI - Expected_RI) / (max(RI) - Expected_RI)

The adjusted Rand index is thus ensured to have a value close to 0.0 for random labeling independently of the number of clusters and samples and exactly 1.0 when the clusterings are identical (up to a permutation).

ARI is a symmetric measure:

\[
\text{adjusted_rand_score}(a, b) = \text{adjusted_rand_score}(b, a)
\]

Read more in the User Guide.

Parameters:

- `labels_true`: int array, shape = [n_samples]
  Ground truth class labels to be used as a reference
- `labels_pred`: array, shape = [n_samples]
  Cluster labels to evaluate

Returns:

- `ari`: float
  Similarity score between -1.0 and 1.0. Random labelings have an ARI close to 0.0. 1.0 stands for perfect match.

See also:

- `adjusted_mutual_info_score`: Adjusted Mutual Information

References

[Hubert1985], [wk]

Examples

Perfectly matching labelings have a score of 1 even

```python
>>> from sklearn.metrics.cluster import adjusted_rand_score
>>> adjusted_rand_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
```

Labelings that assign all classes members to the same clusters are complete but not always pure, hence penalized:

```python
>>> adjusted_rand_score([0, 0, 1, 2], [0, 0, 1, 1])
0.57...
```

ARI is symmetric, so labelings that have pure clusters with members coming from the same classes but unnecessary splits are penalized:

```python
>>> adjusted_rand_score([0, 0, 1, 1], [0, 0, 1, 2])
0.57...
```

If classes members are completely split across different clusters, the assignment is totally incomplete, hence the ARI is very low:
Examples using `sklearn.metrics.adjusted_rand_score`:

- Adjustment for chance in clustering performance evaluation
- Demo of affinity propagation clustering algorithm
- Demo of DBSCAN clustering algorithm
- A demo of K-Means clustering on the handwritten digits data
- Clustering text documents using k-means

`sklearn.metrics.calinski_harabaz_score`

`sklearn.metrics.calinski_harabaz_score(X, labels)`

Compute the Calinski and Harabaz score. The score is defined as ratio between the within-cluster dispersion and the between-cluster dispersion. Read more in the User Guide.

**Parameters**

- `X`: array-like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.
- `labels`: array-like, shape (n_samples,)
  Predicted labels for each sample.

**Returns**

- `score`: float
  The resulting Calinski-Harabaz score.

**References**

[R198]

`sklearn.metrics.completeness_score`

`sklearn.metrics.completeness_score(labels_true, labels_pred)`

Completeness metric of a cluster labeling given a ground truth. A clustering result satisfies completeness if all the data points that are members of a given class are elements of the same cluster.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is not symmetric: switching `label_true` with `label_pred` will return the `homogeneity_score` which will be different in general.

Read more in the User Guide.

**Parameters**

- `labels_true`: int array, shape = [n_samples]
ground truth class labels to be used as a reference

labels_pred : array, shape = [n_samples]
    cluster labels to evaluate

Returns
completeness: float :
    score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

See also:
homogeneity_score, v_measure_score

References

[R54]

Examples

Perfect labelings are complete:

```python
>>> from sklearn.metrics.cluster import completeness_score
>>> completeness_score([[0, 0, 1, 1], [1, 1, 0, 0]])
1.0
```

Non-perfect labelings that assign all classes members to the same clusters are still complete:

```python
>>> print(completeness_score([[0, 0, 1, 1], [0, 0, 0, 0]]))
1.0
>>> print(completeness_score([[0, 1, 2, 3], [0, 0, 1, 1]]))
1.0
```

If classes members are split across different clusters, the assignment cannot be complete:

```python
>>> print(completeness_score([[0, 0, 1, 1], [0, 1, 0, 1]]))
0.0
>>> print(completeness_score([[0, 0, 0, 0], [0, 1, 2, 3]]))
0.0
```

Examples using sklearn.metrics.completeness_score

- Demo of affinity propagation clustering algorithm
- Demo of DBSCAN clustering algorithm
- A demo of K-Means clustering on the handwritten digits data
- Clustering text documents using k-means

sklearn.metrics.fowlkes_mallows_score

sklearn.metrics.fowlkes_mallows_score(labels_true, labels_pred, sparse=False)
Measure the similarity of two clusterings of a set of points.

The Fowlkes-Mallows index (FMI) is defined as the geometric mean between of the precision and recall:
\[
\text{FMI} = \frac{TP}{\sqrt{(TP + FP) \times (TP + FN)}}
\]

Where \(TP\) is the number of **True Positive** (i.e. the number of pair of points that belongs in the same clusters in both \(\text{labels\_true}\) and \(\text{labels\_pred}\), \(FP\) is the number of **False Positive** (i.e. the number of pair of points that belongs in the same clusters in \(\text{labels\_true}\) and not in \(\text{labels\_pred}\)) and \(FN\) is the number of **False Negative** (i.e the number of pair of points that belongs in the same clusters in \(\text{labels\_pred}\) and not in \(\text{labels\_true}\)).

The score ranges from 0 to 1. A high value indicates a good similarity between two clusters.

Read more in the *User Guide*.

**Parameters**

- \(\text{labels\_true}\) : int array, shape = (\(n\_samples\),)
  A clustering of the data into disjoint subsets.

- \(\text{labels\_pred}\) : array, shape = (\(n\_samples\),)
  A clustering of the data into disjoint subsets.

**Return**

- \(\text{score}\) : float
  The resulting Fowlkes-Mallows score.

**References**

[R208], [R209]

**Examples**

Perfect labelings are both homogeneous and complete, hence have score 1.0:

```python
>>> from sklearn.metrics.cluster import fowlkes_mallows_score
>>> fowlkes_mallows_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> fowlkes_mallows_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

If classes members are completely split across different clusters, the assignment is totally random, hence the FMI is null:

```python
>>> fowlkes_mallows_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

**sklearn.metrics.homogeneity_completeness_v_measure**

\[\text{sklearn.metrics.homogeneity_completeness_v_measure}\]

\(\text{sklearn.metrics.homogeneity_completeness_v_measure}(\text{labels\_true}, \text{labels\_pred})\)

Compute the homogeneity and completeness and V-Measure scores at once.

Those metrics are based on normalized conditional entropy measures of the clustering labeling to evaluate given the knowledge of a Ground Truth class labels of the same samples.

A clustering result satisfies homogeneity if all of its clusters contain only data points which are members of a single class.

A clustering result satisfies completeness if all the data points that are members of a given class are elements of the same cluster.
Both scores have positive values between 0.0 and 1.0, larger values being desirable.

Those 3 metrics are independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score values in any way.

V-Measure is furthermore symmetric: swapping `labels_true` and `label_pred` will give the same score. This does not hold for homogeneity and completeness.

Read more in the User Guide.

**Parameters**

- `labels_true`: int array, shape = [n_samples]  
  ground truth class labels to be used as a reference

- `labels_pred`: array, shape = [n_samples]  
  cluster labels to evaluate

**Returns**

- `homogeneity`: float  
  score between 0.0 and 1.0. 1.0 stands for perfectly homogeneous labeling

- `completeness`: float  
  score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

- `v_measure`: float  
  harmonic mean of the first two

**See also:**

- `homogeneity_score`
- `completeness_score`
- `v_measure_score`

```
scikit-learn.metrics.homogeneity_score
```

`scikit-learn.metrics.homogeneity_score (labels_true, labels_pred)`  
Homogeneity metric of a cluster labeling given a ground truth.

A clustering result satisfies homogeneity if all of its clusters contain only data points which are members of a single class.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is not symmetric: switching `label_true` with `label_pred` will return the `completeness_score` which will be different in general.

Read more in the User Guide.

**Parameters**

- `labels_true`: int array, shape = [n_samples]  
  ground truth class labels to be used as a reference

- `labels_pred`: array, shape = [n_samples]  
  cluster labels to evaluate

**Returns**

- `homogeneity`: float  
  score between 0.0 and 1.0. 1.0 stands for perfectly homogeneous labeling

**See also:**

- `completeness_score`
- `v_measure_score`
References

[R59]

Examples

Perfect labelings are homogeneous:

```python
>>> from sklearn.metrics.cluster import homogeneity_score
>>> homogeneity_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

Non-perfect labelings that further split classes into more clusters can be perfectly homogeneous:

```python
>>> print("%.6f" % homogeneity_score([0, 0, 1, 1], [0, 0, 1, 2]))
... 1.0...
>>> print("%.6f" % homogeneity_score([0, 0, 1, 1], [0, 1, 2, 3]))
... 1.0...
```

Clusters that include samples from different classes do not make for an homogeneous labeling:

```python
>>> print("%.6f" % homogeneity_score([0, 0, 1, 1], [0, 1, 0, 1]))
... 0.0...
>>> print("%.6f" % homogeneity_score([0, 0, 1, 1], [0, 0, 0, 0]))
... 0.0...
```

Examples using `sklearn.metrics.homogeneity_score`

- Demo of affinity propagation clustering algorithm
- Demo of DBSCAN clustering algorithm
- A demo of K-Means clustering on the handwritten digits data
- Clustering text documents using k-means

`sklearn.metrics.mutual_info_score`

`sklearn.metrics.mutual_info_score(labels_true, labels_pred, contingency=None)`  
Mutual Information between two clusterings.

The Mutual Information is a measure of the similarity between two labels of the same data. Where $P(i)$ is the probability of a random sample occurring in cluster $U_i$ and $P'(j)$ is the probability of a random sample occurring in cluster $V_j$, the Mutual Information between clusterings $U$ and $V$ is given as:

$$MI(U, V) = \sum_{i=1}^{R} \sum_{j=1}^{C} P(i, j) \log \frac{P(i, j)}{P(i)P'(j)}$$

This is equal to the Kullback-Leibler divergence of the joint distribution with the product distribution of the marginals.
This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching `label_true` with `label_pred` will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

Read more in the *User Guide*.

**Parameters**

- `labels_true`: int array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

- `labels_pred`: array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

- `contingency`: {None, array, sparse matrix}, shape = [n_classes_true, n_classes_pred]
  A contingency matrix given by the `contingency_matrix` function. If value is `None`, it will be computed, otherwise the given value is used, with `labels_true` and `labels_pred` ignored.

**Returns**

- `mi`: float
  Mutual information, a non-negative value

See also:

- `adjusted_mutual_info_score` Adjusted against chance Mutual Information
- `normalized_mutual_info_score` Normalized Mutual Information

**Examples using sklearn.metrics.mutual_info_score**

- *Adjustment for chance in clustering performance evaluation*

sklearn.metrics.normalized_mutual_info_score

```python
sklearn.metrics.normalized_mutual_info_score(labels_true, labels_pred)
```

Normalized Mutual Information (NMI) is an normalization of the Mutual Information (MI) score to scale the results between 0 (no mutual information) and 1 (perfect correlation). In this function, mutual information is normalized by \( \sqrt{H(labels_true) \times H(labels_pred)} \)

This measure is not adjusted for chance. Therefore `adjusted_mutual_info_score` might be preferred.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching `label_true` with `label_pred` will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

Read more in the *User Guide*.

**Parameters**

- `labels_true`: int array, shape = [n_samples]
  A clustering of the data into disjoint subsets.
labels_pred : array, shape = [n_samples]
A clustering of the data into disjoint subsets.

Returns nmi : float
score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

See also:

adjusted_rand_score Adjusted Rand Index
adjusted_mutual_info_score Adjusted Mutual Information (adjusted against chance)

Examples

Perfect labelings are both homogeneous and complete, hence have score 1.0:

>>> from sklearn.metrics.cluster import normalized_mutual_info_score
>>> normalized_mutual_info_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> normalized_mutual_info_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0

If classes members are completely split across different clusters, the assignment is totally incomplete, hence the NMI is null:

>>> normalized_mutual_info_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0

sklearn.metrics.silhouette_score

sklearn.metrics.silhouette_score (X, labels, metric='euclidean', sample_size=None, random_state=None, **kwds)

Compute the mean Silhouette Coefficient of all samples.

The Silhouette Coefficient is calculated using the mean intra-cluster distance (a) and the mean nearest-cluster distance (b) for each sample. The Silhouette Coefficient for a sample is (b -a) / max(a,b). To clarify, b is the distance between a sample and the nearest cluster that the sample is not a part of. Note that Silhouette Coefficient is only defined if number of labels is 2 <= n_labels <= n_samples - 1.

This function returns the mean Silhouette Coefficient over all samples. To obtain the values for each sample, use silhouette_samples.

The best value is 1 and the worst value is -1. Values near 0 indicate overlapping clusters. Negative values generally indicate that a sample has been assigned to the wrong cluster, as a different cluster is more similar.

Read more in the User Guide.

Parameters:

X : array [n_samples_a, n_samples_a] if metric == “precomputed”, or [n_samples_a, n_features] otherwise
Array of pairwise distances between samples, or a feature array.

labels : array, shape = [n_samples]
Predicted labels for each sample.

metric : string, or callable
The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by `metrics.pairwise.pairwise_distances`. If X is the distance array itself, use `metric="precomputed"`.

**sample_size** : int or None

The size of the sample to use when computing the Silhouette Coefficient on a random subset of the data. If `sample_size` is None, no sampling is used.

**random_state** : integer or numpy.RandomState, optional

The generator used to randomly select a subset of samples if `sample_size` is not None. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**kwds** : optional keyword parameters

Any further parameters are passed directly to the distance function. If using a scipy.spatial.distance metric, the parameters are still metric dependent. See the scipy docs for usage examples.

Returns **silhouette** : float

Mean Silhouette Coefficient for all samples.

**References**

[R64], [R65]

**Examples using `sklearn.metrics.silhouette_score`**

- Demo of affinity propagation clustering algorithm
- Demo of DBSCAN clustering algorithm
- A demo of K-Means clustering on the handwritten digits data
- Selecting the number of clusters with silhouette analysis on KMeans clustering
- Clustering text documents using k-means

**sklearn.metrics.silhouette_samples**

`sklearn.metrics.silhouette_samples(X, labels, metric='euclidean', **kwds)`

Compute the Silhouette Coefficient for each sample.

The Silhouette Coefficient is a measure of how well samples are clustered with samples that are similar to themselves. Clustering models with a high Silhouette Coefficient are said to be dense, where samples in the same cluster are similar to each other, and well separated, where samples in different clusters are not very similar to each other.

The Silhouette Coefficient is calculated using the mean intra-cluster distance (a) and the mean nearest-cluster distance (b) for each sample. The Silhouette Coefficient for a sample is \((b - a) / \max(a, b)\). Note that Silhouette Coefficient is only defined if number of labels is \(2 \leq n\_labels \leq n\_samples - 1\).

This function returns the Silhouette Coefficient for each sample.

The best value is 1 and the worst value is -1. Values near 0 indicate overlapping clusters.
Read more in the *User Guide*.

**Parameters**

- **X**: array `[n_samples_a, n_samples_a]` if metric == “precomputed”, or, `[n_samples_a, n_features]` otherwise
  
  Array of pairwise distances between samples, or a feature array.

- **labels**: array, shape = `[n_samples]`
  
  Label values for each sample

- **metric**: string, or callable
  
  The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by `sklearn.metrics.pairwise.pairwise_distances`. If X is the distance array itself, use “precomputed” as the metric.

- ****kwds**: optional keyword parameters
  
  Any further parameters are passed directly to the distance function. If using a `scipy.spatial.distance` metric, the parameters are still metric dependent. See the scipy docs for usage examples.

**Returns**

- **silhouette**: array, shape = `[n_samples]`
  
  Silhouette Coefficient for each samples.

**References**

[R62], [R63]

**Examples using sklearn.metrics.silhouette_samples**

- Selecting the number of clusters with silhouette analysis on KMeans clustering

**sklearn.metrics.v_measure_score**

`sklearn.metrics.v_measure_score(labels_true, labels_pred)`

V-measure cluster labeling given a ground truth.

This score is identical to `normalized_mutual_info_score`.

The V-measure is the harmonic mean between homogeneity and completeness:

\[
V = \frac{2 \times (\text{homogeneity} \times \text{completeness})}{\text{homogeneity} + \text{completeness}}
\]

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching `label_true` with `label_pred` will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

Read more in the *User Guide*.

**Parameters**

- **labels_true**: int array, shape = `[n_samples]`
  
  Ground truth class labels to be used as a reference
labels_pred : array, shape = [n_samples]
    cluster labels to evaluate

Returns
v_measure : float
    score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

See also:
    homogeneity_score, completeness_score

References

[R66]

Examples

Perfect labelings are both homogeneous and complete, hence have score 1.0:

```python
>>> from sklearn.metrics.cluster import v_measure_score
>>> v_measure_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> v_measure_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

Labelings that assign all classes members to the same clusters are complete be not homogeneous, hence penalized:

```python
>>> print("%.6f" % v_measure_score([0, 0, 1, 2], [0, 0, 1, 1]))
... 0.8...
>>> print("%.6f" % v_measure_score([0, 1, 2, 3], [0, 0, 1, 1]))
... 0.66...
```

Labelings that have pure clusters with members coming from the same classes are homogeneous but unnecessary splits harms completeness and thus penalize V-measure as well:

```python
>>> print("%.6f" % v_measure_score([0, 0, 1, 1], [0, 0, 1, 2]))
... 0.8...
>>> print("%.6f" % v_measure_score([0, 0, 1, 1], [0, 1, 2, 3]))
... 0.66...
```

If classes members are completely split across different clusters, the assignment is totally incomplete, hence the V-Measure is null:

```python
>>> print("%.6f" % v_measure_score([0, 0, 0, 0], [0, 1, 2, 3]))
... 0.0...
```

Clusters that include samples from totally different classes totally destroy the homogeneity of the labeling, hence:
Examples using `sklearn.metrics.v_measure_score`

- Biclustering documents with the Spectral Co-clustering algorithm
- Adjustment for chance in clustering performance evaluation
- Demo of affinity propagation clustering algorithm
- Demo of DBSCAN clustering algorithm
- A demo of K-Means clustering on the handwritten digits data
- Clustering text documents using k-means

### 29.20.6 Biclustering metrics

See the Biclustering evaluation section of the user guide for further details.

```python
metrics.consensus_score(a, b[, similarity])
```

The similarity of two sets of biclusters.

**sklearn.metrics.consensus_score**

```python
sklearn.metrics.consensus_score(a, b, similarity='jaccard')
```

The similarity of two sets of biclusters.

Similarity between individual biclusters is computed. Then the best matching between sets is found using the Hungarian algorithm. The final score is the sum of similarities divided by the size of the larger set.

Read more in the User Guide.

**Parameters**

- `a`: (rows, columns)
  - Tuple of row and column indicators for a set of biclusters.
- `b`: (rows, columns)
  - Another set of biclusters like `a`.
- `similarity`: string or function, optional, default: “jaccard”
  - May be the string “jaccard” to use the Jaccard coefficient, or any function that takes four arguments, each of which is a 1d indicator vector: (a_rows, a_columns, b_rows, b_columns).

**References**

- Hochreiter, Bodenhofer, et. al., 2010. FABIA: factor analysis for bicluster acquisition.

Examples using `sklearn.metrics.consensus_score`

- A demo of the Spectral Biclustering algorithm
## 29.20.7 Pairwise metrics

See the *Pairwise metrics, Affinities and Kernels* section of the user guide for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td><code>metrics.pairwise.additive_chi2_kernel(X[, Y])</code></td>
<td>Computes the additive chi-squared kernel between observations in X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.chi2_kernel(X[, Y, gamma])</code></td>
<td>Computes the exponential chi-squared kernel X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.distance_metrics()</code></td>
<td>Valid metrics for pairwise_distances.</td>
</tr>
<tr>
<td><code>metrics.pairwise.euclidean_distances(X[, Y, ...])</code></td>
<td>Considering the rows of X (and Y=X) as vectors, compute the distance matrix between each pair of vectors.</td>
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<tr>
<td><code>metrics.pairwise.kernel_metrics()</code></td>
<td>Valid metrics for pairwise_kernels</td>
</tr>
<tr>
<td><code>metrics.pairwise.linear_kernel(X[, Y])</code></td>
<td>Compute the linear kernel between X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.manhattan_distances(X[, Y, ...])</code></td>
<td>Compute the L1 distances between the vectors in X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.pairwise_distances(X[, Y, ...])</code></td>
<td>Compute the distance matrix from a vector array X and optional Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.pairwise_kernels(X[, Y, ...])</code></td>
<td>Compute the kernel between arrays X and optional array Y.</td>
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<tr>
<td><code>metrics.pairwise.polynomial_kernel(X[, Y, ...])</code></td>
<td>Compute the polynomial kernel between X and Y:</td>
</tr>
<tr>
<td><code>metrics.pairwise.rbf_kernel(X[, Y, gamma])</code></td>
<td>Compute the rbf (gaussian) kernel between X and Y:</td>
</tr>
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<td><code>metrics.pairwise.sigmoid_kernel(X[, Y, ...])</code></td>
<td>Compute the sigmoid kernel between X and Y:</td>
</tr>
<tr>
<td><code>metrics.pairwise.cosine_similarity(X[, Y, ...])</code></td>
<td>Compute cosine similarity between samples in X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.laplacian_kernel(X[, Y, gamma])</code></td>
<td>Compute the laplacian kernel between samples in X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.cosine_distances(X[, Y])</code></td>
<td>Compute cosine distance between samples in X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.paired_distances(X, Y[, metric])</code></td>
<td>Computes the paired distances between X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.paired_euclidean_distances(X[, Y])</code></td>
<td>Computes the paired euclidean distances between X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.paired_manhattan_distances(Y)</code></td>
<td>Computes the L1 distances between the vectors in X and Y.</td>
</tr>
<tr>
<td><code>metrics.pairwise.paired_cosine_distances(X[, Y])</code></td>
<td>Computes the paired cosine distances between X and Y.</td>
</tr>
</tbody>
</table>

### sklearn.metrics.pairwise.additive_chi2_kernel

**Function**

- **`sklearn.metrics.pairwise.additive_chi2_kernel(X, Y=None)`**
- Computes the additive chi-squared kernel between observations in X and Y.

  The chi-squared kernel is computed between each pair of rows in X and Y. X and Y have to be non-negative. This kernel is most commonly applied to histograms.
The chi-squared kernel is given by:

$$k(x, y) = \exp\left(-\gamma \sum \frac{(x - y)^2}{(x + y)}\right)$$

It can be interpreted as a weighted difference per entry.

Read more in the User Guide.

**Parameters**

- **X**: array-like of shape (n_samples_X, n_features)
- **Y**: array of shape (n_samples_Y, n_features)

**Returns**

- **kernel_matrix**: array of shape (n_samples_X, n_samples_Y)

See also:

- `chi2_kernel`: The exponentiated version of the kernel, which is usually preferable.
- `sklearn.kernel_approximation.AdditiveChi2Sampler`: A Fourier approximation to this kernel.

**Notes**

As the negative of a distance, this kernel is only conditionally positive definite.

**References**


**sklearn.metrics.pairwise.chi2_kernel**

Computes the exponential chi-squared kernel X and Y.

The chi-squared kernel is computed between each pair of rows in X and Y. X and Y have to be non-negative. This kernel is most commonly applied to histograms.

The chi-squared kernel is given by:

$$k(x, y) = \exp(-\gamma \sum \frac{(x - y)^2}{(x + y)})$$

It can be interpreted as a weighted difference per entry.

Read more in the User Guide.

**Parameters**

- **X**: array-like of shape (n_samples_X, n_features)
- **Y**: array of shape (n_samples_Y, n_features)
- **gamma**: float, default=1.0
  Scaling parameter of the chi2 kernel.

**Returns**

- **kernel_matrix**: array of shape (n_samples_X, n_samples_Y)

See also:

- `additive_chisquared_kernel`: The additive version of this kernel
sklearn.kernel_approximation.AdditiveChi2Sampler

A Fourier approximation to the additive version of this kernel.

References


sklearn.metrics.pairwise.distance_metrics

sklearn.metrics.pairwise.distance_metrics()

Valid metrics for pairwise_distances.

This function simply returns the valid pairwise distance metrics. It exists to allow for a description of the mapping for each of the valid strings.

The valid distance metrics, and the function they map to, are:

<table>
<thead>
<tr>
<th>metric</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>'cityblock'</td>
<td>metrics.pairwise.manhattan_distances</td>
</tr>
<tr>
<td>'cosine'</td>
<td>metrics.pairwise.cosine_distances</td>
</tr>
<tr>
<td>'euclidean'</td>
<td>metrics.pairwise.euclidean_distances</td>
</tr>
<tr>
<td>'l1'</td>
<td>metrics.pairwise.manhattan_distances</td>
</tr>
<tr>
<td>'l2'</td>
<td>metrics.pairwise.euclidean_distances</td>
</tr>
<tr>
<td>'manhattan'</td>
<td>metrics.pairwise.manhattan_distances</td>
</tr>
</tbody>
</table>

Read more in the User Guide.

sklearn.metrics.pairwise.euclidean_distances

sklearn.metrics.pairwise.euclidean_distances(X, Y=None, Y_norm_squared=None, squared=False, X_norm_squared=None)

Considering the rows of X (and Y=X) as vectors, compute the distance matrix between each pair of vectors.

For efficiency reasons, the euclidean distance between a pair of row vector x and y is computed as:

\[ \text{dist}(x, y) = \sqrt{\text{dot}(x, x) - 2 \times \text{dot}(x, y) + \text{dot}(y, y)} \]

This formulation has two advantages over other ways of computing distances. First, it is computationally efficient when dealing with sparse data. Second, if one argument varies but the other remains unchanged, then \( \text{dot}(x, x) \) and/or \( \text{dot}(y, y) \) can be pre-computed.

However, this is not the most precise way of doing this computation, and the distance matrix returned by this function may not be exactly symmetric as required by, e.g., scipy.spatial.distance functions.

Read more in the User Guide.

Parameters

- **X**: {array-like, sparse matrix}, shape (n_samples_1, n_features)
- **Y**: {array-like, sparse matrix}, shape (n_samples_2, n_features)
- **Y_norm_squared**: array-like, shape (n_samples_2,), optional
  Pre-computed dot-products of vectors in Y (e.g., \((Y**2).sum(axis=1))
- **squared**: boolean, optional
Return squared Euclidean distances.

**X_norm_squared** : array-like, shape = [n_samples_1], optional

Pre-computed dot-products of vectors in X (e.g., \((X**2).sum(axis=1)\))

**Returns**

**distances** : {array, sparse matrix}, shape (n_samples_1, n_samples_2)

See also:

**paired_distances** distances between pairs of elements of X and Y.

### Examples

```python
>>> from sklearn.metrics.pairwise import euclidean_distances

>>> X = [[0, 1], [1, 1]]
>>> # distance between rows of X
>>> euclidean_distances(X, X)
array([[ 0. ,  1. ],
       [ 1. ,  0. ]])

>>> # get distance to origin
>>> euclidean_distances(X, [[0, 0]])
array([[ 1.41421356]])
```

### Valid metrics for pairwise_kernels

This function simply returns the valid pairwise distance metrics. It exists, however, to allow for a verbose description of the mapping for each of the valid strings.

<table>
<thead>
<tr>
<th>metric</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘additive_chi2’</td>
<td>sklearn.pairwise.additive_chi2_kernel</td>
</tr>
<tr>
<td>‘chi2’</td>
<td>sklearn.pairwise.chi2_kernel</td>
</tr>
<tr>
<td>‘linear’</td>
<td>sklearn.pairwise.linear_kernel</td>
</tr>
<tr>
<td>‘polynomial’</td>
<td>sklearn.pairwise.polynomial_kernel</td>
</tr>
<tr>
<td>‘rbf’</td>
<td>sklearn.pairwise.rbf_kernel</td>
</tr>
<tr>
<td>‘laplacian’</td>
<td>sklearn.pairwise.laplacian_kernel</td>
</tr>
<tr>
<td>‘sigmoid’</td>
<td>sklearn.pairwise.sigmoid_kernel</td>
</tr>
<tr>
<td>‘cosine’</td>
<td>sklearn.pairwise.cosine_similarity</td>
</tr>
</tbody>
</table>

Read more in the User Guide.

### Compute the linear kernel between X and Y.

#### Parameters

**X** : array of shape (n_samples_1, n_features)

**Y** : array of shape (n_samples_2, n_features)

Read more in the User Guide.
**Returns**

**Gram matrix**: array of shape (n_samples_1, n_samples_2)

---

### sklearn.metrics.pairwise.manhattan_distances

**sklearn.metrics.pairwise.manhattan_distances**

**X** : array_like

An array with shape (n_samples_X, n_features).

**Y** : array_like, optional

An array with shape (n_samples_Y, n_features).

**sum_over_features** : bool, default=True

If True the function returns the pairwise distance matrix else it returns the componentwise L1 pairwise-distances. Not supported for sparse matrix inputs.

**size_threshold** : int, default=5e8

Unused parameter.

**Returns**

If sum_over_features is False shape is (n_samples_X * n_samples_Y, n_features) and D contains the componentwise L1 pairwise-distances (ie. absolute difference), else shape is (n_samples_X, n_samples_Y) and D contains the pairwise L1 distances.

### Examples

```python
>>> from sklearn.metrics.pairwise import manhattan_distances
>>> manhattan_distances([[3], [3]])
aarray([[ 0. ]])
>>> manhattan_distances([[3], [2]])
aarray([[ 1. ]])
>>> manhattan_distances([[2], [3]])
aarray([[ 1. ]])
>>> manhattan_distances([[1, 2], [3, 4]], [[1, 2], [0, 3]])
aarray([[ 0., 2.],
        [ 4., 4.]])
>>> import numpy as np
>>> X = np.ones((1, 2))
>>> y = 2 * np.ones((2, 2))
>>> manhattan_distances(X, y, sum_over_features=False)
aarray([[ 1., 1.],
        [ 1., 1.]],
        [ 1., 1.])
```

### sklearn.metrics.pairwise.pairwise_distances

**sklearn.metrics.pairwise.pairwise_distances**

**sklearn.metrics.pairwise.pairwise_distances** *(X, Y=None, metric='euclidean', n_jobs=1, 
**kwds)**

Compute the distance matrix from a vector array X and optional Y.
This method takes either a vector array or a distance matrix, and returns a distance matrix. If the input is a vector array, the distances are computed. If the input is a distances matrix, it is returned instead.

This method provides a safe way to take a distance matrix as input, while preserving compatibility with many other algorithms that take a vector array.

If Y is given (default is None), then the returned matrix is the pairwise distance between the arrays from both X and Y.

Valid values for metric are:

- From scikit-learn: ['cityblock', 'cosine', 'euclidean', 'l1', 'l2', 'manhattan']. These metrics support sparse matrix inputs.
- From scipy.spatial.distance: ['braycurtis', 'canberra', 'chebyshev', 'correlation', 'dice', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'yule'] See the documentation for scipy.spatial.distance for details on these metrics. These metrics do not support sparse matrix inputs.

Note that in the case of ‘cityblock’, ‘cosine’ and ‘euclidean’ (which are valid scipy.spatial.distance metrics), the scikit-learn implementation will be used, which is faster and has support for sparse matrices (except for ‘cityblock’). For a verbose description of the metrics from scikit-learn, see the __doc__ of the sklearn.pairwise.distance_metrics function.

Read more in the User Guide.

**Parameters**

- **X**: array [n_samples_a, n_samples_a] if metric == “precomputed”, or, [n_samples_a, n_features] otherwise
  - Array of pairwise distances between samples, or a feature array.

- **Y**: array [n_samples_b, n_features], optional
  - An optional second feature array. Only allowed if metric != “precomputed”.

- **metric**: string, or callable
  - The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by scipy.spatial.distance.pdist for its metric parameter, or a metric listed in pairwise.PAIRWISE_DISTANCE_FUNCTIONS. If metric is “precomputed”, X is assumed to be a distance matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from X as input and return a value indicating the distance between them.

- **n_jobs**: int
  - The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.
  - If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

- ****kwds**: optional keyword parameters
  - Any further parameters are passed directly to the distance function. If using a scipy.spatial.distance metric, the parameters are still metric dependent. See the scipy docs for usage examples.

**Returns**

- **D**: array [n_samples_a, n_samples_a] or [n_samples_a, n_samples_b]
A distance matrix $D$ such that $D_{i, j}$ is the distance between the $i$th and $j$th vectors of the given matrix $X$, if $Y$ is None. If $Y$ is not None, then $D_{i, j}$ is the distance between the $i$th array from $X$ and the $j$th array from $Y$.

**sklearn.metrics.pairwise.pairwise_kernels**

```
sklearn.metrics.pairwise.pairwise_kernels(X, Y=None, metric='linear', filter_params=False, n_jobs=1, **kwds)
```

Compute the kernel between arrays $X$ and optional array $Y$.

This method takes either a vector array or a kernel matrix, and returns a kernel matrix. If the input is a vector array, the kernels are computed. If the input is a kernel matrix, it is returned instead.

This method provides a safe way to take a kernel matrix as input, while preserving compatibility with many other algorithms that take a vector array.

If $Y$ is given (default is None), then the returned matrix is the pairwise kernel between the arrays from both $X$ and $Y$.

**Valid values for metric are:** ['rbf', 'sigmoid', 'polynomial', 'poly', 'linear', 'cosine']

Read more in the User Guide.

**Parameters**

- **X** : array $[n_{samples_a}, n_{samples_a}]$ if metric == “precomputed”, or, $[n_{samples_a}, n_{features}]$ otherwise
  - Array of pairwise kernels between samples, or a feature array.

- **Y** : array $[n_{samples_b}, n_{features}]$
  - A second feature array only if $X$ has shape $[n_{samples_a}, n_{features}]$.

- **metric** : string, or callable
  - The metric to use when calculating kernel between instances in a feature array. If metric is a string, it must be one of the metrics in pairwise.PAIRWISE_KERNEL_FUNCTIONS. If metric is “precomputed”, $X$ is assumed to be a kernel matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from $X$ as input and return a value indicating the distance between them.

- **n_jobs** : int
  - The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.
  - If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, $(n_{cpus} + 1 + n_{jobs})$ are used. Thus for n_jobs = -2, all CPUs but one are used.

- **filter_params** : boolean
  - Whether to filter invalid parameters or not.

- ****kwds** : optional keyword parameters
  - Any further parameters are passed directly to the kernel function.

**Returns**

- **K** : array $[n_{samples_a}, n_{samples_a}]$ or $[n_{samples_a}, n_{samples_b}]$
  - A kernel matrix $K$ such that $K_{i, j}$ is the kernel between the $i$th and $j$th vectors of the given matrix $X$, if $Y$ is None. If $Y$ is not None, then $K_{i, j}$ is the kernel between the $i$th array from $X$ and the $j$th array from $Y$. 

Notes

If metric is ‘precomputed’, Y is ignored and X is returned.

sklearn.metrics.pairwise.polynomial_kernel

sklearn.metrics.pairwise.polynomial_kernel(X, Y=None, degree=3, gamma=None, coef0=1)

Compute the polynomial kernel between X and Y:

\[ K(X, Y) = (\gamma \langle X, Y \rangle + \text{coef0})^{\text{degree}} \]

Read more in the User Guide.

Parameters

- **X**: ndarray of shape (n_samples_1, n_features)
- **Y**: ndarray of shape (n_samples_2, n_features)
- **degree**: int, default 3
- **gamma**: float, default None
  - If None, defaults to 1.0 / n_samples_1
- **coef0**: int, default 1

Returns

- **Gram matrix**: array of shape (n_samples_1, n_samples_2)

sklearn.metrics.pairwise.rbf_kernel

sklearn.metrics.pairwise.rbf_kernel(X, Y=None, gamma=None)

Compute the rbf (gaussian) kernel between X and Y:

\[ K(x, y) = \exp(-\gamma \|x-y\|^2) \]

for each pair of rows x in X and y in Y.

Read more in the User Guide.

Parameters

- **X**: array of shape (n_samples_X, n_features)
- **Y**: array of shape (n_samples_Y, n_features)
- **gamma**: float, default None
  - If None, defaults to 1.0 / n_samples_X

Returns

- **kernel_matrix**: array of shape (n_samples_X, n_samples_Y)

sklearn.metrics.pairwise.sigmoid_kernel

sklearn.metrics.pairwise.sigmoid_kernel(X, Y=None, gamma=None, coef0=1)

Compute the sigmoid kernel between X and Y:

\[ K(X, Y) = \tanh(\gamma \langle X, Y \rangle + \text{coef0}) \]

Read more in the User Guide.
Parameters

$X$ : ndarray of shape (n_samples_1, n_features)

$Y$ : ndarray of shape (n_samples_2, n_features)

$\text{gamma}$ : float, default None

If None, defaults to $1.0 / n_{\text{samples}_1}$

$\text{coef0}$ : int, default 1

Returns

Gramp matrix : array of shape (n_samples_1, n_samples_2)

sklearn.metrics.pairwise.cosine_similarity

sklearn.metrics.pairwise.cosine_similarity($X, Y=None, dense_output=True$)

Compute cosine similarity between samples in $X$ and $Y$.

Cosine similarity, or the cosine kernel, computes similarity as the normalized dot product of $X$ and $Y$:

$K(X, Y) = \langle X, Y \rangle / \|X\| \|Y\|$

On L2-normalized data, this function is equivalent to linear_kernel.

Read more in the User Guide.

Parameters

$X$ : ndarray or sparse array, shape: (n_samples_X, n_features)

Input data.

$Y$ : ndarray or sparse array, shape: (n_samples_Y, n_features)

Input data. If None, the output will be the pairwise similarities between all samples in $X$.

dense_output : boolean (optional), default True

Whether to return dense output even when the input is sparse. If False, the output is sparse if both input arrays are sparse.

New in version 0.17: parameter dense_output for dense output.

Returns

Kernel matrix : array

An array with shape (n_samples_X, n_samples_Y).

sklearn.metrics.pairwise.cosine_distances

sklearn.metrics.pairwise.cosine_distances($X, Y=None$)

Compute cosine distance between samples in $X$ and $Y$.

Cosine distance is defined as 1.0 minus the cosine similarity.

Read more in the User Guide.

Parameters

$X$ : array_like, sparse matrix

with shape (n_samples_X, n_features).

$Y$ : array_like, sparse matrix (optional)

with shape (n_samples_Y, n_features).

Returns

Distance matrix : array

An array with shape (n_samples_X, n_samples_Y).
sklearn.metrics.pairwise.laplacian_kernel

`sklearn.metrics.pairwise.laplacian_kernel(X, Y=None, gamma=None)`

Compute the laplacian kernel between X and Y.

The laplacian kernel is defined as:

\[ K(x, y) = \exp(-\gamma \|x-y\|_1) \]

for each pair of rows x in X and y in Y. Read more in the User Guide.

New in version 0.17.

Parameters:
- `X`: array of shape (n_samples_X, n_features)
- `Y`: array of shape (n_samples_Y, n_features)
- `gamma`: float, default None
  If None, defaults to 1.0 / n_samples_X

Returns:
- `kernel_matrix`: array of shape (n_samples_X, n_samples_Y)

sklearn.metrics.pairwise_distances

`sklearn.metrics.pairwise_distances(X, Y=None, metric='euclidean', n_jobs=1, **kwds)`

Compute the distance matrix from a vector array X and optional Y.

This method takes either a vector array or a distance matrix, and returns a distance matrix. If the input is a vector array, the distances are computed. If the input is a distances matrix, it is returned instead.

This method provides a safe way to take a distance matrix as input, while preserving compatibility with many other algorithms that take a vector array.

If Y is given (default is None), then the returned matrix is the pairwise distance between the arrays from both X and Y.

Valid values for metric are:

- From scikit-learn: ['cityblock', 'cosine', 'euclidean', 'l1', 'l2', 'manhattan']. These metrics support sparse matrix inputs.
- From scipy.spatial.distance: ['braycurtis', 'canberra', 'chebyshev', 'correlation', 'dice', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'yule'] See the documentation for scipy.spatial.distance for details on these metrics. These metrics do not support sparse matrix inputs.

Note that in the case of 'cityblock', 'cosine' and 'euclidean' (which are valid scipy.spatial.distance metrics), the scikit-learn implementation will be used, which is faster and has support for sparse matrices (except for 'cityblock'). For a verbose description of the metrics from scikit-learn, see the __doc__ of the sklearn.pairwise.distance_metrics function.

Read more in the User Guide.

Parameters:
- `X`: array [n_samples_a, n_samples_a] if metric == "precomputed", or, [n_samples_a, n_features] otherwise

  Array of pairwise distances between samples, or a feature array.
Y : array [n_samples_b, n_features], optional

An optional second feature array. Only allowed if metric != “precomputed”.

metric : string, or callable

The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by scipy.spatial.distance.pdist for its metric parameter, or a metric listed in pairwise.PAIRWISE_DISTANCE_FUNCTIONS. If metric is “precomputed”, X is assumed to be a distance matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from X as input and return a value indicating the distance between them.

n_jobs : int

The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**kwds** : optional keyword parameters

Any further parameters are passed directly to the distance function. If using a scipy.spatial.distance metric, the parameters are still metric dependent. See the scipy docs for usage examples.

ReturnsD : array [n_samples_a, n_samples_a] or [n_samples_a, n_samples_b]

A distance matrix D such that D_{i, j} is the distance between the ith and jth vectors of the given matrix X, if Y is None. If Y is not None, then D_{i, j} is the distance between the ith array from X and the jth array from Y.

Examples using sklearn.metrics.pairwise_distances

- Agglomerative clustering with different metrics

sklearn.metrics.pairwise_distances_argmin

sklearn.metrics.pairwise_distances_argmin(X, Y, axis=1, metric='euclidean', batch_size=500, metric_kwarg=None)

Compute minimum distances between one point and a set of points.

This function computes for each row in X, the index of the row of Y which is closest (according to the specified distance).

This is mostly equivalent to calling:

pairwise_distances(X, Y=Y, metric=metric).argmin(axis=axis)

but uses much less memory, and is faster for large arrays.

This function works with dense 2D arrays only.

ParametersX : array-like

Arrays containing points. Respective shapes (n_samples1, n_features) and (n_samples2, n_features)
Y : array-like

Arrays containing points. Respective shapes (n_samples1, n_features) and (n_samples2, n_features)

batch_size : integer

To reduce memory consumption over the naive solution, data are processed in batches, comprising batch_size rows of X and batch_size rows of Y. The default value is quite conservative, but can be changed for fine-tuning. The larger the number, the larger the memory usage.

metric : string or callable

metric to use for distance computation. Any metric from scikit-learn or scipy.spatial.distance can be used.

If metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays as input and return one value indicating the distance between them. This works for Scipy’s metrics, but is less efficient than passing the metric name as a string.

Distance matrices are not supported.

Valid values for metric are:

• from scikit-learn: ['cityblock', 'cosine', 'euclidean', 'l1', 'l2', 'manhattan']

• from scipy.spatial.distance: ['braycurtis', 'canberra', 'chebyshev', 'correlation', 'dice', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'yule']

See the documentation for scipy.spatial.distance for details on these metrics.

metric_kwargs : dict

keyword arguments to pass to specified metric function.

axis : int, optional, default 1

Axis along which the argmin and distances are to be computed.

Returnsargmin : numpy.ndarray

Y[argmin[i], :] is the row in Y that is closest to X[i, :].

See also:

sklearn.metrics.pairwise_distances, sklearn.metrics.pairwise_distances_argmin_min

Examples using sklearn.metrics.pairwise_distances_argmin

• Color Quantization using K-Means

• Comparison of the K-Means and MiniBatchKMeans clustering algorithms

sklearn.metrics.pairwise_distances_argmin_min

sklearn.metrics.pairwise_distances_argmin_min(X, Y, axis=1, metric='euclidean', batch_size=500, metric_kwargs=None)

Compute minimum distances between one point and a set of points.
This function computes for each row in \( X \), the index of the row of \( Y \) which is closest (according to the specified distance). The minimal distances are also returned.

This is mostly equivalent to calling:

\[
(\text{pairwise_distances}(X, Y=Y, \text{metric}=\text{metric}).\text{argmin}(\text{axis}=\text{axis}), \text{pairwise_distances}(X, Y=Y, \text{metric}=\text{metric}).\text{min}(\text{axis}=\text{axis}))
\]

but uses much less memory, and is faster for large arrays.

**Parameters**

- **\( X, Y \) : \{array-like, sparse matrix\}**
  - Arrays containing points. Respective shapes \((n_{samples1}, n_{features})\) and \((n_{samples2}, n_{features})\)

- **\( \text{batch\_size} \) : integer**
  - To reduce memory consumption over the naive solution, data are processed in batches, comprising \( \text{batch\_size} \) rows of \( X \) and \( \text{batch\_size} \) rows of \( Y \). The default value is quite conservative, but can be changed for fine-tuning. The larger the number, the larger the memory usage.

- **\( \text{metric} \) : string or callable, default ‘euclidean’**
  - metric to use for distance computation. Any metric from scikit-learn or scipy.spatial.distance can be used.

  If metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays as input and return one value indicating the distance between them. This works for Scipy’s metrics, but is less efficient than passing the metric name as a string.

  Distance matrices are not supported.

  Valid values for metric are:


  See the documentation for scipy.spatial.distance for details on these metrics.

- **\( \text{metric\_kwargs} \) : dict, optional**
  - Keyword arguments to pass to specified metric function.

- **\( \text{axis} \) : int, optional, default 1**
  - Axis along which the \text{argmin} and distances are to be computed.

**Returns**

- **\( \text{argmin} \) : numpy.ndarray**
  - \( Y[\text{argmin[i]}, :] \) is the row in \( Y \) that is closest to \( X[i, :] \).

- **\( \text{distances} \) : numpy.ndarray**
  - \( \text{distances[i]} \) is the distance between the \( i \)-th row in \( X \) and the \( \text{argmin[i]} \)-th row in \( Y \).

**See also:**

sklearn.metrics.pairwise_distances, sklearn.metrics.pairwise_distances_argmin
sklearn.metrics.pairwise.paired_euclidean_distances

Computes the paired euclidean distances between X and Y

Read more in the User Guide.

Parameters

X : array-like, shape (n_samples, n_features)
Y : array-like, shape (n_samples, n_features)

Returns
distances : ndarray (n_samples, )

sklearn.metrics.pairwise.paired_manhattan_distances

Compute the L1 distances between the vectors in X and Y.

Read more in the User Guide.

Parameters

X : array-like, shape (n_samples, n_features)
Y : array-like, shape (n_samples, n_features)

Returns
distances : ndarray (n_samples, )

sklearn.metrics.pairwise.paired_cosine_distances

Computes the paired cosine distances between X and Y

Read more in the User Guide.

Parameters

X : array-like, shape (n_samples, n_features)
Y : array-like, shape (n_samples, n_features)

Returns
distances : ndarray, shape (n_samples, )

Notes

The cosine distance is equivalent to the half the squared euclidean distance if each sample is normalized to unit norm

sklearn.metrics.pairwise.paired_distances

Computes the paired distances between X and Y.

Read more in the User Guide.

Parameters

X : ndarray (n_samples, n_features)
Y : ndarray (n_samples, n_features)

Notes

The cosines distance is equivalent to half the squared euclidean distance if each sample is normalized to unit norm
metric : string or callable

The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options specified in PAIRED_DISTANCES, including “euclidean”, “manhattan”, or “cosine”. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from X as input and return a value indicating the distance between them.

Returns:
distances : ndarray (n_samples,)

See also:

pairwise_distances: pairwise distances.

Examples

```python
>>> from sklearn.metrics.pairwise import paired_distances
>>> X = [[0, 1], [1, 1]]
>>> Y = [[0, 1], [2, 1]]
>>> paired_distances(X, Y)
array([[0., 1.])
```

29.21 sklearn.mixture: Gaussian Mixture Models

The sklearn.mixture module implements mixture modeling algorithms.

User guide: See the Gaussian mixture models section for further details.

`mixture.GaussianMixture([n_components, ...])` Gaussian Mixture.

`mixture.BayesianGaussianMixture([...])` Variational Bayesian estimation of a Gaussian mixture.

29.21.1 sklearn.mixture.GaussianMixture

class sklearn.mixture.GaussianMixture

Gaussian Mixture.

Representation of a Gaussian mixture model probability distribution. This class allows to estimate the parameters of a Gaussian mixture distribution.

New in version 0.18.

`GaussianMixture`.

Read more in the User Guide.

Parameters:

- `n_components` : int, defaults to 1.
  
The number of mixture components.
covariance_type : \{'full', 'tied', 'diag', 'spherical'\},
                        
    defaults to 'full'.

String describing the type of covariance parameters to use. Must be one of:

- 'full': (each component has its own general covariance matrix),
- 'tied': (all components share the same general covariance matrix),
- 'diag': (each component has its own diagonal covariance matrix),
- 'spherical': (each component has its own single variance).

tol : float, defaults to 1e-3.

    The convergence threshold. EM iterations will stop when the lower bound average gain
    is below this threshold.

reg_covar : float, defaults to 0.

    Non-negative regularization added to the diagonal of covariance. Allows to assure that
    the covariance matrices are all positive.

max_iter : int, defaults to 100.

    The number of EM iterations to perform.

n_init : int, defaults to 1.

    The number of initializations to perform. The best results are kept.

init_params : \{'kmeans', 'random'\}, defaults to 'kmeans'.

    The method used to initialize the weights, the means and the precisions. Must be one
    of:

- 'kmeans': responsibilities are initialized using kmeans.
- 'random': responsibilities are initialized randomly.

weights_init : array-like, shape (n_components, ), optional

    The user-provided initial weights, defaults to None. If it None, weights are initialized
    using the init_params method.

means_init: array-like, shape (n_components, n_features), optional :

    The user-provided initial means, defaults to None, If it None, means are initialized using
    the init_params method.

precisions_init: array-like, optional. :

    The user-provided initial precisions (inverse of the covariance matrices), defaults to
    None. If it None, precisions are initialized using the 'init_params' method. The shape
    depends on 'covariance_type':

    (n_components,) if 'spherical',
    (n_features, n_features) if 'tied',
    (n_components, n_features) if 'diag',
    (n_components, n_features, n_features) if 'full'

random_state : RandomState or an int seed, defaults to None.

    A random number generator instance.

warm_start : bool, default to False.
If ‘warm_start’ is True, the solution of the last fitting is used as initialization for the next call of fit(). This can speed up convergence when fit is called several time on similar problems.

verbose : int, default to 0.

Enable verbose output. If 1 then it prints the current initialization and each iteration step. If greater than 1 then it prints also the log probability and the time needed for each step.

verbose_interval : int, default to 10.

Number of iteration done before the next print.

Attributes weights_ : array-like, shape (n_components,)

The weights of each mixture components.

means_ : array-like, shape (n_components, n_features)

The mean of each mixture component.

covariances_ : array-like

The covariance of each mixture component. The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>Shape</th>
<th>covariance_type</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_components,)</td>
<td>if 'spherical',</td>
</tr>
<tr>
<td>(n_features, n_features)</td>
<td>if 'tied',</td>
</tr>
<tr>
<td>(n_components, n_features)</td>
<td>if 'diag',</td>
</tr>
<tr>
<td>(n_components, n_features, n_features)</td>
<td>if 'full'</td>
</tr>
</tbody>
</table>

precisions_ : array-like

The precision matrices for each component in the mixture. A precision matrix is the inverse of a covariance matrix. A covariance matrix is symmetric positive definite so the mixture of Gaussian can be equivalently parameterized by the precision matrices. Storing the precision matrices instead of the covariance matrices makes it more efficient to compute the log-likelihood of new samples at test time. The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>Shape</th>
<th>covariance_type</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_components,)</td>
<td>if 'spherical',</td>
</tr>
<tr>
<td>(n_features, n_features)</td>
<td>if 'tied',</td>
</tr>
<tr>
<td>(n_components, n_features)</td>
<td>if 'diag',</td>
</tr>
<tr>
<td>(n_components, n_features, n_features)</td>
<td>if 'full'</td>
</tr>
</tbody>
</table>

precisions_cholesky_ : array-like

The cholesky decomposition of the precision matrices of each mixture component. A precision matrix is the inverse of a covariance matrix. A covariance matrix is symmetric positive definite so the mixture of Gaussian can be equivalently parameterized by the precision matrices. Storing the precision matrices instead of the covariance matrices makes it more efficient to compute the log-likelihood of new samples at test time. The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>Shape</th>
<th>covariance_type</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_components,)</td>
<td>if 'spherical',</td>
</tr>
<tr>
<td>(n_features, n_features)</td>
<td>if 'tied',</td>
</tr>
<tr>
<td>(n_components, n_features)</td>
<td>if 'diag',</td>
</tr>
<tr>
<td>(n_components, n_features, n_features)</td>
<td>if 'full'</td>
</tr>
</tbody>
</table>

converged_ : bool

True when convergence was reached in fit(), False otherwise.
n_iter_ : int
Number of step used by the best fit of EM to reach the convergence.

lower_bound_ : float
Log-likelihood of the best fit of EM.

See also:

BayesianGaussianMixture Gaussian mixture model fit with a variational inference.

Methods

aic(X) Akaike information criterion for the current model on the input X.
bic(X) Bayesian information criterion for the current model on the input X.
fit(X[, y]) Estimate model parameters with the EM algorithm.
get_params([deep]) Get parameters for this estimator.
predict(X[, y]) Predict the labels for the data samples in X using trained model.
predict_proba(X) Predict posterior probability of data per each component.
sample([n_samples]) Generate random samples from the fitted Gaussian distribution.
score(X[, y]) Compute the per-sample average log-likelihood of the given data X.
score_samples(X) Compute the weighted log probabilities for each sample.
set_params(**params) Set the parameters of this estimator.

__init__(n_components=1, covariance_type='full', tol=0.001, reg_covar=1e-06, max_iter=100,
         n_init=1, init_params='kmeans', weights_init=None, means_init=None,
         precisions_init=None, random_state=None, warm_start=False, verbose=0,
         verbose_interval=10)
aic(X) Akaike information criterion for the current model on the input X.

Parameters X : array of shape (n_samples, n_dimensions)

Returns aic : float
The lower the better.

bic(X) Bayesian information criterion for the current model on the input X.

Parameters X : array of shape (n_samples, n_dimensions)

Returns bic : float
The lower the better.

fit (X, y=None) Estimate model parameters with the EM algorithm.
The method fit the model $n_{\text{init}}$ times and set the parameters with which the model has the largest likelihood or lower bound. Within each trial, the method iterates between E-step and M-step for $max_{\text{iter}}$ times until the change of likelihood or lower bound is less than $tol$, otherwise, a ConvergenceWarning is raised.

**Parameters**

$X$ : array-like, shape (n_samples, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

self :

get_params ($deep=True$)

Get parameters for this estimator.

**Parameters**

$deep$ : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

$params$ : mapping of string to any

Parameter names mapped to their values.

**predict** ($X$, $y=None$)

Predict the labels for the data samples in $X$ using trained model.

**Parameters**

$X$ : array-like, shape (n_samples, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

$labels$ : array, shape (n_samples,)

Component labels.

**predict_proba** ($X$)

Predict posterior probability of data per each component.

**Parameters**

$X$ : array-like, shape (n_samples, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

$resp$ : array, shape (n_samples, n_components)

Returns the probability of the sample for each Gaussian (state) in the model.

**sample** ($n_{\text{samples}}=1$)

Generate random samples from the fitted Gaussian distribution.

**Parameters**

$n_{\text{samples}}$ : int, optional

Number of samples to generate. Defaults to 1.

**Returns**

$X$ : array, shape (n_samples, n_features)

Randomly generated sample

$y$ : array, shape (n_samples,)

Component labels

**score** ($X$, $y=None$)

Compute the per-sample average log-likelihood of the given data $X$.

**Parameters**

$X$ : array-like, shape (n_samples, n_dimensions)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

$log_{\text{likelihood}}$ : float

Log likelihood of the Gaussian mixture given $X$. 

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**score_samples**(*X*)
Compute the weighted log probabilities for each sample.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **log_prob**: array, shape (n_samples,)
  Log probabilities of each data point in X.

**set_params**(**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**Examples using sklearn.mixture.GaussianMixture**

- Gaussian Mixture Model Ellipsoids
- GMM covariances
- Density Estimation for a Gaussian mixture
- Gaussian Mixture Model Selection
- Gaussian Mixture Model Sine Curve

### 29.21.2 sklearn.mixture.BayesianGaussianMixture

class **sklearn.mixture.BayesianGaussianMixture**

Variational Bayesian estimation of a Gaussian mixture.

This class allows to infer an approximate posterior distribution over the parameters of a Gaussian mixture distribution. The effective number of components can be inferred from the data.

This class implements two types of prior for the weights distribution: a finite mixture model with Dirichlet distribution and an infinite mixture model with the Dirichlet Process. In practice Dirichlet Process inference algorithm is approximated and uses a truncated distribution with a fixed maximum number of components (called the Stick-breaking representation). The number of components actually used almost always depends on the data.

New in version 0.18.

*BayesianGaussianMixture.*

Parameters

- **n_components**: int, defaults to 1.
  - The number of mixture components. Depending on the data and the value of the weight_concentration_prior the model can decide to not use all the components by setting some component weights to values very close to zero. The number of effective components is therefore smaller than n_components.

- **covariance_type**: {'full', 'tied', 'diag', 'spherical'}, defaults to ‘full’
  - String describing the type of covariance parameters to use. Must be one of:
    - 'full' (each component has its own general covariance matrix),
    - 'tied' (all components share the same general covariance matrix),
    - 'diag' (each component has its own diagonal covariance matrix),
    - 'spherical' (each component has its own single variance).

- **tol**: float, defaults to 1e-3.
  - The convergence threshold. EM iterations will stop when the lower bound average gain on the likelihood (of the training data with respect to the model) is below this threshold.

- **reg_covar**: float, defaults to 1e-6.
  - Non-negative regularization added to the diagonal of covariance. Allows to assure that the covariance matrices are all positive.

- **max_iter**: int, defaults to 100.
  - The number of EM iterations to perform.

- **n_init**: int, defaults to 1.
  - The number of initializations to perform. The result with the highest lower bound value on the likelihood is kept.

- **init_params**: {'kmeans', 'random'}, defaults to ‘kmeans’.
  - The method used to initialize the weights, the means and the covariances. Must be one of:
    - 'kmeans' : responsibilities are initialized using kmeans.
    - 'random' : responsibilities are initialized randomly.

- **weight_concentration_prior_type**: str, defaults to ‘dirichlet_process’.
  - String describing the type of the weight concentration prior. Must be one of:
    - 'dirichlet_process' (using the Stick-breaking representation),
    - 'dirichlet_distribution' (can favor more uniform weights).

- **weight_concentration_prior**: float | None, optional.
  - The dirichlet concentration of each component on the weight distribution (Dirichlet). The higher concentration puts more mass in the center and will lead to more components being active, while a lower concentration parameter will lead to more mass at the edge of the mixture weights simplex. The value of the parameter must be greater than 0. If it is None, it’s set to 1 / n_components.

- **mean_precision_prior**: float | None, optional.
  - The precision prior on the mean distribution (Gaussian). Controls the extend to where means can be placed. Smaller values concentrate the means of each clusters around
mean prior. The value of the parameter must be greater than 0. If it is None, it’s set to 1.

mean prior : array-like, shape (n_features,), optional

The prior on the mean distribution (Gaussian). If it is None, it’s set to the mean of X.

degrees_of_freedom_prior : float | None, optional.

The prior of the number of degrees of freedom on the covariance distributions (Wishart).
If it is None, it’s set to n_features.

covariance_prior : float or array-like, optional

The prior on the covariance distribution (Wishart). If it is None, the empirical covariance prior is initialized using the covariance of X. The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>(n_features, n_features)</th>
<th>if 'full',</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_features, n_features)</td>
<td>if 'tied',</td>
</tr>
<tr>
<td>(n_features)</td>
<td>if 'diag',</td>
</tr>
<tr>
<td>float</td>
<td>if 'spherical'</td>
</tr>
</tbody>
</table>

random_state: RandomState or an int seed, defaults to None.

A random number generator instance.

warm_start : bool, default to False.

If 'warm_start' is True, the solution of the last fitting is used as initialization for the next call of fit(). This can speed up convergence when fit is called several times on similar problems.

verbose : int, default to 0.

Enable verbose output. If 1 then it prints the current initialization and each iteration step. If greater than 1 then it prints also the log probability and the time needed for each step.

verbose_interval : int, default to 10.

Number of iteration done before the next print.

Attributes:

weights_ : array-like, shape (n_components,)

The weights of each mixture component.

means_ : array-like, shape (n_components, n_features)

The mean of each mixture component.

covariances_ : array-like

The covariance of each mixture component. The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>(n_components,)</th>
<th>if 'spherical',</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_features, n_features)</td>
<td>if 'tied',</td>
</tr>
<tr>
<td>(n_components, n_features)</td>
<td>if 'diag',</td>
</tr>
<tr>
<td>(n_components, n_features, n_features)</td>
<td>if 'full'</td>
</tr>
</tbody>
</table>

precisions_ : array-like

The precision matrices for each component in the mixture. A precision matrix is the inverse of a covariance matrix. A covariance matrix is symmetric positive definite so the mixture of Gaussian can be equivalently parameterized by the precision matrices.
Storing the precision matrices instead of the covariance matrices makes it more efficient to compute the log-likelihood of new samples at test time. The shape depends on `covariance_type`:

```
(n_components,) if 'spherical',
(n_features, n_features) if 'tied',
(n_components, n_features) if 'diag',
(n_components, n_features, n_features) if 'full'
```

`precisions_cholesky_`: array-like

The cholesky decomposition of the precision matrices of each mixture component. A precision matrix is the inverse of a covariance matrix. A covariance matrix is symmetric positive definite so the mixture of Gaussian can be equivalently parameterized by the precision matrices. Storing the precision matrices instead of the covariance matrices makes it more efficient to compute the log-likelihood of new samples at test time. The shape depends on `covariance_type`:

```
(n_components,) if 'spherical',
(n_features, n_features) if 'tied',
(n_components, n_features) if 'diag',
(n_components, n_features, n_features) if 'full'
```

`converged_`: bool

True when convergence was reached in fit(), False otherwise.

`n_iter_`: int

Number of step used by the best fit of inference to reach the convergence.

`lower_bound_`: float

Lower bound value on the likelihood (of the training data with respect to the model) of the best fit of inference.

`weight_concentration_prior_`: tuple or float

The dirichlet concentration of each component on the weight distribution (Dirichlet). The type depends on `weight_concentration_prior_type`:

```
(float, float) if 'dirichlet_process' (Beta parameters),
float if 'dirichlet_distribution' (Dirichlet parameters).
```

The higher concentration puts more mass in the center and will lead to more components being active, while a lower concentration parameter will lead to more mass at the edge of the simplex.

`weight_concentration_`: array-like, shape (n_components,)

The dirichlet concentration of each component on the weight distribution (Dirichlet).

`mean_precision_prior`: float

The precision prior on the mean distribution (Gaussian). Controls the extend to where means can be placed. Smaller values concentrate the means of each clusters around `mean_prior`.

`mean_precision_`: array-like, shape (n_components,)

The precision of each components on the mean distribution (Gaussian).
means_prior_: array-like, shape (n_features,)

The prior on the mean distribution (Gaussian).

degrees_of_freedom_prior_: float

The prior of the number of degrees of freedom on the covariance distributions (Wishart).

degrees_of_freedom_: array-like, shape (n_components,)

The number of degrees of freedom of each components in the model.

covariance_prior_: float or array-like

The prior on the covariance distribution (Wishart). The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>covariance_type</th>
<th>shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>'full'</td>
<td>(n_features, n_features)</td>
</tr>
<tr>
<td>'tied'</td>
<td>(n_features, n_features)</td>
</tr>
<tr>
<td>'diag'</td>
<td>(n_features)</td>
</tr>
<tr>
<td>'spherical'</td>
<td>float</td>
</tr>
</tbody>
</table>

See also:

GaussianMixture: Finite Gaussian mixture fit with EM.

References

[R67], [R68], [R69]

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X[, y])</td>
<td>Estimate model parameters with the EM algorithm.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X[, y])</td>
<td>Predict the labels for the data samples in X using trained model.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict posterior probability of data per each component.</td>
</tr>
<tr>
<td>sample([n_samples])</td>
<td>Generate random samples from the fitted Gaussian distribution.</td>
</tr>
<tr>
<td>score(X[, y])</td>
<td>Compute the per-sample average log-likelihood of the given data X.</td>
</tr>
<tr>
<td>score_samples(X)</td>
<td>Compute the weighted log probabilities for each sample.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ (n_components=1, covariance_type='full', tol=0.001, reg_covar=1e-06, max_iter=100, n_init=1, init_params='kmeans', weight_concentration_prior_type='dirichlet_process', weight_concentration_prior=None, mean_precision_prior=None, mean_prior=None, degrees_of_freedom_prior=None, covariance_prior=None, random_state=None, warm_start=False, verbose=0, verbose_interval=10)

fit (X, y=None)

Estimate model parameters with the EM algorithm.

The method fit the model n_init times and set the parameters with which the model has the largest likeli-
hood or lower bound. Within each trial, the method iterates between E-step and M-step for max_iter times until the change of likelihood or lower bound is less than tol, otherwise, a ConvergenceWarning is raised.

Parameters:

**X**: array-like, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns:

**self**: 

get_params (deep=True)
Get parameters for this estimator.

Parameters:

**deep**: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

**params**: mapping of string to any
Parameter names mapped to their values.

predict (X, y=None)
Predict the labels for the data samples in X using trained model.

Parameters:

**X**: array-like, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns:

**labels**: array, shape (n_samples,)
Component labels.

predict_proba (X)
Predict posterior probability of data per each component.

Parameters:

**X**: array-like, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns:

**resp**: array, shape (n_samples, n_components)
Returns the probability of the sample for each Gaussian (state) in the model.

sample (n_samples=1)
Generate random samples from the fitted Gaussian distribution.

Parameters:

**n_samples**: int, optional
Number of samples to generate. Defaults to 1.

Returns:

**X**: array, shape (n_samples, n_features)
Randomly generated sample

**y**: array, shape (n_samples,)
Component labels

score (X, y=None)
Compute the per-sample average log-likelihood of the given data X.

Parameters:

**X**: array-like, shape (n_samples, n_dimensions)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns:

**log_likelihood**: float
Log likelihood of the Gaussian mixture given X.
**score_samples** *(X)*
Compute the weighted log probabilities for each sample.

**Parameters**
- **X**: array-like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**
- **log_prob**: array, shape (n_samples,)
  Log probabilities of each data point in X.

**set_params** *(**params**)*
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**
- **self**: 

**Examples using sklearn.mixture.BayesianGaussianMixture**

- Concentration Prior Type Analysis of Variation Bayesian Gaussian Mixture
- Gaussian Mixture Model Ellipsoids
- Gaussian Mixture Model Sine Curve

**29.22 sklearn.multiclass: Multiclass and multilabel classification**

**29.22.1 Multiclass and multilabel classification strategies**

This module implements multiclass learning algorithms:

- one-vs-the-rest / one-vs-all
- one-vs-one
- error correcting output codes

The estimators provided in this module are meta-estimators: they require a base estimator to be provided in their constructor. For example, it is possible to use these estimators to turn a binary classifier or a regressor into a multiclass classifier. It is also possible to use these estimators with multiclass estimators in the hope that their accuracy or runtime performance improves.

All classifiers in scikit-learn implement multiclass classification; you only need to use this module if you want to experiment with custom multiclass strategies.

The one-vs-the-rest meta-classifier also implements a `predict_proba` method, so long as such a method is implemented by the base classifier. This method returns probabilities of class membership in both the single label and multilabel case. Note that in the multilabel case, probabilities are the marginal probability that a given sample falls in the given class. As such, in the multilabel case the sum of these probabilities over all possible labels for a given sample will *not* sum to unity, as they do in the single label case.

**User guide:** See the *Multiclass and multilabel algorithms* section for further details.
multiclass.OneVsRestClassifier(estimator[, ...
...])

One-vs-the-rest (OvR) multiclass/multilabel strategy

multiclass.OneVsOneClassifier(estimator[, ...
...])

One-vs-one multiclass strategy

multiclass.OutputCodeClassifier(estimator[, ...
...])

(Error-Correcting) Output-Code multiclass strategy

29.22.2 sklearn.multiclass.OneVsRestClassifier

class sklearn.multiclass.OneVsRestClassifier (estimator, n_jobs=1)

One-vs-the-rest (OvR) multiclass/multilabel strategy

Also known as one-vs-all, this strategy consists in fitting one classifier per class. For each classifier, the class
is fitted against all the other classes. In addition to its computational efficiency (only n_classes classifiers are
needed), one advantage of this approach is its interpretability. Since each class is represented by one and one
classifier only, it is possible to gain knowledge about the class by inspecting its corresponding classifier. This is
the most commonly used strategy for multiclass classification and is a fair default choice.

This strategy can also be used for multilabel learning, where a classifier is used to predict multiple labels for
instance, by fitting on a 2-d matrix in which cell [i, j] is 1 if sample i has label j and 0 otherwise.

In the multilabel learning literature, OvR is also known as the binary relevance method.

Read more in the User Guide.

Parameters

estimator : estimator object

An estimator object implementing fit and one of decision_function or predict_proba.

n_jobs : int, optional, default: 1

The number of jobs to use for the computation. If -1 all CPUs are used. If 1 is given, no
parallel computing code is used at all, which is useful for debugging. For n_jobs below
-1, (n_cpus + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

Attributes

estimators_ : list of n_classes estimators

Estimators used for predictions.

classes_ : array, shape = [n_classes]

Class labels.

label_binarizer_ : LabelBinarizer object

Object used to transform multiclass labels to binary labels and vice-versa.

multilabel_ : boolean

Whether a OneVsRestClassifier is a multilabel classifier.

Methods

decision_function(X) Returns the distance of each sample from the decision
boundary for each class.

fit(X, y) Fit underlying estimators.

get_params([deep]) Get parameters for this estimator.

Continued on next page
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<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>partial_fit</code></td>
<td>Partially fit underlying estimators</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>Predict multi-class targets using underlying estimators.</td>
</tr>
<tr>
<td><code>predict_proba</code></td>
<td>Probability estimates.</td>
</tr>
<tr>
<td><code>score</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```
__init__(estimator, n_jobs=1)
```

```
decision_function(X)
```
Returns the distance of each sample from the decision boundary for each class. This can only be used with estimators which implement the decision_function method.

Parameters:
- `X`: array-like, shape = [n_samples, n_features]

Returns:
- `T`: array-like, shape = [n_samples, n_classes]

```
fit(X, y)
```
Fit underlying estimators.

Parameters:
- `X`: (sparse) array-like, shape = [n_samples, n_features]
  - Data.
- `y`: (sparse) array-like, shape = [n_samples, ], [n_samples, n_classes]
  - Multi-class targets. An indicator matrix turns on multilabel classification.

Returns:
- `self`

```
get_params(deep=True)
```
Get parameters for this estimator.

Parameters:
- `deep`: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- `params`: mapping of string to any
  - Parameter names mapped to their values.

```
multilabel
```
Whether this is a multilabel classifier

```
partial_fit(X, y, classes=None)
```
Partially fit underlying estimators

Should be used when memory is inefficient to train all data. Chunks of data can be passed in several iteration.

Parameters:
- `X`: (sparse) array-like, shape = [n_samples, n_features]
  - Data.
- `y`: (sparse) array-like, shape = [n_samples, ], [n_samples, n_classes]
  - Multi-class targets. An indicator matrix turns on multilabel classification.
- `classes`: array, shape (n_classes, )
  - Classes across all calls to partial_fit. Can be obtained via `np.unique(y_all)`, where `y_all` is the target vector of the entire dataset. This argument is only required in the first call of partial_fit and can be omitted in the subsequent calls.
**Returns** self:

**predict** *(X)*

Predict multi-class targets using underlying estimators.

**Parameters**

- **X**: (sparse) array-like, shape = [n_samples, n_features]
  Data.

**Returns**

- **y**: (sparse) array-like, shape = [n_samples, ], [n_samples, n_classes].
  Predicted multi-class targets.

**predict_proba** *(X)*

Probability estimates.

The returned estimates for all classes are ordered by label of classes.

Note that in the multilabel case, each sample can have any number of labels. This returns the marginal probability that the given sample has the label in question. For example, it is entirely consistent that two labels both have a 90% probability of applying to a given sample.

In the single label multiclass case, the rows of the returned matrix sum to 1.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **T**: (sparse) array-like, shape = [n_samples, n_classes]
  Returns the probability of the sample for each class in the model, where classes are ordered as they are in `self.classes_`.

**score** *(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.

- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.

**sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **score**: float
  Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

**Examples using** `sklearn.multiclass.OneVsRestClassifier`

- **Multilabel classification**
• Precision-Recall
• Receiver Operating Characteristic (ROC)

29.22.3 sklearn.multiclass.OneVsOneClassifier

class sklearn.multiclass.OneVsOneClassifier (estimator, n_jobs=1)

One-vs-one multiclass strategy

This strategy consists in fitting one classifier per class pair. At prediction time, the class which received the most votes is selected. Since it requires to fit $n_{classes} * (n_{classes} - 1) / 2$ classifiers, this method is usually slower than one-vs-the-rest, due to its $O(n_{classes}^2)$ complexity. However, this method may be advantageous for algorithms such as kernel algorithms which don’t scale well with $n_{samples}$. This is because each individual learning problem only involves a small subset of the data whereas, with one-vs-the-rest, the complete dataset is used $n_{classes}$ times.

Read more in the User Guide.

Parameter estimator : estimator object
An estimator object implementing fit and one of decision_function or predict_proba.

n_jobs : int, optional, default: 1
The number of jobs to use for the computation. If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, $(n\_cpus + 1 + n\_jobs)$ are used. Thus for n_jobs = -2, all CPUs but one are used.

Attributes estimators_ : list of $n_{classes} * (n_{classes} - 1) / 2$ estimators
Estimators used for predictions.

classes_ : numpy array of shape [n_classes]
Array containing labels.

Methods

decision_function(X) Decision function for the OneVsOneClassifier.

fit(X, y) Fit underlying estimators.

get_params([deep]) Get parameters for this estimator.

partial_fit(X, y[, classes]) Partially fit underlying estimators

predict(X) Estimate the best class label for each sample in X.

cscore(X, y[, sample_weight]) Returns the mean accuracy on the given test data and labels.

set_params(**params) Set the parameters of this estimator.

__init__(estimator, n_jobs=1)

decision_function(X)
Decision function for the OneVsOneClassifier.

The decision values for the samples are computed by adding the normalized sum of pair-wise classification confidence levels to the votes in order to disambiguate between the decision values when the votes for all the classes are equal leading to a tie.

Parameters

X : array-like, shape = [n_samples, n_features]
Returns $Y$ : array-like, shape = [n_samples, n_classes]

**fit** ($X$, $y$)
Fit underlying estimators.

**Parameters**

$X$ : (sparse) array-like, shape = [n_samples, n_features]
Data.

$y$ : array-like, shape = [n_samples]
Multi-class targets.

**Returns**

**self** :

**get_params** ($deep=True$)
Get parameters for this estimator.

**Parameters**

$deep$ : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

$param$ : mapping of string to any
Parameter names mapped to their values.

**partial_fit** ($X$, $y$, $classes=None$)
Partially fit underlying estimators
Should be used when memory is inefficient to train all data. Chunks of data can be passed in several iteration, where the first call should have an array of all target variables.

**Parameters**

$X$ : (sparse) array-like, shape = [n_samples, n_features]
Data.

$y$ : array-like, shape = [n_samples]
Multi-class targets.

$classes$ : array, shape (n_classes, )
Classes across all calls to partial_fit. Can be obtained via `np.unique(y_all)`, where $y_all$ is the target vector of the entire dataset. This argument is only required in the first call of partial_fit and can be omitted in the subsequent calls.

**Returns**

**self** :

**predict** ($X$)
Estimate the best class label for each sample in X.
This is implemented as argmax(decision_function($X$), axis=1) which will return the label of the class with most votes by estimators predicting the outcome of a decision for each possible class pair.

**Parameters**

$X$ : (sparse) array-like, shape = [n_samples, n_features]
Data.

**Returns**

$s$ : numpy array of shape [n_samples]
Predicted multi-class targets.

**score** ($X$, $y$, $sample_weight=None$)
Returns the mean accuracy on the given test data and labels.
In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.
Parameters

**X**: array-like, shape = (n_samples, n_features)
Test samples.

**y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
True labels for X.

**sample_weight**: array-like, shape = [n_samples], optional
Sample weights.

Returns

**score**: float
Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns

**self**

29.22.4 sklearn.multiclass.OutputCodeClassifier

```python
class sklearn.multiclass.OutputCodeClassifier(estimator, code_size=1.5, random_state=None, n_jobs=1)
```

(Errcor-Correcting) Output-Code multiclass strategy

Output-code based strategies consist in representing each class with a binary code (an array of 0s and 1s). At fitting time, one binary classifier per bit in the codebook is fitted. At prediction time, the classifiers are used to project new points in the class space and the class closest to the points is chosen. The main advantage of these strategies is that the number of classifiers used can be controlled by the user, either for compressing the model (0 < code_size < 1) or for making the model more robust to errors (code_size > 1). See the documentation for more details.

Read more in the *User Guide.*

**estimator**: estimator object
An estimator object implementing `fit` and one of `decision_function` or `predict_proba`.

**code_size**: float
Percentage of the number of classes to be used to create the code book. A number between 0 and 1 will require fewer classifiers than one-vs-the-rest. A number greater than 1 will require more classifiers than one-vs-the-rest.

**random_state**: numpy.RandomState, optional
The generator used to initialize the codebook. Defaults to numpy.random.

**n_jobs**: int, optional, default: 1
The number of jobs to use for the computation. If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpu + 1 + n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**Attributes**

**estimators_**: list of int(n_classes * code_size) estimators
Estimators used for predictions.

**classes_**: numpy array of shape [n_classes]
Array containing labels.

\texttt{code_book} : numpy array of shape \([n\_classes, code\_size]\)

Binary array containing the code of each class.

\textbf{References}

[R234], [R235], [R236]

\textbf{Methods}

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit underlying estimators.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict multi-class targets using underlying estimators.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

\texttt{__init__} (estimator, code_size=1.5, random_state=None, n_jobs=1)

\texttt{fit} \((X, y)\)

Fit underlying estimators.

\textbf{Parameters}

- \texttt{X} : (sparse) array-like, shape = \([n\_samples, n\_features]\)
  
  Data.

- \texttt{y} : numpy array of shape \([n\_samples]\)
  
  Multi-class targets.

\textbf{Returns}

- \texttt{self} :

\texttt{get_params} (\texttt{deep}=True)

Get parameters for this estimator.

\textbf{Parameters}

- \texttt{deep} : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

\textbf{Returns}

- \texttt{params} : mapping of string to any
  
  Parameter names mapped to their values.

\texttt{predict} \((X)\)

Predict multi-class targets using underlying estimators.

\textbf{Parameters}

- \texttt{X} : (sparse) array-like, shape = \([n\_samples, n\_features]\)
  
  Data.

\textbf{Returns}

- \texttt{y} : numpy array of shape \([n\_samples]\)
  
  Predicted multi-class targets.

\texttt{score} \((X, y, sample\_weight=None)\)

Returns the mean accuracy on the given test data and labels.

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In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  - Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**

- **score**: float
  - Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Return**

- **self**

---

### 29.23 sklearn.multioutput: Multioutput regression and classification

This module implements multioutput regression and classification.

The estimators provided in this module are meta-estimators: they require a base estimator to be provided in their constructor. The meta-estimator extends single output estimators to multioutput estimators.

**User guide**: See the *Multiclass and multilabel algorithms* section for further details.

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---

#### 29.23.1 sklearn.multioutput.MultiOutputRegressor

**class** `sklearn.multioutput.MultiOutputRegressor(estimator, n_jobs=1)`

Multi target regression

This strategy consists of fitting one regressor per target. This is a simple strategy for extending regressors that do not natively support multi-target regression.

**Parameters**

- **estimator**: estimator object
  - An estimator object implementing `fit` and `predict`.
- **n_jobs**: int, optional, default=1
  - The number of jobs to run in parallel for `fit`. If -1, then the number of jobs is set to the number of cores. When individual estimators are fast to train or predict using `n_jobs>1` can result in slower performance due to the overhead of spawning processes.
Methods

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<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<td><code>predict(X)</code></td>
<td>Predict multi-output variable using a model trained for each target variable.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
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</table>

__init__(estimator, n_jobs=1)

`fit(X, y, sample_weight=None)`
Fit the model to data. Fit a separate model for each output variable.

Parameters:
- `X` : (sparse) array-like, shape (n_samples, n_features)
  Data.
- `y` : (sparse) array-like, shape (n_samples, n_outputs)
  Multi-output targets. An indicator matrix turns on multilabel estimation.
- `sample_weight` : array-like, shape = (n_samples) or None
  Sample weights. If None, then samples are equally weighted. Only supported if the underlying regressor supports sample weights.

Returns:
- `self` : object
  Returns self.

`get_params` (deep=True)
Get parameters for this estimator.

Parameters:
- `deep` : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- `params` : mapping of string to any
  Parameter names mapped to their values.

`predict(X)`
Predict multi-output variable using a model trained for each target variable.

Parameters:
- `X` : (sparse) array-like, shape (n_samples, n_features)
  Data.

Returns:
- `y` : (sparse) array-like, shape (n_samples, n_outputs)
  Multi-output targets predicted across multiple predictors. Note: Separate models are generated for each predictor.

`score(X, y, sample_weight=None)`
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})^2).sum$ and $v$ is the residual sum of squares $((y_{true} - y_{true}.mean())^2).sum$. Best possible score
is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \( y \), disregarding the input features, would get a \( R^2 \) score of 0.0.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  
  Test samples.

- **y**: array-like, shape (n_samples) or (n_samples, n_outputs)
  
  True values for X.

- **sample_weight**: array-like, shape [n_samples], optional
  
  Sample weights.

**Returns**

- **score**: float
  
  \( R^2 \) of self.predict(X) wrt. \( y \).

**Notes**

\( R^2 \) is calculated by weighting all the targets equally using `multioutput='uniform_average'`.

```
set_params(**params)
```

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

```
Returns
```

```
self :
```

**Examples using sklearn.multioutput.MultiOutputRegressor**

- *Comparing random forests and the multi-output meta estimator*

**29.23.2 sklearn.multioutput.MultiOutputClassifier**

```
class sklearn.multioutput.MultiOutputClassifier (estimator, n_jobs=1)
```

Multi target classification

This strategy consists of fitting one classifier per target. This is a simple strategy for extending classifiers that do not natively support multi-target classification

**Parameters**

- **estimator**: estimator object
  
  An estimator object implementing `fit`, `score` and `predict_proba`.

- **n_jobs**: int, optional, default=1
  
  The number of jobs to use for the computation. If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For `n_jobs` below -1, `(n_cpus + 1 + n_jobs)` are used. Thus for `n_jobs = -2`, all CPUs but one are used. The number of jobs to use for the computation. It does each target variable in \( y \) in parallel.

**Attributes**

- **estimators_**: list of \( n_output \) estimators
  
  Estimators used for predictions.
## Methods

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit the model to data.</td>
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<td><code>predict(X)</code></td>
<td>Predict multi-output variable using a model trained for each target variable.</td>
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<tr>
<td><code>predict_proba(X)</code></td>
<td>Probability estimates.</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>&quot;Returns the mean accuracy on the given test data and labels.&quot;</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

### `__init__`(estimator, n_jobs=1)

**fit**(X, y, sample_weight=None)

Fit the model to data. Fit a separate model for each output variable.

**Parameters**

- **X**: (sparse) array-like, shape (n_samples, n_features)
  Data.
- **y**: (sparse) array-like, shape (n_samples, n_outputs)
  Multi-output targets. An indicator matrix turns on multilabel estimation.
- **sample_weight**: array-like, shape = (n_samples) or None
  Sample weights. If None, then samples are equally weighted. Only supported if the underlying regressor supports sample weights.

**Returns**

- **self**: object
  Returns self.

### `get_params`(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

### `predict`(X)

Predict multi-output variable using a model trained for each target variable.

**Parameters**

- **X**: (sparse) array-like, shape (n_samples, n_features)
  Data.

### `predict_proba`(X)

Probability estimates. Returns prediction probabilities for each class of each output.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
Data

**Returns**

- : (sparse) array-like, shape = (n_samples, n_classes, n_outputs)

  The class probabilities of the samples for each of the outputs

**score** (*X*, *y*)

"Returns the mean accuracy on the given test data and labels.

**Parameters**

- : array-like, shape [n_samples, n_features]
  Test samples
- : array-like, shape [n_samples, n_outputs]
  True values for *X*

**Returns**

- : float
  accuracy_score of self.predict(*X*) versus *y"

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- : self

## 29.24 sklearn.naive_bayes: Naive Bayes

The sklearn.naive_bayes module implements Naive Bayes algorithms. These are supervised learning methods based on applying Bayes’ theorem with strong (naive) feature independence assumptions.

**User guide:** See the Naive Bayes section for further details.

### 29.24.1 sklearn.naive_bayes.GaussianNB

**class** sklearn.naive_bayes.GaussianNB (*priors=None*)

Gaussian Naive Bayes (GaussianNB)

Can perform online updates to model parameters via *partial_fit* method. For details on algorithm used to update feature means and variance online, see Stanford CS tech report STAN-CS-79-773 by Chan, Golub, and LeVeque:


Read more in the User Guide.

**Parameters**

- : array-like, shape (n_classes,)
  Prior probabilities of the classes. If specified the priors are not adjusted according to the data.

**Attributes**

- : array, shape (n_classes,)
  probability of each class.
class_count_ : array, shape (n_classes,)
    number of training samples observed in each class.
theta_ : array, shape (n_classes, n_features)
    mean of each feature per class
sigma_ : array, shape (n_classes, n_features)
    variance of each feature per class

Examples

```python
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> Y = np.array([1, 1, 1, 2, 2, 2])
>>> from sklearn.naive_bayes import GaussianNB
>>> clf = GaussianNB()
>>> clf.fit(X, Y)
GaussianNB(priors=None)
>>> print(clf.predict([[-0.8, -1]]))
[1]
>>> clf_pf = GaussianNB()
>>> clf_pf.partial_fit(X, Y, np.unique(Y))
GaussianNB(priors=None)
>>> print(clf_pf.predict([[-0.8, -1]]))
[1]
```

Methods

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<td>fit(X, y, sample_weight)</td>
<td>Fit Gaussian Naive Bayes according to X, y</td>
</tr>
<tr>
<td>get_params(deep)</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>partial_fit(X, y, classes, sample_weight)</td>
<td>Incremental fit on a batch of samples.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Perform classification on an array of test vectors X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Return log-probability estimates for the test vector X.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Return probability estimates for the test vector X.</td>
</tr>
<tr>
<td>score(X, y, sample_weight)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__ (priors=None)

fit (X, y, sample_weight=None)
    Fit Gaussian Naive Bayes according to X, y

Parameters

X : array-like, shape (n_samples, n_features)
    Training vectors, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape (n_samples,)
    Target values.

sample_weight : array-like, shape (n_samples,), optional (default=None)
    Sample weights.
Weights applied to individual samples (1. for unweighted).

New in version 0.17: Gaussian Naive Bayes supports fitting with `sample_weight`.

Returns: object

```
get_params (deep=True)
```
Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params**: mapping of string to any
  - Parameter names mapped to their values.

```
partial_fit (X, y, classes=None, sample_weight=None)
```
Incremental fit on a batch of samples.

This method is expected to be called several times consecutively on different chunks of a dataset so as to implement out-of-core or online learning.

This is especially useful when the whole dataset is too big to fit in memory at once.

This method has some performance and numerical stability overhead, hence it is better to call `partial_fit` on chunks of data that are as large as possible (as long as fitting in the memory budget) to hide the overhead.

Parameters:
- **X**: array-like, shape (n_samples, n_features)
  - Training vectors, where n_samples is the number of samples and n_features is the number of features.
- **y**: array-like, shape (n_samples,)
  - Target values.
- **classes**: array-like, shape (n_classes,), optional (default=None)
  - List of all the classes that can possibly appear in the y vector.
  - Must be provided at the first call to `partial_fit`, can be omitted in subsequent calls.
- **sample_weight**: array-like, shape (n_samples,), optional (default=None)
  - Weights applied to individual samples (1. for unweighted).
  - New in version 0.17.

Returns: object

```
predict (X)
```
Perform classification on an array of test vectors X.

Parameters:
- **X**: array-like, shape = [n_samples, n_features]

Returns:
- **C**: array, shape = [n_samples]
  - Predicted target values for X

```
predict_log_proba (X)
```
Return log-probability estimates for the test vector X.

Parameters:
- **X**: array-like, shape = [n_samples, n_features]
ReturnsC : array-like, shape = [n_samples, n_classes]

Returns the log-probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute classes_.

**predict_proba** *(X)*

Return probability estimates for the test vector X.

**Parameters**

\[X\] : array-like, shape = [n_samples, n_features]  
\[C\] : array-like, shape = [n_samples, n_classes]  

Returns the probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute classes_.

**score** *(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

\[X\] : array-like, shape = (n_samples, n_features)  
\[y\] : array-like, shape = (n_samples) or (n_samples, n_outputs)  
\[sample_weight\] : array-like, shape = [n_samples], optional

Sample weights.

**Returns**

score : float  
Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

self : 

---

**Examples using sklearn.naive_bayes.GaussianNB**

- *Probability calibration of classifiers*
- *Probability Calibration curves*
- *Comparison of Calibration of Classifiers*
- *Classifier comparison*
- *Plot class probabilities calculated by the VotingClassifier*
- *Plotting Learning Curves*
29.24.2 sklearn.naive_bayes.MultinomialNB

class sklearn.naive_bayes.MultinomialNB (alpha=1.0, fit_prior=True, class_prior=None)

Naive Bayes classifier for multinomial models

The multinomial Naive Bayes classifier is suitable for classification with discrete features (e.g., word counts for text classification). The multinomial distribution normally requires integer feature counts. However, in practice, fractional counts such as tf-idf may also work.

Read more in the User Guide.

Parameters

alpha : float, optional (default=1.0)

Additive (Laplace/Lidstone) smoothing parameter (0 for no smoothing).

fit_prior : boolean, optional (default=True)

Whether to learn class prior probabilities or not. If false, a uniform prior will be used.

class_prior : array-like, size (n_classes,), optional (default=None)

Prior probabilities of the classes. If specified the priors are not adjusted according to the data.

Attributes

class_log_prior_ : array, shape (n_classes, )

Smoothed empirical log probability for each class.

intercept_ : property

Mirrors class_log_prior_ for interpreting MultinomialNB as a linear model.

feature_log_prob_ : array, shape (n_classes, n_features)

Empirical log probability of features given a class, \( P(x_i | y) \).

coef_ : property

Mirrors feature_log_prob_ for interpreting MultinomialNB as a linear model.

class_count_ : array, shape (n_classes,)

Number of samples encountered for each class during fitting. This value is weighted by the sample weight when provided.

feature_count_ : array, shape (n_classes, n_features)

Number of samples encountered for each (class, feature) during fitting. This value is weighted by the sample weight when provided.

Notes

For the rationale behind the names \( coef \) and \( intercept \), i.e. naive Bayes as a linear classifier, see J. Rennie et al. (2003), Tackling the poor assumptions of naive Bayes text classifiers, ICML.

References

Examples

```python
>>> import numpy as np
>>> X = np.random.randint(5, size=(6, 100))
>>> y = np.array([1, 2, 3, 4, 5, 6])
>>> from sklearn.naive_bayes import MultinomialNB
>>> clf = MultinomialNB()
>>> clf.fit(X, y)
MultinomialNB(alpha=1.0, class_prior=None, fit_prior=True)
>>> print(clf.predict(X[2:3]))
[3]
```

Methods

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit Naive Bayes classifier according to X, y</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>partial_fit(X, y[, classes, sample_weight])</code></td>
<td>Incremental fit on a batch of samples.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Perform classification on an array of test vectors X.</td>
</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>Return log-probability estimates for the test vector X.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Return probability estimates for the test vector X.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(alpha=1.0, fit_prior=True, class_prior=None)
```

**Parameters**

- `X` : array-like, sparse matrix, shape = [n_samples, n_features]
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

- `y` : array-like, shape = [n_samples]
  
  Target values.

- `sample_weight` : array-like, shape = [n_samples], optional (default=None)
  
  Weights applied to individual samples (1. for unweighted).

**Return**

- `self`
  
  Returns self.

```python
get_params(deep=True)
```

**Parameters**

- `deep` : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- `params` : mapping of string to any
  
  Parameter names mapped to their values.
**partial_fit** (*X*, *y*, *classes=None*, *sample_weight=None*)
Incremental fit on a batch of samples.

This method is expected to be called several times consecutively on different chunks of a dataset so as to implement out-of-core or online learning.

This is especially useful when the whole dataset is too big to fit in memory at once.

This method has some performance overhead hence it is better to call partial_fit on chunks of data that are as large as possible (as long as fitting in the memory budget) to hide the overhead.

**Parameters**
- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  Training vectors, where n_samples is the number of samples and n_features is the number of features.
- **y**: array-like, shape = [n_samples]
  Target values.
- **classes**: array-like, shape = [n_classes], optional (default=None)
  List of all the classes that can possibly appear in the y vector.
  Must be provided at the first call to partial_fit, can be omitted in subsequent calls.
- **sample_weight**: array-like, shape = [n_samples], optional (default=None)
  Weights applied to individual samples (1. for unweighted).

**Returns**
- **self**: object
  Returns self.

**predict** (*X*)
Perform classification on an array of test vectors X.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]

**Returns**
- **C**: array, shape = [n_samples]
  Predicted target values for X

**predict_log_proba** (*X*)
Return log-probability estimates for the test vector X.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]

**Returns**
- **C**: array-like, shape = [n_samples, n_classes]
  Returns the log-probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`.

**predict_proba** (*X*)
Return probability estimates for the test vector X.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]

**Returns**
- **C**: array-like, shape = [n_samples, n_classes]
  Returns the probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`.

**score** (*X*, *y*, *sample_weight=None*)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.
**Parameters**

- **X**: array-like, shape = (n_samples, n_features)  
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)  
  True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional  
  Sample weights.

**Returns**

- **score**: float  
  Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Return**

**self**

---

**Examples using sklearn.naive_bayes.MultinomialNB**

- Out-of-core classification of text documents
- Classification of text documents using sparse features
- Classification of text documents: using a MLComp dataset

### 29.24.3 sklearn.naive_bayes.BernoulliNB

**class** `sklearn.naive_bayes.BernoulliNB(alpha=1.0, binarize=0.0, fit_prior=True, class_prior=None)`

Naive Bayes classifier for multivariate Bernoulli models.

Like MultinomialNB, this classifier is suitable for discrete data. The difference is that while MultinomialNB works with occurrence counts, BernoulliNB is designed for binary/boolean features.

Read more in the **User Guide**.

**Parameters**

- **alpha**: float, optional (default=1.0)  
  Additive (Laplace/Lidstone) smoothing parameter (0 for no smoothing).
- **binarize**: float or None, optional (default=0.0)  
  Threshold for binarizing (mapping to booleans) of sample features. If None, input is presumed to already consist of binary vectors.
- **fit_prior**: boolean, optional (default=True)  
  Whether to learn class prior probabilities or not. If false, a uniform prior will be used.
- **class_prior**: array-like, size=[n_classes,], optional (default=None)  
  Prior probabilities of the classes. If specified the priors are not adjusted according to the data.

**Attributes**

- **class_log_prior_**: array, shape = [n_classes]  
  Log probability of each class (smoothed).
**feature_log_prob_**: array, shape = [n_classes, n_features]

Empirical log probability of features given a class, P(x_i|y).

**class_count_**: array, shape = [n_classes]

Number of samples encountered for each class during fitting. This value is weighted by the sample weight when provided.

**feature_count_**: array, shape = [n_classes, n_features]

Number of samples encountered for each (class, feature) during fitting. This value is weighted by the sample weight when provided.

### References


### Examples

```python
>>> import numpy as np
>>> X = np.random.randint(2, size=(6, 100))
>>> Y = np.array([1, 2, 3, 4, 4, 5])
>>> from sklearn.naive_bayes import BernoulliNB
>>> clf = BernoulliNB()
>>> clf.fit(X, Y)
BernoulliNB(alpha=1.0, binarize=0.0, class_prior=None, fit_prior=True)
>>> print(clf.predict(X[2:3]))
[3]
```

### Methods

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit Naive Bayes classifier according to X, y</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>partial_fit(X, y[, classes, sample_weight])</code></td>
<td>Incremental fit on a batch of samples.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Perform classification on an array of test vectors X.</td>
</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>Return log-probability estimates for the test vector X.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Return probability estimates for the test vector X.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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**__init__**(alpha=1.0, binarize=0.0, fit_prior=True, class_prior=None)

**fit**(X, y, sample_weight=None)

Fit Naive Bayes classifier according to X, y
Parameters

X: {array-like, sparse matrix}, shape = [n_samples, n_features]
   Training vectors, where n_samples is the number of samples and n_features is the number of features.

y: array-like, shape = [n_samples]
   Target values.

sample_weight: array-like, shape = [n_samples], optional (default=None)
   Weights applied to individual samples (1. for unweighted).

Returns

self: object
   Returns self.

get_params (deep=True)
   Get parameters for this estimator.

Parameters

deep: boolean, optional
   If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

params: mapping of string to any
   Parameter names mapped to their values.

partial_fit (X, y, classes=None, sample_weight=None)
   Incremental fit on a batch of samples.

   This method is expected to be called several times consecutively on different chunks of a dataset so as to
   implement out-of-core or online learning.

   This is especially useful when the whole dataset is too big to fit in memory at once.

   This method has some performance overhead hence it is better to call partial_fit on chunks of data that are
   as large as possible (as long as fitting in the memory budget) to hide the overhead.

   Parameters

   X: {array-like, sparse matrix}, shape = [n_samples, n_features]
      Training vectors, where n_samples is the number of samples and n_features is the number of features.

   y: array-like, shape = [n_samples]
      Target values.

   classes: array-like, shape = [n_classes], optional (default=None)
      List of all the classes that can possibly appear in the y vector.
      Must be provided at the first call to partial_fit, can be omitted in subsequent calls.

   sample_weight: array-like, shape = [n_samples], optional (default=None)
      Weights applied to individual samples (1. for unweighted).

Returns

self: object
   Returns self.

predict (X)
   Perform classification on an array of test vectors X.

   Parameters

   X: array-like, shape = [n_samples, n_features]

   Returns

   C: array, shape = [n_samples]
Predicted target values for \( X \)

**predict_log_proba** \((X)\)
Return log-probability estimates for the test vector \( X \).

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]

**Returns**
- **C**: array-like, shape = [n_samples, n_classes]

Returns the log-probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`.

**predict_proba** \((X)\)
Return probability estimates for the test vector \( X \).

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]

**Returns**
- **C**: array-like, shape = [n_samples, n_classes]

Returns the probability of the samples for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`.

**score** \((X, y, \text{sample_weight}=\text{None})\)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**
- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for \( X \).
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**
- **score**: float
  Mean accuracy of self.predict(X) wrt. \( y \).

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- **self**: 

Examples using `sklearn.naive_bayes.BernoulliNB`

- Hashing feature transformation using Totally Random Trees
- Classification of text documents using sparse features

**29.25 sklearn.neighbors: Nearest Neighbors**

The `sklearn.neighbors` module implements the k-nearest neighbors algorithm.
User guide: See the Nearest Neighbors section for further details.

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### 29.25.1 sklearn.neighbors.NearestNeighbors

**class** `sklearn.neighbors.NearestNeighbors` (n_neighbors=5, radius=1.0, algorithm='auto', leaf_size=30, metric='minkowski', p=2, metric_params=None, n_jobs=1, **kwargs)

Unsupervised learner for implementing neighbor searches.

Read more in the [User Guide](#).

**Parameters**

- `n_neighbors` : int, optional (default = 5)
  Number of neighbors to use by default for `k_neighbors` queries.
- `radius` : float, optional (default = 1.0)
  Range of parameter space to use by default for `radius_neighbors` queries.
- `algorithm` : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
  Algorithm used to compute the nearest neighbors:
  - 'ball_tree' will use `BallTree`
  - 'kd_tree' will use `KDTree`
  - 'brute' will use a brute-force search.
  - 'auto' will attempt to decide the most appropriate algorithm based on the values passed to `fit` method.

  Note: fitting on sparse input will override the setting of this parameter, using brute force.
- `leaf_size` : int, optional (default = 30)
  Leaf size passed to BallTree or KDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.
- `p` : integer, optional (default = 2) :
  Parameter for the Minkowski metric from `sklearn.metrics.pairwise.pairwise_distances`. When p = 1, this is equivalent to using `manhattan_distance` (l1), and `euclidean_distance` (l2) for p = 2. For arbitrary p, `minkowski_distance` (l_p) is used.
metric : string or callable, default ‘minkowski’

metric to use for distance computation. Any metric from scikit-learn or scipy.spatial.distance can be used.

If metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays as input and return one value indicating the distance between them. This works for Scipy’s metrics, but is less efficient than passing the metric name as a string.

Distance matrices are not supported.

Valid values for metric are:

• from scikit-learn: ['cityblock', 'cosine', 'euclidean', 'l1', 'l2', 'manhattan']
• from scipy.spatial.distance: ['braycurtis', 'canberra', 'chebyshev', 'correlation', 'dice', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'yule']

See the documentation for scipy.spatial.distance for details on these metrics.

metric_params : dict, optional (default = None)

Additional keyword arguments for the metric function.

n_jobs : int, optional (default = 1)

The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores. Affects only k_neighbors and kneighbors_graph methods.

See also:

KNeighborsClassifier, RadiusNeighborsClassifier, KNeighborsRegressor, RadiusNeighborsRegressor, BallTree

Notes

See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.


Examples

```python
>>> import numpy as np
data = np.array([[0, 0, 2], [1, 0, 0], [0, 0, 1]])

>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(2, 0.4)
>>> neigh.fit(data)
NearestNeighbors(...)

>>> neigh.kneighbors([[0, 0, 1.3]], 2, return_distance=False)
... array([[2, 0]], ...)
```
Methods

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<td>Fit the model using X as training data</td>
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<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<td><code>kneighbors([X, n_neighbors, return_distance])</code></td>
<td>Finds the K-neighbors of a point.</td>
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<tr>
<td><code>kneighbors_graph([X, n_neighbors, mode])</code></td>
<td>Computes the (weighted) graph of k-Neighbors for points in X</td>
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<tr>
<td><code>radius_neighbors([X, radius, return_distance])</code></td>
<td>Finds the neighbors within a given radius of a point or points.</td>
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<tr>
<td><code>radius_neighbors_graph([X, radius, mode])</code></td>
<td>Computes the (weighted) graph of Neighbors for points in X</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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</table>

__init__ (n_neighbors=5, radius=1.0, algorithm='auto', leaf_size=30, metric='minkowski', p=2, metric_params=None, n_jobs=1, **kwargs)

fit (X, y=None)
Fit the model using X as training data

Parameters:
- **X**: {array-like, sparse matrix, BallTree, KDTree}
  - Training data. If array or matrix, shape [n_samples, n_features], or [n_samples, n_samples] if metric == 'precomputed'.

get_params (deep=True)
Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params**: mapping of string to any
  - Parameter names mapped to their values.

kneighbors (X=None, n_neighbors=None, return_distance=True)
Finds the K-neighbors of a point.

Parameters:
- **X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
  - The query point or points. If not provided, neighbors of each indexed point are returned.
  - In this case, the query point is not considered its own neighbor.
- **n_neighbors**: int
  - Number of neighbors to get (default is the value passed to the constructor).
- **return_distance**: boolean, optional. Defaults to True.
  - If False, distances will not be returned.
**Returns**: 
- **dist**: array
  - Array representing the lengths to points, only present if return_distance=True
- **ind**: array
  - Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=1)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.kneighbors([[1., 1., 1.]]))
(array([[ 0.5]]), array([[2]]...))
```

As you can see, it returns [[0.5]], and [[2]], which means that the element is at distance 0.5 and is the third element of samples (indexes start at 0). You can also query for multiple points:

```
>>> X = [[0., 1., 0.], [1., 0., 1.]]
>>> neigh.kneighbors(X, return_distance=False)
array([[1], [2]...])
```

**kneighbors_graph**

```
(X=None, n_neighbors=None, mode='connectivity')
```

Computes the (weighted) graph of k-Neighbors for points in X

**Parameters**
- **X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == ‘precomputed’
  - The query point or points. If not provided, neighbors of each indexed point are returned.
  - In this case, the query point is not considered its own neighbor.
- **n_neighbors**: int
  - Number of neighbors for each sample. (default is value passed to the constructor).
- **mode**: {'connectivity', 'distance'}, optional
  - Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**
- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples_fit]
  - n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.

**See also**:

`NearestNeighbors.radius_neighbors_graph`
Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
>>> A.toarray()
array([[ 1., 0., 1.],
       [ 0., 1., 1.],
       [ 1., 0., 1.]])
```

radius_neighbors (X=None, radius=None, return_distance=True)
Finds the neighbors within a given radius of a point or points.

Return the indices and distances of each point from the dataset lying in a ball with size radius around the points of the query array. Points lying on the boundary are included in the results.

The result points are not necessarily sorted by distance to their query point.

Parameters:
- **X** : array-like, (n_samples, n_features), optional
  The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.
- **radius** : float
  Limiting distance of neighbors to return. (default is the value passed to the constructor).
- **return_distance** : boolean, optional. Defaults to True.
  If False, distances will not be returned

Returns:
- **dist** : array, shape (n_samples,) of arrays
  Array representing the distances to each point, only present if return_distance=True. The distance values are computed according to the metric constructor parameter.
- **ind** : array, shape (n_samples,) of arrays
  An array of arrays of indices of the approximate nearest points from the population matrix that lie within a ball of size radius around the query points.

Notes

Because the number of neighbors of each point is not necessarily equal, the results for multiple query points cannot be fit in a standard data array. For efficiency, radius_neighbors returns arrays of objects, where each object is a 1D array of indices or distances.

Examples

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1, 1, 1]:

```python
>>> import numpy as np
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
```
The first array returned contains the distances to all points which are closer than 1.6, while the second array returned contains their indices. In general, multiple points can be queried at the same time.

**radius_neighbors_graph** *(X=None, radius=None, mode='connectivity')*

Computes the (weighted) graph of Neighbors for points in X

Neighborhoods are restricted the points at a distance lower than radius.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features], optional
  
  The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

- **radius**: float
  
  Radius of neighborhoods. (default is the value passed to the constructor).

- **mode**: {'connectivity', 'distance'}, optional
  
  Type of returned matrix: 'connectivity' will return the connectivity matrix with ones and zeros, in 'distance' the edges are Euclidean distance between points.

**Returns**

- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples]
  
  A[i, j] is assigned the weight of edge that connects i to j.

**See also:**

* kneighbors_graph

**Examples**

```python
tests = [[0], [3], [1]]
from sklearn.neighbors import NearestNeighbors
neigh = NearestNeighbors(radius=1.5)
neigh.fit(tests)
A = neigh.radius_neighbors_graph(tests)
A.toarray()
```

```python
array([[ 1., 0., 1.],
       [ 0., 1., 0.],
       [ 1., 0., 1.]])
```

**set_params (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**:
sklearn.neighbors.KNeighborsClassifier

class sklearn.neighbors.KNeighborsClassifier(n_neighbors=5, weights='uniform', algorithm='auto', leaf_size=30, p=2, metric='minkowski', metric_params=None, n_jobs=1, **kwargs)

Classifier implementing the k-nearest neighbors vote.

Read more in the User Guide.

Parameters

- n_neighbors : int, optional (default = 5)
  Number of neighbors to use by default for k_neighbors queries.

- weights : str or callable, optional (default = 'uniform')
  weight function used in prediction. Possible values:
  - 'uniform' : uniform weights. All points in each neighborhood are weighted equally.
  - 'distance' : weight points by the inverse of their distance. in this case, closer neigh-
    bors of a query point will have a greater influence than neighbors which are further
    away.
  - [callable] : a user-defined function which accepts an array of distances, and returns
    an array of the same shape containing the weights.

- algorithm : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
  Algorithm used to compute the nearest neighbors:
  - 'ball_tree' will use BallTree
  - 'kd_tree' will use KDTree
  - 'brute' will use a brute-force search.
  - 'auto' will attempt to decide the most appropriate algorithm based on the values
    passed to fit method.

  Note: fitting on sparse input will override the setting of this parameter, using brute force.

- leaf_size : int, optional (default = 30)
  Leaf size passed to BallTree or KDTree. This can affect the speed of the construction
  and query, as well as the memory required to store the tree. The optimal value depends
  on the nature of the problem.

- metric : string or DistanceMetric object (default = 'minkowski')
  the distance metric to use for the tree. The default metric is minkowski, and with p=2
  is equivalent to the standard Euclidean metric. See the documentation of the Distance-
  Metric class for a list of available metrics.

- p : integer, optional (default = 2)
Power parameter for the Minkowski metric. When $p = 1$, this is equivalent to using manhattan_distance ($l_1$), and euclidean_distance ($l_2$) for $p = 2$. For arbitrary $p$, minkowski_distance ($l_p$) is used.

**metric_params**: dict, optional (default = None)

Additional keyword arguments for the metric function.

**n_jobs**: int, optional (default = 1)

The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores. Doesn’t affect fit method.

See also:

RadiusNeighborsClassifier, KNeighborsRegressor, RadiusNeighborsRegressor, NearestNeighbors

Notes

See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.

**Warning:** Regarding the Nearest Neighbors algorithms, if it is found that two neighbors, neighbor $k+1$ and $k$, have identical distances but different labels, the results will depend on the ordering of the training data.


Examples

```python
>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import KNeighborsClassifier
>>> neigh = KNeighborsClassifier(n_neighbors=3)
>>> neigh.fit(X, y)
KNeighborsClassifier...
>>> print(neigh.predict([[1.1]]))
[0]
>>> print(neigh.predict_proba([[0.9]]))
[[ 0.66666667 0.33333333]]
```

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<td>get_params([deep])</td>
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<tr>
<td>kneighbors([X, n_neighbors, return_distance])</td>
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<td>Computes the (weighted) graph of k-Neighbors for points in X</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict the class labels for the provided data</td>
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<tr>
<td>predict_proba(X)</td>
<td>Return probability estimates for the test data X.</td>
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<td>Returns the mean accuracy on the given test data and labels.</td>
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<tr>
<td><strong>set_params</strong></td>
<td>Set the parameters of this estimator.</td>
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```
__init__ (n_neighbors=5, weights='uniform', algorithm='auto', leaf_size=30, p=2, metric='minkowski', metric_params=None, n_jobs=1, **kwargs)
```

```
fitted (X, y)
    Fit the model using X as training data and y as target values
```

```
Parameters

X : {array-like, sparse matrix, BallTree, KDTree}
    Training data. If array or matrix, shape [n_samples, n_features], or [n_samples, n_samples] if metric='precomputed'.

y : {array-like, sparse matrix}
    Target values of shape = [n_samples] or [n_samples, n_outputs]
```

```
get_params (deep=True)
    Get parameters for this estimator.
```

```
Parameters

deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.
```

```
Returns

params : mapping of string to any
    Parameter names mapped to their values.
```

```
kneighbors (X=None, n_neighbors=None, return_distance=True)
    Finds the K-neighbors of a point.
```

```
Returns

dist : array
    Array representing the lengths to points, only present if return_distance=True

ind : array
    Indices of the nearest points in the population matrix.
```

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who's the closest point to [1,1,1]
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=1)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.kneighbors([[1., 1., 1.]]))
(array([[ 0.5]]), array([[2]]))...

As you can see, it returns [[0.5]], and [2]], which means that the element is at distance 0.5 and is the third element of samples (indexes start at 0). You can also query for multiple points:

>>> X = [[0., 1., 0.], [1., 0., 1.]]
>>> neigh.kneighbors(X, return_distance=False)
array([[[1],
       [2]]...])

kneighbors_graph (X=None, n_neighbors=None, mode='connectivity')
Computes the (weighted) graph of k-Neighbors for points in X

Parameters:
- **X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
  The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.
- **n_neighbors**: int
  Number of neighbors for each sample. (default is value passed to the constructor).
- **mode**: {'connectivity', 'distance'}, optional
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

Returns:
- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples_fit]
  n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.

See also:
NearestNeighbors.radius_neighbors_graph

Examples

>>> X = [[0.], [3.], [1.]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
>>> A.toarray()
array([[ 1., 0., 1.],
       [ 0., 1., 1.],
       [ 1., 0., 1.]])

predict (X)
Predict the class labels for the provided data
Parameters

**X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'

Test samples.

**Returns**

**y**: array of shape [n_samples] or [n_samples, n_outputs]

Class labels for each data sample.

**predict_proba** (*X*)

Return probability estimates for the test data *X*.

**Parameters**

**X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'

Test samples.

**Returns**

**p**: array of shape = [n_samples, n_classes], or a list of n_outputs of such arrays if n_outputs > 1. The class probabilities of the input samples. Classes are ordered by lexicographic order.

**score** (*X*, *y*, **sample_weight**=None)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

**X**: array-like, shape = (n_samples, n_features)

Test samples.

**y**: array-like, shape = (n_samples) or (n_samples, n_outputs)

True labels for *X*.

**sample_weight**: array-like, shape = [n_samples], optional

Sample weights.

**Returns**

**score**: float

Mean accuracy of self.predict(*X*) wrt. *y*.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self**:

Examples using *sklearn.neighbors.KNeighborsClassifier*

- Classifier comparison
- Plot the decision boundaries of a VotingClassifier
- Digits Classification Exercise
- Nearest Neighbors Classification
- Robust Scaling on Toy Data
- Classification of text documents using sparse features
29.25.3 sklearn.neighbors.RadiusNeighborsClassifier

Classifier implementing a vote among neighbors within a given radius

Read more in the User Guide.

Parameters:
- radius : float, optional (default = 1.0)
  Range of parameter space to use by default for `radius_neighbors` queries.

- weights : str or callable
  weight function used in prediction. Possible values:
  - 'uniform' : uniform weights. All points in each neighborhood are weighted equally.
  - 'distance' : weight points by the inverse of their distance. in this case, closer neighbors of a query point will have a greater influence than neighbors which are further away.
  - [callable] : a user-defined function which accepts an array of distances, and returns an array of the same shape containing the weights.

Uniform weights are used by default.

- algorithm : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
  Algorithm used to compute the nearest neighbors:
  - 'ball_tree' will use BallTree
  - 'kd_tree' will use KDtree
  - 'brute' will use a brute-force search.
  - 'auto' will attempt to decide the most appropriate algorithm based on the values passed to `fit` method.

Note: fitting on sparse input will override the setting of this parameter, using brute force.

- leaf_size : int, optional (default = 30)
  Leaf size passed to BallTree or KDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

- metric : string or DistanceMetric object (default='minkowski')
  the distance metric to use for the tree. The default metric is minkowski, and with p=2 is equivalent to the standard Euclidean metric. See the documentation of the DistanceMetric class for a list of available metrics.

- p : integer, optional (default = 2)
  Power parameter for the Minkowski metric. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

- outlier_label : int, optional (default = None)
  Label, which is given for outlier samples (samples with no neighbors on given radius). If set to None, ValueError is raised, when outlier is detected.
**metric_params** : dict, optional (default = None)

Additional keyword arguments for the metric function.

**See also:**

*KNeighborsClassifier, RadiusNeighborsRegressor, KNeighborsRegressor, NearestNeighbors*

**Notes**

See *Nearest Neighbors* in the online documentation for a discussion of the choice of *algorithm* and *leaf_size*.


**Examples**

```python
g{}X = [[0], [1], [2], [3]]
g{}y = [0, 0, 1, 1]
g{}from sklearn.neighbors import RadiusNeighborsClassifier
g{}neigh = RadiusNeighborsClassifier(radius=1.0)
g{}neigh.fit(X, y)
g{}print(neigh.predict([[1.5]]))
[0]
```

**Methods**

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<td>Get parameters for this estimator.</td>
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<td><strong>predict</strong>(X)</td>
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<tr>
<td><strong>radius_neighbors</strong>(X, radius, return_distance)</td>
<td>Finds the neighbors within a given radius of a point or points.</td>
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<td><strong>radius_neighbors_graph</strong>(X, radius)</td>
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<tr>
<td><strong>score</strong>(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><strong>set_params</strong>(<strong>params</strong>)</td>
<td>Set the parameters of this estimator.</td>
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```python
def __init__(radius=1.0, weights='uniform', algorithm='auto', leaf_size=30, p=2, metric='minkowski', outlier_label=None, metric_params=None, **kwargs):
    self.radius = radius
    self.weights = weights
    self.algorithm = algorithm
    self.leaf_size = leaf_size
    self.p = p
    self.metric = metric
    self.outlier_label = outlier_label
    self.metric_params = metric_params

self.fit(X, y)
```

**Parameters**

*X* : {array-like, sparse matrix, BallTree, KDTree}

Training data. If array or matrix, shape [n_samples, n_features], or [n_samples, n_samples] if metric='precomputed'.

*y* : {array-like, sparse matrix}
Target values of shape = [n_samples] or [n_samples, n_outputs]

**get_params**(deep=True)
Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

**predict**(X)
Predict the class labels for the provided data

**Parameters**

- **X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
  Test samples.

**Returns**

- **y**: array of shape [n_samples] or [n_samples, n_outputs]
  Class labels for each data sample.

**radius_neighbors**(X=None, radius=None, return_distance=True)
Finds the neighbors within a given radius of a point or points.

Return the indices and distances of each point from the dataset lying in a ball with size radius around the points of the query array. Points lying on the boundary are included in the results.

The result points are *not* necessarily sorted by distance to their query point.

**Parameters**

- **X**: array-like, (n_samples, n_features), optional
  The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

- **radius**: float
  Limiting distance of neighbors to return. (default is the value passed to the constructor).

- **return_distance**: boolean, optional. Defaults to True.
  If False, distances will not be returned

**Returns**

- **dist**: array, shape (n_samples,) of arrays
  Array representing the distances to each point, only present if return_distance=True. The distance values are computed according to the metric constructor parameter.

- **ind**: array, shape (n_samples,) of arrays
  An array of arrays of indices of the approximate nearest points from the population matrix that lie within a ball of size radius around the query points.

**Notes**

Because the number of neighbors of each point is not necessarily equal, the results for multiple query points cannot be fit in a standard data array. For efficiency, radius_neighbors returns arrays of objects, where each object is a 1D array of indices or distances.
Examples

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1, 1, 1]:

```python
>>> import numpy as np
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.6)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> rng = neigh.radius_neighbors([[1., 1., 1.]])
>>> print(np.asarray(rng[0][0]))
[ 1.5 0.5]
>>> print(np.asarray(rng[1][0]))
[1 2]
```

The first array returned contains the distances to all points which are closer than 1.6, while the second array returned contains their indices. In general, multiple points can be queried at the same time.

**radius_neighbors_graph**

*(X=None, radius=None, mode='connectivity')*

Computes the (weighted) graph of Neighbors for points in X

Neighborhoods are restricted the points at a distance lower than radius.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features], optional
  
The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

- **radius**: float
  
  Radius of neighborhoods. (default is the value passed to the constructor).

- **mode**: {'connectivity', 'distance'}, optional
  
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples]
  
  A[i, j] is assigned the weight of edge that connects i to j.

See also:

kneighbors_graph

Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.5)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.radius_neighbors_graph(X)
>>> A.toarray()
array([[ 1., 0., 1.],
       [ 0., 1., 0.],
       [ 1., 0., 1.]])
```
**score** (*X*, *y*, *sample_weight=None*)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X** : array-like, shape = (n_samples, n_features)
  
  Test samples.

- **y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
  
  True labels for *X*.

- **sample_weight** : array-like, shape = [n_samples], optional
  
  Sample weights.

**Returns**

- **score** : float
  
  Mean accuracy of self.predict(*X*) wrt. *y*.

---

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :
• ‘ball_tree’ will use BallTree
• ‘kd_tree’ will use KDTree
• ‘brute’ will use a brute-force search.
• ‘auto’ will attempt to decide the most appropriate algorithm based on the values passed to fit method.

Note: fitting on sparse input will override the setting of this parameter, using brute force.

leaf_size : int, optional (default = 30)

Leaf size passed to BallTree or KDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

metric : string or DistanceMetric object (default='minkowski')

the distance metric to use for the tree. The default metric is minkowski, and with p=2 is equivalent to the standard Euclidean metric. See the documentation of the DistanceMetric class for a list of available metrics.

p : integer, optional (default = 2)

Power parameter for the Minkowski metric. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

metric_params : dict, optional (default = None)

Additional keyword arguments for the metric function.

n_jobs : int, optional (default = 1)

The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores. Doesn’t affect fit method.

See also:

NearestNeighbors, RadiusNeighborsRegressor, KNeighborsClassifier, RadiusNeighborsClassifier

Notes

See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.

Warning: Regarding the Nearest Neighbors algorithms, if it is found that two neighbors, neighbor $k+1$ and $k$, have identical distances but but different labels, the results will depend on the ordering of the training data.


Examples
```python
>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import KNeighborsRegressor
>>> neigh = KNeighborsRegressor(n_neighbors=2)
>>> neigh.fit(X, y)
KNeighborsRegressor(...)
>>> print(neigh.predict([[1.5]]))
[ 0.5]
```

**Methods**

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<td><code>kneighbors([X, n_neighbors, return_distance])</code></td>
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<td><code>predict(X)</code></td>
<td>Predict the target for the provided data</td>
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<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__**(n_neighbors=5, weights='uniform', algorithm='auto', leaf_size=30, p=2, metric='minkowski', metric_params=None, n_jobs=1, **kwargs)

**fit**(X, y)
Fit the model using X as training data and y as target values

**Parameters**
- **X**: array-like, sparse matrix, BallTree, KDTree
  - Training data. If array or matrix, shape [n_samples, n_features], or [n_samples, n_samples] if metric='precomputed'.
- **y**: array-like, sparse matrix
  - Target values, array of float values, shape = [n_samples] or [n_samples, n_outputs]

**get_params**(deep=True)
Get parameters for this estimator.

**Parameters**
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params**: mapping of string to any
  - Parameter names mapped to their values.

**kneighbors**(X=None, n_neighbors=None, return_distance=True)
Finds the K-neighbors of a point.

**Parameters**
- **X**: array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
  - Training data. If array or matrix, shape [n_samples, n_features], or [n_samples, n_samples] if metric='precomputed'.
- **n_neighbors**: int or None
  - Number of neighbors to look for. If None, defaults to n_neighbors.
- **return_distance**: bool, optional
  - Whether or not to return the distances.

**Returns**
- **indices**: array, shape (n_samples, n_neighbors)
  - Indices of neighbors.
- **distances**: array, shape (n_samples, n_neighbors) if return_distance is True
  - Distances to neighbors.

---

29.25. sklearn.neighbors: Nearest Neighbors

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The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

**n_neighbors** : int

Number of neighbors to get (default is the value passed to the constructor).

**return_distance** : boolean, optional. Defaults to True.

If False, distances will not be returned

**Returns**

**dist** : array

Array representing the lengths to points, only present if return_distance=True

**ind** : array

Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```python
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=1)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.kneighbors([[1., 1., 1.]]))
(array([[ 0.5]]), array([[2]]))
```

As you can see, it returns [[0.5]], and [[2]], which means that the element is at distance 0.5 and is the third element of samples (indexes start at 0). You can also query for multiple points:

```python
>>> X = [[0., 1., 0.], [1., 0., 1.]]
>>> neigh.kneighbors(X, return_distance=False)
array([[1],
       [2]])
```

**kneighbors_graph** (X=None, n_neighbors=None, mode='connectivity')

Computes the (weighted) graph of k-Neighbors for points in X

**Parameters**

- **X** : array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
  The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

- **n_neighbors** : int
  Number of neighbors for each sample. (default is value passed to the constructor).

- **mode** : {'connectivity', 'distance'}, optional
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A** : sparse matrix in CSR format, shape = [n_samples, n_samples_fit]
  n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.
See also:

`NearestNeighbors.radius_neighbors_graph`

Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
>>> A.toarray()
array([[1., 0., 1.],
       [0., 1., 1.],
       [1., 0., 1.]])
```

**predict** *(X)*

Predict the target for the provided data

**Parameters**

- **X** : array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
  
  Test samples.

**Returns**

- **y** : array of int, shape = [n_samples] or [n_samples, n_outputs]
  
  Target values

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred}) ** 2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true}.mean()) ** 2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X** : array-like, shape = (n_samples, n_features)
  
  Test samples.

- **y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
  
  True values for X.

- **sample_weight** : array-like, shape = [n_samples], optional
  
  Sample weights.

**Returns**

- **score** : float
  
  $R^2$ of self.predict(X) wrt. y.

**set_params** (****params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :
Examples using sklearn.neighbors.KNeighborsRegressor

• Face completion with a multi-output estimators
• Nearest Neighbors regression

29.25.5 sklearn.neighbors.RadiusNeighborsRegressor

class sklearn.neighbors.RadiusNeighborsRegressor (radius=1.0, weights='uniform', algorithm='auto', leaf_size=30, p=2, metric='minkowski', metric_params=None, **kwargs)

Regression based on neighbors within a fixed radius.

The target is predicted by local interpolation of the targets associated of the nearest neighbors in the training set.

Read more in the User Guide.

Parameters:
  radius : float, optional (default = 1.0)
    Range of parameter space to use by default for radius_neighbors queries.
  weights : str or callable
    weight function used in prediction. Possible values:
    • 'uniform' : uniform weights. All points in each neighborhood are weighted equally.
    • 'distance' : weight points by the inverse of their distance. in this case, closer neighbors of a query point will have a greater influence than neighbors which are further away.
    • [callable] : a user-defined function which accepts an array of distances, and returns an array of the same shape containing the weights.

Uniform weights are used by default.

  algorithm : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
    Algorithm used to compute the nearest neighbors:
    • 'ball_tree' will use BallTree
    • 'kd_tree' will use KDtree
    • 'brute' will use a brute-force search.
    • 'auto' will attempt to decide the most appropriate algorithm based on the values passed to fit method.

Note: fitting on sparse input will override the setting of this parameter, using brute force.

  leaf_size : int, optional (default = 30)
    Leaf size passed to BallTree or KDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

  metric : string or DistanceMetric object (default='minkowski')
    the distance metric to use for the tree. The default metric is minkowski, and with p=2 is equivalent to the standard Euclidean metric. See the documentation of the DistanceMetric class for a list of available metrics.

  p : integer, optional (default = 2)
Power parameter for the Minkowski metric. When $p = 1$, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for $p = 2$. For arbitrary $p$, minkowski_distance ($l_p$) is used.

**metric_params**: dict, optional (default = None)

Additional keyword arguments for the metric function.

See also:

NearestNeighbors, KNeighborsRegressor, KNeighborsClassifier, RadiusNeighborsClassifier

Notes

See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.


Examples

```python
>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import RadiusNeighborsRegressor
>>> neigh = RadiusNeighborsRegressor(radius=1.0)
>>> neigh.fit(X, y)
RadiusNeighborsRegressor(...)
>>> print(neigh.predict([[1.5]]))
[ 0.5]
```

Methods

- **fit**(X, y) Fit the model using X as training data and y as target values
- **get_params**(deep) Get parameters for this estimator.
- **predict**(X) Predict the target for the provided data
- **radius_neighbors**(X, radius, return_distance) Finds the neighbors within a given radius of a point or points.
- **radius_neighbors_graph**(X, radius, mode) Computes the (weighted) graph of Neighbors for points in X
- **score**(X, y[, sample_weight]) Returns the coefficient of determination $R^2$ of the prediction.
- **set_params**(**kwargs) Set the parameters of this estimator.

```
__init__(radius=1.0, weights='uniform', algorithm='auto', leaf_size=30, p=2, metric='minkowski',
       metric_params=None, **kwargs)

fit(X, y)
Fit the model using X as training data and y as target values

Parameters:

- **X**: {array-like, sparse matrix, BallTree, KDTree}
Training data. If array or matrix, shape [n_samples, n_features], or [n_samples, n_samples] if metric='precomputed'.

y : {array-like, sparse matrix}
 Target values, array of float values, shape = [n_samples] or [n_samples, n_outputs]

get_params (deep=True)
Get parameters for this estimator.

Parameters:
  deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are
    estimators.

Returns:
  params : mapping of string to any
    Parameter names mapped to their values.

predict (X)
Predict the target for the provided data

Parameters:
  X : array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric ==
    'precomputed'
    Test samples.

Returns:
  y : array of int, shape = [n_samples] or [n_samples, n_outputs]
    Target values

radius_neighbors (X=None, radius=None, return_distance=True)
Finds the neighbors within a given radius of a point or points.
Return the indices and distances of each point from the dataset lying in a ball with size radius around
the points of the query array. Points lying on the boundary are included in the results.
The result points are not necessarily sorted by distance to their query point.

Parameters:
  X : array-like, (n_samples, n_features), optional
    The query point or points. If not provided, neighbors of each indexed point are returned.
    In this case, the query point is not considered its own neighbor.

  radius : float
    Limiting distance of neighbors to return. (default is the value passed to the constructor).

  return_distance : boolean, optional. Defaults to True.
    If False, distances will not be returned

Returns:
  dist : array, shape (n_samples,) of arrays
    Array representing the distances to each point, only present if return_distance=True.
    The distance values are computed according to the metric constructor parameter.

  ind : array, shape (n_samples,) of arrays
    An array of arrays of indices of the approximate nearest points from the population
    matrix that lie within a ball of size radius around the query points.
Notes

Because the number of neighbors of each point is not necessarily equal, the results for multiple query points cannot be fit in a standard data array. For efficiency, \texttt{radius_neighbors} returns arrays of objects, where each object is a 1D array of indices or distances.

Examples

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to \([1, 1, 1]\):

```python
>>> import numpy as np
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.6)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> rng = neigh.radius_neighbors([[1., 1., 1.]])
>>> print(np.asarray(rng[0][0]))
[ 1.5 0.5]
>>> print(np.asarray(rng[1][0]))
[1 2]
```

The first array returned contains the distances to all points which are closer than 1.6, while the second array returned contains their indices. In general, multiple points can be queried at the same time.

\texttt{radius_neighbors_graph} \(X=\text{None}, \ radius=\text{None}, \ mode='\text{connectivity}'\)

Computes the (weighted) graph of Neighbors for points in \textit{X}
Neighborhoods are restricted the points at a distance lower than \textit{radius}.

\textbf{Parameters}:
\begin{itemize}
  \item \textit{X} : array-like, shape = [n_samples, n_features], optional
    The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.
  \item \textit{radius} : float
    Radius of neighborhoods. (default is the value passed to the constructor).
  \item \textit{mode} : {‘connectivity’, ‘distance’}, optional
    Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.
\end{itemize}

\textbf{Returns}:
\begin{itemize}
  \item \textit{A} : sparse matrix in CSR format, shape = [n_samples, n_samples]
    \(A[i, j]\) is assigned the weight of edge that connects \textit{i} to \textit{j}.
\end{itemize}

\textbf{See also}:
\textit{kneighbors_graph}

\textbf{Examples}

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.5)
```
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.radius_neighbors_graph(X)
>>> A.toarray()
array([[ 1.,  0.,  1.],
       [ 0.,  1.,  0.],
       [ 1.,  0.,  1.]])

score (X, y, sample_weight=None)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y_{true} - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true}.mean())^2).sum()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.

- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.

- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns

- **score**: float
  $R^2$ of self.predict(X) wrt. y.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns

- **self**: 

29.25.6 sklearn.neighbors.NearestCentroid

class sklearn.neighbors.NearestCentroid (metric='euclidean', shrink_threshold=None)
Nearest centroid classifier.

Each class is represented by its centroid, with test samples classified to the class with the nearest centroid.

Read more in the User Guide.

Parameters

- **metric**: string, or callable
  The metric to use when calculating distance between instances in a feature array. If metric is a string or callable, it must be one of the options allowed by metrics.pairwise.pairwise_distances for its metric parameter. The centroids for the samples corresponding to each class is the point from which the sum of the distances (according to the metric) of all samples that belong to that particular class are minimized. If the “manhattan” metric is provided, this centroid is the median and for all other metrics, the centroid is now set to be the mean.

- **shrink_threshold**: float, optional (default = None)
Threshold for shrinking centroids to remove features.

Attributes

```
centroids_ : array-like, shape = [n_classes, n_features]
```

Centroid of each class

See also:

`sklearn.neighbors.KNeighborsClassifier` nearest neighbors classifier

Notes

When used for text classification with tf-idf vectors, this classifier is also known as the Rocchio classifier.

References


Examples

```python
>>> from sklearn.neighbors.nearest_centroid import NearestCentroid
>>> import numpy as np
>>> X = np.array(
[[-1, -1],
[-2, -1],
[-3, -2],
[1, 1],
[2, 1],
[3, 2]])
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = NearestCentroid()
>>> clf.fit(X, y)
NearestCentroid(metric='euclidean', shrink_threshold=None)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

Methods

```
fit(X, y)       Fit the NearestCentroid model according to the given training data.
get_params([deep])       Get parameters for this estimator.
predict(X)       Perform classification on an array of test vectors X.
score(X, y[, sample_weight])       Returns the mean accuracy on the given test data and labels.
set_params(**params)       Set the parameters of this estimator.
```

```python
__init__(metric='euclidean', shrink_threshold=None)
```
Target values (integers)

get_params (deep=True)
Get parameters for this estimator.

Parameters
depth : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

Returns
params : mapping of string to any
Parameter names mapped to their values.

predict (X)
Perform classification on an array of test vectors X.
The predicted class C for each sample in X is returned.

Parameters
X : array-like, shape = [n_samples, n_features]

Returns
C : array, shape = [n_samples]

Notes
If the metric constructor parameter is “precomputed”, X is assumed to be the distance matrix between the
data to be predicted and self.centroids_.

score (X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.
In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each
sample that each label set be correctly predicted.

Parameters
X : array-like, shape = (n_samples, n_features)
Test samples.
y : array-like, shape = (n_samples) or (n_samples, n_outputs)
True labels for X.
sample_weight : array-like, shape = [n_samples], optional
Sample weights.

Returns
score : float
Mean accuracy of self.predict(X) wrt. y.

set_params (**params)
Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns
self :
Examples using `sklearn.neighbors.NearestCentroid`

- Nearest Centroid Classification
- Classification of text documents using sparse features

### 29.25.7 sklearn.neighbors.BallTree

**class sklearn.neighbors.BallTree**

BallTree for fast generalized N-point problems

BallTree(X, leaf_size=40, metric='minkowski', **kwargs)

**Parameters**

- `X` : array-like, shape = [n_samples, n_features]
  
  n_samples is the number of points in the data set, and n_features is the dimension of the parameter space. Note: if X is a C-contiguous array of doubles then data will not be copied. Otherwise, an internal copy will be made.

- `leaf_size` : positive integer (default = 40)
  
  Number of points at which to switch to brute-force. Changing leaf_size will not affect the results of a query, but can significantly impact the speed of a query and the memory required to store the constructed tree. The amount of memory needed to store the tree scales as approximately n_samples / leaf_size. For a specified leaf_size, a leaf node is guaranteed to satisfy leaf_size <= n_points <= 2 * leaf_size, except in the case that n_samples < leaf_size.

- `metric` : string or DistanceMetric object
  
  the distance metric to use for the tree. Default='minkowski' with p=2 (that is, a euclidean metric). See the documentation of the DistanceMetric class for a list of available metrics. ball_tree.valid_metrics gives a list of the metrics which are valid for BallTree.

**Attributes**

- `data` : np.ndarray
  
  The training data

**Examples**

Query for k-nearest neighbors

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = BallTree(X, leaf_size=2)
>>> dist, ind = tree.query([X[0]], k=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[0. 0.19662693 0.29473397]
```

Pickle and Unpickle a tree. Note that the state of the tree is saved in the pickle operation: the tree needs not be rebuilt upon unpickling.
```python
>>> import numpy as np
>>> import pickle

np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = BallTree(X, leaf_size=2)
>>> s = pickle.dumps(tree)
>>> tree_copy = pickle.loads(s)

>>> dist, ind = tree_copy.query(X[0], k=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[ 0. 0.19662693 0.29473397]

Query for neighbors within a given radius

```python
>>> import numpy as np

np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = BallTree(X, leaf_size=2)

>>> print tree.query_radius(X[0], r=0.3, count_only=True)
3
>>> ind = tree.query_radius(X[0], r=0.3)
>>> print ind  # indices of neighbors within distance 0.3
[3 0 1]

Compute a gaussian kernel density estimate:

```python
>>> import numpy as np

np.random.seed(1)
>>> X = np.random.random((100, 3))
>>> tree = BallTree(X)

>>> tree.kernel_density(X[:3], h=0.1, kernel='gaussian')
array([ 6.94114649, 7.83281226, 7.2071716 ])

Compute a two-point auto-correlation function

```python
>>> import numpy as np

np.random.seed(0)
>>> X = np.random.random((30, 3))
>>> r = np.linspace(0, 1, 5)
>>> tree = BallTree(X)

>>> tree.two_point_correlation(X, r)
array([30, 62, 278, 580, 820])

Methods

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```python
__init__(self)  
x.__init__(...) initializes x; see help(type(x)) for signature

kernel_density(self, X, h, kernel='gaussian', atol=0, rtol=1E-8, breadth_first=True, return_log=False)
Compute the kernel density estimate at points X with the given kernel, using the distance metric specified at tree creation.

Parameters:
- X : array_like
  An array of points to query. Last dimension should match dimension of training data.
- h : float
  the bandwidth of the kernel
- kernel : string
  specify the kernel to use. Options are - `gaussian` - `tophat` - `epanechnikov` - `exponential` - `linear` - `cosine` Default is kernel = `gaussian`
- atol, rtol : float (default = 0)
  Specify the desired relative and absolute tolerance of the result. If the true result is K_true, then the returned result K_ret satisfies abs(K_true - K_ret) < atol + rtol * K_ret The default is zero (i.e. machine precision) for both.
- breadth_first : boolean (default = False)
  if True, use a breadth-first search. If False (default) use a depth-first search. Breadth-first is generally faster for compact kernels and/or high tolerances.
- return_log : boolean (default = False)
  return the logarithm of the result. This can be more accurate than returning the result itself for narrow kernels.

Returns:
- density : ndarray
  The array of (log)-density evaluations, shape = X.shape[:-1]

Examples

Compute a gaussian kernel density estimate:

```python
>>> import numpy as np
>>> np.random.seed(1)
>>> X = np.random.random((100, 3))
>>> tree = BinaryTree(X)
>>> tree.kernel_density(X[:3], h=0.1, kernel='gaussian')
array([ 6.94114649, 7.83281226, 7.2071716 ])

query(X, k=1, return_distance=True, dualtree=False, breadth_first=False)
query the tree for the k nearest neighbors

Parameters:
- X : array-like, last dimension self.dim
```
An array of points to query

\textbf{k} : integer (default = 1)

The number of nearest neighbors to return

\textbf{return\_distance} : boolean (default = True)

if True, return a tuple (d, i) of distances and indices if False, return array i

\textbf{dualtree} : boolean (default = False)

if True, use the dual tree formalism for the query: a tree is built for the query points, and the pair of trees is used to efficiently search this space. This can lead to better performance as the number of points grows large.

\textbf{breadth\_first} : boolean (default = False)

if True, then query the nodes in a breadth-first manner. Otherwise, query the nodes in a depth-first manner.

\textbf{sort\_results} : boolean (default = True)

if True, then distances and indices of each point are sorted on return, so that the first column contains the closest points. Otherwise, neighbors are returned in an arbitrary order.

\textbf{Returns}:

\textbf{i} : if return\_distance == False

(d, i) : if return\_distance == True

d : array of doubles - shape: x.shape[:-1] + (k,)
each entry gives the list of distances to the neighbors of the corresponding point

i : array of integers - shape: x.shape[:-1] + (k,)
each entry gives the list of indices of neighbors of the corresponding point

\section*{Examples}

Query for k-nearest neighbors

\begin{verbatim}
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = BinaryTree(X, leaf_size=2)
>>> dist, ind = tree.query(X[0], k=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[ 0. 0.19662693 0.29473397]
\end{verbatim}

\textbf{query\_radius}()

\textbf{query\_radius}(self, X, r, count\_only = False):

query the tree for neighbors within a radius r

\textbf{Parameters}:

\textbf{X} : array-like, last dimension self.dim

An array of points to query

\textbf{r} : distance within which neighbors are returned
r can be a single value, or an array of values of shape x.shape[:-1] if different radii are desired for each point.

**return_distance** : boolean (default = False)

if True, return distances to neighbors of each point if False, return only neighbors Note that unlike the query() method, setting return_distance=True here adds to the computation time. Not all distances need to be calculated explicitly for return_distance=False. Results are not sorted by default: see sort_results keyword.

**count_only** : boolean (default = False)

if True, return only the count of points within distance r if False, return the indices of all points within distance r If return_distance==True, setting count_only=True will result in an error.

**sort_results** : boolean (default = False)

if True, the distances and indices will be sorted before being returned. If False, the results will not be sorted. If return_distance == False, setting sort_results = True will result in an error.

**Returns**

- **count** : if count_only == True
- **ind** : if count_only == False and return_distance == False
- **(ind, dist)** : if count_only == False and return_distance == True

**count** : array of integers, shape = X.shape[:-1]

each entry gives the number of neighbors within a distance r of the corresponding point.

**ind** : array of objects, shape = X.shape[:-1]

each element is a numpy integer array listing the indices of neighbors of the corresponding point. Note that unlike the results of a k-neighbors query, the returned neighbors are not sorted by distance by default.

**dist** : array of objects, shape = X.shape[:-1]

each element is a numpy double array listing the distances corresponding to indices in i.

**Examples**

Query for neighbors in a given radius

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = BinaryTree(X, leaf_size=2)
>>> print tree.query_radius(X[0], r=0.3, count_only=True)
3
>>> ind = tree.query_radius(X[0], r=0.3)
>>> print ind  # indices of neighbors within distance 0.3
[3 0 1]
```

two_point_correlation() Compute the two-point correlation function

**Parameters**

- **X** : array_like
An array of points to query. Last dimension should match dimension of training data.

\(r\) : array_like

A one-dimensional array of distances

\(\text{dualtree}\) : boolean (default = False)

If true, use a dualtree algorithm. Otherwise, use a single-tree algorithm. Dual tree algorithms can have better scaling for large \(N\).

Returns

\(\text{counts}\) : ndarray

\(\text{counts}[i]\) contains the number of pairs of points with distance less than or equal to \(r[i]\)

Examples

Compute the two-point autocorrelation function of \(X\):

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((30, 3))
>>> r = np.linspace(0, 1, 5)
>>> tree = BinaryTree(X)
>>> tree.two_point_correlation(X, r)
array([ 30, 62, 278, 580, 820])
```

29.25.8 `sklearn.neighbors.KDTree`

class `sklearn.neighbors.KDTree`

KDTree for fast generalized N-point problems

KDTree(X, leaf_size=40, metric='minkowski', **kwargs)

Parameters

\(X\) : array-like, shape = \([n_{\text{samples}}, n_{\text{features}}]\)

\(n_{\text{samples}}\) is the number of points in the data set, and \(n_{\text{features}}\) is the dimension of the parameter space. Note: if \(X\) is a C-contiguous array of doubles then data will not be copied. Otherwise, an internal copy will be made.

\(\text{leaf\_size}\) : positive integer (default = 40)

Number of points at which to switch to brute-force. Changing \(\text{leaf\_size}\) will not affect the results of a query, but can significantly impact the speed of a query and the memory required to store the constructed tree. The amount of memory needed to store the tree scales as approximately \(n_{\text{samples}} / \text{leaf\_size}\). For a specified \(\text{leaf\_size}\), a leaf node is guaranteed to satisfy \(\text{leaf\_size} \leq n_{\text{points}} \leq 2 \times \text{leaf\_size}\), except in the case that \(n_{\text{samples}} < \text{leaf\_size}\).

\(\text{metric}\) : string or DistanceMetric object

the distance metric to use for the tree. Default='minkowski' with \(p=2\) (that is, a euclidean metric). See the documentation of the DistanceMetric class for a list of available metrics. \text{kd\_tree.valid\_metrics} gives a list of the metrics which are valid for KDTree.

Additional keywords are passed to the distance metric class.

Attributes

\(\text{data}\) : np.ndarray

The training data
## Examples

### Query for \(k\)-nearest neighbors

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = KDTree(X, leaf_size=2)
>>> dist, ind = tree.query([X[0]], k=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[ 0. 0.19662693 0.29473397]
```

Pickle and Unpickle a tree. Note that the state of the tree is saved in the pickle operation: the tree needs not be rebuilt upon unpickling.

```python
>>> import numpy as np
>>> import pickle

>>> np.random.seed(0)

>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = KDTree(X, leaf_size=2)
>>> s = pickle.dumps(tree)
>>> tree_copy = pickle.loads(s)
>>> dist, ind = tree_copy.query(X[0], k=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[ 0. 0.19662693 0.29473397]
```

### Query for neighbors within a given radius

```python
>>> import numpy as np
>>> np.random.seed(0)

>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = KDTree(X, leaf_size=2)
>>> print tree.query_radius(X[0], r=0.3, count_only=True)
3
>>> ind = tree.query_radius(X[0], r=0.3)
>>> print ind  # indices of neighbors within distance 0.3
[3 0 1]
```

### Compute a gaussian kernel density estimate:

```python
>>> import numpy as np
>>> np.random.seed(1)

>>> X = np.random.random((100, 3))
>>> tree = KDTree(X)
>>> tree.kernel_density(X[:3], h=0.1, kernel='gaussian')
array([ 6.94114649, 7.83281226, 7.2071716 ])
```

### Compute a two-point auto-correlation function

```python
>>> import numpy as np
>>> np.random.seed(0)

>>> X = np.random.random((30, 3))
>>> r = np.linspace(0, 1, 5)
>>> tree = KDTree(X)
```
>>> tree.two_point_correlation(X, r)  
array([ 30,  62, 278, 580, 820])

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__init__()  
_x.__init__(...) initializes x; see help(type(x)) for signature

kernel_density(self, X, h[, kernel, atol, ...])  
Compute the kernel density estimate at points X with the given kernel, using the distance metric specified at tree creation.

Parameters:

X : array_like  
An array of points to query. Last dimension should match dimension of training data.

h : float  
the bandwidth of the kernel

kernel : string  
specify the kernel to use. Options are - 'gaussian' - 'tophat' - 'epanechnikov' - 'exponential' - 'linear' - 'cosine' Default is kernel = 'gaussian'

atol, rtol : float (default = 0)  
Specify the desired relative and absolute tolerance of the result. If the true result is K_true, then the returned result K_ret satisfies abs(K_true - K_ret) < atol + rtol * K_ret. The default is zero (i.e. machine precision) for both.

breadth_first : boolean (default = False)  
if True, use a breadth-first search. If False (default) use a depth-first search. Breadth-first is generally faster for compact kernels and/or high tolerances.

return_log : boolean (default = False)  
return the logarithm of the result. This can be more accurate than returning the result itself for narrow kernels.

Returns:

density : ndarray  
The array of (log)-density evaluations, shape = X.shape[:-1]
Examples

Compute a gaussian kernel density estimate:

```python
>>> import numpy as np

>>> np.random.seed(1)

>>> X = np.random.random((100, 3))

>>> tree = BinaryTree(X)

>>> tree.kernel_density(X[:3], h=0.1, kernel='gaussian')
array([ 6.94114649, 7.83281226, 7.2071716])
```

query \( (X, k=1, \text{return\_distance}=\text{True}, \text{dualtree}=\text{False}, \text{breadth\_first}=\text{False}) \)
query the tree for the \( k \) nearest neighbors

Parameters

- **X**: array-like, last dimension self.dim
  
  An array of points to query

- **k**: integer (default = 1)
  
  The number of nearest neighbors to return

- **return\_distance**: boolean (default = True)
  
  if True, return a tuple \((d, i)\) of distances and indices
  
  if False, return array \(i\)

- **dualtree**: boolean (default = False)
  
  if True, use the dual tree formalism for the query: a tree is built for the query points,
  
  and the pair of trees is used to efficiently search this space. This can lead to better
  
  performance as the number of points grows large.

- **breadth\_first**: boolean (default = False)
  
  if True, then query the nodes in a breadth-first manner.
  
  Otherwise, query the nodes in a depth-first manner.

- **sort\_results**: boolean (default = True)
  
  if True, then distances and indices of each point are sorted on return, so that the first
  
  column contains the closest points.
  
  Otherwise, neighbors are returned in an arbitrary order.

Returns

- **i**: if return\_distance == False
  
  \((d, i)\): if return\_distance == True

- **d**: array of doubles - shape: \(x.shape[:-1] + (k,)\)
  
  each entry gives the list of distances to the neighbors of the corresponding point

- **i**: array of integers - shape: \(x.shape[:-1] + (k,)\)
  
  each entry gives the list of indices of neighbors of the corresponding point

Examples

Query for k-nearest neighbors

```python
>>> import numpy as np

>>> np.random.seed(0)

>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions

>>> tree = BinaryTree(X, leaf_size=2)
```
```python
>>> dist, ind = tree.query(X[0], k=3)
>>> print ind # indices of 3 closest neighbors
[0 3 1]
>>> print dist # distances to 3 closest neighbors
[ 0. 0.19662693 0.29473397]
```

**query_radius()**

query_radius(self, X, r, count_only = False):

query the tree for neighbors within a radius r

**Parameters**

X : array-like, last dimension self.dim

An array of points to query

r : distance within which neighbors are returned

r can be a single value, or an array of values of shape X.shape[:-1] if different radii are desired for each point.

return_distance : boolean (default = False)

if True, return distances to neighbors of each point if False, return only neighbors Note that unlike the query() method, setting return_distance=True here adds to the computation time. Not all distances need to be calculated explicitly for return_distance=False. Results are not sorted by default: see sort_results keyword.

count_only : boolean (default = False)

if True, return only the count of points within distance r if False, return the indices of all points within distance r If return_distance==True, setting count_only=True will result in an error.

sort_results : boolean (default = False)

if True, the distances and indices will be sorted before being returned. If False, the results will not be sorted. If return_distance == False, setting sort_results = True will result in an error.

**Returns**

count : if count_only == True

ind : if count_only == False and return_distance == False

(ind, dist) : if count_only == False and return_distance == True

count : array of integers, shape = X.shape[:-1]

each entry gives the number of neighbors within a distance r of the corresponding point.

ind : array of objects, shape = X.shape[:-1]

each element is a numpy integer array listing the indices of neighbors of the corresponding point. Note that unlike the results of a k-neighbors query, the returned neighbors are not sorted by distance by default.

dist : array of objects, shape = X.shape[:-1]

each element is a numpy double array listing the distances corresponding to indices in ind.

**Examples**

Query for neighbors in a given radius
```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10, 3))  # 10 points in 3 dimensions
>>> tree = BinaryTree(X, leaf_size=2)
>>> print tree.query_radius(X[0], r=0.3, count_only=True)
3
>>> ind = tree.query_radius(X[0], r=0.3)
>>> print ind  # indices of neighbors within distance 0.3
[3 0 1]
```

### two_point_correlation()

Compute the two-point correlation function

**Parameters**
- `X` : array_like
  An array of points to query. Last dimension should match dimension of training data.
- `r` : array_like
  A one-dimensional array of distances
- `dualtree` : boolean (default = False)
  If true, use a dualtree algorithm. Otherwise, use a single-tree algorithm. Dual tree algorithms can have better scaling for large N.

**Returns**
- `counts` : ndarray
  counts[i] contains the number of pairs of points with distance less than or equal to r[i]

**Examples**

Compute the two-point autocorrelation function of X:

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((30, 3))
>>> r = np.linspace(0, 1, 5)
>>> tree = BinaryTree(X)
>>> tree.two_point_correlation(X, r)
array([ 30, 62, 278, 580, 820])
```

### 29.25.9 sklearn.neighbors.LSHForest

**class** `sklearn.neighbors.LSHForest` *(n_estimators=10, radius=1.0, n_candidates=50, n_neighbors=5, min_hash_match=4, radius_cutoff_ratio=0.9, random_state=None)*

Performs approximate nearest neighbor search using LSH forest.

LSH Forest: Locality Sensitive Hashing forest [1] is an alternative method for vanilla approximate nearest neighbor search methods. LSH forest data structure has been implemented using sorted arrays and binary search and 32 bit fixed-length hashes. Random projection is used as the hash family which approximates cosine distance.

The cosine distance is defined as $1 - \text{cosine_similarity}$: the lowest value is 0 (identical point) but it is bounded above by 2 for the farthest points. Its value does not depend on the norm of the vector points but only on their relative angles.
Read more in the *User Guide*.

**Parameters**  

- **n_estimators**: int (default = 10)
  
  Number of trees in the LSH Forest.

- **min_hash_match**: int (default = 4)
  
  Lowest hash length to be searched when candidate selection is performed for nearest neighbors.

- **n_candidates**: int (default = 10)
  
  Minimum number of candidates evaluated per estimator, assuming enough items meet the *min_hash_match* constraint.

- **n_neighbors**: int (default = 5)
  
  Number of neighbors to be returned from query function when it is not provided to the *kneighbors* method.

- **radius**: float, optional (default = 1.0)
  
  Radius from the data point to its neighbors. This is the parameter space to use by default for the :meth:`radius_neighbors` queries.

- **radius_cutoff_ratio**: float, optional (default = 0.9)
  
  A value ranges from 0 to 1. Radius neighbors will be searched until the ratio between total neighbors within the radius and the total candidates becomes less than this value unless it is terminated by hash length reaching *min_hash_match*.

- **random_state**: int, RandomState instance or None, optional (default=None)
  
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by *np.random*.

**Attributes**

- **hash_functions_**: list of GaussianRandomProjectionHash objects
  
  Hash function \( g(p,x) \) for a tree is an array of 32 randomly generated float arrays with the same dimension as the data set. This array is stored in GaussianRandomProjectionHash object and can be obtained from *components_* attribute.

- **trees_**: array, shape (n_estimators, n_samples)
  
  Each tree (corresponding to a hash function) contains an array of sorted hashed values. The array representation may change in future versions.

- **original_indices_**: array, shape (n_estimators, n_samples)
  
  Original indices of sorted hashed values in the fitted index.

**References**

[R70]

**Examples**

```python
>>> from sklearn.neighbors import LSHForest
```
>>> X_train = [[5, 5, 2], [21, 5, 5], [1, 1, 1], [8, 9, 1], [6, 10, 2]]
>>> X_test = [[9, 1, 6], [3, 1, 10], [7, 10, 3]]
>>> lshf = LSHForest(random_state=42)
>>> lshf.fit(X_train)
LSHForest(min_hash_match=4, n_candidates=50, n_estimators=10,
         n_neighbors=5, radius=1.0, radius_cutoff_ratio=0.9,
         random_state=42)
>>> distances, indices = lshf.kneighbors(X_test, n_neighbors=2)
>>> distances
array([[ 0.069..., 0.149...],
       [ 0.229..., 0.481...],
       [ 0.004..., 0.014...]])
>>> indices
array([[1, 2],
       [2, 0],
       [4, 0]])

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>fit</strong>(X[, y])</td>
<td>Fit the LSH forest on the data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>kneighbors(X[, n_neighbors, return_distance])</td>
<td>Returns n_neighbors of approximate nearest neighbors.</td>
</tr>
<tr>
<td>kneighbors_graph(X[, n_neighbors, mode])</td>
<td>Computes the (weighted) graph of k-Neighbors for points in X</td>
</tr>
<tr>
<td>partial_fit(X[, y])</td>
<td>Inserts new data into the already fitted LSH Forest.</td>
</tr>
<tr>
<td>radius_neighbors(X[, radius, return_distance])</td>
<td>Finds the neighbors within a given radius of a point or points.</td>
</tr>
<tr>
<td>radius_neighbors_graph(X[, radius, mode])</td>
<td>Computes the (weighted) graph of Neighbors for points in X</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__**(n_estimators=10, radius=1.0, n_candidates=50, n_estimators=5, min_hash_match=4, radius_cutoff_ratio=0.9, random_state=None)

**fit**(X, y=None)
Fit the LSH forest on the data.
This creates binary hashes of input data points by getting the dot product of input points and hash_function then transforming the projection into a binary string array based on the sign (positive/negative) of the projection. A sorted array of binary hashes is created.

Parameters

- **X** : array_like or sparse (CSR) matrix, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

- **y** : object
  Returns self.

get_params(deep=True)
Get parameters for this estimator.

Parameters

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.
**Returns**

**params** : mapping of string to any
Parameter names mapped to their values.

**kneighbors** *(X, n_neighbors=None, return_distance=True)*
Returns n_neighbors of approximate nearest neighbors.

**Parameters**

**X** : array_like or sparse (CSR) matrix, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single query.

**n_neighbors** : int, optional (default = None)
Number of neighbors required. If not provided, this will return the number specified at the initialization.

**return_distance** : boolean, optional (default = True)
Returns the distances of neighbors if set to True.

**Returns**

**dist** : array, shape (n_samples, n_neighbors)
Array representing the cosine distances to each point, only present if return_distance=True.

**ind** : array, shape (n_samples, n_neighbors)
Indices of the approximate nearest points in the population matrix.

**kneighbors_graph** *(X=None, n_neighbors=None, mode='connectivity')*
Computes the (weighted) graph of k-Neighbors for points in X

**Parameters**

**X** : array-like, shape (n_query, n_features), or (n_query, n_indexed) if metric == 'precomputed'
The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

**n_neighbors** : int
Number of neighbors for each sample. (default is value passed to the constructor).

**mode** : {'connectivity', 'distance'}, optional
Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

**A** : sparse matrix in CSR format, shape = [n_samples, n_samples_fit]
n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.

**See also:**

*NearestNeighbors.radius_neighbors_graph*

**Examples**

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
```
>>> A.toarray()
array([[ 1., 0., 1.],
       [ 0., 1., 1.],
       [ 1., 0., 1.]])

**partial_fit** *(X, y=None)*

Inserts new data into the already fitted LSH Forest. Cost is proportional to new total size, so additions should be batched.

**Parameters**

- **X**: array_like or sparse (CSR) matrix, shape (n_samples, n_features)
  
  New data point to be inserted into the LSH Forest.

**radius_neighbors** *(X, radius=None, return_distance=True)*

Finds the neighbors within a given radius of a point or points.

Return the indices and distances of some points from the dataset lying in a ball with size radius around the points of the query array. Points lying on the boundary are included in the results.

The result points are not necessarily sorted by distance to their query point.

LSH Forest being an approximate method, some true neighbors from the indexed dataset might be missing from the results.

**Parameters**

- **X**: array_like or sparse (CSR) matrix, shape (n_samples, n_features)
  
  List of n_features-dimensional data points. Each row corresponds to a single query.

- **radius**: float
  
  Limiting distance of neighbors to return. (default is the value passed to the constructor).

- **return_distance**: boolean, optional (default = False)
  
  Returns the distances of neighbors if set to True.

**Returns**

- **dist**: array, shape (n_samples,) of arrays
  
  Each element is an array representing the cosine distances to some points found within radius of the respective query. Only present if return_distance=True.

- **ind**: array, shape (n_samples,) of arrays
  
  Each element is an array of indices for neighbors within radius of the respective query.

**radius_neighbors_graph** *(X=None, radius=None, mode='connectivity')*

Computes the (weighted) graph of Neighbors for points in X

Neighborhoods are restricted the points at a distance lower than radius.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features], optional
  
  The query point or points. If not provided, neighbors of each indexed point are returned. In this case, the query point is not considered its own neighbor.

- **radius**: float
  
  Radius of neighborhoods. (default is the value passed to the constructor).

- **mode**: {'connectivity', 'distance'}, optional
  
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples]
A[i, j] is assigned the weight of edge that connects i to j.

See also:

`kneighbors_graph`

Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.5)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.radius_neighbors_graph(X)
>>> A.toarray()
array([[1., 0., 1.],
       [0., 1., 0.],
       [1., 0., 1.]])
```

`set_params(**params)`

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

`self`

Examples using `sklearn.neighbors.LSHForest`

- Hyper-parameters of Approximate Nearest Neighbors
- Scalability of Approximate Nearest Neighbors

29.25.10 `sklearn.neighbors.DistanceMetric`

class `sklearn.neighbors.DistanceMetric`

DistanceMetric class

This class provides a uniform interface to fast distance metric functions. The various metrics can be accessed via the `get_metric` class method and the metric string identifier (see below). For example, to use the Euclidean distance:

```python
>>> dist = DistanceMetric.get_metric('euclidean')
>>> X = [[0, 1, 2],
      [3, 4, 5]]
>>> dist.pairwise(X)
array([[ 0.        , 5.19615242],
       [5.19615242, 0.        ]])
```

Available Metrics The following lists the string metric identifiers and the associated distance metric classes:

Metrics intended for real-valued vector spaces:
<table>
<thead>
<tr>
<th>identifier</th>
<th>class name</th>
<th>args</th>
<th>distance function</th>
</tr>
</thead>
<tbody>
<tr>
<td>“euclidean”</td>
<td>EuclideanDistance</td>
<td>•</td>
<td>sqrt(sum(((x - y)^2)))</td>
</tr>
<tr>
<td>“manhattan”</td>
<td>ManhattanDistance</td>
<td>•</td>
<td>sum(</td>
</tr>
<tr>
<td>“chebyshev”</td>
<td>ChebyshevDistance</td>
<td>•</td>
<td>max(</td>
</tr>
<tr>
<td>“minkowski”</td>
<td>MinkowskiDistance</td>
<td>p</td>
<td>sum(</td>
</tr>
<tr>
<td>“wminkowski”</td>
<td>WMinkowskiDistance</td>
<td>p, w</td>
<td>sum(w *</td>
</tr>
<tr>
<td>“seuclidean”</td>
<td>SEuclideanDistance</td>
<td>V</td>
<td>sqrt(sum((x - y)^2 / V))</td>
</tr>
<tr>
<td>“mahalanobis”</td>
<td>MahalanobisDistance</td>
<td>V or VI</td>
<td>sqrt((x - y)' V^-1 (x - y))</td>
</tr>
</tbody>
</table>

**Metrics intended for two-dimensional vector spaces:** Note that the haversine distance metric requires data in the form of [latitude, longitude] and both inputs and outputs are in units of radians.

<table>
<thead>
<tr>
<th>identifier</th>
<th>class name</th>
<th>distance function</th>
</tr>
</thead>
<tbody>
<tr>
<td>“haversine”</td>
<td>HaversineDistance</td>
<td>2 arcsin(sqrt(sin^2(0.5<em>dx) • cos(x1)cos(x2)sin^2(0.5</em>dy)))</td>
</tr>
</tbody>
</table>

**Metrics intended for integer-valued vector spaces:** Though intended for integer-valued vectors, these are also valid metrics in the case of real-valued vectors.

<table>
<thead>
<tr>
<th>identifier</th>
<th>class name</th>
<th>distance function</th>
</tr>
</thead>
<tbody>
<tr>
<td>“hamming”</td>
<td>HammingDistance</td>
<td>N_unequal(x,y) / N_tot</td>
</tr>
<tr>
<td>“canberra”</td>
<td>CanberraDistance</td>
<td>sum(</td>
</tr>
<tr>
<td>“braycurtis”</td>
<td>BrayCurtisDistance</td>
<td>sum(</td>
</tr>
</tbody>
</table>

**Metrics intended for boolean-valued vector spaces:** Any nonzero entry is evaluated to “True”. In the listings below, the following abbreviations are used:

- N : number of dimensions
- NTT : number of dims in which both values are True
- NTF : number of dims in which the first value is True, second is False
- NFT : number of dims in which the first value is False, second is True
- NFF : number of dims in which both values are False
- NNEQ : number of non-equal dimensions, NNEQ = NTF + NFT
- NNZ : number of nonzero dimensions, NNZ = NTF + NFT + NTT
<table>
<thead>
<tr>
<th>identifier</th>
<th>class name</th>
<th>distance function</th>
</tr>
</thead>
<tbody>
<tr>
<td>“jaccard”</td>
<td>JaccardDistance</td>
<td>NNEQ / NNZ</td>
</tr>
<tr>
<td>“matching”</td>
<td>MatchingDistance</td>
<td>NNEQ / N</td>
</tr>
<tr>
<td>“dice”</td>
<td>DiceDistance</td>
<td>NNEQ / (NTT + NNZ)</td>
</tr>
<tr>
<td>“kulsinski”</td>
<td>KulinskiDistance</td>
<td>(NNEQ + N - NTT) / (NNEQ + N)</td>
</tr>
<tr>
<td>“rogerstanimoto”</td>
<td>RogersTanimotoDistance</td>
<td>2 * NNEQ / (N + NNEQ)</td>
</tr>
<tr>
<td>“russellrao”</td>
<td>RussellRaoDistance</td>
<td>NNZ / N</td>
</tr>
<tr>
<td>“sokalmichener”</td>
<td>SokalMichenerDistance</td>
<td>2 * NNEQ / (N + NNEQ)</td>
</tr>
<tr>
<td>“sokalsneath”</td>
<td>SokalSneathDistance</td>
<td>NNEQ / (NNEQ + 0.5 * NTT)</td>
</tr>
</tbody>
</table>

User-defined distance:

<table>
<thead>
<tr>
<th>identifier</th>
<th>class name</th>
<th>args</th>
</tr>
</thead>
<tbody>
<tr>
<td>“pyfunc”</td>
<td>PyFuncDistance</td>
<td>func</td>
</tr>
</tbody>
</table>

Here `func` is a function which takes two one-dimensional numpy arrays, and returns a distance. Note that in order to be used within the BallTree, the distance must be a true metric: i.e. it must satisfy the following properties:

1. Non-negativity: \( d(x, y) \geq 0 \)
2. Identity: \( d(x, y) = 0 \) if and only if \( x == y \)
3. Symmetry: \( d(x, y) = d(y, x) \)
4. Triangle Inequality: \( d(x, y) + d(y, z) \geq d(x, z) \)

Because of the Python object overhead involved in calling the python function, this will be fairly slow, but it will have the same scaling as other distances.

Methods

- `dist_to_rdist()`: Convert the true distance to the reduced distance.
- `get_metric()`: Get the given distance metric from the string identifier.
- `pairwise()`: Compute the pairwise distances between X and Y.
- `rdist_to_dist()`: Convert the Reduced distance to the true distance.

```python
__init__()
```
x.__init__(...) initializes x; see help(type(x)) for signature

```python
dist_to_rdist()
```
Convert the true distance to the reduced distance.

The reduced distance, defined for some metrics, is a computationally more efficient measure which preserves the rank of the true distance. For example, in the Euclidean distance metric, the reduced distance is the squared-euclidean distance.

```python
get_metric()
```
Get the given distance metric from the string identifier.

See the docstring of DistanceMetric for a list of available metrics.

**Parameters**: `metric` : string or class name

The distance metric to use

**kwargs**

additional arguments will be passed to the requested metric
pairwise()  
Compute the pairwise distances between X and Y

This is a convenience routine for the sake of testing. For many metrics, the utilities in scipy.spatial.distance.cdist and scipy.spatial.distance.pdist will be faster.

**Parameters**

- **X** : array_like
  Array of shape (Nx, D), representing Nx points in D dimensions.

- **Y** : array_like (optional)
  Array of shape (Ny, D), representing Ny points in D dimensions. If not specified, then Y=X.

**Returns**

- **dist** : ndarray
  The shape (Nx, Ny) array of pairwise distances between points in X and Y.

rdist_to_dist()  
Convert the Reduced distance to the true distance.

The reduced distance, defined for some metrics, is a computationally more efficient measure which preserves the rank of the true distance. For example, in the Euclidean distance metric, the reduced distance is the squared-euclidean distance.

### 29.25.11 sklearn.neighbors.KernelDensity

**class sklearn.neighbors.KernelDensity**

```python
class sklearn.neighbors.KernelDensity(bandwidth=1.0, algorithm='auto', kernel='gaussian', metric='euclidean', atol=0, rtol=0, breadth_first=True, leaf_size=40, metric_params=None)
```

Kernel Density Estimation

Read more in the [User Guide](#).

**Parameters**

- **bandwidth** : float
  The bandwidth of the kernel.

- **algorithm** : string
  The tree algorithm to use. Valid options are ['kd_tree', 'ball_tree', 'auto']. Default is 'auto'.

- **kernel** : string
  The kernel to use. Valid kernels are ['gaussian', 'tophat', 'epanechnikov', 'exponential', 'linear', 'cosine']. Default is 'gaussian'.

- **metric** : string
  The distance metric to use. Note that not all metrics are valid with all algorithms. Refer to the documentation of BallTree and KDTree for a description of available algorithms. Note that the normalization of the density output is correct only for the Euclidean distance metric. Default is 'euclidean'.

- **atol** : float
  The desired absolute tolerance of the result. A larger tolerance will generally lead to faster execution. Default is 0.

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---
**rtol** : float
The desired relative tolerance of the result. A larger tolerance will generally lead to faster execution. Default is 1E-8.

**breadth_first** : boolean
If true (default), use a breadth-first approach to the problem. Otherwise use a depth-first approach.

**leaf_size** : int
Specify the leaf size of the underlying tree. See BallTree or KDTree for details. Default is 40.

**metric_params** : dict
Additional parameters to be passed to the tree for use with the metric. For more information, see the documentation of BallTree or KDTree.

## Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>fit</strong>(X[, y])</td>
<td>Fit the Kernel Density model on the data.</td>
</tr>
<tr>
<td><strong>get_params</strong>(deep)</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><strong>sample</strong>(n_samples[, random_state])</td>
<td>Generate random samples from the model.</td>
</tr>
<tr>
<td><strong>score</strong>(X[, y])</td>
<td>Compute the total log probability under the model.</td>
</tr>
<tr>
<td><strong>score_samples</strong>(X)</td>
<td>Evaluate the density model on the data.</td>
</tr>
<tr>
<td><strong>set_params</strong>(<strong>params</strong>)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

### __init__

` bandwidth=1.0, algorithm='auto', kernel='gaussian', metric='euclidean', atol=0, rtol=0, breadth_first=True, leaf_size=40, metric_params=None`

**fit**(X, y=None)
Fit the Kernel Density model on the data.

**Parameters**

- **X** : array_like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**get_params**(deep=True)
Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  Parameter names mapped to their values.

**sample**(n_samples=1, random_state=None)
Generate random samples from the model.

Currently, this is implemented only for gaussian and tophat kernels.

**Parameters**

- **n_samples** : int, optional
  Number of samples to generate. Defaults to 1.

  **random_state** : RandomState or an int seed (0 by default)
A random number generator instance.

**Returns**

X : array_like, shape (n_samples, n_features)
List of samples.

score (X, y=None)
Compute the total log probability under the model.

**Parameters**

X : array_like, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns

logprob : float
Total log-likelihood of the data in X.

score_samples (X)
Evaluate the density model on the data.

**Parameters**

X : array_like, shape (n_samples, n_features)
An array of points to query. Last dimension should match dimension of training data (n_features).

Returns

density : ndarray, shape (n_samples,)
The array of log(density) evaluations.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns

self :

Examples using sklearn.neighbors.KernelDensity

- Kernel Density Estimation
- Simple 1D Kernel Density Estimation
- Kernel Density Estimate of Species Distributions

| neighbors.kneighbors_graph (X, n_neighbors[, ...]) | Computes the (weighted) graph of k-Neighbors for points in X |
| neighbors.radius_neighbors_graph (X, radius) | Computes the (weighted) graph of Neighbors for points in X |

29.25.12 sklearn.neighbors.kneighbors_graph

sklearn.neighbors.kneighbors_graph (X, n_neighbors, mode='connectivity', metric='minkowski', p=2, metric_params=None, include_self=False, n_jobs=1)

Computes the (weighted) graph of k-Neighbors for points in X

Read more in the User Guide.

**Parameters**

X : array-like or BallTree, shape = [n_samples, n_features]
Sample data, in the form of a numpy array or a precomputed BallTree.

**n_neighbors** : int

Number of neighbors for each sample.

**mode** : {'connectivity', 'distance'}, optional

Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, and ‘distance’ will return the distances between neighbors according to the given metric.

**metric** : string, default ‘minkowski’

The distance metric used to calculate the k-Neighbors for each sample point. The DistanceMetric class gives a list of available metrics. The default distance is ‘euclidean’ (‘minkowski’ metric with the p param equal to 2.)

**include_self** : bool, default=False.

Whether or not to mark each sample as the first nearest neighbor to itself. If `None`, then True is used for mode='connectivity' and False for mode='distance' as this will preserve backwards compatibility.

**p** : int, default 2

Power parameter for the Minkowski metric. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

**metric_params** : dict, optional

Additional keyword arguments for the metric function.

**n_jobs** : int, optional (default = 1)

The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores.

Returns

A sparse matrix in CSR format, shape = [n_samples, n_samples]

A[i, j] is assigned the weight of edge that connects i to j.

See also:

radius_neighbors_graph

Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import kneighbors_graph
>>> A = kneighbors_graph(X, 2, mode='connectivity', include_self=True)
>>> A.todense()
array([[1., 0., 1.],
       [0., 1., 1.],
       [1., 0., 1.]]
```

Examples using sklearn.neighbors.kneighbors_graph

• Agglomerative clustering with and without structure
Comparing different clustering algorithms on toy datasets

Hierarchical clustering: structured vs unstructured ward

29.25.13 sklearn.neighbors.radius_neighbors_graph

sklearn.neighbors.radius_neighbors_graph(X, radius, mode='connectivity', metric='minkowski', p=2, metric_params=None, include_self=False, n_jobs=1)

Computes the (weighted) graph of Neighbors for points in X.

Neighborhoods are restricted the points at a distance lower than radius.

Read more in the User Guide.

Parameters:

X : array-like or BallTree, shape = [n_samples, n_features]
    Sample data, in the form of a numpy array or a precomputed BallTree.

radius : float
    Radius of neighborhoods.

mode : {'connectivity', 'distance'}, optional
    Type of returned matrix: 'connectivity' will return the connectivity matrix with ones and zeros, and 'distance' will return the distances between neighbors according to the given metric.

metric : string, default 'minkowski'
    The distance metric used to calculate the neighbors within a given radius for each sample point. The DistanceMetric class gives a list of available metrics. The default distance is 'euclidean' ('minkowski' metric with the param equal to 2.)

include_self : bool, default=False
    Whether or not to mark each sample as the first nearest neighbor to itself. If None, then True is used for mode='connectivity' and False for mode='distance' as this will preserve backwards compatibility.

p : int, default 2
    Power parameter for the Minkowski metric. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

metric_params : dict, optional
    additional keyword arguments for the metric function.

n_jobs : int, optional (default = 1)
    The number of parallel jobs to run for neighbors search. If -1, then the number of jobs is set to the number of CPU cores.

Returns:

A : sparse matrix in CSR format, shape = [n_samples, n_samples]
    A[i, j] is assigned the weight of edge that connects i to j.

See also:

kneighbors_graph
Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import radius_neighbors_graph
>>> A = radius_neighbors_graph(X, 1.5, mode='connectivity', include_self=True)
>>> A.toarray()
array([[ 1.,  0.,  1.],
       [ 0.,  1.,  0.],
       [ 1.,  0.,  1.]])
```

29.26 sklearn.neural_network: Neural network models

The `sklearn.neural_network` module includes models based on neural networks.


<table>
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<th>Class</th>
<th>Description</th>
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<tr>
<td><code>neural_network.MLPClassifier</code></td>
<td>Multi-layer Perceptron classifier.</td>
</tr>
<tr>
<td><code>neural_network.MLPRegressor</code></td>
<td>Multi-layer Perceptron regressor.</td>
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</table>

### 29.26.1 sklearn.neural_network.BernoulliRBM

Class `sklearn.neural_network.BernoulliRBM`:

```python
class sklearn.neural_network.BernoulliRBM(n_components=256, learning_rate=0.1, batch_size=10, n_iter=10, verbose=0, random_state=None)
```

Bernoulli Restricted Boltzmann Machine (RBM).

A Restricted Boltzmann Machine with binary visible units and binary hidden units. Parameters are estimated using Stochastic Maximum Likelihood (SML), also known as Persistent Contrastive Divergence (PCD) [2].

The time complexity of this implementation is $O(d \times n)$ assuming $d \sim n_{features} \sim n_{components}$.


**Parameters**

- **n_components** : int, optional
  Number of binary hidden units.

- **learning_rate** : float, optional
  The learning rate for weight updates. It is highly recommended to tune this hyperparameter. Reasonable values are in the $10^{[0., -3.]}$ range.

- **batch_size** : int, optional
  Number of examples per minibatch.

- **n_iter** : int, optional
  Number of iterations/sweeps over the training dataset to perform during training.

- **verbose** : int, optional
  The verbosity level. The default, zero, means silent mode.

- **random_state** : integer or numpy.RandomState, optional
A random number generator instance to define the state of the random permutations generator. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**Attributes**

- `intercept_hidden_`: array-like, shape (n_components,)
  Biases of the hidden units.
- `intercept_visible_`: array-like, shape (n_features,)
  Biases of the visible units.
- `components_`: array-like, shape (n_components, n_features)
  Weight matrix, where n_features in the number of visible units and n_components is the number of hidden units.

**References**


**Examples**

```python
>>> from sklearn.neural_network import BernoulliRBM
>>> X = np.array([[0, 0, 0], [0, 1, 1], [1, 0, 1], [1, 1, 1]])
>>> model = BernoulliRBM(n_components=2)
>>> model.fit(X)
```

**Methods**

- `fit(X[, y])` Fit the model to the data X.
- `fit_transform(X[, y])` Fit to data, then transform it.
- `get_params([deep])` Get parameters for this estimator.
- `gibbs(v)` Perform one Gibbs sampling step.
- `partial_fit(X[, y])` Fit the model to the data X which should contain a partial segment of the data.
- `score_samples(X)` Compute the pseudo-likelihood of X.
- `set_params(**params)` Set the parameters of this estimator.
- `transform(X)` Compute the hidden layer activation probabilities, P(h=1|v=X).

```python
>>> fit(X, y=None)
```

```python
>>> init(n_components=256, learning_rate=0.1, batch_size=10, n_iter=10, verbose=0, random_state=None)
```

```python
>>> fit(X, y=None)
```

Fit the model to the data X.
Parameters

\texttt{X} : \{array-like, sparse matrix\} shape (n\_samples, n\_features)

Training data.

Returns

\texttt{self} : BernoulliRBM

The fitted model.

\textbf{\texttt{fit\_transform} \((X, y=\text{None}, **\text{fit\_params})\)}

Fit to data, then transform it.

Fits transformer to \(X\) and \(y\) with optional parameters \texttt{fit\_params} and returns a transformed version of \(X\).

\begin{itemize}
  \item \textbf{Parameters} \(X\) : numpy array of shape \([n\_samples, n\_features]\)
    \begin{itemize}
      \item Training set.
    \end{itemize}
  \item \(y\) : numpy array of shape \([n\_samples]\)
    \begin{itemize}
      \item Target values.
    \end{itemize}
\end{itemize}

Returns

\(X\_new\) : numpy array of shape \([n\_samples, n\_features\_new]\)

Transformed array.

\textbf{\texttt{get\_params} \((\text{deep=}}\text{True})\)}

Get parameters for this estimator.

\begin{itemize}
  \item \textbf{Parameters} \texttt{deep} : boolean, optional
    \begin{itemize}
      \item If True, will return the parameters for this estimator and contained subobjects that are estimators.
    \end{itemize}
\end{itemize}

Returns

\texttt{params} : mapping of string to any

Parameter names mapped to their values.

\textbf{\texttt{gibbs} \((v)\)}

Perform one Gibbs sampling step.

\begin{itemize}
  \item \textbf{Parameters} \(v\) : array-like, shape \([n\_samples, n\_features]\)
    \begin{itemize}
      \item Values of the visible layer to start from.
    \end{itemize}
  \end{itemize}

Returns

\(v\_new\) : array-like, shape \([n\_samples, n\_features]\)

Values of the visible layer after one Gibbs step.

\textbf{\texttt{partial\_fit} \((X, y=\text{None})\)}

Fit the model to the data \(X\) which should contain a partial segment of the data.

\begin{itemize}
  \item \textbf{Parameters} \(X\) : array-like, shape \([n\_samples, n\_features]\)
    \begin{itemize}
      \item Training data.
    \end{itemize}
  \end{itemize}

Returns

\texttt{self} : BernoulliRBM

The fitted model.

\textbf{\texttt{score\_samples} \((X)\)}

Compute the pseudo-likelihood of \(X\).

\begin{itemize}
  \item \textbf{Parameters} \(X\) : \{array-like, sparse matrix\} shape \([n\_samples, n\_features]\)
    \begin{itemize}
      \item Values of the visible layer. Must be all-boolean (not checked).
    \end{itemize}
  \end{itemize}

Returns

\texttt{pseudo\_likelihood} : array-like, shape \([n\_samples,]\)

Value of the pseudo-likelihood (proxy for likelihood).
Notes

This method is not deterministic: it computes a quantity called the free energy on X, then on a randomly corrupted version of X, and returns the log of the logistic function of the difference.

**set_params(**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns

**transform**(X)
Compute the hidden layer activation probabilities, P(h=1|v=X).

Parameters

- **X**: {array-like, sparse matrix} shape (n_samples, n_features)
  The data to be transformed.

Returns

- **h**: array, shape (n_samples, n_components)
  Latent representations of the data.

Examples using `sklearn.neural_network.BernoulliRBM`

- Restricted Boltzmann Machine features for digit classification

29.26.2 `sklearn.neural_network.MLPClassifier`

Class `sklearn.neural_network.MLPClassifier` (hidden_layer_sizes=(100, ), activation='relu', solver='adam', alpha=0.0001, batch_size='auto', learning_rate='constant', learning_rate_init=0.001, power_t=0.5, max_iter=200, shuffle=True, random_state=None, tol=0.0001, verbose=False, warm_start=False, momentum=0.9, nesterovs_momentum=True, early_stopping=False, validation_fraction=0.1, beta_1=0.9, beta_2=0.999, epsilon=1e-08)

Multi-layer Perceptron classifier.

This model optimizes the log-loss function using LBFGS or stochastic gradient descent.

New in version 0.18.

Parameters

- **hidden_layer_sizes**: tuple, length = n_layers - 2, default (100,)
  The ith element represents the number of neurons in the ith hidden layer.

- **activation**: ['identity', 'logistic', 'tanh', 'relu'], default 'relu'
  Activation function for the hidden layer.

  - ‘identity’, no-op activation, useful to implement linear bottleneck, returns f(x) = x
  - ‘logistic’, the logistic sigmoid function, returns f(x) = 1 / (1 + exp(-x)).
  - ‘tanh’, the hyperbolic tan function, returns f(x) = tanh(x).
• 'relu', the rectified linear unit function, returns \( f(x) = \max(0, x) \)

**solver**: {‘lbfgs’, ‘sgd’, ‘adam’}, default ‘adam’

The solver for weight optimization.

• ‘lbfgs’ is an optimizer in the family of quasi-Newton methods.

• ‘sgd’ refers to stochastic gradient descent.

• ‘adam’ refers to a stochastic gradient-based optimizer proposed by Kingma, Diederik, and Jimmy Ba

Note: The default solver ‘adam’ works pretty well on relatively large datasets (with thousands of training samples or more) in terms of both training time and validation score. For small datasets, however, ‘lbfgs’ can converge faster and perform better.

**alpha**: float, optional, default 0.0001

L2 penalty (regularization term) parameter.

**batch_size**: int, optional, default ‘auto’

Size of minibatches for stochastic optimizers. If the solver is ‘lbfgs’, the classifier will not use minibatch. When set to ‘auto’, \( \text{batch\_size} = \min(200, n\_samples) \)

**learning_rate**: {‘constant’, ‘invscaling’, ‘adaptive’}, default ‘constant’

Learning rate schedule for weight updates.

• ‘constant’ is a constant learning rate given by ‘learning_rate_init’.

• ‘invscaling’ gradually decreases the learning rate \( \text{learning\_rate\_init} \) at each time step \( t \) using an inverse scaling exponent of \( \text{power\_t} \). effective_learning_rate = \( \frac{\text{learning\_rate\_init}}{\text{pow}(t, \text{power\_t})} \)

• ‘adaptive’ keeps the learning rate constant to ‘learning_rate_init’ as long as training loss keeps decreasing. Each time two consecutive epochs fail to decrease training loss by at least tol, or fail to increase validation score by at least tol if ‘early_stopping’ is on, the current learning rate is divided by 5.

Only used when solver=’sgd’.

**max_iter**: int, optional, default 200

Maximum number of iterations. The solver iterates until convergence (determined by ‘tol’) or this number of iterations.

**random_state**: int or RandomState, optional, default None

State or seed for random number generator.

**shuffle**: bool, optional, default True

Whether to shuffle samples in each iteration. Only used when solver=’sgd’ or ‘adam’.

**tol**: float, optional, default 1e-4

Tolerance for the optimization. When the loss or score is not improving by at least tol for two consecutive iterations, unless learning_rate is set to ‘adaptive’, convergence is considered to be reached and training stops.

**learning_rate_init**: double, optional, default 0.001

The initial learning rate used. It controls the step-size in updating the weights. Only used when solver=’sgd’ or ‘adam’.
**power_t** : double, optional, default 0.5

The exponent for inverse scaling learning rate. It is used in updating effective learning rate when the learning_rate is set to ‘invscaling’. Only used when solver=’sgd’.

**verbose** : bool, optional, default False

Whether to print progress messages to stdout.

**warm_start** : bool, optional, default False

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

**momentum** : float, default 0.9

Momentum for gradient descent update. Should be between 0 and 1. Only used when solver=’sgd’.

**nesterovs_momentum** : boolean, default True

Whether to use Nesterov’s momentum. Only used when solver=’sgd’ and momentum > 0.

**early_stopping** : bool, default False

Whether to use early stopping to terminate training when validation score is not improving. If set to true, it will automatically set aside 10% of training data as validation and terminate training when validation score is not improving by at least tol for two consecutive epochs. Only effective when solver=’sgd’ or ‘adam’

**validation_fraction** : float, optional, default 0.1

The proportion of training data to set aside as validation set for early stopping. Must be between 0 and 1. Only used if early_stopping is True

**beta_1** : float, optional, default 0.9

Exponential decay rate for estimates of first moment vector in adam, should be in [0, 1). Only used when solver=’adam’

**beta_2** : float, optional, default 0.999

Exponential decay rate for estimates of second moment vector in adam, should be in [0, 1). Only used when solver=’adam’

**epsilon** : float, optional, default 1e-8

Value for numerical stability in adam. Only used when solver=’adam’

**Attributes**

**classes_** : array or list of array of shape (n_classes,)

Class labels for each output.

**loss_** : float

The current loss computed with the loss function.

**coefs_** : list, length n_layers - 1

The ith element in the list represents the weight matrix corresponding to layer i.

**intercepts_** : list, length n_layers - 1

The ith element in the list represents the bias vector corresponding to layer i + 1.

**n_iter_** : int,
The number of iterations the solver has ran.

\texttt{n\_layers\_} : int
Number of layers.

\texttt{\textasciitilde n\_outputs\_} : int
Number of outputs.

\texttt{\textasciitilde out\_activation\_} : string
Name of the output activation function.

**Notes**

MLPClassifier trains iteratively since at each time step the partial derivatives of the loss function with respect to the model parameters are computed to update the parameters.

It can also have a regularization term added to the loss function that shrinks model parameters to prevent overfitting.

This implementation works with data represented as dense numpy arrays or sparse scipy arrays of floating point values.

**References**


**Methods**

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<td>\texttt{fit(X, y)}</td>
<td>Fit the model to data matrix X and target y.</td>
</tr>
<tr>
<td>\texttt{get_params([deep])}</td>
<td>Get parameters for this estimator.</td>
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<td>\texttt{predict(X)}</td>
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<td>\texttt{predict_log_proba(X)}</td>
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<tr>
<td>\texttt{predict_proba(X)}</td>
<td>Probability estimates.</td>
</tr>
<tr>
<td>\texttt{score(X, y[, sample_weight])}</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>\texttt{set_params(**params)}</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

\texttt{\_init\_} (hidden\_layer\_sizes=(100, ), activation=\textquotesingle relu\textquotesingle, solver=\textquotesingle adam\textquotesingle, alpha=0.0001, batch\_size=\textquotesingle auto\textquotesingle, learning\_rate=\textquotesingle constant\textquotesingle, learning\_rate\_init=0.001, power\_t=0.5, max\_iter=200, shuffle=True, random\_state=None, tol=0.0001, verbose=False, warm\_start=False, momentum=0.9, nesterovs\_momentum=True, early\_stopping=False, validation\_fraction=0.1, beta\_1=0.9, beta\_2=0.999, epsilon=1e-08)

\texttt{fit(X, y)}
Fit the model to data matrix X and target y.

- **Parameters**
  - **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
    - The input data.
  - **y**: array-like, shape (n_samples,)
    - The target values.

- **Returns**
  - **self**: returns a trained MLP model.

**get_params** *(deep=True)*
Get parameters for this estimator.

- **Parameters**
  - **deep**: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**
  - **params**: mapping of string to any
    - Parameter names mapped to their values.

**partial_fit**
Fit the model to data matrix X and target y.

- **Parameters**
  - **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
    - The input data.
  - **y**: array-like, shape (n_samples,)
    - The target values.
  - **classes**: array, shape (n_classes)
    - Classes across all calls to partial_fit. Can be obtained via `np.unique(y_all)`, where `y_all` is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in classes.

- **Returns**
  - **self**: returns a trained MLP model.

**predict** *(X)*
Predict using the multi-layer perceptron classifier.

- **Parameters**
  - **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
    - The input data.

- **Returns**
  - **y**: array-like, shape (n_samples, n_classes)
    - The predicted classes.

**predict_log_proba** *(X)*
Return the log of probability estimates.

- **Parameters**
  - **X**: array-like, shape (n_samples, n_features)
    - The input data.

- **Returns**
  - **log_y_prob**: array-like, shape (n_samples, n_classes)
    - The predicted log-probability of the sample for each class in the model, where classes are ordered as they are in `self.classes_`. Equivalent to log(predict_proba(X))
**predict_proba** *(X)*
Probability estimates.

**Parameters**
- **X** : {array-like, sparse matrix}, shape (n_samples, n_features)
  The input data.

**Returns**
- **y_prob** : array-like, shape (n_samples, n_classes)
  The predicted probability of the sample for each class in the model, where classes are ordered as they are in `self.classes_`.

**score** *(X, y, sample_weight=None)*
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**
- **X** : array-like, shape (n_samples, n_features)
  Test samples.
- **y** : array-like, shape (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight** : array-like, shape [n_samples], optional
  Sample weights.

**Returns**
- **score** : float
  Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- **self** :

**Examples using sklearn.neural_network.MLPClassifier**

- Classifier comparison
- Varying regularization in Multi-layer Perceptron
- Compare Stochastic learning strategies for MLPClassifier
- Visualization of MLP weights on MNIST
Multi-layer Perceptron regressor.

This model optimizes the squared-loss using LBFGS or stochastic gradient descent.

New in version 0.18.

Parameters:

- **hidden_layer_sizes**: tuple, length = n_layers - 2, default (100,)
  - The ith element represents the number of neurons in the ith hidden layer.

- **activation**: {'identity', 'logistic', 'tanh', 'relu'}, default 'relu'
  - Activation function for the hidden layer.
    - 'identity', no-op activation, useful to implement linear bottleneck, returns f(x) = x
    - 'logistic', the logistic sigmoid function, returns f(x) = 1 / (1 + exp(-x)).
    - 'tanh', the hyperbolic tan function, returns f(x) = tanh(x).
    - 'relu', the rectified linear unit function, returns f(x) = max(0, x)

- **solver**: {'lbfgs', 'sgd', 'adam'}, default 'adam'
  - The solver for weight optimization.
    - 'lbfgs' is an optimizer in the family of quasi-Newton methods.
    - 'sgd' refers to stochastic gradient descent.
    - 'adam' refers to a stochastic gradient-based optimizer proposed by Kingma, Diederik, and Jimmy Ba

Note: The default solver ‘adam’ works pretty well on relatively large datasets (with thousands of training samples or more) in terms of both training time and validation score. For small datasets, however, ‘lbfgs’ can converge faster and perform better.

- **alpha**: float, optional, default 0.0001
  - L2 penalty (regularization term) parameter.

- **batch_size**: int, optional, default 'auto'
  - Size of minibatches for stochastic optimizers. If the solver is ‘lbfgs’, the classifier will not use minibatch. When set to “auto”, batch_size=min(200, n_samples)

- **learning_rate**: {'constant', 'invscaling', 'adaptive'}, default 'constant'
  - Learning rate schedule for weight updates.
    - 'constant' is a constant learning rate given by ‘learning_rate_init’.
    - 'invscaling' gradually decreases the learning rate learning_rate_ at each time step 't' using an inverse scaling exponent of 'power_t'. effective_learning_rate = learning_rate_init / pow(t, power_t)
• 'adaptive' keeps the learning rate constant to 'learning_rate_init' as long as training loss keeps decreasing. Each time two consecutive epochs fail to decrease training loss by at least tol, or fail to increase validation score by at least tol if 'early_stopping' is on, the current learning rate is divided by 5.

Only used when solver='sgd'.

max_iter : int, optional, default 200

Maximum number of iterations. The solver iterates until convergence (determined by 'tol') or this number of iterations.

random_state : int or RandomState, optional, default None

State or seed for random number generator.

shuffle : bool, optional, default True

Whether to shuffle samples in each iteration. Only used when solver='sgd' or 'adam'.

tol : float, optional, default 1e-4

Tolerance for the optimization. When the loss or score is not improving by at least tol for two consecutive iterations, unless learning_rate is set to 'adaptive', convergence is considered to be reached and training stops.

learning_rate_init : double, optional, default 0.001

The initial learning rate used. It controls the step-size in updating the weights. Only used when solver='sgd' or 'adam'.

power_t : double, optional, default 0.5

The exponent for inverse scaling learning rate. It is used in updating effective learning rate when the learning_rate is set to 'invscaling'. Only used when solver='sgd'.

verbose : bool, optional, default False

Whether to print progress messages to stdout.

warm_start : bool, optional, default False

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

momentum : float, default 0.9

Momentum for gradient descent update. Should be between 0 and 1. Only used when solver='sgd'.

nesterovs_momentum : boolean, default True

Whether to use Nesterov’s momentum. Only used when solver='sgd' and momentum > 0.

early_stopping : bool, default False

Whether to use early stopping to terminate training when validation score is not improving. If set to true, it will automatically set aside 10% of training data as validation and terminate training when validation score is not improving by at least tol for two consecutive epochs. Only effective when solver='sgd' or 'adam'

validation_fraction : float, optional, default 0.1

The proportion of training data to set aside as validation set for early stopping. Must be between 0 and 1. Only used if early_stopping is True
**beta_1**: float, optional, default 0.9

Exponential decay rate for estimates of first moment vector in adam, should be in [0, 1). Only used when solver='adam'

**beta_2**: float, optional, default 0.999

Exponential decay rate for estimates of second moment vector in adam, should be in [0, 1). Only used when solver='adam'

**epsilon**: float, optional, default 1e-8

Value for numerical stability in adam. Only used when solver='adam'

**Attributes**

- **loss_**: float
  
  The current loss computed with the loss function.

- **coefs_***: list, length n_layers - 1
  
  The ith element in the list represents the weight matrix corresponding to layer i.

- **intercepts_***: list, length n_layers - 1
  
  The ith element in the list represents the bias vector corresponding to layer i + 1.

- **n_iter_***: int
  
  The number of iterations the solver has ran.

- **n_layers_**: int
  
  Number of layers.

- **n_outputs_***: int
  
  Number of outputs.

- **out_activation_***: string
  
  Name of the output activation function.

**Notes**

MLPRegressor trains iteratively since at each time step the partial derivatives of the loss function with respect to the model parameters are computed to update the parameters.

It can also have a regularization term added to the loss function that shrinks model parameters to prevent overfitting.

This implementation works with data represented as dense and sparse numpy arrays of floating point values.

**References**


Methods

**fit**\((X, y)\) Fit the model to data matrix \(X\) and target \(y\).

**get_params**\([\text{deep}]\) Get parameters for this estimator.

**predict**\((X)\) Predict using the multi-layer perceptron model.

**score**\((X, y[, \text{sample_weight}])\) Returns the coefficient of determination \(R^2\) of the prediction.

**set_params**\((**params**)\) Set the parameters of this estimator.

```python
__init__ (hidden_layer_sizes=(100, ), activation='relu', solver='adam', alpha=0.0001,
            batch_size='auto', learning_rate='constant', learning_rate_init=0.001, power_t=0.5,
            max_iter=200, shuffle=True, random_state=None, tol=0.0001, verbose=False,
            early_stopping=False, validation_fraction=0.1, beta_1=0.9, beta_2=0.999,
            epsilon=1e-08)
```

**fit** \((X, y)\)
Fit the model to data matrix \(X\) and target \(y\).

**Parameters**
- **X** : \{array-like, sparse matrix\}, shape (n_samples, n_features)
  The input data.
- **y** : array-like, shape (n_samples,)
  The target values.

**Returns**
- **self** : returns a trained MLP model.

**get_params** \((\text{deep}=\text{True})\)
Get parameters for this estimator.

**Parameters**
- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params** : mapping of string to any
  Parameter names mapped to their values.

**partial_fit**
Fit the model to data matrix \(X\) and target \(y\).

**Parameters**
- **X** : \{array-like, sparse matrix\}, shape (n_samples, n_features)
  The input data.
- **y** : array-like, shape (n_samples,)
  The target values.

**Returns**
- **self** : returns a trained MLP model.

**predict** \((X)\)
Predict using the multi-layer perceptron model.

**Parameters**
- **X** : \{array-like, sparse matrix\}, shape (n_samples, n_features)
  The input data.

**Returns**
- **y** : array-like, shape (n_samples, n_outputs)
  The predicted values.
score$(X, y, \text{sample\_weight=None})$

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y\_true - y\_pred)^2).\text{sum}()$ and $v$ is the residual sum of squares $((y\_true - y\_true.mean())^2).\text{sum}()$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- $X$ : array-like, shape = (n_samples, n_features)
  
  Test samples.

- $y$ : array-like, shape = (n_samples) or (n_samples, n_outputs)
  
  True values for $X$.

- $\text{sample\_weight}$ : array-like, shape = [n_samples], optional
  
  Sample weights.

**Returns**

- $\text{score}$ : float
  
  $R^2$ of self.predict($X$) wrt. $y$.

**set_params**(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- $\text{self}$:

## 29.27 sklearn.calibration: Probability Calibration

Calibration of predicted probabilities.

**User guide:** See the *Probability calibration* section for further details.

<table>
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<tr>
<th>calibration.CalibratedClassifierCV(...)</th>
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## 29.27.1 sklearn.calibration.CalibratedClassifierCV

class sklearn.calibration.CalibratedClassifierCV($base\_estimator=None$, $method='sigmoid', cv=3$)

Probability calibration with isotonic regression or sigmoid.

With this class, the base_classifier is fit on the train set of the cross-validation generator and the test set is used for calibration. The probabilities for each of the folds are then averaged for prediction. In case that cv=”prefit” is passed to __init__, it is assumed that base_estimator has been fitted already and all data is used for calibration. Note that data for fitting the classifier and for calibrating it must be disjoint.

Read more in the *User Guide*.

**Parameters**

- $base\_estimator$ : instance BaseEstimator
  
  The classifier whose output decision function needs to be calibrated to offer more accurate predict_proba outputs. If cv=prefit, the classifier must have been fit already on data.
**method** : ‘sigmoid’ or ‘isotonic’

The method to use for calibration. Can be ‘sigmoid’ which corresponds to Platt’s method or ‘isotonic’ which is a non-parametric approach. It is not advised to use isotonic calibration with too few calibration samples (< 1000) since it tends to overfit. Use sigmoids (Platt’s calibration) in this case.

**cv** : integer, cross-validation generator, iterable or “prefit”, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

• None, to use the default 3-fold cross-validation,
• integer, to specify the number of folds.
• An object to be used as a cross-validation generator.
• An iterable yielding train/test splits.

For integer/None inputs, if y is binary or multiclass, sklearn.model_selection.StratifiedKFold is used. If y is neither binary nor multiclass, sklearn.model_selection.KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

If “prefit” is passed, it is assumed that base_estimator has been fitted already and all data is used for calibration.

**Attributes**

`classes_` : array, shape (n_classes)

The class labels.

`calibrated_classifiers_` : list (len() equal to cv or 1 if cv == “prefit”) :

The list of calibrated classifiers, one for each crossvalidation fold, which has been fitted on all but the validation fold and calibrated on the validation fold.

**References**

[R1], [R2], [R3], [R4]

**Methods**

- `fit(X, y[, sample_weight])` Fit the calibrated model
- `get_params([deep])` Get parameters for this estimator.
- `predict(X)` Predict the target of new samples.
- `predict_proba(X)` Posterior probabilities of classification
- `score(X, y[, sample_weight])` Returns the mean accuracy on the given test data and labels.
- `set_params(**params)` Set the parameters of this estimator.

__init__(base_estimator=None, method='sigmoid', cv=3)

fit (X, y, sample_weight=None) 

Fit the calibrated model

**Parameters**

`X` : array-like, shape (n_samples, n_features)

Training data.
y : array-like, shape (n_samples,)
    Target values.

sample_weight : array-like, shape = [n_samples] or None
    Sample weights. If None, then samples are equally weighted.

Returns
self : object
    Returns an instance of self.

get_params (deep=True)
    Get parameters for this estimator.

Parameters
deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns
params : mapping of string to any
    Parameter names mapped to their values.

predict (X)
    Predict the target of new samples. Can be different from the prediction of the uncalibrated classifier.

Parameters
X : array-like, shape (n_samples, n_features)
    The samples.

Returns
C : array, shape (n_samples,)
    The predicted class.

predict_proba (X)
    Posterior probabilities of classification

This function returns posterior probabilities of classification according to each class on an array of test vectors X.

Parameters
X : array-like, shape (n_samples, n_features)
    The samples.

Returns
C : array, shape (n_samples, n_classes)
    The predicted probas.

score (X, y, sample_weight=None)
    Returns the mean accuracy on the given test data and labels.
    In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters
X : array-like, shape = (n_samples, n_features)
    Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)
    True labels for X.

sample_weight : array-like, shape = [n_samples], optional
    Sample weights.

Returns
score : float
Mean accuracy of self.predict(X) wrt. y.

**set_params**( **params**)  
Set the parameters of this estimator.  
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.  

Returns: self

Examples using `sklearn.calibration.CalibratedClassifierCV`

- Probability calibration of classifiers  
- Probability Calibration curves  
- Probability Calibration for 3-class classification

```
calibration.calibration_curve(y_true, y_prob)  
```

Compute true and predicted probabilities for a calibration curve.  

29.27.2 sklearn.calibration.calibration_curve

**sklearn.calibration.calibration_curve**( y_true, y_prob, normalize=False, n_bins=5)  

Compute true and predicted probabilities for a calibration curve.  

Read more in the User Guide.

**Parameters**

- **y_true**: array, shape (n_samples,)  
  True targets.  
- **y_prob**: array, shape (n_samples,)  
  Probabilities of the positive class.  
- **normalize** : bool, optional, default=False  
  Whether y_prob needs to be normalized into the bin [0, 1], i.e. is not a proper probability. If True, the smallest value in y_prob is mapped onto 0 and the largest one onto 1.  
- **n_bins** : int  
  Number of bins. A bigger number requires more data.  

**Returns**

- **prob_true**: array, shape (n_bins,)  
  The true probability in each bin (fraction of positives).  
- **prob_pred**: array, shape (n_bins,)  
  The mean predicted probability in each bin.

**References**

Examples using sklearn.calibration.calibration_curve

- Probability Calibration curves
- Comparison of Calibration of Classifiers

29.28 sklearn.cross_decomposition: Cross decomposition

User guide: See the Cross decomposition section for further details.

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29.28.1 sklearn.cross_decomposition.PLSRegression

class sklearn.cross_decomposition.PLSRegression(n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)

PLS regression

PLSRegression implements the PLS 2 blocks regression known as PLS2 or PLS1 in case of one dimensional response. This class inherits from _PLS with mode="A", deflation_mode="regression", norm_y_weights=False and algorithm="nipals".

Read more in the User Guide.

Parameters

- **n_components**: int, (default 2)
  Number of components to keep.
- **scale**: boolean, (default True)
  whether to scale the data
- **max_iter**: an integer, (default 500)
  the maximum number of iterations of the NIPALS inner loop (used only if algorithm="nipals")
- **tol**: non-negative real
  Tolerance used in the iterative algorithm default 1e-06.
- **copy**: boolean, default True
  Whether the deflation should be done on a copy. Let the default value to True unless you don’t care about side effect

Attributes

- **x_weights_**: array, [p, n_components]
  X block weights vectors.
- **y_weights_**: array, [q, n_components]
  Y block weights vectors.
x_loadings_ : array, [p, n_components]
X block loadings vectors.
y_loadings_ : array, [q, n_components]
Y block loadings vectors.
x_scores_ : array, [n_samples, n_components]
X scores.
y_scores_ : array, [n_samples, n_components]
Y scores.
x_rotations_ : array, [p, n_components]
X block to latents rotations.
y_rotations_ : array, [q, n_components]
Y block to latents rotations.
coef_ : array, [p, q]:
The coefficients of the linear model: \( Y = X \ coef_ + \ Err \)
n_iter_ : array-like
Number of iterations of the NIPALS inner loop for each component.

Notes
Matrices:

T: x_scores_
U: y_scores_
W: x_weights_
C: y_weights_
P: x_loadings_
Q: y_loadings_

Are computed such that:

\[
X = T P.T + \text{Err} \quad \text{and} \quad Y = U Q.T + \text{Err}
\]

\[
T[:, k] = Xk W[:, k] \quad \text{for} \quad k \ \text{in} \ \text{range}(n\text{\_components})
\]

\[
U[:, k] = Yk C[:, k] \quad \text{for} \quad k \ \text{in} \ \text{range}(n\text{\_components})
\]

\[
x\_\text{rotations}_\_ = W (P.T W)^{-1}
\]

\[
y\_\text{rotations}_\_ = C (Q.T C)^{-1}
\]

where Xk and Yk are residual matrices at iteration k.


For each component k, find weights u, v that optimizes: \( \max \ corr(Xk u, Yk v) \times \text{std}(Xk u) \text{std}(Yk u) \), such that \(|u| = 1\)

Note that it maximizes both the correlations between the scores and the intra-block variances.

The residual matrix of X (Xk+1) block is obtained by the deflation on the current X score: x_score.

The residual matrix of Y (Yk+1) block is obtained by deflation on the current X score. This performs the PLS regression known as PLS2. This mode is prediction oriented.
This implementation provides the same results that 3 PLS packages provided in the R language (R-project):

- “mixOmics” with function pls(X, Y, mode = “regression”)
- “plspm ” with function plsreg2(X, Y)
- “pls” with function oscorespls.fit(X, Y)

References


Examples

```python
>>> from sklearn.cross_decomposition import PLSRegression
>>> X = [[0., 0., 1.], [1.,0.,0.], [2.,2.,2.], [2.,5.,4.]]
>>> Y = [[0.1, -0.2], [0.9, 1.1], [6.2, 5.9], [11.9, 12.3]]
>>> pls2 = PLSRegression(n_components=2)
>>> pls2.fit(X, Y)
... PLSRegression(copy=True, max_iter=500, n_components=2, scale=True, tol=1e-06)
>>> Y_pred = pls2.predict(X)
```

Methods

- `fit(X, Y)` Fit model to data.
- `fit_transform(X[, y])` Learn and apply the dimension reduction on the train data.
- `get_params([deep])` Get parameters for this estimator.
- `predict(X[, copy])` Apply the dimension reduction learned on the train data.
- `score(X, y[, sample_weight])` Returns the coefficient of determination \( R^2 \) of the prediction.
- `set_params(**params)` Set the parameters of this estimator.
- `transform(X[, Y , copy])` Apply the dimension reduction learned on the train data.

```python
__init__ (n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)

fit (X, Y)
Fit model to data.

Parameters:

- `X` : array-like, shape = [n_samples, n_features]
  Training vectors, where n_samples in the number of samples and n_features is the number of predictors.

- `Y` : array-like of response, shape = [n_samples, n_targets]
  Target vectors, where n_samples in the number of samples and n_targets is the number of response variables.
```
fit_transform \((X, y=None, **fit_params)\)
Learn and apply the dimension reduction on the train data.

Parameters

- \(X\) : array-like of predictors, shape = \([n_samples, p]\)
  
  Training vectors, where \(n_samples\) in the number of samples and \(p\) is the number of predictors.

- \(Y\) : array-like of response, shape = \([n_samples, q]\), optional
  
  Training vectors, where \(n_samples\) in the number of samples and \(q\) is the number of response variables.

- \(copy\) : boolean, default True
  
  Whether to copy \(X\) and \(Y\), or perform in-place normalization.

Returns

- \(x_scores\) if \(Y\) is not given,
  
  \((x_scores, y_scores)\) otherwise.

get_params \((deep=True)\)
Get parameters for this estimator.

Parameters

- \(deep\) : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

- \(params\) : mapping of string to any
  
  Parameter names mapped to their values.

predict \((X, copy=True)\)
Apply the dimension reduction learned on the train data.

Parameters

- \(X\) : array-like of predictors, shape = \([n_samples, p]\)
  
  Training vectors, where \(n_samples\) in the number of samples and \(p\) is the number of predictors.

- \(copy\) : boolean, default True
  
  Whether to copy \(X\) and \(Y\), or perform in-place normalization.

Notes

This call requires the estimation of a \(p \times q\) matrix, which may be an issue in high dimensional space.

score \((X, y, sample_weight=None)\)
Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y_true - y_pred) ** 2).sum()\) and \(v\) is the residual sum of squares \(((y_true - y_true.mean()) ** 2).sum()\). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \(y\), disregarding the input features, would get a \(R^2\) score of 0.0.

Parameters

- \(X\) : array-like, shape = \((n_samples, n_features)\)
  
  Test samples.

- \(y\) : array-like, shape = \((n_samples)\) or \((n_samples, n_outputs)\)
  
  True values for \(X\).

- \(sample_weight\) : array-like, shape = \([n_samples]\), optional
Sample weights.

**Return** score : float

R^2 of self.predict(X) wrt. y.

**set_params (**)params**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Return** self :

**transform (X, Y=None, copy=True)**

Apply the dimension reduction learned on the train data.

**Parameters**

X : array-like of predictors, shape = [n_samples, p]

Training vectors, where n_samples in the number of samples and p is the number of predictors.

Y : array-like of response, shape = [n_samples, q], optional

Training vectors, where n_samples in the number of samples and q is the number of response variables.

copy : boolean, default True

Whether to copy X and Y, or perform in-place normalization.

**Returns**

x_scores if Y is not given, (x_scores, y_scores) otherwise. :

Examples using sklearn.cross_decomposition.PLSRegression

• Compare cross decomposition methods

29.28.2 sklearn.cross_decomposition.PLSCanonical

class sklearn.cross_decomposition.PLSCanonical (n_components=2, scale=True, algorithm='nipals', max_iter=500, tol=1e-06, copy=True)

PLSCanonical implements the 2 blocks canonical PLS of the original Wold algorithm [Tenenhaus 1998] p.204, referred as PLS-C2A in [Wegelin 2000].

This class inherits from PLS with mode=”A” and deflation_mode=”canonical”, norm_y_weights=True and algorithm=”nipals”, but svd should provide similar results up to numerical errors.

Read more in the User Guide.

**Parameters**

scale : boolean, scale data? (default True)

algorithm : string, “nipals” or “svd”

The algorithm used to estimate the weights. It will be called n_components times, i.e. once for each iteration of the outer loop.

max_iter : an integer, (default 500)

the maximum number of iterations of the NIPALS inner loop (used only if algorithm=”nipals”)
tol : non-negative real, default 1e-06
   the tolerance used in the iterative algorithm

copy : boolean, default True
   Whether the deflation should be done on a copy. Let the default value to True unless
   you don’t care about side effect

n_components : int, number of components to keep. (default 2).

Attributes

x_weights_ : array, shape = [p, n_components]
   X block weights vectors.

y_weights_ : array, shape = [q, n_components]
   Y block weights vectors.

x_loadings_ : array, shape = [p, n_components]
   X block loadings vectors.

y_loadings_ : array, shape = [q, n_components]
   Y block loadings vectors.

x_scores_ : array, shape = [n_samples, n_components]
   X scores.

y_scores_ : array, shape = [n_samples, n_components]
   Y scores.

x_rotations_ : array, shape = [p, n_components]
   X block to latents rotations.

y_rotations_ : array, shape = [q, n_components]
   Y block to latents rotations.

n_iter_ : array-like
   Number of iterations of the NIPALS inner loop for each component. Not useful if the
   algorithm provided is “svd”.

See also:

CCA, PLSSVD

Notes

Matrices:

T: x_scores_
U: y_scores_
W: x_weights_
C: y_weights_
P: x_loadings_
Q: y_loadings_

Are computed such that:
\[ X = TP^T + \text{Err} \quad \text{and} \quad Y = UQ^T + \text{Err} \]

\[ T[:, k] = X_k W[:, k] \quad \text{for} \quad k \in \text{range}(n_{\text{components}}) \]

\[ U[:, k] = Y_k C[:, k] \quad \text{for} \quad k \in \text{range}(n_{\text{components}}) \]

\[ x_{\text{rotations}}_ = W \quad (P^T W)^{-1} \]

\[ y_{\text{rotations}}_ = C \quad (Q^T C)^{-1} \]

where \( X_k \) and \( Y_k \) are residual matrices at iteration \( k \).


For each component \( k \), find weights \( u, v \) that optimize:

\[
\max \text{corr}(X_k u, Y_k v) \times \text{std}(X_k u) \text{std}(Y_k u), \text{such that } \|u\| = \|v\| = 1'
\]

Note that it maximizes both the correlations between the scores and the intra-block variances.

The residual matrix of \( X \) (\( X_k+1 \)) block is obtained by the deflation on the current \( X \) score: \( x_{\text{score}} \).

The residual matrix of \( Y \) (\( Y_k+1 \)) block is obtained by deflation on the current \( Y \) score. This performs a canonical symmetric version of the PLS regression. But slightly different than the CCA. This is mostly used for modeling.

This implementation provides the same results that the “plspm” package provided in the R language (R-project), using the function plsca(X, Y). Results are equal or collinear with the function pls(...) of the “mixOmics” package. The difference relies in the fact that mixOmics implementation does not exactly implement the Wold algorithm since it does not normalize \( y_{\text{weights}} \) to one.

References


Examples

```python
>>> from sklearn.cross_decomposition import PLSCanonical
>>> X = [[0., 0., 1.], [1.,0.,0.], [2.,2.,2.], [2.,5.,4.]]
>>> Y = [[0.1, -0.2], [0.9, 1.1], [6.2, 5.9], [11.9, 12.3]]
>>> plsca = PLSCanonical(n_components=2)
>>> plsca.fit(X, Y)
...
PLSCanonical(algorithm='nipals', copy=True, max_iter=500, n_components=2,
            scale=True, tol=1e-06)
>>> X_c, Y_c = plsca.transform(X, Y)
```

Methods

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<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
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<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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<td><code>transform(X[, Y, copy])</code></td>
<td>Apply the dimension reduction learned on the train data.</td>
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```python
__init__(n_components=2, scale=True, algorithm='nipals', max_iter=500, tol=1e-06, copy=True)
```

`fit(X, Y)`

Fit model to data.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training vectors, where n_samples is the number of samples and n_features is the number of predictors.
- **Y**: array-like of response, shape = [n_samples, n_targets]
  - Target vectors, where n_samples is the number of samples and n_targets is the number of response variables.

```python
fit_transform(X, y=None, **fit_params)
```

Learn and apply the dimension reduction on the train data.

**Parameters**

- **X**: array-like of predictors, shape = [n_samples, p]
  - Training vectors, where n_samples is the number of samples and p is the number of predictors.
- **Y**: array-like of response, shape = [n_samples, q], optional
  - Training vectors, where n_samples is the number of samples and q is the number of response variables.
- **copy**: boolean, default True
  - Whether to copy X and Y, or perform in-place normalization.

**Returns**

- **x_scores** if Y is not given, (**x_scores**, **y_scores**) otherwise.

```python
get_params(deep=True)
```

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

```python
predict(X, copy=True)
```

Apply the dimension reduction learned on the train data.

**Parameters**

- **X**: array-like of predictors, shape = [n_samples, p]
  - Training vectors, where n_samples is the number of samples and p is the number of predictors.
- **copy**: boolean, default True
  - Whether to copy X and Y, or perform in-place normalization.
Notes

This call requires the estimation of a p x q matrix, which may be an issue in high dimensional space.

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $\sum((y_{true} - y_{pred})^2)$ and $v$ is the residual sum of squares $\sum((y_{true} - y_{true.mean})^2)$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Returns**

- **score**: float
  $R^2$ of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**:

**transform** *(X, Y=None, copy=True)*

Apply the dimension reduction learned on the train data.

**Parameters**

- **X**: array-like of predictors, shape = [n_samples, p]
  Training vectors, where n_samples in the number of samples and p is the number of predictors.
- **Y**: array-like of response, shape = [n_samples, q], optional
  Training vectors, where n_samples in the number of samples and q is the number of response variables.
- **copy**: boolean, default True
  Whether to copy X and Y, or perform in-place normalization.

**Returns**

- **x_scores** if **Y** is not given, (**x_scores, y_scores**) otherwise.

**Examples using** **sklearn.cross_decomposition.PLSCanonical**

- Compare cross decomposition methods
29.28.3 `sklearn.cross_decomposition.CCA`

class `sklearn.cross_decomposition.CCA`(n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)

CCA Canonical Correlation Analysis.
CCA inherits from PLS with mode=”B” and deflation_mode=“canonical”.
Read more in the User Guide.

**Parameters**

- **n_components** : int, (default 2).
  number of components to keep.
- **scale** : boolean, (default True)
  whether to scale the data?
- **max_iter** : an integer, (default 500)
  the maximum number of iterations of the NIPALS inner loop
- **tol** : non-negative real, default 1e-06.
  the tolerance used in the iterative algorithm
- **copy** : boolean
  Whether the deflation be done on a copy. Let the default value to True unless you don’t care about side effects

**Attributes**

- **x_weights_** : array, [p, n_components]
  X block weights vectors.
- **y_weights_** : array, [q, n_components]
  Y block weights vectors.
- **x_loadings_** : array, [p, n_components]
  X block loadings vectors.
- **y_loadings_** : array, [q, n_components]
  Y block loadings vectors.
- **x_scores_** : array, [n_samples, n_components]
  X scores.
- **y_scores_** : array, [n_samples, n_components]
  Y scores.
- **x_rotations_** : array, [p, n_components]
  X block to latents rotations.
- **y_rotations_** : array, [q, n_components]
  Y block to latents rotations.
- **n_iter_** : array-like
  Number of iterations of the NIPALS inner loop for each component.

See also:

`PLSCanonical, PLSSVD`
Notes

For each component k, find the weights u, v that maximizes max corr(Xk u, Yk v), such that |u| = |v| = 1

Note that it maximizes only the correlations between the scores.

The residual matrix of X (Xk+1) block is obtained by the deflation on the current X score: x_score.

The residual matrix of Y (Yk+1) block is obtained by deflation on the current Y score.

References


Examples

```python
>>> from sklearn.cross_decomposition import CCA
>>> X = [[0., 0., 1.], [1.,0.,0.], [2.,2.,2.], [3.,5.,4.]]
>>> Y = [[0.1, -0.2], [0.9, 1.1], [6.2, 5.9], [11.9, 12.3]]
>>> cca = CCA(n_components=1)
>>> cca.fit(X, Y)
... CCA(copy=True, max_iter=500, n_components=1, scale=True, tol=1e-06)
>>> X_c, Y_c = cca.transform(X, Y)
```

Methods

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<td>fit(X, Y)</td>
<td>Fit model to data.</td>
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<td>get_params([deep])</td>
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<td>predict(X, copy)</td>
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</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the dimension reduction learned on the train data.</td>
</tr>
<tr>
<td>set_params(V&quot;n&quot;params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X, [Y, copy])</td>
<td>Apply the dimension reduction learned on the train data.</td>
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</table>

```python
>>> from sklearn.cross_decomposition import CCA
>>> X = [[0., 0., 1.], [1.,0.,0.], [2.,2.,2.], [3.,5.,4.]]
>>> Y = [[0.1, -0.2], [0.9, 1.1], [6.2, 5.9], [11.9, 12.3]]
>>> cca = CCA(n_components=1)
>>> cca.fit(X, Y)
... CCA(copy=True, max_iter=500, n_components=1, scale=True, tol=1e-06)
>>> X_c, Y_c = cca.transform(X, Y)
```
Target vectors, where n_samples in the number of samples and n_targets is the number of response variables.

**fit_transform** *(X, y=None, **fit_params)*
Learn and apply the dimension reduction on the train data.

- **Parameters**
  - *X*: array-like of predictors, shape = [n_samples, p]
    Training vectors, where n_samples in the number of samples and p is the number of predictors.
  - *Y*: array-like of response, shape = [n_samples, q], optional
    Training vectors, where n_samples in the number of samples and q is the number of response variables.
  - *copy*: boolean, default True
    Whether to copy X and Y, or perform in-place normalization.

- **Returns**
  - *x_scores* if *Y* is not given, *(x_scores, y_scores)* otherwise.

**get_params** *(deep=True)*
Get parameters for this estimator.

- **Parameters**
  - *deep*: boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **Returns**
  - *params*: mapping of string to any
    Parameter names mapped to their values.

**predict** *(X, copy=True)*
Apply the dimension reduction learned on the train data.

- **Parameters**
  - *X*: array-like of predictors, shape = [n_samples, p]
    Training vectors, where n_samples in the number of samples and p is the number of predictors.
  - *copy*: boolean, default True
    Whether to copy X and Y, or perform in-place normalization.

**Notes**
This call requires the estimation of a p x q matrix, which may be an issue in high dimensional space.

**score** *(X, y, sample_weight=None)*
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $1 - \frac{u}{v}$, where $u$ is the regression sum of squares ($(y_{true} - y_{pred})^2$).sum() and $v$ is the residual sum of squares ($(y_{true} - y_{true.mean})^2$).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

- **Parameters**
  - *X*: array-like, shape = (n_samples, n_features)
    Test samples.
  - *y*: array-like, shape = (n_samples) or (n_samples, n_outputs)
    True values for X.
**sample_weight** : array-like, shape = [n_samples], optional

Sample weights.

**Returnsscore** : float

R^2 of self.predict(X) wrt. y.

**set_params (**)params**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returnssel**: 

**transform (X, Y=None, copy=True)**

Apply the dimension reduction learned on the train data.

**Parameters**

- **X** : array-like of predictors, shape = [n_samples, p]
  
  Training vectors, where n_samples is the number of samples and p is the number of predictors.

- **Y** : array-like of response, shape = [n_samples, q], optional
  
  Training vectors, where n_samples is the number of samples and q is the number of response variables.

- **copy** : boolean, default True
  
  Whether to copy X and Y, or perform in-place normalization.

**Returns**

- **x_scores** if Y is not given, (x_scores, y_scores) otherwise.

**Examples using sklearn.cross_decomposition.CCA**

- Multilabel classification

  - Compare cross decomposition methods

**29.28.4 sklearn.cross_decomposition.PLSSVD**

class sklearn.cross_decomposition.PLSSVD (n_components=2, scale=True, copy=True)

Partial Least Square SVD

Simply perform a svd on the crosscovariance matrix: X'Y There are no iterative deflation here.

Read more in the User Guide.

**Parameters**

- **n_components** : int, default 2

  Number of components to keep.

- **scale** : boolean, default True

  Whether to scale X and Y.

- **copy** : boolean, default True

  Whether to copy X and Y, or perform in-place computations.

**Attributes**

- **x_weights_** : array, [p, n_components]
X block weights vectors.

y_weights_ : array, [q, n_components]
Y block weights vectors.

x_scores_ : array, [n_samples, n_components]
X scores.

y_scores_ : array, [n_samples, n_components]
Y scores.

See also:
PLSCanonical, CCA

Methods

```
fit(X, Y)
fit_transform(X[, y])
get_params([deep])
set_params(**params)
transform(X[, Y])
```

_init_ (n_components=2, scale=True, copy=True)

fit_transform (X, y=None, **fit_params)
Learn and apply the dimension reduction on the train data.

Parameters

X : array-like of predictors, shape = [n_samples, p]
Training vectors, where n_samples in the number of samples and p is the number of predictors.

Y : array-like of response, shape = [n_samples, q], optional
Training vectors, where n_samples in the number of samples and q is the number of response variables.

Returns

x_scores if Y is not given, (x_scores, y_scores) otherwise.

get_params (deep=True)
Get parameters for this estimator.

Parameters

deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

params : mapping of string to any
Parameter names mapped to their values.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.
Returns self:

**transform** *(X, Y=None)*

Apply the dimension reduction learned on the train data.

## 29.29 sklearn.pipeline: Pipeline

The `sklearn.pipeline` module implements utilities to build a composite estimator, as a chain of transforms and estimators.

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<td>Concatenates results of multiple transformer objects.</td>
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### 29.29.1 sklearn.pipeline.Pipeline

The `sklearn.pipeline.Pipeline` class

**sklearn.pipeline.Pipeline**(steps)

Pipeline of transforms with a final estimator.

Sequentially apply a list of transforms and a final estimator. Intermediate steps of the pipeline must be ‘transforms’, that is, they must implement fit and transform methods. The final estimator only needs to implement fit.

The purpose of the pipeline is to assemble several steps that can be cross-validated together while setting different parameters. For this, it enables setting parameters of the various steps using their names and the parameter name separated by a ‘__', as in the example below. A step’s estimator may be replaced entirely by setting the parameter with its name to another estimator, or a transformer removed by setting to None.


**Parameters**

**steps** : list

List of (name, transform) tuples (implementing fit/transform) that are chained, in the order in which they are chained, with the last object an estimator.

**Attributes**

**named_steps** : dict

Read-only attribute to access any step parameter by user given name. Keys are step names and values are steps parameters.

### Examples

```python
>>> from sklearn import svm
>>> from sklearn.datasets import samples_generator
>>> from sklearn.feature_selection import SelectKBest
>>> from sklearn.feature_selection import f_regression
>>> from sklearn.pipeline import Pipeline

# generate some data to play with
>>> X, y = samples_generator.make_classification(...
... n_informative=5, n_redundant=0, random_state=42)

# ANOVA SVM-C
>>>anova_filter = SelectKBest(f_regression, k=5)
>>>clf = svm.SVC(kernel='linear')
>>>anova_svm = Pipeline([('anova', anova_filter), ('svc', clf)])

# You can set the parameters using the names issued
>>>anova_filter__k = 10

# For instance, fit using a k of 10 in the SelectKBest
```
```python
>>> # and a parameter 'C' of the svm
>>> anova_svm.set_params(anova__k=10, svc__C=.1).fit(X, y)
... Pipeline(steps=[...])
>>> prediction = anova_svm.predict(X)
>>> anova_svm.score(X, y)
0.829...
>>> # getting the selected features chosen by anova_filter
>>> anova_svm.named_steps['anova'].get_support()
... array([False, False,  True,  True, False, False,  True,  True, False,
          False, False,  True, False,  True,  True, False, True, False,
          True, False], dtype=bool)
```

### Methods

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<td>Fit the model</td>
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<tr>
<td><code>fit_predict</code></td>
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<tr>
<td><code>fit_transform</code></td>
<td>Fit the model and transform with the final estimator</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>Apply transforms to the data, and predict with the final estimator</td>
</tr>
<tr>
<td><code>predict_log_proba</code></td>
<td>Apply transforms, and predict_log_proba of the final estimator</td>
</tr>
<tr>
<td><code>predict_proba</code></td>
<td>Apply transforms, and predict_proba of the final estimator</td>
</tr>
<tr>
<td><code>score</code></td>
<td>Apply transforms, and score with the final estimator</td>
</tr>
<tr>
<td><code>set_params(**kwargs)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

### `__init__(steps)`

**decision_function** (*args, **kwargs)

Apply transforms, and decision_function of the final estimator

**Parameters**

- **X** : iterable
  
  Data to predict on. Must fulfill input requirements of first step of the pipeline.

- **Returns**
  
  **y_score** : array-like, shape = [n_samples, n_classes]

**fit**(X, y=None, **fit_params)

Fit the model

Fit all the transforms one after the other and transform the data, then fit the transformed data using the final estimator.

**Parameters**

- **X** : iterable
  
  Training data. Must fulfill input requirements of first step of the pipeline.

- **y** : iterable, default=None
  
  Training targets. Must fulfill label requirements for all steps of the pipeline.
**fit_params**: dict of string -> object

Parameters passed to the fit method of each step, where each parameter name is prefixed such that parameter p for step s has key s__p.

Returns self: Pipeline

This estimator

**fit_predict** (*args, **kwargs)

Applies fit_predict of last step in pipeline after transforms.

Applies fit_transforms of a pipeline to the data, followed by the fit_predict method of the final estimator in the pipeline. Valid only if the final estimator implements fit_predict.

Parameters

**X**: iterable

Training data. Must fulfill input requirements of first step of the pipeline.

**y**: iterable, default=None

Training targets. Must fulfill label requirements for all steps of the pipeline.

**fit_params**: dict of string -> object

Parameters passed to the fit method of each step, where each parameter name is prefixed such that parameter p for step s has key s__p.

Returns y_pred: array-like

fit_transform (**X**, y=None, **fit_params)

Fit the model and transform with the final estimator

Fits all the transforms one after the other and transforms the data, then uses fit_transform on transformed data with the final estimator.

Parameters

**X**: iterable

Training data. Must fulfill input requirements of first step of the pipeline.

**y**: iterable, default=None

Training targets. Must fulfill label requirements for all steps of the pipeline.

**fit_params**: dict of string -> object

Parameters passed to the fit method of each step, where each parameter name is prefixed such that parameter p for step s has key s__p.

Returns Xt: array-like, shape = [n_samples, n_transformed_features]

Transformed samples

get_params (**deep**=True)

Get parameters for this estimator.

Parameters **deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns params: mapping of string to any

Parameter names mapped to their values.

inverse_transform

Apply inverse transformations in reverse order
All estimators in the pipeline must support `inverse_transform`.

**Parameters**

`Xt`: array-like, shape = [n_samples, n_transformed_features]

Data samples, where `n_samples` is the number of samples and `n_features` is the number of features. Must fulfill input requirements of last step of pipeline’s `inverse_transform` method.

**Returns**

`Xt`: array-like, shape = [n_samples, n_features]

**predict** (*args, **kwargs)

Apply transforms to the data, and predict with the final estimator

**Parameters**

`X`: iterable

Data to predict on. Must fulfill input requirements of first step of the pipeline.

**Returns**

`y_pred`

**predict_log_proba** (*args, **kwargs)

Apply transforms, and predict_log_proba of the final estimator

**Parameters**

`X`: iterable

Data to predict on. Must fulfill input requirements of first step of the pipeline.

**Returns**

`y_score`

**predict_proba** (*args, **kwargs)

Apply transforms, and predict_proba of the final estimator

**Parameters**

`X`: iterable

Data to predict on. Must fulfill input requirements of first step of the pipeline.

**Returns**

`y_proba`

**score** (*args, **kwargs)

Apply transforms, and score with the final estimator

**Parameters**

`X`: iterable

Data to predict on. Must fulfill input requirements of first step of the pipeline.

`y`: iterable, default=None

Targets used for scoring. Must fulfill label requirements for all steps of the pipeline.

**Returns**

`socre`

**set_params** (**kwargs)

Set the parameters of this estimator.

Valid parameter keys can be listed with `get_params()`.

**Returns**

`self`

**transform**

Apply transforms, and transform with the final estimator

This also works where final estimator is `None`: all prior transformations are applied.

**Parameters**

`X`: iterable

Data to transform. Must fulfill input requirements of first step of the pipeline.

**Returns**

`Xt`: array-like, shape = [n_samples, n_transformed_features]
Examples using `sklearn.pipeline.Pipeline`

- Concatenating multiple feature extraction methods
- Feature Union with Heterogeneous Data Sources
- Imputing missing values before building an estimator
- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Pipelining: chaining a PCA and a logistic regression
- Explicit feature map approximation for RBF kernels
- Feature agglomeration vs. univariate selection
- Sample pipeline for text feature extraction and evaluation
- Underfitting vs. Overfitting
- Restricted Boltzmann Machine features for digit classification
- SVM-Anova: SVM with univariate feature selection
- Classification of text documents using sparse features

29.29.2 `sklearn.pipeline.FeatureUnion`

class `sklearn.pipeline.FeatureUnion(transformer_list, n_jobs=1, transformer_weights=None)`

Concatenates results of multiple transformer objects.

This estimator applies a list of transformer objects in parallel to the input data, then concatenates the results. This is useful to combine several feature extraction mechanisms into a single transformer.

Parameters of the transformers may be set using its name and the parameter name separated by a ‘__’. A transformer may be replaced entirely by setting the parameter with its name to another transformer, or removed by setting to None.

Read more in the User Guide.

**Parameterstransformer_list: list of (string, transformer) tuples:**

List of transformer objects to be applied to the data. The first half of each tuple is the name of the transformer.

**n_jobs: int, optional:**

Number of jobs to run in parallel (default 1).

**transformer_weights: dict, optional:**

Multiplicative weights for features per transformer. Keys are transformer names, values the weights.

**Methods**

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<th>Description</th>
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<td><code>fit(X[, y])</code></td>
<td>Fit all transformers using X.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit all transformers, transform the data and concatenate results.</td>
</tr>
<tr>
<td><code>get_feature_names()</code></td>
<td>Get feature names from all transformers.</td>
</tr>
</tbody>
</table>
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<tr>
<th>Method</th>
<th>Description</th>
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<td>get_params(deep)</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>set_params(**kwargs)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform X separately by each transformer, concatenate results.</td>
</tr>
</tbody>
</table>

__init__(transformer_list, n_jobs=1, transformer_weights=None)

fit(X, y=None)
Fit all transformers using X.

Parameters:

- X : iterable or array-like, depending on transformers
  Input data, used to fit transformers.
- y : array-like, shape (n_samples, ...), optional
  Targets for supervised learning.

Returns:

- self : FeatureUnion
  This estimator

fit_transform(X, y=None, **fit_params)
Fit all transformers, transform the data and concatenate results.

Parameters:

- X : iterable or array-like, depending on transformers
  Input data to be transformed.
- y : array-like, shape (n_samples, ...), optional
  Targets for supervised learning.

Returns:

- X_t : array-like or sparse matrix, shape (n_samples, sum_n_components)
  hstack of results of transformers. sum_n_components is the sum of n_components (output dimension) over transformers.

get_feature_names()
Get feature names from all transformers.

Returns:

- feature_names : list of strings
  Names of the features produced by transform.

get_params(deep=True)
Get parameters for this estimator.

Parameters:

- deep: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

- params : mapping of string to any
  Parameter names mapped to their values.

set_params(**kwargs)
Set the parameters of this estimator.

Valid parameter keys can be listed with get_params().

Returns:

- self :
transform \((X)\)
Transform \(X\) separately by each transformer, concatenate results.

**Parameters**

\(X\) : iterable or array-like, depending on transformers
Input data to be transformed.

**Returns**

\(X_t\) : array-like or sparse matrix, shape \((n\_samples, \sum\_n\_components)\)
htstack of results of transformers. \(\sum\_n\_components\) is the sum of \(n\_components\) (output dimension) over transformers.

Examples using sklearn.pipeline.FeatureUnion

- Concatenating multiple feature extraction methods
- Feature Union with Heterogeneous Data Sources

pipeline.make_pipeline(*steps)  
Construct a Pipeline from the given estimators.

pipeline.make_union(*transformers)  
Construct a FeatureUnion from the given transformers.

29.29.3 sklearn.pipeline.make_pipeline

sklearn.pipeline.make_pipeline(*steps)  
Construct a Pipeline from the given estimators.

This is a shorthand for the Pipeline constructor; it does not require, and does not permit, naming the estimators. Instead, their names will be set to the lowercase of their types automatically.

**Returns**

\(p\) : Pipeline

Examples

```python
>>> from sklearn.naive_bayes import GaussianNB
>>> from sklearn.preprocessing import StandardScaler
>>> make_pipeline(StandardScaler(), GaussianNB(priors=None))
...  Pipeline(steps=[('standardscaler', StandardScaler(copy=True, with_mean=True, with_std=True)), ('gaussiannb', GaussianNB(priors=None))])
```

Examples using sklearn.pipeline.make_pipeline

- Feature transformations with ensembles of trees
- Pipeline Anova SVM
- Polynomial interpolation
- Robust linear estimator fitting
- Using FunctionTransformer to select columns
- Clustering text documents using k-means
29.29.4 sklearn.pipeline.make_union

sklearn.pipeline.make_union(*transformers)
Construct a FeatureUnion from the given transformers.

This is a shorthand for the FeatureUnion constructor; it does not require, and does not permit, naming the transformers. Instead, they will be given names automatically based on their types. It also does not allow weighting.

Returns: FeatureUnion

Examples

```python
>>> from sklearn.decomposition import PCA, TruncatedSVD
>>> make_union(PCA(), TruncatedSVD())
FeatureUnion(n_jobs=1,
    transformer_list=[('pca',
        PCA(copy=True, iterated_power='auto',
            n_components=None, random_state=None,
            svd_solver='auto', tol=0.0, whiten=False)),
        ('truncatedsvd',
            TruncatedSVD(algorithm='randomized',
            n_components=2, n_iter=5,
            random_state=None, tol=0.0))],
    transformer_weights=None)
```

29.30 sklearn.preprocessing: Preprocessing and Normalization

The sklearn.preprocessing module includes scaling, centering, normalization, binarization and imputation methods.

User guide: See the Preprocessing data section for further details.

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<td>Binarize data (set feature values to 0 or 1) according to a threshold</td>
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<td>Constructs a transformer from an arbitrary callable.</td>
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<td>Generate polynomial and interaction features.</td>
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<th>Preprocessing Class</th>
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<td>preprocessing.RobustScaler([with_centering, ...])</td>
<td>Scale features using statistics that are robust to outliers.</td>
</tr>
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<td>preprocessing.StandardScaler([copy, ...])</td>
<td>Standardize features by removing the mean and scaling to unit variance</td>
</tr>
</tbody>
</table>

29.30.1 sklearn.preprocessing.Binarizer

class sklearn.preprocessing.Binarizer (threshold=0.0, copy=True)

Binarize data (set feature values to 0 or 1) according to a threshold

Values greater than the threshold map to 1, while values less than or equal to the threshold map to 0. With the default threshold of 0, only positive values map to 1.

Binarization is a common operation on text count data where the analyst can decide to only consider the presence or absence of a feature rather than a quantified number of occurrences for instance.

It can also be used as a pre-processing step for estimators that consider boolean random variables (e.g. modelled using the Bernoulli distribution in a Bayesian setting).

Read more in the User Guide.

Parametersthreshold : float, optional (0.0 by default)

Feature values below or equal to this are replaced by 0, above it by 1. Threshold may not be less than 0 for operations on sparse matrices.

copy : boolean, optional, default True

set to False to perform inplace binarization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix).

See also:

binarize Equivalent function without the object oriented API.

Notes

If the input is a sparse matrix, only the non-zero values are subject to update by the Binarizer class.

This estimator is stateless (besides constructor parameters), the fit method does nothing but is useful when used in a pipeline.

Methods

<table>
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<tr>
<th>Method</th>
<th>Description</th>
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<td>fit(X[, y])</td>
<td>Do nothing and return the estimator unchanged</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X[, y, copy])</td>
<td>Binarize each element of X</td>
</tr>
</tbody>
</table>

__init__ (threshold=0.0, copy=True)

fit (X, y=None)

Do nothing and return the estimator unchanged
This method is just there to implement the usual API and hence work in pipelines.

**fit_transform** \((X, y=None, **fit_params)\)

Fit to data, then transform it.

Fits transformer to \(X\) and \(y\) with optional parameters \(fit\_params\) and returns a transformed version of \(X\).

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  
  Training set.

- **y**: numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**get_params** \((deep=True)\)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  
  Parameter names mapped to their values.

**set_params** \(**params\)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \(<component>__<parameter>\) so that it’s possible to update each component of a nested object.

**Returns**

- **self**

**transform** \((X, y=None, copy=None)\)

Binarize each element of \(X\)

**Parameters**

- **X**: {array-like, sparse matrix}, shape [n_samples, n_features]
  
  The data to binarize, element by element. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.

---

### 29.30.2 sklearn.preprocessing.FunctionTransformer

**class sklearn.preprocessing.FunctionTransformer** \((func=None, inverse_func=None, validate=True, accept_sparse=False, pass_y=False, kw_args=None, inv_kw_args=None)\)

Constructs a transformer from an arbitrary callable.

A FunctionTransformer forwards its \(X\) (and optionally \(y\)) arguments to a user-defined function or function object and returns the result of this function. This is useful for stateless transformations such as taking the log of frequencies, doing custom scaling, etc.

A FunctionTransformer will not do any checks on its function’s output.

Note: If a lambda is used as the function, then the resulting transformer will not be pickleable.

New in version 0.17.
Parameters

**func** : callable, optional default=None

The callable to use for the transformation. This will be passed the same arguments as transform, with args and kwargs forwarded. If func is None, then func will be the identity function.

**inverse_func** : callable, optional default=None

The callable to use for the inverse transformation. This will be passed the same arguments as inverse_transform, with args and kwargs forwarded. If inverse_func is None, then inverse_func will be the identity function.

**validate** : bool, optional default=True

Indicate that the input X array should be checked before calling func. If validate is false, there will be no input validation. If it is true, then X will be converted to a 2-dimensional NumPy array or sparse matrix. If this conversion is not possible or X contains NaN or infinity, an exception is raised.

**accept_sparse** : boolean, optional

Indicate that func accepts a sparse matrix as input. If validate is False, this has no effect. Otherwise, if accept_sparse is False, sparse matrix inputs will cause an exception to be raised.

**pass_y** : bool, optional default=False

Indicate that transform should forward the y argument to the inner callable.

**kw_args** : dict, optional

Dictionary of additional keyword arguments to pass to func.

**inv_kw_args** : dict, optional

Dictionary of additional keyword arguments to pass to inverse_func.

Methods

```python
fit(X[, y])
fit_transform(X[, y])
get_params([deep])
inverse_transform(X[, y])
set_params(**params)
transform(X[, y])
```

**init** *(func=None, inverse_func=None, validate=True, accept_sparse=False, pass_y=False, kw_args=None, inv_kw_args=None)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

**X** : numpy array of shape [n_samples, n_features]

Training set.

**y** : numpy array of shape [n_samples]
Target values.

Returns:

$X_{new}$ : numpy array of shape [n_samples, n_features_new]

Transformed array.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

params : mapping of string to any
Parameter names mapped to their values.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns:

self

Examples using sklearn.preprocessing.FunctionTransformer

- Using FunctionTransformer to select columns

29.30.3 sklearn.preprocessing.Imputer

class sklearn.preprocessing.Imputer (missing_values='NaN', strategy='mean', axis=0, verbose=0, copy=True)

Imputation transformer for completing missing values.

Read more in the User Guide.

Parameters:

missing_values : integer or “NaN”, optional (default=”NaN”)

The placeholder for the missing values. All occurrences of missing_values will be imputed. For missing values encoded as np.nan, use the string value “NaN”.

strategy : string, optional (default=”mean”)

The imputation strategy.

- If “mean”, then replace missing values using the mean along the axis.
- If “median”, then replace missing values using the median along the axis.
- If “most_frequent”, then replace missing using the most frequent value along the axis.

axis : integer, optional (default=0)

The axis along which to impute.

- If axis=0, then impute along columns.
- If axis=1, then impute along rows.

verbose : integer, optional (default=0)

Controls the verbosity of the imputer.
copy : boolean, optional (default=True)

If True, a copy of X will be created. If False, imputation will be done in-place whenever possible. Note that, in the following cases, a new copy will always be made, even if copy=False:

• If X is not an array of floating values;
• If X is sparse and missing_values=0;
• If axis=0 and X is encoded as a CSR matrix;
• If axis=1 and X is encoded as a CSC matrix.

Attributes

statistics_ : array of shape (n_features,

The imputation fill value for each feature if axis == 0.

Notes

• When axis=0, columns which only contained missing values at fit are discarded upon transform.
• When axis=1, an exception is raised if there are rows for which it is not possible to fill in the missing values (e.g., because they only contain missing values).

Methods

fit(X[, y]) Fit the imputer on X.

fit_transform(X[, y]) Fit to data, then transform it.

get_params([deep]) Get parameters for this estimator.

set_params(**params) Set the parameters of this estimator.

transform(X) Impute all missing values in X.

__init__ (missing_values='NaN', strategy='mean', axis=0, verbose=0, copy=True)

fit (X, y=None)

Fit the imputer on X.

Parameters

X : {array-like, sparse matrix}, shape (n_samples, n_features)

Input data, where n_samples is the number of samples and n_features is the number of features.

Returns

self : object

Returns self.

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

X : numpy array of shape [n_samples, n_features]

Training set.

y : numpy array of shape [n_samples]

Target values.
Returns X_new : numpy array of shape [n_samples, n_features_new]

Transformed array.

get_params (deep=True)

Get parameters for this estimator.

Parameters:

deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

params : mapping of string to any
    Parameter names mapped to their values.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:

self

transform (X)

Impute all missing values in X.

Parameters:

X : {array-like, sparse matrix}, shape = [n_samples, n_features]
    The input data to complete.

Examples using sklearn.preprocessing.Imputer

• Imputing missing values before building an estimator

29.30.4 sklearn.preprocessing.KernelCenterer

class sklearn.preprocessing.KernelCenterer

Center a kernel matrix

Let K(x, z) be a kernel defined by phi(x)^T phi(z), where phi is a function mapping x to a Hilbert space. KernelCenterer centers (i.e., normalize to have zero mean) the data without explicitly computing phi(x). It is equivalent to centering phi(x) with sklearn.preprocessing.StandardScaler(with_std=False).

Read more in the User Guide.

Methods

<table>
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<td>Fit KernelCenterer</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(K[, y, copy])</td>
<td>Center kernel matrix.</td>
</tr>
</tbody>
</table>

__init__()  
x.__init__(...) initializes x; see help(type(x)) for signature
**fit** *(K, y=None)*  
Fit KernelCenterer  

**Parameters**

- **K**: numpy array of shape [n_samples, n_samples]  
  Kernel matrix.

**Returns**

- **self**: returns an instance of self.

**fit_transform** *(X, y=None, **fit_params)*  
Fit to data, then transform it.  
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]  
  Training set.
- **y**: numpy array of shape [n_samples]  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]  
  Transformed array.

**get_params** *(deep=True)*  
Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any  
  Parameter names mapped to their values.

**set_params** *(**params)*  
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**transform** *(K, y=None, copy=True)*  
Center kernel matrix.

**Parameters**

- **K**: numpy array of shape [n_samples1, n_samples2]  
  Kernel matrix.
- **copy**: boolean, optional, default True  
  Set to False to perform inplace computation.

**Returns**

- **K_new**: numpy array of shape [n_samples1, n_samples2]

### 29.30.5 sklearn.preprocessing.LabelBinarizer

**class** `sklearn.preprocessing.LabelBinarizer(neg_label=0, pos_label=1, sparse_output=False)`  
Binarize labels in a one-vs-all fashion

29.30. sklearn.preprocessing: Preprocessing and Normalization 1815
Several regression and binary classification algorithms are available in the scikit. A simple way to extend these algorithms to the multi-class classification case is to use the so-called one-vs-all scheme.

At learning time, this simply consists in learning one regressor or binary classifier per class. In doing so, one needs to convert multi-class labels to binary labels (belong or does not belong to the class). LabelBinarizer makes this process easy with the transform method.

At prediction time, one assigns the class for which the corresponding model gave the greatest confidence. LabelBinarizer makes this easy with the inverse_transform method.

Read more in the User Guide.

**Parameters**

- `neg_label` : int (default: 0)
  
  Value with which negative labels must be encoded.

- `pos_label` : int (default: 1)
  
  Value with which positive labels must be encoded.

- `sparse_output` : boolean (default: False)
  
  True if the returned array from transform is desired to be in sparse CSR format.

**Attributes**

- `classes_` : array of shape [n_class]
  
  Holds the label for each class.

- `y_type_` : str,
  

- `sparse_input_` : boolean,
  
  True if the input data to transform is given as a sparse matrix, False otherwise.

**See also:**

- `label_binarize` function to perform the transform operation of LabelBinarizer with fixed classes.
- `sklearn.preprocessing.OneHotEncoder` encode categorical integer features using a one-hot aka one-of-K scheme.

**Examples**

```python
>>> from sklearn import preprocessing
>>> lb = preprocessing.LabelBinarizer()
>>> lb.fit([1, 2, 6, 4, 2])
LabelBinarizer(neg_label=0, pos_label=1, sparse_output=False)
>>> lb.classes_
array([1, 2, 4, 6])
>>> lb.transform([1, 6])
array([[1, 0, 0, 0],
       [0, 0, 0, 1]])
```

Binary targets transform to a column vector

```python
>>> lb = preprocessing.LabelBinarizer()
>>> lb.fit_transform(['yes', 'no', 'no', 'yes'])
array([[1],
       [1],
       [1],
       [1]])
```
Passing a 2D matrix for multilabel classification

```python
>>> import numpy as np
>>> lb.fit(np.array([[0, 1, 1], [1, 0, 0]]))
LabelBinarizer(neg_label=0, pos_label=1, sparse_output=False)
>>> lb.classes_
array([0, 1, 2])
>>> lb.transform([0, 1, 2, 1])
array([[1, 0, 0],
       [0, 1, 0],
       [0, 0, 1],
       [0, 1, 0]])
```

### Methods

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<td>fit(y)</td>
<td>Fit label binarizer</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>inverse_transform(Y[, threshold])</td>
<td>Transform binary labels back to multi-class labels</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(y)</td>
<td>Transform multi-class labels to binary labels.</td>
</tr>
</tbody>
</table>

#### __init__ (neg_label=0, pos_label=1, sparse_output=False)

**fit** (y)

Fit label binarizer

**Parameters**

- **y**: numpy array of shape (n_samples,) or (n_samples, n_classes)
  
  Target values. The 2-d matrix should only contain 0 and 1, represents multilabel classification.

**Returns**

- **self**: returns an instance of self.

**fit_transform** (X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  
  Training set.

- **y**: numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

#### get_params (deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns: params : mapping of string to any
Parameter names mapped to their values.

inverse_transform (Y, threshold=None)
Transform binary labels back to multi-class labels

Parameters:
Y : numpy array or sparse matrix with shape [n_samples, n_classes]
Target values. All sparse matrices are converted to CSR before inverse transformation.
threshold : float or None
Threshold used in the binary and multi-label cases.

Use 0 when:
• Y contains the output of decision_function (classifier)
Use 0.5 when:
• Y contains the output of predict_proba
If None, the threshold is assumed to be half way between neg_label and pos_label.

Returns:
y : numpy array or CSR matrix of shape [n_samples] Target values.

Notes
In the case when the binary labels are fractional (probabilistic), inverse_transform chooses the class with the greatest value. Typically, this allows to use the output of a linear model’s decision_function method directly as the input of inverse_transform.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns: self

transform (y)
Transform multi-class labels to binary labels

The output of transform is sometimes referred to by some authors as the 1-of-K coding scheme.

Parameters:
y : numpy array or sparse matrix of shape (n_samples,) or
(n_samples, n_classes) Target values. The 2-d matrix should only contain 0 and 1, represents multilabel classification. Sparse matrix can be CSR, CSC, COO, DOK, or LIL.

Returns:
Y : numpy array or CSR matrix of shape [n_samples, n_classes]
Shape will be [n_samples, 1] for binary problems.
29.30.6 sklearn.preprocessing.LabelEncoder

**Class sklearn.preprocessing.LabelEncoder**

Encode labels with value between 0 and n_classes-1.

Read more in the User Guide.

**Attributes**

- **classes_**: array of shape (n_class,)
  Holds the label for each class.

**See also:**

- `sklearn.preprocessing.OneHotEncoder` encode categorical integer features using a one-hot aka one-of-K scheme.

**Examples**

`LabelEncoder` can be used to normalize labels.

```python
>>> from sklearn import preprocessing
>>> le = preprocessing.LabelEncoder()
>>> le.fit([1, 2, 2, 6])
LabelEncoder()
>>> le.classes_
array([1, 2, 6])
>>> le.transform([1, 1, 2, 6])
array([0, 0, 1, 2])
>>> le.inverse_transform([0, 0, 1, 2])
array([1, 1, 2, 6])
```

It can also be used to transform non-numerical labels (as long as they are hashable and comparable) to numerical labels.

```python
>>> le = preprocessing.LabelEncoder()
>>> le.fit(["paris", "paris", "tokyo", "amsterdam"])
LabelEncoder()
>>> list(le.classes_)
['amsterdam', 'paris', 'tokyo']
>>> le.transform(["tokyo", "tokyo", "paris"])
array([2, 2, 1])
>>> list(le.inverse_transform([2, 2, 1]))
['tokyo', 'tokyo', 'paris']
```

**Methods**

<table>
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<th>Description</th>
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<td><strong>fit(y)</strong></td>
<td>Fit label encoder</td>
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<tr>
<td><strong>fit_transform(y)</strong></td>
<td>Fit label encoder and return encoded labels</td>
</tr>
<tr>
<td><strong>get_params</strong></td>
<td>Get parameters for this estimator.</td>
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<td><strong>inverse_transform(y)</strong></td>
<td>Transform labels back to original encoding.</td>
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<tr>
<td><strong>set_params</strong></td>
<td>Set the parameters of this estimator.</td>
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<tr>
<td><strong>transform(y)</strong></td>
<td>Transform labels to normalized encoding.</td>
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</table>

29.30. sklearn.preprocessing: Preprocessing and Normalization
__init__()
    x.__init__(...) initializes x; see help(type(x)) for signature

fit (y)
    Fit label encoder
    Parameters
    y : array-like of shape (n_samples,)
        Target values.
    Returns
    self : returns an instance of self.

fit_transform (y)
    Fit label encoder and return encoded labels
    Parameters
    y : array-like of shape [n_samples]
        Target values.
    Returns
    y : array-like of shape [n_samples]

get_params (deep=True)
    Get parameters for this estimator.
    Parameters
    deep : boolean, optional
        If True, will return the parameters for this estimator and contained subobjects that are estimators.
    Returns
    params : mapping of string to any
        Parameter names mapped to their values.

inverse_transform (y)
    Transform labels back to original encoding.
    Parameters
    y : numpy array of shape [n_samples]
        Target values.
    Returns
    y : numpy array of shape [n_samples]

set_params (**params)
    Set the parameters of this estimator.
    The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.
    Returns
    self :

transform (y)
    Transform labels to normalized encoding.
    Parameters
    y : array-like of shape [n_samples]
        Target values.
    Returns
    y : array-like of shape [n_samples]

29.30.7 sklearn.preprocessing.MultiLabelBinarizer

class sklearn.preprocessing.MultiLabelBinarizer (classes=None, sparse_output=False)
    Transform between iterable of iterables and a multilabel format
Although a list of sets or tuples is a very intuitive format for multilabel data, it is unwieldy to process. This transformer converts between this intuitive format and the supported multilabel format: a (samples x classes) binary matrix indicating the presence of a class label.

**Parameters**

- **classes**: array-like of shape [n_classes] (optional)
  
  Indicates an ordering for the class labels

- **sparse_output**: boolean (default: False).
  
  Set to true if output binary array is desired in CSR sparse format

**Attributes**

- **classes_**: array of labels

  A copy of the classes parameter where provided, or otherwise, the sorted set of classes found when fitting.

See also:

**sklearn.preprocessing.OneHotEncoder**

encode categorical integer features using a one-hot aka one-of-K scheme.

**Examples**

```python
>>> from sklearn.preprocessing import MultiLabelBinarizer
>>> mlb = MultiLabelBinarizer()
>>> mlb.fit_transform(((1, 2), (3,)))
array([[1, 1, 0],
       [0, 0, 1]])
>>> mlb.classes_
array([1, 2, 3])
```

```python
>>> mlb.fit_transform([set(['sci-fi', 'thriller']), set(['comedy'])])
array([[0, 1, 1],
       [1, 0, 0]])
```

```python
list(mlb.classes_)
['comedy', 'sci-fi', 'thriller']
```

**Methods**

- **fit**(y)
  
  Fit the label sets binarizer, storing classes_

- **fit_transform**(y)
  
  Fit the label sets binarizer and transform the given label sets

- **get_params**(deep)
  
  Get parameters for this estimator.

- **inverse_transform**(yt)
  
  Transform the given indicator matrix into label sets

- **set_params**(**\*\*params)
  
  Set the parameters of this estimator.

- **transform**(y)
  
  Transform the given label sets

**__init__**(classes=None, sparse_output=False)

**fit**(y)

Fit the label sets binarizer, storing classes_

**Parameters**

- **y**: iterable of iterables

29.30. sklearn.preprocessing: Preprocessing and Normalization
A set of labels (any orderable and hashable object) for each sample. If the classes parameter is set, y will not be iterated.

Returns: returns this MultiLabelBinarizer instance

**fit_transform** *(y)*
Fit the label sets binarizer and transform the given label sets

Parameters:
- y: iterable of iterables
  A set of labels (any orderable and hashable object) for each sample. If the classes parameter is set, y will not be iterated.

Returns:
- y_indicator: array or CSR matrix, shape (n_samples, n_classes)
  A matrix such that y_indicator[i, j] = 1 iff classes[j] is in y[i], and 0 otherwise.

**get_params** *(deep=True)*
Get parameters for this estimator.

Parameters:
- deep: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params: mapping of string to any
  Parameter names mapped to their values.

**inverse_transform** *(yt)*
Transform the given indicator matrix into label sets

Parameters:
- yt: array or sparse matrix of shape (n_samples, n_classes)
  A matrix containing only 1s and 0s.

Returns:
- y: list of tuples
  The set of labels for each sample such that y[i] consists of classes[j] for each yt[i, j] == 1.

**set_params** **(**params**)**
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>_<parameter>` so that it's possible to update each component of a nested object.

Returns: self

**transform** *(y)*
Transform the given label sets

Parameters:
- y: iterable of iterables
  A set of labels (any orderable and hashable object) for each sample. If the classes parameter is set, y will not be iterated.

Returns:
- y_indicator: array or CSR matrix, shape (n_samples, n_classes)
  A matrix such that y_indicator[i, j] = 1 iff classes[j] is in y[i], and 0 otherwise.
29.30.8 sklearn.preprocessing.MaxAbsScaler

**class sklearn.preprocessing.MaxAbsScaler (copy=True)**

Scale each feature by its maximum absolute value.

This estimator scales and translates each feature individually such that the maximal absolute value of each feature in the training set will be 1.0. It does not shift/center the data, and thus does not destroy any sparsity.

This scaler can also be applied to sparse CSR or CSC matrices.

New in version 0.17.

**Parameters**

- **copy** : boolean, optional, default is True
  Set to False to perform inplace scaling and avoid a copy (if the input is already a numpy array).

**Attributes**

- **scale_** : ndarray, shape (n_features,)
  Per feature relative scaling of the data.
  New in version 0.17: scale_ attribute.

- **max_abs_** : ndarray, shape (n_features,)
  Per feature maximum absolute value.

- **n_samples_seen_** : int
  The number of samples processed by the estimator. Will be reset on new calls to fit, but increments across partial_fit calls.

**See also:**

- maxabs_scale Equivalent function without the object oriented API.

**Methods**

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<tr>
<td>fit (X, y=None)</td>
<td>Compute the maximum absolute value to be used for later scaling.</td>
</tr>
<tr>
<td>fit_transform (X, y)</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params (deep)</td>
<td>Get parameters for this estimator.</td>
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<tr>
<td>inverse_transform (X)</td>
<td>Scale back the data to the original representation</td>
</tr>
<tr>
<td>partial_fit (X, y)</td>
<td>Online computation of max absolute value of X for later scaling.</td>
</tr>
<tr>
<td>set_params (X=**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform (X, y)</td>
<td>Scale the data</td>
</tr>
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## Examples

**fit**

```python
sklearn.preprocessing.MaxAbsScaler(copy=True).fit(X)
```

**fit_transform**

```python
sklearn.preprocessing.MaxAbsScaler(copy=True).fit_transform(X)
```

**__init__**

```python
sklearn.preprocessing.MaxAbsScaler(copy=True)
```

**fit**

```python
sklearn.preprocessing.MaxAbsScaler(copy=True).fit(X)
```

**fit_transform**

```python
sklearn.preprocessing.MaxAbsScaler(copy=True).fit_transform(X)
```
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X** : numpy array of shape [n_samples, n_features]
  
  Training set.

- **y** : numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new** : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**get_params** (*deep=True*)

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any

  Parameter names mapped to their values.

**inverse_transform(X)**

Scale back the data to the original representation

**Parameters**

- **X** : {array-like, sparse matrix}

  The data that should be transformed back.

**partial_fit(X, y=None)**

Online computation of max absolute value of X for later scaling. All of X is processed as a single batch. This is intended for cases when fit is not feasible due to very large number of n_samples or because X is read from a continuous stream.

**Parameters**

- **X** : {array-like, sparse matrix}, shape [n_samples, n_features]

  The data used to compute the mean and standard deviation used for later scaling along the features axis.

- **y** : Passthrough for “Pipeline“ compatibility.

**set_params(**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

- **self** :

**transform(X, y=None)**

Scale the data

**Parameters**

- **X** : {array-like, sparse matrix}

  The data that should be scaled.

### 29.30.9 sklearn.preprocessing.MinMaxScaler

class sklearn.preprocessing.MinMaxScaler (*feature_range=(0, 1), copy=True*)

Transforms features by scaling each feature to a given range.
This estimator scales and translates each feature individually such that it is in the given range on the training set, i.e. between zero and one.

The transformation is given by:

\[
X_{\text{std}} = (X - X_{\text{min}}(\text{axis}=0)) / (X_{\text{max}}(\text{axis}=0) - X_{\text{min}}(\text{axis}=0))
\]

\[
X_{\text{scaled}} = X_{\text{std}} \times (\text{max} - \text{min}) + \text{min}
\]

where min, max = feature_range.

This transformation is often used as an alternative to zero mean, unit variance scaling.

Read more in the User Guide.

**Parameters**

- **feature_range**: tuple (min, max), default=(0, 1)
  Desired range of transformed data.

- **copy**: boolean, optional, default True
  Set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array).

**Attributes**

- **min_**: ndarray, shape (n_features,)
  Per feature adjustment for minimum.

- **scale_**: ndarray, shape (n_features,)
  Per feature relative scaling of the data.
  New in version 0.17: scale_ attribute.

- **data_min_**: ndarray, shape (n_features,)
  Per feature minimum seen in the data
  New in version 0.17: data_min_ instead of deprecated data_min.

- **data_max_**: ndarray, shape (n_features,)
  Per feature maximum seen in the data
  New in version 0.17: data_max_ instead of deprecated data_max.

- **data_range_**: ndarray, shape (n_features,)
  Per feature range (data_max_ - data_min_) seen in the data
  New in version 0.17: data_range_ instead of deprecated data_range.

**See also:**

- **minmax_scale**: Equivalent function without the object oriented API.

**Methods**

- **fit(X[, y])** Compute the minimum and maximum to be used for later scaling.
- **fit_transform(X[, y])** Fit to data, then transform it.
- **get_params**(deep) Get parameters for this estimator.
- **inverse_transform(X)** Undo the scaling of X according to feature_range.
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<td>partial_fit(X[, y])</td>
<td>Online computation of min and max on X for later scaling.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Scaling features of X according to feature_range.</td>
</tr>
</tbody>
</table>

__init__(feature_range=(0, 1), copy=True)

data_min
DEPRECATED: Attribute data_min will be removed in 0.19. Use data_min_ instead

data_range
DEPRECATED: Attribute data_range will be removed in 0.19. Use data_range_ instead

fit (X, y=None)
Compute the minimum and maximum to be used for later scaling.

Parameters

- **X**: array-like, shape [n_samples, n_features]
  The data used to compute the per-feature minimum and maximum used for later scaling along the features axis.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

- **X**: numpy array of shape [n_samples, n_features]
  Training set.
- **y**: numpy array of shape [n_samples]
  Target values.

Returns

- **X_new**: numpy array of shape [n_samples, n_features_new]
  Transformed array.

get_params (deep=True)
Get parameters for this estimator.

Parameters

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

- **params**: mapping of string to any
  Parameter names mapped to their values.

inverse_transform (X)
Undo the scaling of X according to feature_range.

Parameters

- **X**: array-like, shape [n_samples, n_features]
  Input data that will be transformed. It cannot be sparse.

partial_fit (X, y=None)
Online computation of min and max on X for later scaling. All of X is processed as a single batch. This is intended for cases when fit is not feasible due to very large number of n_samples or because X is read from a continuous stream.

Parameters

- **X**: array-like, shape [n_samples, n_features]
The data used to compute the mean and standard deviation used for later scaling along the features axis.

\[ y \]: Passthrough for `Pipeline` compatibility.

`set_params(**params)`
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns `self`:

`transform(X)`
Scaling features of \(X\) according to feature_range.

Parameters:

\[ X \]: array-like, shape [n_samples, n_features]
Input data that will be transformed.

Examples using `sklearn.preprocessing.MinMaxScaler`

- Compare Stochastic learning strategies for `MLPClassifier`

### 29.30.10 `sklearn.preprocessing.Normalizer`

```python
class sklearn.preprocessing.Normalizer(norm='l2', copy=True)
```

Normalizes samples individually to unit norm.

Each sample (i.e. each row of the data matrix) with at least one non zero component is rescaled independently of other samples so that its norm (11 or 12) equals one.

This transformer is able to work both with dense numpy arrays and scipy.sparse matrix (use CSR format if you want to avoid the burden of a copy / conversion).

Scaling inputs to unit norms is a common operation for text classification or clustering for instance. For instance the dot product of two l2-normalized TF-IDF vectors is the cosine similarity of the vectors and is the base similarity metric for the Vector Space Model commonly used by the Information Retrieval community.

Read more in the `User Guide`.

Parameters:

\[ \text{norm} \]: ‘l1’, ‘l2’, or ‘max’, optional (‘l2’ by default)
The norm to use to normalize each non zero sample.

\[ \text{copy} \]: boolean, optional, default True
set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix).

See also:

- `normalize`: Equivalent function without the object oriented API.

Notes

This estimator is stateless (besides constructor parameters), the fit method does nothing but is useful when used in a pipeline.
## Methods

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<td><code>fit(X[, y])</code></td>
<td>Do nothing and return the estimator unchanged</td>
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<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
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<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(X[, y, copy])</code></td>
<td>Scale each non zero row of X to unit norm</td>
</tr>
</tbody>
</table>

```python
__init__(norm='l2', copy=True)
```

```python
fit (X, y=None)
```

Do nothing and return the estimator unchanged

This method is just there to implement the usual API and hence work in pipelines.

```python
fit_transform (X, y=None, **fit_params)
```

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X** : numpy array of shape [n_samples, n_features]
  
  Training set.

- **y** : numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new** : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

```python
get_params (deep=True)
```

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

```python
set_params (**params)
```

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self** :

```python
transform (X, y=None, copy=None)
```

Scale each non zero row of X to unit norm

**Parameters**

- **X** : {array-like, sparse matrix}, shape [n_samples, n_features]
  
  The data to normalize, row by row. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.
Examples using `sklearn.preprocessing.Normalizer`

- Clustering text documents using k-means

### 29.30.11 sklearn.preprocessing.OneHotEncoder

class sklearn.preprocessing.OneHotEncoder(n_values='auto', categorical_features='all', dtype=\<numpy.float64\>, sparse=True, handle_unknown='error')

Encode categorical integer features using a one-hot aka one-of-K scheme.

The input to this transformer should be a matrix of integers, denoting the values taken on by categorical (discrete) features. The output will be a sparse matrix where each column corresponds to one possible value of one feature. It is assumed that input features take on values in the range [0, n_values).

This encoding is needed for feeding categorical data to many scikit-learn estimators, notably linear models and SVMs with the standard kernels.

Note: a one-hot encoding of y labels should use a LabelBinarizer instead.

Read more in the User Guide.

Parameters:

- `n_values`: 'auto', int or array of ints
  Number of values per feature.
  - 'auto': determine value range from training data.
  - int[number of categorical values per feature.]
  - array[n_values[i] is the number of categorical values in X[:,i].]

- `categorical_features`: “all” or array of indices or mask
  Specify what features are treated as categorical.
  - 'all' (default): All features are treated as categorical.
  - array of indices: Array of categorical feature indices.
  - mask: Array of length n_features and with dtype=bool.
  Non-categorical features are always stacked to the right of the matrix.

- `dtype`: number type, default=np.float
  Desired dtype of output.

- `sparse`: boolean, default=True
  Will return sparse matrix if set True else will return an array.

- `handle_unknown`: str, ‘error’ or ‘ignore’
  Whether to raise an error or ignore if a unknown categorical feature is present during transform.

Attributes:

- `active_features_`: array
  Indices for active features, meaning values that actually occur in the training set. Only available when n_values is 'auto'.

- `feature_indices_`: array of shape (n_features,)
Indices to feature ranges. Feature \( i \) in the original data is mapped to features from `feature_indices_[i]` to `feature_indices_[i+1]` (and then potentially masked by `active_features_` afterwards).

**n_values_** : array of shape (n_features,)

Maximum number of values per feature.

See also:

- `scikit-learn.feature_extraction.DictVectorizer` performs a one-hot encoding of dictionary items (also handles string-valued features).
- `scikit-learn.feature_extraction.FeatureHasher` performs an approximate one-hot encoding of dictionary items or strings.
- `scikit-learn.preprocessing.LabelBinarizer` binarizes labels in a one-vs-all fashion.
- `scikit-learn.preprocessing.MultiLabelBinarizer` transforms between iterable of iterables and a multilabel format, e.g. a (samples x classes) binary matrix indicating the presence of a class label.
- `scikit-learn.preprocessing.LabelEncoder` encodes labels with values between 0 and \( n \text{\_classes}-1 \).

**Examples**

Given a dataset with three features and two samples, we let the encoder find the maximum value per feature and transform the data to a binary one-hot encoding.

```python
>>> from sklearn.preprocessing import OneHotEncoder
>>> enc = OneHotEncoder()
>>> enc.fit([[0, 0, 3], [1, 1, 0], [0, 2, 1], [1, 0, 2]])
OneHotEncoder(categorical_features='all', dtype=<... 'numpy.float64'>,
handle_unknown='error', n_values='auto', sparse=True)
>>> enc.n_values_  
array([2, 3, 4])
>>> enc.feature_indices_  
array([0, 2, 5, 9])
>>> enc.transform([[0, 1, 1]]).toarray()
array([[ 1., 0., 0., 1., 0., 0., 1., 0., 0.]])
```

### Methods

- `fit(X[, y])` Fit OneHotEncoder to X.
- `fit_transform(X[, y])` Fit OneHotEncoder to X, then transform X.
- `get_params([deep])` Get parameters for this estimator.
- `set_params(**params)` Set the parameters of this estimator.
- `transform(X)` Transform X using one-hot encoding.

**__init__**(n_values='auto', categorical_features='all', dtype=<type 'numpy.float64'>, sparse=True, handle_unknown='error')

**fit**(X, y=None)

Fit OneHotEncoder to X.

**Parameters**

- **X** : array-like, shape [n_samples, n_feature]
Input array of type int.

Returns:

**fit_transform** *(X, y=None)*
Fit OneHotEncoder to X, then transform X.

Equivalent to self.fit(X).transform(X), but more convenient and more efficient. See fit for the parameters, transform for the return value.

**get_params**(deep=True)
Get parameters for this estimator.

Parameters:

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:

- **params** : mapping of string to any
  Parameter names mapped to their values.

**set_params**(**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

- **transform** *(X)*
  Transform X using one-hot encoding.

Parameters:

- **X** : array-like, shape [n_samples, n_features]
  Input array of type int.

Returns:

- **X_out** : sparse matrix if sparse=True else a 2-d array, dtype=int
  Transformed input.

Examples using `sklearn.preprocessing.OneHotEncoder`

- Feature transformations with ensembles of trees

29.30.12 `sklearn.preprocessing.PolynomialFeatures`

class `sklearn.preprocessing.PolynomialFeatures`(degree=2, interaction_only=False, include_bias=True)

Generate polynomial and interaction features.

Generate a new feature matrix consisting of all polynomial combinations of the features with degree less than or equal to the specified degree. For example, if an input sample is two dimensional and of the form [a, b], the degree-2 polynomial features are [1, a, b, a^2, ab, b^2].

Parameters:

- **degree** : integer
  The degree of the polynomial features. Default = 2.

- **interaction_only** : boolean, default = False
  if True, will return the parameters for this estimator and contained subobjects that are estimators.
If true, only interaction features are produced: features that are products of at most degree distinct input features (so not \( x[1] ** 2, x[0] * x[2] ** 3 \), etc.).

**include_bias**: boolean

If True (default), then include a bias column, the feature in which all polynomial powers are zero (i.e. a column of ones - acts as an intercept term in a linear model).

**Attributes**

- **powers_**: array, shape \((n_output_features, n_input_features)\)
  
  \( \text{powers}_{[i, j]} \) is the exponent of the jth input in the ith output.

- **n_input_features_**: int
  
  The total number of input features.

- **n_output_features_**: int
  
  The total number of polynomial output features. The number of output features is computed by iterating over all suitably sized combinations of input features.

**Notes**

Be aware that the number of features in the output array scales polynomially in the number of features of the input array, and exponentially in the degree. High degrees can cause overfitting.

See examples/linear_model/plot_polynomial_interpolation.py

**Examples**

```python
>>> X = np.arange(6).reshape(3, 2)
>>> X
array([[0, 1],
       [2, 3],
       [4, 5]])
>>> poly = PolynomialFeatures(2)
>>> poly.fit_transform(X)
array([[ 1., 0., 1., 0., 0., 1.],
       [ 1., 2., 3., 4., 6., 9.],
       [ 1., 4., 5., 16., 20., 25.]])
>>> poly = PolynomialFeatures(interaction_only=True)
>>> poly.fit_transform(X)
array([[ 1., 0., 1., 0.],
       [ 1., 2., 3., 6.],
       [ 1., 4., 5., 20.]])
```

**Methods**

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<td><code>fit(X[, y])</code></td>
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<tr>
<td><code>transform(X[, y])</code></td>
<td>Transform data to polynomial features</td>
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</table>
**__init__** *(degree=2, interaction_only=False, include_bias=True)*

**fit** *(X, y=None)*

Compute number of output features.

**fit_transform** *(X, y=None, **fit_params)*

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

*Parameters*

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

*Returns*

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_feature_names** *(input_features=None)*

Return feature names for output features

*Parameters*

- **input_features**: list of string, length n_features, optional
  - String names for input features if available. By default, “x0”, “x1”, ... “xn_features” is used.

*Returns*

- **output_feature_names**: list of string, length n_output_features

**get_params** *(deep=True)*

Get parameters for this estimator.

*Parameters*

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

*Returns*

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**set_params** *(**params)*

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

*Returns*

- **self**: self

**transform** *(X, y=None)*

Transform data to polynomial features

*Parameters*

- **X**: array-like, shape [n_samples, n_features]
  - The data to transform, row by row.

*Returns*

- **XP**: np.ndarray shape [n_samples, NP]
  - The matrix of features, where NP is the number of polynomial features generated from the combination of inputs.
Examples using `sklearn.preprocessing.PolynomialFeatures`

- Polynomial interpolation
- Robust linear estimator fitting
- Underfitting vs. Overfitting

### 29.30.13 `sklearn.preprocessing.RobustScaler`

**class** `sklearn.preprocessing.RobustScaler(with_centering=True, with_scaling=True, quantile_range=(25.0, 75.0), copy=True)`

Scale features using statistics that are robust to outliers.

This Scaler removes the median and scales the data according to the quantile range (defaults to IQR: Interquartile Range). The IQR is the range between the 1st quartile (25th quantile) and the 3rd quartile (75th quantile).

Centering and scaling happen independently on each feature (or each sample, depending on the `axis` argument) by computing the relevant statistics on the samples in the training set. Median and interquartile range are then stored to be used on later data using the `transform` method.

Standardization of a dataset is a common requirement for many machine learning estimators. Typically this is done by removing the mean and scaling to unit variance. However, outliers can often influence the sample mean / variance in a negative way. In such cases, the median and the interquartile range often give better results.

New in version 0.17.


**Parameters**

- `with_centering` : boolean, True by default
  If True, center the data before scaling. This does not work (and will raise an exception) when attempted on sparse matrices, because centering them entails building a dense matrix which in common use cases is likely to be too large to fit in memory.

- `with_scaling` : boolean, True by default
  If True, scale the data to interquartile range.

- `quantile_range` : tuple (q_min, q_max), 0.0 < q_min < q_max < 100.0
  Default: (25.0, 75.0) = (1st quantile, 3rd quantile) = IQR Quantile range used to calculate `scale_`. New in version 0.18.

- `copy` : boolean, optional, default is True
  If False, try to avoid a copy and do inplace scaling instead. This is not guaranteed to always work inplace; e.g. if the data is not a NumPy array or scipy.sparse CSR matrix, a copy may still be returned.

**Attributes**

- `center_` : array of floats
  The median value for each feature in the training set.

- `scale_` : array of floats
  The (scaled) interquartile range for each feature in the training set.

  New in version 0.17: `scale_` attribute.

**See also:**
robust_scale Equivalent function without the object oriented API.

sklearn.decomposition.PCA Further removes the linear correlation across features with 'whiten=True'.

Notes

See examples/preprocessing/plot_robust_scaling.py for an example.

Methods

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<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
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<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
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<tr>
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<td>Scale back the data to the original representation</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X[, y])</td>
<td>Center and scale the data</td>
</tr>
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</table>

__init__ (with_centering=True, with_scaling=True, quantile_range=(25.0, 75.0), copy=True)

`fit (X, y=None)`
Compute the median and quantiles to be used for scaling.

Parameters:

- `X` : array-like, shape [n_samples, n_features]
  The data used to compute the median and quantiles used for later scaling along the
  features axis.

`fit_transform (X, y=None, **fit_params)`
Fit to data, then transform it.
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters:

- `X` : numpy array of shape [n_samples, n_features]
  Training set.
- `y` : numpy array of shape [n_samples]
  Target values.

Returns:

- `X_new` : numpy array of shape [n_samples, n_features_new]
  Transformed array.

`get_params (deep=True)`
Get parameters for this estimator.

Parameters:

- `deep` : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are
  estimators.

Returns:

- `params` : mapping of string to any
  Parameter names mapped to their values.
inverse_transform(X)
   Scale back the data to the original representation

   Parameters:
   X : array-like
      The data used to scale along the specified axis.

set_params(**params)
   Set the parameters of this estimator.

   The method works on simple estimators as well as on nested objects (such as pipelines). The latter have
   parameters of the form <component>__<parameter> so that it's possible to update each component
   of a nested object.

   Returns:
   self

transform(X, y=None)
   Center and scale the data

   Parameters:
   X : array-like
      The data used to scale along the specified axis.

Examples using sklearn.preprocessing.RobustScaler

   * Robust Scaling on Toy Data

29.30.14 sklearn.preprocessing.StandardScaler

class sklearn.preprocessing.StandardScaler(copy=True, with_mean=True, with_std=True)
   Standardize features by removing the mean and scaling to unit variance

   Centering and scaling happen independently on each feature by computing the relevant statistics on the samples
   in the training set. Mean and standard deviation are then stored to be used on later data using the transform
   method.

   Standardization of a dataset is a common requirement for many machine learning estimators: they might behave
   badly if the individual feature do not more or less look like standard normally distributed data (e.g. Gaussian
   with 0 mean and unit variance).

   For instance many elements used in the objective function of a learning algorithm (such as the RBF kernel of
   Support Vector Machines or the L1 and L2 regularizers of linear models) assume that all features are centered
   around 0 and have variance in the same order. If a feature has a variance that is orders of magnitude larger
   that others, it might dominate the objective function and make the estimator unable to learn from other features
   correctly as expected.

   This scaler can also be applied to sparse CSR or CSC matrices by passing with_mean=False to avoid breaking
   the sparsity structure of the data.

   Read more in the User Guide.

   Parameters:
   with_mean : boolean, True by default
      If True, center the data before scaling. This does not work (and will raise an exception)
      when attempted on sparse matrices, because centering them entails building a dense
      matrix which in common use cases is likely to be too large to fit in memory.

   with_std : boolean, True by default
      If True, scale the data to unit variance (or equivalently, unit standard deviation).
copy : boolean, optional, default True

If False, try to avoid a copy and do inplace scaling instead. This is not guaranteed to always work inplace; e.g. if the data is not a NumPy array or scipy.sparse CSR matrix, a copy may still be returned.

Attributes

scale_ : ndarray, shape (n_features,)

Per feature relative scaling of the data.

New in version 0.17: scale_ is recommended instead of deprecated std_.

mean_ : array of floats with shape [n_features]

The mean value for each feature in the training set.

var_ : array of floats with shape [n_features]

The variance for each feature in the training set. Used to compute scale_.

n_samples_seen_ : int

The number of samples processed by the estimator. Will be reset on new calls to fit, but increments across partial_fit calls.

See also:

scale

Equivalent function without the object oriented API.

sklearn.decomposition.PCA

Further removes the linear correlation across features with ‘whiten=True’.

Methods

* fit(X, y)

  Compute the mean and std to be used for later scaling.

* fit_transform(X, y)

  Fit to data, then transform it.

* get_params([deep])

  Get parameters for this estimator.

* inverse_transform(X[, copy])

  Scale back the data to the original representation

* partial_fit(X[, y])

  Online computation of mean and std on X for later scaling.

* set_params(**params)

  Set the parameters of this estimator.

* transform(X[, y, copy])

  Perform standardization by centering and scaling

* __init__(copy=True, with_mean=True, with_std=True)

  fit (X, y=None)

  Compute the mean and std to be used for later scaling.

  Parameters

  X : {array-like, sparse matrix}, shape [n_samples, n_features]

  The data used to compute the mean and standard deviation used for later scaling along the features axis.

  y: Passthrough for **Pipeline** compatibility.

  fit_transform (X, y=None, **fit_params)

  Fit to data, then transform it.

  Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

  Parameters

  X : nump array of shape [n_samples, n_features]
Training set.

\[ y \]: numpy array of shape [n_samples]

Target values.

**Returns**

\[ X_{\text{new}} \]: numpy array of shape [n_samples, n_features_new]

Transformed array.

**get_params** (*deep=True*)

Get parameters for this estimator.

**Parameters**

\[ deep \]: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

\[ params \]: mapping of string to any

Parameter names mapped to their values.

**inverse_transform** (*X*, *copy=None*)

Scale back the data to the original representation

**Parameters**

\[ X \]: array-like, shape [n_samples, n_features]

The data used to scale along the features axis.

**partial_fit** (*X*, *y=None*)

Online computation of mean and std on X for later scaling. All of X is processed as a single batch. This is intended for cases when fit is not feasible due to very large number of n_samples or because X is read from a continuous stream.

The algorithm for incremental mean and std is given in Equation 1.5a,b in Chan, Tony F., Gene H. Golub, and Randall J. LeVeque. “Algorithms for computing the sample variance: Analysis and recommendations.” The American Statistician 37.3 (1983): 242-247:

**Parameters**

\[ X \]: {array-like, sparse matrix}, shape [n_samples, n_features]

The data used to compute the mean and standard deviation used for later scaling along the features axis.

\[ y \]: Passthrough for “Pipeline“ compatibility. :

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

\[ self \]:

**std**

DEPRECATED: Attribute std_ will be removed in 0.19. Use scale_ instead

**transform** (*X*, *y=None, copy=None*)

Perform standardization by centering and scaling

**Parameters**

\[ X \]: array-like, shape [n_samples, n_features]

The data used to scale along the features axis.
Examples using `sklearn.preprocessing.StandardScaler`

- Prediction Latency
- Classifier comparison
- Comparing different clustering algorithms on toy datasets
- Demo of DBSCAN clustering algorithm
- L1 Penalty and Sparsity in Logistic Regression
- Sparse recovery: feature selection for sparse linear models
- Varying regularization in Multi-layer Perceptron
- Robust Scaling on Toy Data
- RBF SVM parameters

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<tr>
<td><code>preprocessing.binarize(X[, threshold, copy])</code></td>
<td>Boolean thresholding of array-like or scipy.sparse matrix</td>
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<tr>
<td><code>preprocessing.label_binarize(y, classes[, ...])</code></td>
<td>Binarize labels in a one-vs-all fashion</td>
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<tr>
<td><code>preprocessing.maxabs_scale(X[, axis, copy])</code></td>
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<tr>
<td><code>preprocessing.minmax_scale(X[, ...])</code></td>
<td>Transforms features by scaling each feature to a given range.</td>
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<tr>
<td><code>preprocessing.normalize(X[, norm, axis, ...])</code></td>
<td>Scale input vectors individually to unit norm (vector length).</td>
</tr>
<tr>
<td><code>preprocessing.robust_scale(X[, norm, axis, ...])</code></td>
<td>Standardize a dataset along any axis</td>
</tr>
<tr>
<td><code>preprocessing.scale(X[, axis, with_mean, ...])</code></td>
<td>Standardize a dataset along any axis</td>
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29.30.15 `sklearn.preprocessing.add_dummy_feature`

`sklearn.preprocessing.add_dummy_feature(X, value=1.0)`
Augment dataset with an additional dummy feature.

This is useful for fitting an intercept term with implementations which cannot otherwise fit it directly.

**Parameters**

- `X` : {array-like, sparse matrix}, shape [n_samples, n_features]
  Data.

- `value` : float
  Value to use for the dummy feature.

**Returns**

- `X` : {array, sparse matrix}, shape [n_samples, n_features + 1]
  Same data with dummy feature added as first column.

**Examples**

```python
>>> from sklearn.preprocessing import add_dummy_feature
>>> add_dummy_feature([[0, 1], [1, 0]])
array([[ 1.,  0.,  1.],
       [ 1.,  1.,  0.]])
```
29.30.16 sklearn.preprocessing.binarize

sklearn.preprocessing.binarize(X, threshold=0.0, copy=True)

Boolean thresholding of array-like or scipy.sparse matrix

Parameters:

X : {array-like, sparse matrix}, shape [n_samples, n_features]
    The data to binarize, element by element. scipy.sparse matrices should be in CSR or
    CSC format to avoid an unnecessary copy.

threshold : float, optional (0.0 by default)
    Feature values below or equal to this are replaced by 0, above it by 1. Threshold may
    not be less than 0 for operations on sparse matrices.

copy : boolean, optional, default True
    Set to False to perform inplace binarization and avoid a copy (if the input is already a
    numpy array or a scipy.sparse CSR / CSC matrix and if axis is 1).

See also:

Binarizer Performs binarization using the Transformer API (e.g. as part of a preprocessing
sklearn.pipeline.Pipeline).

29.30.17 sklearn.preprocessing.label_binarize

sklearn.preprocessing.label_binarize(y, classes, neg_label=0, pos_label=1, sparse_output=False)

Binarize labels in a one-vs-all fashion

Several regression and binary classification algorithms are available in the scikit. A simple way to extend these
algorithms to the multi-class classification case is to use the so-called one-vs-all scheme.

This function makes it possible to compute this transformation for a fixed set of class labels known ahead of
time.

Parameters:

y : array-like
    Sequence of integer labels or multilabel data to encode.

classes : array-like of shape [n_classes]
    Uniquely holds the label for each class.

neg_label : int (default: 0)
    Value with which negative labels must be encoded.

pos_label : int (default: 1)
    Value with which positive labels must be encoded.

sparse_output : boolean (default: False),
    Set to true if output binary array is desired in CSR sparse format

Returns:

Y : numpy array or CSR matrix of shape [n_samples, n_classes]
    Shape will be [n_samples, 1] for binary problems.

See also:
**LabelBinarizer** class used to wrap the functionality of label_binarize and allow for fitting to classes independently of the transform operation

**Examples**

```python
>>> from sklearn.preprocessing import label_binarize
>>> label_binarize([1, 6], classes=[1, 2, 4, 6])
array([[1, 0, 0, 0],
       [0, 0, 0, 1]])
```

The class ordering is preserved:

```python
>>> label_binarize([1, 6], classes=[1, 6, 4, 2])
array([[1, 0, 0, 0],
       [0, 1, 0, 0]])
```

Binary targets transform to a column vector

```python
>>> label_binarize(['yes', 'no', 'no', 'yes'], classes=['no', 'yes'])
array([[1],
       [0],
       [0],
       [1]])
```

**Examples using sklearn.preprocessing.label_binarize**

- Precision-Recall
- Receiver Operating Characteristic (ROC)

### 29.30.18 sklearn.preprocessing.maxabs_scale

`sklearn.preprocessing.maxabs_scale(X, axis=0, copy=True)`

Scale each feature to the [-1, 1] range without breaking the sparsity.

This estimator scales each feature individually such that the maximal absolute value of each feature in the training set will be 1.0.

This scaler can also be applied to sparse CSR or CSC matrices.

**Parameters**

- `axis` : int (0 by default)
  
  axis used to scale along. If 0, independently scale each feature, otherwise (if 1) scale each sample.

- `copy` : boolean, optional, default is True
  
  Set to False to perform inplace scaling and avoid a copy (if the input is already a numpy array).

**See also:**

- `MaxAbsScaler` performs scaling to the [-1, 1] range using the ‘‘Transformer‘‘ API (e.g. as part of a preprocessing `sklearn.pipeline.Pipeline`).
### 29.30.19 sklearn.preprocessing.minmax_scale

`sklearn.preprocessing.minmax_scale(X, feature_range=(0, 1), axis=0, copy=True)`

Transforms features by scaling each feature to a given range.

This estimator scales and translates each feature individually such that it is in the given range on the training set, i.e. between zero and one.

The transformation is given by:

\[
X_{\text{std}} = \frac{(X - \text{min}(axis=0))}{\text{max}(axis=0) - \text{min}(axis=0)}
\]

\[
X_{\text{scaled}} = X_{\text{std}} \times (\text{max} - \text{min}) + \text{min}
\]

where min, max = feature_range.

This transformation is often used as an alternative to zero mean, unit variance scaling.

Read more in the User Guide.

New in version 0.17: `minmax_scale` function interface to `sklearn.preprocessing.MinMaxScaler`.

**Parameters**

- `feature_range`: tuple (min, max), default=(0, 1)
  - Desired range of transformed data.
- `axis`: int (0 by default)
  - axis used to scale along. If 0, independently scale each feature, otherwise (if 1) scale each sample.
- `copy`: boolean, optional, default is True
  - Set to False to perform inplace scaling and avoid a copy (if the input is already a numpy array).

**See also:**

`MinMaxScaler` Performs scaling to a given range using the ‘Transformer’ API (e.g. as part of a preprocessing `sklearn.pipeline.Pipeline`).

### 29.30.20 sklearn.preprocessing.normalize

`sklearn.preprocessing.normalize(X, norm='l2', axis=1, copy=True, return_norm=False)`

Scale input vectors individually to unit norm (vector length).

Read more in the User Guide.

**Parameters**

- `X`: {array-like, sparse matrix}, shape [n_samples, n_features]
  - The data to normalize, element by element. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.
- `norm`: ‘l1’, ‘l2’, or ‘max’, optional (‘l2’ by default)
  - The norm to use to normalize each non zero sample (or each non-zero feature if axis is 0).
- `axis`: 0 or 1, optional (1 by default)
  - axis used to normalize the data along. If 1, independently normalize each sample, otherwise (if 0) normalize each feature.
- `copy`: boolean, optional, default True
set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix and if axis is 1).

**return_norm**: boolean, default False

whether to return the computed norms

See also:

*Normalizer* Performs normalization using the Transformer API (e.g. as part of a preprocessing *sklearn.pipeline.Pipeline*).

**29.30.21 sklearn.preprocessing.robust_scale**

`sklearn.preprocessing.robust_scale(X, axis=0, with_centering=True, with_scaling=True, quantile_range=(25.0, 75.0), copy=True)`

Standardize a dataset along any axis

Center to the median and component wise scale according to the interquartile range.

Read more in the *User Guide*.

**Parameters**

- **X**: array-like
  The data to center and scale.

- **axis**: int (0 by default)
  axis used to compute the medians and IQR along. If 0, independently scale each feature, otherwise (if 1) scale each sample.

- **with_centering**: boolean, True by default
  If True, center the data before scaling.

- **with_scaling**: boolean, True by default
  If True, scale the data to unit variance (or equivalently, unit standard deviation).

- **quantile_range**: tuple (q_min, q_max), 0.0 < q_min < q_max < 100.0
  Default: (25.0, 75.0) = (1st quantile, 3rd quantile) = IQR Quantile range used to calculate `scale_`.
  New in version 0.18.

- **copy**: boolean, optional, default is True
  set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix and if axis is 1).

See also:

*RobustScaler* Performs centering and scaling using the Transformer API (e.g. as part of a preprocessing *sklearn.pipeline.Pipeline*).

**Notes**

This implementation will refuse to center scipy.sparse matrices since it would make them non-sparse and would potentially crash the program with memory exhaustion problems.
Instead the caller is expected to either set explicitly \texttt{with_centering=False} (in that case, only variance scaling will be performed on the features of the CSR matrix) or to call \texttt{X.toarray()} if he/she expects the materialized dense array to fit in memory.

To avoid memory copy the caller should pass a CSR matrix.

### 29.30.22 \texttt{sklearn.preprocessing.scale}

\texttt{sklearn.preprocessing.scale (X, axis=0, with_mean=True, with_std=True, copy=True)}

Standardize a dataset along any axis

Center to the mean and component wise scale to unit variance.

Read more in the \textit{User Guide}.

**Parameters**

\texttt{X} : {array-like, sparse matrix}

The data to center and scale.

\texttt{axis} : int (0 by default)

axis used to compute the means and standard deviations along. If 0, independently standardize each feature, otherwise (if 1) standardize each sample.

\texttt{with_mean} : boolean, True by default

If True, center the data before scaling.

\texttt{with_std} : boolean, True by default

If True, scale the data to unit variance (or equivalently, unit standard deviation).

\texttt{copy} : boolean, optional, default True

set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSC matrix and if axis is 1).

**See also:**

\texttt{StandardScaler} Performs scaling to unit variance using the ‘‘Transformer’’ API (e.g. as part of a pre-processing \texttt{sklearn.pipeline.Pipeline}).

**Notes**

This implementation will refuse to center scipy.sparse matrices since it would make them non-sparse and would potentially crash the program with memory exhaustion problems.

Instead the caller is expected to either set explicitly \texttt{with_mean=False} (in that case, only variance scaling will be performed on the features of the CSC matrix) or to call \texttt{X.toarray()} if he/she expects the materialized dense array to fit in memory.

To avoid memory copy the caller should pass a CSC matrix.

**Examples using \texttt{sklearn.preprocessing.scale}**

- \textit{A demo of K-Means clustering on the handwritten digits data}
29.31 sklearn.random_projection: Random projection

Random Projection transformers

Random Projections are a simple and computationally efficient way to reduce the dimensionality of the data by trading a controlled amount of accuracy (as additional variance) for faster processing times and smaller model sizes.

The dimensions and distribution of Random Projections matrices are controlled so as to preserve the pairwise distances between any two samples of the dataset.

The main theoretical result behind the efficiency of random projection is the Johnson-Lindenstrauss lemma (quoting Wikipedia):

In mathematics, the Johnson-Lindenstrauss lemma is a result concerning low-distortion embeddings of points from high-dimensional into low-dimensional Euclidean space. The lemma states that a small set of points in a high-dimensional space can be embedded into a space of much lower dimension in such a way that distances between the points are nearly preserved. The map used for the embedding is at least Lipschitz, and can even be taken to be an orthogonal projection.

User guide: See the Random Projection section for further details.

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29.31.1 sklearn.random_projection.GaussianRandomProjection

class sklearn.random_projection.GaussianRandomProjection (n_components='auto', eps=0.1, random_state=None)

Reduce dimensionality through Gaussian random projection

The components of the random matrix are drawn from N(0, 1 / n_components).

Read more in the User Guide.

Parameters

- n_components : int or 'auto', optional (default = 'auto')
  - Dimensionality of the target projection space.
  - n_components can be automatically adjusted according to the number of samples in the dataset and the bound given by the Johnson-Lindenstrauss lemma. In that case the quality of the embedding is controlled by the eps parameter.
  - It should be noted that Johnson-Lindenstrauss lemma can yield very conservative estimated of the required number of components as it makes no assumption on the structure of the dataset.
- eps : strictly positive float, optional (default=0.1)
  - Parameter to control the quality of the embedding according to the Johnson-Lindenstrauss lemma when n_components is set to ‘auto’.
  - Smaller values lead to better embedding and higher number of dimensions (n_components) in the target projection space.
- random_state : integer, RandomState instance or None (default=None)
  - Control the pseudo random number generator used to generate the matrix at fit time.

Attributes

- n_components_ : int
Concrete number of components computed when n_components=“auto”.

components_: numpy array of shape [n_components, n_features]

Random matrix used for the projection.

See also:
SparseRandomProjection

Methods

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</table>

__init__(n_components='auto', eps=0.1, random_state=None)

fit (X, y=None)
Generate a sparse random projection matrix

Parameters:
- X : numpy array or scipy.sparse of shape [n_samples, n_features]
  Training set: only the shape is used to find optimal random matrix dimensions based on the theory referenced in the afore mentioned papers.
- y : is not used: placeholder to allow for usage in a Pipeline.

Returns:
- self

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters:
- X : numpy array of shape [n_samples, n_features]
  Training set.
- y : numpy array of shape [n_samples]
  Target values.

Returns:
- X_new : numpy array of shape [n_samples, n_features_new]
  Transformed array.

get_params (deep=True)
Get parameters for this estimator.

Parameters:
- deep : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params : mapping of string to any
  Parameter names mapped to their values.
**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns:** self

**transform** (X, y=None)
Project the data by using matrix product with the random matrix

**Parameters**

- **X**: numpy array or scipy.sparse of shape [n_samples, n_features]
  
The input data to project into a smaller dimensional space.

- **y**: is not used: placeholder to allow for usage in a Pipeline.

**Returns**

- **X_new**: numpy array or scipy sparse of shape [n_samples, n_components]
  
  Projected array.

### 29.31.2 sklearn.random_projection.SparseRandomProjection

class sklearn.random_projection.SparseRandomProjection (n_components='auto', density='auto', eps=0.1, dense_output=False, random_state=None)

Reduce dimensionality through sparse random projection

Sparse random matrix is an alternative to dense random projection matrix that guarantees similar embedding quality while being much more memory efficient and allowing faster computation of the projected data.

If we note $s = 1 / \text{density}$ the components of the random matrix are drawn from:

- $-\sqrt{s} / \sqrt{n\_\text{components}}$ with probability $1 / 2s$
- $0$ with probability $1 - 1 / s$
- $+\sqrt{s} / \sqrt{n\_\text{components}}$ with probability $1 / 2s$

Read more in the User Guide.

**Parameters**

- **n_components**: int or ‘auto’, optional (default = ‘auto’)
  
  Dimensionality of the target projection space.

  n_components can be automatically adjusted according to the number of samples in the dataset and the bound given by the Johnson-Lindenstrauss lemma. In that case the quality of the embedding is controlled by the eps parameter.

  It should be noted that Johnson-Lindenstrauss lemma can yield very conservative estimated of the required number of components as it makes no assumption on the structure of the dataset.

- **density**: float in range [0, 1], optional (default=’auto’)
  
  Ratio of non-zero component in the random projection matrix.

  If density = ‘auto’, the value is set to the minimum density as recommended by Ping Li et al.: $1 / \sqrt{n\_\text{features}}$.

  Use density = $1 / 3.0$ if you want to reproduce the results from Achlioptas, 2001.

- **eps**: strictly positive float, optional, (default=0.1)
Parameter to control the quality of the embedding according to the Johnson-Lindenstrauss lemma when n_components is set to ‘auto’.

Smaller values lead to better embedding and higher number of dimensions (n_components) in the target projection space.

dense_output : boolean, optional (default=False)

If True, ensure that the output of the random projection is a dense numpy array even if the input and random projection matrix are both sparse. In practice, if the number of components is small the number of zero components in the projected data will be very small and it will be more CPU and memory efficient to use a dense representation.

If False, the projected data uses a sparse representation if the input is sparse.

random_state : integer, RandomState instance or None (default=None)

Control the pseudo random number generator used to generate the matrix at fit time.

Attributes

n_component_ : int

Concrete number of components computed when n_components=“auto”.

components_ : CSR matrix with shape [n_components, n_features]

Random matrix used for the projection.

density_ : float in range 0.0 - 1.0

Concrete density computed from when density = “auto”.

See also:

GaussianRandomProjection

References

[R71], [R72]

Methods

fit(X[, y])

Generate a sparse random projection matrix

fit_transform(X[, y])

Fit to data, then transform it.

get_params([deep])

Get parameters for this estimator.

set_params(**params)

Set the parameters of this estimator.

transform(X[, y])

Project the data by using matrix product with the random matrix

__init__(n_components='auto', density='auto', eps=0.1, dense_output=False, random_state=None)

Generate a sparse random projection matrix

Parameters

X : numpy array or scipy.sparse of shape [n_samples, n_features]

Training set: only the shape is used to find optimal random matrix dimensions based on the theory referenced in the afore mentioned papers.

y : is not used: placeholder to allow for usage in a Pipeline.
Returns:

**fit_transform**(X, y=None, **fit_params)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**set_params**(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form **<component>__<parameter>** so that it’s possible to update each component of a nested object.

**Returns**

- **self**:

**transform**(X, y=None)

Project the data by using matrix product with the random matrix

**Parameters**

- **X**: numpy array or scipy.sparse of shape [n_samples, n_features]
  - The input data to project into a smaller dimensional space.
- **y**: is not used: placeholder to allow for usage in a Pipeline.

**Returns**

- **X_new**: numpy array or scipy sparse of shape [n_samples, n_components]
  - Projected array.

**Examples using** `sklearn.random_projection.SparseRandomProjection`

- *The Johnson-Lindenstrauss bound for embedding with random projections*
- *Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...*

---

29.31. `sklearn.random_projection`: Random projection

1849
29.31.3 sklearn.random_projection.johnson_lindenstrauss_min_dim

sklearn.random_projection.johnson_lindenstrauss_min_dim(n_samples, eps=0.1)

Find a ‘safe’ number of components to randomly project to

The distortion introduced by a random projection \( p \) only changes the distance between two points by a factor \((1 + \varepsilon)\) in an euclidean space with good probability. The projection \( p \) is an \( \varepsilon \)-embedding as defined by:

\[
(1 - \varepsilon) \|u - v\|^2 < \|p(u) - p(v)\|^2 < (1 + \varepsilon) \|u - v\|^2
\]

Where \( u \) and \( v \) are any rows taken from a dataset of shape \([n_{\text{samples}}, n_{\text{features}}]\), \( \varepsilon \) is in \([0, 1]\) and \( p \) is a projection by a random Gaussian \( N(0, 1) \) matrix with shape \([n_{\text{components}}, n_{\text{features}}]\) (or a sparse Achlioptas matrix).

The minimum number of components to guarantee the \( \varepsilon \)-embedding is given by:

\[
n_{\text{components}} \geq \frac{4 \log(n_{\text{samples}})}{(\varepsilon^2 / 2 - \varepsilon^3 / 3)}
\]

Read more in the User Guide.

Parameters

- **n_samples** : int or numpy array of int greater than 0,
  Number of samples. If an array is given, it will compute a safe number of components array-wise.

- **eps** : float or numpy array of float in \([0, 1]\), optional (default=0.1)
  Maximum distortion rate as defined by the Johnson-Lindenstrauss lemma. If an array is given, it will compute a safe number of components array-wise.

Returns

- **n_components** : int or numpy array of int,
  The minimal number of components to guarantee with good probability an \( \varepsilon \)-embedding with \( n_{\text{samples}} \).

References

[R73], [R74]

Examples

```python
>>> johnson_lindenstrauss_min_dim(1e6, eps=0.5)
663
```

```python
>>> johnson_lindenstrauss_min_dim(1e6, eps=[0.5, 0.1, 0.01])
array([663, 11841, 1112658])
```

```python
>>> johnson_lindenstrauss_min_dim([1e4, 1e5, 1e6], eps=0.1)
array([7894, 9868, 11841])
```

Examples using sklearn.random_projection.johnson_lindenstrauss_min_dim

- *The Johnson-Lindenstrauss bound for embedding with random projections*
29.32 sklearn.semi_supervised Semi-Supervised Learning

The `sklearn.semi_supervised` module implements semi-supervised learning algorithms. These algorithms utilized small amounts of labeled data and large amounts of unlabeled data for classification tasks. This module includes Label Propagation.

**User guide:** See the [Semi-Supervised](https://docs.scikit-learn.org/stable/glossary.html#term-semi-supervised) section for further details.

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### 29.32.1 sklearn.semi_supervised.LabelPropagation

```python
class sklearn.semi_supervised.LabelPropagation (kernel='rbf', gamma=20, n_neighbors=7,
alpha=1, max_iter=30, tol=0.001,
n_jobs=1)
```

Label Propagation classifier

Read more in the *User Guide*.

**Parameters**

- `kernel`: {'knn', 'rbf'}
  
  String identifier for kernel function to use. Only ’rbf’ and ‘knn’ kernels are currently supported.

- `gamma`: float
  
  Parameter for rbf kernel

- `n_neighbors`: integer > 0
  
  Parameter for knn kernel

- `alpha`: float
  
  Clamping factor

- `max_iter`: float
  
  Change maximum number of iterations allowed

- `tol`: float
  
  Convergence tolerance: threshold to consider the system at steady state

**Attributes**

- `X_`: array, shape = [n_samples, n_features]
  
  Input array.

- `classes_`: array, shape = [n_classes]
  
  The distinct labels used in classifying instances.

- `label_distributions_`: array, shape = [n_samples, n_classes]
  
  Categorical distribution for each item.

- `transduction_`: array, shape = [n_samples]
  
  Label assigned to each item via the transduction.

- `n_iter_`: int
Number of iterations run.

See also:

*LabelSpreading* Alternate label propagation strategy more robust to noise

References


Examples

```python
>>> from sklearn import datasets
>>> from sklearn.semi_supervised import LabelPropagation
>>> label_prop_model = LabelPropagation()
>>> iris = datasets.load_iris()
>>> random_unlabeled_points = np.where(np.random.randint(0, 2,...
... size=len(iris.target)))
>>> labels = np.copy(iris.target)
>>> labels[random_unlabeled_points] = -1
>>> label_prop_model.fit(iris.data, labels)
...
LabelPropagation(...)
```

Methods

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<th>Method</th>
<th>Description</th>
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<td><code>fit(X, y)</code></td>
<td>Fit a semi-supervised label propagation model based</td>
</tr>
<tr>
<td><code>get_params</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Performs inductive inference across the model.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Predict probability for each possible outcome.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

__init__ *(kernel='rbf', gamma=20, n_neighbors=7, alpha=1, max_iter=30, tol=0.001, n_jobs=1)*

**fit** *(X, y)*

Fit a semi-supervised label propagation model based.

All the input data is provided matrix X (labeled and unlabeled) and corresponding label matrix y with a dedicated marker value for unlabeled samples.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  A [n_samples by n_samples] size matrix will be created from this

- **y**: array_like, shape = [n_samples]
  
  n_labeled_samples (unlabeled points are marked as -1) All unlabeled samples will be transductively assigned labels

**Returns**

- **self**: returns an instance of self.
get_params (deep=True)
Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params**: mapping of string to any
  Parameter names mapped to their values.

predict (X)
Performs inductive inference across the model.

Parameters:
- **X**: array_like, shape = [n_samples, n_features]

Returns:
- **y**: array_like, shape = [n_samples]
  Predictions for input data

predict_proba (X)
Predict probability for each possible outcome.

Parameters:
- **X**: array_like, shape = [n_samples, n_features]

Returns:
- **probabilities**: array, shape = [n_samples, n_classes]
  Normalized probability distributions across class labels

score (X, y, sample_weight=None)
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters:
- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns:
- **score**: float
  Mean accuracy of self.predict(X) wrt. y.

set_params (**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:
- **self**:
29.32.2 sklearn.semi_supervised.LabelSpreading

class sklearn.semi_supervised.LabelSpreading

LabelSpreading model for semi-supervised learning

This model is similar to the basic Label Propagation algorithm, but uses affinity matrix based on the normalized graph Laplacian and soft clamping across the labels.

Read more in the User Guide.

Parameters

- **kernel**: {'knn', 'rbf'}
  
  String identifier for kernel function to use. Only 'rbf' and 'knn' kernels are currently supported.

- **gamma**: float
  
  Parameter for rbf kernel

- **n_neighbors**: integer > 0
  
  Parameter for knn kernel

- **alpha**: float
  
  Clamping factor

- **max_iter**: float
  
  Maximum number of iterations allowed

- **tol**: float
  
  Convergence tolerance: threshold to consider the system at steady state

- **n_jobs**: int, optional (default = 1)
  
  The number of parallel jobs to run. If -1, then the number of jobs is set to the number of CPU cores.

Attributes

- **X_**: array, shape = [n_samples, n_features]
  
  Input array.

- **classes_**: array, shape = [n_classes]
  
  The distinct labels used in classifying instances.

- **label_distributions_**: array, shape = [n_samples, n_classes]
  
  Categorical distribution for each item.

- **transduction_**: array, shape = [n_samples]
  
  Label assigned to each item via the transduction.

- **n_iter_**: int
  
  Number of iterations run.

See also:

*LabelPropagation* Unregularized graph based semi-supervised learning
References


Examples

```python
>>> from sklearn import datasets
>>> from sklearn.semi_supervised import LabelSpreading
>>> label_prop_model = LabelSpreading()
>>> iris = datasets.load_iris()
>>> random_unlabeled_points = np.where(np.random.randint(0, 2,
... size=len(iris.target)))
>>> labels = np.copy(iris.target)
>>> labels[random_unlabeled_points] = -1
>>> label_prop_model.fit(iris.data, labels)
... LabelSpreading(...)
```

Methods

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<td>fit(X, y)</td>
<td>Fit a semi-supervised label propagation model based on input data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Performs inductive inference across the model.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict probability for each possible outcome.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(*params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(kernel='rbf', gamma=20, n_neighbors=7, alpha=0.2, max_iter=30, tol=0.001, n_jobs=1)
fit (X, y)
    Fit a semi-supervised label propagation model based on input data.
    All the input data is provided matrix X (labeled and unlabeled) and corresponding label matrix y with a dedicated marker value for unlabeled samples.
    ParametersX : array-like, shape = [n_samples, n_features]
        A [n_samples by n_features] size matrix will be created from this
    y : array-like, shape = [n_samples]
        n_labeled_samples (unlabeled points are marked as -1) All unlabeled samples will be transductively assigned labels
    Returnsself : returns an instance of self.
get_params (deep=True)
    Get parameters for this estimator.
    Parametersdeep : boolean, optional
        If True, will return the parameters for this estimator and contained subobjects that are estimators.
```
Parameters

**predict** *(X)*

Performs inductive inference across the model.

**Parameters**

* X : array_like, shape = [n_samples, n_features]

**Returns**

* y : array_like, shape = [n_samples]

Predictions for input data

**predict_proba** *(X)*

Predict probability for each possible outcome.

Compute the probability estimates for each single sample in X and each possible outcome seen during training (categorical distribution).

**Parameters**

* X : array_like, shape = [n_samples, n_features]

**Returns**

* probabilities : array, shape = [n_samples, n_classes]

Normalized probability distributions across class labels

**score** *(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

* X : array-like, shape = (n_samples, n_features)

Test samples.

* y : array-like, shape = (n_samples) or (n_samples, n_outputs)

True labels for X.

* sample_weight : array-like, shape = [n_samples], optional

Sample weights.

**Returns**

* score : float

Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

* self : object

Examples using `sklearn.semi_supervised.LabelSpreading`

- `Label Propagation digits: Demonstrating performance`
- `Label Propagation digits active learning`
- `Label Propagation learning a complex structure`
- `Decision boundary of label propagation versus SVM on the Iris dataset`
### 29.33 sklearn.svm: Support Vector Machines

The sklearn.svm module includes Support Vector Machine algorithms.

**User guide:** See the Support Vector Machines section for further details.

#### 29.33.1 Estimators

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<td>C-Support Vector Classification.</td>
</tr>
<tr>
<td>svm.LinearSVC(penalty, loss, dual, tol, C, ...)</td>
<td>Linear Support Vector Classification.</td>
</tr>
<tr>
<td>svm.NuSVC(nu, kernel, degree, gamma, ...)</td>
<td>Nu-Support Vector Classification.</td>
</tr>
<tr>
<td>svm.SVR(kernel, degree, gamma, coef0, tol, ...)</td>
<td>Epsilon-Support Vector Regression.</td>
</tr>
<tr>
<td>svm.LinearSVR(epsilon, tol, C, loss, ...)</td>
<td>Linear Support Vector Regression.</td>
</tr>
<tr>
<td>svm.NuSVR(nu, C, kernel, degree, gamma, ...)</td>
<td>Nu Support Vector Regression.</td>
</tr>
<tr>
<td>svm.OneClassSVM(kernel, degree, gamma, ...)</td>
<td>Unsupervised Outlier Detection.</td>
</tr>
</tbody>
</table>

**sklearn.svm.SVC**

**class** sklearn.svm.SVC(\(C=1.0, \text{kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, class_weight=None, verbose=False, max_iter=-1, decision_function_shape=None, random_state=None}\) )

C-Support Vector Classification.

The implementation is based on libsvm. The fit time complexity is more than quadratic with the number of samples which makes it hard to scale to dataset with more than a couple of 10000 samples.

The multiclass support is handled according to a one-vs-one scheme.

For details on the precise mathematical formulation of the provided kernel functions and how \(\text{gamma, coef0 and degree}\) affect each other, see the corresponding section in the narrative documentation: **Kernel functions**.

Read more in the **User Guide**.

**Parameters**

- **C** : float, optional (default=1.0)
  
  Penalty parameter C of the error term.

- **kernel** : string, optional (default=’rbf’)
  
  Specifies the kernel type to be used in the algorithm. It must be one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’ or a callable. If none is given, ‘rbf’ will be used. If a callable is given it is used to pre-compute the kernel matrix from data matrices; that matrix should be an array of shape \((n_samples, n_samples)\).

- **degree** : int, optional (default=3)
  
  Degree of the polynomial kernel function (‘poly’). Ignored by all other kernels.

- **gamma** : float, optional (default=’auto’)
  
  Kernel coefficient for ‘rbf’, ‘poly’ and ‘sigmoid’. If gamma is ‘auto’ then 1/n_features will be used instead.

- **coef0** : float, optional (default=0.0)
  
  Independent term in kernel function. It is only significant in ‘poly’ and ‘sigmoid’.

- **probability** : boolean, optional (default=False)
Whether to enable probability estimates. This must be enabled prior to calling fit, and will slow down that method.

**shrinking**: boolean, optional (default=True)
Whether to use the shrinking heuristic.

**tol**: float, optional (default=1e-3)
Tolerance for stopping criterion.

**cache_size**: float, optional
Specify the size of the kernel cache (in MB).

**class_weight**: {dict, ‘balanced’}, optional
Set the parameter C of class i to class_weight[i]*C for SVC. If not given, all classes are supposed to have weight one. The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as n_samples / (n_classes * np.bincount(y))

**verbose**: bool, default: False
Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

**max_iter**: int, optional (default=-1)
Hard limit on iterations within solver, or -1 for no limit.

**decision_function_shape**: ‘ovo’, ‘ovr’ or None, default=None
Whether to return a one-vs-rest (‘ovr’) decision function of shape (n_samples, n_classes) as all other classifiers, or the original one-vs-one (‘ovo’) decision function of libsvm which has shape (n_samples, n_classes * (n_classes - 1) / 2). The default of None will currently behave as ‘ovo’ for backward compatibility and raise a deprecation warning, but will change ‘ovr’ in 0.19.

New in version 0.17: decision_function_shape=’ovr’ is recommended.

Changed in version 0.17: Deprecated decision_function_shape=’ovo’ and None.

**random_state**: int seed, RandomState instance, or None (default)
The seed of the pseudo random number generator to use when shuffling the data for probability estimation.

**Attributes**

- **support_**: array-like, shape = [n_SV]
  Indices of support vectors.

- **support_vectors_**: array-like, shape = [n_SV, n_features]
  Support vectors.

- **n_support_**: array-like, dtype=int32, shape = [n_class]
  Number of support vectors for each class.

- **dual_coef_**: array, shape = [n_class-1, n_SV]
  Coefficients of the support vector in the decision function. For multiclass, coefficient for all 1-vs-1 classifiers. The layout of the coefficients in the multiclass case is somewhat non-trivial. See the section about multi-class classification in the SVM section of the User Guide for details.
coef_ : array, shape = [n_class-1, n_features]
Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.

coef_ is a readonly property derived from dual_coef_ and support_vectors_.

intercept_ : array, shape = [n_class * (n_class-1) / 2]
Constants in decision function.

See also:

LinearSVC: Scalable Linear Support Vector Machine for classification implemented using liblinear. Check the See also section of LinearSVC for more comparison element.

Examples

```python
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> y = np.array([1, 1, 2, 2])
>>> from sklearn.svm import SVC
>>> clf = SVC()
>>> clf.fit(X, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
   decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
   max_iter=-1, probability=False, random_state=None, shrinking=True,
   tol=0.001, verbose=False)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

Methods

decision_function(X) Distance of the samples X to the separating hyperplane.

fit(X, y[, sample_weight]) Fit the SVM model according to the given training data.

get_params([deep]) Get parameters for this estimator.

predict(X) Perform classification on samples in X.

score(X, y[, sample_weight]) Returns the mean accuracy on the given test data and labels.

set_params(**params) Set the parameters of this estimator.

__init__(C=1.0, kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, class_weight=None, verbose=False, max_iter=-1, decision_function_shape='ovr', random_state=None) Distance of the samples X to the separating hyperplane.

ParametersX : array-like, shape (n_samples, n_features)

ReturnsX : array-like, shape (n_samples, n_classes * (n_classes-1) / 2)

Returns the decision function of the sample for each class in the model. If decision_function_shape='ovr', the shape is (n_samples, n_classes)
**fit** *(X, y, sample_weight=None)*

Fit the SVM model according to the given training data.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  - Training vectors, where n_samples is the number of samples and n_features is the number of features. For kernel="precomputed", the expected shape of X is (n_samples, n_samples).

- **y**: array-like, shape (n_samples,)
  - Target values (class labels in classification, real numbers in regression)

- **sample_weight**: array-like, shape (n_samples,)
  - Per-sample weights. Rescale C per sample. Higher weights force the classifier to put more emphasis on these points.

**Returns**

- **self**: object
  - Returns self.

**Notes**

- If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.
- If X is a dense array, then the other methods will not support sparse matrices as input.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**predict** *(X)*

Perform classification on samples in X.

- For an one-class model, +1 or -1 is returned.

**predict_log_proba**

Compute log probabilities of possible outcomes for samples in X.

- The model need to have probability information computed at training time: fit with attribute probability set to True.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)
  - For kernel="precomputed", the expected shape of X is [n_samples_test, n_samples_train]
Returns $T$ : array-like, shape (n_samples, n_classes)

Returns the log-probabilities of the sample for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute classes_.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

predict_proba

Compute probabilities of possible outcomes for samples in X.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters $X$ : array-like, shape (n_samples, n_features)

For kernel="precomputed", the expected shape of $X$ is [n_samples_test, n_samples_train]

Returns $T$ : array-like, shape (n_samples, n_classes)

Returns the probability of the sample for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute classes_.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

score ($X$, $y$, sample_weight=None)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters $X$ : array-like, shape = (n_samples, n_features)

Test samples.

$y$ : array-like, shape = (n_samples) or (n_samples, n_outputs)

True labels for $X$.

sample_weight : array-like, shape = [n_samples], optional

Sample weights.

Returns score : float

Mean accuracy of self.predict($X$) wrt. $y$.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns self :
Examples using `sklearn.svm.SVC`

- Concatenating multiple feature extraction methods
- Feature Union with Heterogeneous Data Sources
- Explicit feature map approximation for RBF kernels
- Multilabel classification
- Faces recognition example using eigenfaces and SVMs
- Libsvm GUI
- Plot classification probability
- Classifier comparison
- Recognizing hand-written digits
- Plot the decision boundaries of a VotingClassifier
- Cross-validation on Digits Dataset Exercise
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- Receiver Operating Characteristic (ROC) with cross validation
- Plotting Validation Curves
- Decision boundary of label propagation versus SVM on the Iris dataset
- SVM with custom kernel
- Plot different SVM classifiers in the iris dataset
- RBF SVM parameters
- SVM: Maximum margin separating hyperplane
- SVM: Separating hyperplane for unbalanced classes
- SVM-Anova: SVM with univariate feature selection
- SVM-Kernels
- SVM Margins Example
- SVM: Weighted samples
sklearn.svm.LinearSVC

class sklearn.svm.LinearSVC (penalty='l2', loss='squared_hinge', dual=True, tol=0.0001, C=1.0, multi_class='ovr', fit_intercept=True, intercept_scaling=1, class_weight=None, verbose=0, random_state=None, max_iter=1000)

Linear Support Vector Classification.

Similar to SVC with parameter kernel='linear', but implemented in terms of liblinear rather than libsvm, so it has more flexibility in the choice of penalties and loss functions and should scale better to large numbers of samples.

This class supports both dense and sparse input and the multiclass support is handled according to a one-vs-the-rest scheme.

Read more in the User Guide.

Parameters

C : float, optional (default=1.0)

Penalty parameter C of the error term.

loss : string, ‘hinge’ or ‘squared_hinge’ (default=’squared_hinge’)

Specifies the loss function. ‘hinge’ is the standard SVM loss (used e.g. by the SVC class) while ‘squared_hinge’ is the square of the hinge loss.

penalty : string, ‘l1’ or ‘l2’ (default=’l2’)

Specifies the norm used in the penalization. The ‘l2’ penalty is the standard used in SVC. The ‘l1’ leads to coef_ vectors that are sparse.

dual : bool, (default=True)

Select the algorithm to either solve the dual or primal optimization problem. Prefer dual=False when n_samples > n_features.

tol : float, optional (default=1e-4)

Tolerance for stopping criteria.

multi_class : string, ‘ovr’ or ‘crammer_singer’ (default=’ovr’):

Determines the multi-class strategy if y contains more than two classes. "ovr" trains n_classes one-vs-rest classifiers, while "crammer_singer" optimizes a joint objective over all classes. While crammer_singer is interesting from a theoretical perspective as it is consistent, it is seldom used in practice as it rarely leads to better accuracy and is more expensive to compute. If "crammer_singer" is chosen, the options loss, penalty and dual will be ignored.

fit_intercept : boolean, optional (default=True)

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (i.e. data is expected to be already centered).

intercept_scaling : float, optional (default=1)

When self.fit_intercept is True, instance vector x becomes [x, self.intercept_scaling], i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic feature weight Note! the synthetic feature weight is subject to l1/l2 regularization as all other features. To lessen the effect of regularization on synthetic feature weight (and therefore on the intercept) intercept_scaling has to be increased.

class_weight : {dict, ‘balanced’}, optional
Set the parameter C of class to `class_weight[i] * C` for SVC. If not given, all classes are supposed to have weight one. The “balanced” mode uses the values of `y` to automatically adjust weights inversely proportional to class frequencies in the input data as `n_samples / (n_classes * np.bincount(y))`.

**verbose** : int, (default=0)

Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in liblinear that, if enabled, may not work properly in a multithreaded context.

**random_state** : int seed, RandomState instance, or None (default=None)

The seed of the pseudo random number generator to use when shuffling the data.

**max_iter** : int, (default=1000)

The maximum number of iterations to be run.

**Attributes**

**coef_** : array, shape = [n_features] if n_classes == 2 else [n_classes, n_features]

Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.

`coef_` is a readonly property derived from `raw_coef_` that follows the internal memory layout of liblinear.

**intercept_** : array, shape = [1] if n_classes == 2 else [n_classes]

Constants in decision function.

**See also:**

**SVC**

Implementation of Support Vector Machine classifier using libsvm: the kernel can be non-linear but its SMO algorithm does not scale to large number of samples as LinearSVC does. Furthermore SVC multi-class mode is implemented using one vs one scheme while LinearSVC uses one vs the rest. It is possible to implement one vs the rest with SVC by using the `sklearn.multiclass.OneVsRestClassifier` wrapper. Finally SVC can fit dense data without memory copy if the input is C-contiguous. Sparse data will still incur memory copy though.

**sklearn.linear_model.SGDClassifier**

SGDClassifier can optimize the same cost function as LinearSVC by adjusting the penalty and loss parameters. In addition it requires less memory, allows incremental (online) learning, and implements various loss functions and regularization regimes.

**Notes**

The underlying C implementation uses a random number generator to select features when fitting the model. It is thus not uncommon to have slightly different results for the same input data. If that happens, try with a smaller `tol` parameter.

The underlying implementation, liblinear, uses a sparse internal representation for the data that will incur a memory copy.

Predict output may not match that of standalone liblinear in certain cases. See `differences from liblinear` in the narrative documentation.

**References**

**LIBLINEAR: A Library for Large Linear Classification**
Methods

decision_function(X)  
Predict confidence scores for samples.

densify()  
Convert coefficient matrix to dense array format.

fit(X, y[, sample_weight])  
Fit the model according to the given training data.

fit_transform(X[, y])  
Fit to data, then transform it.

get_params([deep])  
Get parameters for this estimator.

predict(X)  
Predict class labels for samples in X.

score(X, y[, sample_weight])  
Returns the mean accuracy on the given test data and labels.

set_params(**params)  
Set the parameters of this estimator.

sparsify()  
Convert coefficient matrix to sparse format.

transform(*args, **kwargs)  
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.

__init__ (penalty='l2', loss='squared_hinge', dual=True, tol=0.0001, C=1.0, multi_class='ovr', fit_intercept=True, intercept_scaling=1, class_weight=None, verbose=0, random_state=None, max_iter=1000)

decision_function (X)  
Predict confidence scores for samples.

The confidence score for a sample is the signed distance of that sample to the hyperplane.

ParametersX : {array-like, sparse matrix}, shape = (n_samples, n_features)

Samples.

Returnsarray, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes) :

Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

densify ()  
Convert coefficient matrix to dense array format.

Converts the coef_ member (back) to a numpy.ndarray. This is the default format of coef_ and is required for fitting, so calling this method is only required on models that have previously been sparsified; otherwise, it is a no-op.

Returnsself: estimator :

fit (X, y, sample_weight=None)  
Fit the model according to the given training data.

ParametersX : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training vector, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]

Target vector relative to X

sample_weight : array-like, shape = [n_samples], optional

Array of weights that are assigned to individual samples. If not provided, then each sample is given unit weight.

Returnsself : object
Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters `fit_params` and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.

**predict** *(X)*

Predict class labels for samples in X.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  - Samples.

**Returns**

- **C**: array, shape = [n_samples]
  - Predicted class label per sample.

**score** *(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  - Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**

- **score**: float
  - Mean accuracy of self.predict(X) wrt. y.

**set_params** *(**params)*

Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns:**

```python
self
```

**sparsify()**

Convert coefficient matrix to sparse format.

Converts the `coef_` member to a scipy.sparse matrix, which for L1-regularized models can be much more memory- and storage-efficient than the usual numpy.ndarray representation.

The `intercept_` member is not converted.

**Returns:**

```python
self
```

**Notes**

For non-sparse models, i.e. when there are not many zeros in `coef_`, this may actually *increase* memory usage, so use this method with care. A rule of thumb is that the number of zero elements, which can be computed with `(coef_ == 0).sum()`, must be more than 50% for this to provide significant benefits.

After calling this method, further fitting with the partial_fit method (if any) will not work until you call densify.

**transform(*args, **kwargs)**

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

**Parameters**

```python
X : array or scipy sparse matrix of shape [n_samples, n_features]
The input samples.
```

```python
threshold : [string, float or None, optional (default=None)]
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.
```

**Returns**

```python
X_r : array of shape [n_samples, n_selected_features]
The input samples with only the selected features.
```

**Examples using `sklearn.svm.LinearSVC`**

- Selecting dimensionality reduction with Pipeline and GridSearchCV
- Explicit feature map approximation for RBF kernels
- Probability Calibration curves
- Comparison of Calibration of Classifiers
• *Plot different SVM classifiers in the iris dataset*
• *Scaling the regularization parameter for SVCs*
• *Classification of text documents using sparse features*

**sklearn.svm.NuSVC**

class sklearn.svm.NuSVC (nu=0.5, kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, class_weight=None, verbose=False, max_iter=-1, decision_function_shape=None, random_state=None)

Nu-Support Vector Classification.

Similar to SVC but uses a parameter to control the number of support vectors.

The implementation is based on libsvm.

Read more in the *User Guide*.

**Parameters**

- **nu**: float, optional (default=0.5)
  An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval (0, 1].

- **kernel**: string, optional (default='rbf')
  Specifies the kernel type to be used in the algorithm. It must be one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’ or a callable. If none is given, ‘rbf’ will be used. If a callable is given it is used to precompute the kernel matrix.

- **degree**: int, optional (default=3)
  Degree of the polynomial kernel function (‘poly’). Ignored by all other kernels.

- **gamma**: float, optional (default='auto')
  Kernel coefficient for ‘rbf’, ‘poly’ and ‘sigmoid’. If gamma is ‘auto’ then 1/n_features will be used instead.

- **coef0**: float, optional (default=0.0)
  Independent term in kernel function. It is only significant in ‘poly’ and ‘sigmoid’.

- **probability**: boolean, optional (default=False)
  Whether to enable probability estimates. This must be enabled prior to calling *fit*, and will slow down that method.

- **shrinking**: boolean, optional (default=True)
  Whether to use the shrinking heuristic.

- **tol**: float, optional (default=1e-3)
  Tolerance for stopping criterion.

- **cache_size**: float, optional
  Specify the size of the kernel cache (in MB).

- **class_weight**: {dict, ‘balanced’}, optional
Set the parameter C of class i to class_weight[i]*C for SVC. If not given, all classes are supposed to have weight one. The “balanced” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies as \( \frac{n_{\text{samples}}}{n_{\text{classes}} \times \text{np.bincount}(y)} \)

**verbose**: bool, default: False

Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

**max_iter**: int, optional (default=-1)

Hard limit on iterations within solver, or -1 for no limit.

**decision_function_shape**: ‘ovo’, ‘ovr’ or None, default=None

Whether to return a one-vs-rest (‘ovr’) decision function of shape (n_samples, n_classes) as all other classifiers, or the original one-vs-one (‘ovo’) decision function of libsvm which has shape (n_samples, n_classes * (n_classes - 1) / 2). The default of None will currently behave as ‘ovo’ for backward compatibility and raise a deprecation warning, but will change ‘ovr’ in 0.19.

New in version 0.17: \( \text{decision_function_shape='ovr'} \) is recommended.

Changed in version 0.17: Deprecated \( \text{decision_function_shape='ovo'} \) and None.

**random_state**: int seed, RandomState instance, or None (default)

The seed of the pseudo random number generator to use when shuffling the data for probability estimation.

**Attributes**:

- **support_**: array-like, shape = [n_SV]
  
  Indices of support vectors.

- **support_vectors_**: array-like, shape = [n_SV, n_features]
  
  Support vectors.

- **n_support_**: array-like, dtype=int32, shape = [n_class]
  
  Number of support vectors for each class.

- **dual_coef_**: array, shape = [n_class-1, n_SV]
  
  Coefficients of the support vector in the decision function. For multiclass, coefficient for all 1-vs-1 classifiers. The layout of the coefficients in the multiclass case is somewhat non-trivial. See the section about multi-class classification in the SVM section of the User Guide for details.

- **coef_**: array, shape = [n_class-1, n_features]
  
  Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.

  coef_ is readonly property derived from dual_coef_ and support_vectors_.

- **intercept_**: array, shape = [n_class * (n_class-1) / 2]
  
  Constants in decision function.

**See also**:


Examples

```python
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> y = np.array([1, 1, 2, 2])
>>> from sklearn.svm import NuSVC
>>> clf = NuSVC()
>>> clf.fit(X, y)
NuSVC(cache_size=200, class_weight=None, coef0=0.0,
      decision_function_shape=None, degree=3, gamma='auto', kernel='rbf',
      max_iter=-1, nu=0.5, probability=False, random_state=None,
      shrinking=True, tol=0.001, verbose=False)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

Methods

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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit the SVM model according to the given training data.</td>
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<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<td>Perform classification on samples in X.</td>
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<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
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</table>

```python
__init__(nu=0.5, kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, class_weight=None, verbose=False, max_iter=-1, decision_function_shape=None, random_state=None)
```

**decision_function (X)**

Distance of the samples X to the separating hyperplane.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)

**Returns**

- **X**: array-like, shape (n_samples, n_classes * (n_classes-1) / 2)

Returns the decision function of the sample for each class in the model. If decision_function_shape='ovr', the shape is (n_samples, n_classes)

**fit (X, y, sample_weight=None)**

Fit the SVM model according to the given training data.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features. For kernel="precomputed", the expected shape of X is (n_samples, n_samples).

- **y**: array-like, shape (n_samples,)
  
  Target values (class labels in classification, real numbers in regression)

- **sample_weight**: array-like, shape (n_samples,)
  
  Per-sample weights. Rescale C per sample. Higher weights force the classifier to put more emphasis on these points.
Returns the model建成后，预测

**Notes**

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

**get_params** *(deep=True)*

Get parameters for this estimator.

Parameters

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

**predict** *(X)*

Perform classification on samples in X.

For an one-class model, +1 or -1 is returned.

Parameters

- **X** : {array-like, sparse matrix}, shape (n_samples, n_features)
  
  For kernel="precomputed", the expected shape of X is [n_samples_test, n_samples_train]

Returns

- **y_pred** : array, shape (n_samples,)
  
  Class labels for samples in X.

**predict_log_proba**

Compute log probabilities of possible outcomes for samples in X.

The model need to have probability information computed at training time: fit with attribute `probability` set to True.

Parameters

- **X** : array-like, shape (n_samples, n_features)
  
  For kernel="precomputed", the expected shape of X is [n_samples_test, n_samples_train]

Returns

- **T** : array-like, shape (n_samples, n_classes)
  
  Returns the log-probabilities of the sample for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

**predict_proba**

Compute probabilities of possible outcomes for samples in X.
The model needs to have probability information computed at training time: fit with attribute `probability` set to True.

**Parameters**
- `X` : array-like, shape (n_samples, n_features)
  - For kernel="precomputed", the expected shape of X is [n_samples_test, n_samples_train]

**Returns**
- `T` : array-like, shape (n_samples, n_classes)
  - Returns the probability of the sample for each class in the model. The columns correspond to the classes in sorted order, as they appear in the attribute `classes_`.

**Notes**

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

**score***(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**
- `X` : array-like, shape = (n_samples, n_features)
  - Test samples.
- `y` : array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True labels for X.
- `sample_weight` : array-like, shape = [n_samples], optional
  - Sample weights.

**Returns**
- `score` : float
  - Mean accuracy of self.predict(X) wrt. y.

**set_params***(**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**
- `self` : 

**Examples using sklearn.svm.NuSVC**

- **Non-linear SVM**

**sklearn.svm.SVR**

**class** sklearn.svm.SVR*(kernel='rbf', degree=3, gamma='auto', coef0=0.0, tol=0.001, C=1.0, epsilon=0.1, shrinking=True, cache_size=200, verbose=False, max_iter=-1)*

Epsilon-Support Vector Regression.

The free parameters in the model are C and epsilon.
The implementation is based on libsvm.

Read more in the User Guide.

**Parameters**

- **C** : float, optional (default=1.0)
  
  Penalty parameter C of the error term.

- **epsilon** : float, optional (default=0.1)
  
  Epsilon in the epsilon-SVR model. It specifies the epsilon-tube within which no penalty is associated in the training loss function with points predicted within a distance epsilon from the actual value.

- **kernel** : string, optional (default='rbf')
  
  Specifies the kernel type to be used in the algorithm. It must be one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’ or a callable. If none is given, ‘rbf’ will be used. If a callable is given it is used to precompute the kernel matrix.

- **degree** : int, optional (default=3)
  
  Degree of the polynomial kernel function (‘poly’). Ignored by all other kernels.

- **gamma** : float, optional (default='auto')
  
  Kernel coefficient for ‘rbf’, ‘poly’ and ‘sigmoid’. If gamma is ‘auto’ then 1/n_features will be used instead.

- **coef0** : float, optional (default=0.0)
  
  Independent term in kernel function. It is only significant in ‘poly’ and ‘sigmoid’.

- **shrinking** : boolean, optional (default=True)
  
  Whether to use the shrinking heuristic.

- **tol** : float, optional (default=1e-3)
  
  Tolerance for stopping criterion.

- **cache_size** : float, optional
  
  Specify the size of the kernel cache (in MB).

- **verbose** : bool, default: False
  
  Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

- **max_iter** : int, optional (default=-1)
  
  Hard limit on iterations within solver, or -1 for no limit.

**Attributes**

- **support_** : array-like, shape = [n_SV]
  
  Indices of support vectors.

- **support_vectors_** : array-like, shape = [nSV, n_features]
  
  Support vectors.

- **dual_coef_** : array, shape = [1, n_SV]
  
  Coefficients of the support vector in the decision function.

- **coef_** : array, shape = [1, n_features]
Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.

coeff_ is readonly property derived from dual_coef_ and support_vectors_.

**intercept_** : array, shape = [1]

Constants in decision function.

**sample_weight** : array-like, shape = [n_samples]

Individual weights for each sample

See also:

*NuSVR* Support Vector Machine for regression implemented using libsvm using a parameter to control the number of support vectors.

*LinearSVR* Scalable Linear Support Vector Machine for regression implemented using liblinear.

Examples

```python
>>> from sklearn.svm import SVR
>>> import numpy as np

>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)

>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)

>>> clf = SVR(C=1.0, epsilon=0.2)
>>> clf.fit(X, y)

SVR(C=1.0, cache_size=200, coef0=0.0, degree=3, epsilon=0.2, gamma='auto',
    kernel='rbf', max_iter=-1, shrinking=True, tol=0.001, verbose=False)
```

Methods

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<td>decision_function(*args, **kwargs)</td>
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<tr>
<td>fit(X, y[, sample_weight])</td>
<td>Fit the SVM model according to the given training data.</td>
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<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
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<tr>
<td>predict(X)</td>
<td>Perform regression on samples in X.</td>
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<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
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</table>

__init__(kernel=’rbf’, degree=3, gamma=’auto’, coef0=0.0, tol=0.001, C=1.0, epsilon=0.1, shrinking=True, cache_size=200, verbose=False, max_iter=-1)

decision_function(*args, **kwargs)

DEPRECATED: and will be removed in 0.19

Distance of the samples X to the separating hyperplane.

**Parameters**

- **X** : array-like, shape (n_samples, n_features)
  
  For kernel="precomputed", the expected shape of X is [n_samples_test, n_samples_train].

**Returns**

- **X** : array-like, shape (n_samples, n_class * (n_class-1) / 2)
Returns the decision function of the sample for each class in the model.

**fit** (*X, y, sample_weight=None*)

Fit the SVM model according to the given training data.

**Parameters**

*X*: array-like, sparse matrix, shape (n_samples, n_features)

Training vectors, where n_samples is the number of samples and n_features is the number of features. For kernel="precomputed", the expected shape of X is (n_samples, n_samples).

*y*: array-like, shape (n_samples,)

Target values (class labels in classification, real numbers in regression)

**sample_weight**: array-like, shape (n_samples,)

Per-sample weights. Rescale C per sample. Higher weights force the classifier to put more emphasis on these points.

**Returns**

*self* : object

Returns self.

**Notes**

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

**get_params** (*deep=True*)

Get parameters for this estimator.

**Parameters**

*deep*: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

*params* : mapping of string to any

Parameter names mapped to their values.

**predict** (*X*)

Perform regression on samples in X.

For an one-class model, +1 or -1 is returned.

**Parameters**

*X*: array-like, sparse matrix, shape (n_samples, n_features)

For kernel="precomputed", the expected shape of X is (n_samples_test, n_samples_train).

**Returns**

*y_pred* : array, shape (n_samples,)

**score** (*X, y, sample_weight=None*)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

**Parameters**

*X*: array-like, shape = (n_samples, n_features)
Test samples.

\( y \): array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for X.

**sample_weight**: array-like, shape = [n_samples], optional

Sample weights.

**Returns**

\( score \): float

R\(^2\) of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

self:

Examples using sklearn.svm.SVR

- Comparison of kernel ridge regression and SVR
- Prediction Latency
- Support Vector Regression (SVR) using linear and non-linear kernels

**sklearn.svm.LinearSVR**

_class sklearn.svm.LinearSVR(epsilon=0.0, tol=0.0001, C=1.0, loss='epsilon_insensitive', fit_intercept=True, intercept_scaling=1.0, dual=True, verbose=0, random_state=None, max_iter=1000)_

Linear Support Vector Regression.

Similar to SVR with parameter kernel='linear', but implemented in terms of liblinear rather than libsvm, so it has more flexibility in the choice of penalties and loss functions and should scale better to large numbers of samples.

This class supports both dense and sparse input.

Read more in the User Guide.

**Parameters**

- **C**: float, optional (default=1.0)

  Penalty parameter C of the error term. The penalty is a squared L2 penalty. The bigger this parameter, the less regularization is used.

- **loss**: string, ‘epsilon_insensitive’ or ‘squared_epsilon_insensitive’ (default=’epsilon_insensitive’)

  Specifies the loss function. ‘11’ is the epsilon-insensitive loss (standard SVR) while ‘12’ is the squared epsilon-insensitive loss.

- **epsilon**: float, optional (default=0.1)

  Epsilon parameter in the epsilon-insensitive loss function. Note that the value of this parameter depends on the scale of the target variable y. If unsure, set epsilon=0.
Select the algorithm to either solve the dual or primal optimization problem. Prefer dual=False when n_samples > n_features.

tol : float, optional (default=1e-4)
    Tolerance for stopping criteria.

fit_intercept : boolean, optional (default=True)
    Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (i.e. data is expected to be already centered).

intercept_scaling : float, optional (default=1)
    When self.fit_intercept is True, instance vector x becomes [x, self.intercept_scaling], i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic feature weight Note! the synthetic feature weight is subject to l1/l2 regularization as all other features. To lessen the effect of regularization on synthetic feature weight (and therefore on the intercept) intercept_scaling has to be increased.

verbose : int, (default=0)
    Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in liblinear that, if enabled, may not work properly in a multithreaded context.

random_state : int seed, RandomState instance, or None (default=None)
    The seed of the pseudo random number generator to use when shuffling the data.

max_iter : int, (default=1000)
    The maximum number of iterations to be run.

Attributes

coef_ : array, shape = [n_features] if n_classes == 2 else [n_classes, n_features]
    Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.
    
    coef_ is a readonly property derived from raw_coef_ that follows the internal memory layout of liblinear.

intercept_ : array, shape = [1] if n_classes == 2 else [n_classes]
    Constants in decision function.

See also:

LinearSVC
Implementation of Support Vector Machine classifier using the same library as this class (liblinear).

SVR
Implementation of Support Vector Machine regression using libsvm: the kernel can be non-linear but its SMO algorithm does not scale to large number of samples as LinearSVC does.

sklearn.linear_model.SGDRegressor
SGDRegressor can optimize the same cost function as LinearSVR by adjusting the penalty and loss parameters. In addition it requires less memory, allows incremental (online) learning, and implements various loss functions and regularization regimes.

Methods

decision_function(*args, **kwargs)
    DEPRECATED: and will be removed in 0.19.
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<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit the model according to the given training data.</td>
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<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<td><code>predict(X)</code></td>
<td>Predict using the linear model</td>
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<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__** (epsilon=0.0, tol=0.0001, C=1.0, loss='epsilon_insensitive', fit_intercept=True, intercept_scaling=1.0, dual=True, verbose=0, random_state=None, max_iter=1000)

**decision_function** (*args, **kwargs)

DEPRECATED: and will be removed in 0.19.

Decision function of the linear model.

**Parameters**
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.
- **Returns**
  - **C**: array, shape = (n_samples,)
  - Returns predicted values.

**fit (X, y, sample_weight=\text{None})**

Fit the model according to the given training data.

**Parameters**
- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  - Training vector, where n_samples in the number of samples and n_features is the number of features.
- **y**: array-like, shape = [n_samples]
  - Target vector relative to X
- **sample_weight**: array-like, shape = [n_samples], optional
  - Array of weights that are assigned to individual samples. If not provided, then each sample is given unit weight.

**Returns**
- **self**: object
  - Returns self.

**get_params (deep=\text{True})**

Get parameters for this estimator.

**Parameters**
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params**: mapping of string to any
  - Parameter names mapped to their values.

**predict (X)**

Predict using the linear model

**Parameters**
- **X**: {array-like, sparse matrix}, shape = (n_samples, n_features)
  - Samples.

**Returns**
- **C**: array, shape = (n_samples,)
Returns predicted values.

score(X, y, sample_weight=None)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $1 - \frac{u}{v}$, where $u$ is the regression sum of squares $\sum((y_{true} - y_{pred})^2)$ and $v$ is the residual sum of squares $\sum((y_{true} - y_{true.mean})^2)$. Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of $y$, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters:
- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns:
- **score**: float
  $R^2$ of self.predict(X) wrt. y.

set_params(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns:
- **self**: 

**sklearn.svm.NuSVR**

class sklearn.svm.NuSVR(nu=0.5, C=1.0, kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, tol=0.001, cache_size=200, verbose=False, max_iter=-1)

Nu Support Vector Regression.

Similar to NuSVC, for regression, uses a parameter nu to control the number of support vectors. However, unlike NuSVC, where nu replaces C, here nu replaces the parameter epsilon of epsilon-SVR.

The implementation is based on libsvm.

Read more in the User Guide.

Parameters:
- **C**: float, optional (default=1.0)
  Penalty parameter C of the error term.
- **nu**: float, optional
  An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval (0, 1]. By default 0.5 will be taken.
- **kernel**: string, optional (default='rbf')
  Specifies the kernel type to be used in the algorithm. It must be one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’ or a callable. If none is given, ‘rbf’ will be used. If a callable is given it is used to precompute the kernel matrix.
- **degree**: int, optional (default=3)
Degree of the polynomial kernel function (`poly`). Ignored by all other kernels.

- **gamma**: float, optional (default=`'auto'`)
  
  Kernel coefficient for `'rbf'`, `'poly'` and `'sigmoid'`. If gamma is `'auto'` then `1/n_features` will be used instead.

- **coef0**: float, optional (default=0.0)
  
  Independent term in kernel function. It is only significant in `'poly'` and `'sigmoid'`.

- **shrinking**: boolean, optional (default=True)
  
  Whether to use the shrinking heuristic.

- **tol**: float, optional (default=1e-3)
  
  Tolerance for stopping criterion.

- **cache_size**: float, optional
  
  Specify the size of the kernel cache (in MB).

- **verbose**: bool, default: False
  
  Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

- **max_iter**: int, optional (default=-1)
  
  Hard limit on iterations within solver, or -1 for no limit.

**Attributes**

- **support_**: array-like, shape = [n_SV]
  
  Indices of support vectors.

- **support_vectors_**: array-like, shape = [nSV, n_features]
  
  Support vectors.

- **dual_coef_**: array, shape = [1, n_SV]
  
  Coefficients of the support vector in the decision function.

- **coef_**: array, shape = [1, n_features]
  
  Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.

  `coef_` is readonly property derived from `dual_coef_` and `support_vectors_`.

- **intercept_**: array, shape = [1]
  
  Constants in decision function.

**See also:**

- **NuSVC**: Support Vector Machine for classification implemented with libsvm with a parameter to control the number of support vectors.

- **SVR**: epsilon Support Vector Machine for regression implemented with libsvm.

**Examples**
```python
>>> from sklearn.svm import NuSVR
>>> import numpy as np

>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)

>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)

>>> clf = NuSVR(C=1.0, nu=0.1)

>>> clf.fit(X, y)
NuSVR(C=1.0, cache_size=200, coef0=0.0, degree=3, gamma='auto',
    kernel='rbf', max_iter=-1, nu=0.1, shrinking=True, tol=0.001,
    verbose=False)
```

**Methods**

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<td><code>decision_function</code></td>
<td>Distance of the samples $X$ to the separating hyperplane.</td>
</tr>
<tr>
<td><code>fit</code></td>
<td>Fit the SVM model according to the given training data.</td>
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<tr>
<td><code>get_params</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>Perform regression on samples in $X$.</td>
</tr>
<tr>
<td><code>score</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__**

```python
__init__(nu=0.5, C=1.0, kernel='rbf', degree=3, gamma='auto', coef0=0.0, shrinking=True, tol=0.001, cache_size=200, verbose=False, max_iter=-1)
```

**decision_function**

```python
decision_function(*args, **kwargs)
```

DEPRECATED: and will be removed in 0.19

Distance of the samples $X$ to the separating hyperplane.

**Parameters**

- $X$: array-like, shape (n_samples, n_features)
  
  For kernel="precomputed", the expected shape of $X$ is [n_samples_test, n_samples_train].

**Returns**

- $X$: array-like, shape (n_samples, n_class * (n_class-1) / 2)
  
  Returns the decision function of the sample for each class in the model.

**fit**

```python
fit(X, y, sample_weight=None)
```

Fit the SVM model according to the given training data.

**Parameters**

- $X$: array-like, sparse matrix, shape (n_samples, n_features)
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features. For kernel="precomputed", the expected shape of $X$ is (n_samples, n_samples).

- $y$: array-like, shape (n_samples,)
  
  Target values (class labels in classification, real numbers in regression)

- `sample_weight`: array-like, shape (n_samples,)
  
  Per-sample weights. Rescale $C$ per sample. Higher weights force the classifier to put more emphasis on these points.

**Return**

- `self`: object

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Returns self.

Notes

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

get_params (deep=True)

Get parameters for this estimator.

Parameters:
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- params: mapping of string to any
  - Parameter names mapped to their values.

predict (X)

Perform regression on samples in X.

For an one-class model, +1 or -1 is returned.

Parameters:
- X: {array-like, sparse matrix}, shape (n_samples, n_features)
  - For kernel=’precomputed’, the expected shape of X is (n_samples_test, n_samples_train).

Returns:
- y_pred: array, shape (n_samples,)

score (X, y, sample_weight=None)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ($(y_{true} - y_{pred})^2$).sum() and $v$ is the residual sum of squares ($(y_{true} - y_{true}.mean())^2$).sum(). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y, disregarding the input features, would get a $R^2$ score of 0.0.

Parameters:
- X: array-like, shape = (n_samples, n_features)
  - Test samples.
- y: array-like, shape = (n_samples) or (n_samples, n_outputs)
  - True values for X.
- sample_weight: array-like, shape = [n_samples], optional
  - Sample weights.

Returns:
- score: float
  - $R^2$ of self.predict(X) wrt. y.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:
- self:
Examples using `sklearn.svm.NuSVR`

- *Model Complexity Influence*

```python
sklearn.svm.OneClassSVM
```

Parameters:

- `kernel`: string, optional (default='rbf')
  
  Specifies the kernel type to be used in the algorithm. It must be one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’ or a callable. If none is given, ‘rbf’ will be used. If a callable is given it is used to precompute the kernel matrix.

- `nu`: float, optional
  
  An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval (0, 1]. By default 0.5 will be taken.

- `degree`: int, optional (default=3)
  
  Degree of the polynomial kernel function (‘poly’). Ignored by all other kernels.

- `gamma`: float, optional (default='auto')
  
  Kernel coefficient for ‘rbf’, ‘poly’ and ‘sigmoid’. If gamma is ‘auto’ then 1/n_features will be used instead.

- `coef0`: float, optional (default=0.0)
  
  Independent term in kernel function. It is only significant in ‘poly’ and ‘sigmoid’.

- `tol`: float, optional
  
  Tolerance for stopping criterion.

- `shrinking`: boolean, optional
  
  Whether to use the shrinking heuristic.

- `cache_size`: float, optional
  
  Specify the size of the kernel cache (in MB).

- `verbose`: bool, default: False
  
  Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

- `max_iter`: int, optional (default=-1)
  
  Hard limit on iterations within solver, or -1 for no limit.

- `random_state`: int seed, RandomState instance, or None (default)
  
  The seed of the pseudo random number generator to use when shuffling the data for probability estimation.
Attributes

- `support_` : array-like, shape = [n_SV]
  Indices of support vectors.

- `support_vectors_` : array-like, shape = [n_SV, n_features]
  Support vectors.

- `dual_coef_` : array, shape = [n_classes-1, n_SV]
  Coefficients of the support vectors in the decision function.

- `coef_` : array, shape = [n_classes-1, n_features]
  Weights assigned to the features (coefficients in the primal problem). This is only available in the case of a linear kernel.
  
  `coef_` is readonly property derived from `dual_coef_` and `support_vectors_`

- `intercept_` : array, shape = [n_classes-1]
  Constants in decision function.

Methods

- `decision_function(X)`
  Distance of the samples X to the separating hyperplane.

- `fit(X[, y, sample_weight])`
  Detects the soft boundary of the set of samples X.

- `get_params([deep])`
  Get parameters for this estimator.

- `predict(X)`
  Perform regression on samples in X.

- `set_params(**params)`
  Set the parameters of this estimator.

`__init__(kernel='rbf', degree=3, gamma='auto', coef0=0.0, tol=0.001, nu=0.5, shrinking=True, cache_size=200, verbose=False, max_iter=-1, random_state=None)`

- `decision_function(X)`
  Distance of the samples X to the separating hyperplane.

  Parameters:
  - X : array-like, shape (n_samples, n_features)

  Returns:
  - X : array-like, shape (n_samples,)
    Returns the decision function of the samples.

- `fit(X, y=None, sample_weight=None, **params)`
  Detects the soft boundary of the set of samples X.

  Parameters:
  - X : {array-like, sparse matrix}, shape (n_samples, n_features)
    Set of samples, where n_samples is the number of samples and n_features is the number of features.

  - sample_weight : array-like, shape (n_samples,)
    Per-sample weights. Rescale C per sample. Higher weights force the classifier to put more emphasis on these points.

  Returns:
  - self : object
    Returns self.
Notes

If X is not a C-ordered contiguous array it is copied.

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

**predict**(X)

Perform regression on samples in X.

For an one-class model, +1 or -1 is returned.

**Parameters**

- **X**: {array-like, sparse matrix}, shape (n_samples, n_features)
  For kernel="precomputed", the expected shape of X is (n_samples_test, n_samples_train).

**Returns**

- **y_pred**: array, shape (n_samples,)

**set_params**(**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

- **self**:

Examples using **sklearn.svm.OneClassSVM**

- Outlier detection on a real data set
- Species distribution modeling
- Libsvm GUI
- Outlier detection with several methods.
- One-class SVM with non-linear kernel (RBF)

**sklearn.svm.l1_min_c**

Return the lowest bound for C such that for C in (l1_min_C, infinity) the model is guaranteed not to be empty.

```
svm.l1_min_c(X, y[, loss, fit_intercept, ...])
```

**sklearn.svm.l1_min_c**

Return the lowest bound for C such that for C in (l1_min_C, infinity) the model is guaranteed not to be empty. This applies to l1 penalized classifiers, such as LinearSVC with penalty='l1' and linear_model.LogisticRegression with penalty='l1'.

This value is valid if class_weight parameter in fit() is not set.
Parameters

- **X**: array-like or sparse matrix, shape = [n_samples, n_features]
  Training vector, where n_samples is the number of samples and n_features is the number of features.

- **y**: array, shape = [n_samples]
  Target vector relative to X

- **loss**: {'squared_hinge', 'log'}, default ‘squared_hinge’
  Specifies the loss function. With ‘squared_hinge’ it is the squared hinge loss (a.k.a. L2 loss). With ‘log’ it is the loss of logistic regression models. ‘l2’ is accepted as an alias for ‘squared_hinge’, for backward compatibility reasons, but should not be used in new code.

- **fit_intercept**: bool, default: True
  Specifies if the intercept should be fitted by the model. It must match the fit() method parameter.

- **intercept_scaling**: float, default: 1
  when fit_intercept is True, instance vector x becomes [x, intercept_scaling], i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. It must match the fit() method parameter.

Returns

- **C**: float
  minimum value for C

Examples using `sklearn.svm.l1_min_c`

- *Path with L1- Logistic Regression*

29.33.2 Low-level methods

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>svm.libsvm.fit</code></td>
<td>Train the model using libsvm (low-level method)</td>
</tr>
<tr>
<td><code>svm.libsvm.decision_function</code></td>
<td>Predict margin (libsvm name for this is predict_values)</td>
</tr>
<tr>
<td><code>svm.libsvm.predict</code></td>
<td>Predict target values of X given a model (low-level method)</td>
</tr>
<tr>
<td><code>svm.libsvm.predict_proba</code></td>
<td>Predict probabilities</td>
</tr>
<tr>
<td><code>svm.libsvm.cross_validation</code></td>
<td>Binding of the cross-validation routine (low-level routine)</td>
</tr>
</tbody>
</table>

`sklearn.svm.libsvm.fit`

`sklearn.svm.libsvm.fit()`
Train the model using libsvm (low-level method)

- **Parameters**
  - **X**: array-like, dtype=object, size=[n_samples, n_features]
    Training vector, where n_samples is the number of samples and n_features is the number of features.
  - **y**: array, dtype=object, size=[n_samples]
    Target vector
  - **svm_type**: {0, 1, 2, 3, 4}, optional
    Type of SVM: C_SVC, NuSVC, OneClassSVM, EpsilonSVR or NuSVR respectively. 0 by default.
**kernel**: {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}, optional

Kernel to use in the model: linear, polynomial, RBF, sigmoid or precomputed. 'rbf' by default.

**degree**: int32, optional

Degree of the polynomial kernel (only relevant if kernel is set to polynomial), 3 by default.

**gamma**: float64, optional

Gamma parameter in RBF kernel (only relevant if kernel is set to RBF). 0.1 by default.

**coef0**: float64, optional

Independent parameter in poly/sigmoid kernel. 0 by default.

**tol**: float64, optional

Numeric stopping criterion (WRITEME). 1e-3 by default.

**C**: float64, optional

C parameter in C-Support Vector Classification. 1 by default.

**nu**: float64, optional

0.5 by default.

**epsilon**: double, optional

0.1 by default.

**class_weight**: array, dtype float64, shape (n_classes,), optional

np.empty(0) by default.

**sample_weight**: array, dtype float64, shape (n_samples,), optional

np.empty(0) by default.

**shrinking**: int, optional

1 by default.

**probability**: int, optional

0 by default.

**cache_size**: float64, optional

Cache size for gram matrix columns (in megabytes). 100 by default.

**max_iter**: int (-1 for no limit), optional.

Stop solver after this many iterations regardless of accuracy (XXX Currently there is no API to know whether this kicked in.) -1 by default.

**random_seed**: int, optional

Seed for the random number generator used for probability estimates. 0 by default.

**Returnssupport**: array, shape=[n_support]

index of support vectors

**support_vectors**: array, shape=[n_support, n_features]
support vectors (equivalent to X[support]). Will return an empty array in the case of precomputed kernel.

**n_class_SV** : array
number of support vectors in each class.

**sv_coef** : array
coefficients of support vectors in decision function.

**intercept** : array
intercept in decision function

**probA, probB** : array
probability estimates, empty array for probability=False

### sklearn.svm.libsvm.decision_function

**sklearn.svm.libsvm.decision_function()**
Predict margin (libsvm name for this is predict_values)
We have to reconstruct model and parameters to make sure we stay in sync with the python object.

### sklearn.svm.libsvm.predict

**sklearn.svm.libsvm.predict()**
Predict target values of X given a model (low-level method)

**Parameters**:

- **X**: array-like, dtype=float, size=[n_samples, n_features]
- **svm_type**: {0, 1, 2, 3, 4}
  Type of SVM: C SVC, nu SVC, one class, epsilon SVR, nu SVR
- **kernel** : {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}
  Type of kernel.
- **degree** : int
  Degree of the polynomial kernel.
- **gamma** : float
  Gamma parameter in RBF kernel.
- **coef0** : float
  Independent parameter in poly/sigmoid kernel.

**Returns**:

- **dec_values** : array
  predicted values.

### sklearn.svm.libsvm.predict_proba

**sklearn.svm.libsvm.predict_proba()**
Predict probabilities

svm_model stores all parameters needed to predict a given value.
For speed, all real work is done at the C level in function copy_predict (libsvm_helper.c).
We have to reconstruct model and parameters to make sure we stay in sync with the python object.
See sklearn.svm.predict for a complete list of parameters.

**Parameters**

- **X**: array-like, dtype=float
  - **kernel**: {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}

**Returns**

- **dec_values**: array
  - predicted values.

---

**sklearn.svm.libsvm.cross_validation**

`sklearn.svm.libsvm.cross_validation()`

Binding of the cross-validation routine (low-level routine)

**Parameters**

- **X**: array-like, dtype=float, size=[n_samples, n_features]
- **Y**: array, dtype=float, size=[n_samples]
- **svm_type**: {0, 1, 2, 3, 4}
  - Type of SVM: C SVC, nu SVC, one class, epsilon SVR, nu SVR
- **kernel**: {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}
  - Kernel to use in the model: linear, polynomial, RBF, sigmoid or precomputed.
- **degree**: int
  - Degree of the polynomial kernel (only relevant if kernel is set to polynomial)
- **gamma**: float
  - Gamma parameter in RBF kernel (only relevant if kernel is set to RBF)
- **coef0**: float
  - Independent parameter in poly/sigmoid kernel.
- **tol**: float
  - Stopping criteria.
- **C**: float
  - C parameter in C-Support Vector Classification
- **nu**: float
- **cache_size**: float
- **random_seed**: int, optional
  - Seed for the random number generator used for probability estimates. 0 by default.

**Returns**

- **target**: array, float
29.34 sklearn.tree: Decision Trees

The `sklearn.tree` module includes decision tree-based models for classification and regression.

User guide: See the Decision Trees section for further details.

```
<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>tree.DecisionTreeClassifier([criterion, ...])</code></td>
<td>A decision tree classifier.</td>
</tr>
<tr>
<td><code>tree.DecisionTreeRegressor([criterion, ...])</code></td>
<td>A decision tree regressor.</td>
</tr>
<tr>
<td><code>tree.ExtraTreeClassifier([criterion, ...])</code></td>
<td>An extremely randomized tree classifier.</td>
</tr>
<tr>
<td><code>tree.ExtraTreeRegressor([criterion, ...])</code></td>
<td>An extremely randomized tree regressor.</td>
</tr>
</tbody>
</table>
```

29.34.1 sklearn.tree.DecisionTreeClassifier

```
class sklearn.tree.DecisionTreeClassifier (criterion='gini', splitter='best', max_depth=None,
                                        min_samples_split=2, min_samples_leaf=1,
                                        min_weight_fraction_leaf=0.0,
                                        max_features=None, random_state=None,
                                        max_leaf_nodes=None, min_impurity_split=1e-07,
                                        class_weight=None, presort=False)
```

A decision tree classifier.

Read more in the User Guide.

**Parameters**

- **criterion**: string, optional (default="gini")
  - The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain.

- **splitter**: string, optional (default="best")
  - The strategy used to choose the split at each node. Supported strategies are “best” to choose the best split and “random” to choose the best random split.

- **max_features**: int, float, string or None, optional (default=None)
  - The number of features to consider when looking for the best split:
    - If int, then consider `max_features` features at each split.
    - If float, then `max_features` is a percentage and `int(max_features * n_features)` features are considered at each split.
    - If “auto”, then `max_features`=`sqrt(n_features)`.
    - If “sqrt”, then `max_features`=`sqrt(n_features)`.
    - If “log2”, then `max_features`=`log2(n_features)`.
    - If None, then `max_features`=`n_features`.
  - Note: the search for a split does not stop until at least one valid partition of the node samples is found, even if it requires to effectively inspect more than `max_features` features.

- **max_depth**: int or None, optional (default=None)
  - The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than `min_samples_split` samples.

- **min_samples_split**: int, float, optional (default=2)
  - The minimum number of samples required to split an internal node:
• If int, then consider \( \text{min\_samples\_split} \) as the minimum number.

• If float, then \( \text{min\_samples\_split} \) is a percentage and \( \text{ceil}(\text{min\_samples\_split} \times n\_samples) \) are the minimum number of samples for each split.

Changed in version 0.18: Added float values for percentages.

**min\_samples\_leaf**: int, float, optional (default=1)

The minimum number of samples required to be at a leaf node:

• If int, then consider \( \text{min\_samples\_leaf} \) as the minimum number.

• If float, then \( \text{min\_samples\_leaf} \) is a percentage and \( \text{ceil}(\text{min\_samples\_leaf} \times n\_samples) \) are the minimum number of samples for each node.

Changed in version 0.18: Added float values for percentages.

**min\_weight\_fraction\_leaf**: float, optional (default=0.0)

The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.

**max\_leaf\_nodes**: int or None, optional (default=None)

Grow a tree with \( \text{max\_leaf\_nodes} \) in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

**class\_weight**: dict, list of dicts, “balanced” or None, optional (default=None)

Weights associated with classes in the form \{\text{class\_label}: \text{weight}\}. If not given, all classes are supposed to have weight one. For multi-output problems, a list of dicts can be provided in the same order as the columns of \( y \).

The “balanced” mode uses the values of \( y \) to automatically adjust weights inversely proportional to class frequencies in the input data as \( n\_samples / (n\_classes \times \text{np.bincount}(y)) \)

For multi-output, the weights of each column of \( y \) will be multiplied.

Note that these weights will be multiplied with sample_weight (passed through the fit method) if sample_weight is specified.

**random\_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**min\_impurity\_split**: float, optional (default=1e-7)

Threshold for early stopping in tree growth. A node will split if its impurity is above the threshold, otherwise it is a leaf.

New in version 0.18.

**presort**: bool, optional (default=False)

Whether to presort the data to speed up the finding of best splits in fitting. For the default settings of a decision tree on large datasets, setting this to true may slow down the training process. When using either a smaller dataset or a restricted depth, this may speed up the training.

**Attributes**

- \text{classes\_}: array of shape = [n\_classes] or a list of such arrays
The classes labels (single output problem), or a list of arrays of class labels (multi-output problem).

**feature_importances_** : array of shape = [n_features]

The feature importances. The higher, the more important the feature. The importance of a feature is computed as the (normalized) total reduction of the criterion brought by that feature. It is also known as the Gini importance [R78].

**max_features_** : int,

The inferred value of max_features.

**n_classes_** : int or list

The number of classes (for single output problems), or a list containing the number of classes for each output (for multi-output problems).

**n_features_** : int

The number of features when fit is performed.

**n_outputs_** : int

The number of outputs when fit is performed.

**tree_** : Tree object

The underlying Tree object.

See also:

*DecisionTreeRegressor*

References

[R75], [R76], [R77], [R78]

Examples

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn.model_selection import cross_val_score
>>> from sklearn.tree import DecisionTreeClassifier
>>> clf = DecisionTreeClassifier(random_state=0)
>>> iris = load_iris()
>>> cross_val_score(clf, iris.data, iris.target, cv=10)
... array([1. , 0.93..., 0.86..., 0.93..., 0.93..., 0.93..., 0.93..., 1. , 0.93..., 1. ])
```

Methods

<table>
<thead>
<tr>
<th>apply(X[, check_input])</th>
<th>Returns the index of the leaf that each sample is predicted as.</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_path(X[, check_input])</td>
<td>Return the decision path in the tree</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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<tbody>
<tr>
<td><code>fit(X, y[, sample_weight, check_input, ...])</code></td>
<td>Build a decision tree classifier from the training set (X, y).</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit data, then transform it.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td><code>predict(X[, check_input])</code></td>
<td>Predict class or regression value for X.</td>
</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>Predict class log-probabilities of the input samples X.</td>
</tr>
<tr>
<td><code>predict_proba(X[, check_input])</code></td>
<td>Predict class probabilities of the input samples X.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(*args, **kwargs)</code></td>
<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
</tr>
</tbody>
</table>

```python
__init__(criterion='gini', splitter='best', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features=None, random_state=None, max_leaf_nodes=None, min_impurity_split=1e-07, class_weight=None, presort=False)
```

**apply** *(X, check_input=True)*

Returns the index of the leaf that each sample is predicted as.

New in version 0.17.

**Parameters**

- **X**: array_like or sparse matrix, shape = [n_samples, n_features]
  - The input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csr_matrix`.

- **check_input**: boolean, (default=True)
  - Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Returns**

- **X_leaves**: array_like, shape = [n_samples,]
  - For each datapoint x in X, return the index of the leaf x ends up in. Leaves are numbered within [0; self.tree_.node_count), possibly with gaps in the numbering.

**decision_path** *(X, check_input=True)*

Return the decision path in the tree

New in version 0.18.

**Parameters**

- **X**: array_like or sparse matrix, shape = [n_samples, n_features]
  - The input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csr_matrix`.

- **check_input**: boolean, (default=True)
  - Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Returns**

- **indicator**: sparse csr array, shape = [n_samples, n_nodes]
  - Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

**feature_importances_**

Return the feature importances.
The importance of a feature is computed as the (normalized) total reduction of the criterion brought by that feature. It is also known as the Gini importance.

**Returns** feature_importances_: array, shape = [n_features]

**fit** (X, y, sample_weight=None, check_input=True, X_idx_sorted=None)
Build a decision tree classifier from the training set (X, y).

**Parameters**

- **X**: array-like or sparse matrix, shape = [n_samples, n_features]
  - The training input samples. Internally, it will be converted to dtype=np.float32 and if a sparse matrix is provided to a sparse csc_matrix.

- **y**: array-like, shape = [n_samples] or [n_samples, n_outputs]
  - The target values (class labels) as integers or strings.

- **sample_weight**: array-like, shape = [n_samples] or None
  - Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node. Splits are also ignored if they would result in any single class carrying a negative weight in either child node.

- **check_input**: boolean, (default=True)
  - Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

- **X_idx_sorted**: array-like, shape = [n_samples, n_features], optional
  - The indexes of the sorted training input samples. If many tree are grown on the same dataset, this allows the ordering to be cached between trees. If None, the data will be sorted here. Don’t use this parameter unless you know what to do.

**Returns** self: object
Returns self.

**fit_transform** (X, y=None, **fit_params)
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.

- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**get_params** (deep=True)
Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  - Parameter names mapped to their values.
**predict** \( (X, \text{check\_input}=\text{True}) \)

Predict class or regression value for \( X \).

For a classification model, the predicted class for each sample in \( X \) is returned. For a regression model, the predicted value based on \( X \) is returned.

**Parameters**

- **X** : array-like or sparse matrix of shape \([n\_samples, n\_features]\)
  The input samples. Internally, it will be converted to \( \text{dtype} = \text{np\_float32} \) and if a sparse matrix is provided to a sparse \( \text{csr\_matrix} \).

- **check\_input** : boolean, default=True
  Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Returns**

- **y** : array of shape \([n\_samples]\) or \([n\_samples, n\_outputs]\)
  The predicted classes, or the predict values.

**predict\_log\_proba** \( (X) \)

Predict class log-probabilities of the input samples \( X \).

**Parameters**

- **X** : array-like or sparse matrix of shape \([n\_samples, n\_features]\)
  The input samples. Internally, it will be converted to \( \text{dtype} = \text{np\_float32} \) and if a sparse matrix is provided to a sparse \( \text{csr\_matrix} \).

- **Returns**
  - **p** : array of shape \([n\_samples, n\_classes]\), or a list of \( n\_outputs \) such arrays if \( n\_outputs > 1 \). The class log-probabilities of the input samples. The order of the classes corresponds to that in the attribute \( \text{classes\_} \).

**predict\_proba** \( (X, \text{check\_input}=\text{True}) \)

Predict class probabilities of the input samples \( X \).

The predicted class probability is the fraction of samples of the same class in a leaf.

**Parameters**

- **X** : array-like or sparse matrix of shape \([n\_samples, n\_features]\)
  The input samples. Internally, it will be converted to \( \text{dtype} = \text{np\_float32} \) and if a sparse matrix is provided to a sparse \( \text{csr\_matrix} \).

- **Returns**
  - **p** : array of shape \([n\_samples, n\_classes]\), or a list of \( n\_outputs \) such arrays if \( n\_outputs > 1 \). The class probabilities of the input samples. The order of the classes corresponds to that in the attribute \( \text{classes\_} \).

**score** \( (X, y, \text{sample\_weight}=\text{None}) \)

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**

- **X** : array-like, shape \((n\_samples, n\_features)\)
  Test samples.

- **y** : array-like, shape \((n\_samples)\) or \((n\_samples, n\_outputs)\)
  True labels for \( X \).

- **sample\_weight** : array-like, shape \([n\_samples]\), optional
Sample weights.

**Returns**

score : float
Mean accuracy of self.predict(X) wrt. y.

**set_params** (**params**)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

self :

**transform** (*args, **kwargs*)
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses coef_ or feature_importances_ to determine the most important features. For models with a coef_ for each class, the absolute sum over the classes is used.

**Parameters**

X : array or scipy sparse matrix of shape [n_samples, n_features]
The input samples.

threshold [string, float or None, optional (default=None)] The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

X_r : array of shape [n_samples, n_selected_features]
The input samples with only the selected features.

Examples using sklearn.tree.DecisionTreeClassifier

- Classifier comparison
- Discrete versus Real AdaBoost
- Multi-class AdaBoosted Decision Trees
- Two-class AdaBoost
- Plot the decision surfaces of ensembles of trees on the iris dataset
- Plot the decision boundaries of a VotingClassifier
- Plot the decision surface of a decision tree on the iris dataset
- Understanding the decision tree structure
29.34.2 sklearn.tree.DecisionTreeRegressor

class sklearn.tree.DecisionTreeRegressor(criterion='mse', splitter='best',
                                       max_depth=None,
                                       min_samples_split=2, min_samples_leaf=1,
                                       min_weight_fraction_leaf=0.0, max_features=None,
                                       random_state=None, max_leaf_nodes=None,
                                       min_impurity_split=1e-07, presort=False)

A decision tree regressor.

Read more in the User Guide.

Parameters:

criterion : string, optional (default="mse")

The function to measure the quality of a split. Supported criteria are “mse” for the
mean squared error, which is equal to variance reduction as feature selection criterion,
and “mae” for the mean absolute error.

New in version 0.18: Mean Absolute Error (MAE) criterion.

splitter : string, optional (default="best")

The strategy used to choose the split at each node. Supported strategies are “best” to
choose the best split and “random” to choose the best random split.

max_features : int, float, string or None, optional (default=None)

The number of features to consider when looking for the best split:

• If int, then consider max_features features at each split.
• If float, then max_features is a percentage and int(max_features * n_features) features
  are considered at each split.
• If “auto”, then max_features=n_features.
• If “sqrt”, then max_features=sqrt(n_features).
• If “log2”, then max_features=log2(n_features).
• If None, then max_features=n_features.

Note: the search for a split does not stop until at least one valid partition of the node
samples is found, even if it requires to effectively inspect more than max_features
features.

max_depth : int or None, optional (default=None)

The maximum depth of the tree. If None, then nodes are expanded until all leaves are
pure or until all leaves contain less than min_samples_split samples.

min_samples_split : int, float, optional (default=2)

The minimum number of samples required to split an internal node:

• If int, then consider min_samples_split as the minimum number.
• If float, then min_samples_split is a percentage and ceil(min_samples_split * n_samples)
  are the minimum number of samples for each split.

Changed in version 0.18: Added float values for percentages.

min_samples_leaf : int, float, optional (default=1)

The minimum number of samples required to be at a leaf node:

• If int, then consider min_samples_leaf as the minimum number.
• If float, then \( \text{min\_samples\_leaf} \) is a percentage and \( \text{cei}(\text{min\_samples\_leaf} * n\_samples) \) are the minimum number of samples for each node.

Changed in version 0.18: Added float values for percentages.

\textbf{min\_weight\_fraction\_leaf} : float, optional (default=0.)

The minimum weighted fraction of the sum total of weights (of all the input samples) required to be at a leaf node. Samples have equal weight when sample_weight is not provided.

\textbf{max\_leaf\_nodes} : int or None, optional (default=None)

Grow a tree with \text{max\_leaf\_nodes} in best-first fashion. Best nodes are defined as relative reduction in impurity. If None then unlimited number of leaf nodes.

\textbf{random\_state} : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, \text{random\_state} is the random number generator; If None, the random number generator is the RandomState instance used by \text{np.random}.

\textbf{min\_impurity\_split} : float, optional (default=1e-7)

Threshold for early stopping in tree growth. If the impurity of a node is below the threshold, the node is a leaf.

New in version 0.18.

\textbf{presort} : bool, optional (default=False)

Whether to presort the data to speed up the finding of best splits in fitting. For the default settings of a decision tree on large datasets, setting this to true may slow down the training process. When using either a smaller dataset or a restricted depth, this may speed up the training.

\textbf{Attributes feature\_importances\_} : array of shape = [n\_features]

The feature importances. The higher, the more important the feature. The importance of a feature is computed as the (normalized) total reduction of the criterion brought by that feature. It is also known as the Gini importance \cite{R82}.

\textbf{max\_features\_} : int,

The inferred value of max_features.

\textbf{n\_features\_} : int

The number of features when \text{fit} is performed.

\textbf{n\_outputs\_} : int

The number of outputs when \text{fit} is performed.

\textbf{tree\_} : Tree object

The underlying Tree object.

\textbf{See also:}

\textit{DecisionTreeClassifier}

\textbf{References}

\cite{R79}, \cite{R80}, \cite{R81}, \cite{R82}
## Examples

```python
>>> from sklearn.datasets import load_boston
>>> from sklearn.model_selection import cross_val_score
>>> from sklearn.tree import DecisionTreeRegressor

>>> boston = load_boston()
>>> regressor = DecisionTreeRegressor(random_state=0)
>>> cross_val_score(regressor, boston.data, boston.target, cv=10)

...  
...  
array([ 0.61..., 0.57..., -0.34..., 0.41..., 0.75..., 0.07..., 0.29..., 0.33..., -1.42..., -1.77...])
```

## Methods

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<td>Returns the index of the leaf that each sample is predicted as.</td>
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<td><code>decision_path(X[, check_input])</code></td>
<td>Return the decision path in the tree.</td>
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<tr>
<td><code>fit(X, y[, sample_weight, check_input, ...])</code></td>
<td>Build a decision tree regressor from the training set (X, y).</td>
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<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
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<td><code>predict(X[, check_input])</code></td>
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<td><code>score(X, y[, sample_weight])</code></td>
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<td><code>set_params(**params)</code></td>
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<td><code>transform(**args, **kwargs)</code></td>
<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
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### `__init__` (criterion='mse', splitter='best', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features=None, random_state=None, max_leaf_nodes=None, min_impurity_split=1e-07, presort=False)

**apply** (X, check_input=True)

Returns the index of the leaf that each sample is predicted as.

New in version 0.17.

**Parameters**

- **X**: array_like or sparse matrix, shape = [n_samples, n_features]
  
  The input samples. Internally, it will be converted to dtype=np.float32 and if a sparse matrix is provided to a sparse csr_matrix.

- **check_input**: boolean, (default=True)
  
  Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Returns**

- **X_leaves**: array_like, shape = [n_samples,]
  
  For each datapoint x in X, return the index of the leaf x ends up in. Leaves are numbered within [0; self.tree_.node_count), possibly with gaps in the numbering.

### `decision_path` (X, check_input=True)

Return the decision path in the tree

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New in version 0.18.

**Parameters**

- **X**: array_like or sparse matrix, shape = [n_samples, n_features]
  
  The input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csr_matrix`.

- **check_input**: boolean, (default=True)
  
  Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Returns**

- **indicator**: sparse csr array, shape = [n_samples, n_nodes]
  
  Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

**feature_importances_**

Return the feature importances.

The importance of a feature is computed as the (normalized) total reduction of the criterion brought by that feature. It is also known as the Gini importance.

**Returns**

- **feature_importances_**: array, shape = [n_features]

**fit** *(X, y, sample_weight=None, check_input=True, X_idx_sorted=None)*

Build a decision tree regressor from the training set (X, y).

**Parameters**

- **X**: array-like or sparse matrix, shape = [n_samples, n_features]
  
  The training input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csc_matrix`.

- **y**: array-like, shape = [n_samples] or [n_samples, n_outputs]
  
  The target values (real numbers). Use `dtype=np.float64` and `order='C'` for maximum efficiency.

- **sample_weight**: array-like, shape = [n_samples] or None
  
  Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node.

- **check_input**: boolean, (default=True)
  
  Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

- **X_idx_sorted**: array-like, shape = [n_samples, n_features], optional
  
  The indexes of the sorted training input samples. If many tree are grown on the same dataset, this allows the ordering to be cached between trees. If None, the data will be sorted here. Don’t use this parameter unless you know what to do.

**Returns**

- **self**: object
  
  Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it.

Fits transformer to X and y with optional parameters `fit_params` and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  
  Training set.
y : numpy array of shape [n_samples]
    Target values.

Returns

X_new : numpy array of shape [n_samples, n_features_new]
    Transformed array.

get_params (deep=True)
    Get parameters for this estimator.

Parameters

deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are
    estimators.

Returns

params : mapping of string to any
    Parameter names mapped to their values.

predict (X, check_input=True)
    Predict class or regression value for X.

    For a classification model, the predicted class for each sample in X is returned. For a regression model,
    the predicted value based on X is returned.

Parameters

X : array-like or sparse matrix of shape = [n_samples, n_features]
    The input samples. Internally, it will be converted to dtype=np.float32 and if a
    sparse matrix is provided to a sparse csr_matrix.

check_input : boolean, (default=True)
    Allow to bypass several input checking. Don’t use this parameter unless you know what
    you do.

Returns

y : array of shape = [n_samples] or [n_samples, n_outputs]
    The predicted classes, or the predict values.

score (X, y, sample_weight=None)
    Returns the coefficient of determination R^2 of the prediction.

    The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) **
    2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score
    is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always
    predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

Parameters

X : array-like, shape = (n_samples, n_features)
    Test samples.

y : array-like, shape = (n_samples) or (n_samples, n_outputs)
    True values for X.

sample_weight : array-like, shape = [n_samples], optional
    Sample weights.

Returns

score : float
    R^2 of self.predict(X) wrt. y.

set_params (**params)
    Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**: `self`  
`transform(*args, **kwargs)`  
DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce X to its most important features.

Uses `coef_` or `feature_importances_` to determine the most important features. For models with a `coef_` for each class, the absolute sum over the classes is used.

**Parameters**

- `X` : array or scipy sparse matrix of shape [n_samples, n_features]  
The input samples.

- `threshold` [string, float or None, optional (default=None)]  
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns** `X_r` : array of shape [n_samples, n_selected_features]  
The input samples with only the selected features.

**Examples using sklearn.tree.DecisionTreeRegressor**

- Decision Tree Regression with AdaBoost
- Single estimator versus bagging: bias-variance decomposition
- Decision Tree Regression
- Multi-output Decision Tree Regression

**29.34.3 sklearn.tree.ExtraTreeClassifier**

```python  
class sklearn.tree.ExtraTreeClassifier (criterion='gini', splitter='random', max_depth=None,  
min_samples_split=2, min_samples_leaf=1,  
min_weight_fraction_leaf=0.0, max_features='auto',  
random_state=None, max_leaf_nodes=None,  
min_impurity_split=1e-07, class_weight=None)  
```

An extremely randomized tree classifier.

Extra-trees differ from classic decision trees in the way they are built. When looking for the best split to separate the samples of a node into two groups, random splits are drawn for each of the `max_features` randomly selected features and the best split among those is chosen. When `max_features` is set 1, this amounts to building a totally random decision tree.

Warning: Extra-trees should only be used within ensemble methods.

Read more in the **User Guide**.

See also:
References

[R250]

Methods

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<thead>
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<th>apply(X[, check_input])</th>
<th>Returns the index of the leaf that each sample is predicted as.</th>
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<td>decision_path(X[, check_input])</td>
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</tr>
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<td>fit(X, y[, sample_weight, check_input, ...])</td>
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<td>Fit to data, then transform it.</td>
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<td>Get parameters for this estimator.</td>
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<tr>
<td>predict(X[, check_input])</td>
<td>Predict class or regression value for X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Predict class log-probabilities of the input samples X.</td>
</tr>
<tr>
<td>predict_proba(X[, check_input])</td>
<td>Predict class probabilities of the input samples X.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(*args, **kwargs)</td>
<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
</tr>
</tbody>
</table>

__init__(criterion='gini', splitter='random', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', random_state=None, max_leaf_nodes=None, min_impurity_split=1e-07, class_weight=None)

apply (X, check_input=True)
Returns the index of the leaf that each sample is predicted as.

New in version 0.17.

Parameters

X : array_like or sparse matrix, shape = [n_samples, n_features]
The input samples. Internally, it will be converted to dtype=np.float32 and if a sparse matrix is provided to a sparse csr_matrix.

check_input : boolean, (default=True)
Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

Returns

X_leaves : array_like, shape = [n_samples,]
For each datapoint x in X, return the index of the leaf x ends up in. Leaves are numbered within [0; self.tree_.node_count), possibly with gaps in the numbering.

decision_path (X, check_input=True)
Return the decision path in the tree

New in version 0.18.

Parameters

X : array_like or sparse matrix, shape = [n_samples, n_features]
The input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csr_matrix`.

**check_input** : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**Returns**

**indicator** : sparse csr array, shape = [n_samples, n_nodes]

Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

**feature_importances_**

Return the feature importances.

The importance of a feature is computed as the (normalized) total reduction of the criterion brought by that feature. It is also known as the Gini importance.

**Returns**

**feature_importances_** : array, shape = [n_features]

**fit(X, y, sample_weight=None, check_input=True, X_idx_sorted=None)**

Build a decision tree classifier from the training set (X, y).

**Parameters**

**X** : array-like or sparse matrix, shape = [n_samples, n_features]

The training input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csc_matrix`.

**y** : array-like, shape = [n_samples] or [n_samples, n_outputs]

The target values (class labels) as integers or strings.

**sample_weight** : array-like, shape = [n_samples] or None

Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node. Splits are also ignored if they would result in any single class carrying a negative weight in either child node.

**check_input** : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

**X_idx_sorted** : array-like, shape = [n_samples, n_features], optional

The indexes of the sorted training input samples. If many tree are grown on the same dataset, this allows the ordering to be cached between trees. If None, the data will be sorted here. Don’t use this parameter unless you know what to do.

**Returns**

**self** : object

Returns self.

**fit_transform(X, y=None, **fit_params)**

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

**X** : numpy array of shape [n_samples, n_features]

Training set.

**y** : numpy array of shape [n_samples]

Target values.
**Returns**

`X_new` : numpy array of shape [n_samples, n_features_new]

Transformed array.

**get_params** *(deep=True)*

Get parameters for this estimator.

**Parameters**

`deep` : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

**Returns**

`params` : mapping of string to any

Parameter names mapped to their values.

**predict** *(X, check_input=True)*

Predict class or regression value for X.

For a classification model, the predicted class for each sample in X is returned. For a regression model,
the predicted value based on X is returned.

**Parameters**

`X` : array-like or sparse matrix of shape = [n_samples, n_features]

The input samples. Internally, it will be converted to dtype=np.float32 and if a
sparse matrix is provided to a sparse csr_matrix.

`check_input` : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what
you do.

**Returns**

`y` : array of shape = [n_samples] or [n_samples, n_outputs]

The predicted classes, or the predict values.

**predict_log_proba** *(X)*

Predict class log-probabilities of the input samples X.

**Parameters**

`X` : array-like or sparse matrix of shape = [n_samples, n_features]

The input samples. Internally, it will be converted to dtype=np.float32 and if a
sparse matrix is provided to a sparse csr_matrix.

**Returns**

`p` : array of shape = [n_samples, n_classes], or a list of n_outputs

such arrays if n_outputs > 1. The class log-probabilities of the input samples. The order
of the classes corresponds to that in the attribute `classes_`.

**predict_proba** *(X, check_input=True)*

Predict class probabilities of the input samples X.

The predicted class probability is the fraction of samples of the same class in a leaf.

`check_input` : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what
you do.

**Parameters**

`X` : array-like or sparse matrix of shape = [n_samples, n_features]

The input samples. Internally, it will be converted to dtype=np.float32 and if a
sparse matrix is provided to a sparse csr_matrix.

**Returns**

`p` : array of shape = [n_samples, n_classes], or a list of n_outputs

such arrays if n_outputs > 1. The class probabilities of the input samples. The order
of the classes corresponds to that in the attribute `classes_`.
score$(X, y, \text{sample\_weight}=\text{None})$

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

Parameters:
- $X$ : array-like, shape = (n_samples, n_features)
  Test samples.
- $y$ : array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for $X$.
- $\text{sample\_weight}$ : array-like, shape = [n_samples], optional
  Sample weights.

Returns:
- $\text{score}$ : float
  Mean accuracy of self.predict$(X)$ wrt. $y$.

set_params (**params)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form $\text{<component>\_<parameter>}$ so that it’s possible to update each component of a nested object.

Returns:
- $\text{self}$

transform(*args, **kwargs)**

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce $X$ to its most important features.

Uses $\text{coef\_}$ or $\text{feature\_importances\_}$ to determine the most important features. For models with a $\text{coef\_}$ for each class, the absolute sum over the classes is used.

Parameters:
- $X$ : array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.
- $\text{threshold}$ [string, float or None, optional (default=None)]
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute $\text{threshold}$ is used. Otherwise, “mean” is used by default.

Returns:
- $X_r$ : array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.
### 29.34.4 `sklearn.tree.ExtraTreeRegressor`

Class `sklearn.tree.ExtraTreeRegressor`(`criterion='mse', splitter='random', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', random_state=None, min_impurity_split=1e-07, max_leaf_nodes=None)``

An extremely randomized tree regressor.

Extra-trees differ from classic decision trees in the way they are built. When looking for the best split to separate the samples of a node into two groups, random splits are drawn for each of the `max_features` randomly selected features and the best split among those is chosen. When `max_features` is set 1, this amounts to building a totally random decision tree.

Warning: Extra-trees should only be used within ensemble methods.

Read more in the User Guide.

See also: `ExtraTreeClassifier`, `ExtraTreesClassifier`, `ExtraTreesRegressor`

References

[R251]

Methods

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<td><code>apply(X[, check_input])</code></td>
<td>Returns the index of the leaf that each sample is predicted as.</td>
</tr>
<tr>
<td><code>decision_path(X[, check_input])</code></td>
<td>Return the decision path in the tree</td>
</tr>
<tr>
<td><code>fit(X, y[, sample_weight, check_input, ...])</code></td>
<td>Build a decision tree regressor from the training set (X, y).</td>
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<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
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<td><code>get_params([deep])</code></td>
<td>Get parameters for this estimator.</td>
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<tr>
<td><code>predict(X[, check_input])</code></td>
<td>Predict class or regression value for X.</td>
</tr>
<tr>
<td><code>score(X, y[, sample_weight])</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td><code>transform(**args, **kwargs)</code></td>
<td>DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19.</td>
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</tbody>
</table>

__init__(`criterion='mse', splitter='random', max_depth=None, min_samples_split=2, min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', random_state=None, min_impurity_split=1e-07, max_leaf_nodes=None)``

``apply`(X, check_input=True)

Returns the index of the leaf that each sample is predicted as.

New in version 0.17.

Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td><code>X</code></td>
<td>array_like or sparse matrix, shape = [n_samples, n_features]</td>
</tr>
</tbody>
</table>

The input samples. Internally, it will be converted to `dtype=np.float32` and if a sparse matrix is provided to a sparse `csr_matrix`. 

---

29.34. `sklearn.tree`: Decision Trees

1907
check_input : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

ReturnsX_leaves : array_like, shape = [n_samples,]

For each datapoint x in X, return the index of the leaf x ends up in. Leaves are numbered within [0; self.tree_.node_count), possibly with gaps in the numbering.

decision_path (X, check_input=True)

Return the decision path in the tree

New in version 0.18.

ParametersX : array_like or sparse matrix, shape = [n_samples, n_features]

The input samples. Internally, it will be converted to dtype=np.float32 and if a sparse matrix is provided to a sparse csr_matrix.

check_input : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

Returnsindicator : sparse csr array, shape = [n_samples, n_nodes]

Return a node indicator matrix where non zero elements indicates that the samples goes through the nodes.

feature_importances_

Return the feature importances.

The importance of a feature is computed as the (normalized) total reduction of the criterion brought by that feature. It is also known as the Gini importance.

Returnsfeature_importances_ : array, shape = [n_features]

fit (X, y, sample_weight=None, check_input=True, X_idx_sorted=None)

Build a decision tree regressor from the training set (X, y).

ParametersX : array-like or sparse matrix, shape = [n_samples, n_features]

The training input samples. Internally, it will be converted to dtype=np.float32 and if a sparse matrix is provided to a sparse csc_matrix.

y : array-like, shape = [n_samples] or [n_samples, n_outputs]

The target values (real numbers). Use dtype=np.float64 and order='C' for maximum efficiency.

sample_weight : array-like, shape = [n_samples] or None

Sample weights. If None, then samples are equally weighted. Splits that would create child nodes with net zero or negative weight are ignored while searching for a split in each node.

check_input : boolean, (default=True)

Allow to bypass several input checking. Don’t use this parameter unless you know what you do.

X_idx_sorted : array-like, shape = [n_samples, n_features], optional
The indexes of the sorted training input samples. If many tree are grown on the same
dataset, this allows the ordering to be cached between trees. If None, the data will be
sorted here. Don’t use this parameter unless you know what to do.

Returns

*object*:

Returns self.

**fit_transform** (*X*, *y=None, **fit_params*)

Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

* X : numpy array of shape [n_samples, n_features]
  
  Training set.

* y : numpy array of shape [n_samples]
  
  Target values.

**Returns**

* X_new : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**get_params** (*deep=True*)

Get parameters for this estimator.

**Parameters**

* deep : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are
  estimators.

**Returns**

* params : mapping of string to any
  
  Parameter names mapped to their values.

**predict** (*X*, *check_input=True*)

Predict class or regression value for X.

For a classification model, the predicted class for each sample in X is returned. For a regression model,
the predicted value based on X is returned.

**Parameters**

* X : array-like or sparse matrix of shape = [n_samples, n_features]
  
  The input samples. Internally, it will be converted to dtype=np.float32 and if a
  sparse matrix is provided to a sparse csr_matrix.

* check_input : boolean, (default=True)
  
  Allow to bypass several input checking. Don’t use this parameter unless you know what
  you do.

**Returns**

* y : array of shape = [n_samples] or [n_samples, n_outputs]
  
  The predicted classes, or the predict values.

**score** (*X*, *y*, *sample_weight=None*)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred) **
2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score
is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always
predicts the expected value of y, disregarding the input features, would get a R^2 score of 0.0.

**Parameters**

* X : array-like, shape = (n_samples, n_features)
Test samples.

\(y\): array-like, shape = (n_samples) or (n_samples, n_outputs)

True values for \(X\).

\textbf{sample_weight}: array-like, shape = [n_samples], optional

Sample weights.

Returns \textbf{score}: float

\(R^2\) of \(\text{self.predict}(X)\) wrt. \(y\).

\textbf{set_params} (**\text{params})

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form \(<\text{component}>__<\text{parameter}>\) so that it’s possible to update each component of a nested object.

Returns \textbf{self}:

\textbf{transform} (*\text{args}, **\text{kwargs})

DEPRECATED: Support to use estimators as feature selectors will be removed in version 0.19. Use SelectFromModel instead.

Reduce \(X\) to its most important features.

Uses \texttt{coef_} or \texttt{feature_importances_} to determine the most important features. For models with a \texttt{coef_} for each class, the absolute sum over the classes is used.

\textbf{Parameters} \(X\) : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

\textbf{threshold}[string, float or None, optional (default=None)] The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute \texttt{threshold} is used. Otherwise, “mean” is used by default.

Returns \(X_r\): array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

\texttt{tree.export_graphviz}

Export a decision tree in DOT format.

\texttt{29.34.5 sklearn.tree.export_graphviz}

\texttt{sklearn.tree.export_graphviz()}

Export a decision tree in DOT format.

This function generates a GraphViz representation of the decision tree, which is then written into \texttt{out_file}. Once exported, graphical renderings can be generated using, for example:

\begin{verbatim}
$ dot -Tps tree.dot -o tree.ps (PostScript format)
$ dot -Tpng tree.dot -o tree.png (PNG format)
\end{verbatim}
The sample counts that are shown are weighted with any sample_weights that might be present.

Read more in the User Guide.

**Parameters**

- **decision_tree**: decision tree classifier
  The decision tree to be exported to GraphViz.

- **out_file**: file object or string, optional (default='tree.dot')
  Handle or name of the output file. If None, the result is returned as a string. This will default from version 0.20.

- **max_depth**: int, optional (default=None)
  The maximum depth of the representation. If None, the tree is fully generated.

- **feature_names**: list of strings, optional (default=None)
  Names of each of the features.

- **class_names**: list of strings, bool or None, optional (default=None)
  Names of each of the target classes in ascending numerical order. Only relevant for classification and not supported for multi-output. If True, shows a symbolic representation of the class name.

- **label**: {'all', 'root', 'none'}, optional (default='all')
  Whether to show informative labels for impurity, etc. Options include 'all' to show at every node, 'root' to show only at the top root node, or 'none' to not show at any node.

- **filled**: bool, optional (default=False)
  When set to True, paint nodes to indicate majority class for classification, extremity of values for regression, or purity of node for multi-output.

- **leaves_parallel**: bool, optional (default=False)
  When set to True, draw all leaf nodes at the bottom of the tree.

- **impurity**: bool, optional (default=True)
  When set to True, show the impurity at each node.

- **node_ids**: bool, optional (default=False)
  When set to True, show the ID number on each node.

- **proportion**: bool, optional (default=False)
  When set to True, change the display of 'values' and/or 'samples' to be proportions and percentages respectively.

- **rotate**: bool, optional (default=False)
  When set to True, orient tree left to right rather than top-down.

- **rounded**: bool, optional (default=False)
  When set to True, draw node boxes with rounded corners and use Helvetica fonts instead of Times-Roman.

- **special_characters**: bool, optional (default=False)
  When set to False, ignore special characters for PostScript compatibility.

**Returns**

dot_data: string
String representation of the input tree in GraphViz dot format. Only returned if `out_file` is None.
New in version 0.18.

**Examples**

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn import tree

>>> clf = tree.DecisionTreeClassifier()
>>> iris = load_iris()

>>> clf = clf.fit(iris.data, iris.target)
>>> tree.export_graphviz(clf,
... out_file='tree.dot')
```

### 29.35 sklearn.utils: Utilities

The `sklearn.utils` module includes various utilities.

**Developer guide:** See the *Utilities for Developers* page for further details.

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<td>Turn seed into a np.random.RandomState instance</td>
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<td><code>sklearn.utils.estimator_checks.check_estimator(Estimator)</code></td>
<td>Check if estimator adheres to scikit-learn conventions.</td>
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<td><code>sklearn.utils.resample(*arrays, **options)</code></td>
<td>Resample arrays or sparse matrices in a consistent way</td>
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<tr>
<td><code>sklearn.utils.shuffle(*arrays, **options)</code></td>
<td>Shuffle arrays or sparse matrices in a consistent way</td>
</tr>
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#### 29.35.1 sklearn.utils.check_random_state

`sklearn.utils.check_random_state(seed)`

Turn seed into a np.random.RandomState instance

If seed is None, return the RandomState singleton used by np.random. If seed is an int, return a new RandomState instance seeded with seed. If seed is already a RandomState instance, return it. Otherwise raise ValueError.

**Examples using sklearn.utils.check_random_state**

- Isotonic Regression
- Face completion with a multi-output estimators
- Empirical evaluation of the impact of k-means initialization
- Manifold Learning methods on a severed sphere
- Scaling the regularization parameter for SVCs
**29.35.2 sklearn.utils.estimator_checks.check_estimator**

`sklearn.utils.estimator_checks.check_estimator(Estimator)`

Check if estimator adheres to scikit-learn conventions.

This estimator will run an extensive test-suite for input validation, shapes, etc. Additional tests for classifiers, regressors, clustering or transformers will be run if the Estimator class inherits from the corresponding mixin from sklearn.base.

**Parameters**

- `Estimator` : class
  Class to check. Estimator is a class object (not an instance).

**29.35.3 sklearn.utils.resample**

`sklearn.utils.resample(*arrays, **options)`

Resample arrays or sparse matrices in a consistent way.

The default strategy implements one step of the bootstrapping procedure.

**Parameters**

- `arrays` : sequence of indexable data-structures
  Indexable data-structures can be arrays, lists, dataframes or scipy sparse matrices with consistent first dimension.

- `replace` : boolean, True by default
  Implements resampling with replacement. If False, this will implement (sliced) random permutations.

- `n_samples` : int, None by default
  Number of samples to generate. If left to None this is automatically set to the first dimension of the arrays. If replace is False it should not be larger than the length of arrays.

- `random_state` : int or RandomState instance
  Control the shuffling for reproducible behavior.

**Returns**

- `resampled_arrays` : sequence of indexable data-structures
  Sequence of resampled views of the collections. The original arrays are not impacted.

**See also:**

`sklearn.utils.shuffle`

**Examples**

It is possible to mix sparse and dense arrays in the same run:

```python
>>> X = np.array([[1., 0.], [2., 1.], [0., 0.]])
>>> y = np.array([0, 1, 2])
>>> from scipy.sparse import coo_matrix
>>> X_sparse = coo_matrix(X)
>>> from sklearn.utils import resample
>>> X, X_sparse, y = resample(X, X_sparse, y, random_state=0)
>>> X
```
array([[1., 0.],
       [2., 1.],
       [1., 0.]]))

>>> X_sparse
<3x2 sparse matrix of type '<... 'numpy.float64'>'
   with 4 stored elements in Compressed Sparse Row format>

>>> X_sparse.toarray()
array([[1., 0.],
       [2., 1.],
       [1., 0.]]))

>>> y
array([0, 1, 0])

>>> resample(y, n_samples=2, random_state=0)
array([0, 1])

29.35.4 sklearn.utils.shuffle

sklearn.utils.shuffle(*arrays, **options)

Shuffle arrays or sparse matrices in a consistent way

This is a convenience alias to resample(*arrays, replace=False) to do random permutations of the collections.

Parameters*arrays : sequence of indexable data-structures

Indexable data-structures can be arrays, lists, dataframes or scipy sparse matrices with consistent first dimension.

random_state : int or RandomState instance

Control the shuffling for reproducible behavior.

n_samples : int, None by default

Number of samples to generate. If left to None this is automatically set to the first dimension of the arrays.

Returnsshuffled_arrays : sequence of indexable data-structures

Sequence of shuffled views of the collections. The original arrays are not impacted.

See also:

sklearn.utils.resample

Examples

It is possible to mix sparse and dense arrays in the same run:

```python
>>> X = np.array([[1., 0.], [2., 1.], [0., 0.]])
>>> y = np.array([0, 1, 2])
>>> from scipy.sparse import coo_matrix
>>> X_sparse = coo_matrix(X)
```
from sklearn.utils import shuffle

X, X_sparse, y = shuffle(X, X_sparse, y, random_state=0)

X
array([[ 0.,  0.],
       [ 2.,  1.],
       [ 1.,  0.]])

X_sparse
<3x2 sparse matrix of type '<... 'numpy.float64'>'
with 3 stored elements in Compressed Sparse Row format>

X_sparse.toarray()
array([[ 0.,  0.],
       [ 2.,  1.],
       [ 1.,  0.]])

y
array([2, 1, 0])

shuffle(y, n_samples=2, random_state=0)
array([0, 1])

Examples using sklearn.utils.shuffle

- Model Complexity Influence
- Prediction Latency
- Color Quantization using K-Means
- Empirical evaluation of the impact of k-means initialization
- Gradient Boosting regression

29.36 Recently deprecated

29.36.1 To be removed in 0.19

| sklearn.lda.LDA(solver, shrinkage, priors, ...) | Alias for sklearn.discriminant_analysis.LinearDiscriminantAnalysis. |
| sklearn.qda.QDA(priors, reg_param, ...) | Alias for sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis. |

sklearn.lda.LDA

Warning: DEPRECATED

class sklearn.lda.LDA (solver='svd', shrinkage=None, priors=None, n_components=None, store_covariance=False, tol=0.0001)
    Alias for sklearn.discriminant_analysis.LinearDiscriminantAnalysis.

DeprecationWarning: This class will be removed in 0.19. Use sklearn.discriminant_analysis.LinearDiscriminantAnalysis instead.

29.36. Recently deprecated 1915
Methods

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<td>decision_function(X)</td>
<td>Predict confidence scores for samples.</td>
</tr>
<tr>
<td>fit(X, y[, store_covariance, tol])</td>
<td>Fit LinearDiscriminantAnalysis model according to the given training data and parameters.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict class labels for samples in X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Estimate log probability.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Estimate probability.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Project data to maximize class separation.</td>
</tr>
</tbody>
</table>

__init__(solver='svd', shrinkage=None, priors=None, n_components=None, store_covariance=False, tol=0.0001)

decision_function(X)
   Predict confidence scores for samples.
   The confidence score for a sample is the signed distance of that sample to the hyperplane.

Parameters:

- X : {array-like, sparse matrix}, shape = (n_samples, n_features)
  Samples.

Returns:

- array, shape=(n_samples,) if n_classes == 2 else (n_samples, n_classes) :
  Confidence scores per (sample, class) combination. In the binary case, confidence score for self.classes_[1] where >0 means this class would be predicted.

fit(X, y, store_covariance=None, tol=None)
   Fit LinearDiscriminantAnalysis model according to the given training data and parameters.
   Changed in version 0.17: Deprecated store_covariance have been moved to main constructor.
   Changed in version 0.17: Deprecated tol have been moved to main constructor.

Parameters:

- X : array-like, shape (n_samples, n_features)
  Training data.

- y : array, shape (n_samples,)
  Target values.

fit_transform(X, y=None, **fit_params)
   Fit to data, then transform it.
   Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters:

- X : numpy array of shape [n_samples, n_features]
  Training set.

- y : numpy array of shape [n_samples]
  Target values.

Returns:

- X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

**get_params** *(deep=True)*
Get parameters for this estimator.

**Parameters**
- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**
- **params** : mapping of string to any
  Parameter names mapped to their values.

**predict** *(X)*
Predict class labels for samples in X.

**Parameters**
- **X** : {array-like, sparse matrix}, shape = [n_samples, n_features]
  Samples.

**Returns**
- **C** : array, shape = [n_samples]
  Predicted class label per sample.

**predict_log_proba** *(X)*
Estimate log probability.

**Parameters**
- **X** : array-like, shape (n_samples, n_features)
  Input data.

**Returns**
- **C** : array, shape (n_samples, n_classes)
  Estimated log probabilities.

**predict_proba** *(X)*
Estimate probability.

**Parameters**
- **X** : array-like, shape (n_samples, n_features)
  Input data.

**Returns**
- **C** : array, shape (n_samples, n_classes)
  Estimated probabilities.

**score** *(X, y, sample_weight=None)*
Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.

**Parameters**
- **X** : array-like, shape = (n_samples, n_features)
  Test samples.
- **y** : array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight** : array-like, shape = [n_samples], optional
  Sample weights.

**Returns**
- **score** : float
  Mean accuracy of self.predict(X) wrt. y.

29.36. Recently deprecated
**set_params(**params)**
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns:

**transform**(X)
Project data to maximize class separation.

Parameters:

X : array-like, shape (n_samples, n_features)
Input data.

Returns:

X_new : array, shape (n_samples, n_components)
Transformed data.

**sklearn.qda.QDA**

Warning: DEPRECATED

**class sklearn.qda.QDA(priors=None, reg_param=0.0, store_covariances=False, tol=0.0001)**

Alias for `sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis`.

Deprecated since version 0.17: This class will be removed in 0.19. Use `sklearn.discriminant_analysis.QuadraticDiscriminantAnalysis` instead.

**Methods**

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<thead>
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<tr>
<td>decision_function</td>
<td>Apply decision function to an array of samples.</td>
</tr>
<tr>
<td>fit(X, y[, store_covariances, tol])</td>
<td>Fit the model according to the given training data and parameters.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Perform classification on an array of test vectors X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Return posterior probabilities of classification.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Return posterior probabilities of classification.</td>
</tr>
<tr>
<td>score(X, y[, sample_weight])</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**__init__**(priors=None, reg_param=0.0, store_covariances=False, tol=0.0001)

**decision_function**(X)
Apply decision function to an array of samples.

Parameters:

X : array-like, shape = [n_samples, n_features]
Array of samples (test vectors).

Returns:

C : array, shape = [n_samples, n_classes] or [n_samples,]
Decision function values related to each class, per sample. In the two-class case, the
shape is [n_samples,], giving the log likelihood ratio of the positive class.

**fit** *(X, y, store_covariances=None, tol=None)*

Fit the model according to the given training data and parameters.

Changed in version 0.17: Deprecated `store_covariance` have been moved to main constructor.

Changed in version 0.17: Deprecated `tol` have been moved to main constructor.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Training vector, where n_samples in the number of samples and n_features is the number of features.

- **y**: array, shape = [n_samples]
  
  Target values (integers)

**get_params**(deep=True)

Get parameters for this estimator.

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

**predict** *(X)*

Perform classification on an array of test vectors X.

The predicted class C for each sample in X is returned.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **C** : array, shape = [n_samples]

**predict_log_proba** *(X)*

Return posterior probabilities of classification.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Array of samples/test vectors.

**Returns**

- **C** : array, shape = [n_samples, n_classes]
  
  Posterior log-probabilities of classification per class.

**predict_proba** *(X)*

Return posterior probabilities of classification.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Array of samples/test vectors.

**Returns**

- **C** : array, shape = [n_samples, n_classes]
  
  Posterior probabilities of classification per class.

**score** *(X, y, sample_weight=None)*

Returns the mean accuracy on the given test data and labels.

In multi-label classification, this is the subset accuracy which is a harsh metric since you require for each sample that each label set be correctly predicted.
Parameters:
- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True labels for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

Returns:
- **score**: float
  Mean accuracy of self.predict(X) wrt. y.

**set_params(**params**)**

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns:
- **self**: object

---

**sklearn.datasets.load_lfw_pairs**

**Warning**: DEPRECATED

**sklearn.datasets.load_lfw_pairs(**args, **kwargs**)

DEPRECATED: Function ‘load_lfw_pairs’ has been deprecated in 0.17 and will be removed in 0.19. Use `fetch_lfw_pairs(download_if_missing=False)` instead.

Alias for `fetch_lfw_pairs(download_if_missing=False)`

Deprecated since version 0.17: This function will be removed in 0.19. Use `sklearn.datasets.fetch_lfw_pairs` with parameter `download_if_missing=False` instead.

Check `fetch_lfw_pairs`.__doc__ for the documentation and parameter list.

**sklearn.datasets.load_lfw_people**

**Warning**: DEPRECATED

**sklearn.datasets.load_lfw_people(**args, **kwargs**)

DEPRECATED: Function ‘load_lfw_people’ has been deprecated in 0.17 and will be removed in 0.19. Use `fetch_lfw_people(download_if_missing=False)` instead.
Alias for fetch_lfw_people(download_if_missing=False)

Deprecated since version 0.17: This function will be removed in 0.19. Use sklearn.datasets.fetch_lfw_people with parameter download_if_missing=False instead.

Check fetch_lfw_people.__doc__ for the documentation and parameter list.

29.36.2 To be removed in 0.20

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**sklearn.grid_search.ParameterGrid**

**Warning:** DEPRECATED

**class** sklearn.grid_search.ParameterGrid(param_grid)

Grid of parameters with a discrete number of values for each.

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.ParameterGrid instead.

Can be used to iterate over parameter value combinations with the Python built-in function iter.

Read more in the [User Guide](#).

29.36. Recently deprecated
Parameters

param_grid : dict of string to sequence, or sequence of such

The parameter grid to explore, as a dictionary mapping estimator parameters to sequences of allowed values.

An empty dict signifies default parameters.

A sequence of dicts signifies a sequence of grids to search, and is useful to avoid exploring parameter combinations that make no sense or have no effect. See the examples below.

See also:

GridSearchCV uses ParameterGrid to perform a full parallelized parameter search.

Examples

>>> from sklearn.grid_search import ParameterGrid
>>> param_grid = {'a': [1, 2], 'b': [True, False]}
>>> list(ParameterGrid(param_grid)) ==
... [{'a': 1, 'b': True}, {'a': 1, 'b': False},
... {'a': 2, 'b': True}, {'a': 2, 'b': False}]
True

>>> grid = [{'kernel': ['linear']}, {'kernel': ['rbf'], 'gamma': [1, 10]}]
>>> list(ParameterGrid(grid)) == [{'kernel': 'linear'},
... {'kernel': 'rbf', 'gamma': 1},
... {'kernel': 'rbf', 'gamma': 10}]
True
>>> ParameterGrid(grid)[1] == {'kernel': 'rbf', 'gamma': 1}
True
.. automethod:: __init__

sklearn.grid_search.ParameterSampler

Warning: DEPRECATED

class sklearn.grid_search.ParameterSampler(param_distributions, n_iter, random_state=None)

Generator on parameters sampled from given distributions.

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.ParameterSampler instead.

Non-deterministic iterable over random candidate combinations for hyper-parameter search. If all parameters are presented as a list, sampling without replacement is performed. If at least one parameter is given as a distribution, sampling with replacement is used. It is highly recommended to use continuous distributions for continuous parameters.

Note that as of SciPy 0.12, the scipy.stats.distributions do not accept a custom RNG instance and always use the singleton RNG from numpy.random. Hence setting random_state will not guarantee a deterministic iteration whenever scipy.stats distributions are used to define the parameter search space.

Read more in the User Guide.
Parameters:

**param_distributions**: dict

Dictionary where the keys are parameters and values are distributions from which a parameter is to be sampled. Distributions either have to provide a `rvs` function to sample from them, or can be given as a list of values, where a uniform distribution is assumed.

**n_iter**: integer

Number of parameter settings that are produced.

**random_state**: int or RandomState

Pseudo random number generator state used for random uniform sampling from lists of possible values instead of scipy.stats distributions.

Returns:

**params**: dict of string to any

Yields dictionaries mapping each estimator parameter to as sampled value.

Examples:

```python
>>> from sklearn.grid_search import ParameterSampler
>>> from scipy.stats.distributions import expon
>>> import numpy as np

>>> np.random.seed(0)

>>> param_grid = {'a':[1, 2], 'b': expon()}
>>> param_list = list(ParameterSampler(param_grid, n_iter=4))

>>> rounded_list = [{k: round(v, 6) for k, v in d.items()} for d in param_list]

>>> rounded_list == [{'b': 0.89856, 'a': 1},
                      {'b': 0.923223, 'a': 1},
                      {'b': 1.878964, 'a': 2},
                      {'b': 1.038159, 'a': 2}]
True
```

**sklearn.grid_search.GridSearchCV**

Warning: DEPRECATED

class sklearn.grid_search.GridSearchCV (estimator, param_grid, scoring=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, pre_dispatch='2*n_jobs', error_score='raise')

Exhaustive search over specified parameter values for an estimator.

Deprecation since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.GridSearchCV instead.

Important members are fit, predict.

GridSearchCV implements a “fit” and a “score” method. It also implements “predict”, “predict_proba”, “decision_function”, “transform” and “inverse_transform” if they are implemented in the estimator used.

The parameters of the estimator used to apply these methods are optimized by cross-validated grid-search over a parameter grid.
Read more in the *User Guide*.

**Parameters**

- **estimator**: estimator object.
  
  A object of that type is instantiated for each grid point. This is assumed to implement the scikit-learn estimator interface. Either estimator needs to provide a `score` function, or `scoring` must be passed.

- **param_grid**: dict or list of dictionaries
  
  Dictionary with parameters names (string) as keys and lists of parameter settings to try as values, or a list of such dictionaries, in which case the grids spanned by each dictionary in the list are explored. This enables searching over any sequence of parameter settings.

- **scoring**: string, callable or None, default=None
  
  A string (see model evaluation documentation) or a scorer callable object / function with signature `scorer(estimator, X, y)`. If None, the `score` method of the estimator is used.

- **fit_params**: dict, optional
  
  Parameters to pass to the fit method.

- **n_jobs**: int, default=1
  
  Number of jobs to run in parallel.

  Changed in version 0.17: Upgraded to joblib 0.9.3.

- **pre_dispatch**: int, or string, optional
  
  Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

  - None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
  - An int, giving the exact number of total jobs that are spawned
  - A string, giving an expression as a function of `n_jobs`, as in `2*n_jobs`

- **iid**: boolean, default=True
  
  If True, the data is assumed to be identically distributed across the folds, and the loss minimized is the total loss per sample, and not the mean loss across the folds.

- **cv**: int, cross-validation generator or an iterable, optional
  
  Determines the cross-validation splitting strategy. Possible inputs for cv are:

  - None, to use the default 3-fold cross-validation,
  - integer, to specify the number of folds.
  - An object to be used as a cross-validation generator.
  - An iterable yielding train/test splits.

  For integer/None inputs, if the estimator is a classifier and `y` is either binary or multiclass, `sklearn.model_selection.StratifiedKFold` is used. In all other cases, `sklearn.model_selection.KFold` is used.

  Refer *User Guide* for the various cross-validation strategies that can be used here.
**refit** : boolean, default=True

Refit the best estimator with the entire dataset. If “False”, it is impossible to make predictions using this GridSearchCV instance after fitting.

**verbose** : integer

Controls the verbosity: the higher, the more messages.

**error_score** : ‘raise’ (default) or numeric

Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.

**Attributes**

**grid_scores_** : list of named tuples

Contains scores for all parameter combinations in param_grid. Each entry corresponds to one parameter setting. Each named tuple has the attributes:

• **parameters**, a dict of parameter settings

• **mean_validation_score**, the mean score over the cross-validation folds

• **cv_validation_scores**, the list of scores for each fold

**best_estimator_** : estimator

Estimator that was chosen by the search, i.e. estimator which gave highest score (or smallest loss if specified) on the left out data. Not available if refit=False.

**best_score_** : float

Score of best_estimator on the left out data.

**best_params_** : dict

Parameter setting that gave the best results on the hold out data.

**scorer_** : function

Scorer function used on the held out data to choose the best parameters for the model.

See also:

**ParameterGrid** generates all the combinations of a hyperparameter grid.

**sklearn.cross_validation.train_test_split** utility function to split the data into a development set usable for fitting a GridSearchCV instance and an evaluation set for its final evaluation.

**sklearn.metrics.make_scorer** Make a scorer from a performance metric or loss function.

**Notes**

The parameters selected are those that maximize the score of the left out data, unless an explicit score is passed in which case it is used instead.

If **n_jobs** was set to a value higher than one, the data is copied for each point in the grid (and not **n_jobs** times).

This is done for efficiency reasons if individual jobs take very little time, but may raise errors if the dataset is large and not enough memory is available. A workaround in this case is to set **pre_dispatch**. Then, the memory is copied only **pre_dispatch** many times. A reasonable value for **pre_dispatch** is \(2 \times n_{\text{jobs}}\).
Examples

```python
>>> from sklearn import svm, grid_search, datasets
>>> iris = datasets.load_iris()
>>> parameters = {'kernel':('linear', 'rbf'), 'C':[1, 10]}
>>> svr = svm.SVC()
>>> clf = grid_search.GridSearchCV(svr, parameters)
>>> clf.fit(iris.data, iris.target)
... GridSearchCV(cv=None, error_score=..., estimator=SVC(C=1.0, cache_size=..., class_weight=..., coef0=..., decision_function_shape=None, degree=..., gamma=..., kernel='rbf', max_iter=-1, probability=False, random_state=None, shrinking=True, tol=..., verbose=False),
  fit_params={}, iid=..., n_jobs=1, param_grid=..., pre_dispatch=..., refit=..., scoring=..., verbose=...)
```

Methods

```python
decision_function(*args, **kwargs) Call decision_function on the estimator with the best found parameters.
fit(X[, y]) Run fit with all sets of parameters.
get_params([deep]) Get parameters for this estimator.
inverse_transform(*args, **kwargs) Call inverse_transform on the estimator with the best found parameters.
predict(*args, **kwargs) Call predict on the estimator with the best found parameters.
predict_log_proba(*args, **kwargs) Call predict_log_proba on the estimator with the best found parameters.
predict_proba(*args, **kwargs) Call predict_proba on the estimator with the best found parameters.
score(X[, y]) Returns the score on the given data, if the estimator has been refit.
set_params(**params) Set the parameters of this estimator.
transform(*args, **kwargs) Call transform on the estimator with the best found parameters.
```

__init__(estimator, param_grid, scoring=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, pre_dispatch='2*n_jobs', error_score='raise')

decision_function(*args, **kwargs) Call decision_function on the estimator with the best found parameters.

Only available if refit=True and the underlying estimator supports decision_function.

ParametersX : indexable, length n_samples

Must fulfill the input assumptions of the underlying estimator.

fit(X, y=None) Run fit with all sets of parameters.

ParametersX : array-like, shape = [n_samples, n_features]
Training vector, where \(n_{\text{samples}}\) is the number of samples and \(n_{\text{features}}\) is the number of features.

\[ y : \text{array-like, shape } = [n_{\text{samples}}] \text{ or } [n_{\text{samples}}, n_{\text{output}}], \text{optional} \]

Target relative to \(X\) for classification or regression; None for unsupervised learning.

**get_params** (*deep=True*)
Get parameters for this estimator.

Parameters:
- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
- **params** : mapping of string to any
  Parameter names mapped to their values.

**inverse_transform** (*args, **kwargs*)
Call inverse_transform on the estimator with the best found parameters.
Only available if the underlying estimator implements inverse_transform and refit=True.

Parameters:
- **Xt** : indexable, length \(n_{\text{samples}}\)
  Must fulfill the input assumptions of the underlying estimator.

**predict** (*args, **kwargs*)
Call predict on the estimator with the best found parameters.
Only available if refit=True and the underlying estimator supports predict.

Parameters:
- **X** : indexable, length \(n_{\text{samples}}\)
  Must fulfill the input assumptions of the underlying estimator.

**predict_log_proba** (*args, **kwargs*)
Call predict_log_proba on the estimator with the best found parameters.
Only available if refit=True and the underlying estimator supports predict_log_proba.

Parameters:
- **X** : indexable, length \(n_{\text{samples}}\)
  Must fulfill the input assumptions of the underlying estimator.

**predict_proba** (*args, **kwargs*)
Call predict_proba on the estimator with the best found parameters.
Only available if refit=True and the underlying estimator supports predict_proba.

Parameters:
- **X** : indexable, length \(n_{\text{samples}}\)
  Must fulfill the input assumptions of the underlying estimator.

**score** (*X*, \(y=\text{None}\))
Returns the score on the given data, if the estimator has been refit.
This uses the score defined by scoring where provided, and the best_estimator_.score method otherwise.

Parameters:
- **X** : array-like, shape = [\(n_{\text{samples}}, n_{\text{features}}\)]
  Input data, where \(n_{\text{samples}}\) is the number of samples and \(n_{\text{features}}\) is the number of features.
- \(y : \text{array-like, shape } = [n_{\text{samples}}] \text{ or } [n_{\text{samples}}, n_{\text{output}}], \text{optional}\)
Target relative to X for classification or regression; None for unsupervised learning.

Returns: score

Notes

• The long-standing behavior of this method changed in version 0.16.
• It no longer uses the metric provided by estimator.score if the scoring parameter was set when fitting.

set_params(**params)
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns: self

transform(*args, **kwargs)
Call transform on the estimator with the best found parameters.

Only available if the underlying estimator supports transform and refit=True.

Parameters:

X: indexable, length n_samples
Must fulfill the input assumptions of the underlying estimator.
**Parameter**{estimator} : estimator object.

A object of that type is instantiated for each grid point. This is assumed to implement the scikit-learn estimator interface. Either estimator needs to provide a score function, or scoring must be passed.

**param_distributions** : dict

Dictionary with parameters names (string) as keys and distributions or lists of parameters to try. Distributions must provide a rvs method for sampling (such as those from scipy.stats.distributions). If a list is given, it is sampled uniformly.

**n_iter** : int, default=10

Number of parameter settings that are sampled. n_iter trades off runtime vs quality of the solution.

**scoring** : string, callable or None, default=None

A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y). If None, the score method of the estimator is used.

**fit_params** : dict, optional

Parameters to pass to the fit method.

**n_jobs** : int, default=1

Number of jobs to run in parallel.

**pre_dispatch** : int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

• None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs

• An int, giving the exact number of total jobs that are spawned

• A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

**iid** : boolean, default=True

If True, the data is assumed to be identically distributed across the folds, and the loss minimized is the total loss per sample, and not the mean loss across the folds.

**cv** : int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

• None, to use the default 3-fold cross-validation,

• integer, to specify the number of folds.

• An object to be used as a cross-validation generator.

• An iterable yielding train/test splits.

For integer/None inputs, if the estimator is a classifier and y is either binary or multiclass, sklearn.model_selection.StratifiedKFold is used. In all other cases, sklearn.model_selection.KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

29.36. Recently deprecated
refit : boolean, default=True

Refit the best estimator with the entire dataset. If “False”, it is impossible to make predictions using this RandomizedSearchCV instance after fitting.

verbose : integer

Controls the verbosity: the higher, the more messages.

random_state : int or RandomState

Pseudo random number generator state used for random uniform sampling from lists of possible values instead of scipy.stats distributions.

error_score : ‘raise’ (default) or numeric

Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.

Attributes

grid_scores_ : list of named tuples

Contains scores for all parameter combinations in param_grid. Each entry corresponds to one parameter setting. Each named tuple has the attributes:

• parameters, a dict of parameter settings
• mean_validation_score, the mean score over the cross-validation folds
• cv_validation_scores, the list of scores for each fold

best_estimator_ : estimator

Estimator that was chosen by the search, i.e. estimator which gave highest score (or smallest loss if specified) on the left out data. Not available if refit=False.

best_score_ : float

Score of best_estimator on the left out data.

best_params_ : dict

Parameter setting that gave the best results on the hold out data.

See also:

GridSearchCV Does exhaustive search over a grid of parameters.

ParameterSampler A generator over parameter settings, constructed from param_distributions.

Notes

The parameters selected are those that maximize the score of the held-out data, according to the scoring parameter.

If n_jobs was set to a value higher than one, the data is copied for each parameter setting(and not n_jobs times). This is done for efficiency reasons if individual jobs take very little time, but may raise errors if the dataset is large and not enough memory is available. A workaround in this case is to set pre_dispatch. Then, the memory is copied only pre_dispatch many times. A reasonable value for pre_dispatch is 2 * n_jobs.

Methods
scikit-learn user guide, Release 0.18.2

---

**decision_function**(*args, **kwargs)**
Call decision_function on the estimator with the best found parameters.

**fit**(X[, y])
Run fit on the estimator with randomly drawn parameters.

**get_params**(deep)
Get parameters for this estimator.

**inverse_transform**(*args, **kwargs)**
Call inverse_transform on the estimator with the best found parameters.

**predict**(*args, **kwargs)**
Call predict on the estimator with the best found parameters.

**predict_log_proba**(*args, **kwargs)**
Call predict_log_proba on the estimator with the best found parameters.

**predict_proba**(*args, **kwargs)**
Call predict_proba on the estimator with the best found parameters.

**score**(X[, y])
Returns the score on the given data, if the estimator has been refit.

**set_params**(**params)**
Set the parameters of this estimator.

**transform**(*args, **kwargs)**
Call transform on the estimator with the best found parameters.

---

**__init__**(estimator, param_distributions, n_iter=10, scoring=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, error_score='raise', pre_dispatch='2*n_jobs', random_state=None)

**decision_function**(*args, **kwargs)**
Call decision_function on the estimator with the best found parameters.

Only available if refit=True and the underlying estimator supports decision_function.

- **Parameters**
  - X: indexable, length n_samples
    Must fulfill the input assumptions of the underlying estimator.

- **fit**(X, y=None)
  Run fit on the estimator with randomly drawn parameters.

  - **Parameters**
    - X: array-like, shape = [n_samples, n_features]
      Training vector, where n_samples in the number of samples and n_features is the number of features.
    - y: array-like, shape = [n_samples] or [n_samples, n_output], optional
      Target relative to X for classification or regression; None for unsupervised learning.

- **get_params**(deep=True)
  Get parameters for this estimator.

  - **Parameters**
    - deep: boolean, optional
      If True, will return the parameters for this estimator and contained subobjects that are estimators.

  - **Returns**
    - params: mapping of string to any
      Parameter names mapped to their values.

- **inverse_transform**(*args, **kwargs)**
  Call inverse_transform on the estimator with the best found parameters.

  Only available if the underlying estimator implements inverse_transform and refit=True.

---

29.36. Recently deprecated 1931
Parameters:

- **predict**: Call predict on the estimator with the best found parameters.
  
  Only available if refit=True and the underlying estimator supports predict.

  Parameters:
  
  - **X**: indexable, length n_samples
    
    Must fulfill the input assumptions of the underlying estimator.

- **predict_log_proba**: Call predict_log_proba on the estimator with the best found parameters.
  
  Only available if refit=True and the underlying estimator supports predict_log_proba.

  Parameters:
  
  - **X**: indexable, length n_samples
    
    Must fulfill the input assumptions of the underlying estimator.

- **predict_proba**: Call predict_proba on the estimator with the best found parameters.
  
  Only available if refit=True and the underlying estimator supports predict_proba.

  Parameters:
  
  - **X**: indexable, length n_samples
    
    Must fulfill the input assumptions of the underlying estimator.

- **score**: Returns the score on the given data, if the estimator has been refit.
  
  This uses the score defined by scoring where provided, and the best_estimator_.score method otherwise.

  Parameters:
  
  - **X**: array-like, shape = [n_samples, n_features]
    
    Input data, where n_samples is the number of samples and n_features is the number of features.
  
  - **y**: array-like, shape = [n_samples] or [n_samples, n_output], optional
    
    Target relative to X for classification or regression; None for unsupervised learning.

  Returns:
  
  score : float

Notes

- The long-standing behavior of this method changed in version 0.16.

- It no longer uses the metric provided by estimator.score if the scoring parameter was set when fitting.

set_params (**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:

self :
transform(*args, **kwargs)

Call transform on the estimator with the best found parameters.

Only available if the underlying estimator supports transform and refit=True.

Parameters:

- **X**: indexable, length n_samples
  
  Must fulfill the input assumptions of the underlying estimator.

---

**sklearn.cross_validation.LeaveOneOut**

**Warning**: DEPRECATED

class sklearn.cross_validation.LeaveOneOut(n)

Leave-One-Out cross validation iterator.

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.LeaveOneOut instead.

Provides train/test indices to split data in train test sets. Each sample is used once as a test set (singleton) while the remaining samples form the training set.

Note: LeaveOneOut(n) is equivalent to KFold(n, n_folds=n) and LeavePOut(n, p=1).

Due to the high number of test sets (which is the same as the number of samples) this cross validation method can be very costly. For large datasets one should favor KFold, StratifiedKFold or ShuffleSplit.

Read more in the User Guide.

Parameters:

- **n**: int
  
  Total number of elements in dataset.

See also:

LeaveOneLabelOut, domain-specific

Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4]])
>>> y = np.array([1, 2])
>>> loo = cross_validation.LeaveOneOut(2)
>>> len(loo)
2
>>> print(loo)
sklearn.cross_validation.LeaveOneOut(n=2)

>>> for train_index, test_index in loo:
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print(X_train, X_test, y_train, y_test)
TRAIN: [1] TEST: [0]
[[3 4]] [[1 2]] [2] [1]
TRAIN: [0] TEST: [1]
[[1 2]] [[3 4]] [1] [2]
```

29.36. Recently deprecated
sklearn.cross_validation.LeavePOut

Warning: DEPRECATED

class sklearn.cross_validation.LeavePOut(n, p)

Leave-P-Out cross validation iterator

Deprecation since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.LeavePOut instead.

Provides train/test indices to split data in train test sets. This results in testing on all distinct samples of size p, while the remaining n - p samples form the training set in each iteration.

Note: LeavePOut(n,p) is NOT equivalent to KFold(n,n_folds=n // p) which creates non-overlapping test sets.

Due to the high number of iterations which grows combinatorically with the number of samples this cross validation method can be very costly. For large datasets one should favor KFold, StratifiedKFold or ShuffleSplit.

Read more in the User Guide.

Parameters

- n : int
  Total number of elements in dataset.

- p : int
  Size of the test sets.

Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 3, 4])
>>> lpo = cross_validation.LeavePOut(4, 2)
>>> len(lpo)
6
>>> print(lpo)
sklearn.cross_validation.LeavePOut(n=4, p=2)
>>> for train_index, test_index in lpo:
...     print("TRAIN: "{train_index}, "TEST: "{test_index})
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [2 3], TEST: [0 1]
TRAIN: [1 3], TEST: [0 2]
TRAIN: [1 2], TEST: [0 3]
TRAIN: [0 3], TEST: [1 2]
TRAIN: [0 2], TEST: [1 3]
TRAIN: [0 1], TEST: [2 3]
.. automethod:: __init__
```
**sklearn.cross_validation.KFold**

**Warning**: DEPRECATED

**class** `sklearn.cross_validation.KFold(n, n_folds=3, shuffle=False, random_state=None)`

K-Folds cross validation iterator.

Deprecated since version 0.18: This module will be removed in 0.20. Use `sklearn.model_selection.KFold` instead.

Provides train/test indices to split data in train test sets. Split dataset into k consecutive folds (without shuffling by default).

Each fold is then used as a validation set once while the k - 1 remaining fold(s) form the training set.

Read more in the User Guide.

**Parameters**:

- `n` : int
  Total number of elements.
- `n_folds` : int, default=3
  Number of folds. Must be at least 2.
- `shuffle` : boolean, optional
  Whether to shuffle the data before splitting into batches.
- `random_state` : None, int or RandomState
  When shuffle=True, pseudo-random number generator state used for shuffling. If None, use default numpy RNG for shuffling.

See also:

- `StratifiedKFold`, `folds`, `classification`
- `LabelKFold` K-fold iterator variant with non-overlapping labels.

**Notes**

The first n % n_folds folds have size n // n_folds + 1, other folds have size n // n_folds.

**Examples**

```python
>>> from sklearn.cross_validation import KFold
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([1, 2, 3, 4])
>>> kf = KFold(4, n_folds=2)
>>> len(kf)
2
>>> print(kf)
sklearn.cross_validation.KFold(n=4, n_folds=2, shuffle=False, random_state=None)

>>> for train_index, test_index in kf:
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
```

29.36. Recently deprecated
.. automethod:: __init__

sklearn.cross_validation.LabelKFold

**Warning:** DEPRECATED

```python
from sklearn.cross_validation import LabelKFold
X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
y = np.array([1, 2, 3, 4])
labels = np.array([0, 0, 2, 2])
label_kfold = LabelKFold(labels, n_folds=2)
for train_index, test_index in label_kfold:
    print("TRAIN: \[0 1\], TEST: \[2 3\]")
    X_train, X_test = X[train_index], X[test_index]
    y_train, y_test = y[train_index], y[test_index]
    print(X_train, X_test, y_train, y_test)
```

```python
TRAIN: [0 1] TEST: [2 3]
[[1 2] [3 4]] [[5 6]]
```
Leave-One-Label-Out cross-validation iterator

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.LeaveOneGroupOut instead.

Provides train/test indices to split data according to a third-party provided label. This label information can be used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the labels could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

Read more in the User Guide.

**Parameters**

- **labels**: array-like of int with shape (n_samples,)
  Arbitrary domain-specific stratification of the data to be used to draw the splits.

**See also:**

*LabelKFold*K-fold iterator variant with non-overlapping labels.

**Examples**

```python
cross_validation
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 1, 2])
>>> labels = np.array([1, 1, 2, 2])
>>> lol = cross_validation.LeaveOneLabelOut(labels)
>>> len(lol)
2
>>> print(lol)
klearn.cross_validation.LeaveOneLabelOut(labels=[1 1 2 2])
>>> for train_index, test_index in lol:
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print(X_train, X_test, y_train, y_test)
TRAIN: [2 3] TEST: [0 1]
[[5 6]
 [7 8]] [[1 2]
 [3 4]] [1 2] [1 2]
```
class sklearn.cross_validation.LeavePLabelOut (labels, p)

Leave-P-Label_Out cross-validation iterator

Deprecated since version 0.18: This module will be removed in 0.20. Use
sklearn.model_selection.LeavePGroupsOut instead.

Provides train/test indices to split data according to a third-party provided label. This label information can be
used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the labels could be the year of collection of the samples and thus allow for cross-validation against
time-based splits.

The difference between LeavePLabelOut and LeaveOneLabelOut is that the former builds the test sets with all
the samples assigned to \( p \) different values of the labels while the latter uses samples all assigned the same labels.

Read more in the User Guide.

Parameters:

- **labels**: array-like of int with shape (n_samples,)
  
  Arbitrary domain-specific stratification of the data to be used to draw the splits.

- **p**: int
  
  Number of samples to leave out in the test split.

See also:

- LabelKFold K-fold iterator variant with non-overlapping labels.

Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [5, 6]])
>>> y = np.array([1, 2, 1])
>>> labels = np.array([1, 2, 3])
>>> lpl = cross_validation.LeavePLabelOut(labels, p=2)
>>> len(lpl)
3
>>> print(lpl)
sklearn.cross_validation.LeavePLabelOut(labels=[1 2 3], p=2)
>>> for train_index, test_index in lpl:
...     print("TRAIN:", train_index, "TEST:", test_index)
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print(X_train, X_test, y_train, y_test)
TRAIN: [2] TEST: [0 1]
```
sklearn.cross_validation.LabelShuffleSplit

**Warning:** DEPRECATED

class sklearn.cross_validation.LabelShuffleSplit (labels, n_iter=5, test_size=0.2, train_size=None, random_state=None)

Shuffle-Labels-Out cross-validation iterator

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.GroupShuffleSplit instead.

Provides randomized train/test indices to split data according to a third-party provided label. This label information can be used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the labels could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

The difference between LeavePLabelOut and LabelShuffleSplit is that the former generates splits using all subsets of size \( p \) unique labels, whereas LabelShuffleSplit generates a user-determined number of random test splits, each with a user-determined fraction of unique labels.

For example, a less computationally intensive alternative to LeavePLabelOut(labels, p=10) would be LabelShuffleSplit(labels, test_size=10, n_iter=100).

Note: The parameters test_size and train_size refer to labels, and not to samples, as in ShuffleSplit.

New in version 0.17.

**Parameters**

**labels** : array, [n_samples]

Labels of samples

**n_iter** : int (default 5)

Number of re-shuffling and splitting iterations.

**test_size** : float (default 0.2), int, or None

If float, should be between 0.0 and 1.0 and represent the proportion of the labels to include in the test split. If int, represents the absolute number of test labels. If None, the value is automatically set to the complement of the train size.

**train_size** : float, int, or None (default is None)

If float, should be between 0.0 and 1.0 and represent the proportion of the labels to include in the train split. If int, represents the absolute number of train labels. If None, the value is automatically set to the complement of the test size.

**random_state** : int or RandomState
Pseudo-random number generator state used for random sampling.

```python
__init__(labels, n_iter=5, test_size=0.2, train_size=None, random_state=None)
```

**sklearn.cross_validation.StratifiedKFold**

---

**Warning:** DEPRECATED

```python
class sklearn.cross_validation.StratifiedKFold(y, n_folds=3, shuffle=False, random_state=None)
```

Stratified K-Folds cross validation iterator

Deprecated since version 0.18: This module will be removed in 0.20. Use `sklearn.model_selection.StratifiedKFold` instead.

Provides train/test indices to split data in train test sets.

This cross-validation object is a variation of KFold that returns stratified folds. The folds are made by preserving the percentage of samples for each class.

Read more in the User Guide.

**Parameters**

- `y` : array-like, [n_samples]
  
  Samples to split in K folds.
- `n_folds` : int, default=3
  
  Number of folds. Must be at least 2.
- `shuffle` : boolean, optional
  
  Whether to shuffle each stratification of the data before splitting into batches.
- `random_state` : None, int or RandomState
  
  When shuffle=True, pseudo-random number generator state used for shuffling. If None, use default numpy RNG for shuffling.

**See also:**

- `LabelKFold` K-fold iterator variant with non-overlapping labels.

**Notes**

All the folds have size trunc(n_samples / n_folds), the last one has the complementary.

**Examples**

```python
>>> from sklearn.cross_validation import StratifiedKFold
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> skf = StratifiedKFold(y, n_folds=2)
>>> len(skf)
2
>>> print(skf)
sklearn.cross_validation.StratifiedKFold(labels=[0 0 1 1], n_folds=2,
... 
```
```python
for train_index, test_index in skf:
    print("TRAIN:", train_index, "TEST:", test_index)
    X_train, X_test = X[train_index], X[test_index]
    y_train, y_test = y[train_index], y[test_index]
```

Random permutation cross-validation iterator.

**Parameters**
- **n**: int
  - Total number of elements in the dataset.
- **n_iter**: int (default 10)
  - Number of re-shuffling & splitting iterations.
- **test_size**: float (default 0.1), int, or None
  - If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples. If None, the value is automatically set to the complement of the train size.
- **train_size**: float, int, or None (default is None)
  - If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test size.
- **random_state**: int or RandomState
  - Pseudo-random number generator state used for random sampling.

**Examples**
```python
from sklearn import cross_validation
rs = cross_validation.ShuffleSplit(4, n_iter=3, test_size=.25, random_state=0)
len(rs)
```
ShuffleSplit(4, n_iter=3, test_size=0.25, ...)  
>>> for train_index, test_index in rs:
...      print("TRAIN:", train_index, "TEST:", test_index)
...
TRAIN: [3 1 0] TEST: [2]
TRAIN: [2 1 3] TEST: [0]
TRAIN: [0 2 1] TEST: [3]

>>> rs = cross_validation.ShuffleSplit(4, n_iter=3, train_size=0.5, test_size=.25, random_state=0)
>>> for train_index, test_index in rs:
...      print("TRAIN:", train_index, "TEST:", test_index)
...
TRAIN: [3 1] TEST: [2]
TRAIN: [2 1] TEST: [0]
TRAIN: [0 2] TEST: [3]
.. automethod:: __init__

sklearn.cross_validation.StratifiedShuffleSplit

Warning: DEPRECATED

class sklearn.cross_validation.StratifiedShuffleSplit(y, n_iter=10, test_size=0.1, train_size=None, random_state=None)

Stratified ShuffleSplit cross validation iterator

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.StratifiedShuffleSplit instead.

Provides train/test indices to split data in train test sets.

This cross-validation object is a merge of StratifiedKFold and ShuffleSplit, which returns stratified randomized folds. The folds are made by preserving the percentage of samples for each class.

Note: like the ShuffleSplit strategy, stratified random splits do not guarantee that all folds will be different, although this is still very likely for sizeable datasets.

Read more in the User Guide.

Parameters
- **y** : array, [n_samples]
  Labels of samples.
- **n_iter** : int (default 10)
  Number of re-shuffling & splitting iterations.
- **test_size** : float (default 0.1), int, or None
  If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples. If None, the value is automatically set to the complement of the train size.
- **train_size** : float, int, or None (default is None)
If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test size.

random_state : int or RandomState
Pseudo-random number generator state used for random sampling.

Examples

```python
>>> from sklearn.cross_validation import StratifiedShuffleSplit
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> sss = StratifiedShuffleSplit(y, 3, test_size=0.5, random_state=0)
>>> len(sss)
3
>>> print(sss)
StratifiedShuffleSplit(labels=[0 0 1 1], n_iter=3, ...)
>>> for train_index, test_index in sss:
...    print("TRAIN:", train_index, "TEST:", test_index)
...    X_train, X_test = X[train_index], X[test_index]
...    y_train, y_test = y[train_index], y[test_index]
TRAIN: [1 2] TEST: [3 0]
TRAIN: [0 2] TEST: [1 3]
TRAIN: [0 2] TEST: [3 1]
```
```python
>>> ps = PredefinedSplit(test_fold=[0, 1, -1, 1])
>>> len(ps)
2
>>> print(ps)
sklearn.cross_validation.PredefinedSplit(test_fold=[ 0 1 -1 1])
>>> for train_index, test_index in ps:
...    print("TRAIN: \[", train_index, " TEST: \[", test_index)
...    X_train, X_test = X[train_index], X[test_index]
...    y_train, y_test = y[train_index], y[test_index]
TRAIN: [1 2 3] TEST: [0]
TRAIN: [0 2] TEST: [1 3]
```

**sklearn.decomposition.RandomizedPCA**

**Warning:** DEPRECATED

class sklearn.decomposition.RandomizedPCA(*args, **kwargs)

Principal component analysis (PCA) using randomized SVD

Deprecated since version 0.18: This class will be removed in 0.20. Use PCA with parameter svd_solver ‘randomized’ instead. The new implementation DOES NOT store whiten components_. Apply transform to get them.

Linear dimensionality reduction using approximated Singular Value Decomposition of the data and keeping only the most significant singular vectors to project the data to a lower dimensional space.

Read more in the User Guide.

**Parameters**

-n_components : int, optional

Maximum number of components to keep. When not given or None, this is set to n_features (the second dimension of the training data).

-copy : bool

If False, data passed to fit are overwritten and running fit(X).transform(X) will not yield the expected results, use fit_transform(X) instead.

-iterated_power : int, default=2

Number of iterations for the power method.

Changed in version 0.18.

-whiten : bool, optional

When True (False by default) the components_ vectors are multiplied by the square root of (n_samples) and divided by the singular values to ensure uncorrelated outputs with unit component-wise variances.

Whitening will remove some information from the transformed signal (the relative variance scales of the components) but can sometime improve the predictive accuracy of the downstream estimators by making their data respect some hard-wired assumptions.

-random_state : int or RandomState instance or None (default)

Pseudo Random Number generator seed control. If None, use the numpy.random singleton.
Attributes

**components_** : array, [n_components, n_features]

Components with maximum variance.

**explained_variance_ratio_** : array, [n_components]

Percentage of variance explained by each of the selected components. k is not set then all components are stored and the sum of explained variances is equal to 1.0

**mean_** : array, [n_features]

Per-feature empirical mean, estimated from the training set.

See also:

*PCA*, *TruncatedSVD*

References

[Halko2009], [MRT]

Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import RandomizedPCA
>>> X = np.array([[[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]]]
>>> pca = RandomizedPCA(n_components=2)
>>> pca.fit(X)
RandomizedPCA(copy=True, iterated_power=2, n_components=2,
             random_state=None, whiten=False)
>>> print(pca.explained_variance_ratio_)
[ 0.99244... 0.00755...]
```

Methods

**fit**(X[, y])

Fit the model with X by extracting the first principal components.

**fit_transform**(X[, y])

Fit the model with X and apply the dimensionality reduction on X.

**get_params**(deep)

Get parameters for this estimator.

**inverse_transform**(X[, y])

Transform data back to its original space.

**set_params**(**kwargs)

Set the parameters of this estimator.

**transform**(X[, y])

Apply dimensionality reduction on X.

**__init__**(*args, **kwargs)

DEPRECATED: RandomizedPCA was deprecated in 0.18 and will be removed in 0.20. Use PCA(svd_solver='randomized') instead. The new implementation DOES NOT store whiten components_. Apply transform to get them.

**fit**(X, y=None)

Fit the model with X by extracting the first principal components.

Parameters

**X**: array-like, shape (n_samples, n_features) :
Training data, where n_samples in the number of samples and n_features is the number of features.

**Returns**

`self : object`

Returns the instance itself.

### fit_transform(X, y=None)

Fit the model with X and apply the dimensionality reduction on X.

**Parameters**

- `X : array-like, shape (n_samples, n_features)`
  
  New data, where n_samples in the number of samples and n_features is the number of features.

**Returns**

- `X_new : array-like, shape (n_samples, n_components)`

### get_params(deep=True)

Get parameters for this estimator.

**Parameters**

- `deep : boolean, optional`
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- `params : mapping of string to any`
  
  Parameter names mapped to their values.

### inverse_transform(X, y=None)

Transform data back to its original space.

**Parameters**

- `X : array-like, shape (n_samples, n_components)`
  
  New data, where n_samples in the number of samples and n_components is the number of components.

**Returns**

- `X_original : array-like, shape (n_samples, n_features)`

**Notes**

If whitening is enabled, inverse_transform does not compute the exact inverse operation of transform.

### set_params(**params)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- `self`:

### transform(X, y=None)

Apply dimensionality reduction on X.

X is projected on the first principal components previous extracted from a training set.

**Parameters**

- `X : array-like, shape (n_samples, n_features)`
  
  New data, where n_samples in the number of samples and n_features is the number of features.

**Returns**

- `X_new : array-like, shape (n_samples, n_components)`
Warning: DEPRECATED

class sklearn.gaussian_process.GaussianProcess(*args, **kwargs)

The legacy Gaussian Process model class.

Deprecated since version 0.18: This class will be removed in 0.20. Use the GaussianProcessRegressor instead.

Read more in the User Guide.

Parameters:

regr : string or callable, optional

A regression function returning an array of outputs of the linear regression functional basis. The number of observations n_samples should be greater than the size p of this basis. Default assumes a simple constant regression trend. Available built-in regression models are:

'constant', 'linear', 'quadratic'

corr : string or callable, optional

A stationary autocorrelation function returning the autocorrelation between two points x and x'. Default assumes a squared-exponential autocorrelation model. Built-in correlation models are:

'absolute_exponential', 'squared_exponential',
'generalized_exponential', 'cubic', 'linear'

beta0 : double array_like, optional

The regression weight vector to perform Ordinary Kriging (OK). Default assumes Universal Kriging (UK) so that the vector beta of regression weights is estimated using the maximum likelihood principle.

storage_mode : string, optional

A string specifying whether the Cholesky decomposition of the correlation matrix should be stored in the class (storage_mode = 'full') or not (storage_mode = 'light'). Default assumes storage_mode = 'full', so that the Cholesky decomposition of the correlation matrix is stored. This might be a useful parameter when one is not interested in the MSE and only plan to estimate the BLUP, for which the correlation matrix is not required.

verbose : boolean, optional

A boolean specifying the verbose level. Default is verbose = False.

theta0 : double array_like, optional

An array with shape (n_features, ) or (1, ). The parameters in the autocorrelation model. If thetaL and thetaU are also specified, theta0 is considered as the starting point for the maximum likelihood estimation of the best set of parameters. Default assumes isotropic autocorrelation model with theta0 = 1e-1.

thetaL : double array_like, optional
thetaU : double array_like, optional

An array with shape matching theta0’s. Upper bound on the autocorrelation parameters for maximum likelihood estimation. Default is None, so that it skips maximum likelihood estimation and it uses theta0.

thetaU : double array_like, optional

An array with shape matching theta0’s. Upper bound on the autocorrelation parameters for maximum likelihood estimation. Default is None, so that it skips maximum likelihood estimation and it uses theta0.

normalize : boolean, optional

Input X and observations y are centered and reduced wrt means and standard deviations estimated from the n_samples observations provided. Default is normalize = True so that data is normalized to ease maximum likelihood estimation.

nugget : double or ndarray, optional

Introduce a nugget effect to allow smooth predictions from noisy data. If nugget is an ndarray, it must be the same length as the number of data points used for the fit. The nugget is added to the diagonal of the assumed training covariance; in this way it acts as a Tikhonov regularization in the problem. In the special case of the squared exponential correlation function, the nugget mathematically represents the variance of the input values. Default assumes a nugget close to machine precision for the sake of robustness (nugget = 10. * MACHINE_EPSILON).

optimizer : string, optional

A string specifying the optimization algorithm to be used. Default uses ‘fmin_cobyla’ algorithm from scipy.optimize. Available optimizers are:

'fmin_cobyla', 'Welch'

‘Welch’ optimizer is due to Welch et al., see reference [WBSWM1992]. It consists in iterating over several one-dimensional optimizations instead of running one single multi-dimensional optimization.

random_start : int, optional

The number of times the Maximum Likelihood Estimation should be performed from a random starting point. The first MLE always uses the specified starting point (theta0), the next starting points are picked at random according to an exponential distribution (log-uniform on [thetaL, thetaU]). Default does not use random starting point (random_start = 1).

random_state: integer or numpy.RandomState, optional:

The generator used to shuffle the sequence of coordinates of theta in the Welch optimizer. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

theta_ : array

Specified theta OR the best set of autocorrelation parameters (the sought maximizer of the reduced likelihood function).

reduced_likelihood_function_value_ : array

The optimal reduced likelihood function value.
Notes

The presentation implementation is based on a translation of the DACE Matlab toolbox, see reference [NLNS2002].

References

[NLNS2002], [WBSWM1992]

Examples

```python
>>> import numpy as np
>>> from sklearn.gaussian_process import GaussianProcess
>>> X = np.array([[1., 3., 5., 6., 7., 8.]]).T
>>> y = (X * np.sin(X)).ravel()
>>> gp = GaussianProcess(theta0=0.1, thetaL=.001, thetaU=1.)
>>> gp.fit(X, y)
GaussianProcess(beta0=None... ...
```

Methods

- `fit(X, y)` The Gaussian Process model fitting method.
- `get_params([deep])` Get parameters for this estimator.
- `predict(X[, eval_MSE, batch_size])` This function evaluates the Gaussian Process model at x.
- `reduced_likelihood_function([theta])` This function determines the BLUP parameters and evaluates the reduced likelihood function for the given autocorrelation parameters theta.
- `score(X, y[, sample_weight])` Returns the coefficient of determination $R^2$ of the prediction.
- `set_params(**params)` Set the parameters of this estimator.

__init__(*args, **kwargs)

DEPRECATED: GaussianProcess was deprecated in version 0.18 and will be removed in 0.20. Use the GaussianProcessRegressor instead.

```python
>>> fit(X, y)
```

The Gaussian Process model fitting method.

Parameters

- `X` : double array_like
  An array with shape (n_samples, n_features) with the input at which observations were made.
- `y` : double array_like
  An array with shape (n_samples, ) or shape (n_samples, n_targets) with the observations of the output to be predicted.

Returns

- `gp` : self
A fitted Gaussian Process model object awaiting data to perform predictions.

get_params (deep=True)
Get parameters for this estimator.

Parameters:
depth: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns:
params: mapping of string to any
Parameter names mapped to their values.

predict (X, eval_MSE=False, batch_size=None)
This function evaluates the Gaussian Process model at x.

Parameters:
X: array_like
An array with shape (n_eval, n_features) giving the point(s) at which the prediction(s) should be made.

eval_MSE: boolean, optional
A boolean specifying whether the Mean Squared Error should be evaluated or not. Default assumes evalMSE = False and evaluates only the BLUP (mean prediction).

batch_size: integer, optional
An integer giving the maximum number of points that can be evaluated simultaneously (depending on the available memory). Default is None so that all given points are evaluated at the same time.

Returns:
y: array_like, shape (n_samples, ) or (n_samples, n_targets)
An array with shape (n_eval, ) if the Gaussian Process was trained on an array of shape (n_samples, ) or an array with shape (n_eval, n_targets) if the Gaussian Process was trained on an array of shape (n_samples, n_targets) with the Best Linear Unbiased Prediction at x.

MSE: array_like, optional (if eval_MSE == True)
An array with shape (n_eval, ) or (n_eval, n_targets) as with y, with the Mean Squared Error at x.

reduced_likelihood_function (theta=None)
This function determines the BLUP parameters and evaluates the reduced likelihood function for the given autocorrelation parameters theta.

Maximizing this function wrt the autocorrelation parameters theta is equivalent to maximizing the likelihood of the assumed joint Gaussian distribution of the observations y evaluated onto the design of experiments X.

Parameters:
theta: array_like, optional
An array containing the autocorrelation parameters at which the Gaussian Process model parameters should be determined. Default uses the built-in autocorrelation parameters (ie theta = self.theta_).

Returns:
reduced_likelihood_function_value: double
The value of the reduced likelihood function associated to the given autocorrelation parameters theta.

par: dict
A dictionary containing the requested Gaussian Process model parameters:

- **sigma2**: Gaussian Process variance.
- **beta**: Generalized least-squares regression weights for Universal Kriging or given beta0 for Ordinary Kriging.
- **gamma**: Gaussian Process weights.
- **C**: Cholesky decomposition of the correlation matrix [R].
- **Ft**: Solution of the linear equation system: \([R] \times Ft = F\)
- **G**: QR decomposition of the matrix Ft.

**score** *(X, y, sample_weight=None)*

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y_{true} - y_{pred}) ** 2).sum()\) and \(v\) is the residual sum of squares \(((y_{true} - y_{true.mean()}) ** 2).sum()\). Best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of \(y\), disregarding the input features, would get a \(R^2\) score of 0.0.

**Parameters**

- **X**: array-like, shape = (n_samples, n_features)
  Test samples.
- **y**: array-like, shape = (n_samples) or (n_samples, n_outputs)
  True values for X.
- **sample_weight**: array-like, shape = [n_samples], optional
  Sample weights.

**Return**

- **score**: float
  \(R^2\) of self.predict(X) wrt. y.

**set_params** (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Return**

- **self**: 

**sklearn.mixture.GMM**

---

**Warning**: DEPRECATED

**class** `sklearn.mixture.GMM (*args, **kwargs)`

Legacy Gaussian Mixture Model

Deprecated since version 0.18: This class will be removed in 0.20. Use `sklearn.mixture.GaussianMixture` instead.

**Methods**
### Gaussian Mixture Model

#### Methods

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<td>Akaike information criterion for the current model fit and the proposed data.</td>
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<td><code>fit(X[, y])</code></td>
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<td>Return the per-sample likelihood of the data under the model.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of this estimator.</td>
</tr>
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</table>

#### Initialization

```python
__init__(*args, **kwargs)
```

Deprecation: The class GMM is deprecated in 0.18 and will be removed in 0.20. Use class GaussianMixture instead.

#### `aic(X)`

Akaike information criterion for the current model fit and the proposed data.

**Parameters**
- `X`: array of shape(n_samples, n_dimensions)

**Returns**
- `aic`: float (the lower the better)

#### `bic(X)`

Bayesian information criterion for the current model fit and the proposed data.

**Parameters**
- `X`: array of shape(n_samples, n_dimensions)

**Returns**
- `bic`: float (the lower the better)

#### `fit(X, y=None)`

Estimate model parameters with the EM algorithm.

A initialization step is performed before entering the expectation-maximization (EM) algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string "" when creating the GMM object. Likewise, if you would like just to do an initialization, set n_iter=0.

**Parameters**
- `X`: array_like, shape = (n_features)

**Returns**
- `self`

#### `fit_predict(X, y=None)`

Fit and then predict labels for data.

Warning: Due to the final maximization step in the EM algorithm, with low iterations the prediction may not be 100% accurate.

New in version 0.17: `fit_predict` method in Gaussian Mixture Model.

**Parameters**
- `X`: array-like, shape = [n_samples, n_features]

**Returns**
- `C`: array, shape = (n_samples,) component memberships

#### `get_params(deep=True)`

Get parameters for this estimator.
**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params** : mapping of string to any
  
  Parameter names mapped to their values.

**predict** *(X)*

Predict label for data.

- **Parameters**
  
  - **X** : array-like, shape = [n_samples, n_features]

- **Returns**
  
  - **C** : array, shape = (n_samples,) component memberships

**predict_proba** *(X)*

Predict posterior probability of data under each Gaussian in the model.

- **Parameters**
  
  - **X** : array-like, shape = [n_samples, n_features]

- **Returns**
  
  - **responsibilities** : array-like, shape = (n_samples, n_components)
    
    Returns the probability of the sample for each Gaussian (state) in the model.

**sample** *(n_samples=1, random_state=None)*

Generate random samples from the model.

- **Parameters**
  
  - **n_samples** : int, optional
    
    Number of samples to generate. Defaults to 1.

- **Returns**
  
  - **X** : array-like, shape (n_samples, n_features)
    
    List of samples

**score** *(X, y=None)*

Compute the log probability under the model.

- **Parameters**
  
  - **X** : array_like, shape (n_samples, n_features)
    
    List of n_features-dimensional data points. Each row corresponds to a single data point.

- **Returns**
  
  - **logprob** : array_like, shape (n_samples,)
    
    Log probabilities of each data point in X

**score_samples** *(X)*

Return the per-sample likelihood of the data under the model.

Compute the log probability of X under the model and return the posterior distribution (responsibilities) of each mixture component for each element of X.

- **Parameters**
  
  - **X** : array_like, shape (n_samples, n_features)
    
    List of n_features-dimensional data points. Each row corresponds to a single data point.

- **Returns**
  
  - **logprob** : array_like, shape (n_samples,)
    
    Log probabilities of each data point in X.

  - **responsibilities** : array_like, shape (n_samples, n_components)
    
    Posterior probabilities of each mixture component for each observation

**set_params** *(**params)*

Set the parameters of this estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns `self`:

**sklearn.mixture.DPGMM**

---

**Warning:** DEPRECATED

#### sklearn.mixture.DPGMM

**class sklearn.mixture.DPGMM(*args, **kwargs)**

Dirichlet Process Gaussian Mixture Models

Deprecated since version 0.18: This class will be removed in 0.20. Use `sklearn.mixture.BayesianGaussianMixture` with parameter `weight_concentration_prior_type='dirichlet_process'` instead.

Methods

- `aic(X)`
  - Akaike information criterion for the current model fit and the proposed data.
- `bic(X)`
  - Bayesian information criterion for the current model fit and the proposed data.
- `fit(X[, y])`
  - Estimate model parameters with the EM algorithm.
- `fit_predict(X[, y])`
  - Fit and then predict labels for data.
- `get_params([deep])`
  - Get parameters for this estimator.
- `lower_bound(X, z)`
  - Returns a lower bound on model evidence based on X and membership.
- `predict(X)`
  - Predict label for data.
- `predict_proba(X)`
  - Predict posterior probability of data under each Gaussian in the model.
- `sample([n_samples, random_state])`
  - Generate posterior samples from the model.
- `score(X[, y])`
  - Compute the log probability under the model.
- `score_samples(X)`
  - Return the likelihood of the data under the model.
- `set_params(**params)`
  - Set the parameters of this estimator.

---

**DEPRECATED:** The `DPGMM` class is not working correctly and it’s better to use `sklearn.mixture.BayesianGaussianMixture` class with parameter `weight_concentration_prior_type='dirichlet_process'` instead. `DPGMM` is deprecated in 0.18 and will be removed in 0.20.

- `aic(X)`
  - Akaike information criterion for the current model fit and the proposed data.
    
    **Parameters**
    
    `X` : array of shape(n_samples, n_dimensions)

    **Returns**

    `aic`: float (the lower the better)

- `bic(X)`
  - Bayesian information criterion for the current model fit and the proposed data.
Parameters $X$ : array of shape (n_samples, n_dimensions)

Returns $bic$ : float (the lower the better)

$\text{fit}(X, y=\text{None})$
Estimate model parameters with the EM algorithm.

A initialization step is performed before entering the expectation-maximization (EM) algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string ‘’ when creating the GMM object. Likewise, if you would like just to do an initialization, set n_iter=0.

Parameters $X$ : array_like, shape (n, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns self :

$\text{fit_predict}(X, y=\text{None})$
Fit and then predict labels for data.

Warning: Due to the final maximization step in the EM algorithm, with low iterations the prediction may not be 100% accurate.

New in version 0.17: fit_predict method in Gaussian Mixture Model.

Parameters $X$ : array-like, shape = [n_samples, n_features]

Returns $C$ : array, shape = (n_samples,) component memberships

$\text{get_params}(\text{deep}=\text{True})$
Get parameters for this estimator.

Parameters $\text{deep}$ : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns $\text{params}$ : mapping of string to any
Parameter names mapped to their values.

$\text{lower_bound}(X, z)$
returns a lower bound on model evidence based on X and membership

$\text{predict}(X)$
Predict label for data.

Parameters $X$ : array-like, shape = [n_samples, n_features]

Returns $C$ : array, shape = (n_samples,) component memberships

$\text{predict_proba}(X)$
Predict posterior probability of data under each Gaussian in the model.

Parameters $X$ : array-like, shape = [n_samples, n_features]

Returns $\text{responsibilities}$ : array-like, shape = (n_samples, n_components)
Returns the probability of the sample for each Gaussian (state) in the model.

$\text{sample}(n_{\text{samples}}=1, \text{random_state}=\text{None})$
Generate random samples from the model.

Parameters $n_{\text{samples}}$ : int, optional
Number of samples to generate. Defaults to 1.

Returns $X$ : array_like, shape (n_samples, n_features)
score \((X, y=None)\)
Compute the log probability under the model.

**Parameters**

- **X**: array_like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **logprob**: array_like, shape (n_samples,)
  Log probabilities of each data point in X

score_samples \((X)\)
Return the likelihood of the data under the model.

Compute the bound on log probability of X under the model and return the posterior distribution (responsibilities) of each mixture component for each element of X. This is done by computing the parameters for the mean-field of z for each observation.

**Parameters**

- **X**: array_like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **logprob**: array_like, shape (n_samples,)
  Log probabilities of each data point in X
- **responsibilities**: array_like, shape (n_samples, n_components)
  Posterior probabilities of each mixture component for each observation

set_params (**params)**
Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: object

**Warning:** DEPRECATED
Number of mixture components.

covariance_type: string, default ‘diag’:

String describing the type of covariance parameters to use. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.

alpha: float, default 1:

Real number representing the concentration parameter of the dirichlet distribution. Intuitively, the higher the value of alpha the more likely the variational mixture of Gaussians model will use all components it can.

tol: float, default 1e-3

Convergence threshold.

n_iter: int, default 10

Maximum number of iterations to perform before convergence.

params: string, default ‘wmc’

Controls which parameters are updated in the training process. Can contain any combination of ‘w’ for weights, ‘m’ for means, and ‘c’ for covars.

init_params: string, default ‘wmc’

Controls which parameters are updated in the initialization process. Can contain any combination of ‘w’ for weights, ‘m’ for means, and ‘c’ for covars. Defaults to ‘wmc’.

verbose: int, default 0

Controls output verbosity.

Attributes:

covariance_type: string

String describing the type of covariance parameters used by the DP-GMM. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.

n_features: int

Dimensionality of the Gaussians.

n_components: int (read-only)

Number of mixture components.

weights_: array, shape (n_components,)

Mixing weights for each mixture component.

means_: array, shape (n_components, n_features)

Mean parameters for each mixture component.

precs_: array

Precision (inverse covariance) parameters for each mixture component. The shape depends on covariance_type:

<table>
<thead>
<tr>
<th>Shape</th>
<th>covariance_type</th>
</tr>
</thead>
<tbody>
<tr>
<td>('n_components', 'n_features')</td>
<td>if 'spherical',</td>
</tr>
<tr>
<td>('n_features', 'n_features')</td>
<td>if 'tied',</td>
</tr>
<tr>
<td>('n_components', 'n_features')</td>
<td>if 'diag',</td>
</tr>
<tr>
<td>('n_components', 'n_features', 'n_features')</td>
<td>if 'full'</td>
</tr>
</tbody>
</table>

converged_: bool

29.36. Recently deprecated
True when convergence was reached in fit(), False otherwise.

See also:

**GMM** Finite Gaussian mixture model fit with EM

**DPGMM** Infinite Gaussian mixture model, using the dirichlet process, fit with a variational algorithm

**Methods**

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<td>bic(X)</td>
<td>Bayesian information criterion for the current model fit and the proposed data.</td>
</tr>
<tr>
<td>fit(X[, y])</td>
<td>Estimate model parameters with the EM algorithm.</td>
</tr>
<tr>
<td>fit_predict(X[, y])</td>
<td>Fit and then predict labels for data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for this estimator.</td>
</tr>
<tr>
<td>lower_bound(X, z)</td>
<td>returns a lower bound on model evidence based on X and membership</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict label for data.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict posterior probability of data under each Gaussian in the model.</td>
</tr>
<tr>
<td>sample([n_samples, random_state])</td>
<td>Generate random samples from the model.</td>
</tr>
<tr>
<td>score(X[, y])</td>
<td>Compute the log probability under the model.</td>
</tr>
<tr>
<td>score_samples(X)</td>
<td>Return the likelihood of the data under the model.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of this estimator.</td>
</tr>
</tbody>
</table>

**init** (*args, **kwargs)

DEPRECATED: The VBGMM class is not working correctly and it’s better to use sklearn.mixture.BayesianGaussianMixture class with parameter weight_concentration_prior_type='dirichlet_distribution' instead. VBGMM is deprecated in 0.18 and will be removed in 0.20.

**aic**(X)
Akaike information criterion for the current model fit and the proposed data.

**Parameters**

- **X**: array of shape(n_samples, n_dimensions)

**Returns**

- **aic**: float (the lower the better)

**bic**(X)
Bayesian information criterion for the current model fit and the proposed data.

**Parameters**

- **X**: array of shape(n_samples, n_dimensions)

**Returns**

- **bic**: float (the lower the better)

**fit**(X, y=None)
Estimate model parameters with the EM algorithm.

A initialization step is performed before entering the expectation-maximization (EM) algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string ‘’ when creating the GMM object. Likewise, if you would like just to do an initialization, set n_iter=0.

**Parameters**

- **X**: array_like, shape (n, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.
Returns self :

**fit_predict** (*X, y=None*)
Fit and then predict labels for data.

Warning: Due to the final maximization step in the EM algorithm, with low iterations the prediction may not be 100% accurate.

New in version 0.17: *fit_predict* method in Gaussian Mixture Model.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **C**: array, shape = (n_samples,) component memberships

**get_params** (*deep=True*)
Get parameters for this estimator.

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**Returns**

- **params**: mapping of string to any
  Parameter names mapped to their values.

**lower_bound** (*X, z*)
returns a lower bound on model evidence based on X and membership

**predict** (*X*)
Predict label for data.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **C**: array, shape = (n_samples,) component memberships

**predict_proba** (*X*)
Predict posterior probability of data under each Gaussian in the model.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **responsibilities**: array-like, shape = (n_samples, n_components)
  Returns the probability of the sample for each Gaussian (state) in the model.

**sample** (*n_samples=1, random_state=None*)
Generate random samples from the model.

**Parameters**

- **n_samples**: int, optional
  Number of samples to generate. Defaults to 1.

**Returns**

- **X**: array_like, shape (n_samples, n_features)
  List of samples

**score** (*X, y=None*)
Compute the log probability under the model.

**Parameters**

- **X**: array_like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **logprob**: array_like, shape (n_samples,)
  Log probabilities of each data point in X
score_samples (X)

Return the likelihood of the data under the model.

Compute the bound on log probability of X under the model and return the posterior distribution (responsibilities) of each mixture component for each element of X.

This is done by computing the parameters for the mean-field of z for each observation.

**Parameters**

- **X** : array_like, shape (n_samples, n_features)
  
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **logprob** : array_like, shape (n_samples,)
  
  Log probabilities of each data point in X

- **responsibilities** : array_like, shape (n_samples, n_components)
  
  Posterior probabilities of each mixture component for each observation

set_params (**params**)

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The latter have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**

grid_search.fit_grid_point(X, y, estimator, ...)

Run fit on one set of parameters.

Deprecated since version 0.18: This module will be removed in 0.20. Use `sklearn.model_selection.fit_grid_point` instead.

**Parameters**

- **X** : array-like, sparse matrix or list
  
  1960 Chapter 29. API Reference
Input data.

\( y \) : array-like or None

Targets for input data.

\texttt{estimator} : estimator object

A object of that type is instantiated for each grid point. This is assumed to implement the scikit-learn estimator interface. Either estimator needs to provide a \texttt{score} function, or \texttt{scoring} must be passed.

\texttt{parameters} : dict

Parameters to be set on estimator for this grid point.

\texttt{train} : ndarray, dtype int or bool

Boolean mask or indices for training set.

\texttt{test} : ndarray, dtype int or bool

Boolean mask or indices for test set.

\texttt{scorer} : callable or None.

If provided must be a scorer callable object / function with signature \texttt{scorer(estimator,X,y)}.

\texttt{verbose} : int

Verbosity level.

\**\texttt{fit_params}** : kwargs

Additional parameter passed to the fit function of the estimator.

\texttt{error_score} : ‘raise’ (default) or numeric

Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.

\texttt{Returns} \texttt{score} : float

Score of this parameter setting on given training / test split.

\texttt{parameters} : dict

The parameters that have been evaluated.

\texttt{n_samples_test} : int

Number of test samples in this split.

\texttt{sklearn.learning_curve.learning_curve}

\texttt{sklearn.learning_curve.learning_curve}(\texttt{estimator, X, y, train_sizes=\texttt{array([0.1, 0.33, 0.55, 0.78, 1.])}, cv=\texttt{None}, scoring=\texttt{None}, exploit_incremental_learning=\texttt{False}, n_jobs=\texttt{1}, pre_dispatch=\texttt{’all’}, verbose=\texttt{0}, error_score=\texttt{’raise’})

Learning curve.

\texttt{29.36. Recently deprecated}
Determines cross-validated training and test scores for different training set sizes.

A cross-validation generator splits the whole dataset k times in training and test data. Subsets of the training set with varying sizes will be used to train the estimator and a score for each training subset size and the test set will be computed. Afterwards, the scores will be averaged over all k runs for each training subset size.

Read more in the User Guide.

**Parameters**

- **estimator**: object type that implements the “fit” and “predict” methods.
  
  An object of that type which is cloned for each validation.

- **X**: array-like, shape (n_samples, n_features)
  
  Training vector, where n_samples is the number of samples and n_features is the number of features.

- **y**: array-like, shape (n_samples) or (n_samples, n_features), optional
  
  Target relative to X for classification or regression; None for unsupervised learning.

- **train_sizes**: array-like, shape (n_ticks,), dtype float or int
  
  Relative or absolute numbers of training examples that will be used to generate the learning curve. If the dtype is float, it is regarded as a fraction of the maximum size of the training set (that is determined by the selected validation method), i.e. it has to be within (0, 1]. Otherwise it is interpreted as absolute sizes of the training sets. Note that for classification the number of samples usually have to be big enough to contain at least one sample from each class. (default: np.linspace(0.1, 1.0, 5))

- **cv**: int, cross-validation generator or an iterable, optional
  
  Determines the cross-validation splitting strategy. Possible inputs for cv are:

  - None, to use the default 3-fold cross-validation,
  - integer, to specify the number of folds.
  - An object to be used as a cross-validation generator.
  - An iterable yielding train/test splits.

  For integer/None inputs, if the estimator is a classifier and y is either binary or multiclass, sklearn.model_selection.StratifiedKFold is used. In all other cases, sklearn.model_selection.KFold is used.

  Refer User Guide for the various cross-validation strategies that can be used here.

- **scoring**: string, callable or None, optional, default: None

  A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y).

- **exploit_incremental_learning**: boolean, optional, default: False

  If the estimator supports incremental learning, this will be used to speed up fitting for different training set sizes.

- **n_jobs**: integer, optional

  Number of jobs to run in parallel (default 1).

- **pre_dispatch**: integer or string, optional
Number of predispatched jobs for parallel execution (default is all). The option can reduce the allocated memory. The string can be an expression like ‘2*n_jobs’.

**verbose** : integer, optional

Controls the verbosity: the higher, the more messages.

**error_score** : ‘raise’ (default) or numeric

Value to assign to the score if an error occurs in estimator fitting. If set to ‘raise’, the error is raised. If a numeric value is given, FitFailedWarning is raised. This parameter does not affect the refit step, which will always raise the error.

**Return**

Return 

**train_sizes_abs** : array, shape = (n_unique_ticks,), dtype int

Numbers of training examples that has been used to generate the learning curve. Note that the number of ticks might be less than n_ticks because duplicate entries will be removed.

**train_scores** : array, shape (n_ticks, n_cv_folds)

Scores on training sets.

**test_scores** : array, shape (n_ticks, n_cv_folds)

Scores on test set.

**Notes**

See examples/model_selection/plot_learning_curve.py

**sklearn.learning_curve.validation_curve**

**Warning**: DEPRECATED

**sklearn.learning_curve.validation_curve**(estimator, X, y, param_name, param_range, cv=None, scoring=None, n_jobs=1, pre_dispatch='all', verbose=0)

Validation curve.

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.validation_curve instead.

Determine training and test scores for varying parameter values.

Compute scores for an estimator with different values of a specified parameter. This is similar to grid search with one parameter. However, this will also compute training scores and is merely a utility for plotting the results.

Read more in the User Guide.

**Parameters**

**estimator** : object type that implements the “fit” and “predict” methods

An object of that type which is cloned for each validation.

**X** : array-like, shape (n_samples, n_features)

Training vector, where n_samples is the number of samples and n_features is the number of features.
y : array-like, shape (n_samples) or (n_samples, n_features), optional
Target relative to X for classification or regression; None for unsupervised learning.

param_name : string
Name of the parameter that will be varied.

param_range : array-like, shape (n_values,)
The values of the parameter that will be evaluated.

cv : int, cross-validation generator or an iterable, optional
Determines the cross-validation splitting strategy. Possible inputs for cv are:
• None, to use the default 3-fold cross-validation,
• integer, to specify the number of folds.
• An object to be used as a cross-validation generator.
• An iterable yielding train/test splits.
For integer/None inputs, if the estimator is a classifier and y is either binary or multiclass, sklearn.model_selection.StratifiedKFold is used. In all other cases, sklearn.model_selection.KFold is used.
Refer User Guide for the various cross-validation strategies that can be used here.

scoring : string, callable or None, optional, default: None
A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator, X, y).

n_jobs : integer, optional
Number of jobs to run in parallel (default 1).

pre_dispatch : integer or string, optional
Number of predispatched jobs for parallel execution (default is all). The option can reduce the allocated memory. The string can be an expression like ‘2*n_jobs’.

verbose : integer, optional
Controls the verbosity: the higher, the more messages.

Return strain_scores : array, shape (n_ticks, n_cv_folds)
Scores on training sets.

test_scores : array, shape (n_ticks, n_cv_folds)
Scores on test set.

Notes

See examples/model_selection/plot_validation_curve.py
sklearn.cross_validation.cross_val_predict

**Warning: DEPRECATED**

sklearn.cross_validation.cross_val_predict (estimator, X, y=None, cv=None, n_jobs=1, verbose=0, fit_params=None, pre_dispatch='2*n_jobs')

Generate cross-validated estimates for each input data point

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.cross_val_predict instead.

Read more in the User Guide.

**Parameters**

**estimator**: estimator object implementing ‘fit’ and ‘predict’

The object to use to fit the data.

**X**: array-like

The data to fit. Can be, for example a list, or an array at least 2d.

**y**: array-like, optional, default: None

The target variable to try to predict in the case of supervised learning.

**cv**: int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the default 3-fold cross-validation,
- integer, to specify the number of folds.
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.

For integer/None inputs, if the estimator is a classifier and y is either binary or multiclass, StratifiedKFold is used. In all other cases, KFold is used.

Refer User Guide for the various cross-validation strategies that can be used here.

**n_jobs**: integer, optional

The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

**verbose**: integer, optional

The verbosity level.

**fit_params**: dict, optional

Parameters to pass to the fit method of the estimator.

**pre_dispatch**: int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs

29.36. Recently deprecated 1965
• An int, giving the exact number of total jobs that are spawned
• A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

Returns

preds : ndarray
This is the result of calling ‘predict’

Examples

```python
>>> from sklearn import datasets, linear_model
>>> from sklearn.cross_validation import cross_val_predict
>>> diabetes = datasets.load_diabetes()
>>> X = diabetes.data[:150]
>>> y = diabetes.target[:150]
>>> lasso = linear_model.Lasso()
>>> y_pred = cross_val_predict(lasso, X, y)
```

**sklearn.cross_validation.cross_val_score**

**Warning:** DEPRECATED

sklearn.cross_validation.cross_val_score(estimator, X, y=None, scoring=None, cv=None, n_jobs=1, verbose=0, fit_params=None, pre_dispatch='2*n_jobs')

Evaluate a score by cross-validation

Deprecated since version 0.18: This module will be removed in 0.20. Use sklearn.model_selection.cross_val_score instead.

Read more in the User Guide.

**Parameters**

- **estimator** : estimator object implementing ‘fit’
  The object to use to fit the data.
- **X** : array-like
  The data to fit. Can be, for example a list, or an array at least 2d.
- **y** : array-like, optional, default: None
  The target variable to try to predict in the case of supervised learning.
- **scoring** : string, callable or None, optional, default: None
  A string (see model evaluation documentation) or a scorer callable object / function with signature scorer(estimator,X,y).
- **cv** : int, cross-validation generator or an iterable, optional
  Determines the cross-validation splitting strategy. Possible inputs for cv are:
  • None, to use the default 3-fold cross-validation,
  • integer, to specify the number of folds.
  • An object to be used as a cross-validation generator.
  • An iterable yielding train/test splits.
For integer/None inputs, if the estimator is a classifier and \( y \) is either binary or multiclass, \texttt{StratifiedKFold} is used. In all other cases, \texttt{KFold} is used.

Refer \textit{User Guide} for the various cross-validation strategies that can be used here.

\textbf{n\_jobs} : integer, optional

The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

\textbf{verbose} : integer, optional

The verbosity level.

\textbf{fit\_params} : dict, optional

Parameters to pass to the fit method of the estimator.

\textbf{pre\_dispatch} : int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
- An int, giving the exact number of total jobs that are spawned
- A string, giving an expression as a function of n\_jobs, as in ‘2\*n\_jobs’

\textbf{Returnsscores} : array of float, shape=(len(list(cv)),)

Array of scores of the estimator for each run of the cross validation.

\textbf{See also:}

\texttt{sklearn.metrics.make\_scorer} Make a scorer from a performance metric or loss function.

\textbf{Examples}

```python
>>> from sklearn import datasets, linear_model
>>> from sklearn.cross_validation import cross_val_score
>>> diabetes = datasets.load_diabetes()
>>> X = diabetes.data[:150]
>>> y = diabetes.target[:150]
>>> lasso = linear_model.Lasso()
>>> print(cross_val_score(lasso, X, y))
[ 0.33150734 0.08022311 0.03531764]
```

\texttt{sklearn.cross\_validation.check\_cv}

\textbf{Warning: DEPRECATED}

\texttt{sklearn.cross\_validation.check\_cv}(cv, X=None, y=None, classifier=False)

Input checker utility for building a CV in a user friendly way.
scikit-learn user guide, Release 0.18.2

Deprecation since version 0.18: This module will be removed in 0.20. Use `sklearn.model_selection.check_cv` instead.

**Parameters**

**cv**: int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- **None**, to use the default 3-fold cross-validation,
- **integer**, to specify the number of folds.
- **An object to be used as a cross-validation generator**.
- **An iterable yielding train/test splits**.

For integer/None inputs, if classifier is True and `y` is binary or multiclass, `StratifiedKFold` is used. In all other cases, `KFold` is used.

Refer [User Guide](https://scikit-learn.org/stable/modules/generated/sklearn.model_selection.KFold.html) for the various cross-validation strategies that can be used here.

**X**: array-like

The data the cross-val object will be applied on.

**y**: array-like

The target variable for a supervised learning problem.

**classifier**: boolean optional

Whether the task is a classification task, in which case stratified KFold will be used.

**Returns**

**checked_cv**: a cross-validation generator instance.

The return value is guaranteed to be a cv generator instance, whatever the input type.

---

### sklearn.cross_validation.permutation_test_score

**Warning**: DEPRECATED

```
sklearn.cross_validation.permutation_test_score(estimator, X, y, cv=None, n_permutations=100, n_jobs=1, labels=None, random_state=0, verbose=0, scoring=None)
```

Evaluate the significance of a cross-validated score with permutations

Deprecation since version 0.18: This module will be removed in 0.20. Use `sklearn.model_selection.permutation_test_score` instead.


**Parameters**

**estimator**: estimator object implementing ‘fit’

The object to use to fit the data.

**X**: array-like of shape at least 2D

The data to fit.

**y**: array-like

The target variable to try to predict in the case of supervised learning.

**scoring**: string, callable or None, optional, default: None
A string (see model evaluation documentation) or a scorer callable object / function with
signature `scorer(estimator, X, y)`.

**cv**: int, cross-validation generator or an iterable, optional

Determines the cross-validation splitting strategy. Possible inputs for cv are:

- None, to use the default 3-fold cross-validation,
- integer, to specify the number of folds,
- An object to be used as a cross-validation generator.
- An iterable yielding train/test splits.

For integer/None inputs, if the estimator is a classifier and $y$ is either binary or multi-class, `StratifiedKFold` is used. In all other cases, `KFold` is used.

Refer [User Guide](https://scikit-learn.org/stable/user_guide.html) for the various cross-validation strategies that can be used here.

**n_permutations**: integer, optional

Number of times to permute $y$.

**n_jobs**: integer, optional

The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

**labels**: array-like of shape [n_samples] (optional)

Labels constrain the permutation among groups of samples with a same label.

**random_state**: RandomState or an int seed (0 by default)

A random number generator instance to define the state of the random permutations generator.

**verbose**: integer, optional

The verbosity level.

**Return**

- `score`: float
  
  The true score without permuting targets.

- `permutation_scores`: array, shape (n_permutations,)
  
  The scores obtained for each permutations.

- `pvalue`: float
  
  The returned value equals p-value if `scoring` returns bigger numbers for better scores (e.g., accuracy_score). If `scoring` is rather a loss function (i.e. when lower is better such as with `mean_squared_error`) then this is actually the complement of the p-value: 1 - p-value.

**Notes**

This function implements Test 1 in:

sklearn.cross_validation.train_test_split

**Warning: DEPRECATED**

**sklearn.cross_validation.train_test_split (**arrays, **options)**

Split arrays or matrices into random train and test subsets

Deprecation since version 0.18: This module will be removed in 0.20. Use `sklearn.model_selection.train_test_split` instead.

Quick utility that wraps input validation and next(iter(ShuffleSplit(n_samples))) and application to input data into a single call for splitting (and optionally subsampling) data in a oneliner.

Read more in the User Guide.

**Parameters**

- **arrays** : sequence of indexables with same length / shape[0]
  Allowed inputs are lists, numpy arrays, scipy-sparse matrices or pandas dataframes.
- **test_size** : float, int, or None (default is None)
  If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples. If None, the value is automatically set to the complement of the train size. If train size is also None, test size is set to 0.25.
- **train_size** : float, int, or None (default is None)
  If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test size.
- **random_state** : int or RandomState
  Pseudo-random number generator state used for random sampling.
- **stratify** : array-like or None (default is None)
  If not None, data is split in a stratified fashion, using this as the labels array.

**New in version 0.17:** stratify splitting

**Returnssplitting** : list, length = 2 * len(arrays),

List containing train-test split of inputs.

New in version 0.16: If the input is sparse, the output will be a `scipy.sparse.csr_matrix`. Else, output type is the same as the input type.

**Examples**

```python
>>> import numpy as np
>>> from sklearn.cross_validation import train_test_split
>>> X, y = np.arange(10).reshape((5, 2)), range(5)
>>> X
array([[0, 1],
       [2, 3],
       [4, 5],
       [6, 7],
       [8, 9]])
```
```python
>>> list(y)
[0, 1, 2, 3, 4]

>>> X_train, X_test, y_train, y_test = train_test_split(
...     X, y, test_size=0.33, random_state=42)
...
>>> X_train
array([[4, 5],
       [0, 1],
       [6, 7]])
>>> y_train
[2, 0, 3]
>>> X_test
array([[2, 3],
       [8, 9]])
>>> y_test
[1, 4]
```
30.1 Contributing

This project is a community effort, and everyone is welcome to contribute.

The project is hosted on https://github.com/scikit-learn/scikit-learn

Scikit-learn is somewhat selective when it comes to adding new algorithms, and the best way to contribute and to help the project is to start working on known issues. See Issues for New Contributors to get started.

Our community, our values

We are a community based on openness and friendly, didactic, discussions.

We aspire to treat everybody equally, and value their contributions.

Decisions are made based on technical merit and consensus.

Code is not the only way to help the project. Reviewing pull requests, answering questions to help others on mailing lists or issues, organizing and teaching tutorials, working on the website, improving the documentation, are all priceless contributions.

We abide by the principles of openness, respect, and consideration of others of the Python Software Foundation: https://www.python.org/psf/codeofconduct/

30.1.1 Submitting a bug report

In case you experience issues using this package, do not hesitate to submit a ticket to the Bug Tracker. You are also welcome to post feature requests or pull requests.

30.1.2 Retrieving the latest code

We use Git for version control and GitHub for hosting our main repository.

You can check out the latest sources with the command:

git clone git://github.com/scikit-learn/scikit-learn.git
or if you have write privileges:

```
$ git clone git@github.com:scikit-learn/scikit-learn.git
```

If you run the development version, it is cumbersome to reinstall the package each time you update the sources. It is thus preferred that you add the scikit-learn directory to your `PYTHONPATH` and build the extension in place:

```
python setup.py build_ext --inplace
```

Another option is to use the `develop` option if you change your code a lot and do not want to have to reinstall every time. This basically builds the extension in place and creates a link to the development directory (see the `setuptools` docs):

```
python setup.py develop
```

**Note:** if you decide to do that you have to rerun:

```
python setup.py build_ext --inplace
```

every time the source code of a compiled extension is changed (for instance when switching branches or pulling changes from upstream).

On Unix-like systems, you can simply type `make` in the top-level folder to build in-place and launch all the tests. Have a look at the `Makefile` for additional utilities.

### 30.1.3 Contributing code

**Note:** To avoid duplicating work, it is highly advised that you contact the developers on the mailing list before starting work on a non-trivial feature.

https://mail.python.org/mailman/listinfo/scikit-learn

**How to contribute**

The preferred way to contribute to scikit-learn is to fork the main repository on GitHub, then submit a “pull request” (PR):

1. Create an account on GitHub if you do not already have one.
2. Fork the project repository: click on the ‘Fork’ button near the top of the page. This creates a copy of the code under your account on the GitHub server.
3. Clone this copy to your local disk:

```
$ git clone git@github.com:YourLogin/scikit-learn.git
```

4. Create a branch to hold your changes:

```
$ git checkout -b my-feature
```

and start making changes. Never work in the `master` branch!

5. Work on this copy, on your computer, using Git to do the version control. When you’re done editing, do:
to record your changes in Git, then push them to GitHub with:

\$ git push -u origin my-feature

Finally, go to the web page of the your fork of the scikit-learn repo, and click ‘Pull request’ to send your changes to the maintainers for review. You may want to consider sending an email to the mailing list for more visibility.

**Note:** In the above setup, your origin remote repository points to YourLogin/scikit-learn.git. If you wish to fetch/merge from the main repository instead of your forked one, you will need to add another remote to use instead of origin. If we choose the name upstream for it, the command will be:

\$ git remote add upstream https://github.com/scikit-learn/scikit-learn.git

If any of the above seems like magic to you, then look up the Git documentation and the Git development workflow on the web.

In particular, if some conflicts arise between your branch and the master branch, you will need to rebase your branch on master. Please avoid merging master branch into yours. If you did it anyway, you can fix it following this example.

### Contributing pull requests

It is recommended to check that your contribution complies with the following rules before submitting a pull request:

- Follow the [coding-guidelines](http://) (see below). To make sure that your PR does not add PEP8 violations you can run ./build_tools/travis/flake8_diff.sh or make flake8-diff on a Unix-like system.

- When applicable, use the validation tools and other code in the sklearn.utils submodule. A list of utility routines available for developers can be found in the [Utilities for Developers](http://) page.

- If your pull request addresses an issue, please use the title to describe the issue and mention the issue number in the pull request description to ensure a link is created to the original issue.

- All public methods should have informative docstrings with sample usage presented as doctests when appropriate.

- Please prefix the title of your pull request with [MRG] if the contribution is complete and should be subjected to a detailed review. Two core developers will review your code and change the prefix of the pull request to [MRG + 1] and [MRG + 2] on approval, making it eligible for merging. Incomplete contributions should be prefixed [WIP] to indicate a work in progress (and changed to [MRG] when it matures). WIPs may be useful to: indicate you are working on something to avoid duplicated work, request broad review of functionality or API, or seek collaborators. WIPs often benefit from the inclusion of a task list in the PR description.

- All other tests pass when everything is rebuilt from scratch. On Unix-like systems, check with (from the toplevel source folder):

\$ make

- When adding additional functionality, provide at least one example script in the examples/ folder. Have a look at other examples for reference. Examples should demonstrate why the new functionality is useful in practice and, if possible, compare it to other methods available in scikit-learn.

- Documentation and high-coverage tests are necessary for enhancements to be accepted.

30.1. Contributing
• At least one paragraph of narrative documentation with links to references in the literature (with PDF links when possible) and the example. For more details on writing and building the documentation, see the Documentation section.

You can also check for common programming errors with the following tools:

• Code with a good unittest coverage (at least 90%, better 100%), check with:

```bash
$ pip install nose coverage
$ nosetests --with-coverage path/to/tests_for_package
```

see also Testing and improving test coverage

• No pyflakes warnings, check with:

```bash
$ pip install pyflakes
$ pyflakes path/to/module.py
```

• No PEP8 warnings, check with:

```bash
$ pip install pep8
$ pep8 path/to/module.py
```

• AutoPEP8 can help you fix some of the easy redundant errors:

```bash
$ pip install autopep8
$ autopep8 path/to/pep8.py
```

Bonus points for contributions that include a performance analysis with a benchmark script and profiling output (please report on the mailing list or on the GitHub wiki).

Also check out the How to optimize for speed guide for more details on profiling and Cython optimizations.

---

**Note:** The current state of the scikit-learn code base is not compliant with all of those guidelines, but we expect that enforcing those constraints on all new contributions will get the overall code base quality in the right direction.

**Note:** For two very well documented and more detailed guides on development workflow, please pay a visit to the Scipy Development Workflow - and the Astropy Workflow for Developers sections.

---

**Filing Bugs**

We use Github issues to track all bugs and feature requests; feel free to open an issue if you have found a bug or wish to see a feature implemented.

It is recommended to check that your issue complies with the following rules before submitting:

• Verify that your issue is not being currently addressed by other issues or pull requests.

• If you are submitting an algorithm or feature request, please verify that the algorithm fulfills our new algorithm requirements.

• Please ensure all code snippets and error messages are formatted in appropriate code blocks. See Creating and highlighting code blocks.

• Please include your operating system type and version number, as well as your Python, scikit-learn, numpy, and scipy versions. This information can be found by running the following code snippet:
• Please be specific about what estimators and/or functions are involved and the shape of the data, as appropriate; please include a reproducible code snippet or link to a gist. If an exception is raised, please provide the traceback.

Issues for New Contributors

New contributors should look for the following tags when looking for issues. We strongly recommend that new contributors tackle “easy” issues first: this helps the contributor become familiar with the contribution workflow, and for the core devs to become acquainted with the contributor; besides which, we frequently underestimate how easy an issue is to solve!

Easy Tags

A great way to start contributing to scikit-learn is to pick an item from the list of Easy issues in the issue tracker. Resolving these issues allow you to start contributing to the project without much prior knowledge. Your assistance in this area will be greatly appreciated by the more experienced developers as it helps free up their time to concentrate on other issues.

Need Contributor Tags

We often use the Need Contributor tag to mark issues regardless of difficulty. Additionally, we use the Need Contributor tag to mark Pull Requests which have been abandoned by their original contributor and are available for someone to pick up where the original contributor left off. The list of issues with the Need Contributor tag can be found here.

Note that not all issues which need contributors will have this tag.

Documentation

We are glad to accept any sort of documentation: function docstrings, reStructuredText documents (like this one), tutorials, etc. reStructuredText documents live in the source code repository under the doc/ directory.

You can edit the documentation using any text editor, and then generate the HTML output by typing `make html` from the doc/ directory. Alternatively, `make html-noplot` can be used to quickly generate the documentation without the example gallery. The resulting HTML files will be placed in _build/html/ and are viewable in a web browser. See the README file in the doc/ directory for more information.

For building the documentation, you will need sphinx, matplotlib and pillow.

**When you are writing documentation**, it is important to keep a good compromise between mathematical and algorithmic details, and give intuition to the reader on what the algorithm does.

Basically, to elaborate on the above, it is best to always start with a small paragraph with a hand-waving explanation of what the method does to the data. Then, it is very helpful to point out why the feature is useful and when it should be used - the latter also including “big O” \( O(g(n)) \) complexities of the algorithm, as opposed to just rules of thumb, as the latter can be very machine-dependent. If those complexities are not available, then rules of thumb may be provided instead.
Secondly, a generated figure from an example (as mentioned in the previous paragraph) should then be included to further provide some intuition.

Next, one or two small code examples to show its use can be added.

Next, any math and equations, followed by references, can be added to further the documentation. Not starting the documentation with the maths makes it more friendly towards users that are just interested in what the feature will do, as opposed to how it works “under the hood”.

Finally, follow the formatting rules below to make it consistently good:

- Add “See also” in docstrings for related classes/functions.
- “See also” in docstrings should be one line per reference, with a colon and an explanation, for example:

```
See also
--------
SelectKBest : Select features based on the k highest scores.
SelectFpr : Select features based on a false positive rate test.
```

- For unwritten formatting rules, try to follow existing good works:
  - For “References” in docstrings, see the Silhouette Coefficient 
    (sklearn.metrics.silhouette_score).

**Warning: Sphinx version**

While we do our best to have the documentation build under as many version of Sphinx as possible, the different versions tend to behave slightly differently. To get the best results, you should use version 1.0.

**Testing and improving test coverage**

High-quality unit testing is a cornerstone of the scikit-learn development process. For this purpose, we use the nose package. The tests are functions appropriately named, located in tests subdirectories, that check the validity of the algorithms and the different options of the code.

The full scikit-learn tests can be run using ‘make’ in the root folder. Alternatively, running ‘nosetests’ in a folder will run all the tests of the corresponding subpackages.

We expect code coverage of new features to be at least around 90%.

**Note: Workflow to improve test coverage**

To test code coverage, you need to install the coverage package in addition to nose.

1. Run ‘make test-coverage’. The output lists for each file the line numbers that are not tested.
2. Find a low hanging fruit, looking at which lines are not tested, write or adapt a test specifically for these lines.
3. Loop.

**Developers web site**

More information can be found on the developer’s wiki.
**Issue Tracker Tags**

All issues and pull requests on the Github issue tracker should have (at least) one of the following tags:

- **Bug / Crash**  Something is happening that clearly shouldn’t happen. Wrong results as well as unexpected errors from estimators go here.

- **Cleanup / Enhancement**  Improving performance, usability, consistency.

- **Documentation**  Missing, incorrect or sub-standard documentations and examples.

- **New Feature**  Feature requests and pull requests implementing a new feature.

There are three other tags to help new contributors:

- **Easy**  This issue can be tackled by anyone, no experience needed. Ask for help if the formulation is unclear.

- **Moderate**  Might need some knowledge of machine learning or the package, but is still approachable for someone new to the project.

- **Needs Contributor**  This tag marks an issue which currently lacks a contributor or a PR that needs another contributor to take over the work. These issues can range in difficulty, and may not be approachable for new contributors. Note that not all issues which need contributors will have this tag.

### 30.1.4 Other ways to contribute

Code is not the only way to contribute to scikit-learn. For instance, documentation is also a very important part of the project and often doesn’t get as much attention as it deserves. If you find a typo in the documentation, or have made improvements, do not hesitate to send an email to the mailing list or submit a GitHub pull request. Full documentation can be found under the doc/ directory.

It also helps us if you spread the word: reference the project from your blog and articles, link to it from your website, or simply say “I use it”:

### 30.1.5 Coding guidelines

The following are some guidelines on how new code should be written. Of course, there are special cases and there will be exceptions to these rules. However, following these rules when submitting new code makes the review easier so new code can be integrated in less time.

Uniformly formatted code makes it easier to share code ownership. The scikit-learn project tries to closely follow the official Python guidelines detailed in PEP8 that detail how code should be formatted and indented. Please read it and follow it.

In addition, we add the following guidelines:

- Use underscores to separate words in non class names: `n_samples` rather than `nsamples`.

- Avoid multiple statements on one line. Prefer a line return after a control flow statement (if/for).

- Use relative imports for references inside scikit-learn.

- Unit tests are an exception to the previous rule; they should use absolute imports, exactly as client code would. A corollary is that, if `sklearn.foo` exports a class or function that is implemented in `sklearn.foo.bar.baz`, the test should import it from `sklearn.foo`. 
• Please don’t use import * in any case. It is considered harmful by the official Python recommendations. It makes the code harder to read as the origin of symbols is no longer explicitly referenced, but most important, it prevents using a static analysis tool like pyflakes to automatically find bugs in scikit-learn.

• Use the numpy docstring standard in all your docstrings.

A good example of code that we like can be found here.

Input validation

The module sklearn.utils contains various functions for doing input validation and conversion. Sometimes, np.asarray suffices for validation; do not use np.asanyarray or np.atleast_2d, since those let NumPy’s np.matrix through, which has a different API (e.g., * means dot product on np.matrix, but Hadamard product on np.ndarray).

In other cases, be sure to call check_array on any array-like argument passed to a scikit-learn API function. The exact parameters to use depends mainly on whether and which scipy.sparse matrices must be accepted.

For more information, refer to the Utilities for Developers page.

Random Numbers

If your code depends on a random number generator, do not use numpy.random.random() or similar routines. To ensure repeatability in error checking, the routine should accept a keyword random_state and use this to construct a numpy.random.RandomState object. See sklearn.utils.check_random_state in Utilities for Developers.

Here’s a simple example of code using some of the above guidelines:

```python
from sklearn.utils import check_array, check_random_state

def choose_random_sample(X, random_state=0):
    """
    Choose a random point from X

    Parameters
    ----------
    X : array-like, shape = (n_samples, n_features)
        array representing the data
    random_state : RandomState or an int seed (0 by default)
        A random number generator instance to define the state of the
        random permutations generator.

    Returns
    -------
    x : numpy array, shape = (n_features,)
        A random point selected from X
    """
    X = check_array(X)
    random_state = check_random_state(random_state)
    i = random_state.randint(X.shape[0])
    return X[i]
```

If you use randomness in an estimator instead of a freestanding function, some additional guidelines apply.

First off, the estimator should take a random_state argument to its __init__ with a default value of None. It should store that argument’s value, unmodified, in an attribute random_state. fit can call
check_random_state on that attribute to get an actual random number generator. If, for some reason, randomness is needed after fit, the RNG should be stored in an attribute random_state_. The following example should make this clear:

```python
class GaussianNoise(BaseEstimator, TransformerMixin):
    """This estimator ignores its input and returns random Gaussian noise.
    It also does not adhere to all scikit-learn conventions, but showcases how to handle randomness.
    """
    def __init__(self, n_components=100, random_state=None):
        self.random_state = random_state
        # the arguments are ignored anyway, so we make them optional
    def fit(self, X=None, y=None):
        self.random_state_ = check_random_state(self.random_state)
        def transform(self, X):
            n_samples = X.shape[0]
            return self.random_state_.randn(n_samples, n_components)
```

The reason for this setup is reproducibility: when an estimator is fit twice to the same data, it should produce an identical model both times, hence the validation in fit, not __init___.

### Deprecation

If any publicly accessible method, function, attribute or parameter is renamed, we still support the old one for two releases and issue a deprecation warning when it is called/passed/accessed. E.g., if the function zero_one is renamed to zero_one_loss, we add the decorator deprecated (from sklearn.utils) to zero_one and call zero_one_loss from that function:

```python
from ..utils import deprecated

def zero_one_loss(y_true, y_pred, normalize=True):
    # actual implementation
    pass
@deprecated("Function 'zero_one' was renamed to 'zero_one_loss' in version 0.13 and will be removed in release 0.15. 'Default behavior is changed from 'normalize=False' to 'normalize=True'"

def zero_one(y_true, y_pred, normalize=False):
    return zero_one_loss(y_true, y_pred, normalize)
```

If an attribute is to be deprecated, use the decorator deprecated on a property. E.g., renaming an attribute labels_ to classes_ can be done as:

```python
@property
@deprecated("Attribute labels_ was deprecated in version 0.13 and "
            "will be removed in 0.15. Use 'classes_' instead")
def labels_(self):
    return self.classes_
```

If a parameter has to be deprecated, use DeprecationWarning appropriately. In the following example, k is deprecated and renamed to n_clusters:

30.1. Contributing
import warnings

def example_function(n_clusters=8, k=None):
    if k is not None:
        warnings.warn("'k' was renamed to n_clusters in version 0.13 and ", DeprecationWarning)
        n_clusters = k

As in these examples, the warning message should always give both the version in which the deprecation happened and the version in which the old behavior will be removed. If the deprecation happened in version 0.x-dev, the message should say deprecation occurred in version 0.x and the removal will be in 0.(x+2). For example, if the deprecation happened in version 0.18-dev, the message should say it happened in version 0.18 and the old behavior will be removed in version 0.20.

Python 3.x support

All scikit-learn code should work unchanged in both Python 2.[67] and 3.2 or newer. Since Python 3.x is not backwards compatible, that may require changes to code and it certainly requires testing on both 2.6 or 2.7, and 3.2 or newer.

For most numerical algorithms, Python 3.x support is easy: just remember that print is a function and integer division is written //. String handling has been overhauled, though, as have parts of the Python standard library. The six package helps with cross-compatibility and is included in scikit-learn as sklearn.externals.six.

30.1.6 APIs of scikit-learn objects

To have a uniform API, we try to have a common basic API for all the objects. In addition, to avoid the proliferation of framework code, we try to adopt simple conventions and limit to a minimum the number of methods an object must implement.

Different objects

The main objects in scikit-learn are (one class can implement multiple interfaces):

**Estimator** The base object, implements a fit method to learn from data, either:

```python
estimator = obj.fit(data, targets)
```

or:

```python
estimator = obj.fit(data)
```

**Predictor** For supervised learning, or some unsupervised problems, implements:

```python
prediction = obj.predict(data)
```

Classification algorithms usually also offer a way to quantify certainty of a prediction, either using decision_function or predict_proba:

```python
probability = obj.predict_proba(data)
```

**Transformer** For filtering or modifying the data, in a supervised or unsupervised way, implements:

```python
new_data = obj.transform(data)
```
When fitting and transforming can be performed much more efficiently together than separately, implements:

```python
new_data = obj.fit_transform(data)
```

**Model** A model that can give a goodness of fit measure or a likelihood of unseen data, implements (higher is better):

```python
score = obj.score(data)
```

## Estimators

The API has one predominant object: the estimator. A estimator is an object that fits a model based on some training data and is capable of inferring some properties on new data. It can be, for instance, a classifier or a regressor. All estimators implement the fit method:

```python
estimator.fit(X, y)
```

All built-in estimators also have a `set_params` method, which sets data-independent parameters (overriding previous parameter values passed to `__init__`).

All estimators in the main scikit-learn codebase should inherit from `sklearn.base.BaseEstimator`.

### Instantiation

This concerns the creation of an object. The object’s `__init__` method might accept constants as arguments that determine the estimator’s behavior (like the C constant in SVMs). It should not, however, take the actual training data as an argument, as this is left to the `fit()` method:

```python
clf2 = SVC(C=2.3)
clf3 = SVC([[1, 2], [2, 3]], [-1, 1]) # WRONG!
```

The arguments accepted by `__init__` should all be keyword arguments with a default value. In other words, a user should be able to instantiate an estimator without passing any arguments to it. The arguments should all correspond to hyperparameters describing the model or the optimisation problem the estimator tries to solve. These initial arguments (or parameters) are always remembered by the estimator. Also note that they should not be documented under the “Attributes” section, but rather under the “Parameters” section for that estimator.

In addition, **every keyword argument accepted by `__init__` should correspond to an attribute on the instance.** Scikit-learn relies on this to find the relevant attributes to set on an estimator when doing model selection.

To summarize, an `__init__` should look like:

```python
def __init__(self, param1=1, param2=2):
    self.param1 = param1
    self.param2 = param2
```

There should be no logic, not even input validation, and the parameters should not be changed. The corresponding logic should be put where the parameters are used, typically in `fit`. The following is wrong:

```python
def __init__(self, param1=1, param2=2, param3=3):
    # WRONG: parameters should not be modified
    if param1 > 1:
        param2 += 1
    self.param1 = param1
```
# WRONG: the object's attributes should have exactly the name of
# the argument in the constructor
self.param3 = param2

The reason for postponing the validation is that the same validation would have to be performed in set_params, which is used in algorithms like GridSearchCV.

Fitting

The next thing you will probably want to do is to estimate some parameters in the model. This is implemented in the `fit()` method.

The `fit()` method takes the training data as arguments, which can be one array in the case of unsupervised learning, or two arrays in the case of supervised learning.

Note that the model is fitted using X and y, but the object holds no reference to X and y. There are, however, some exceptions to this, as in the case of precomputed kernels where this data must be stored for use by the predict method.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>array-like, with shape = [N, D], where N is the number of samples and D is the number of features.</td>
</tr>
<tr>
<td>y</td>
<td>array, with shape = [N], where N is the number of samples.</td>
</tr>
<tr>
<td>kwargs</td>
<td>optional data-dependent parameters.</td>
</tr>
</tbody>
</table>

`X.shape[0]` should be the same as `y.shape[0]`. If this requisite is not met, an exception of type `ValueError` should be raised.

`y` might be ignored in the case of unsupervised learning. However, to make it possible to use the estimator as part of a pipeline that can mix both supervised and unsupervised transformers, even unsupervised estimators need to accept a `y=None` keyword argument in the second position that is just ignored by the estimator. For the same reason, `fit_predict`, `fit_transform`, `score` and `partial_fit` methods need to accept a `y` argument in the second place if they are implemented.

The method should return the object (`self`). This pattern is useful to be able to implement quick one liners in an IPython session such as:

```python
y_predicted = SVC(C=100).fit(X_train, y_train).predict(X_test)
```

Depending on the nature of the algorithm, `fit` can sometimes also accept additional keywords arguments. However, any parameter that can have a value assigned prior to having access to the data should be an `__init__` keyword argument. **Fit parameters should be restricted to directly data dependent variables.** For instance a Gram matrix or an affinity matrix which are precomputed from the data matrix `X` are data dependent. A tolerance stopping criterion `tol` is not directly data dependent (although the optimal value according to some scoring function probably is).

Estimated Attributes

Attributes that have been estimated from the data must always have a name ending with trailing underscore, for example the coefficients of some regression estimator would be stored in a `coef_` attribute after `fit` has been called.

The last-mentioned attributes are expected to be overridden when you call `fit` a second time without taking any previous value into account: **fit should be idempotent.**

Optional Arguments

In iterative algorithms, the number of iterations should be specified by an integer called `n_iter`. 
30.1.7 Rolling your own estimator

If you want to implement a new estimator that is scikit-learn-compatible, whether it is just for you or for contributing it to scikit-learn, there are several internals of scikit-learn that you should be aware of in addition to the scikit-learn API outlined above. You can check whether your estimator adheres to the scikit-learn interface and standards by running `utils.estimator_checks.check_estimator` on the class:

```python
>>> from sklearn.utils.estimator_checks import check_estimator
>>> from sklearn.svm import LinearSVC
>>> check_estimator(LinearSVC)  # passes
```

The main motivation to make a class compatible to the scikit-learn estimator interface might be that you want to use it together with model assessment and selection tools such as `model_selection.GridSearchCV`.

For this to work, you need to implement the following interface. If a dependency on scikit-learn is okay for your code, you can prevent a lot of boilerplate code by deriving a class from `BaseEstimator` and optionally the mixin classes in `sklearn.base`. E.g., below is a custom classifier. For more information on this example, see scikit-learn-contrib:

```python
>>> import numpy as np
>>> from sklearn.base import BaseEstimator, ClassifierMixin
>>> from sklearn.utils.validation import check_X_y, check_array, check_is_fitted
>>> from sklearn.utils.multiclass import unique_labels
>>> from sklearn.metrics import euclidean_distances
>>> class TemplateClassifier(BaseEstimator, ClassifierMixin):
...     ...     def __init__(self, demo_param='demo'):
...         self.demo_param = demo_param
...     ...     def fit(self, X, y):
...         # Check that X and y have correct shape
...         X, y = check_X_y(X, y)
...         # Store the classes seen during fit
...         self.classes_ = unique_labels(y)
...         self.X_ = X
...         self.y_ = y
...         # Return the classifier
...         return self
...     ...     def predict(self, X):
...         # Check if fit had been called
...         check_is_fitted(self, ['X_', 'y_'])
...         # Input validation
...         X = check_array(X)
...         closest = np.argmin(euclidean_distances(X, self.X_), axis=1)
...         return self.y_[closest]
```

get_params and set_params

All scikit-learn estimators have `get_params` and `set_params` functions. The `get_params` function takes no arguments and returns a dict of the `__init__` parameters of the estimator, together with their values. It must take one keyword argument, `deep`, which receives a boolean value that determines whether the method should return the parameters of sub-estimators (for most estimators, this can be ignored). The default value for `deep` should be true.
The `set_params` on the other hand takes as input a dict of the form `{'parameter': value}` and sets the parameter of the estimator using this dict. Return value must be estimator itself.

While the `get_params` mechanism is not essential (see Cloning below), the `set_params` function is necessary as it is used to set parameters during grid searches.

The easiest way to implement these functions, and to get a sensible `__repr__` method, is to inherit from `sklearn.base.BaseEstimator`. If you do not want to make your code dependent on scikit-learn, the easiest way to implement the interface is:

```python
def get_params(self, deep=True):
    # suppose this estimator has parameters "alpha" and "recursive"
    return {'alpha': self.alpha, 'recursive': self.recursive}

def set_params(self, **parameters):
    for parameter, value in parameters.items():
        selfsetattr(parameter, value)
    return self
```

Parameters and init

As `model_selection.GridSearchCV` uses `set_params` to apply parameter setting to estimators, it is essential that calling `set_params` has the same effect as setting parameters using the `__init__` method. The easiest and recommended way to accomplish this is to **not do any parameter validation in `__init__`**. All logic behind estimator parameters, like translating string arguments into functions, should be done in `fit`. Also it is expected that parameters with trailing `_` are **not to be set inside the `__init__` method**. All and only the public attributes set by `fit` have a trailing `_`. As a result the existence of parameters with trailing `_` is used to check if the estimator has been fitted.

Cloning

For use with the `model_selection` module, an estimator must support the `base.clone` function to replicate an estimator. This can be done by providing a `get_params` method. If `get_params` is present, then `clone(estimator)` will be an instance of `type(estimator)` on which `set_params` has been called with clones of the result of `estimator.get_params()`.

Objects that do not provide this method will be deep-copied (using the Python standard function `copy.deepcopy`) if `safe=False` is passed to `clone`.

Pipeline compatibility

For an estimator to be usable together with `pipeline.Pipeline` in any but the last step, it needs to provide a `fit` or `fit_transform` function. To be able to evaluate the pipeline on any data but the training set, it also needs to provide a `transform` function. There are no special requirements for the last step in a pipeline, except that it has a `fit` function. All `fit` and `fit_transform` functions must take arguments `X, y`, even if `y` is not used. Similarly, for `score` to be usable, the last step of the pipeline needs to have a `score` function that accepts an optional `y`.

Estimator types

Some common functionality depends on the kind of estimator passed. For example, cross-validation in `model_selection.GridSearchCV` and `model_selection.cross_val_score` defaults to being stratified when used on a classifier, but not otherwise. Similarly, scorers for average precision that take a continuous
prediction need to call decision_function for classifiers, but predict for regressors. This distinction between classifiers and regressors is implemented using the _estimator_type attribute, which takes a string value. It should be "classifier" for classifiers and "regressor" for regressors and "clusterer" for clustering methods, to work as expected. Inheriting from ClassifierMixin, RegressorMixin or ClusterMixin will set the attribute automatically.

**Working notes**

For unresolved issues, TODOs, and remarks on ongoing work, developers are advised to maintain notes on the GitHub wiki.

**Specific models**

Classifiers should accept y (target) arguments to fit that are sequences (lists, arrays) of either strings or integers. They should not assume that the class labels are a contiguous range of integers; instead, they should store a list of classes in a classes_ attribute or property. The order of class labels in this attribute should match the order in which predict_proba, predict_log_proba and decision_function return their values. The easiest way to achieve this is to put:

```python
self.classes_, y = np.unique(y, return_inverse=True)
```

in fit. This returns a new y that contains class indexes, rather than labels, in the range \(0, n_{\text{classes}}\).

A classifier's predict method should return arrays containing class labels from classes_. In a classifier that implements decision_function, this can be achieved with:

```python
def predict(self, X):
    D = self.decision_function(X)
    return self.classes_[np.argmax(D, axis=1)]
```

In linear models, coefficients are stored in an array called coef_, and the independent term is stored in intercept_. sklearn.linear_model.base contains a few base classes and mixins that implement common linear model patterns.

The sklearn.utils.multiclass module contains useful functions for working with multiclass and multilabel problems.

**30.2 Developers’ Tips for Debugging**

**30.2.1 Memory errors: debugging Cython with valgrind**

While python/numpy’s built-in memory management is relatively robust, it can lead to performance penalties for some routines. For this reason, much of the high-performance code in scikit-learn is written in cython. This performance gain comes with a tradeoff, however: it is very easy for memory bugs to crop up in cython code, especially in situations where that code relies heavily on pointer arithmetic.

Memory errors can manifest themselves a number of ways. The easiest ones to debug are often segmentation faults and related glibc errors. Uninitialized variables can lead to unexpected behavior that is difficult to track down. A very useful tool when debugging these sorts of errors is valgrind.

Valgrind is a command-line tool that can trace memory errors in a variety of code. Follow these steps:

1. Install valgrind on your system.
2. Download the python valgrind suppression file: valgrind-python.supp.

3. Follow the directions in the README.valgrind file to customize your python suppressions. If you don’t, you will have spurious output coming related to the python interpreter instead of your own code.

4. Run valgrind as follows:

```
$> valgrind -v --suppressions=valgrind-python.supp python my_test_script.py
```

The result will be a list of all the memory-related errors, which reference lines in the C-code generated by cython from your .pyx file. If you examine the referenced lines in the .c file, you will see comments which indicate the corresponding location in your .pyx source file. Hopefully the output will give you clues as to the source of your memory error.

For more information on valgrind and the array of options it has, see the tutorials and documentation on the valgrind web site.

### 30.3 Utilities for Developers

Scikit-learn contains a number of utilities to help with development. These are located in `sklearn.utils`, and include tools in a number of categories. All the following functions and classes are in the module `sklearn.utils`.

**Warning:** These utilities are meant to be used internally within the scikit-learn package. They are not guaranteed to be stable between versions of scikit-learn. Backports, in particular, will be removed as the scikit-learn dependencies evolve.

#### 30.3.1 Validation Tools

These are tools used to check and validate input. When you write a function which accepts arrays, matrices, or sparse matrices as arguments, the following should be used when applicable.

- `assert_all_finite`: Throw an error if array contains NaNs or Infs.
- `as_float_array`: convert input to an array of floats. If a sparse matrix is passed, a sparse matrix will be returned.
- `check_array`: convert input to 2d array, raise error on sparse matrices. Allowed sparse matrix formats can be given optionally, as well as allowing 1d or nd arrays. Calls `assert_all_finite` by default.
- `check_X_y`: check that X and y have consistent length, calls check_array on X, and column_or_1d on y. For multilabel classification or multitarget regression, specify multi_output=True, in which case check_array will be called on y.
- `indexable`: check that all input arrays have consistent length and can be sliced or indexed using safe_index. This is used to validate input for cross-validation.

If your code relies on a random number generator, it should never use functions like `numpy.random.random` or `numpy.random.normal`. This approach can lead to repeatability issues in unit tests. Instead, a `numpy.random.RandomState` object should be used, which is built from a `random_state` argument passed to the class or function. The function `check_random_state`, below, can then be used to create a random number generator object.

- `check_random_state`: create a `np.random.RandomState` object from a parameter `random_state`.
  - If `random_state` is `None` or `np.random`, then a randomly-initialized `RandomState` object is returned.
– If `random_state` is an integer, then it is used to seed a new `RandomState` object.
– If `random_state` is a `RandomState` object, then it is passed through.

For example:

```python
>>> from sklearn.utils import check_random_state
>>> random_state = 0
>>> random_state = check_random_state(random_state)
>>> random_state.rand(4)
array([[ 0.5488135 , 0.71518937, 0.60276338, 0.54488318]])
```

### 30.3.2 Efficient Linear Algebra & Array Operations

- `extmath.randomized_range_finder`: construct an orthonormal matrix whose range approximates the range of the input. This is used in `extmath.randomized_svd`, below.
- `extmath.randomized_svd`: compute the k-truncated randomized SVD. This algorithm finds the exact truncated singular values decomposition using randomization to speed up the computations. It is particularly fast on large matrices on which you wish to extract only a small number of components.
- `arrayfuncs.cholesky_delete`: (used in `sklearn.linear_model.least_angle.lars_path`) Remove an item from a cholesky factorization.
- `arrayfuncs.min_pos`: (used in `sklearn.linear_model.least_angle`) Find the minimum of the positive values within an array.
- `extmath.norm`: computes Euclidean (L2) vector norm by directly calling the BLAS nrm2 function. This is more stable than `scipy.linalg.norm`. See Fabian’s blog post for a discussion.
- `extmath.fast_logdet`: efficiently compute the log of the determinant of a matrix.
- `extmath.density`: efficiently compute the density of a sparse vector
- `extmath.safe_sparse_dot`: dot product which will correctly handle `scipy.sparse` inputs. If the inputs are dense, it is equivalent to `numpy.dot`.
- `extmath.logsumexp`: compute the sum of X assuming X is in the log domain. This is equivalent to calling `np.log(np.sum(np.exp(X)))`, but is robust to overflow/underflow errors. Note that there is similar functionality in `np.logaddexp.reduce`, but because of the pairwise nature of this routine, it is slower for large arrays. Scipy has a similar routine in `scipy.misc.logsumexp` (in scipy versions < 0.10, this is found in `scipy.maxentropy.logsumexp`), but the scipy version does not accept an `axis` keyword.
- `extmath.weighted_mode`: an extension of `scipy.stats.mode` which allows each item to have a real-valued weight.
- `resample`: Resample arrays or sparse matrices in a consistent way. used in `shuffle`, below.
- `shuffle`: Shuffle arrays or sparse matrices in a consistent way. Used in `sklearn.cluster.k_means`.

### 30.3.3 Efficient Random Sampling

- `random.sample_without_replacement`: implements efficient algorithms for sampling n_samples integers from a population of size n_population without replacement.
30.3.4 Efficient Routines for Sparse Matrices

The `sklearn.utils.sparsefuncs` cython module hosts compiled extensions to efficiently process `scipy.sparse` data.

- `sparsefuncs.mean_variance_axis`: compute the means and variances along a specified axis of a CSR matrix. Used for normalizing the tolerance stopping criterion in `sklearn.cluster.k_means_.KMeans`.
- `sparsefuncs.inplace_csr_row_normalize_l1` and `sparsefuncs.inplace_csr_row_normalize_l2`: can be used to normalize individual sparse samples to unit L1 or L2 norm as done in `sklearn.preprocessing.Normalizer`.
- `sparsefuncs.inplace_csr_column_scale`: can be used to multiply the columns of a CSR matrix by a constant scale (one scale per column). Used for scaling features to unit standard deviation in `sklearn.preprocessing.StandardScaler`.

30.3.5 Graph Routines

- `graph.single_source_shortest_path_length`: (not currently used in scikit-learn) Return the shortest path from a single source to all connected nodes on a graph. Code is adapted from `networkx`. If this is ever needed again, it would be far faster to use a single iteration of Dijkstra’s algorithm from `graph_shortest_path`.
- `graph.graph_laplacian`: (used in `sklearn.cluster.spectral.spectral_embedding`) Return the Laplacian of a given graph. There is specialized code for both dense and sparse connectivity matrices.
- `graph_shortest_path.graph_shortest_path`: (used in `sklearn.manifold.Isomap`) Return the shortest path between all pairs of connected points on a directed or undirected graph. Both the Floyd-Warshall algorithm and Dijkstra’s algorithm are available. The algorithm is most efficient when the connectivity matrix is a `scipy.sparse.csr_matrix`.

30.3.6 Backports

- `fixes.expit`: Logistic sigmoid function. Replacement for SciPy 0.10’s `scipy.special.expit`.
- `sparsetools.connected_components` (backported from `scipy.sparse.connected_components` in scipy 0.12). Used in `sklearn.cluster.hierarchical`, as well as in tests for `sklearn.feature_extraction`.

ARPACK

- `arpack.eigs` (backported from `scipy.sparse.linalg.eigs` in scipy 0.10) Sparse non-symmetric eigenvalue decomposition using the Arnoldi method. A limited version of `eigs` is available in earlier scipy versions.
- `arpack.eigsh` (backported from `scipy.sparse.linalg.eigsh` in scipy 0.10) Sparse non-symmetric eigenvalue decomposition using the Arnoldi method. A limited version of `eigsh` is available in earlier scipy versions.
- `arpack.svds` (backported from `scipy.sparse.linalg.svds` in scipy 0.10) Sparse non-symmetric eigenvalue decomposition using the Arnoldi method. A limited version of `svds` is available in earlier scipy versions.
Benchmarks

- bench.total_seconds (back-ported from timedelta.total_seconds in Python 2.7). Used in benchmarks/bench_glm.py.

### 30.3.7 Testing Functions

- testing.assert_in, testing.assert_not_in: Assertions for container membership. Designed for forward compatibility with Nose 1.0.
- testing.assert_raise_message: Assertions for checking the error raise message.
- testing.mock_mldata_urlopen: Mocks the urlopen function to fake requests to mldata.org. Used in tests of sklearn.datasets.
- testing.all_estimators: returns a list of all estimators in scikit-learn to test for consistent behavior and interfaces.

### 30.3.8 Multiclass and multilabel utility function

- multiclass.is_multilabel: Helper function to check if the task is a multi-label classification one.
- multiclass.is_label_indicator_matrix: Helper function to check if a classification output is in label indicator matrix format.
- multiclass.unique_labels: Helper function to extract an ordered array of unique labels from different formats of target.

### 30.3.9 Helper Functions

- gen_even_slices: generator to create n-packs of slices going up to n. Used in sklearn.decomposition.dict_learning and sklearn.cluster.k_means.
- safe_mask: Helper function to convert a mask to the format expected by the numpy array or scipy sparse matrix on which to use it (sparse matrices support integer indices only while numpy arrays support both boolean masks and integer indices).
- safe_sqr: Helper function for unified squaring (**2) of array-likes, matrices and sparse matrices.

### 30.3.10 Hash Functions

- murmurhash3_32 provides a python wrapper for the MurmurHash3_x86_32 C++ non cryptographic hash function. This hash function is suitable for implementing lookup tables, Bloom filters, Count Min Sketch, feature hashing and implicitly defined sparse random projections:

```
>>> from sklearn.utils import murmurhash3_32
>>> murmurhash3_32("some feature", seed=0) == -384616559
True
>>> murmurhash3_32("some feature", seed=0, positive=True) == 3910350737
True
```

The sklearn.utils.murmurhash module can also be “cimported” from other cython modules so as to benefit from the high performance of MurmurHash while skipping the overhead of the Python interpreter.
30.3.11 Warnings and Exceptions

- deprecated: Decorator to mark a function or class as deprecated.

30.4 How to optimize for speed

The following gives some practical guidelines to help you write efficient code for the scikit-learn project.

**Note:** While it is always useful to profile your code so as to check performance assumptions, it is also highly recommended to review the literature to ensure that the implemented algorithm is the state of the art for the task before investing into costly implementation optimization.

Times and times, hours of efforts invested in optimizing complicated implementation details have been rendered irrelevant by the subsequent discovery of simple algorithmic tricks, or by using another algorithm altogether that is better suited to the problem.

The section *A sample algorithmic trick: warm restarts for cross validation* gives an example of such a trick.

30.4.1 Python, Cython or C/C++?

In general, the scikit-learn project emphasizes the readability of the source code to make it easy for the project users to dive into the source code so as to understand how the algorithm behaves on their data but also for ease of maintainability (by the developers).

When implementing a new algorithm is thus recommended to start implementing it in Python using Numpy and Scipy by taking care of avoiding looping code using the vectorized idioms of those libraries. In practice this means trying to replace any nested for loops by calls to equivalent Numpy array methods. The goal is to avoid the CPU wasting time in the Python interpreter rather than crunching numbers to fit your statistical model. It’s generally a good idea to consider NumPy and SciPy performance tips: http://scipy.github.io/old-wiki/pages/PerformanceTips

Sometimes however an algorithm cannot be expressed efficiently in simple vectorized Numpy code. In this case, the recommended strategy is the following:

1. Profile the Python implementation to find the main bottleneck and isolate it in a dedicated module level function. This function will be reimplemented as a compiled extension module.
2. If there exists a well maintained BSD or MIT C/C++ implementation of the same algorithm that is not too big, you can write a Cython wrapper for it and include a copy of the source code of the library in the scikit-learn source tree: this strategy is used for the classes svm.LinearSVC, svm.SVC and linear_model.LogisticRegression (wrappers for liblinear and libsvm).
3. Otherwise, write an optimized version of your Python function using Cython directly. This strategy is used for the linear_model.ElasticNet and linear_model.SGDClassifier classes for instance.
4. Move the Python version of the function in the tests and use it to check that the results of the compiled extension are consistent with the gold standard, easy to debug Python version.
5. Once the code is optimized (not simple bottleneck spottable by profiling), check whether it is possible to have coarse grained parallelism that is amenable to multi-processing by using the joblib.Parallel class.

When using Cython, use either

```
$ python setup.py build_ext -i $ python setup.py install
```
to generate C files. You are responsible for adding .c/.cpp extensions along with build parameters in each submodule setup.py.

C/C++ generated files are embedded in distributed stable packages. The goal is to make it possible to install scikit-learn stable version on any machine with Python, Numpy, Scipy and C/C++ compiler.

30.4.2 Fast matrix multiplications

Matrix multiplications (matrix-matrix and matrix-vector) are usually handled using the NumPy function np.dot, but in versions of NumPy before 1.7.2 this function is suboptimal when the inputs are not both in the C (row-major) layout; in that case, the inputs may be implicitly copied to obtain the right layout. This obviously consumes memory and takes time.

The function fast_dot in sklearn.utils.extmath offers a fast replacement for np.dot that prevents copies from being made in some cases. In all other cases, it dispatches to np.dot and when the NumPy version is new enough, it is in fact an alias for that function, making it a drop-in replacement. Example usage of fast_dot:

```python
>>> import numpy as np
>>> from sklearn.utils.extmath import fast_dot
>>> X = np.random.random_sample([2, 10])
>>> np.allclose(np.dot(X, X.T), fast_dot(X, X.T))
True
```

This function operates optimally on 2-dimensional arrays, both of the same dtype, which should be either single or double precision float. If these requirements aren’t met or the BLAS package is not available, the call is silently dispatched to numpy.dot. If you want to be sure when the original numpy.dot has been invoked in a situation where it is suboptimal, you can activate the related warning:

```python
>>> import warnings
>>> from sklearn.exceptions import NonBLASDotWarning
>>> warnings.simplefilter('always', NonBLASDotWarning)
```

30.4.3 Profiling Python code

In order to profile Python code we recommend to write a script that loads and prepare your data and then use the IPython integrated profiler for interactively exploring the relevant part for the code.

Suppose we want to profile the Non Negative Matrix Factorization module of the scikit. Let us setup a new IPython session and load the digits dataset and as in the Recognizing hand-written digits example:

```python
In [1]: from sklearn.decomposition import NMF
In [2]: from sklearn.datasets import load_digits
In [3]: X = load_digits().data
```

Before starting the profiling session and engaging in tentative optimization iterations, it is important to measure the total execution time of the function we want to optimize without any kind of profiler overhead and save it somewhere for later reference:

```python
In [4]: %timeit NMF(n_components=16, tol=1e-2).fit(X)
1 loops, best of 3: 1.7 s per loop
```

To have a look at the overall performance profile using the %prun magic command:
In [5]: %prun -l nmf.py NMF(n_components=16, tol=le-2).fit(X)

14496 function calls in 1.682 CPU seconds

Ordered by: internal time
List reduced from 90 to 9 due to restriction <'nmf.py'>

ncalls  tottime  percall  cumtime  percall filename:lineno(function)
36    0.609   0.017   1.499  0.042 nmf.py:151(_nls_subproblem)
1263  0.157  0.000   0.157  0.000 nmf.py:18(_pos)
673   0.008  0.000   0.057  0.000 nmf.py:28(norm)
1    0.006  0.006   0.047  0.047 nmf.py:42(_initialize_nmf)
36   0.001  0.000   0.010  0.000 nmf.py:36(_sparseness)
30   0.001  0.000   0.001  0.000 nmf.py:23(_neg)
1    0.000  0.000   0.000  0.000 nmf.py:337(__init__)
1    0.000  0.000   0.000  0.000 nmf.py:461(fit)

The `tottime` column is the most interesting: it gives to total time spent executing the code of a given function ignoring the time spent in executing the sub-functions. The real total time (local code + sub-function calls) is given by the `cumtime` column.

Note the use of the `-l nmf.py` that restricts the output to lines that contains the “nmf.py” string. This is useful to have a quick look at the hotspot of the nmf Python module it-self ignoring anything else.

Here is the beginning of the output of the same command without the `-l nmf.py` filter:

In [5] %prun NMF(n_components=16, tol=le-2).fit(X)

16159 function calls in 1.840 CPU seconds

Ordered by: internal time

ncalls  tottime  percall  cumtime  percall filename:lineno(function)
2833  0.653  0.000   0.653  0.000 {numpy.core._dotblas.dot}
46    0.651  0.014   1.636  0.036 nmf.py:151(_nls_subproblem)
1397  0.171  0.000   0.171  0.000 nmf.py:18(_pos)
2780  0.167  0.000   0.167  0.000 {method 'sum' of 'numpy.ndarray'
  <objects>}
    1    0.064   0.064   1.840   1.840 nmf.py:352(fit_transform)
1542   0.043  0.000   0.043  0.000 {method 'flatten' of 'numpy.ndarray'
  <objects>}
    337  0.019  0.000   0.019  0.000 {method 'all' of 'numpy.ndarray'
  <objects>}
2734   0.011  0.000   0.181  0.000 fromnumeric.py:1185(sum)
    2    0.010  0.005   0.010  0.005 {numpy.linalg.lapack_lite.dgesdd}
748    0.009  0.000   0.065  0.000 nmf.py:28(norm)
...

The above results show that the execution is largely dominated by dot products operations (delegated to blas). Hence there is probably no huge gain to expect by rewriting this code in Cython or C/C++: in this case out of the 1.7s total execution time, almost 0.7s are spent in compiled code we can consider optimal. By rewriting the rest of the Python code and assuming we could achieve a 1000% boost on this portion (which is highly unlikely given the shallowness of the Python loops), we would not gain more than a 2.4x speed-up globally.

Hence major improvements can only be achieved by **algorithmic improvements** in this particular example (e.g. trying to find operation that are both costly and useless to avoid computing them rather than trying to optimize their implementation).

It is however still interesting to check what’s happening inside the `_nls_subproblem` function which is the hotspot if we only consider Python code: it takes around 100% of the accumulated time of the module. In order to better

understand the profile of this specific function, let us install `line-prof` and wire it to IPython:

```bash
$ pip install line-profiler
```

- **Under IPython <= 0.10**, edit `~/.ipython/ipy_user_conf.py` and ensure the following lines are present:

```python
import IPython.ipapi
ip = IPython.ipapi.get()
```

Towards the end of the file, define the `%lprun` magic:

```python
import line_profiler
ip.expose_magic('lprun', line_profiler.magic_lprun)
```

- **Under IPython 0.11+**, first create a configuration profile:

```bash
$ ipython profile create
```

Then create a file named `~/.ipython/extensions/line_profiler_ext.py` with the following content:

```python
import line_profiler
def load_ipython_extension(ip):
    ip.define_magic('lprun', line_profiler.magic_lprun)
```

Then register it in `~/.ipython/profile_default/ipython_config.py`:

```python
c.TerminalIPythonApp.extensions = ['line_profiler_ext',]
c.InteractiveShellApp.extensions = ['line_profiler_ext',]
```

This will register the `%lprun` magic command in the IPython terminal application and the other frontends such as qtconsole and notebook.

Now restart IPython and let us use this new toy:

```python
In [1]: from sklearn.datasets import load_digits
In [2]: from sklearn.decomposition.nmf import _nls_subproblem, NMF
In [3]: X = load_digits().data
In [4]: %lprun _nls_subproblem NMF(n_components=16, tol=1e-2).fit(X)
```

Timer unit: 1e-06 s
File: sklearn/decomposition/nmf.py
Function: _nls_subproblem at line 137
Total time: 1.73153 s

<table>
<thead>
<tr>
<th>Line</th>
<th>Hits</th>
<th>Time</th>
<th>Per Hit</th>
<th>% Time</th>
<th>Line Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>def _nls_subproblem(V, W, H_init,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-tol, max_iter):</td>
</tr>
<tr>
<td>138</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>***Non-negative least square_</td>
</tr>
</tbody>
</table>

30.4. How to optimize for speed 1995
By looking at the top values of the % Time column it is really easy to pin-point the most expensive expressions that would deserve additional care.

### 30.4.4 Memory usage profiling

You can analyze in detail the memory usage of any Python code with the help of memory_profiler. First, install the latest version:

```bash
$ pip install -U memory_profiler
```

Then, setup the magics in a manner similar to line_profiler.

- **Under IPython <= 0.10.** edit `~/.ipython/ipy_user_conf.py` and ensure the following lines are present:

```python
import IPython.ipapi
ip = IPython.ipapi.get()
```

Towards the end of the file, define the `%memit` and `%mprun` magics:

```python
import memory_profiler
ip.expose_magic('%memit', memory_profiler.magic_memit)
ip.expose_magic('%mprun', memory_profiler.magic_mprun)
```
• **Under IPython 0.11+,** first create a configuration profile:

```
$ ipython profile create
```

Then create a file named `~/ipython/extensions/memory_profiler_ext.py` with the following content:

```python
import memory_profiler

def load_ipython_extension(ip):
    ip.define_magic('memit', memory_profiler.magic_memit)
    ip.define_magic('mprun', memory_profiler.magic_mprun)
```

Then register it in `~/ipython/profile_default/ipython_config.py`:

```python
c.TerminalIPythonApp.extensions = [
    'memory_profiler_ext',
]
c.InteractiveShellApp.extensions = [
    'memory_profiler_ext',
]
```

This will register the `%memit` and `%mprun` magic commands in the IPython terminal application and the other frontends such as qtconsole and notebook.

%mprun is useful to examine, line-by-line, the memory usage of key functions in your program. It is very similar to %lprun, discussed in the previous section. For example, from the memory_profiler examples directory:

```
In [1]: from example import my_func
In [2]: %mprun -f my_func my_func()
Filename: example.py
Line #  Mem usage  Increment  Line Contents
==============================================
3 @profile
4  5.97 MB  0.00 MB  def my_func():
5  13.61 MB  7.64 MB  a = [1] * (10 ** 6)
6  166.20 MB  152.59 MB  b = [2] * (2 * 10 ** 7)
7  13.61 MB -152.59 MB  del b
8  13.61 MB  0.00 MB  return a
```

Another useful magic that memory_profiler defines is `%memit`, which is analogous to `%timeit`. It can be used as follows:

```
In [1]: import numpy as np
In [2]: %memit np.zeros(1e7)
maximum of 3: 76.402344 MB per loop
```

For more details, see the docstrings of the magics, using `%memit?` and `%mprun?`.

### 30.4.5 Performance tips for the Cython developer

If profiling of the Python code reveals that the Python interpreter overhead is larger by one order of magnitude or more than the cost of the actual numerical computation (e.g. for loops over vector components, nested evaluation of conditional expression, scalar arithmetic...), it is probably adequate to extract the hotspot portion of the code as a
standalone function in a `.pyx` file, add static type declarations and then use Cython to generate a C program suitable to be compiled as a Python extension module.

The official documentation available at http://docs.cython.org/ contains a tutorial and reference guide for developing such a module. In the following we will just highlight a couple of tricks that we found important in practice on the existing cython codebase in the scikit-learn project.

TODO: html report, type declarations, bound checks, division by zero checks, memory alignment, direct blas calls...

- https://www.youtube.com/watch?v=gMvkiQ-gOW8

### 30.4.6 Profiling compiled extensions

When working with compiled extensions (written in C/C++ with a wrapper or directly as Cython extension), the default Python profiler is useless: we need a dedicated tool to introspect what’s happening inside the compiled extension itself.

#### Using yep and google-perftools

Easy profiling without special compilation options use yep:

- https://pypi.python.org/pypi/yep
- http://fa.bianp.net/blog/2011/a-profiler-for-python-extensions

**Note:** google-perftools provides a nice ‘line by line’ report mode that can be triggered with the `--lines` option. However this does not seem to work correctly at the time of writing. This issue can be tracked on the project issue tracker.

#### Using gprof

In order to profile compiled Python extensions one could use `gprof` after having recompiled the project with `gcc -pg` and using the `python-dbg` variant of the interpreter on debian / ubuntu: however this approach requires to also have `numpy` and `scipy` recompiled with `--pg` which is rather complicated to get working.

Fortunately there exist two alternative profilers that don’t require you to recompile everything.

#### Using valgrind / callgrind / kcachegrind

TODO

### 30.4.7 Multi-core parallelism using `joblib.Parallel`

TODO: give a simple teaser example here.

Checkout the official `joblib` documentation:

- https://pythonhosted.org/joblib
30.4.8 A sample algorithmic trick: warm restarts for cross validation

TODO: demonstrate the warm restart tricks for cross validation of linear regression with Coordinate Descent.

30.5 Advanced installation instructions

There are different ways to get scikit-learn installed:

- Install the version of scikit-learn provided by your operating system or Python distribution. This is the quickest option for those who have operating systems that distribute scikit-learn.

- Install an official release. This is the best approach for users who want a stable version number and aren’t concerned about running a slightly older version of scikit-learn.

- Install the latest development version. This is best for users who want the latest-and-greatest features and aren’t afraid of running brand-new code.

Note: If you wish to contribute to the project, you need to install the latest development version.

30.5.1 Installing an official release

Scikit-learn requires:

- Python (>= 2.6 or >= 3.3),
- NumPy (>= 1.6.1),
- SciPy (>= 0.9).

Mac OSX

Scikit-learn and its dependencies are all available as wheel packages for OSX:

```
pip install -U numpy scipy scikit-learn
```

Linux

At this time scikit-learn does not provide official binary packages for Linux so you have to build from source if you want the latest version. If you don’t need the newest version, consider using your package manager to install scikit-learn. It is usually the easiest way, but might not provide the newest version.

Installing build dependencies

Installing from source requires you to have installed the scikit-learn runtime dependencies, Python development headers and a working C/C++ compiler. Under Debian-based operating systems, which include Ubuntu, if you have Python 2 you can install all these requirements by issuing:

```
sudo apt-get install build-essential python-dev python-setuptools \ python-numpy python-scipy \ libatlas-dev libatlas3gf-base
```
If you have Python 3:

```
sudo apt-get install build-essential python3-dev python3-setuptools 
   python3-numpy python3-scipy 
   libatlas-dev libatlas3gf-base
```

On recent Debian and Ubuntu (e.g. Ubuntu 13.04 or later) make sure that ATLAS is used to provide the implementation of the BLAS and LAPACK linear algebra routines:

```
sudo update-alternatives --set libblas.so.3 
   /usr/lib/atlas-base/atlas/libblas.so.3
sudo update-alternatives --set liblapack.so.3 
   /usr/lib/atlas-base/atlas/liblapack.so.3
```

**Note:** In order to build the documentation and run the example code contains in this documentation you will need matplotlib:

```
sudo apt-get install python-matplotlib
```

**Note:** The above installs the ATLAS implementation of BLAS (the Basic Linear Algebra Subprograms library). Ubuntu 11.10 and later, and recent (testing) versions of Debian, offer an alternative implementation called OpenBLAS. Using OpenBLAS can give speedups in some scikit-learn modules, but can freeze joblib/multiprocessing prior to OpenBLAS version 0.2.8-4, so using it is not recommended unless you know what you’re doing.

If you do want to use OpenBLAS, then replacing ATLAS only requires a couple of commands. ATLAS has to be removed, otherwise NumPy may not work:

```
sudo apt-get remove libatlas3gf-base libatlas-dev
sudo apt-get install libopenblas-dev
```

```
sudo update-alternatives --set libblas.so.3 
   /usr/lib/openblas-base/libopenblas.so.0
sudo update-alternatives --set liblapack.so.3 
   /usr/lib/lapack/liblapack.so.3
```

On Red Hat and clones (e.g. CentOS), install the dependencies using:

```
sudo yum -y install gcc gcc-c++ numpy python-devel scipy
```

**Building scikit-learn with pip**

This is usually the fastest way to install or upgrade to the latest stable release:

```
pip install --user --install-option="--prefix=" -U scikit-learn
```

The `--user` flag asks pip to install scikit-learn in the `~/.local` folder therefore not requiring root permission. This flag should make pip ignore any old version of scikit-learn previously installed on the system while benefiting from system packages for numpy and scipy. Those dependencies can be long and complex to build correctly from source.

The `--install-option="--prefix="` flag is only required if Python has a `distutils.cfg` configuration with a predefined `prefix=` entry.
From source package

download the source package from pypi, unpack the sources and cd into the source directory.
This packages uses distutils, which is the default way of installing python modules. The install command is:

```
python setup.py install
```
or alternatively (also from within the scikit-learn source folder):

```
pip install .
```

**Warning:** Packages installed with the `python setup.py install` command cannot be uninstalled nor upgraded by `pip` later. To properly uninstall scikit-learn in that case it is necessary to delete the `sklearn` folder from your Python `site-packages` directory.

Windows

First, you need to install numpy and scipy from their own official installers.
Wheel packages (.whl files) for scikit-learn from pypi can be installed with the `pip` utility. Open a console and type the following to install or upgrade scikit-learn to the latest stable release:

```
pip install -U scikit-learn
```
If there are no binary packages matching your python, version you might to try to install scikit-learn and its dependencies from christoph gohlke unofficial windows installers or from a `python distribution` instead.

### 30.5.2 Third party distributions of scikit-learn

Some third-party distributions are now providing versions of scikit-learn integrated with their package-management systems.
These can make installation and upgrading much easier for users since the integration includes the ability to automatically install dependencies (numpy, scipy) that scikit-learn requires.
The following is an incomplete list of python and os distributions that provide their own version of scikit-learn.

**MacPorts for Mac OSX**

The MacPorts package is named `py<XY>-scikits-learn`, where XY denotes the Python version. It can be installed by typing the following command:

```
sudo port install py26-scikit-learn
```
or:

```
sudo port install py27-scikit-learn
```
Arch Linux

Arch Linux’s package is provided through the official repositories as python-scikit-learn for Python 3 and python2-scikit-learn for Python 2. It can be installed by typing the following command:

```
# pacman -S python-scikit-learn
```

or:

```
# pacman -S python2-scikit-learn
```

depending on the version of Python you use.

NetBSD

scikit-learn is available via pkgsrc-wip:

http://pkgsrc.se/wip/py-scikit_learn

Fedora

The Fedora package is called python-scikit-learn for the Python 2 version and python3-scikit-learn for the Python 3 version. Both versions can be installed using yum:

```
$ sudo yum install python-scikit-learn
```

or:

```
$ sudo yum install python3-scikit-learn
```

Building on windows

To build scikit-learn on Windows you need a working C/C++ compiler in addition to numpy, scipy and setuptools.

Picking the right compiler depends on the version of Python (2 or 3) and the architecture of the Python interpreter, 32-bit or 64-bit. You can check the Python version by running the following in cmd or powershell console:

```
python --version
```

and the architecture with:

```
python -c "import struct; print(struct.calcsize('P') * 8)"
```

The above commands assume that you have the Python installation folder in your PATH environment variable.

32-bit Python

For 32-bit python it is possible use the standalone installers for microsoft visual c++ express 2008 for Python 2 or Microsoft Visual C++ Express 2010 for Python 3.

Once installed you should be able to build scikit-learn without any particular configuration by running the following command in the scikit-learn folder:
**64-bit Python**

For the 64-bit architecture, you either need the full Visual Studio or the free Windows SDKs that can be downloaded from the links below.

The Windows SDKs include the MSVC compilers both for 32 and 64-bit architectures. They come as a GRMSDKX_EN_DVD.iso file that can be mounted as a new drive with a setup.exe installer in it.

- For Python 2 you need SDK v7.0: MS Windows SDK for Windows 7 and .NET Framework 3.5 SP1
- For Python 3 you need SDK v7.1: MS Windows SDK for Windows 7 and .NET Framework 4

Both SDKs can be installed in parallel on the same host. To use the Windows SDKs, you need to setup the environment of a cmd console launched with the following flags (at least for SDK v7.0):

```
cmd /E:ON /V:ON /K
```

Then configure the build environment with:

```
SET DISTUTILS_USE_SDK=1
SET MSSID=1
"C:\Program Files\Microsoft SDKs\Windows\v7.0\Setup\WindowsSdkVer.exe" -q -version:v7.0
"C:\Program Files\Microsoft SDKs\Windows\v7.0\Bin\SetEnv.cmd" /x64 /release
```

Finally you can build scikit-learn in the same cmd console:

```
python setup.py install
```

Replace `v7.0` by the `v7.1` in the above commands to do the same for Python 3 instead of Python 2.

Replace `/x64` by `/x86` to build for 32-bit Python instead of 64-bit Python.

**Building binary packages and installers**

The `.whl` package and `.exe` installers can be built with:

```
pip install wheel
python setup.py bdist_wheel bdist_wininst -b doc/logos/scikit-learn-logo.bmp
```

The resulting packages are generated in the `dist/` folder.

**Using an alternative compiler**

It is possible to use MinGW (a port of GCC to Windows OS) as an alternative to MSVC for 32-bit Python. Not that extensions built with mingw32 can be redistributed as reusable packages as they depend on GCC runtime libraries typically not installed on end-users environment.

To force the use of a particular compiler, pass the `--compiler` flag to the build step:

```
python setup.py build --compiler=my_compiler install
```

where `my_compiler` should be one of `mingw32` or `msvc`.

30.5. Advanced installation instructions
30.5.3 Bleeding Edge

See section *Retrieving the latest code* on how to get the development version. Then follow the previous instructions to build from source depending on your platform. You will also require Cython >=0.23 in order to build the development version.

30.5.4 Testing

Testing scikit-learn once installed

Testing requires having the `nose` library. After installation, the package can be tested by executing `from outside` the source directory:

```
$ nosetests -v sklearn
```

Under Windows, it is recommended to use the following command (adjust the path to the `python.exe` program) as using the `nosedtests.exe` program can badly interact with tests that use multiprocessing:

```
C:\Python34\python.exe -c "import nose; nose.main()" -v sklearn
```

This should give you a lot of output (and some warnings) but eventually should finish with a message similar to:

```
Ran 3246 tests in 260.618s
OK (SKIP=20)
```

Otherwise, please consider posting an issue into the bug tracker or to the *Mailing List* including the traceback of the individual failures and errors. Please include your operating system, your version of NumPy, SciPy and scikit-learn, and how you installed scikit-learn.

Testing scikit-learn from within the source folder

Scikit-learn can also be tested without having the package installed. For this you must compile the sources inplace from the source directory:

```
python setup.py build_ext --inplace
```

Test can now be run using nosetests:

```
nosetests -v sklearn/
```

This is automated by the commands:

```
make in
```

and:

```
make test
```

You can also install a symlink named `site-packages/scikit-learn.egg-link` to the development folder of scikit-learn with:

```
pip install --editable .
```
30.6 Maintainer / core-developer information

For more information see https://github.com/scikit-learn/scikit-learn/wiki/How-to-make-a-release

30.6.1 Making a release

1. Update docs:
   • edit the doc/whats_new.rst file to add release title and commit statistics. You can retrieve commit statistics with:
     
     ```shell
     $ git shortlog -ns 0.998..
     ```
   • edit the doc/conf.py to increase the version number
   • edit the doc/themes/scikit-learn/layout.html to change the ‘News’ entry of the front page.

2. Update the version number in sklearn/__init__.py, the __version__ variable

3. Create the tag and push it:
   
   ```shell
   $ git tag 0.999
   $ git push origin --tags
   ```

4. create tarballs:
   • Wipe clean your repo:
     
     ```shell
     $ git clean -xfd
     ```
   • Register and upload on PyPI:
     
     ```shell
     $ python setup.py sdist register upload
     ```
   • Upload manually the tarball on SourceForge: https://sourceforge.net/projects/scikit-learn/files/

5. Push the documentation to the website (see README in doc folder)

6. Build binaries for windows and push them to PyPI:
   
   ```shell
   $ python setup.py bdist_wininst upload
   ```

And upload them also to sourceforge
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